On Amplitudes and Field Redefinitions

LHC EFT WG Meeting, Mar 18, 2024

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arXiv: 2008.08597, 2108.03240, 2202.06965, 2312.06748
with Tim Cohen, Nathaniel Craig, and Dave Sutherland

Outline

- Impact of Field Redefinitions on EFTs
- Review: Non-derivative Field Redefinitions
  --- Field space geometry
- Challenge: Derivative Field Redefinitions
  --- How does 1PI Effective Action transform?
  --- All-loop tensor-like recursion relation
  --- Transformation lemma

Field Redefinition Invariance

$$\mathcal{L}(\phi) \quad , \quad \phi = f\left(\tilde{\phi}, \partial_\mu\right) = \tilde{\phi} + \frac{1}{\Lambda}\tilde{\phi}^2 + \frac{1}{\Lambda^3}\left(\partial\tilde{\phi}\right)^2 + \cdots$$

Different Lagrangian
Same on-shell amplitudes

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Different Lagrangian
Same on-shell amplitudes

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 \xrightarrow{\phi = \tilde{\phi} + \frac{1}{\Lambda}\tilde{\phi}^2} \frac{1}{2}\left(\partial\tilde{\phi}\right)^2 - \frac{1}{2}m^2\tilde{\phi}^2 - \frac{m^2}{\Lambda}\tilde{\phi}^3 + \frac{2}{\Lambda}\tilde{\phi}\left(\partial\tilde{\phi}\right)^2 - \frac{m^2}{2\Lambda^2}\tilde{\phi}^4 + \frac{2}{\Lambda^2}\tilde{\phi}^2\left(\partial\tilde{\phi}\right)^2$$

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$$\overline{\mathcal{M}}_3 = \frac{2}{\Lambda}\left(p_1^2 + p_2^2 + p_3^2 - 3m^2\right) \quad \rightarrow \quad 0$$

$$\overline{\mathcal{M}}_4 = -\frac{4}{\Lambda^2}\left[2\left(p_1^2 + p_2^2 - 2m^2\right) + \frac{1}{s_{12} - m^2}\left(p_1^2 + p_2^2 - 2m^2\right)^2\right]_{3\text{perms}} \quad \rightarrow \quad 0$$

Field Redefinition Invariance

$$\mathcal{L}(\phi) \quad , \quad \phi = f\left(\tilde{\phi}, \partial_\mu\right) = \tilde{\phi} + \frac{1}{\Lambda}\tilde{\phi}^2 + \frac{1}{\Lambda^3}\left(\partial\tilde{\phi}\right)^2 + \cdots$$

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Same on-shell amplitudes

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- This could make the following tasks less straightforward:
  - --- Enumerating Independent Operators (EOM Redundancies)
  - --- Distinguishing Linear vs Nonlinear Realizations
  - --- Calculating Amplitudes and RGEs

EOM Redundancies among Operators

$$\mathcal{L}^{(0)} \sim \frac{1}{\Lambda^0} \qquad \mathcal{L}^{(2)} \sim \frac{1}{\Lambda^2}$$

$$\mathcal{L} = \frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{4}\lambda\phi^4 + \frac{c_6}{\Lambda^2}\phi^6 + \frac{c_1}{\Lambda^2}\phi^3\left(\partial^2\phi\right) + \frac{c_2}{\Lambda^2}\left(\partial^2\phi\right)^2 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

Plug in the EOM: $\dfrac{\delta S^{(0)}}{\delta\phi} = -\partial^2\phi - \lambda\phi^3 = 0$

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Make a field redefinition:

$$\phi \to \phi + \frac{c_1}{\Lambda^2}\phi^3 \quad \Rightarrow \quad \mathcal{L} \to \mathcal{L} + \frac{\delta S^{(0)}}{\delta\phi}\frac{c_1}{\Lambda^2}\phi^3 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

$$= \mathcal{L} + \left(-\partial^2\phi - \lambda\phi^3\right)\frac{c_1}{\Lambda^2}\phi^3 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

