

# Probing new physics through entanglement

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THE UNIVERSITY  
*of* EDINBURGH

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Uppsala, Sweden

# “Heart of Quantum Mechanics”

“There is a troubling weirdness about QM. Perhaps its weirdest feature is entanglement, the need to describe even systems that extend over macroscopic distances in ways that are inconsistent with classical ideas.”

- Weinberg

## Quantum superposition

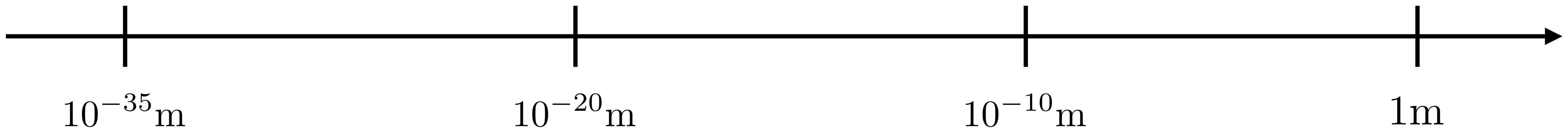
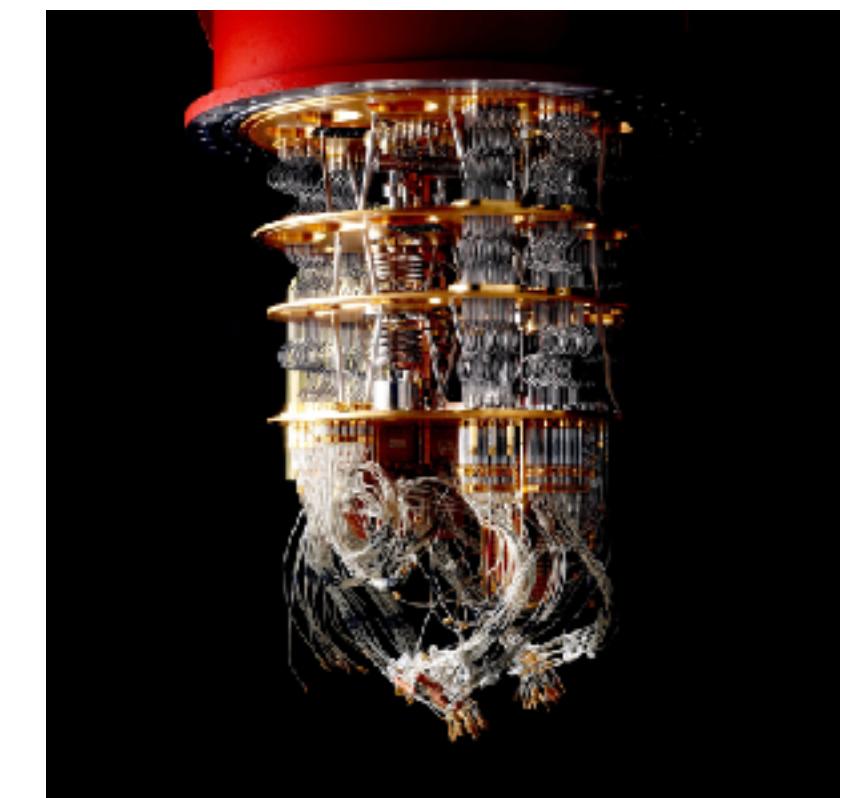
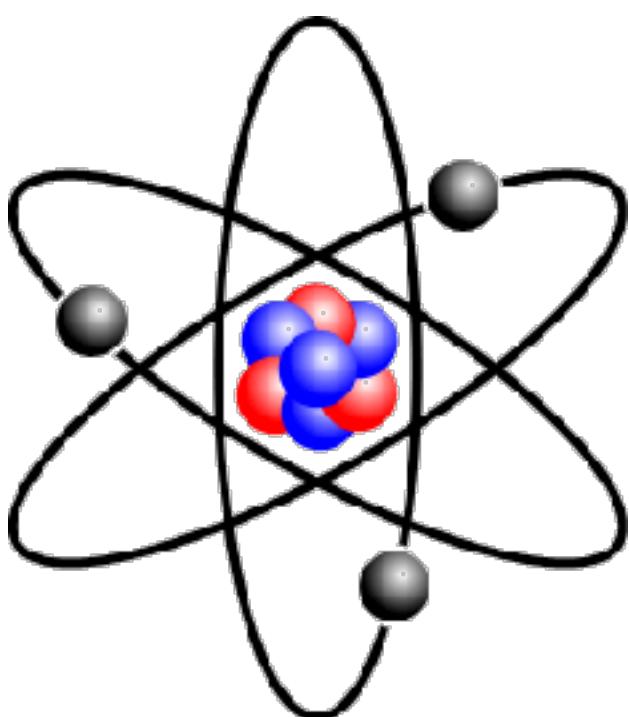
$$|\psi\rangle = \frac{1}{\sqrt{2}} |\text{alive cat}\rangle + \frac{1}{\sqrt{2}} |\text{dead cat}\rangle$$

## Entanglement

$$|\psi_{ab}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \neq |\psi_a\rangle \otimes |\psi_b\rangle$$

