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Jet Bundle Geometry of Scalar EFTs

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[M.Alminawi,I.Brivio and J.Davighi, arXiv:2308.00017]

Scalar Effective Field Theories

$$L = V + \frac{1}{2} g_{ij}(\phi)\partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4)$$

Outline

- Recap of geometric formulations of scalar EFTs
- Motivate and introduce jet bundle formalism
- Derive Feynman rules using jet bundles

Motivation for Geometric Formulation

- The two most popular EFT extensions of the standard model are SMEFT and HEFT

$$\text{SM} \subset \text{SMEFT} \subset \text{HEFT}$$

- Every SMEFT theory can be mapped onto HEFT, but the inverse is not true

- HEFT is needed when an $O(4)$ invariant point does not exist, which may be identified geometrically

  [T.Cohen, N.Craig, X.Lu and D.Sutherland ,arXiv:2008.08597]

  [R.Alonso, E.Jenkins and A.Manohar, arXiv:1605.03602]

Scalar Theories: Geometric Formulation

- Fields ϕ_i are viewed as coordinates on a field space manifold M

  [R. Alonso, E.E. Jenkins and A.V. Manohar, arXiv:1511.00724]

  [T. Cohen, N. Craig, X. Lu and D. Sutherland, arXiv:2108.03240]

- The manifold may be equipped with a Riemannian metric (geometry)

  [R. Alonso, E.E. Jenkins and A.V. Manohar, arXiv:1605.03602]

$$g = g_{ij}(\phi) d\phi^i \otimes d\phi^j$$

Geometric Formulation: Pullback

- The Lagrangian is a function on spacetime Σ while the metric g is a tensor on the manifold M

- The metric must be pulled back to Σ via a map ϕ^*

$$\phi^*\left(g(u)du^i \otimes du^j\right) = g(\phi)\partial_\mu\phi^i\partial_\nu\phi^j dx^\mu \otimes dx^\nu$$

- Contracting with $\eta^{\mu\nu} \frac{\partial}{\partial x^\mu} \otimes \frac{\partial}{\partial x^\nu}$ gives the two derivative terms of the Lagrangian

Motivation for Jet Bundles

- Limitations of the formalism:

  1. The potential V is not obtained from the metric and must be added in by hand
  2. Higher derivative terms $O(\partial^4)$ do not emerge from the geometry

