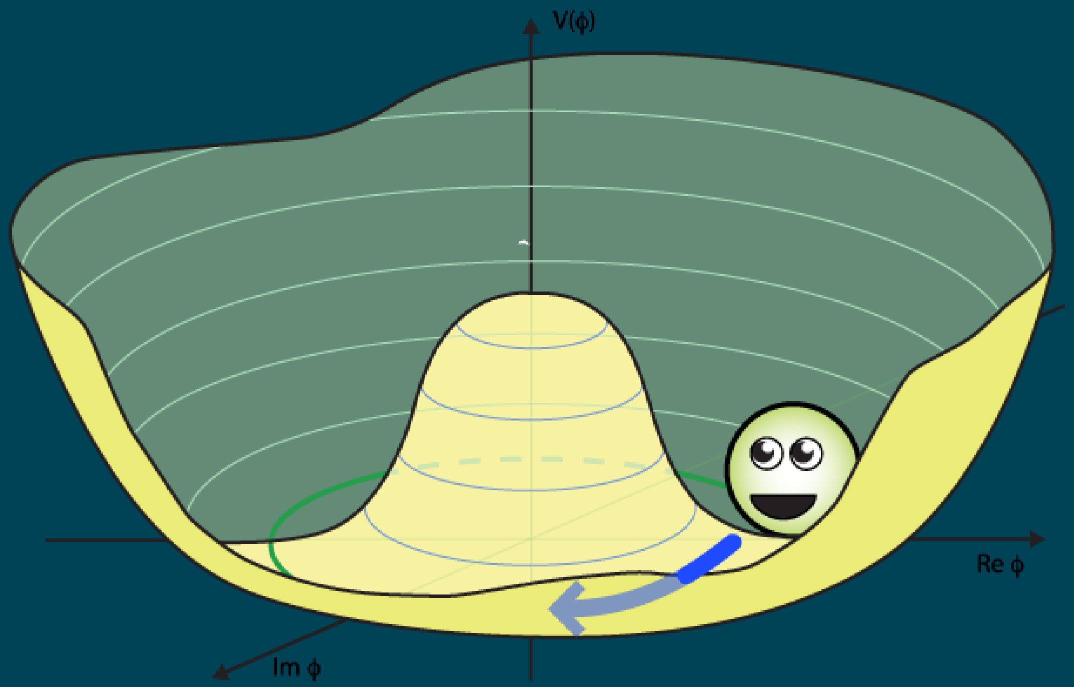
Western Norway University of Applied Sciences

Extended Higgs sectors

Talk given at Higgs 2024,
Uppsala, November 2024



Odd Magne Øgreid

Motivations for extended Higgs sectors:

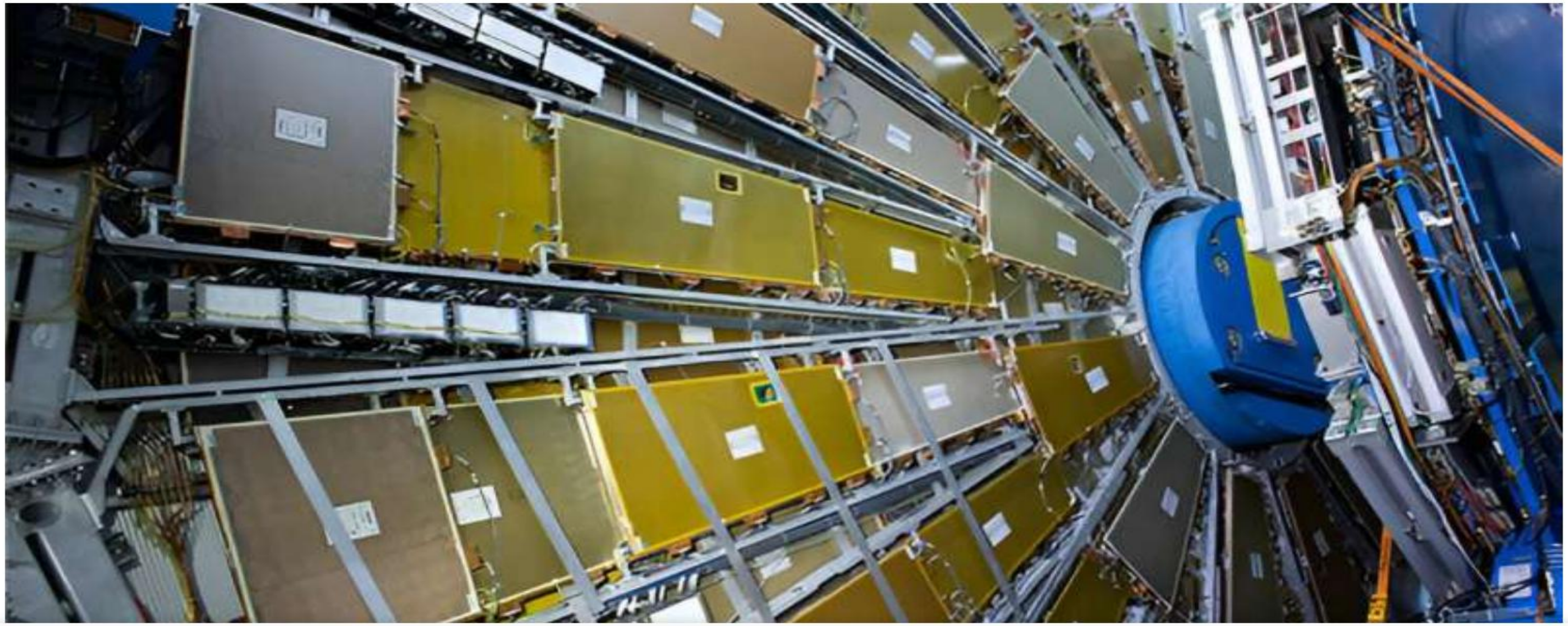
- Shortcomings of the Standard Model of particle physics. Fails to explain phenomena like for instance
  - The Matter-Antimatter asymmetry of the universe
  - Dark matter and dark energy
  - The strong CP problem
  - Neutrino masses and oscillations
  - Does not include gravity
  - It is just a low-energy approximation of the interactions between particles, and becomes invalid at higher energies.
  - Does not explain the large spread of the masses of the elementary particles.
  - Does not explain the difference in strength between the fundamental forces.
- Experimental access to Higgs sector since July 2012. There might be answers to some of these questions hidden in this sector.
- Extended Higgs sectors may allow for
  - Dark matter
  - Additional sources of CP violation
- A CP-conserving 2HDM is embedded in SUSY models
- Provides a rich (but not too rich) particle zoo.
- Expect large portions of parameter space testable at LHC for such models.
- Some popular extensions
  - Extra scalar singlets (complex or real)
  - Extra doublets (NHDMs)
  - Triplets
  - Composite Higgs
  - …

