DI – HIGGS THEORY OVERVIEW

Michael Spira (PSI)

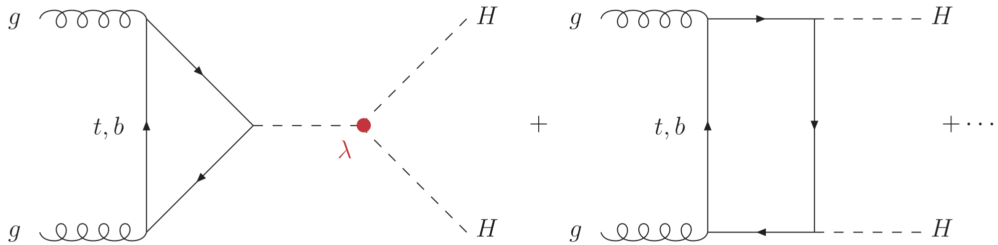
I Introduction

II $gg \to HH$

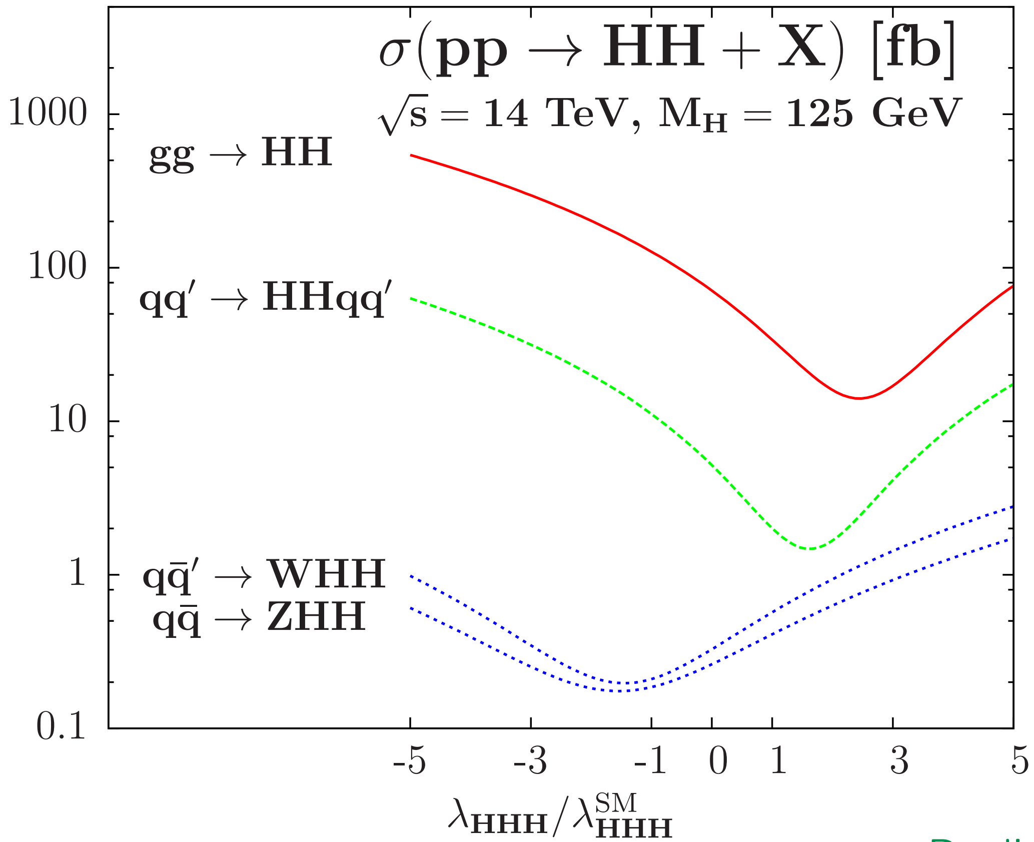
III Other processes

IV Conclusions

I INTRODUCTION



- third generation dominant: $t\,(b)$



$gg \to HH$:

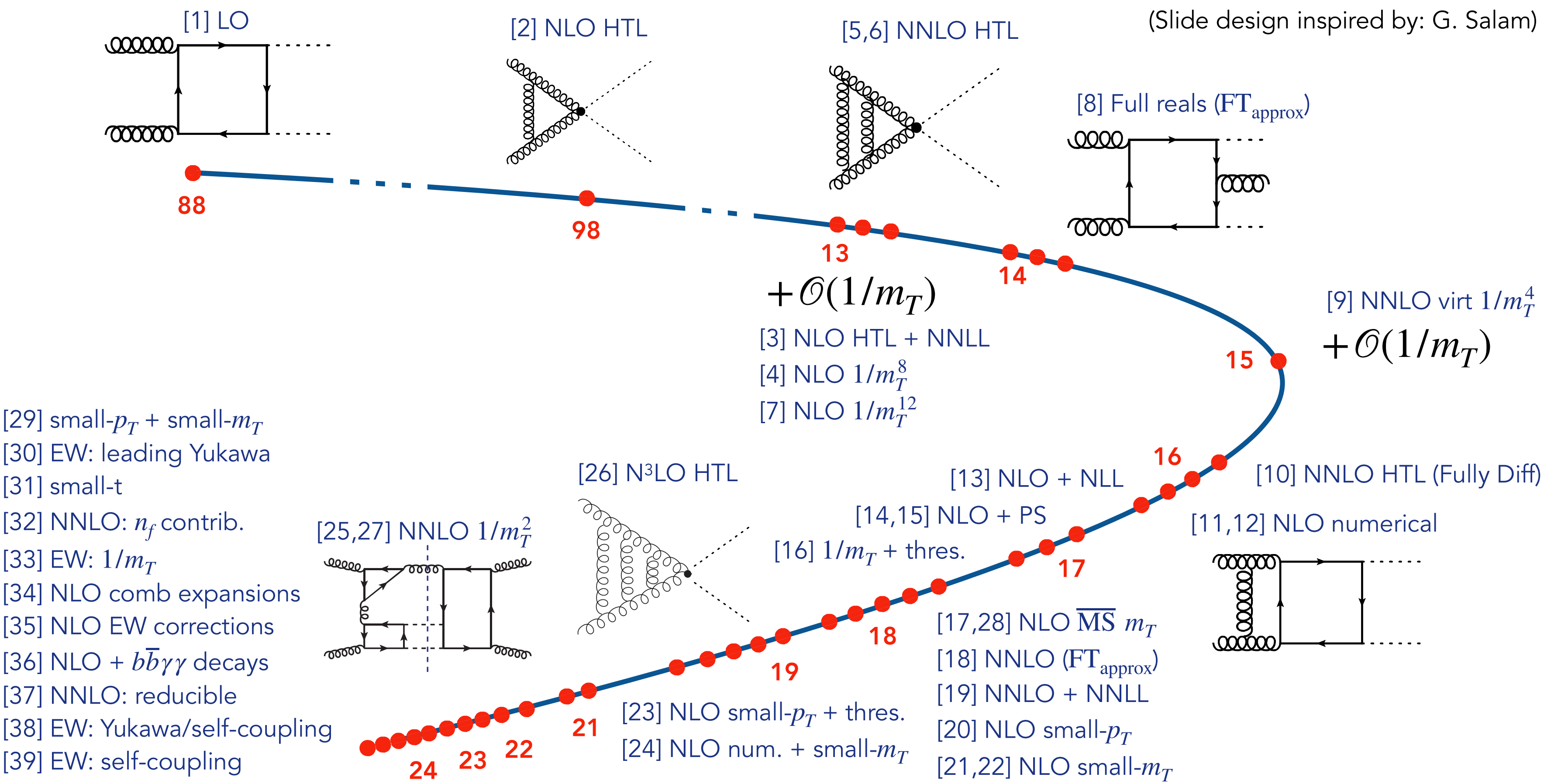
$$\frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$$

Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, S.

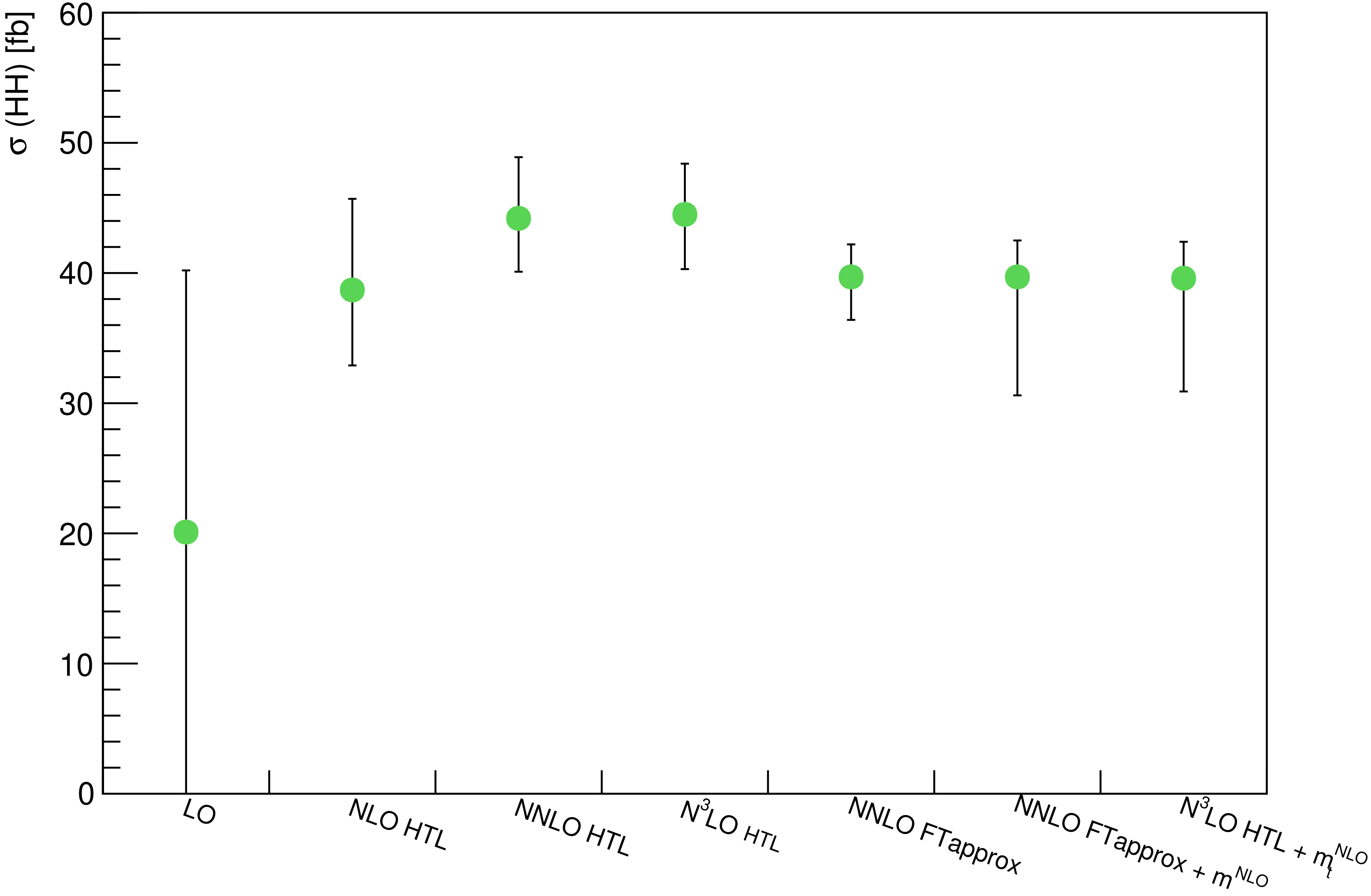
S. Jones

II $gg \to HH$

Overview



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23 [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, SPJ, Kerner, Stone, Vestner [39] Li, Si, Wang, Zhang, Zhao 24