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Impact of Field Redefinitions

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Make a field redefinition:

$$\frac{c_1}{\Lambda^2}\phi^3\frac{\delta^2 S^{(0)}}{\delta\phi^2}\frac{c_1}{\Lambda^2}\phi^3 + \frac{\delta S^{(2)}}{\delta\phi}\frac{c_1}{\Lambda^2}\phi^3$$

$$\phi \to \phi + \frac{c_1}{\Lambda^2}\phi^3 \quad \Rightarrow \quad \mathcal{L} \to \mathcal{L} + \frac{\delta S^{(0)}}{\delta\phi}\frac{c_1}{\Lambda^2}\phi^3 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

$$= \mathcal{L} + \left(-\partial^2\phi - \lambda\phi^3\right)\frac{c_1}{\Lambda^2}\phi^3 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

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~~Plug in the EOM~~: $\frac{\delta S^{(0)}}{\delta\phi} = -\partial^2\phi - \lambda\phi^3 = 0$

$$\frac{c_1}{\Lambda^2}\phi^3\left(\partial^2\phi\right) \to -\frac{c_1}{\Lambda^2}\lambda\phi^6 \qquad \frac{c_2}{\Lambda^2}\left(\partial^2\phi\right)^2 \to \frac{c_2}{\Lambda^2}\lambda^2\phi^6$$

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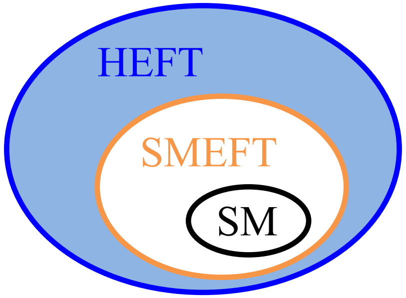
Linear vs Nonlinear Realizations



$$\mathcal{L}_{\text{HEFT}}(h,U) = K^2 \frac{1}{2}(\partial h)^2 + \frac{1}{2} v^2 F^2 \frac{1}{2} \text{tr}\left[\left(D_\mu U\right)^\dagger \left(D^\mu U\right)\right] + \cdots$$

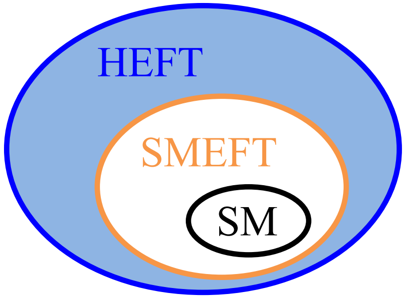
nonlinear realization

$$\left(i\sigma^2 H^*,\, H\right) = \frac{1}{\sqrt{2}}(v+h)\,U$$

$$\frac{1}{2}(v+h)^2 = |H|^2$$

$U = e^{i\pi^a \sigma^a / v}$: Goldstones

Linear vs Nonlinear Realizations



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nonlinear realization

$$\frac{1}{2}(\partial h)^2 = \frac{1}{4|H|^2}\left(\partial|H|^2\right)^2$$

$$\frac{1}{2}(v+h)^2 \frac{1}{2}\text{tr}\left[\left(D_\mu U\right)^\dagger \left(D^\mu U\right)\right] = |DH|^2 - \frac{1}{4|H|^2}\left(\partial|H|^2\right)^2$$

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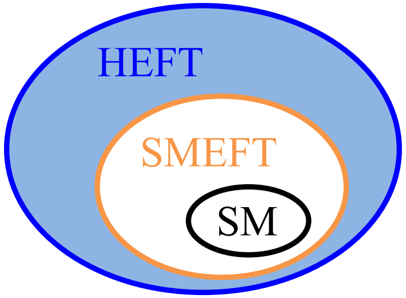

