CLS scale setting from f_π, f_K

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Overview

A precise scale is needed for almost all projects.
E.g. the coupling:

Coupling

- Finite volume: $L_{\rm had}\Lambda^{(3)}_{\overline{\rm MS}}$
  with $\bar{g}^2_{\rm GF}(L^{-1}_{\rm had}) = 11.31$
- For further steps $L_{\rm had}\Lambda^{(3)}_{\overline{\rm MS}} \to \Lambda^{(3)}_{\overline{\rm MS}}$ [MeV] $\to \Lambda^{(5)}_{\overline{\rm MS}} \to \alpha_s(M_Z)$
  a scale is necessary.

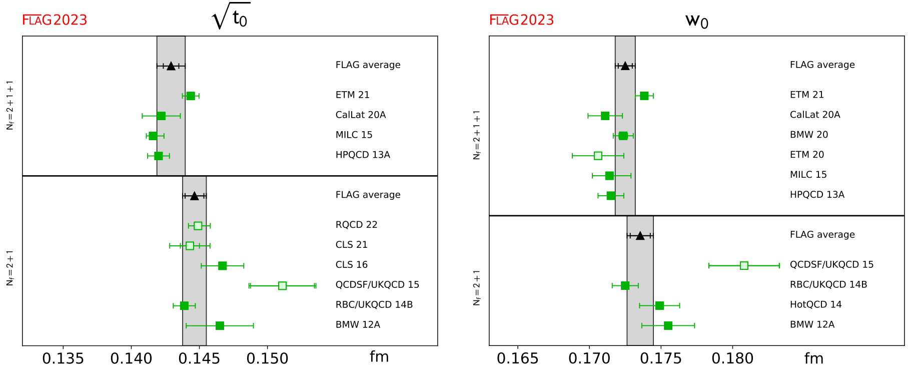
Two ingredients

- $1/\sqrt{t_0^\star}$ [MeV]
- $t_0^\star/a^2$ at β values where $L_{\rm had} \in \{8, 10, 12, 16, \ldots\}$
  $\to [\sqrt{t_0^\star}/L_{\rm had}]^{\rm cont}$ can be determined

(The second requirement is why $t_0^\star$ is better than $t_0^{\rm phys}$)

The World

Most groups compute t_0^{phys} or w_0^{phys} but not $t_0^\star$



[FLAG Review website]

Notable exception: RQCD, who compute also $t_0^\star$

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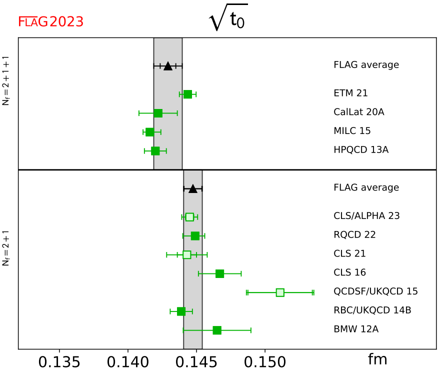


(unofficial update)

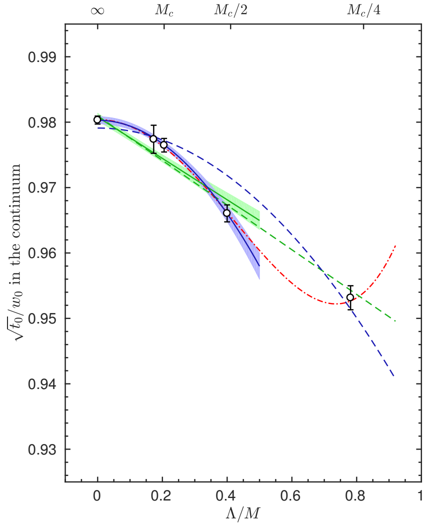
Notable exception: RQCD, who compute also $t_0^\star$

Discrepancies

What causes the wide spread of results?

- Frozen topology?
- Rooted staggered vs Wilson?
- Different inputs, e.g.
  physical vs iso-QCD
- $N_\mathrm{f} = 2+1$ vs $N_\mathrm{f} = 2+1+1$?