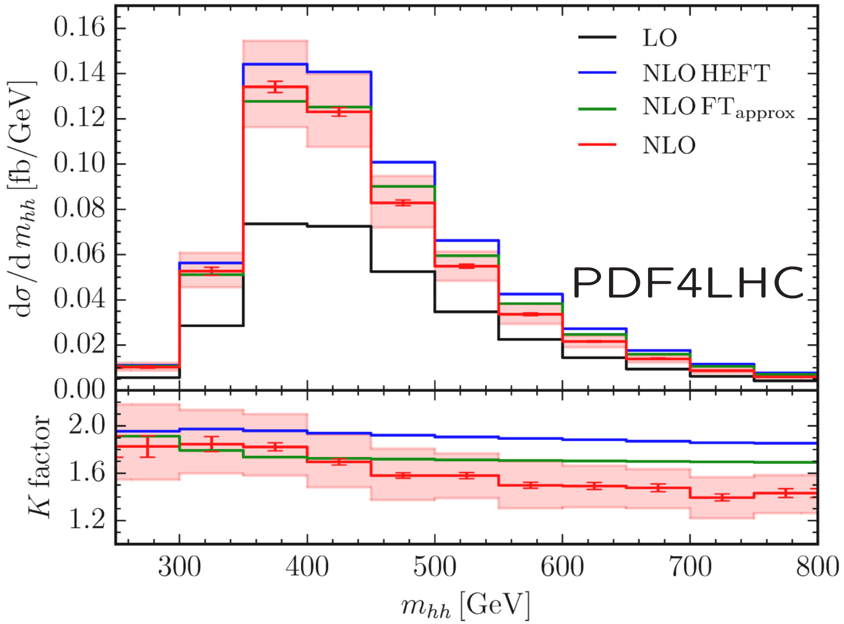


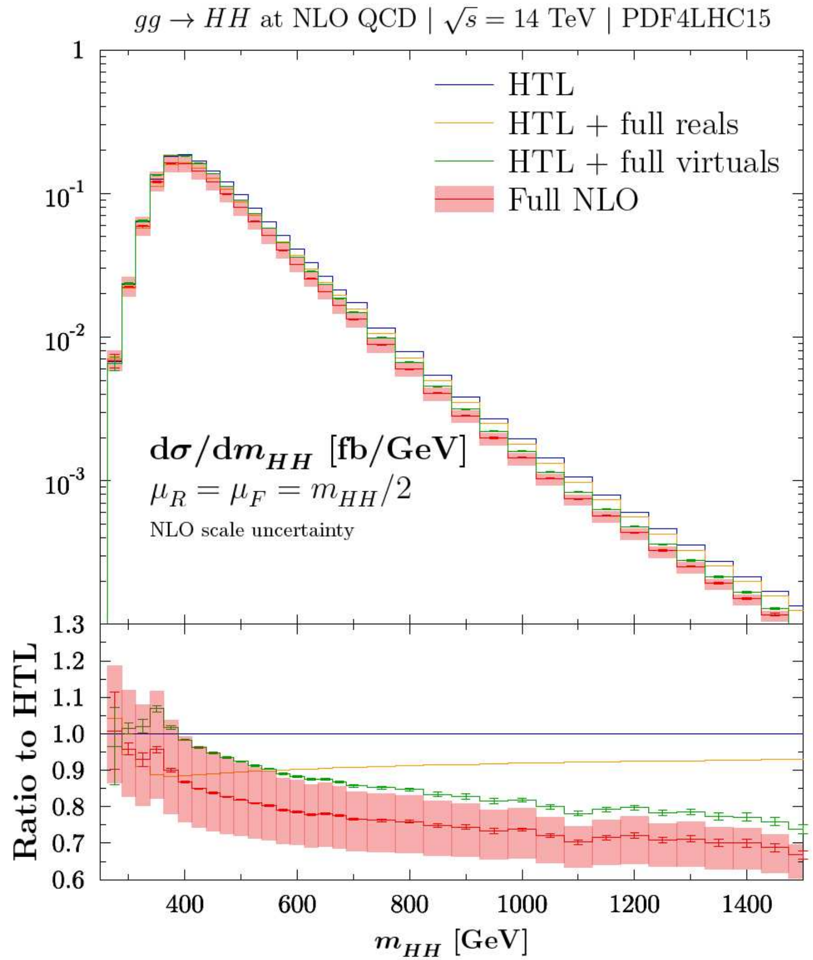
σ (HH) [fb]
60
50
40
30
20
10
0
LO
NLO HTL
NNLO HTL
N3LO HTL
NNLO FTapprox
NNLO FTapprox + m NLO
N3LO HTL + m t NLO

Full NLO calculation: top only, numerical integration

Borowka *et al.*	Baglio *et al.*
tensor reduction	no tensor reduction
sector decomposition	IR, end-point subtraction
contour deformation	IBP, Richardson extrapolation
$m_t = 173$ GeV	$m_t = 172.5$ GeV

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke
Baglio, Campanario, Glaus, Mühlleitner, Ronca, S., Streicher



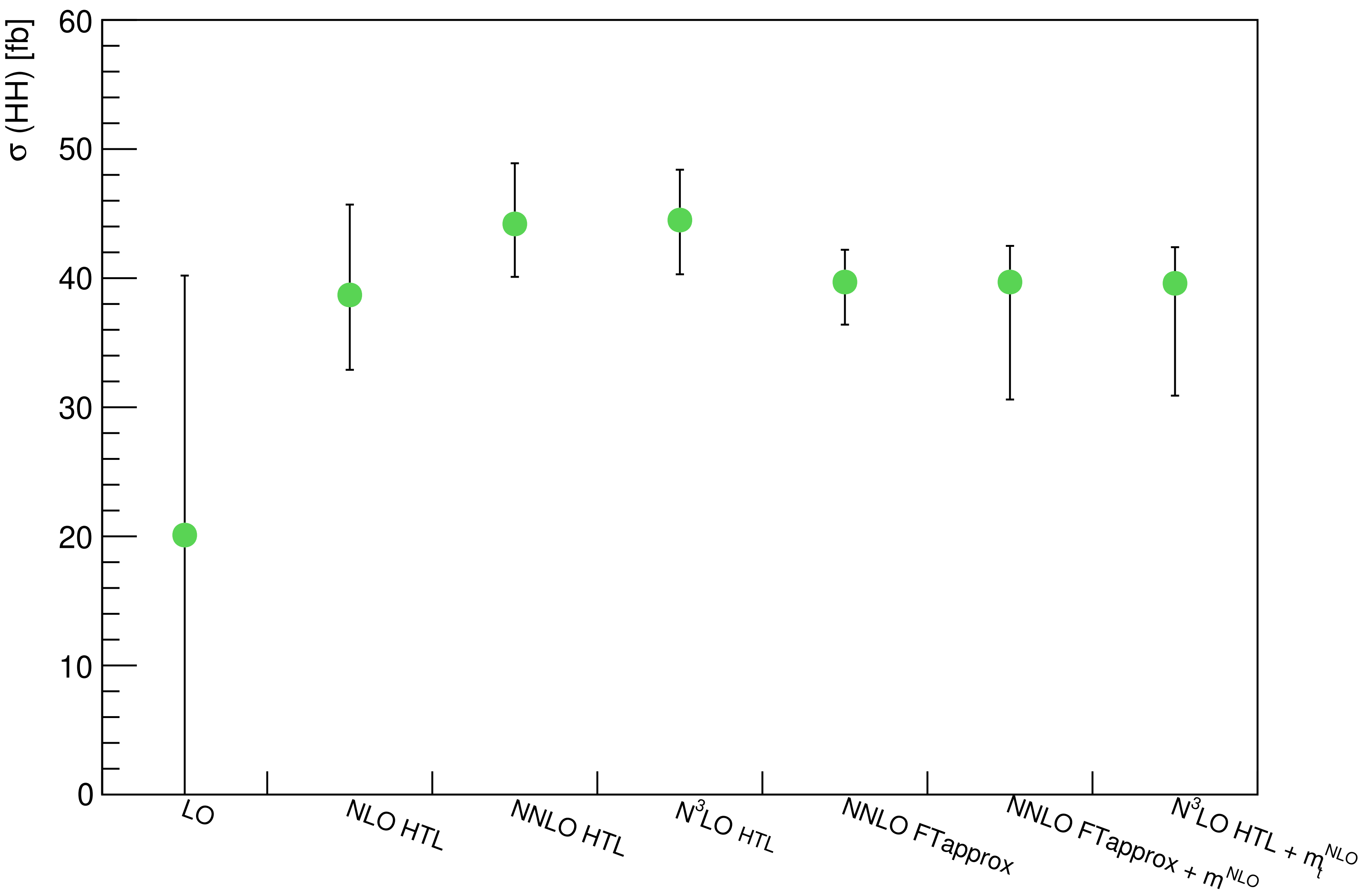


Borowka, Greiner, Heinrich, Jones, Kerner
Schlenk, Schubert, Zirke

Baglio, Campanario, Glaus,
Mühlleitner, Ronca, S., Streicher

$\sigma_{NLO} =$	$32.91(10)^{+13.8\%}_{-12.8\%}\ fb$	$32.81(7)^{+13.5\%}_{-12.5\%}\ fb$
$\sigma^{HTL}_{NLO} =$	$38.75^{+18\%}_{-15\%}\ fb$	$38.66^{+18\%}_{-15\%}\ fb$
$m_t =$	173 GeV	172.5 GeV

$\Rightarrow$ -15% mass effects on top of LO



σ (HH) [fb]
60
50
40
30
20
10
0
LO
NLO HTL
NNLO HTL
N3LO HTL
NNLO FTapprox
NNLO FTapprox + m_t^NLO
N3LO HTL + m_t^NLO

uncertainties due to m_t

- use m_t, $\overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q \rightarrow$ uncertainty = envelope:

$$\frac{d\sigma(gg \rightarrow HH)}{dQ}|_{Q=300 \text{ GeV}} = 0.02978(7)^{+6\%}_{-34\%} \text{ fb/GeV},$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ}|_{Q=400 \text{ GeV}} = 0.1609(4)^{+0\%}_{-13\%} \text{ fb/GeV},$$

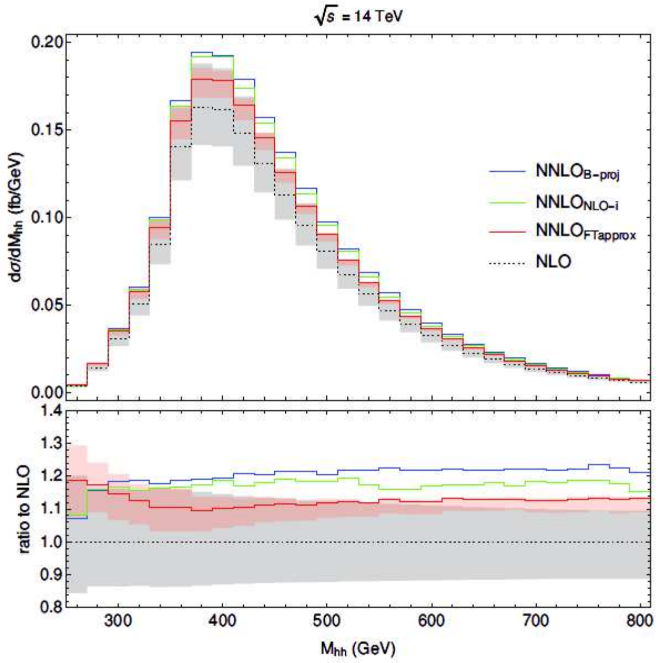
$$\frac{d\sigma(gg \rightarrow HH)}{dQ}|_{Q=600 \text{ GeV}} = 0.03204(9)^{+0\%}_{-30\%} \text{ fb/GeV},$$