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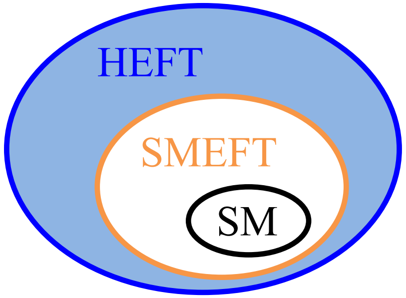
$$V(H) \propto \frac{1}{4}(v+h)^2 + \frac{1}{4v}(v+h)^3 + \frac{1}{16v^2}(v+h)^4$$

$$= \frac{1}{4}\left(2H^\dagger H\right) + \frac{1}{4v}\left(\sqrt{2H^\dagger H}\right)^3 + \frac{1}{16v^2}\left(2H^\dagger H\right)^2$$

HEFT?

Linear vs Nonlinear Realizations



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nonlinear realization

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$$(i\sigma^2 H^*, H) = \frac{1}{\sqrt{2}}(v+h) U$$

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$U = e^{i\pi^a \sigma^a / v}$: Goldstones

$$V(H) \propto \frac{1}{4}(v+h)^2 + \frac{1}{4v}(v+h)^3 + \frac{1}{16v^2}(v+h)^4$$

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HEFT?

$$= \left[\frac{1}{2}(v+h) + \frac{1}{4v}(v+h)^2\right]^2$$

$$= (v_1 + h_1)^2 = 2H_1^\dagger H_1$$

SMEFT

Field redefinition $h_1 \equiv h + \frac{1}{4v} h^2$

RG Ambiguities and Infinite RGEs

arXiv > hep-th > arXiv:2104.07037

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High Energy Physics – Theory

[Submitted on 14 Apr 2021 (v1), last revised 3 May 2021 (this version, v2)]

On Ambiguities and Divergences in Perturbative Renormalization Group Functions

Florian Herren, Anders Eller Thomsen

There is an ambiguity in choosing field-strength renormalization factors in the $\overline{\mathrm{MS}}$ scheme starting from the 3-loop order in perturbation theory. More concerning, trivially choosing Hermitian factors has been shown to produce divergent renormalization group functions, which are commonly understood to be finite quantities. We demonstrate that the divergences of the RG functions are such that they vanish in the RG equation due to the Ward identity associated with the flavor symmetry. It turns out that any such divergences can be removed using the renormalization ambiguity and that the use of the flavor-improved β-function is preferred. We show how our observations resolve the issue of divergences appearing in previous calculations of the 3-loop SM Yukawa β-functions and provide the first calculation of the flavor-improved 3-loop SM β-functions in the gaugeless limit.

arXiv > hep-ph > arXiv:2402.08715

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High Energy Physics – Phenomenology

[Submitted on 13 Feb 2024]

Field Redefinitions and Infinite Field Anomalous Dimensions

Aneesh V. Manohar, Julie Pagès, Jasper Roosmale Nepveu

Field redefinitions are commonly used to reduce the number of operators in the Lagrangian by removing redundant operators and transforming to a minimal operator basis. We give a general argument that such field redefinitions, while leaving the S-matrix invariant and consequently finite, lead not only to infinite Green's functions, but also to infinite field anomalous dimensions γ_ϕ. These divergences cannot be removed by counterterms without reintroducing redundant operators.

Field Space Geometry

$$\mathcal{L} = \frac{1}{2} g_{ab}(\phi)\left(\partial_\mu \phi^a\right)\left(\partial^\mu \phi^b\right) - V(\phi)$$

$$\Gamma^c_{ab} = \frac{1}{2} g^{ck}\left(g_{ka,b} + g_{kb,a} - g_{ab,k}\right) \quad \Rightarrow \quad \nabla_a$$

$$\phi = f\left(\tilde{\phi}, \cancel{\partial_\mu}\right) \quad \Rightarrow \quad \left(\partial_\mu \phi^a\right) = \frac{\partial \phi^a}{\partial \tilde{\phi}^b}\left(\partial_\mu \tilde{\phi}^b\right) \quad \Rightarrow \quad \tilde{g}_{cd}\left(\tilde{\phi}\right) = \frac{\partial \phi^a}{\partial \tilde{\phi}^c} \frac{\partial \phi^b}{\partial \tilde{\phi}^d} g_{ab}(\phi)$$