AUGUST 22, 2024

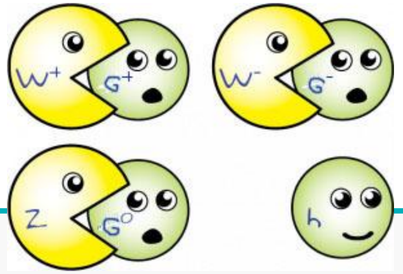
Editors' notes

Deviations in particle interactions point to the existence of new bosons

by Wits University



The Higgs-Doublet of the Standard Model



- In SM we have only one Higgs-Doublet $\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} G^+ \\ v + h + iG^0 \end{pmatrix}$

- Kinetic term: $\mathcal{L}_{\text{kin}} = (D^\mu \Phi)^\dagger (D_\mu \Phi)$

Free-field + interactions with Gauge bosons

- Potential: $-\mathcal{L}_V = \mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$

Mass terms + self interactions

- Yukawas: $$-\mathcal{L}_Y = \overline{Q^0_L}\tilde{\Phi}\eta^{U,0}U^0_R + \overline{Q^0_L}\Phi\left(\eta^{D,0}\right)^\dagger D^0_R + \overline{L^0_L}\Phi\left(\eta^{E,0}\right)^\dagger E^0_R + \text{h.c.}$$

Fermion masses + fermion interactions

The general 2HDM potential

$$V = V_2 + V_4$$

$$V_2 = -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[\underline{m_{12}^2}\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\}$$

$$V_4 = \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

$$+\frac{1}{2}\left[\underline{\lambda_5}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\underline{\lambda_6}(\Phi_1^\dagger\Phi_1) + \underline{\lambda_7}(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\}$$

- 14 parameters (reducible to 11)
- 4 **complex** parameters
- Model implies flavor-changing neutral currents - not observed. Remedy: Constrain model by imposing a symmetry (Z_2)
- Leads to study of allowed symmetries of the 2HDM potential extensible to Lagrangian
- Six (seven?) transformations on the doublets are known that leave both the potential and kinetic terms unchanged. In addition, there are custodial symmetries.

Three Higgs-Family-symmetries: Z_2, U(1) and SO(3)

Z_2: Invariance under the transformation

$$\Phi_1 \to \Phi_1, \qquad \Phi_2 \to -\Phi_2$$

U(1): Invariance under the transformation

$$\Phi_1 \to e^{-i\theta}\Phi_1, \qquad \Phi_2 \to e^{i\theta}\Phi_2$$

SO(3): Invariance under the transformation

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \to \begin{pmatrix} e^{-i\alpha}\cos\theta & e^{-i\beta}\sin\theta \\ -e^{i\beta}\sin\theta & e^{i\alpha}\cos\theta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

Three CP-symmetries: CP1, CP2 and CP3

CP1: Invariance under the transformation

$$\Phi_1 \to \Phi_1^*, \qquad \Phi_2 \to \Phi_2^*$$

CP2: Invariance under the transformation

$$\Phi_1 \to \Phi_2^*, \qquad \Phi_2 \to -\Phi_1^*$$

CP3: Invariance under the transformation

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \Phi_1^* \\ \Phi_2^* \end{pmatrix}$$

Each of these symmetries constrains the 2HDM, leaving us with fewer parameters, and constraints upon observable masses and couplings of the model.

Also possible to break these symmetries softly or spontaneously.

Implications for the potential parameters in the symmetry basis

	V_2			V_4						
Symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
CP1			real					real	real	real
Z_2			0						0	0
U(1)			0					0	0	0
CP2		m_{11}^2	0		λ_1					$-\lambda_6$
CP3		m_{11}^2	0		λ_1			$\lambda_1 - \lambda_3 - \lambda_4$ (real)	0	0
SO(3)		m_{11}^2	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

- Symmetries may or may not be spontaneously broken by the vacuum.
- Possible to introduce soft breaking terms in V_2.
- Many different models possible.
- How to distinguish between all these models? They all imply different physics.
- Introduce new parameters (masses and couplings)

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{e^{i\xi}}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

The physical parameter set $\mathcal{P}$ and counting of parameters.

- Potential has initially 14 parameters
- Exploit the freedom to change basis and reduce to 11 independent parameters.
- Traditional approach:
  Work out masses and couplings expressed in terms of the initial 14 (or 11) parameters of the potential (exchange some for VEVs).
- Alternative approach:
  Work the other way around. Pick a set of 11 independent physical masses and couplings (all invariants) and express the initial 14 parameters in terms of these
- If we choose our set of 11 independent parameters to consist of:
  - Four squared masses
  - Three gauge couplings
  - Four scalar couplings

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

- Observables from the potential (invariants) expressible through these.
- Trilinear and quadrilinear scalar couplings expressible through these.

$$\begin{aligned} e_i &\equiv \frac{2}{g^2}\mathrm{Coefficient}(\mathcal{L}, H_i W^- W^+) \\ q_i &\equiv \mathrm{Coefficient}(V, H_i H^- H^+) \\ q &\equiv \mathrm{Coefficient}(V, H^- H^- H^+ H^+). \end{aligned}$$

Satisfying: $v^2 = e_1^2 + e_2^2 + e_3^2$

Description of translation process:
Ogreid: PoS CORFU2017 (2018) 065

Remaining scalar couplings expressible in terms of $\mathcal{P}$:
Grzadkowski, Haber, Ogreid & Osland: JHEP 12 (2018) 056

Symmetries of potential (exact, spontaneously broken or softly broken all described in terms of $\mathcal{P}$:
Ferreira, Grzadkowski, Ogreid & Osland: JHEP 02 (2021) 196
Ferreira, Grzadkowski, Ogreid & Osland: JHEP 01 (2023) 143

Results for exact symmetries

Symmetry	Conditions	Remark
CP1	$e_k = q_k = 0$	H_k is CP-odd
Z_2	$e_j = e_k = q_j = q_k = 0$	Inert Doublet Model (IDM)
U(1)	$M_j = M_k, \quad e_j = q_j = e_k = q_k = 0$	IDM with mass degeneracy
CP2	Not possible	
CP3	$M_j = M_k, \quad e_j = q_j = e_k = q_k = 0,$ $2M_{H^\pm}^2 = e_i q_i - M_i^2, \quad 2v^2 q = M_i^2$	IDM with mass degeneracy plus constraints also on charged mass and quadrilinear scalar coupling
SO(3)	Not possible	

More solutions were found. These were mass-degenerate cases which turned out to be RGE-unstable and therefore not included here.