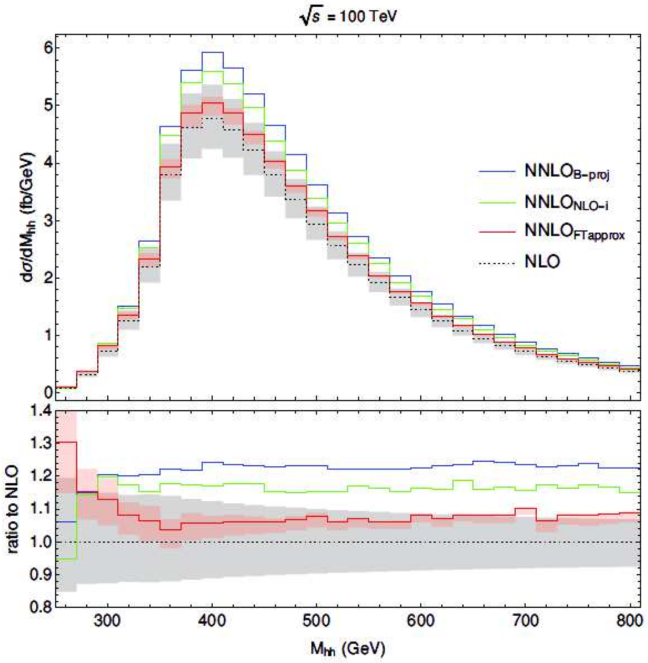
$$\frac{d\sigma(gg \rightarrow HH)}{dQ}|_{Q=1200 \text{ GeV}} = 0.000435(4)^{+0\%}_{-35\%} \text{ fb/GeV}$$

- bin-by-bin interpolation:

$$\sigma(gg \rightarrow HH) = 32.81^{+4\%}_{-18\%} \text{ fb}$$

• NNLO MC: full top-mass effects @ NLO & double real @ NNLO





Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli

⇒ 20% effects beyond NLO

• NLO: matching to parton showers Heinrich, Jones, Kerner, Luisoni, Vryonidou

• combination of full NLO and small mass expansion
Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann

final combined ren./fac. scale and m_t scale/scheme unc. @ NNLO$_{FTapprox}$:

$$\sqrt{s} = 13 \text{ TeV}: \quad \sigma_{tot} = 31.05^{+6\%}_{-23\%} \text{ fb}$$
$$\sqrt{s} = 14 \text{ TeV}: \quad \sigma_{tot} = 36.69^{+6\%}_{-23\%} \text{ fb}$$
$$\sqrt{s} = 27 \text{ TeV}: \quad \sigma_{tot} = 139.9^{+5\%}_{-22\%} \text{ fb}$$
$$\sqrt{s} = 100 \text{ TeV}: \quad \sigma_{tot} = 1224^{+4\%}_{-21\%} \text{ fb}$$

• expansion methods $\rightarrow$ fully differential Monte Carlo

Bonciani, Degrassi, Giardino, Gröber
Bagnaschi, Degrassi, Gröber

• 2HDM: $gg \rightarrow hh, hH, HH, AA$ available, too

• combined uncertainties @ NNLO$_{FTapprox}$ [$\mu_R = \mu_F = M_{HH}/2$]:

$$\kappa_\lambda = -10: \quad \sigma_{tot} = 1680^{+13\%}_{-14\%} \text{ fb}$$
$$\kappa_\lambda = -5: \quad \sigma_{tot} = 598.9^{+13\%}_{-15\%} \text{ fb}$$
$$\kappa_\lambda = -1: \quad \sigma_{tot} = 131.9^{+11\%}_{-16\%} \text{ fb}$$
$$\kappa_\lambda = 0: \quad \sigma_{tot} = 70.38^{+8\%}_{-18\%} \text{ fb}$$
$$\kappa_\lambda = 1: \quad \sigma_{tot} = 31.05^{+6\%}_{-23\%} \text{ fb}$$
$$\kappa_\lambda = 2: \quad \sigma_{tot} = 13.81^{+3\%}_{-28\%} \text{ fb}$$
$$\kappa_\lambda = 2.4: \quad \sigma_{tot} = 13.10^{+6\%}_{-27\%} \text{ fb}$$
$$\kappa_\lambda = 3: \quad \sigma_{tot} = 18.67^{+12\%}_{-22\%} \text{ fb}$$
$$\kappa_\lambda = 5: \quad \sigma_{tot} = 94.82^{+18\%}_{-13\%} \text{ fb}$$
$$\kappa_\lambda = 10: \quad \sigma_{tot} = 672.2^{+16\%}_{-13\%} \text{ fb}$$

Is this everything?

Is this everything?

No. . .

Is this everything?

No. . .

electroweak corrections. . .

(i) y_t: HTL for $ggH(H)$ coupling + full corrections to HHH vertex

Mühlleitner, Schlenk, S.

(ii) y_t: analytical results for $ggHH$ coupling in the HEL

Davies, Mishima, Schönwald, Steinhauser, Zhang

and close to the production threshold

Davies, Schönwald, Steinhauser, Zhang

(iii) λ: elw. corrections due to the Higgs self-interactions

Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao

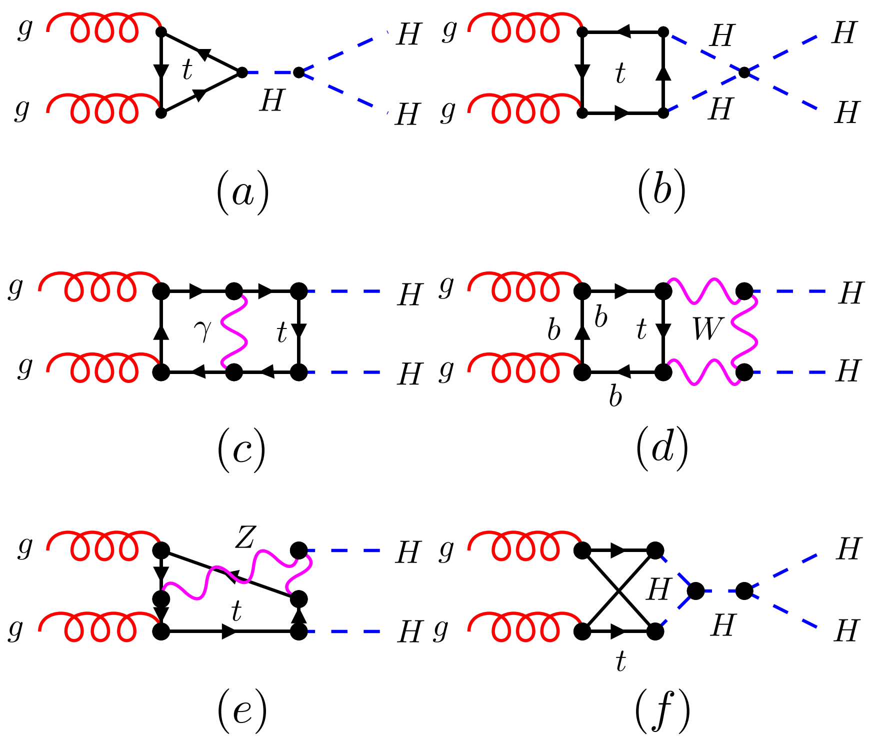
(iv) g_t, λ: elw. corrections due to the top Yukawa and Higgs self-interactions [only Higgs exchange diagrams]

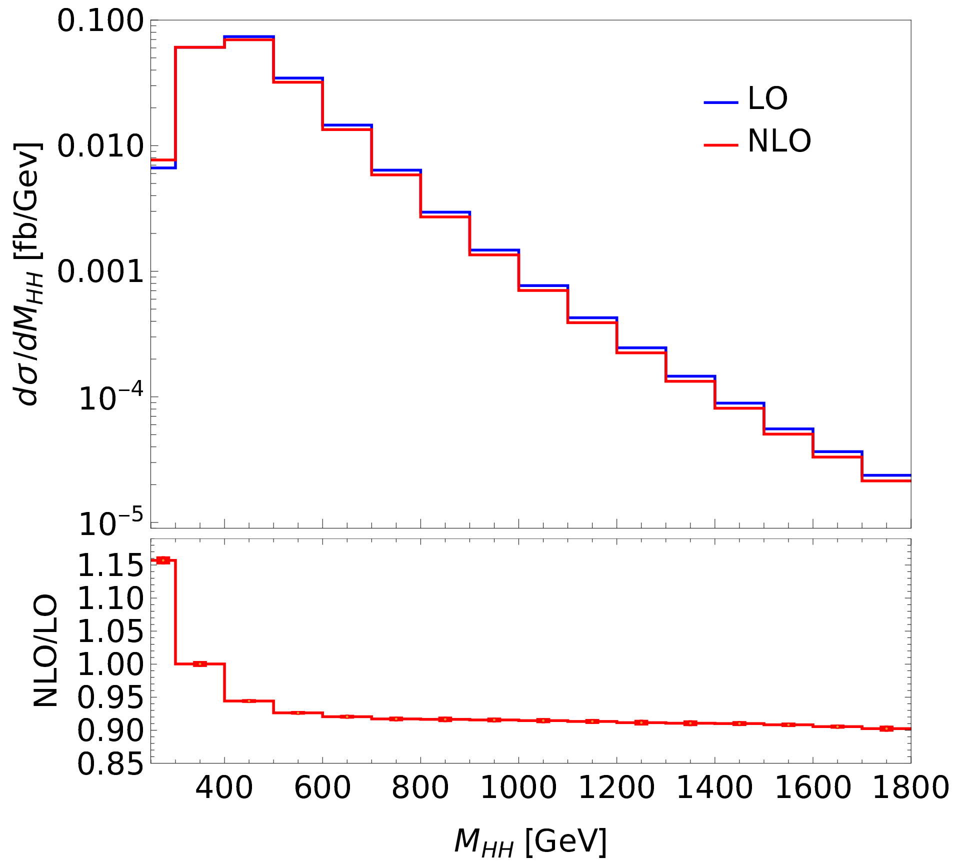
Heinrich, Jones, Kerner, Stone, Vestner

(v) full elw. corrections ($\leftarrow$ to be checked)

Bi, Huang, Huang, Ma, Yu

Full electroweak corrections



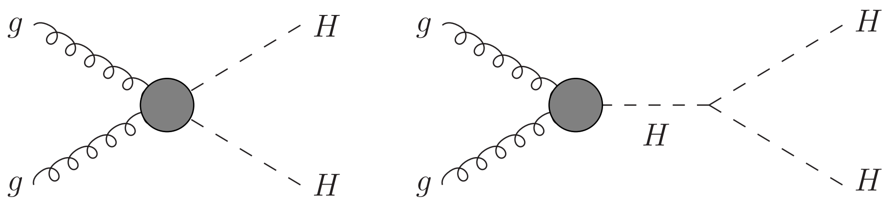


- −4.2% for total cross section

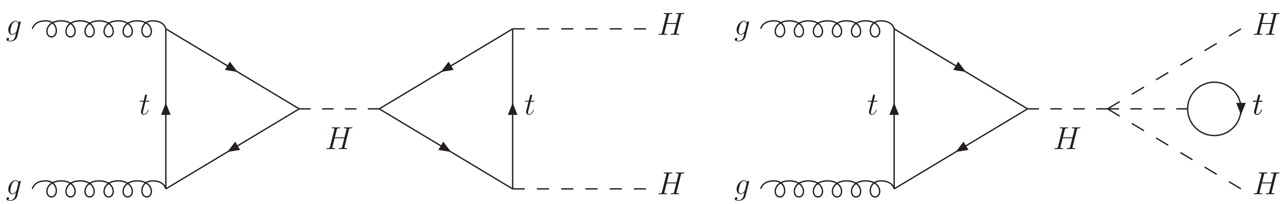
Bi, Huang, Huang, Ma, Yu

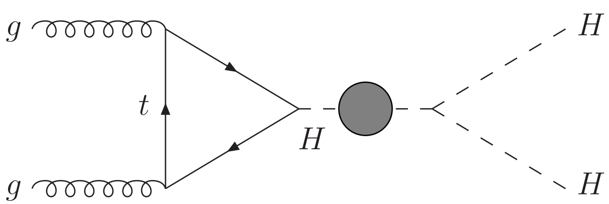
Top-Yukawa-induced elw. corrections

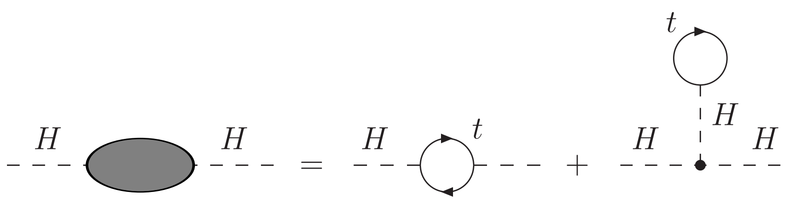
Mühlleitner, Schlenk, S.