Field Space Geometry

$$\mathcal{L} = \frac{1}{2} g_{ab}(\phi)\left(\partial_\mu \phi^a\right)\left(\partial^\mu \phi^b\right) - V(\phi) \qquad \Gamma^c_{ab} = \frac{1}{2} g^{ck}\left(g_{ka,b} + g_{kb,a} - g_{ab,k}\right) \quad \Rightarrow \quad \nabla_a$$

$$\phi = f\left(\tilde{\phi}, \partial_\mu\right) \quad \Rightarrow \quad \left(\partial_\mu \phi^a\right) = \frac{\partial \phi^a}{\partial \tilde{\phi}^b}\left(\partial_\mu \tilde{\phi}^b\right) \quad \Rightarrow \quad \tilde{g}_{cd}\left(\tilde{\phi}\right) = \frac{\partial \phi^a}{\partial \tilde{\phi}^c} \frac{\partial \phi^b}{\partial \tilde{\phi}^d} g_{ab}(\phi)$$

$$\overline{\mathcal{M}}_{a_1 a_2 a_3} = -\bar{V}_{;(a_1 a_2 a_3)} \quad , \quad \overline{\mathcal{M}}_{a_1 a_2 a_3 a_4} = -\bar{V}_{;(a_1 a_2 a_3 a_4)} - \left[\frac{2}{3} \bar{R}_{a_1 (a_3 a_4) a_2} s_{12}\right]_{3\,\text{terms}} - \left[\bar{V}_{;(a_1 a_2 b)} \frac{\bar{g}^{bc}}{s_{12} - m_b^2} \bar{V}_{;(a_3 a_4 c)}\right]_{3\,\text{terms}}$$

Field Space Geometry

$$\mathcal{L} = \frac{1}{2} g_{ab}(\phi)\left(\partial_\mu \phi^a\right)\left(\partial^\mu \phi^b\right) - V(\phi) \qquad \Gamma^c_{ab} = \frac{1}{2} g^{ck}\left(g_{ka,b} + g_{kb,a} - g_{ab,k}\right) \quad \Rightarrow \quad \nabla_a$$

$$\phi = f\left(\tilde{\phi}, \cancel{\partial_\mu}\right) \quad \Rightarrow \quad \left(\partial_\mu \phi^a\right) = \frac{\partial \phi^a}{\partial \tilde{\phi}^b}\left(\partial_\mu \tilde{\phi}^b\right) \quad \Rightarrow \quad \tilde{g}_{cd}\left(\tilde{\phi}\right) = \frac{\partial \phi^a}{\partial \tilde{\phi}^c} \frac{\partial \phi^b}{\partial \tilde{\phi}^d} g_{ab}(\phi)$$

$$\overline{\mathcal{M}}_{a_1 a_2 a_3} = -\overline{V}_{;(a_1 a_2 a_3)} \quad , \quad \overline{\mathcal{M}}_{a_1 a_2 a_3 a_4} = -\overline{V}_{;(a_1 a_2 a_3 a_4)} - \left[\frac{2}{3} \overline{R}_{a_1 (a_3 a_4) a_2} s_{12}\right]_{\text{3 terms}} - \left[\overline{V}_{;(a_1 a_2 b)} \frac{\overline{g}^{bc}}{s_{12} - m_b^2} \overline{V}_{;(a_3 a_4 c)}\right]_{\text{3 terms}}$$

- Distinguishing Linear vs Nonlinear Realizations

  Alonso, Jenkins, and Manohar, arXiv: 1511.00724, 1605.03602

  Cohen, Craig, **XL**, and Sutherland, arXiv: 2008.08597

- Calculating Amplitudes and RGEs

  Cohen, Craig, **XL**, and Sutherland, arXiv: 2108.03240

  Alonso and West, arXiv: 2109.13290

  Helset, Jenkins, and Manohar, arXiv: 2210.08000, 2212.03253

  Assi, Helset, Manohar, Pagès, and Shen, arXiv: 2307.03187

  Jenkins, Manohar, Naterop, and Pagès, arXiv: 2308.06315, 2310.19883

Derivative Field Redefinitions?