Results for spontaneously broken symmetries

Symmetry	Conditions	Remark
CP1	$\frac{M_{H^\pm}^2}{v^2} = \frac{e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_1^2 M_3^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2}{2(e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2)},$ $q = \frac{(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2}{2(e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2)}$	Constraint on charged mass and quadrilinear ch. scalar coupling
Z_2	$M_{H^\pm}^2 = v^2 \frac{e_j q_j M_i^2 + e_i q_i M_j^2 - M_i^2 M_j^2}{e_j^2 M_i^2 + e_i^2 M_j^2},$ $q = \frac{(e_j q_i - e_i q_j)^2 + M_i^2 M_j^2}{2(e_j^2 M_i^2 + e_i^2 M_j^2)}, \quad e_k = q_k = 0$	H_k is CP-odd
U(1)	$M_{H^\pm}^2 = v^2 \frac{e_j q_j M_i^2 + e_i q_i M_j^2 - M_i^2 M_j^2}{2(e_j^2 M_i^2 + e_i^2 M_j^2)},$ $q = \frac{(e_j q_i - e_i q_j)^2 + M_i^2 M_j^2}{2(e_j^2 M_i^2 + e_i^2 M_j^2)}, \quad e_k = q_k = 0, \quad M_k = 0$	H_k is CP-odd and massless
CP2	$2M_{H^\pm}^2 = e_i q_i - M_i^2, \quad 2v^2 q = M_i^2,$ $e_j = q_j = e_k = q_k = 0$	IDM with two extra constraints
CP3	$2M_{H^\pm}^2 = e_i q_i - M_i^2, \quad 2v^2 q = M_i^2,$ $e_j = q_j = e_k = q_k = 0, \quad M_k = 0$	IDM with a massless scalar plus two extra constraints
SO(3)	$2M_{H^\pm}^2 = e_i q_i - M_i^2, \quad 2v^2 q = M_i^2,$ $e_j = q_j = e_k = q_k = 0, \quad M_j = M_k = 0$	IDM with two massless scalars plus two extra constraints

More solutions were found. These were mass-degenerate cases which turned out to be RGE-unstable and therefore not included here.

Results for softly broken CP1

- Demanding invariance of V_4 under CP1
- Equivalent to I_{6Z}= 0 (Gunion & Haber)

$$I_{6Z} = c_{21}M^4_{H^\pm}q + c_{20}M^4_{H^\pm} + c_{12}M^2_{H^\pm}q^2 + c_{11}M^2_{H^\pm}q + c_{10}M^2_{H^\pm} + c_{03}q^3 + c_{02}q^2 + c_{01}q + c_{00} = 0$$

- Here, c_{ij} are polynomials in the parameters $\{M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3\}$
- 10 free parameters

Results for softly broken Z_2

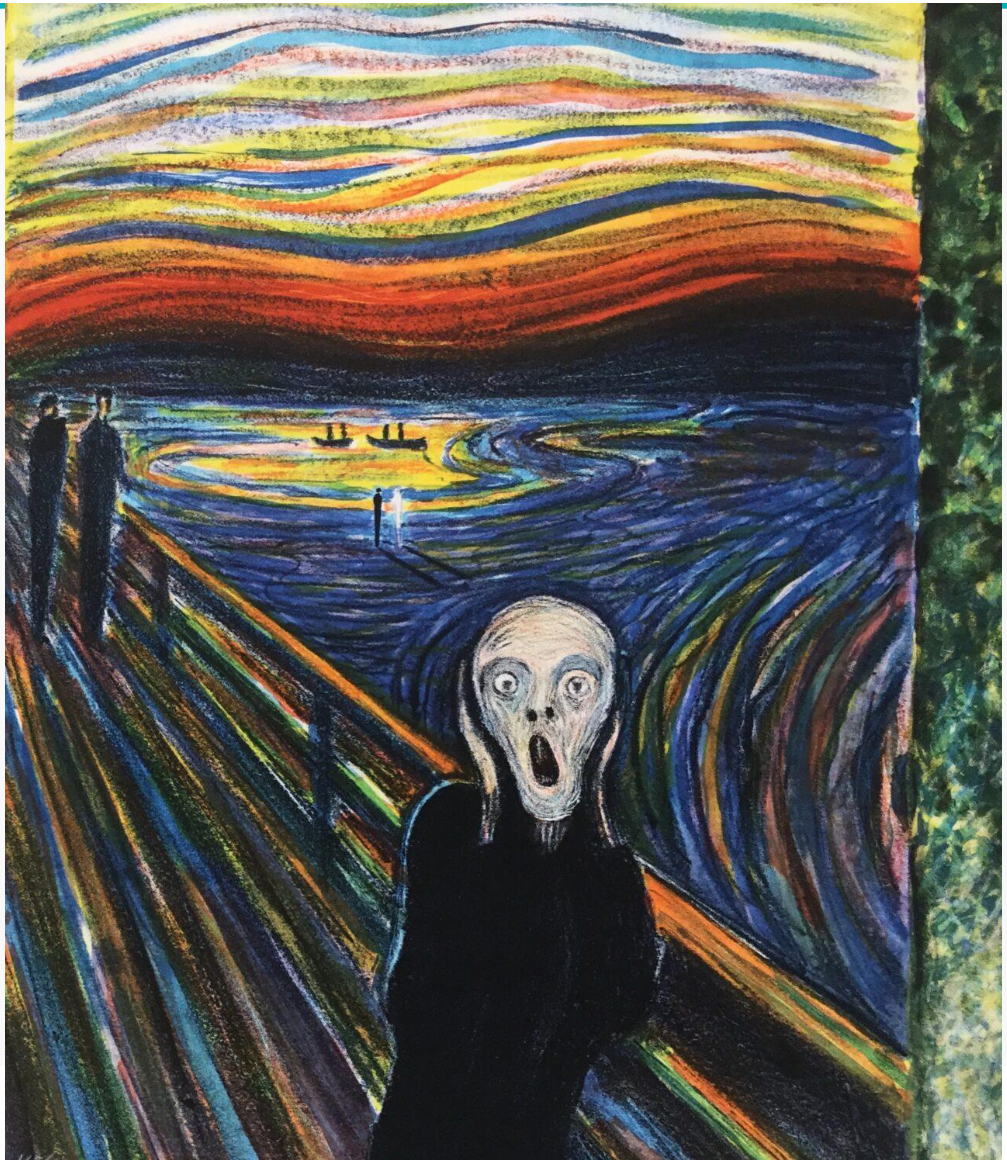
- Demanding invariance of V_4 under Z_2
- Equivalent to $[Z^{(1)}, Z^{(11)}]$= 0 (Davidson & Haber)