← HTL

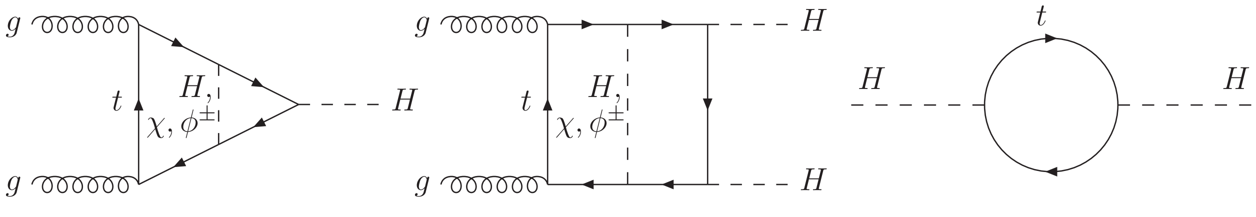






(i) effective $ggH(H)$ couplings:

$$\mathcal{L}_{eff} = C_1 \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} \log\left(1 + C_2 \frac{H}{v}\right)$$



(C_1) (C_2)

- $C_1 = 1 - 3x_t$: genuine vertex corrections $[x_t = G_F m_t^2/(8\sqrt{2}\pi^2)]$

Djoaudi, Gambino
Chetyrkin, Kniehl, Steinhauser

- $C_2 = 1 + 7x_t/2$ $[= 1 + \delta Z_H/2 - \delta v/v]$: universal corrections

Kniehl, Spira
Kwiatkowski, Steinhauser

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} \left\{ (1 + \delta_1) \frac{H}{v} + (1 + \eta_1) \frac{H^2}{2v^2} + \mathcal{O}(H^3) \right\}$$

$$\delta_1 = \frac{x_t}{2} + \mathcal{O}(x_t^2) \qquad \eta_1 = 4x_t + \mathcal{O}(x_t^2)$$

elw. gaugeless limit + QCD = top-Yukawa model + QCD

$$\phi = \begin{pmatrix} G^+ \\ \frac{v + H + iG^0}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L} = -\frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \bar{t} i \not{D} t + |\partial_\mu \phi|^2 - V(\phi) - g_t \bar{Q}_L \phi^c t_R$$

$$V(\phi) = -\mu^2 |\phi|^2 + \frac{\lambda}{2} |\phi|^4$$

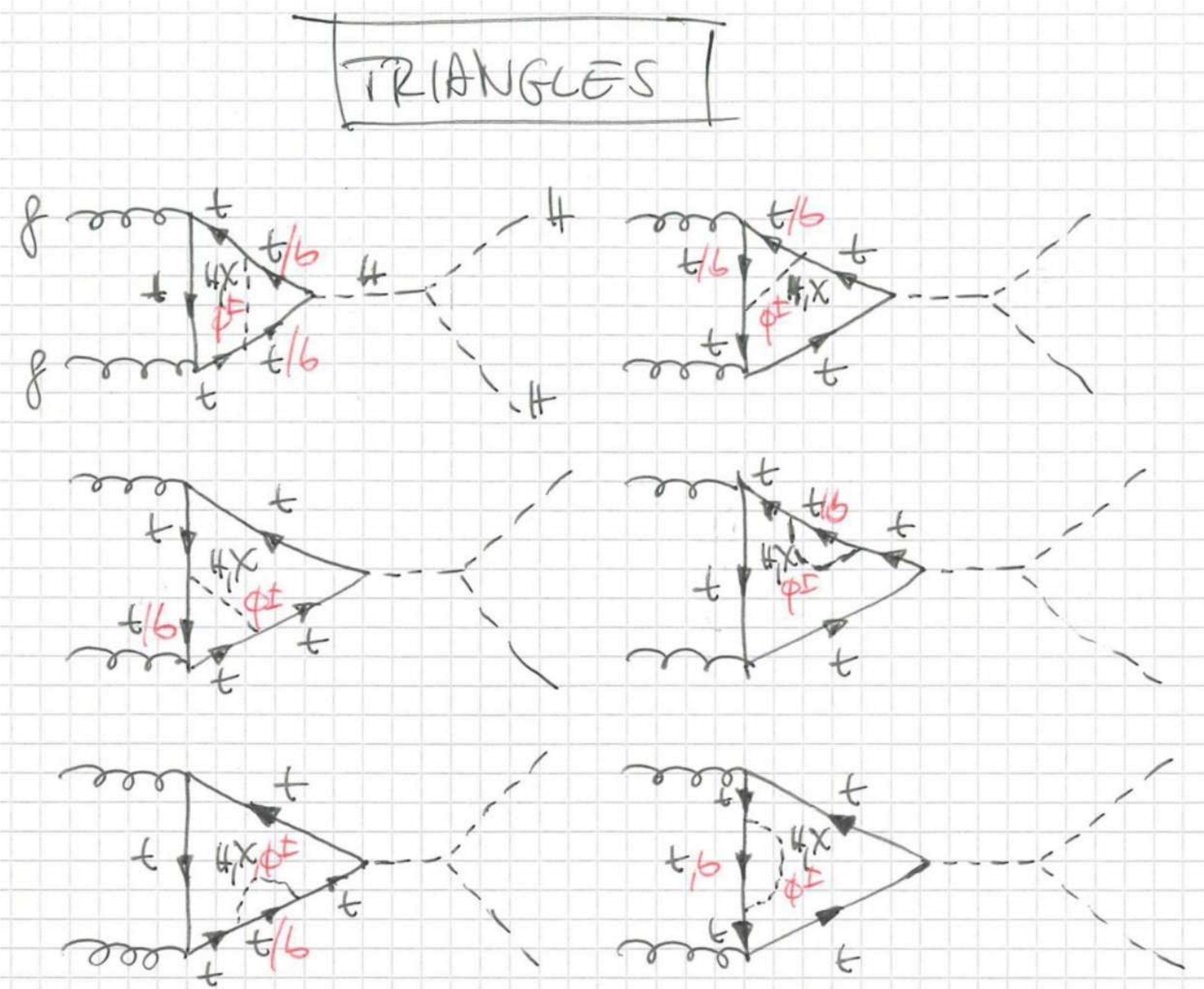
$$= -\frac{M_H^2}{8} v^2 + \frac{M_H^2}{2} H^2 + \frac{M_H^2}{v} \left[\frac{H^3}{2} + \frac{H}{2} (G^0)^2 + H G^+ G^- \right]$$

$$+ \frac{M_H^2}{2v^2} \left[\frac{H^4}{4} + \frac{H^2}{2} (G^0)^2 + H^2 G^+ G^- + (G^+ G^-)^2 + (G^0)^2 G^+ G^- + \frac{(G^0)^4}{4} \right]$$

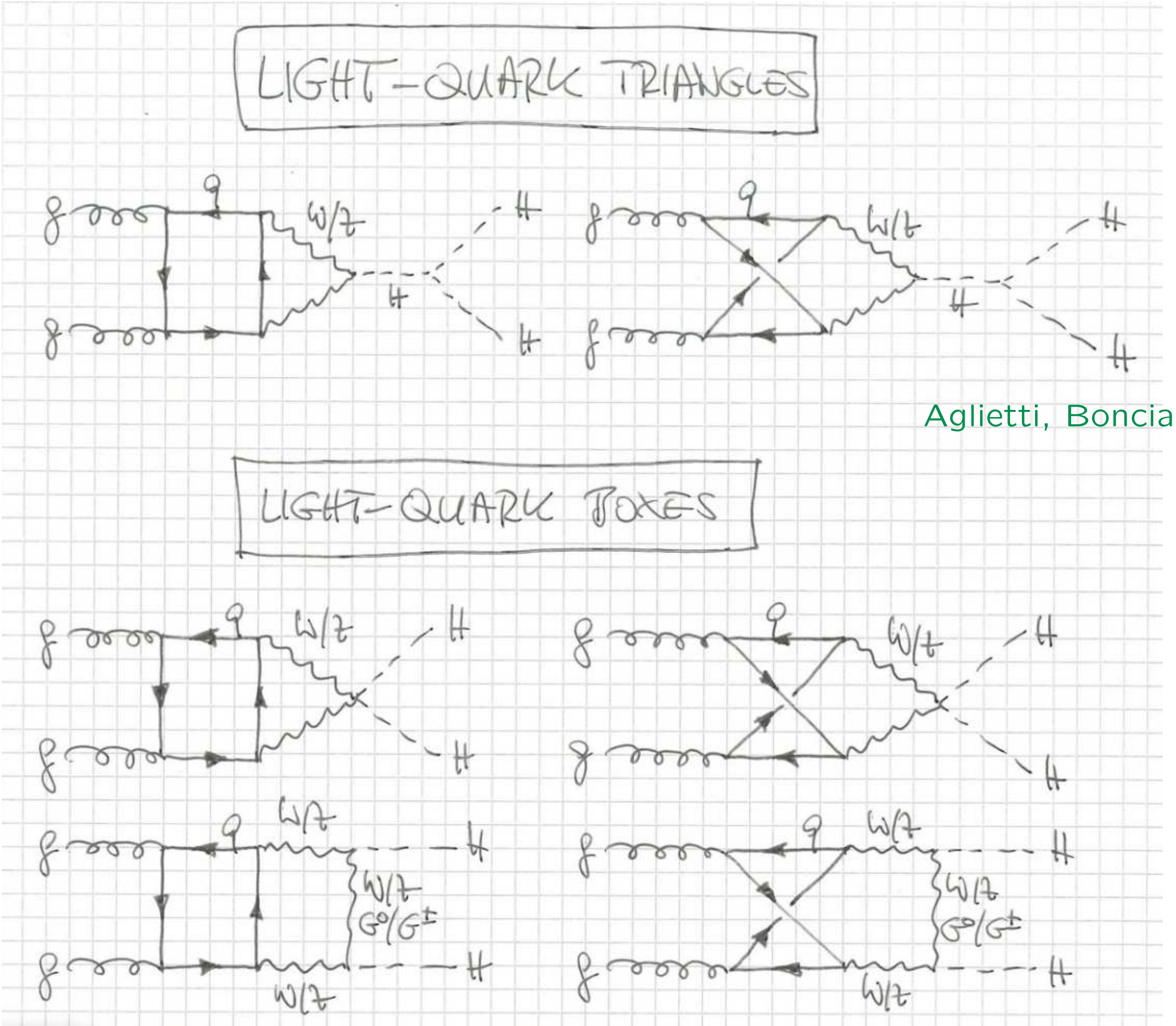
$$\Rightarrow M_{G^0} = M_{G^\pm} = 0$$

<u>INTERMEZZO</u>

Full top-mass dependence (wave-function ren. adjusted appropriately)