- Lagrange Geometry and Jet Bundle Approach

Craig, Lee, **XL**, and Sutherland, arXiv: 2305.09722

Craig and Lee, arXiv: 2307.15742

Alminawi, Brivio, and Davighi, arXiv: 2308.00017

$$\text{Manifold Coordinates}: \quad \phi \;\to\; \left(\phi\,,\partial_{\mu_1}\phi\right) \;\to\; \left(\phi\,,\partial_{\mu_1}\phi\,,\cdots,\partial_{\mu_1}\cdots\partial_{\mu_k}\phi\right)$$

- Functional Geometry Approach

Cohen, Craig, **XL**, and Sutherland, arXiv: 2202.06965, 2312.06748

Infinite dimensional

$$\text{Manifold Coordinates}: \quad \phi \;\to\; \left(\phi\,,\partial_{\mu_1}\phi\,,\cdots,\partial_{\mu_1}\cdots\partial_{\mu_k}\phi\,,\cdots\right) \quad , \qquad \frac{\partial}{\partial\phi^a} \;\to\; \frac{\delta}{\delta\phi^a\left(x\right)}$$

- Geometry-Kinematics Duality

Cheung, Helset, and Parra-Martinez, arXiv: 2202.06972

$$\frac{1}{n!}\bar{g}_{p_1p_2\,,q_1\cdots q_n}\left(2\pi\right)^4\delta^4\left(p_1+p_2+q_1+\cdots+q_n\right)\equiv\frac{1}{-p_1p_2}\frac{1}{\left(n+2\right)!}\frac{\delta^{n+2}S}{\delta\phi\left(p_1\right)\delta\phi\left(p_2\right)\delta\phi\left(q_1\right)\cdots\delta\phi\left(q_n\right)}\bigg|_{\phi=0}$$

Tracking 1PI Effective Action

$$\overline{\mathcal{M}}_{a_1a_2a_3} = -\bar{V}_{;(a_1a_2a_3)} \quad , \quad \overline{\mathcal{M}}_{a_1a_2a_3a_4} = -\bar{V}_{;(a_1a_2a_3a_4)} - \left[\frac{2}{3}\bar{R}_{a_1(a_3a_4)a_2} s_{12}\right]_{3\text{ terms}} - \left[\bar{V}_{;(a_1a_2b)} \frac{\bar{g}^{bc}}{s_{12}-m_b^2} \bar{V}_{;(a_3a_4c)}\right]_{3\text{ terms}}$$

$$i\mathcal{M}_{x_1x_2x_3} = \frac{\delta^3(i\Gamma)}{\delta\varphi^{x_1}\delta\varphi^{x_2}\delta\varphi^{x_3}} \quad , \quad i\mathcal{M}_{x_1x_2x_3x_4} = \frac{\delta^4(i\Gamma)}{\delta\varphi^{x_1}\delta\varphi^{x_2}\delta\varphi^{x_3}\delta\varphi^{x_4}} + \left[\frac{\delta^3(i\Gamma)}{\delta\varphi^{x_1}\delta\varphi^{x_2}\delta\varphi^{y}} D^{yz} \frac{\delta^3(i\Gamma)}{\delta\varphi^{z}\delta\varphi^{x_3}\delta\varphi^{x_4}}\right]_{3\text{ terms}}$$

$$\mathcal{L}(\phi) \longrightarrow \Gamma[\varphi] \longrightarrow \mathcal{M}_{x_1\cdots x_n}$$