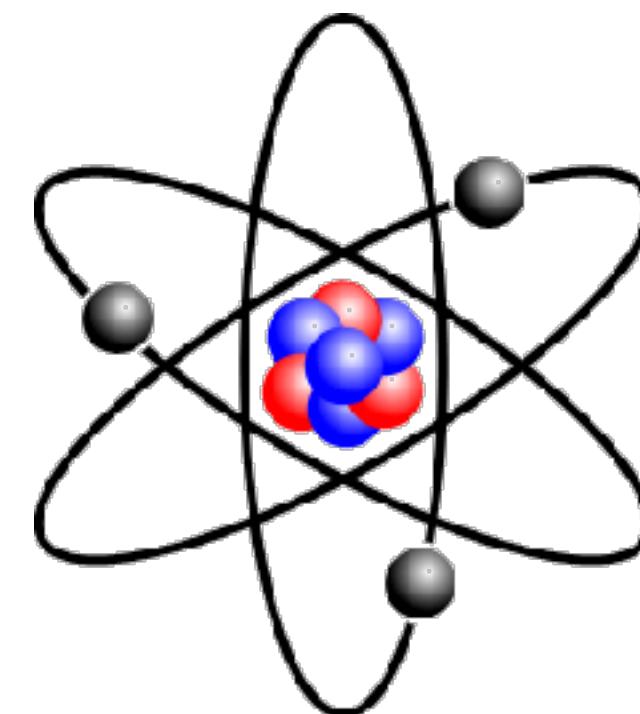
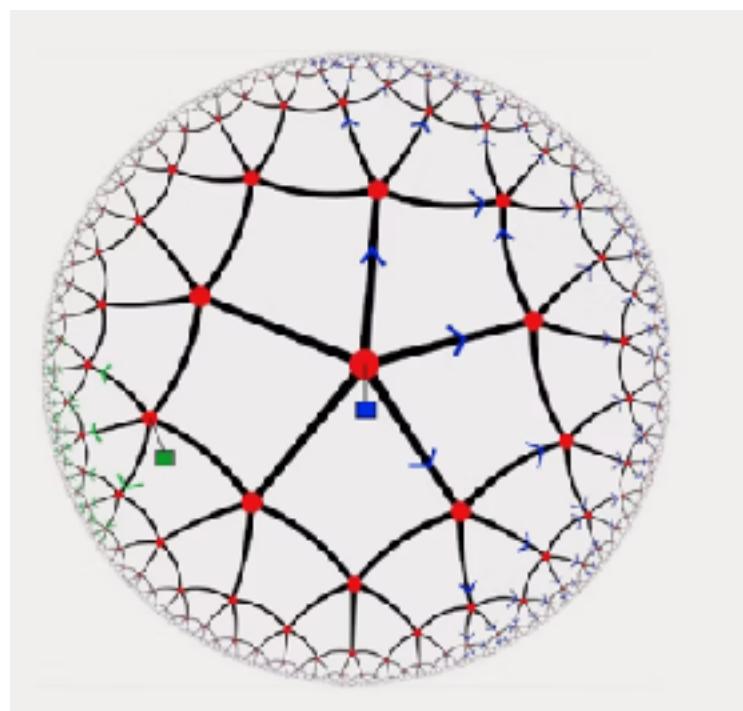
# Borders of the Quantum World: Energy/distance scales

Quantum computers  
Quantum devices,..



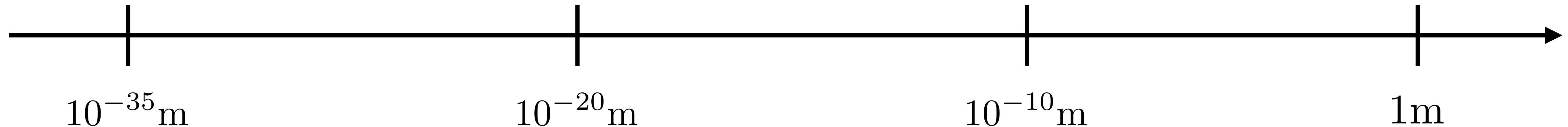
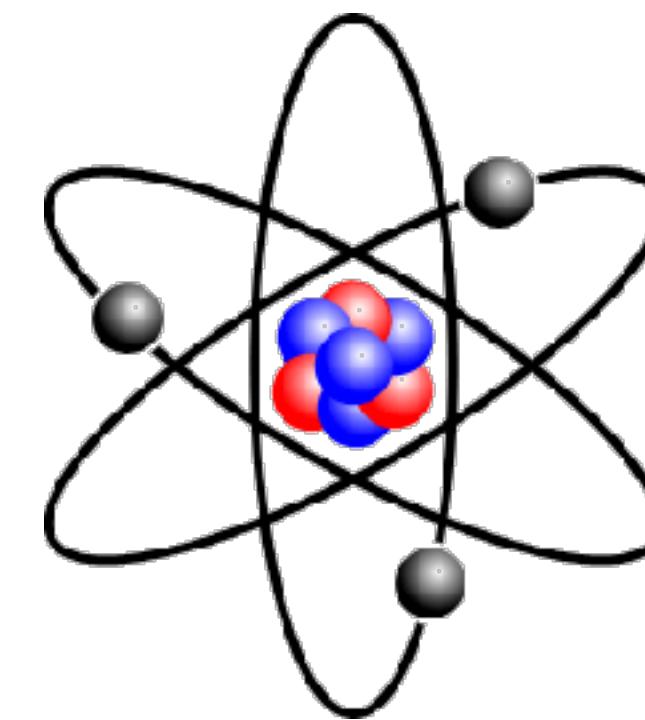
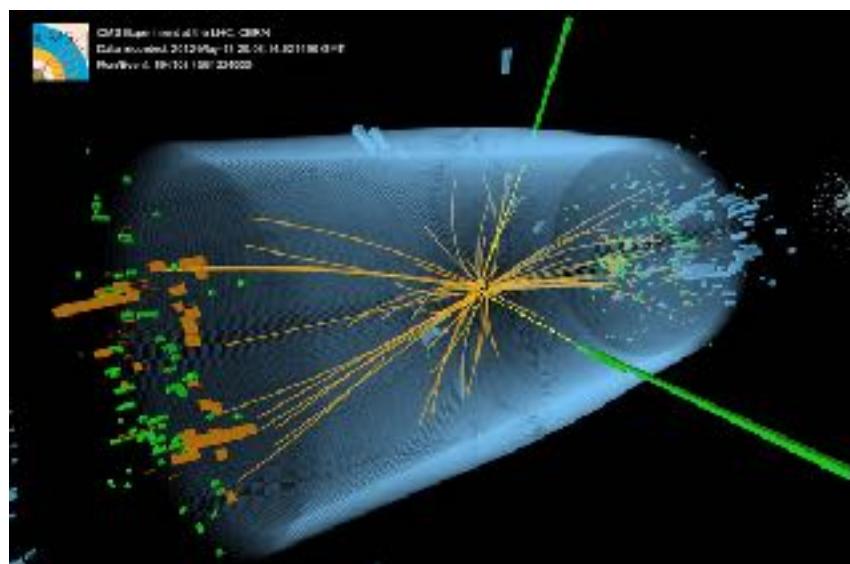
# Borders of the Quantum World: Energy/distance scales