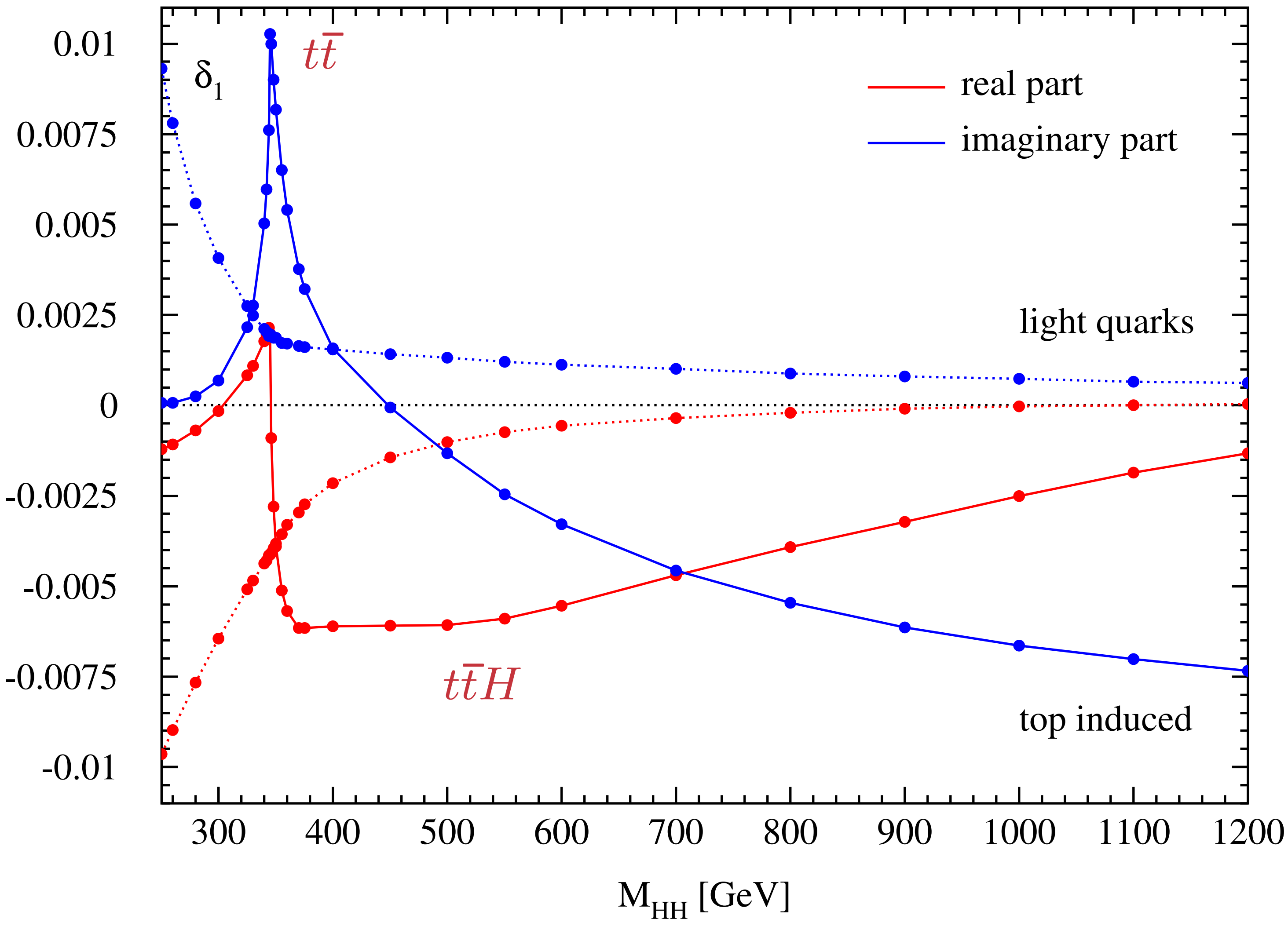


Light-quark loops



Aglietti, Bonciani, Degrassi, Vicini

$$\delta_1 = \delta_{2loop} + \delta Z_H/2 - \delta v/v$$



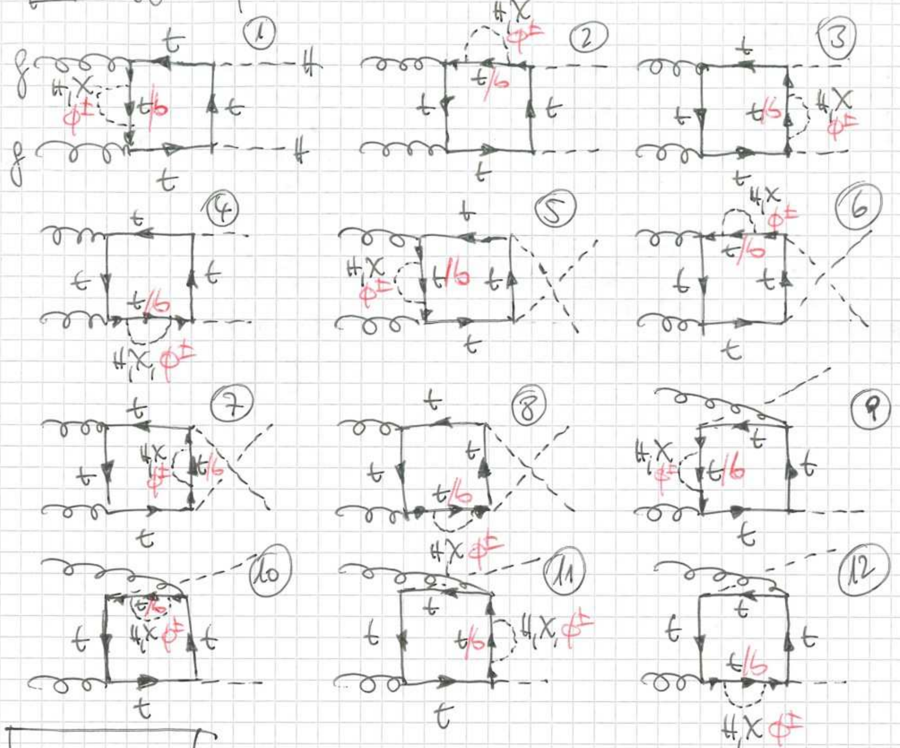
Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, Mühlleitner, Ronca, S.

- box diagrams in the making. . .

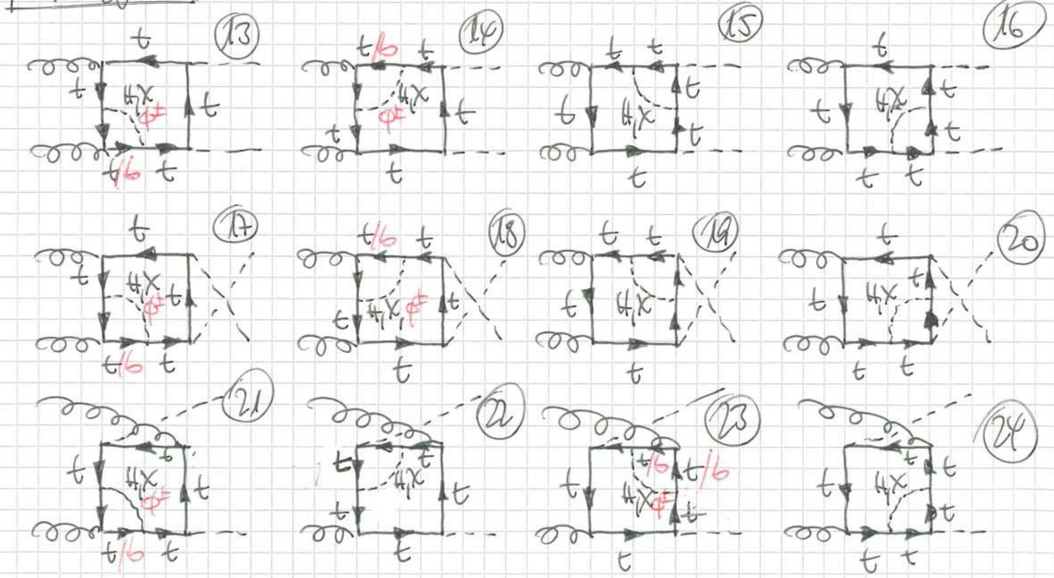
(1)

Topology 1

BOXES

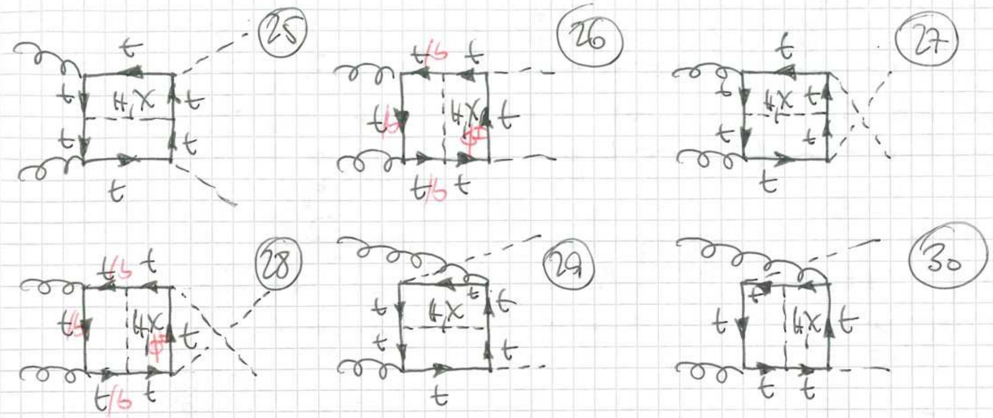


Topology 2



(2)

TOPOLOGY 3



(ii) effective $HHH(H)$ couplings:

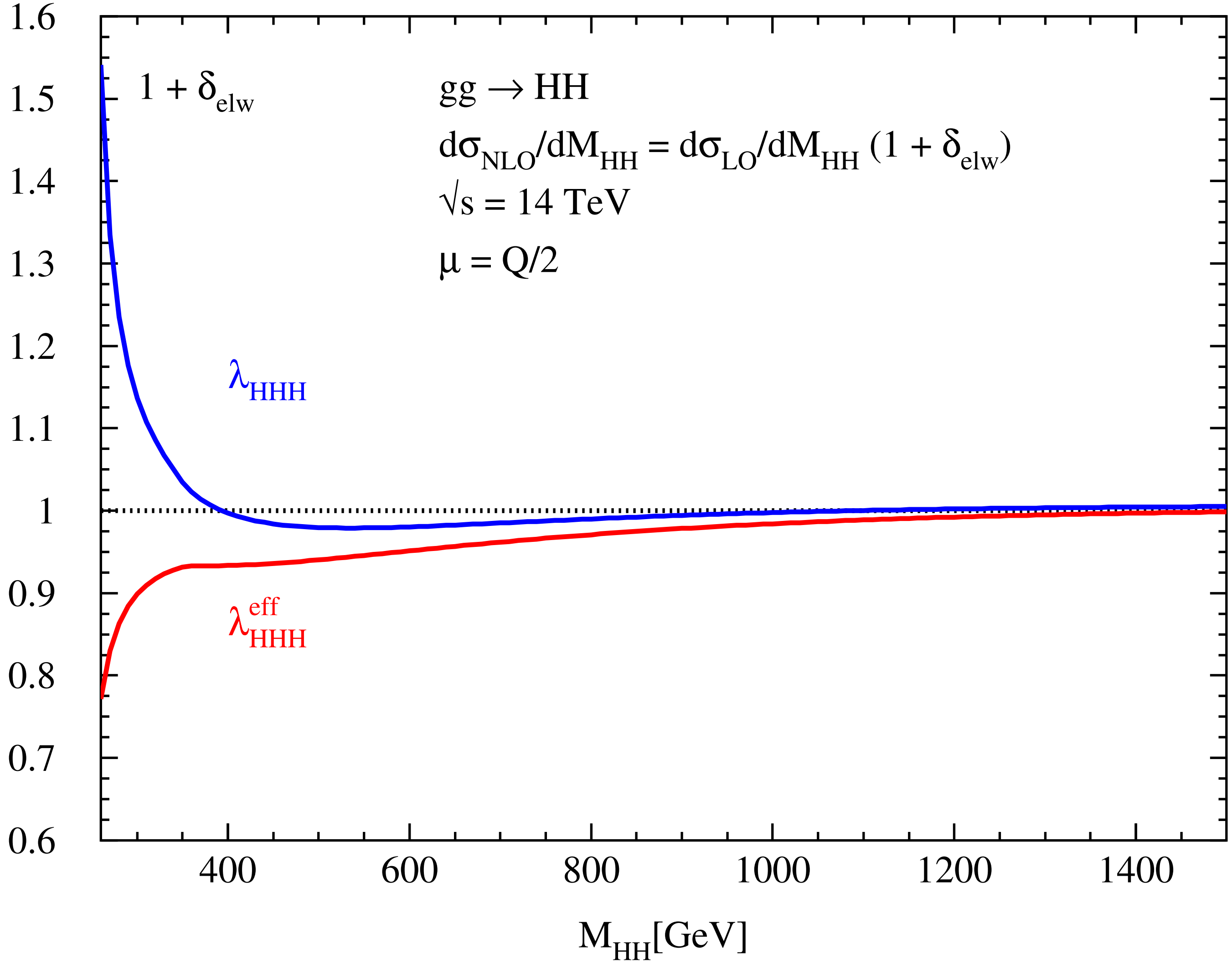
- effective Higgs potential: Coleman, Weinberg

$$
\begin{aligned}
V_{eff} &= V_0 + V_1 \\
V_0 &= \mu_0^2|\phi|^2 + \frac{\lambda_0}{2}|\phi|^4 \\
V_1 &= \frac{3\overline{m}_t^4}{16\pi^2}\Gamma(1+\epsilon)(4\pi^2)^\epsilon\left(\frac{1}{\epsilon} + \log\frac{\bar{\mu}^2}{\overline{m}_t^2} + \frac{3}{2}\right) \\
\phi &= \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v+H \end{array}\right) \qquad \overline{m}_t = m_t\left(1+\frac{H}{v}\right)
\end{aligned}
$$