- Field redefinitions involving derivatives have no geometric interpretation

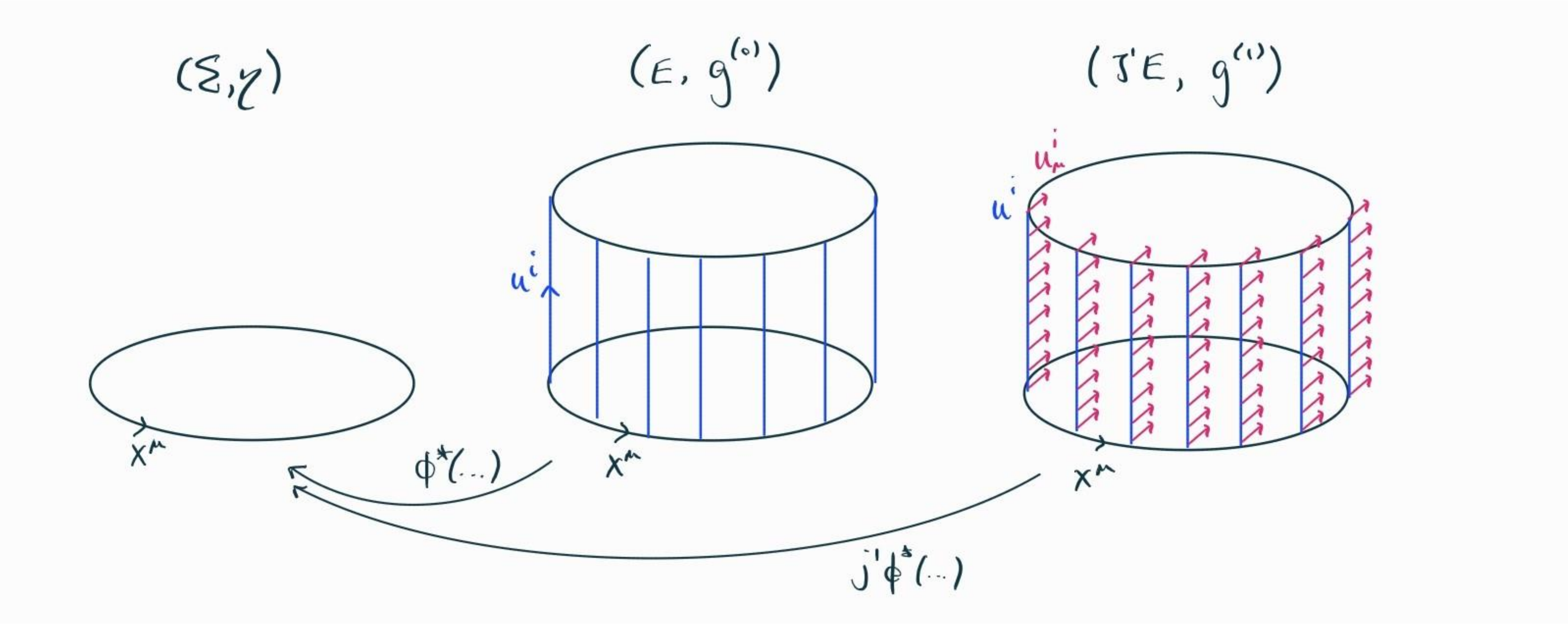
Motivation for Jet Bundles

- $L = V(\phi) + \frac{1}{2} g_{ij}(\phi)\partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4)$ Geometry on M

- $L = V(\phi) + \frac{1}{2} g_{ij}(\phi)\partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4) + O(\partial^6)$ Geometry on $J^1\pi$

- $L = V(\phi) + \frac{1}{2} g_{ij}(\phi)\partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4) + O(\partial^6) + O(\partial^8)$
  Geometry on $J^2\pi$

What is a Jet Bundle?



What is a Jet Bundle?

- The three manifolds Σ, E, J^1E have coordinates (x^μ), (x^μ, u^i) and (x^μ, u^i, u^i_μ) respectively

- Sections $\phi, \psi, \ldots$ are local inverses to the projection map $\pi\colon \Sigma \to E$ and are used to define scalar fields

$$\phi^* u^i = \phi^i$$

- Sections can be prolongated such that $j^1\phi$ is a local inverse to $\pi_1\colon \Sigma \to J^1E$

$$(j^1\phi)^* u^i = \phi^i, \qquad (j^1\phi)^* u^i_\mu = \partial_\mu \phi^i$$

Geometry on Jet Bundles

- A metric can be introduced to any jet manifold:
  - 0-Jet Bundle:

$$\begin{pmatrix} g_{\mu\nu}dx^{\mu} \otimes dx^{\nu} & g_{i\mu}dx^{\mu} \otimes du^{i} \\ g_{i\mu}dx^{\mu} \otimes du^{i} & g_{ij}du^{i} \otimes du^{j} \end{pmatrix}$$

  - 1-Jet Bundle:

$$\begin{pmatrix} g_{\mu\nu}dx^{\mu} \otimes dx^{\nu} & g_{i\mu}dx^{\mu} \otimes du^{i} & g_{i\mu}^{\nu}dx^{\mu} \otimes du_{\nu}^{i} \\ g_{i\mu}dx^{\mu} \otimes du^{i} & g_{ij}du^{i} \otimes du^{j} & g_{ij}^{\mu}du_{\mu}^{i} \otimes du^{j} \\ g_{i\mu}^{\nu}dx^{\mu} \otimes du_{\nu}^{i} & g_{ij}^{\mu}du_{\mu}^{i} \otimes du^{j} & g_{ij}^{\mu\nu}du_{\mu}^{i} \otimes du_{\nu}^{j} \end{pmatrix}$$