$$q = d_{010} - \frac{1}{2}d_{012} - d_{101} - \frac{4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\mathrm{Im}J_{30}}{2\mathrm{Im}J_1},$$

$$\begin{aligned}
&2\mathrm{Im}J_1[2(d_{012} + d_{101} - d_{010})\mathrm{Im}J_1 + 4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\,\mathrm{Im}J_{30}]M^2_{H^\pm} \\
&= v^2\{2(d_{010}d_{012} - d_{010}d_{101} - d_{022} + d_{200})(\mathrm{Im}J_1)^2 \\
&\quad + [4(2d_{101} - d_{010})\mathrm{Im}J_{11} + (d_{012} - 2d_{010} + 3d_{101})\mathrm{Im}J_2 + 2(d_{101} - d_{012})\mathrm{Im}J_{30}]\mathrm{Im}J_1 \\
&\quad + (2\,\mathrm{Im}J_{11} + \mathrm{Im}J_2)(4\,\mathrm{Im}J_{11} + \mathrm{Im}J_2 + 2\,\mathrm{Im}J_{30})\}
\end{aligned}$$

- Here, d_{ijk} and Im J_i are polynomials in the parameters $\{M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3\}$
- 9 free parameters
- Physical description of the C2HDM
- Popular model since FCNC are constrained and CP is broken
- No dark matter

The seventh(?) symmetry of the 2HDM



Working out the constraints for a softly broken CP3, we discovered a model with extraordinary properties.

- It is RG-stable, yet does not correspond to any of the six known symmetries of the 2HDM
- What is going on? The set of constraints

$$\begin{aligned} \lambda_6 + \lambda_7 &= 0, \\ \lambda_2 &= \lambda_1, \\ m_{11}^2 + m_{22}^2 &= 0, \end{aligned}$$

was found to be RGE-stable to all loop orders (using results of Bednyakov).

Bilinears formalism of the 2HDM and the r_0-symmetry

- Potential in bilinear notation

$$V = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

where

$$\begin{aligned} r^\mu &= (r_0\,,\, r_1\,,\, r_2\,,\, r_3) = (r_0\,,\, \vec{r})\,, \\ M^\mu &= (m_{11}^2 + m_{22}^2\,,\, 2\mathrm{Re}(m_{12}^2)\,,\, -2\mathrm{Im}(m_{12}^2)\,,\, m_{22}^2 - m_{11}^2) = (M_0\,,\, \vec{M})\,, \end{aligned}$$

$$\Lambda^{\mu\nu} = \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\lambda_1+\lambda_2)+\lambda_3 & -\mathrm{Re}\,(\lambda_6+\lambda_7) & \mathrm{Im}\,(\lambda_6+\lambda_7) & \frac{1}{2}(\lambda_2-\lambda_1) \\ -\mathrm{Re}\,(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & \mathrm{Re}\,(\lambda_6-\lambda_7) \\ \mathrm{Im}\,(\lambda_6+\lambda_7) & -\mathrm{Im}\,(\lambda_5) & \lambda_4-\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_6-\lambda_7) \\ \frac{1}{2}(\lambda_2-\lambda_1) & \mathrm{Re}\,(\lambda_6-\lambda_7) & -\mathrm{Im}\,(\lambda_6-\lambda_7) & \frac{1}{2}(\lambda_1+\lambda_2)-\lambda_3 \end{pmatrix}.$$

$$\begin{aligned} r_0 &= \frac{1}{2}\left(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2\right), \\ r_1 &= \frac{1}{2}\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) = \mathrm{Re}\left(\Phi_1^\dagger\Phi_2\right), \\ r_2 &= -\frac{i}{2}\left(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1\right) = \mathrm{Im}\left(\Phi_1^\dagger\Phi_2\right), \\ r_3 &= \frac{1}{2}\left(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right). \end{aligned}$$

- The six «old» symmetries can be explained from invariance under $\vec{r} \to S\,\vec{r}$ e.g.

$$S_{Z_2} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\,, \quad S_{CP1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- Can we express the «new symmetry» in the bilinear formalism?
- Turns out that invariance under $r_0 \to -r_0$ leads to our model
- Hence the name r_0-symmetry
- Impossible to change sign of r_0 using HF- or CP-transformations

More on the r_0-symmetry

- Parameterise the two doublets as

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix},$$

then

$$\begin{aligned} r_0 &= \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 + \phi_5^2 + \phi_6^2 + \phi_7^2 + \phi_8^2), \\ r_1 &= \phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8, \\ r_2 &= -\phi_2\phi_5 + \phi_1\phi_6 - \phi_4\phi_7 + \phi_3\phi_8, \\ r_3 &= \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 - \phi_5^2 - \phi_6^2 - \phi_7^2 - \phi_8^2). \end{aligned}$$

- Want to change sign of r_0 while r_1, r_2 , r_3 are unchanged

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}$$

works!!!