It from Qubit



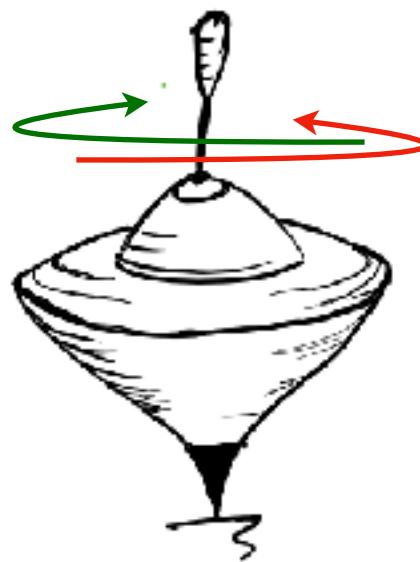
# Borders of the Quantum World: Energy/distance scales

LHC



# Back to the basics... entanglement in spin space

One qubit



$$|\psi\rangle = c_1 |\uparrow\rangle + c_2 |\downarrow\rangle = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

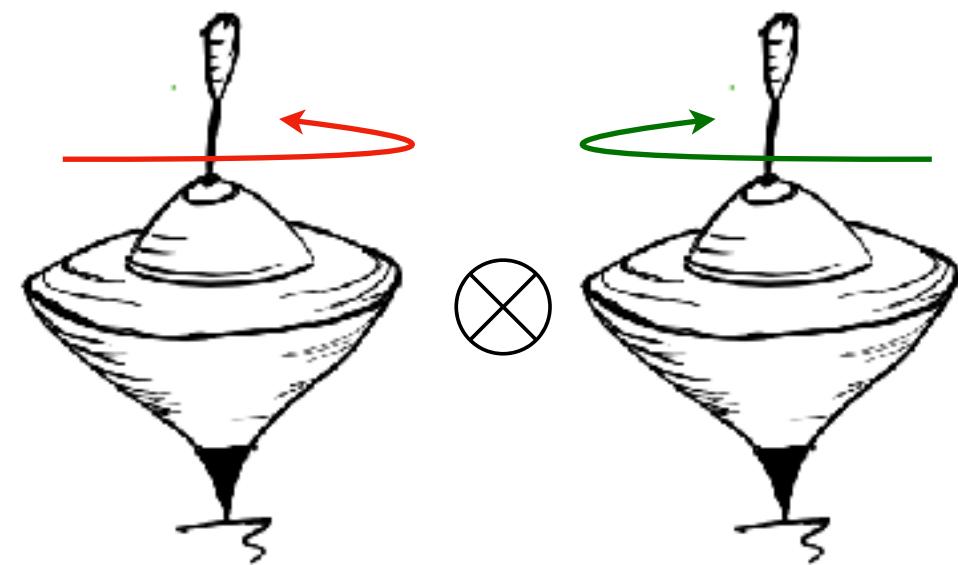
Density matrix:  $\rho_{\text{pure}} = |\psi\rangle\langle\psi| \longrightarrow \rho_{\text{mixed}} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

Gen. decomposition (identity and Pauli's)

$$\rho = \frac{1}{2} (\mathbb{1}_2 + a_i \sigma^i)$$

# Back to the basics... entanglement in spin space

Two qubit



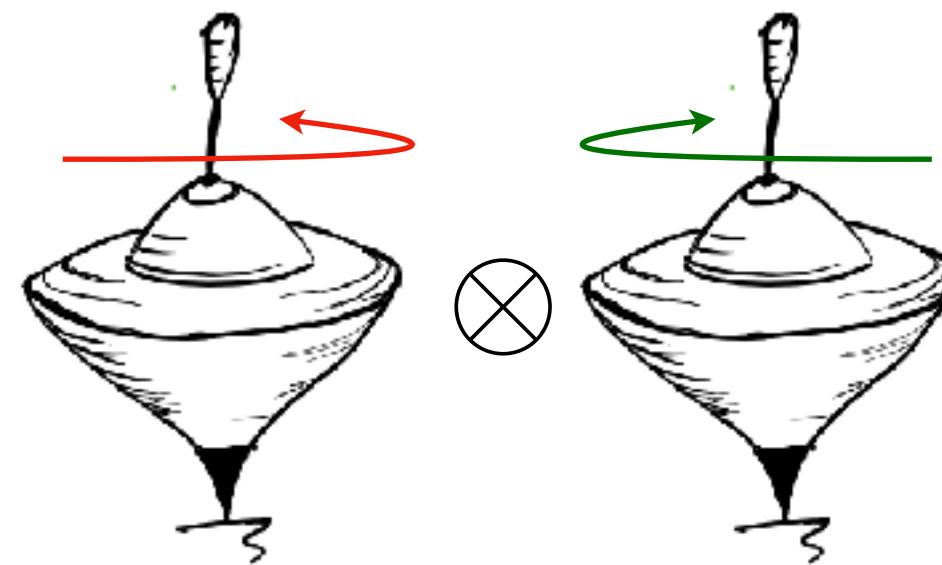
$$|\psi_{ab}\rangle = c_1 |\uparrow\uparrow\rangle + c_2 |\uparrow\downarrow\rangle + c_3 |\downarrow\uparrow\rangle + c_4 |\downarrow\downarrow\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$$