Geometry on Jet Bundles

- A Riemannian metric introduced on the jet manifold j^1E allows us to define a Lagrangian on Σ via

$$L = \frac{1}{2}\langle \eta^{-1}, (j^1\phi)^* g \rangle$$

- Field redefinitions are simply understood as changes of section $j^1\phi \to j^1\psi$

Geometry on Jet Bundles

$$L = \frac{1}{2}\langle \eta^{-1}, (j^1\phi)^* g \rangle$$

$$g = g_{\mu\nu} dx^\mu \otimes dx^\nu + g_{ij} d\phi^i \otimes d\phi^j + g_{ij}^{\mu\nu} d\phi^i_\mu \otimes d\phi^j_\nu$$

$$+ g_{i\mu} d\phi^i \otimes dx^\mu + g^\mu_{ij} d\phi^i_\mu \otimes d\phi^j + g^\mu_{i\nu} d\phi^i_\mu \otimes dx^\nu$$

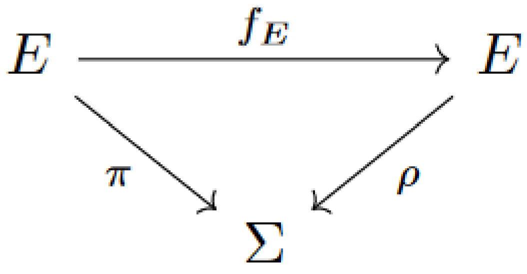
$$g_{ij}\eta^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j \subset L$$

$$g_{\mu\nu}\eta^{\mu\nu} = V(\phi) + \cdots \subset L$$

$$g^{\mu\nu}_{ij}\eta^{\rho\sigma}\partial_\rho\partial_\mu\phi^i\partial_\sigma\partial_\nu\phi^j \subset L$$

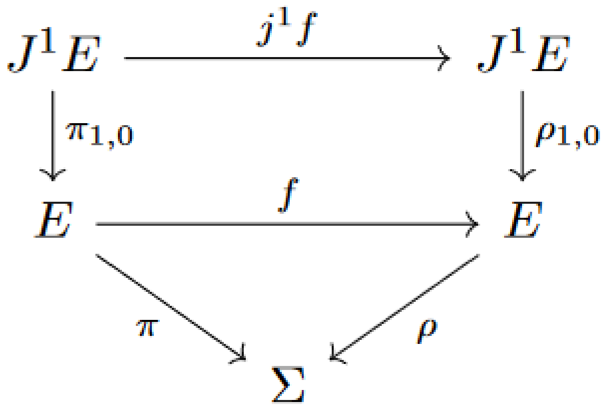
Non-Derivative Field Redefinitions

- In the case of non-derivative field redefinitions bundle morphisms provide an equivalent result to changes of sections



Morphism on 0-Jet Bundle

Equivalent to $\psi = f_E \circ \phi$



Morphism on 1-Jet Bundle

Equivalent to $j^1\psi = j^1 f \circ j^1\phi$

Non-Derivative Field Redefinitions

- Bundle morphisms consist of diffeomorphisms between manifolds
- $j^1 f$ is called a prolongation of f which means that it satisfies certain properties

$$x^\mu \circ j^1 f = x^\mu \circ \rho_1 \circ j^1 f = x^\mu$$
$$u^i \circ j^1 f = u^i \circ \rho_{1,0} \circ j^1 f = u^i \circ f$$

$$(u^i_\mu \circ j^1 f)(j^1_x \phi) = \partial_\mu \left(u^i \circ f\right)|_x + u^j_\mu \frac{\partial (u^i \circ f)}{\partial u^j}|_x$$

Non-Derivative Field Redefinitions

- Consider a single field in one dimensional spacetime, then the coordinates of the jet bundle are (t, u, u_t)
- A field redefinition of the form $\phi \to \phi^2$ can be expressed in terms of a bundle morphism

$$u \to u \circ j^1 f = u^2$$
$$u_t \to u_t \circ j^1 f = 2\, u\, u_t$$

- Equivalent change of section

$$u \circ \phi = \phi \to u \circ j^1 f \circ \phi = u^2 \circ \phi = \phi^2$$
$$u_t \circ \phi = \dot{\phi} \to u_t \circ j^1 f \circ \phi = 2\, u\, u_t \circ \phi = 2\phi\dot{\phi}$$