- What about the kinetic terms? Define

$$D^\mu = \partial^\mu + \frac{ig}{2}\sigma_i W_i^\mu + i\frac{g'}{2}B^\mu,$$

and scalar kinetic terms

$$\mathcal{L}_k = (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2)$$

- Turns out to be invariant under r_0 provided also

$$\partial_\mu \to -i\partial_\mu, \quad B_\mu \to iB_\mu,$$

$$W_{1\mu} \to iW_{1\mu}, \quad W_{2\mu} \to -iW_{2\mu}, \quad W_{3\mu} \to iW_{3\mu}.$$

- Call this "*the extended r_0 transformation*"

Implies $x_\mu \to ix_\mu$, but leaves d^4x invariant

- Gauge kinetic terms $\mathcal{L}^B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{i\mu\nu}W_i^{\mu\nu}$ also invariant.

$$\begin{aligned} B^{\mu\nu} &= \partial^\nu B^\mu - \partial^\mu B^\nu, \\ W_i^{\mu\nu} &= \partial^\nu W_i^\mu - \partial^\mu W_i^\nu + g\epsilon_{ijk}W_j^\mu W_k^\nu, \end{aligned}$$

- Stay tuned…

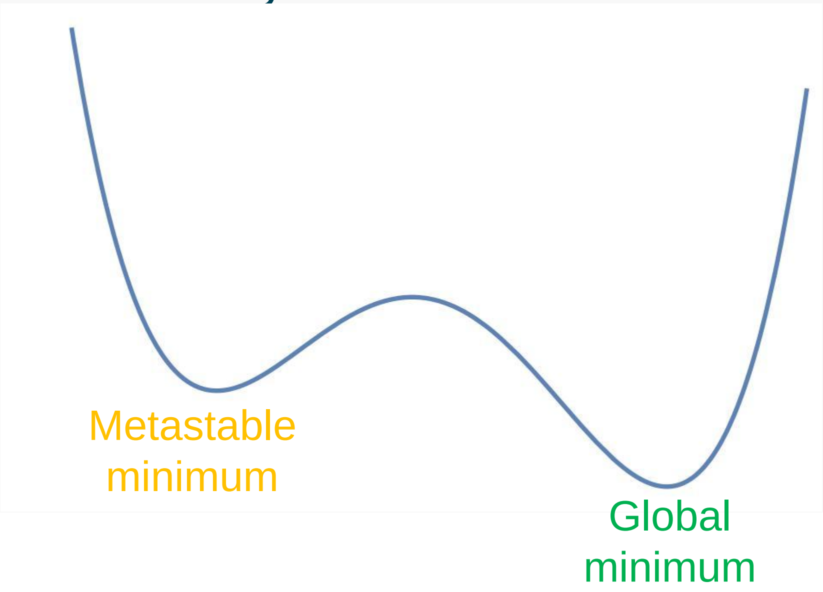
Three Higgs doublet models:

- In 2HDM, one can have Dark Matter OR CP violation, but not both simultaneously
- In 3HDM, one can have both Dark Matter and CP violation simultaneously.
- 3HDM Adds two more neutral scalars and one extra pair of charged scalars to the list.
- Study of allowed symmetries in the 3HDM and their implications
- Weird models with CP-symmetries of order 4.
- Again, symmetries may be broken either spontaneously or softly
- Large zoo of possible symmetry-constrained models available. Pick one you like…
- Some popular models
  - $Z_2 x Z_2$
  - S_3
  - $U(1)_1$ or $U(1)_2$
  - CP4

Constraints for NHDMs

Theoretical constraints

- Positivity of squared masses
- Boundedness from below
- Unitarity at tree level
- Perturbativity
- Metastability



Experimental constraints

- Near-alignment $W^+W^-h_{\mathrm{SM}}$
- CP constraints $h_{\mathrm{SM}} \to \tau\bar{\tau}$
- Electroweak precision observables *S*, *T*, *U*
- Digamma signal strength $h_{\mathrm{SM}} \to \gamma\gamma$
- $\bar{B} \to X_s\gamma$
- Electron EDM

CP-symmetries of the NHDM potential (and the VEV)

Whenever there exists a U(N) matrix X_{ij} so that both the NHDM potential and the VEV is invariant under the transformation

$$\Phi_i \to X_{ij}\Phi_j^*$$

the NHDM is CP-invariant, or **CP-conserving**.

If only the potential is invariant, but not the VEV, then CP is spontaneously broken.

If only V_4 is CP-invariant, but not V_2, then CP is softly broken.

Alternative form for NHDM-potential useful

$$V(\Phi_1,\Phi_2) = Y_{ab}\Phi_a^\dagger\Phi_b + \frac{1}{2}Z_{abcd}(\Phi_a^\dagger\Phi_b)(\Phi_c^\dagger\Phi_d)$$

Tensor constructed from VEVs useful

$$V_{ab} = \frac{v_a v_b^*}{v^2}$$

For 2HDM:

$$Y_{11} = -\frac{m_{11}^2}{2}, \quad Y_{12} = -\frac{m_{12}^2}{2},$$
$$Y_{21} = -\frac{(m_{12}^2)^*}{2}, \quad Y_{22} = -\frac{m_{22}^2}{2},$$

$$\begin{aligned}
&Z_{1111} = \lambda_1, \quad Z_{2222} = \lambda_2, \quad Z_{1122} = Z_{2211} = \lambda_3,\\
&Z_{1221} = Z_{2112} = \lambda_4, \quad Z_{1212} = \lambda_5, \quad Z_{2121} = (\lambda_5)^*,\\
&Z_{1112} = Z_{1211} = \lambda_6, \quad Z_{1121} = Z_{2111} = (\lambda_6)^*,\\
&Z_{1222} = Z_{2212} = \lambda_7, \quad Z_{2122} = Z_{2221} = (\lambda_7)^*.
\end{aligned}$$

CP-symmetry of the 2HDM

CP-properties of the 2HDM determined by three CP-odd invariants, first discovered by Lavoura and Silva. Re-expressed by Gunion and Haber as:

$$
\begin{aligned}
\mathrm{Im}\, J_1 &= -\frac{2}{v^2}\mathrm{Im}\left[V_{da}Y_{ab}Z_{bccd}\right], \\
\mathrm{Im}\, J_2 &= \frac{4}{v^4}\mathrm{Im}\left[V_{ab}V_{dc}Y_{be}Y_{cf}Z_{eafd}\right], \\
\mathrm{Im}\, J_3 &= \mathrm{Im}\left[V_{ab}V_{dc}Z_{bgge}Z_{chhf}Z_{eafd}\right].
\end{aligned}
$$