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$$\mathcal{L}(\phi) \longrightarrow \Gamma[\varphi] \longrightarrow \mathcal{M}_{x_1 \cdots x_n}$$

$$\phi = f\left(\tilde{\phi}, \partial_\mu\right) \qquad \widetilde{\mathcal{M}} \neq \mathcal{M} \quad , \quad \overline{\widetilde{\mathcal{M}}} = \overline{\mathcal{M}}$$

$$\tilde{\Gamma}[\tilde{\varphi}] \overset{?}{\longleftrightarrow} \Gamma[\varphi]$$

Derivative Field Redefinitions

Tracking 1PI Effective Action

$$\mathcal{L}_{\text{EFT}}(\phi) = \sum_i c_i \mathcal{O}_i(\phi) \quad \longleftrightarrow \quad \mathcal{L}_{\text{UV}}(\phi, \Phi)$$

same amplitudes

Functional Matching : $\Gamma_{\text{EFT}}[\varphi] = \Gamma_{\text{UV}}[\varphi] \quad \Rightarrow \quad \mathcal{L}_{\text{EFT}}(\mathcal{L}_{\text{UV}})$

Tracking 1PI Effective Action

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- Two-loop Formalism

Fuentes-Martín, Palavric, and Thomsen, arXiv: 2311.13630

- Automation Packages

Mathematica package: STrEAM.m

Cohen, **XL**, and Zhang, arXiv: 2012.07851

SUPER TRACER MATCHETE

Fuentes-Martín, König, Pagès, Thomsen, and Wilsch, arXiv: 2012.08506, 2212.04510

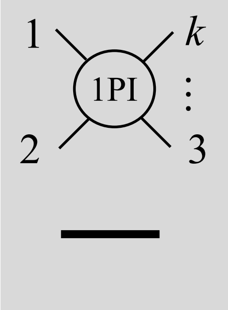
Tracking 1PI Effective Action

- All-loop tensor-like recursion

Cohen, Craig, **XL**, and Sutherland, arXiv: 2202.06965

$$k\text{-pt 1PI vertices}: \quad i\mathcal{V}_{x_1\cdots x_k} = \frac{\delta^k\left(i\Gamma\right)}{\delta\varphi^{x_1}\cdots\delta\varphi^{x_k}}$$

$$\text{full propagator}: \quad D^{xy} = -\left[\frac{\delta^2\left(i\Gamma\right)}{\delta\varphi^x\delta\varphi^y}\right]^{-1}$$



Tracking 1PI Effective Action

- All-loop tensor-like recursion — Cohen, Craig, **XL**, and Sutherland, arXiv: 2202.06965

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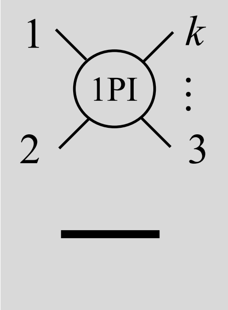

$$i\mathcal{V}_{x_1x_2x_3} = \frac{\delta^3 (i\Gamma)}{\delta\varphi^{x_1}\delta\varphi^{x_2}\delta\varphi^{x_3}} = i\mathcal{M}_{x_1x_2x_3}$$

$$\frac{\delta}{\delta\varphi^{x}} D^{y_1y_2} = D^{y_1z_1}\frac{\delta^3 (i\Gamma)}{\delta\varphi^{z_1}\delta\varphi^{x}\delta\varphi^{z_2}} D^{z_2y_2}$$

$$\mathcal{M}_{x_1\cdots x_n x_{n+1}} = \frac{\delta}{\delta\varphi^{x_{n+1}}}\mathcal{M}_{x_1\cdots x_n} - \sum_{i=1}^{n} G^{y}_{x_{n+1}x_i}\mathcal{M}_{x_1\cdots(x_i\to y)\cdots x_n}$$

$$G^{y}_{x_{n+1}x_i} \equiv -i\mathcal{M}_{x_{n+1}x_i z}D^{zy}$$

Tracking 1PI Effective Action

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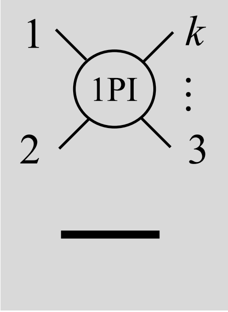