Density matrix is 4x4

$$\rho = \frac{\mathbf{1}_2 \otimes \mathbf{1}_2 + B_i^+ \sigma^i \otimes \mathbf{1}_2 + B_i^- \mathbf{1}_2 \otimes \sigma^i + C_{ij} \sigma^i \otimes \sigma^j}{4}.$$

# Back to the basics... entanglement in spin space

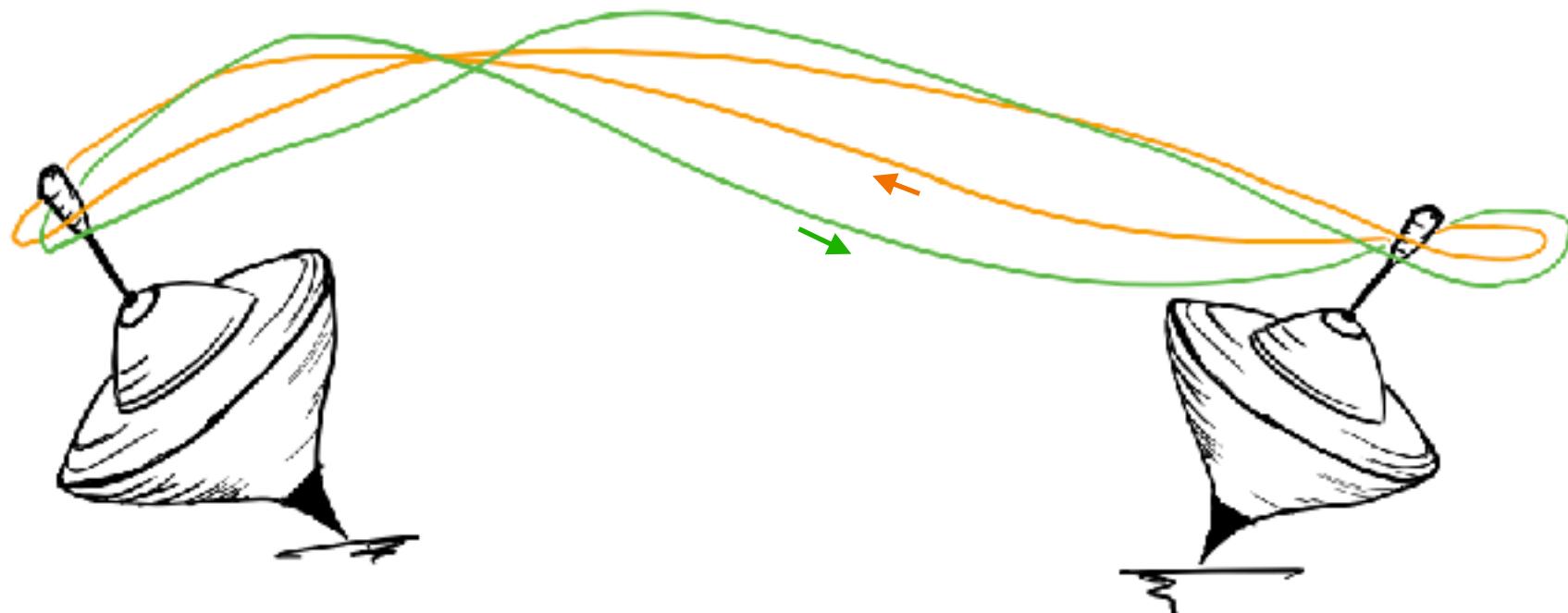
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Entanglement



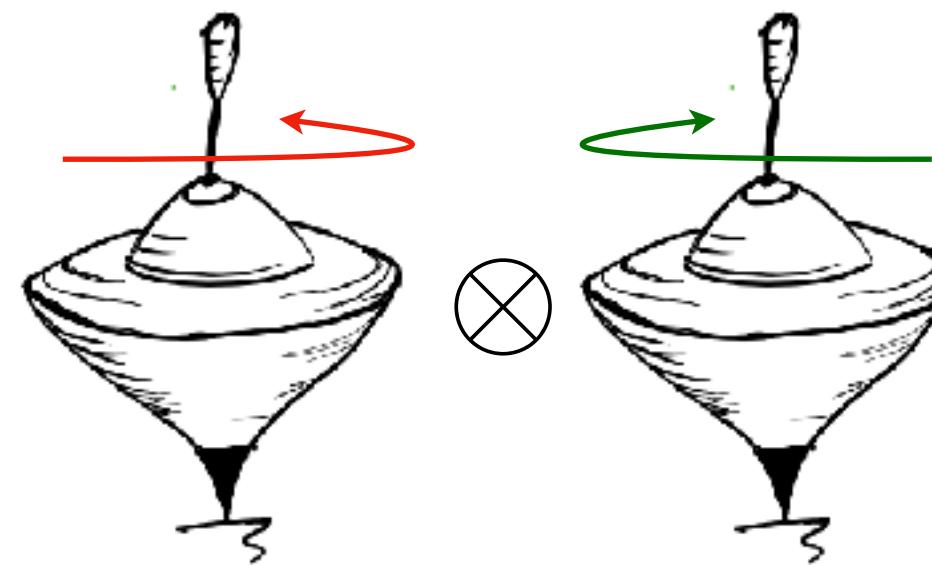
Given a bipartite system  $\mathcal{H}_{ab} = \mathcal{H}_a \otimes \mathcal{H}_b$

Can you write  $|\psi_{ab}\rangle = |\psi_a\rangle \otimes |\psi_b\rangle$  ?

No? Then it is entangled.