2HDM (potential and vacuum) is CP conserving iff

$$\mathrm{Im}\, J_1 = \mathrm{Im}\, J_2 = \mathrm{Im}\, J_3 = 0$$

- Understood in terms of physical parameters

$$e_k = q_k = 0$$

Another set of invariants determine if potential only is CP invariant.

$$
\begin{aligned}
I_{Y3Z} &= \mathrm{Im}\left[Z^{(1)}_{a\bar{c}}Z^{(1)}_{e\bar{b}}Z_{b\bar{e}c\bar{d}}Y_{d\bar{a}}\right], \\
I_{2Y2Z} &= \mathrm{Im}\left[Y_{a\bar{b}}Y_{c\bar{d}}Z_{b\bar{a}d\bar{f}}Z^{(1)}_{f\bar{c}}\right], \\
I_{3Y3Z} &= \mathrm{Im}\left[Z_{a\bar{c}b\bar{d}}Z_{c\bar{e}d\bar{g}}Z_{e\bar{h}f\bar{q}}Y_{g\bar{a}}Y_{h\bar{b}}Y_{q\bar{f}}\right], \\
I_{6Z} &= \mathrm{Im}\left[Z_{a\bar{b}c\bar{d}}Z^{(1)}_{b\bar{f}}Z^{(1)}_{d\bar{h}}Z_{f\bar{a}j\bar{k}}Z_{k\bar{j}m\bar{n}}Z_{n\bar{m}h\bar{c}}\right].
\end{aligned}
$$

2HDM potential is CP invariant iff

$$I_{Y3Z} = I_{2Y2Z} = I_{3Y3Z} = I_{6Z} = 0$$

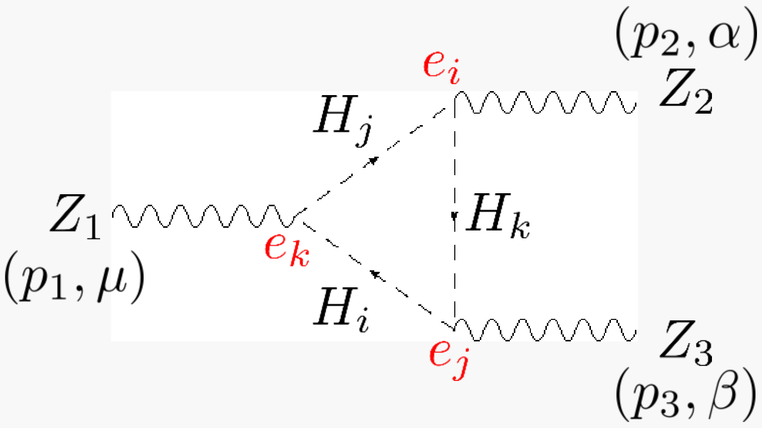
Understood in terms of physical parameters

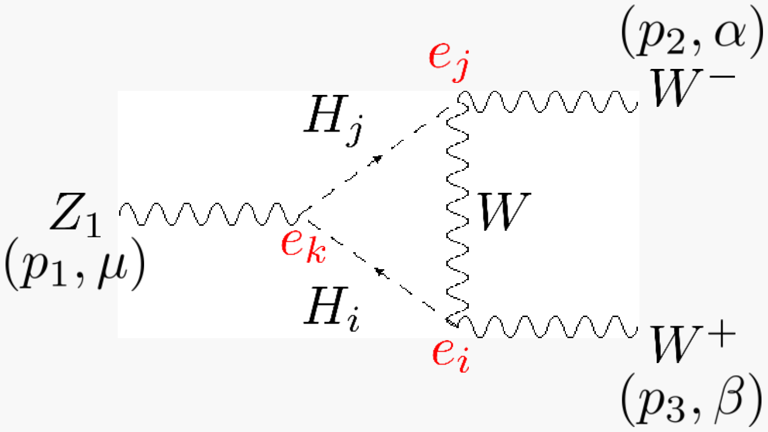
$$
\begin{aligned}
\frac{M^2_{H^\pm}}{v^2} &= \frac{e_1q_1M_2^2M_3^2 + e_2q_2M_1^2M_3^2 + e_3q_3M_1^2M_2^2 - M_1^2M_2^2M_3^2}{2(e_1^2M_2^2M_3^2 + e_2^2M_3^2M_1^2 + e_3^2M_1^2M_2^2)}, \\
q &= \frac{(e_2q_3 - e_3q_2)^2M_1^2 + (e_3q_1 - e_1q_3)^2M_2^2 + (e_1q_2 - e_2q_1)^2M_3^2 + M_1^2M_2^2M_3^2}{2(e_1^2M_2^2M_3^2 + e_2^2M_3^2M_1^2 + e_3^2M_1^2M_2^2)}
\end{aligned}
$$

[Ref: Grzadkowski, Ogreid, Osland, JHEP11 (2014) 084]

Processes containing Im J_2:

- ZZZ vertex and ZWW vertex





- Summing over all possible combinations of *i,j,k*, we find

$$\mathcal{M} \quad \propto \quad \mathrm{Im} J_2$$

- Amplitudes directly proportional to Weak-basis invariant.
  Ideal place to discover CPV from the extra doublet.

CP-symmetry of the 3HDM

3HDMs provide more phases in the potential as well as in the VEV, thereby providing more sources of CP violation.

There exist 3HDMs with complex potential parameters (irremovable phases) that are CP conserving. (unlike the 2HDM).

For the general 3HDM,

- No set of invariants equivalent to a CP conserving 3HDM is known.
- No set of invariants equivalent to a CP-conserving potential is known
- CP-conservation in 3HDM is not yet understood in terms of physical parameters
- Some special cases has been worked out.