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- Transformation lemma — Cohen, **XL**, and Sutherland, arXiv: 2312.06748

$$\tilde{\Gamma}[\tilde{\varphi}] = \Gamma\left[\varphi[\tilde{\varphi}]\right] \quad \Rightarrow \quad \widetilde{\mathcal{M}}_{x_1\cdots x_n} = \frac{\delta\varphi^{y_1}}{\delta\tilde{\varphi}^{x_1}}\cdots\frac{\delta\varphi^{y_n}}{\delta\tilde{\varphi}^{x_n}}\mathcal{M}_{y_1\cdots y_n} + U_{x_1\cdots x_n} \quad \Rightarrow \quad \overline{\widetilde{\mathcal{M}}}_{x_1\cdots x_n} = \overline{\mathcal{M}}_{x_1\cdots x_n}$$

Tracking 1PI Effective Action

- All-loop tensor-like recursion Cohen, Craig, **XL**, and Sutherland, arXiv: 2202.06965

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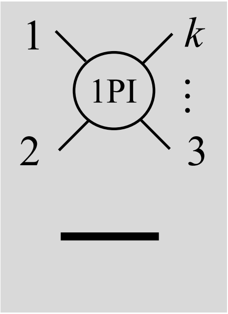

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- Transformation of 1PI Effective Action up to one loop

$$\varphi\left[\tilde{\varphi}\right] = f\left[\tilde{\varphi}\right] + a\left[\tilde{\varphi}\right]$$

$$\phi = f\left[\tilde{\phi}\right] \quad\Rightarrow\quad \tilde{\Gamma}\left[\tilde{\varphi}\right] = \Gamma\left[f\left[\tilde{\varphi}\right]\right] + \left.\frac{\delta S}{\delta\phi^{x}}\right|_{\phi=f\left[\tilde{\varphi}\right]} a^{x}\left[\tilde{\varphi}\right] = \Gamma\left[f\left[\tilde{\varphi}\right] + a\left[\tilde{\varphi}\right]\right]$$

Summary

- Field Redefinitions cause many complications in EFTs
  - --- EOM redundancies
  - --- Obscuring linear vs non-linear realizations
  - --- Complications in calculating amplitudes and RGEs
- Derivative Field Redefinitions are more challenging
- Tracking the transformation of 1PI Effective Action helps
  - --- All-loop tensor-like recursion relation

$$\mathcal{M}_{x_1\cdots x_n x_{n+1}} = \frac{\delta}{\delta\varphi^{x_{n+1}}}\mathcal{M}_{x_1\cdots x_n} - \sum_{i=1}^{n} G^{y}_{x_{n+1}x_i}\mathcal{M}_{x_1\cdots(x_i\to y)\cdots x_n} \qquad G^{y}_{x_{n+1}x_i} \equiv -i\mathcal{M}_{x_{n+1}x_i z}D^{zy}$$

  - --- Transformation lemma

$$\tilde{\Gamma}\left[\tilde{\varphi}\right] = \Gamma\left[\varphi\left[\tilde{\varphi}\right]\right] \quad \Rightarrow \quad \widetilde{\mathcal{M}}_{x_1\cdots x_n} = \frac{\delta\varphi^{y_1}}{\delta\tilde{\varphi}^{x_1}}\cdots\frac{\delta\varphi^{y_n}}{\delta\tilde{\varphi}^{x_n}}\mathcal{M}_{y_1\cdots y_n} + U_{x_1\cdots x_n}$$