- after renormalization

$$
\begin{aligned}
\lambda_{HHH}^{eff} &= 3\frac{M_H^2}{v} + \Delta\lambda_{HHH}\,, \qquad \lambda_{HHHH}^{eff} = 3\frac{M_H^2}{v^2} + \Delta\lambda_{HHHH} \\
\Delta\lambda_{HHH} &= -\frac{3m_t^4}{\pi^2 v^3}\,, \qquad \Delta\lambda_{HHHH} = -\frac{12 m_t^4}{\pi^2 v^4}
\end{aligned}
$$

$$
\boxed{\lambda_{HHH}^{eff} = 3\frac{M_H^2}{v} - \frac{3m_t^4}{\pi^2 v^3} \approx 0.91 \times 3\frac{M_H^2}{v}}
$$



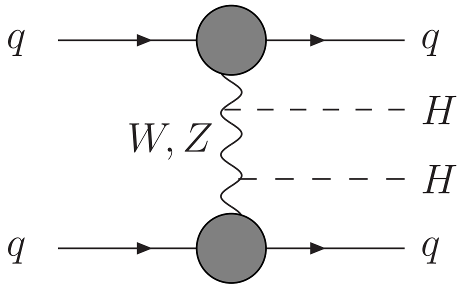
$$\sigma = 1.002 \times \sigma_{LO} \qquad (\lambda_{HHH})$$

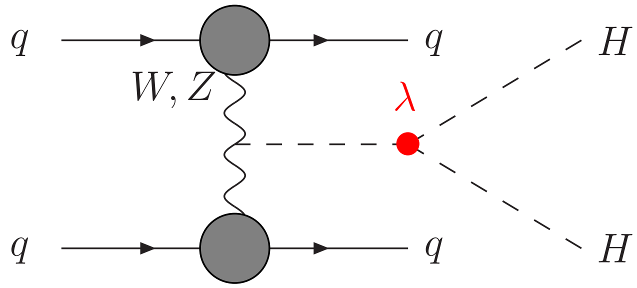
$$\sigma = 0.938 \times \sigma_{LO} \qquad (\lambda^{eff}_{HHH})$$ ← disfavoured

Mühlleitner, Schlenk, S.

III *OTHER PROCESSES*

(i) VBF





- QCD corrections ← DIS (STFU approach)

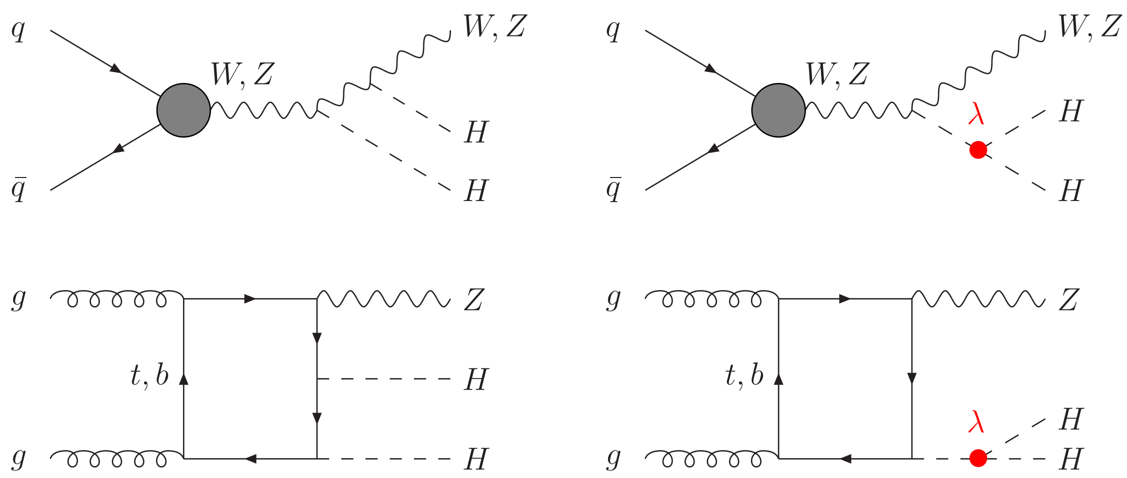
- NLO ∼ 10%, NNLO+N^{3}LO $\lesssim$ 1% $[\mu_R = \mu_F = \sqrt{-q_{1,2}^2}\,(\geq 1\ \mathrm{GeV})]$

Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, S.
Ling, Zhang, Ma, Guo, Li, Li
Dreyer, Karlberg

- differential @ NNLO

Dreyer, Karlberg

(ii) Double Higgs-strahlung



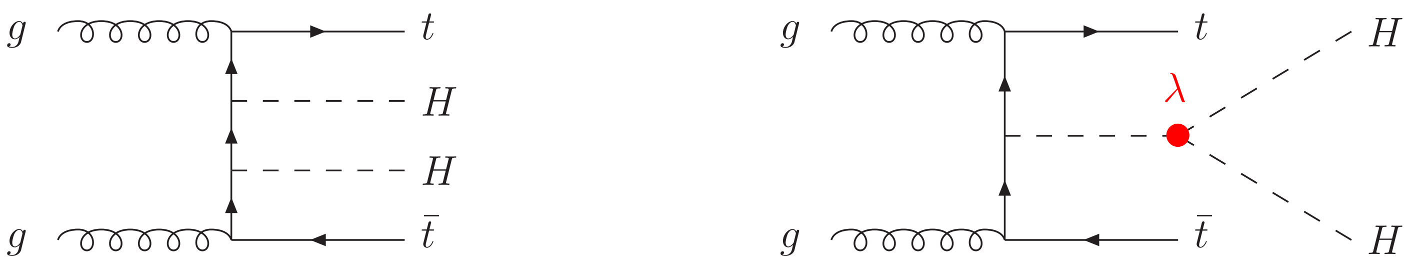
- QCD corrections ← DY
- $gg \to ZHH$: $\sim 30\%$ (LO → NNLO)
- NLO+NNLO $\sim 30\%$ $[\mu_R = \mu_F = M_{HHV}]$

Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, S.
Li, Wang
Li, Li, Wang

- differential @ NNLO

Li, Wang
Li, Li, Wang

(iii) $t\bar{t}HH$



- QCD corrections: MG5_aMC@NLO
- $t\bar{t}HH$: $\sim -20\%$ moderate ($\leftarrow$ single H) $[\mu_R = \mu_F = M_{t\bar{t}}/2]$
- $tjHH$: $\sim +20\%$ moderate $[\mu_R = \mu_F = M_{HH}/2]$

Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro

IV CONCLUSIONS

- Higgs pair production: m_t effects on top of LO $\sim -15\%$ for σ_{tot}
  [larger for distributions]

- uncertainties due to factorization/renormalization scale and m_t scale/scheme choice @NNLO$_{FTapprox}$ $\lesssim 25\%$

- scale and scheme uncertainties due to m_t relevant for large momenta

- combined uncertainties available for λ dependence, too.

- elw. corrections: small for total cxn, larger for distributions

- effective radiatively corrected λ^{eff}_{HHH} disfavoured
  [momentum dependence of same size]

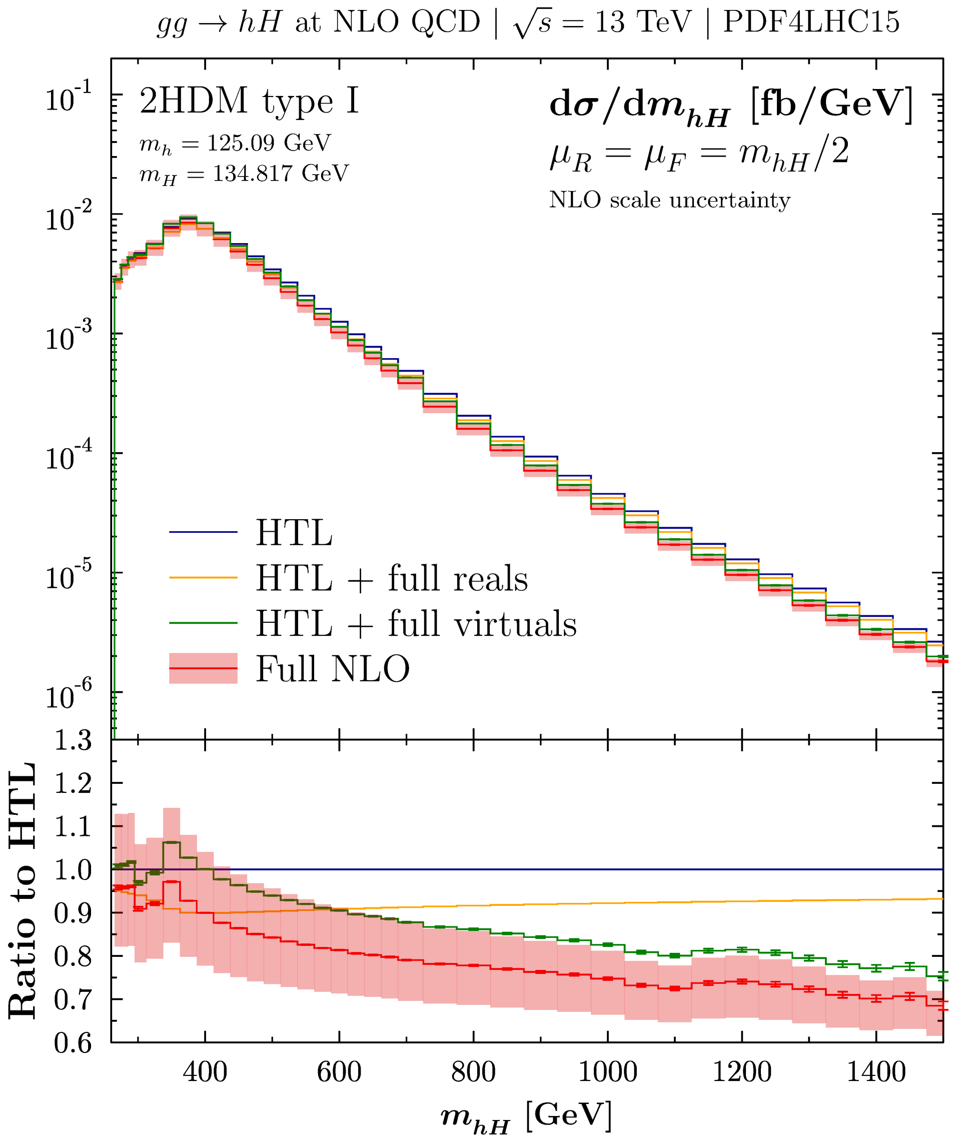
- VBF @ N^{3}LO, VHH @ NNLO, $t\bar{t}HH$ @ NLO