The CP4 3HDM

Only one doublet ϕ_1 has non-vanishing VEV

$$
\begin{aligned}
V = &- m_{11}^2(\phi_1^\dagger\phi_1) - m_{22}^2(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3) \\
&+ \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2[(\phi_2^\dagger\phi_2)^2 + (\phi_3^\dagger\phi_3)^2] + \lambda_3'(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) \\
&+ \lambda_3(\phi_1^\dagger\phi_1)[(\phi_2^\dagger\phi_2) + (\phi_3^\dagger\phi_3)] \\
&+ \lambda_4'(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2) + \lambda_4[(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + (\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1)] \\
&+ \Big\{\lambda_5(\phi_3^\dagger\phi_1)(\phi_2^\dagger\phi_1) + \tfrac{1}{2}\lambda_6[(\phi_2^\dagger\phi_1)^2 - (\phi_1^\dagger\phi_3)^2] \\
&+\lambda_8(\phi_2^\dagger\phi_3)^2 + \lambda_9(\phi_2^\dagger\phi_3)[(\phi_2^\dagger\phi_2) - (\phi_3^\dagger\phi_3)] + \text{h.c.}\Big\}
\end{aligned}
$$

Invariant under CP-transform

$$
\phi_i \to W_{ij}\phi_j^*, \quad W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}
$$

CP-conserving, yet irremovable phases

3HDM with S_3 symmetry

Both Dark matter and CP violation possible

$$\begin{aligned}
V =&\mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2), \\
&+ \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\
&+ \left[\lambda_4 \left\{(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2)\right\} + \text{h.c.}\right] + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) \\
&+ \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] + \left[\lambda_7 \left\{(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2)\right\} + \text{h.c.}\right] \\
&+ \lambda_8 (h_S^\dagger h_S)^2,
\end{aligned}$$

$$\begin{aligned}
I_{5Z}^{(1)} &= \operatorname{Im}\left[Z_{aabc} Z_{cedg} Z_{edfh} Z_{gihb} Z_{ifjj}\right], \\
I_{5Z}^{(2)} &= \operatorname{Im}\left[Z_{abbc} Z_{cedg} Z_{edfh} Z_{giha} Z_{ijjf}\right], \\
I_{6Z}^{(1)} &= \operatorname{Im}\left[Z_{acbd} Z_{cade} Z_{egfh} Z_{gihk} Z_{ibjl} Z_{kflj}\right], \\
I_{6Z}^{(2)} &= \operatorname{Im}\left[Z_{acbe} Z_{cadf} Z_{egfi} Z_{gbhk} Z_{idjl} Z_{khlj}\right], \\
I_{7Z} &= \operatorname{Im}\left[Z_{acbd} Z_{cedf} Z_{egfi} Z_{gahj} Z_{ikjm} Z_{kbln} Z_{mhnl}\right], \\
I_{2Y3Z} &= \operatorname{Im}\left[Y_{ac} Y_{be} Z_{cgdf} Z_{ehfd} Z_{gbha}\right].
\end{aligned}$$

$$\begin{gathered}
Y_{11} = \mu_1^2, \quad Y_{22} = \mu_1^2, \quad Y_{33} = \mu_0^2, \\
Z_{1111} = Z_{2222} = 2\lambda_1 + 2\lambda_3, \quad Z_{3333} = 2\lambda_8, \\
Z_{1122} = Z_{2211} = 2\lambda_1 - 2\lambda_3, \quad Z_{1133} = Z_{2233} = Z_{3311} = Z_{3322} = \lambda_5, \\
Z_{1221} = Z_{2112} = -2\lambda_2 + 2\lambda_3, \quad Z_{1331} = Z_{2332} = Z_{3113} = Z_{3223} = \lambda_6, \\
Z_{1212} = Z_{2121} = 2\lambda_2 + 2\lambda_3, \quad Z_{1313} = Z_{2323} = Z_{3131} = Z_{3232} = 2\lambda_7, \\
Z_{1123} = Z_{1213} = Z_{1312} = Z_{1321} = Z_{2113} = Z_{2311} = \lambda_4^*, \\
Z_{1132} = Z_{1231} = Z_{2131} = Z_{3112} = Z_{3121} = Z_{3211} = \lambda_4, \\
Z_{2223} = Z_{2322} = -\lambda_4^*, \quad Z_{2232} = Z_{3222} = -\lambda_4.
\end{gathered}$$

Potential is CP invariant iff all 6 invariants vanish.

3HDM with Z2 x Z2 symmetry (real parameters, complex VEV)

$$
\begin{aligned}
V = &- \left[m_{11}(\phi_1^\dagger\phi_1) + m_{22}(\phi_2^\dagger\phi_2) + m_{33}(\phi_3^\dagger\phi_3)\right] \\
&+ \lambda_{11}(\phi_1^\dagger\phi_1)^2 + \lambda_{22}(\phi_2^\dagger\phi_2)^2 + \lambda_{33}(\phi_3^\dagger\phi_3)^2 \\
&+ \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_{13}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) \\
&+ \lambda'_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda'_{13}(\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1) + \lambda'_{23}(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2) \\
&+ \lambda_1\left[(\phi_2^\dagger\phi_3)^2 + (\phi_3^\dagger\phi_2)^2\right] + \lambda_2\left[(\phi_3^\dagger\phi_1)^2 + (\phi_1^\dagger\phi_3)^2\right] + \lambda_3\left[(\phi_1^\dagger\phi_2)^2 + (\phi_2^\dagger\phi_1)^2\right].
\end{aligned}
$$

Model is CP conserving iff all 15 invariants vanish.