# Back to the basics... entanglement in spin space

Two qubit

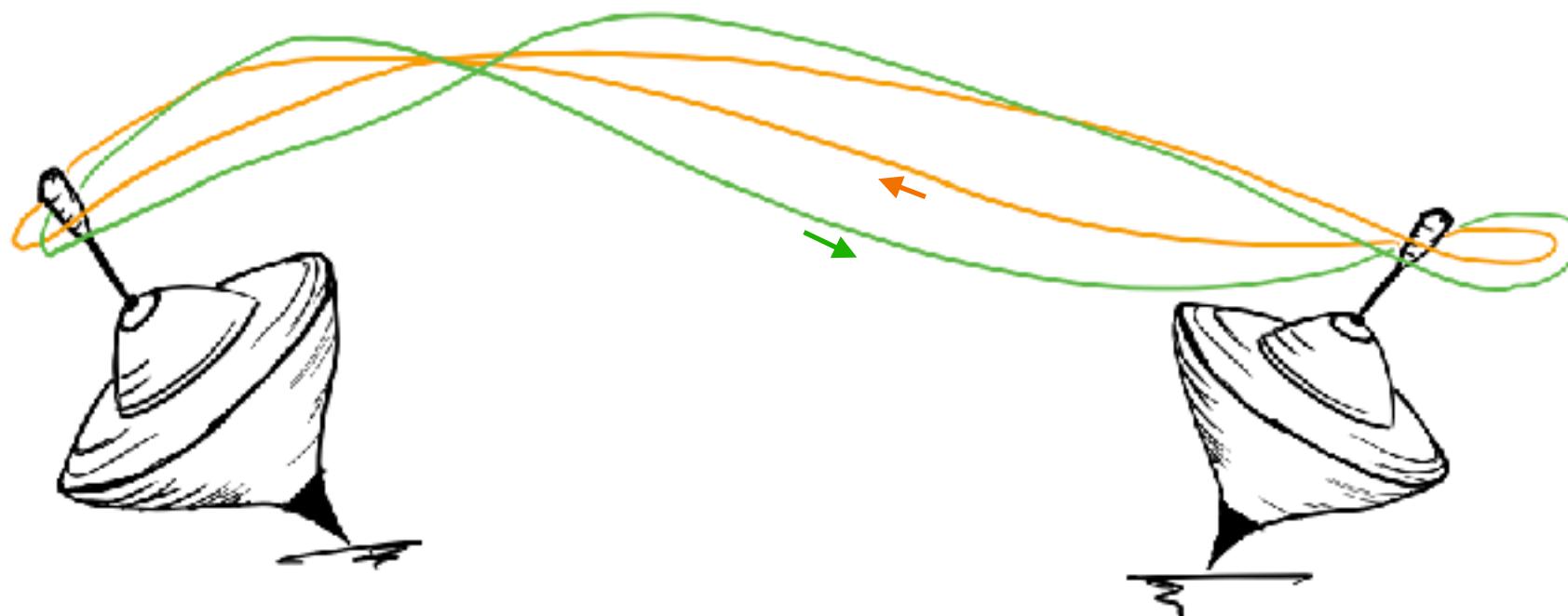


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Entanglement



Peres-Horodecki Criterion:

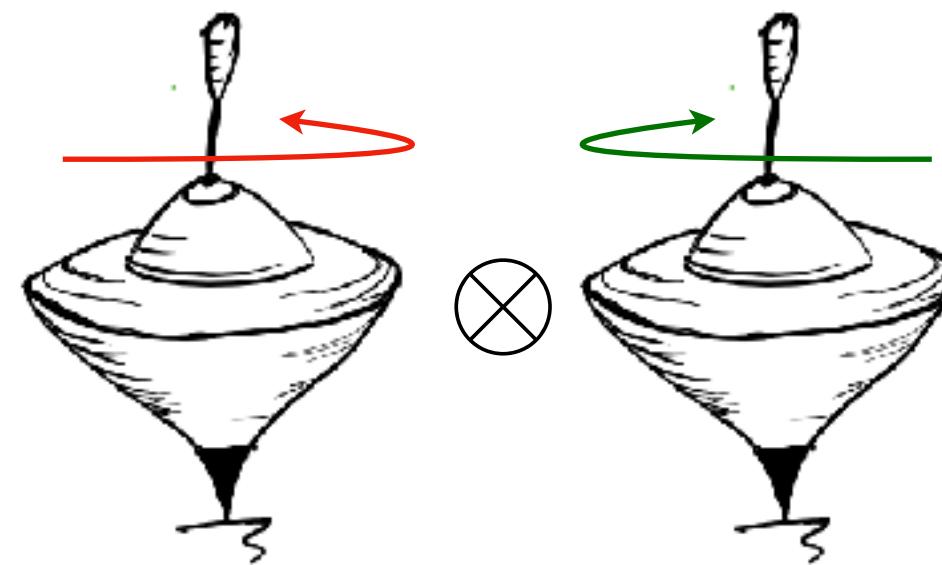
$$\Delta \equiv -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$

Concurrence (max = 1):

$$C[\rho] = \max(\Delta/2, 0)$$

# Back to the basics... entanglement in spin space

Two qubit



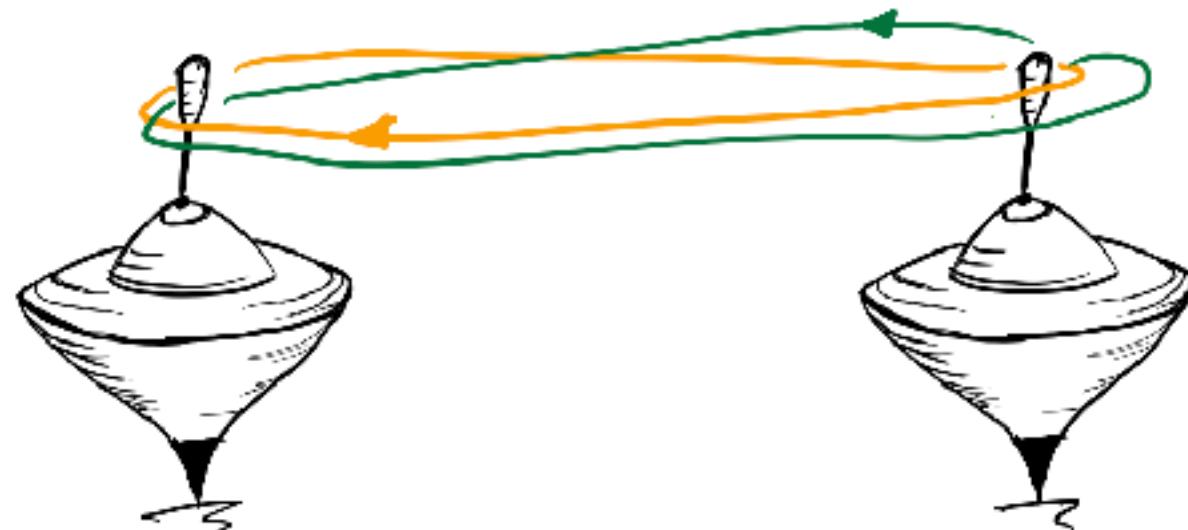
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Entanglement