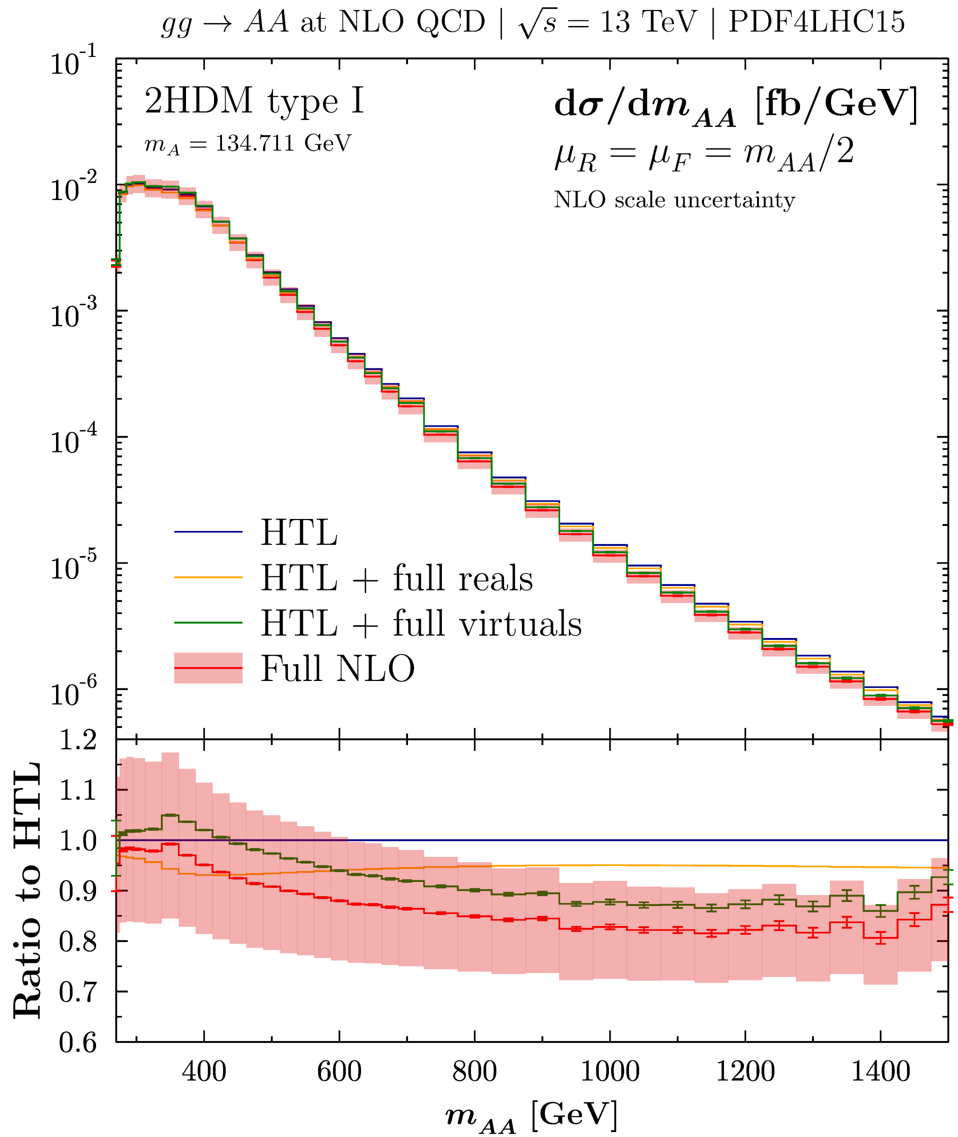
BACKUP SLIDES

2HDM [type I]: $gg \to hh, hH, HH, AA$ [no $hA, HA \to$ DY-like]

$M_h = 125.09$ GeV $\quad M_H = 134.817$ GeV $\quad M_A = 134.711$ GeV

$\mathrm{tg}\beta = 3.759 \quad \alpha = -0.102 \quad m_{12}^2 = 4305\ \mathrm{GeV}^2 \quad \Rightarrow \cos(\beta-\alpha) = 0.157$





Baglio, Campanario, Glaus, Mühlleitner, Ronca, S.

combined uncertainties

$$13\text{ TeV}: \quad \sigma(gg \to hH) = 1.592(1)^{+21\%}_{-24\%}\text{ fb}$$
$$14\text{ TeV}: \quad \sigma(gg \to hH) = 1.876(1)^{+21\%}_{-24\%}\text{ fb}$$
$$27\text{ TeV}: \quad \sigma(gg \to hH) = 7.036(4)^{+18\%}_{-23\%}\text{ fb}$$
$$100\text{ TeV}: \quad \sigma(gg \to hH) = 60.49(4)^{+16\%}_{-25\%}\text{ fb}$$

$$13\text{ TeV}: \quad \sigma(gg \to AA) = 1.643(1)^{+26\%}_{-21\%}\text{ fb}$$
$$14\text{ TeV}: \quad \sigma(gg \to AA) = 1.927(1)^{+26\%}_{-22\%}\text{ fb}$$
$$27\text{ TeV}: \quad \sigma(gg \to AA) = 7.012(4)^{+23\%}_{-21\%}\text{ fb}$$
$$100\text{ TeV}: \quad \sigma(gg \to AA) = 58.12(3)^{+22\%}_{-22\%}\text{ fb}$$

II CALCULATION

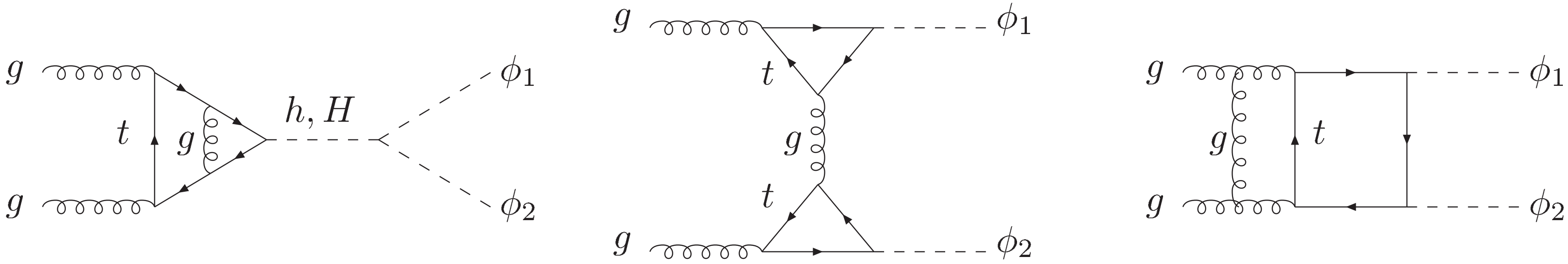
$$\sigma_{\mathrm{NLO}}(pp \to HH + X) = \sigma_{\mathrm{LO}} + \Delta\sigma_{\mathrm{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

$$\begin{aligned}
\sigma_{\mathrm{LO}} &= \int_{\tau_0}^{1} d\tau\, \frac{d\mathcal{L}^{gg}}{d\tau}\, \hat{\sigma}_{\mathrm{LO}}(Q^2 = \tau s) \\
\Delta\sigma_{\mathrm{virt}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau\, \frac{d\mathcal{L}^{gg}}{d\tau}\, \hat{\sigma}_{\mathrm{LO}}(Q^2 = \tau s)\; C \\
\Delta\sigma_{gg} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau\, \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z}\, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s) \left\{ -z P_{gg}(z) \log \frac{M^2}{\tau s} \right. \\
&\qquad \left. + d_{gg}(z) + 6[1 + z^4 + (1-z)^4] \left(\frac{\log(1-z)}{1-z} \right)_+ \right\} \\
\Delta\sigma_{gq} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z}\, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s (1-z)^2} + d_{gq}(z) \right\} \\
\Delta\sigma_{q\bar{q}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z}\, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s)\; d_{q\bar{q}}(z)
\end{aligned}$$

$$C \to \pi^2 + \frac{11}{2} + C_{\triangle\triangle}, \quad d_{gg} \to -\frac{11}{2}(1-z)^3, \quad d_{gq} \to \frac{2}{3}z^2 - (1-z)^2, \quad d_{q\bar{q}} \to \frac{32}{27}(1-z)^3$$

(i) virtual corrections

47 gen. box diags, 8 tria diags (← single Higgs), 1PR (← $H, A \to Z\gamma$)



(i) (ii) (iii)

- two formfactors:

$$\mathcal{A}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu} \qquad F_1 = C_\triangle F_\triangle + F_\Box \qquad F_2 = G_\Box$$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{q_1^\nu q_2^\mu}{(q_1 q_2)},$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{M_H^2 q_1^\nu q_2^\mu}{p_T^2 (q_1 q_2)} - 2\frac{(q_2 p_1) q_1^\nu p_1^\mu}{p_T^2 (q_1 q_2)} - 2\frac{(q_1 p_1) p_1^\nu q_2^\mu}{p_T^2 (q_1 q_2)} + 2\frac{p_1^\nu p_1^\mu}{p_T^2}$$

$$P_1^{\mu\nu} = \frac{(1-\epsilon) T_1^{\mu\nu} + \epsilon T_2^{\mu\nu}}{2(1-2\epsilon)} \qquad P_2^{\mu\nu} = \frac{\epsilon T_1^{\mu\nu} + (1-\epsilon) T_2^{\mu\nu}}{2(1-2\epsilon)}$$

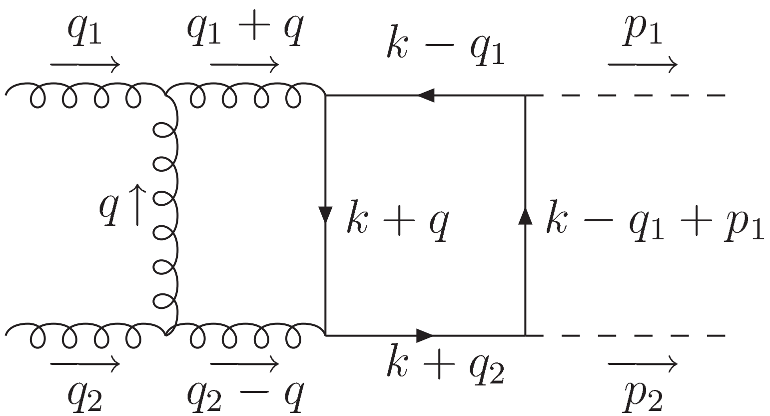
$$P_1^{\mu\nu} \mathcal{A}_{\mu\nu} = F_1 \qquad P_2^{\mu\nu} \mathcal{A}_{\mu\nu} = F_2$$

- full diagram w/o tensor reduction → 6-dim. Feynman integrals

- UV-singularities: end-point subtractions

$$\int_0^1 dx\, \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx\, \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx\, \frac{f(x)-f(1)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx\, \frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- IR-sing.: IR-subtraction (based on struc. of integr. and rel. to HTL)



$$\rho_s = \hat{s}/m_t^2 = Q^2/m_t^2$$
$$\rho_t = (\hat{t} - M_H^2)/m_t^2$$
$$\rho_u = (\hat{u} - M_H^2)/m_t^2$$
$$\rho_H = M_H^2/m_t^2$$
$$d^6x = dx\,dy\,dz\,dr\,ds\,dt$$

$$\Delta F_i = \Gamma(1+2\epsilon)\left(\frac{4\pi\mu_0^2}{m_t^2}\right)^{2\epsilon} \int_0^1 d^6x\, \frac{x^{1+\epsilon}(1-x)^{\epsilon} r^{1+\epsilon} s^{-\epsilon} H_i(\vec{x})}{N^{3+2\epsilon}(\vec{x})}$$

$$N(\vec{x}) = ar^2 + br + c$$

$$a = x(1-x)ys\Big[-\rho_s(1-y-t) + \rho_t yz - \rho_u z(1-y-t) + \rho_H yz^2\Big]$$

$$b = 1 - \rho_s x\Big\{xy(1-y) + (1-x)[(1-s)(1-y-t) + yst]\Big\} - \rho_H xyz(1-xyz)$$
$$-\rho_t xyz[1 - xy - (1-x)(1-s)] - \rho_u xyz[x(1-y) + (1-x)st]$$

$$c = -\rho_s x(1-x)(1-s)t$$

- subtract integrand with linear denominator

$$\Delta F_i = \frac{\alpha_s}{\pi}\Gamma(1+2\epsilon)\left(\frac{4\pi\mu_0^2}{m_t^2}\right)^{2\epsilon}(G_1+G_2)$$

$$G_1 = \int_0^1 d^6x\; x^{1+\epsilon}(1-x)^\epsilon r^{1+\epsilon}s^{-\epsilon}\left\{\frac{H_i(\vec{x})}{N^{3+2\epsilon}(\vec{x})} - \frac{H_i(\vec{x})|_{r=0}}{(br+c)^{3+2\epsilon}}\right\}$$

$$G_2 = \int_0^1 d^6x\; x^{1+\epsilon}(1-x)^\epsilon r^{1+\epsilon}s^{-\epsilon}\frac{H_i(\vec{x})|_{r=0}}{(br+c)^{3+2\epsilon}}$$