Feynman Rules

- The metric permits an expansion around a point p. Notation $g_{ij}(p) = \overline{g_{ij}}$, with derivatives $\partial_k g_{ij}(p) = \overline{g_{ij,k}}$

- We choose the point p to be the vacuum and impose Poincaré invariance

- We pullback the expansion of the metric to spacetime and Fourier transform to obtain the Feynman rules

Feynman Rules

- Propagator:

$$i \text{ - - - - - - - - } j \;=\; \frac{i}{\bar{g}_{ij}p^2 + \frac{\eta^{\mu\nu}}{2}\bar{g}_{\mu\nu,ij}{}^{\rho\sigma}p_\rho p_\sigma + 2\left(\bar{g}_{i\mu,j}{}^{\nu} - \bar{g}^{\nu}_{i\mu,j}\right)p_\nu p^\mu + \frac{\eta^{\mu\nu}}{2}\bar{g}_{\mu\nu,ij} + \bar{g}^{\mu\nu}_{ij}p^2 p_\mu p_\nu}$$

0-Jet terms

1-Jet Terms

- Requiring a canonical kinetic term imposes

$$\bar{g}_{ij} + \frac{1}{2d}\eta^{\mu\nu}\eta_{\rho\sigma}\bar{g}_{\mu\nu,ij}{}^{\rho\sigma} + \frac{2}{d}\delta^{\mu}_{\nu}\left(\bar{g}_{i\mu,j}{}^{\nu} - \bar{g}^{\nu}_{i\mu,j}\right) = \delta_{ij}$$

Geometric Invariants

- When expressing individual Feynman rules, we encounter non-tensorial objects such as $g_{ij,k}$ for example
- Amplitudes constructed from the Feynman rules should be tensors
- It is convenient then to rexpress individual Feynman rules in terms of Christoffel symbols and Riemann tensors

Three Point Amplitude on 0-Jet Bundle

- Consider the 3-point Feynman rule on the 0-Jet bundle

$$(\frac{1}{12}\eta^{\mu\nu}\overline{g_{\mu\,\nu,a_1a_2a_3}} + \frac{1}{2}\overline{g_{a_1a_2,a_3}}\, p_1 \cdot p_2 + \frac{1}{2}\overline{g_{a_1a_3,a_2}}\, p_1 \cdot\, p_3 + \frac{1}{2}\overline{g_{a_2\,a_3,a_1}}\{\, p_2 \cdot\, p_3)$$

- The momenta can be expressed via Mandelstam variable

$$p_i.p_j = \frac{1}{2}(s_{ij} - p_i^2 - p_j^2)$$

- Christoffel symbols are defined by

$$\Gamma^I_{JK} = \frac{1}{2}g^{IM}(g_{JM,K} + g_{KM,J} - g_{JK,M})$$

Three Point Amplitude on 0-Jet Bundle

- The compatibility of the metric tensor with the Levi Civita connection allows us to derive

$$\overline{g_{\mu\nu,a_1a_2a_3}} = \overline{(\partial_{a_3}\partial_{a_2}\Gamma^{\rho}_{a_1\mu})}\eta_{\rho\nu} + \overline{(\partial_{a_3}\partial_{a_2}\Gamma^{\rho}_{a_1\nu})}\eta_{\rho\mu}$$

- Contracting with $\eta^{\mu\nu}$ we get the full expression for the three point amplitude

$$i(\frac{1}{6}\eta^{\mu\nu}\overline{(\partial_{a_3}\partial_{a_2}\Gamma^{\mu}_{a_1\mu})} + \frac{1}{4}p_1^2\overline{\Gamma_{a_1a_2a_3}} + \frac{1}{4}p_2^2\overline{\Gamma_{a_2a_1a_3}} + \frac{1}{4}p_3^2\overline{\Gamma_{a_3a_1a_2}})$$

Thank you for your attention!