Maximally entangled states (e.g Bell states):



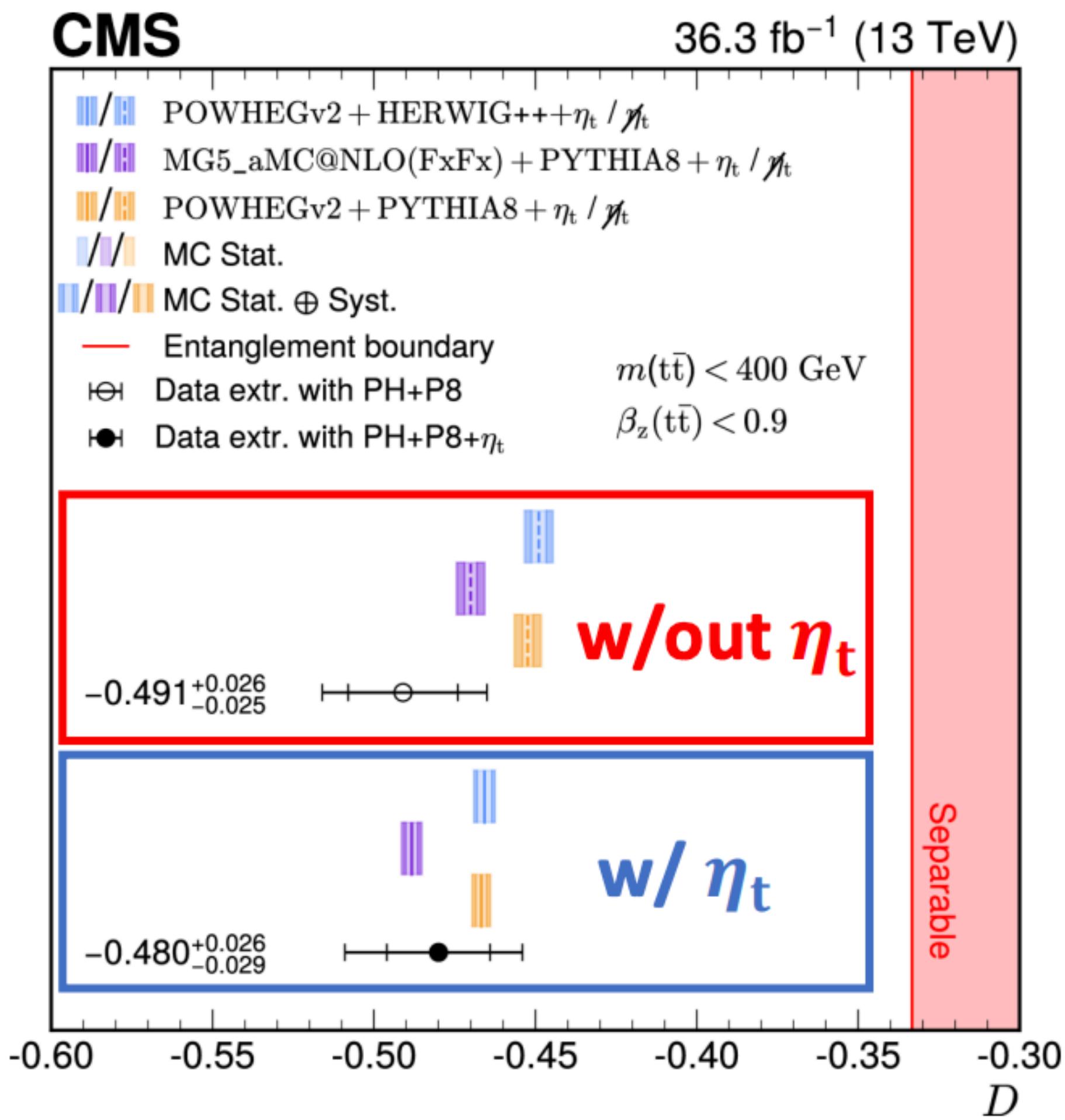
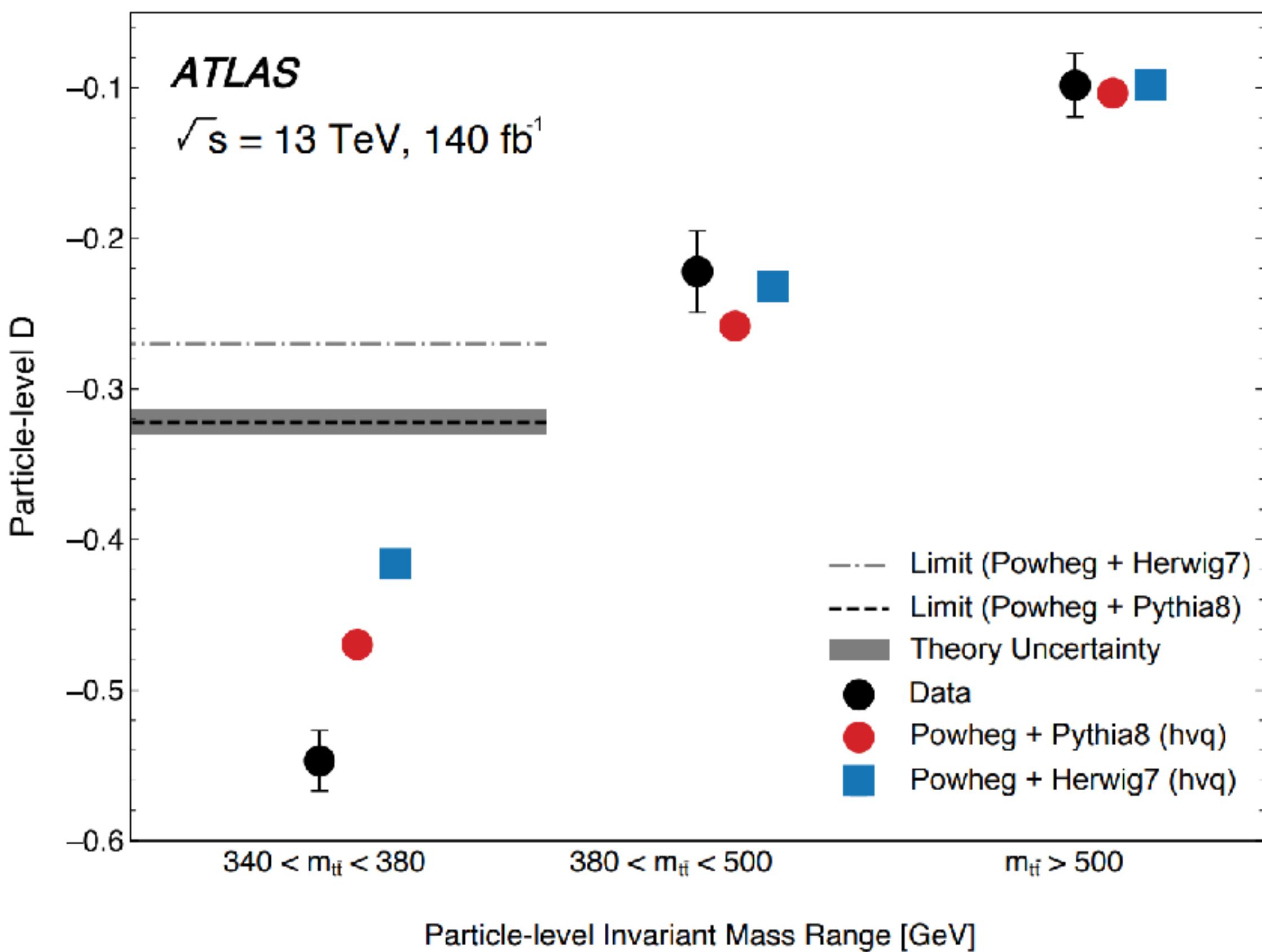
$$|\Phi^\pm\rangle = \frac{|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle}{\sqrt{2}} \quad \text{or} \quad |\Psi^\pm\rangle = \frac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}$$