$$
\begin{aligned}
J_1 &= \mathrm{Im}\{\hat V_{ac}\hat V_{be} Z_{cadf} Z_{edfg} Z_{gbhh}\}, \\
J_2 &= \mathrm{Im}\{\hat V_{ac}\hat V_{be} Z_{cadf} Z_{edfg} Z_{ghhb}\}, \\
J_3 &= \mathrm{Im}\{\hat V_{ac}\hat V_{be} Z_{cadf} Z_{egfd} Z_{gbhh}\}, \\
J_4 &= \mathrm{Im}\{\hat V_{ac}\hat V_{bd} Z_{cedg} Z_{eafh} Z_{gbhf}\}, \\
J_5 &= \mathrm{Im}\{\hat V_{ac}\hat V_{bd} Z_{cedg} Z_{ehfa} Z_{gfhb}\}, \\
J_6 &= \mathrm{Im}\{\hat V_{ac}\hat V_{bd} Z_{cedf} Z_{eafg} Z_{gbhh}\}, \\
J_7 &= \mathrm{Im}\{\hat V_{ad}\hat V_{be}\hat V_{cf} Z_{daeh} Z_{fbgi} Z_{hcig}\}, \\
J_8 &= \mathrm{Im}\{\hat V_{ad}\hat V_{be}\hat V_{cf} Z_{daeh} Z_{figb} Z_{hgic}\}, \\
J_9 &= \mathrm{Im}\{\hat V_{ad}\hat V_{be}\hat V_{cf} Z_{daeg} Z_{fbgh} Z_{hcii}\}, \\
J_{10} &= \mathrm{Im}\{\hat V_{ad}\hat V_{be}\hat V_{cf} Z_{daeg} Z_{fhgi} Z_{hbic}\}, \\
J_{11} &= \mathrm{Im}\{\hat V_{ac}\hat V_{be} Z_{cadg} Z_{edff} Z_{gihh} Z_{ibjj}\}, \\
J_{12} &= \mathrm{Im}\{\hat V_{ac}\hat V_{be} Z_{cadg} Z_{effd} Z_{ghhi} Z_{ijjb}\}, \\
J_{13} &= \mathrm{Im}\{\hat V_{ac}\hat V_{be} Z_{cadf} Z_{edfg} Z_{gihj} Z_{ibjh}\}, \\
J_{14} &= \mathrm{Im}\{\hat V_{ac}\hat V_{bd} Z_{cedf} Z_{eafg} Z_{gihj} Z_{ibjh}\}, \\
v^6 J_{15} &= \mathrm{Im}\{\hat V_{ac}\hat V_{bd} Y_{cf} Y_{dg} Y_{ea} Z_{fbge}\}.
\end{aligned}
$$

Both Dark matter and CP violation possible

3HDM with Z2 x Z2 symmetry (complex parameters)

$$
\begin{aligned}
V \;=\;& -[m_{11}(\phi_1^\dagger\phi_1)+m_{22}(\phi_2^\dagger\phi_2)+m_{33}(\phi_3^\dagger\phi_3)] \\
& +\lambda_{11}(\phi_1^\dagger\phi_1)^2+\lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)+\lambda_{13}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3)+\lambda_{22}(\phi_2^\dagger\phi_2)^2 \\
& +\lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3)+\lambda_{33}(\phi_3^\dagger\phi_3)^2 \\
& +\lambda'_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)+\lambda'_{13}(\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1)+\lambda'_{23}(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2) \\
& +\left\{\lambda_1(\phi_2^\dagger\phi_3)^2+\lambda_2(\phi_3^\dagger\phi_1)^2+\lambda_3(\phi_1^\dagger\phi_2)^2+\ \text{h.c.}\right\}
\end{aligned}
$$

Potential is CP conserving iff all 15 invariants vanish. Spontaneous CP violation may happen.

Soft CP violation may happen if only the first 10 invariants vanish.

$$
\begin{aligned}
I_{5Z}^{(1)} &= \mathrm{Im}\,[Z_{aabc}Z_{cedf}Z_{egfi}Z_{gbhd}Z_{ijjh}]\,, \\
I_{5Z}^{(2)} &= \mathrm{Im}\,[Z_{aabc}Z_{cedf}Z_{egfi}Z_{gjhb}Z_{ihjd}]\,, \\
I_{5Z}^{(3)} &= \mathrm{Im}\,[Z_{abbc}Z_{cedf}Z_{egfi}Z_{gahj}Z_{idjh}]\,, \\
I_{6Z}^{(1)} &= \mathrm{Im}\,[Z_{aabc}Z_{cdde}Z_{egfh}Z_{gihk}Z_{ibjf}Z_{kjll}]\,, \\
I_{6Z}^{(2)} &= \mathrm{Im}\,[Z_{aabc}Z_{cedf}Z_{egfi}Z_{gbhd}Z_{ikjl}Z_{khlj}]\,, \\
I_{6Z}^{(3)} &= \mathrm{Im}\,[Z_{abbc}Z_{cedf}Z_{egfi}Z_{gahd}Z_{ikjl}Z_{khlj}]\,, \\
I_{6Z}^{(4)} &= \mathrm{Im}\,[Z_{acbe}Z_{cfdb}Z_{egfi}Z_{gkha}Z_{iljd}Z_{kjlh}]\,, \\
I_{6Z}^{(5)} &= \mathrm{Im}\,[Z_{acbe}Z_{cfdb}Z_{egfi}Z_{gkhd}Z_{iljh}Z_{kjla}]\,, \\
I_{6Z}^{(6)} &= \mathrm{Im}\,[Z_{acbe}Z_{cfdg}Z_{eifk}Z_{gbhl}Z_{ihjd}Z_{kalj}]\,, \\
I_{9Z} &= \mathrm{Im}\,[Z_{apbm}Z_{crdf}Z_{eifg}Z_{gohe}Z_{icjd}Z_{kqln}Z_{mknb}Z_{oapl}Z_{qjrh}] \\
I_{Y4Z}^{(1)} &= \mathrm{Im}\,[Y_{ab}Z_{bdce}Z_{dfeh}Z_{fagc}Z_{hgii}]\,, \\
I_{Y4Z}^{(2)} &= \mathrm{Im}\,[Y_{ab}Z_{bdce}Z_{dfeh}Z_{fagi}Z_{hcig}]\,, \\
I_{Y4Z}^{(3)} &= \mathrm{Im}\,[Y_{ab}Z_{bdce}Z_{dfeh}Z_{figa}Z_{hgic}]\,, \\
I_{3Y3Z} &= \mathrm{Im}\,[Y_{ab}Y_{bd}Y_{cf}Z_{dceg}Z_{fhgi}Z_{haie}]\,, \\
I_{Y5Z} &= \mathrm{Im}\,[Y_{ab}Z_{bdcf}Z_{dceh}Z_{fagi}Z_{hjik}Z_{jekg}]\,.
\end{aligned}
$$

Summary!

- 2HDM - well understood. Physical implications known. “New symmetry” yet to be understood.
- 3HDM – lots of work in progress. Physical implications partially known.
- NHDM – some studies done. Lots left to do!
- Multi-Higgs models provide DM candidates and extra sources of CP violation.

The future:

Still awaiting the discovery of additional scalar particles (Higgses).

95 GeV

125 GeV

152 GeV

??? GeV

??? GeV