- $G_2 \to$ hypergeom. fct. after r-integration [arg $\to$ 1/arg]

- thresholds: $Q^2 \geq 0, 4m_t^2 \to$ IBP $\to$ reduction of power of denominator $[m_t^2 \to m_t^2(1-ih)]$

$$\int_0^1 dx\, \frac{f(x)}{(a+bx)^3} = \frac{f(0)}{2a^2b} - \frac{f(1)}{2b(a+b)^2} + \int_0^1 dx \frac{f'(x)}{2b(a+bx)^2}$$

- extrapolation to NWA ($h \to 0$): Richardson extrapolation 1911

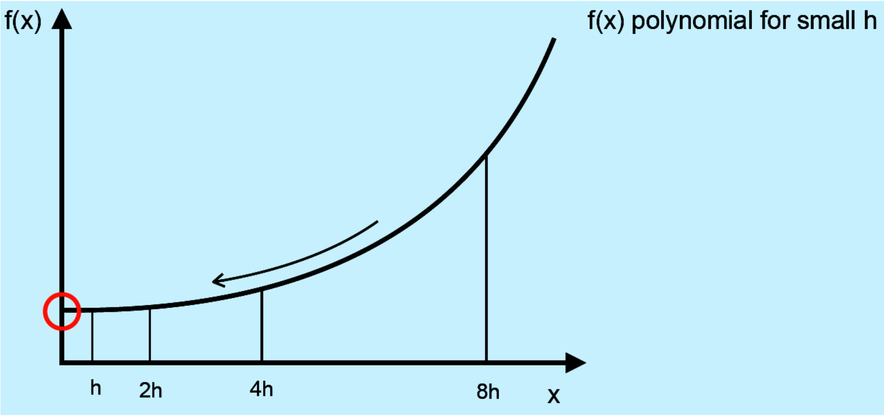
$M_2 = 2f(h) - f(2h) = f(0) + \mathcal{O}(h^2)$

$M_4 = \{8f(h) - 6f(2h) + f(4h)\}/3 = f(0) + \mathcal{O}(h^3)$

$M_8 = \{64f(h) - 56f(2h) + 14f(4h) - f(8h)\}/21 = f(0) + \mathcal{O}(h^4)$

etc.

$[h \geq 0.05]$



- renormalization: α_s: $\overline{\text{MS}}$, 5 flavours
  m_t: on-shell

- PS-integration $\rightarrow$ 7-dim. integrals for $d\sigma/dQ^2$

- subtraction of HTL $\rightarrow$ IR-finite mass effects

- add back HTL results $\leftarrow$ HPAIR

(ii) real corrections

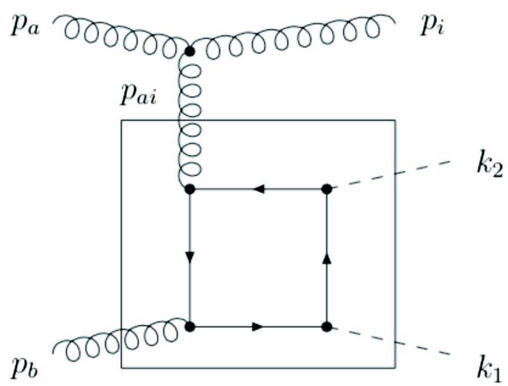
- full matrix elements generated with FeynArts and FormCalc

- matrix elements in HTL involving full LO sub-matrix elements subtracted $\to$ IR-, COLL-finite [adding back HTL results $\leftarrow$ HPAIR]

$$\sum \overline{|\mathcal{M}_{gg}|^2} = \sum \overline{\left|\tilde{\mathcal{M}}_{LO}\right|^2} \frac{24\pi^2}{Q^4}\frac{\alpha_s}{\pi}\left\{\frac{s^4+t^4+u^4+Q^8}{stu} - 4\frac{\epsilon}{1-\epsilon}Q^2\right\}$$

$$\sum \overline{|\mathcal{M}_{gq}|^2} = \sum \overline{\left|\tilde{\mathcal{M}}_{LO}\right|^2} \frac{32\pi^2}{3Q^4}\frac{\alpha_s}{\pi}\left\{\frac{s^2+u^2}{-t} + \epsilon\frac{(s+u)^2}{t}\right\}$$

$$\sum \overline{\left|\mathcal{M}_{q\bar{q}}\right|^2} = \sum \overline{\left|\tilde{\mathcal{M}}_{LO}\right|^2} \frac{256\pi^2}{9Q^4}\frac{\alpha_s}{\pi}(1-\epsilon)\left\{\frac{t^2+u^2}{s} - \epsilon\frac{(t+u)^2}{s}\right\}$$



- m_t scale/scheme uncertainties at LO:

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=300\ \text{GeV}} = 0.01656^{+62\%}_{-2.4\%}\ \text{fb/GeV}$$

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=400\ \text{GeV}} = 0.09391^{+0\%}_{-20\%}\ \text{fb/GeV}$$

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=600\ \text{GeV}} = 0.02132^{+0\%}_{-48\%}\ \text{fb/GeV}$$

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=1200\ \text{GeV}} = 0.0003223^{+0\%}_{-56\%}\ \text{fb/GeV}$$

$$F_i = F_{i,LO} + \Delta F_i$$
$$\Delta F_i = \Delta F_{i,HTL} + \Delta F_{i,mass}$$

- pole mass:

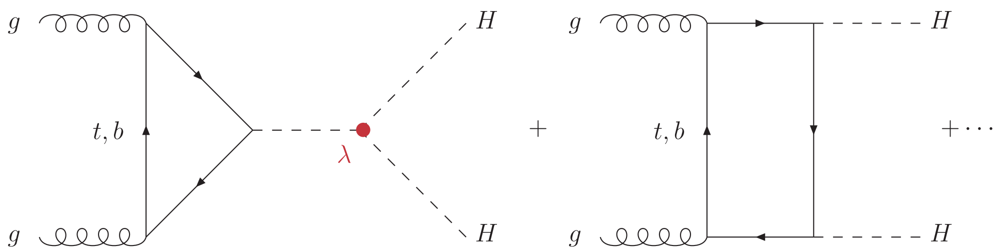
$$F_{1,LO} \to 4\frac{m_t^2}{\hat{s}}$$
$$F_{2,LO} \to -\frac{m_t^2}{\hat{s}\hat{t}(\hat{s}+\hat{t})}\{(\hat{s}+\hat{t})^2 L_{1ts}^2 + \hat{t}^2 L_{ts}^2 + \pi^2[(\hat{s}+\hat{t})^2 + \hat{t}^2]\}$$

- $\overline{\text{MS}}$ mass:

$$F_{1,LO} \to 4\frac{\overline{m}_t^2(\mu_t)}{\hat{s}}$$
$$F_{2,LO} \to -\frac{\overline{m}_t^2(\mu_t)}{\hat{s}\hat{t}(\hat{s}+\hat{t})}\{(\hat{s}+\hat{t})^2 L_{1ts}^2 + \hat{t}^2 L_{ts}^2 + \pi^2[(\hat{s}+\hat{t})^2 + \hat{t}^2]\}$$

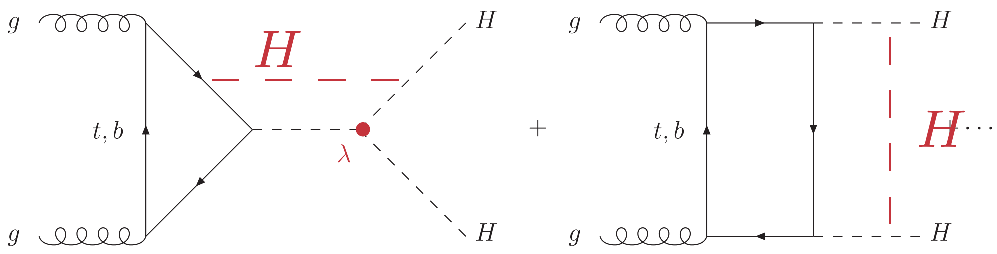
• different scales for y_t in triangle (Q) and box (M_H) diagrams?

$\rightarrow$ has to hold at all orders



- different scales for y_t in triangle (Q) and box (M_H) diagrams?

$\rightarrow$ has to hold at all orders



elw. corrections

$\Rightarrow$ same scales in all diagrams

$$
\begin{aligned}
\sigma_{\mathrm{LO}} &= \int_{\tau_0}^{1} d\tau \; \frac{d\mathcal{L}^{gg}}{d\tau} \; \hat{\sigma}_{\mathrm{LO}}(Q^2 = \tau s) \\
\frac{d\mathcal{L}^{gg}}{d\tau} &= \int_{\tau}^{1} \frac{dx}{x} g(x, \mu_F) g\left(\frac{\tau}{x}, \mu_F\right) \\
\hat{\sigma}_{LO} &= \frac{G_F^2 \alpha_s^2(\mu_R)}{512(2\pi)^3} \int_{\hat{t}_-}^{\hat{t}_+} d\hat{t} [|C_\triangle F_\triangle + F_\Box|^2 + |G_\Box|^2] \\
\hat{t}_\pm &= -\frac{1}{2}\left[Q^2 - 2M_H^2 \mp Q^2 \sqrt{1 - 4\frac{M_H^2}{Q^2}}\right] \\
\lambda_{HHH} &= 3\frac{M_H^2}{v} \\
C_\triangle &= \frac{\lambda_{HHH} v}{(Q^2 - M_H^2)} \\
\text{HTL: } F_\triangle &\to 2/3, \qquad F_\Box \to -2/3, \qquad G_\Box \to 0
\end{aligned}
$$

$$
\begin{aligned}
C_\triangle F_\triangle &\to C_\triangle F_\triangle (1 + \Delta_\triangle) \\
F_\Box &\to F_\Box (1 + \Delta_\Box) \\
\Delta_\triangle &= \delta_1 + \Delta_{HHH} \\
\Delta_\Box &= \eta_1
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{HHH} &= \Delta_{vertex} + \Delta_{self} + \Delta_{CT} \\
\Delta_{vertex} &= \frac{m_t^4}{v^2 M_H^2}\frac{8}{(4\pi)^2}\Big\{B_0(Q^2;m_t,m_t) + 2B_0(M_H^2;m_t,m_t) \\
&\quad + \left(4m_t^2 - \frac{Q^2+2M_H^2}{2}\right) C_0(Q^2,M_H^2,M_H^2;m_t,m_t,m_t)\Big\} + \frac{T_1}{vM_H^2} \\
\Delta_{self} &= \frac{\Sigma_H(Q^2)}{Q^2-M_H^2} + \frac{1}{2}\Sigma'_H(M_H^2) \\
\Delta_{CT} &= \frac{\delta M_H^2}{Q^2-M_H^2} + \frac{\delta\lambda_{HHH}}{\lambda_{HHH}} \\
\Sigma_H(Q^2) &= 3\,\frac{T_1}{v} + 6\,\frac{m_t^2}{(4\pi)^2 v^2}\left\{2A_0(m_t) + (4m_t^2 - Q^2)B_0(Q^2;m_t,m_t)\right\} + \mathcal{O}(m_t^0) \\
\Sigma'_H(Q^2) &= 6\,\frac{m_t^2}{(4\pi)^2 v^2}\left\{(4m_t^2 - Q^2)B'_0(Q^2;m_t,m_t) - B_0(Q^2;m_t,m_t)\right\} + \mathcal{O}(m_t^0) \\
\frac{T_1}{v} &= -12\,\frac{m_t^2}{(4\pi)^2 v^2}A_0(m_t) \\
\frac{\delta\lambda_{HHH}}{\lambda_{HHH}} &= \frac{\delta M_H^2}{M_H^2} + \frac{1}{2}\frac{\Sigma_W(0)}{M_W^2} \\
\frac{\Sigma_W(0)}{M_W^2} &= 2\,\frac{T_1}{vM_H^2} + \frac{2m_t^2}{(4\pi)^2 v^2}\left\{B_0(0;m_t,0) + 2B_0(0;m_t,m_t) + m_t^2 B'_0(0;m_t,0)\right\} + \mathcal{O}(m_t^0) \\
\delta M_H^2 &= -\Sigma_H(M_H^2)
\end{aligned}
$$