Measures:  $\Delta = 2$        $C[\rho] = 1$

# Top-pair entanglement



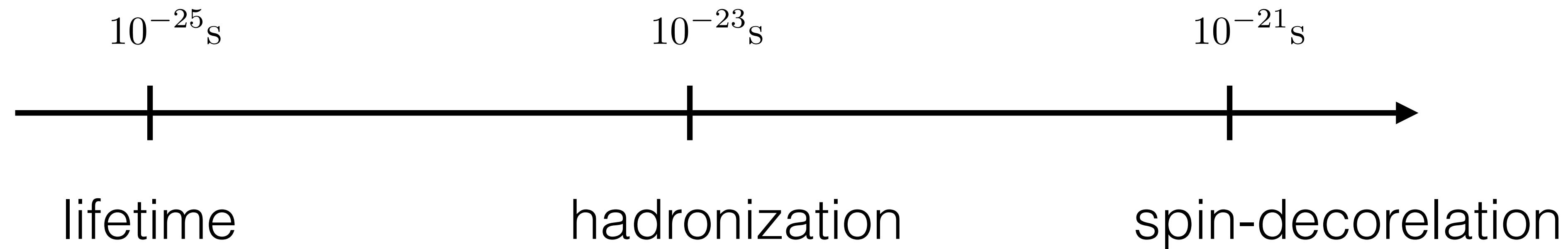
# ATLAS and CMS observed!



$D < -1/3 \longleftrightarrow \text{entangled}$

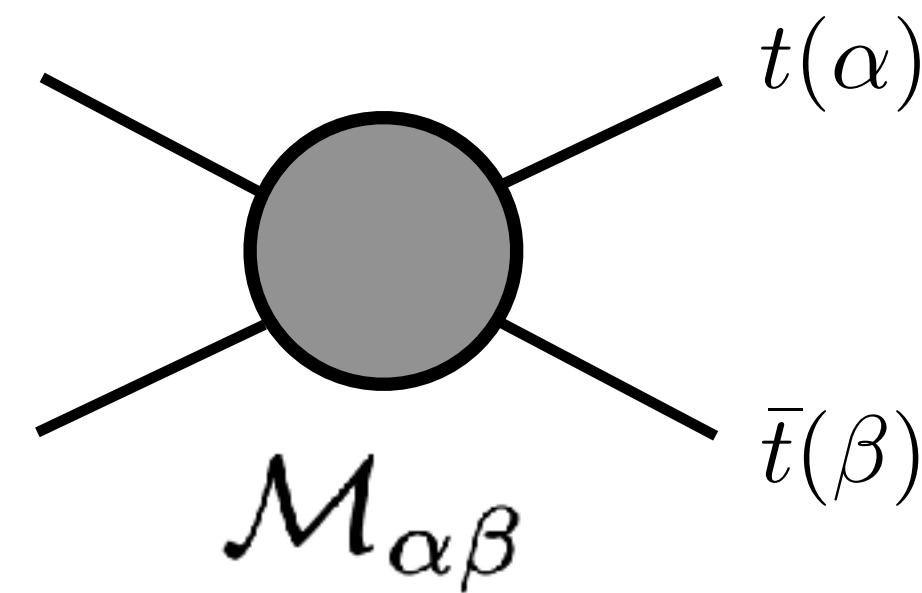
# Why top quarks?

