Spectroscopy with smeared spectral densities

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Challenges

To access $\rho(\omega_n)$ we face a **ill-posed inverse problem**

$$G(a\tau) = \int_{\omega_{\min}}^{\infty} d\omega \ \rho(\omega) e^{-a\omega\tau}$$

- finite L means $\rho(\omega_n)$ is a distribution of $\delta-$functions

$$\rho_L(\omega_n) = \sum_n \frac{|\langle n|\Phi(0)|0\rangle|^2}{2\omega_n(L)} \delta(\omega - \omega_n(L)).$$

- Difficulties depend on the method chosen to tackle the inverse problem. (MEM, BG, BR, Chebychev, . . .)
- $\rho(\omega_n)$ resolution depends on quality of the data and N_t.

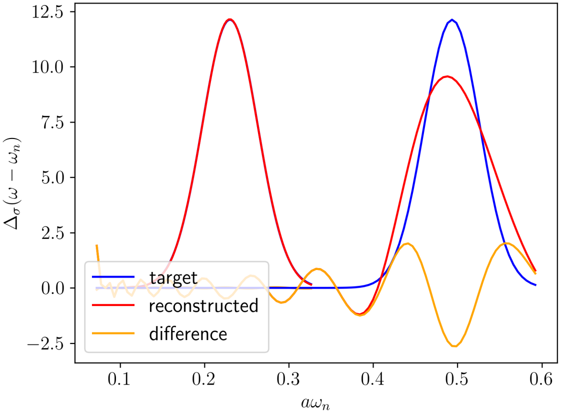
Linear methods

$i.e.$ Backus-Gilbert or Chebyshev polynomials approaches [Backus,Gilbert, R. Soc. '70 - Hansen,Meyer,Robaina, PRD '17 - Hansen,Lupo,Tantalo, PRD '19 - Bailas,Hashimoto,Ishikawa, Prog. Theo. Phys. '20]

- Reconstruct $\rho(\omega_n) = \sum_{\tau=1}^{\tau_{\max}} g_\tau(\omega_n) C(\tau)$, hence size of $g_\tau(\omega_n)$ important
- The introduction of $smearing\ \sigma$ necessary,

  problem: spectral features "washed out"
- Resolution gets progressively worse as we increase ω_n

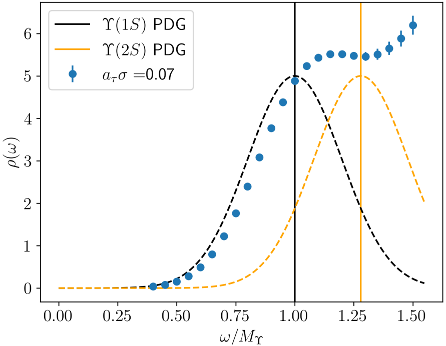
Important: interplay between σ, L and a.



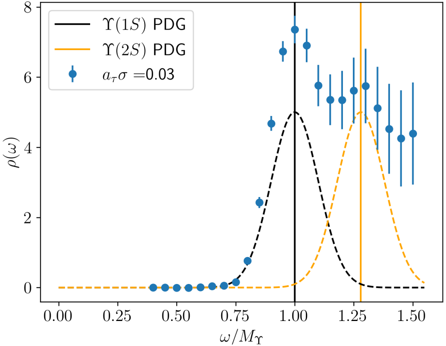
NRQCD Υ spectrum, $N_\tau = 128$, $N_s = 32$,

Resolution

We require $\sigma \gg \frac{1}{L}$ but also small enough to resolve spectral features



NRQCD Υ spectrum,
$N_\tau = 128,\ N_s = 32,\ a_\tau/a_s \simeq 3.5$

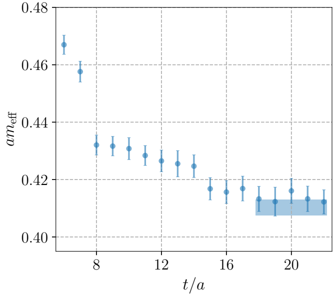


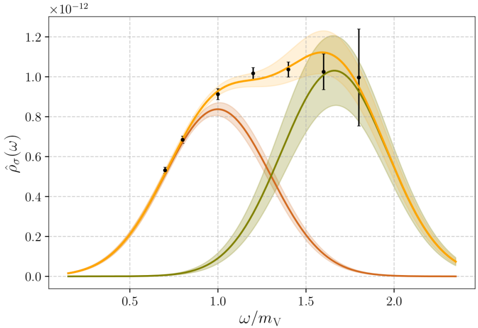
NRQCD Υ spectrum,
$N_\tau = 128,\ N_s = 32,\ a_\tau/a_s \simeq 3.5$

“True” $\rho(\omega_n)$ only recovered after $\lim_{L\to\infty}$ **followed** by $\lim_{\sigma\to 0}$.

Benefits

We can extract spectral densities and compute finite L spectrum.





Taken from 2405.01388, courtesy of Niccolò Forzano

	aE_0 G	aE_0 C	m_C	σ_G/m_C	σ_C/m_C
V	$0.4099(59)$	$0.4083(25)$	$0.4098(25)$	0.30	0.22

Extracted from 2405.01388, [Bennett *et al.*]. Spectral density and standard results compatible