How well does decoupling of a charm quark work for ratios like $\sqrt{t_0}/f_\pi$, $\sqrt{t_0}/M_\Omega, \ldots$?
- For purely gluonic quantities the effect was much smaller.



[ALPHA, Phys.Lett.B 774 (2017)]

Scale Setting 2016

[M.Bruno, T.K., S.Schaefer, Phys.Rev.D 95 (2017)]

- CLS ensembles
  - 13 in total
  - No $\beta = 3.85$
  - Lightest: $m_\pi \approx 200$ MeV @ $\beta = 3.55$
- Compute: am_π, am_K, af_π, af_K, t_0/a^2
  ($O(a)$ improved f_x)
- Compute: Derivatives of ↑ with respect to bare quark masses
- Form dimensionless products
  - $\phi_2 = 8t_0 m_\pi^2$, proxy for light quark mass
  - $\phi_4 = 8t_0 \left(m_K^2 + \frac{1}{2} m_\pi^2\right)$, proxy for tr$[\overline{M}]$
  - $\sqrt{t_0} f_{\pi K} = \sqrt{t_0} \frac{2}{3} \left(f_K + \frac{1}{2} f_\pi\right)$
    (Mild quark-mass dependence along $\phi_4 =$ const. trajectory)

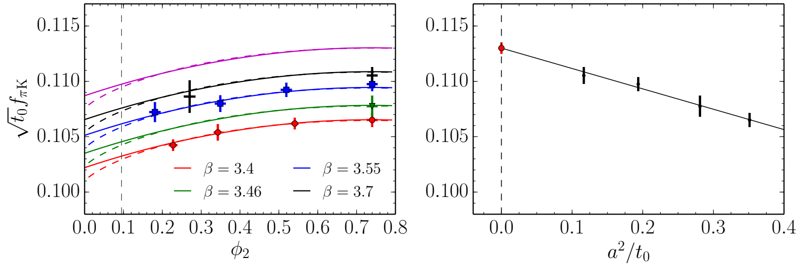
Strategy

- Use Physical inputs (pure iso-QCD):
  - $m_\pi^{phys} = 134.8(3)$ MeV [FLAG '16]
  - $m_K^{phys} = 494.2(3)$ MeV [FLAG '16]
  - $f_\pi^{\text{phys}} = 130.4(2)$ MeV [PDG '16]
  - $f_K^{\text{phys}} = 156.2(7)$ MeV [PDG '16]
- Guess t_0^{phys} [MeV]
- Iterate:
  - Compute ϕ_2^{phys}, ϕ_4^{phys}
  - Use derivatives to shift all quantities to mass point with $\phi_4 = \phi_4^{\text{phys}}$
  - Chiral-Continuum-Extrapolation (global fit) of $\sqrt{t_0} f_{\pi K}$ vs ϕ_2
  - From $a \to 0$, $\phi_2 \to \phi_2^{\text{phys}}$ extrapolation: read off $[\sqrt{t_0} f_{\pi K}]^{\text{phys}}$
  - Divide by $f_{\pi K}^{\text{phys}}$ to obtain new t_0^{phys}

Chiral-Continuum Extrapolations

Two types of fits

- Taylor around $\phi_2 = \phi_2^{\text{sym}}$
- $SU(3) - \chi$PT



$$\rightarrow \sqrt{8t_0^{\text{phys}}} = 0.415(4)(2) \text{ fm}$$

Update 2021

[B.Strassberger et al. PoS LATTICE2021 (2022)]

- Include correlator data from Zeuthen, Mainz and Regensburg
  $\rightarrow$ averaging procedure
  - 20 ensembles in total
  - including $\beta = 3.85$
  - including $m_\pi \approx m_\pi^{\text{phys}}$
- Mass derivatives not available on all ensembles
  $\rightarrow$ global fit for $\left.\frac{\partial X}{\partial \phi_4}\right|_{\text{fixed direction}}$