- it is very heavy (172.5 GeV ~ mass of Rhenium nucleus)



- allows to efficiently reconstruct the spin from decay products
- top spin correlations vastly studied (both in theo and exp):  
D0 and CDF at Tevatron and ATLAS and CMS at LHC
- Observation of entanglement by ATLAS and CMS

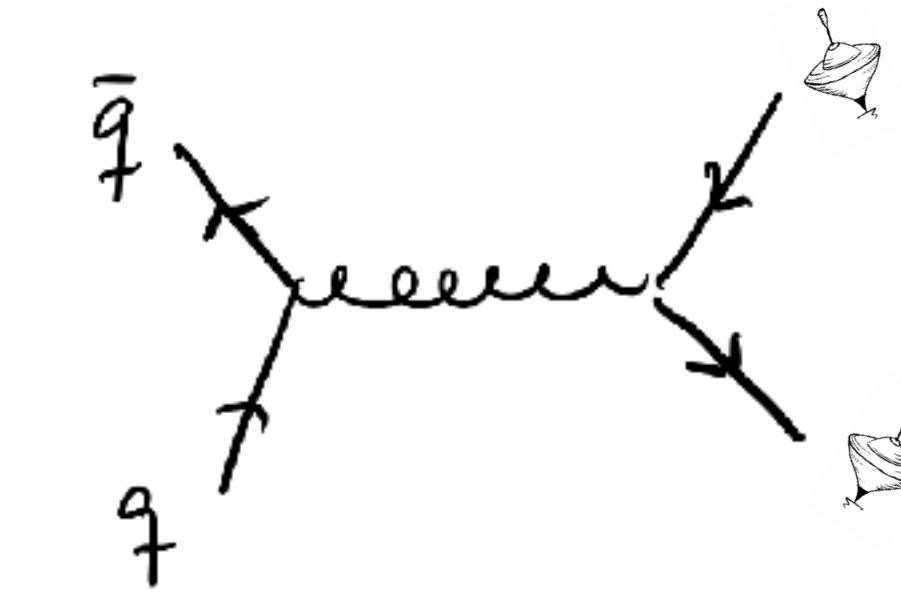
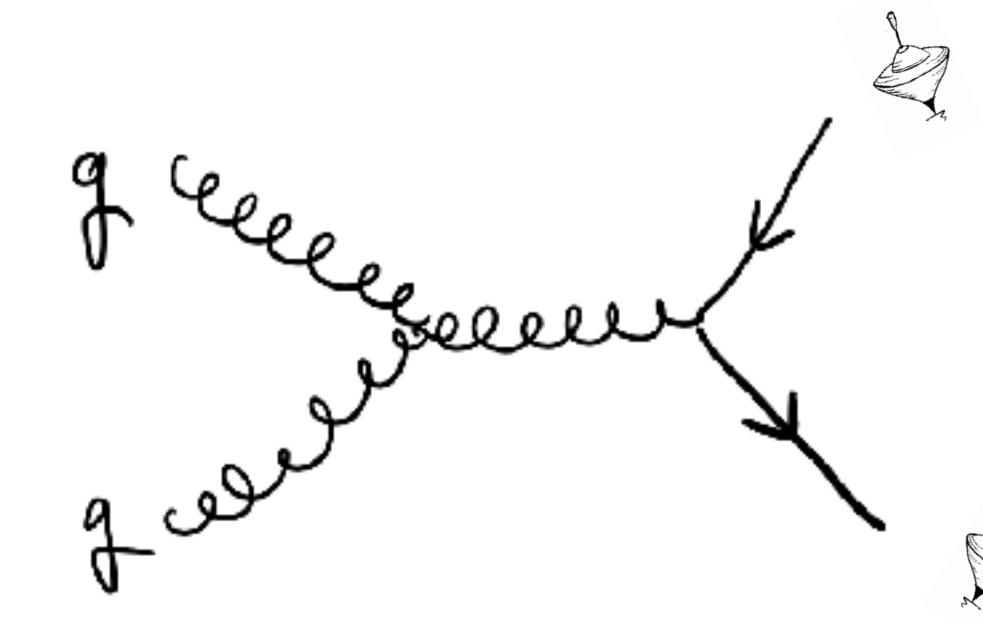
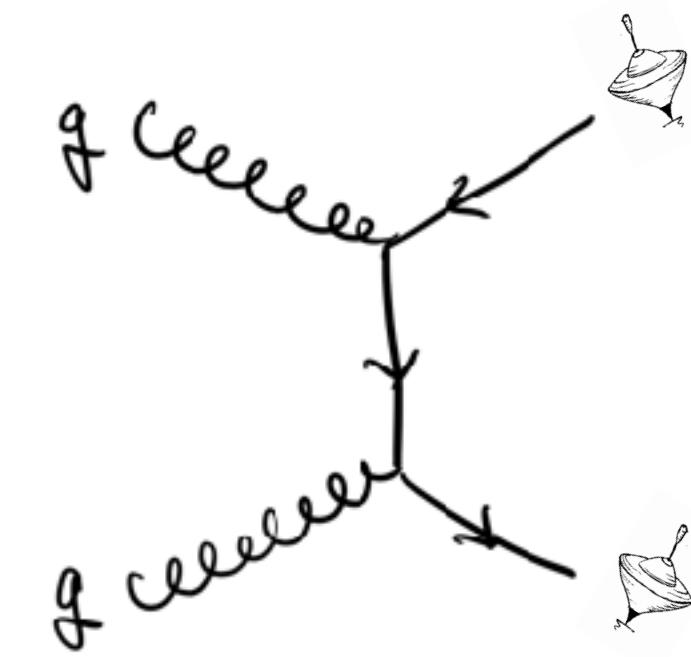
# How to construct the spin density matrix?



$$R^I_{(\alpha_1\beta_1)(\alpha_2\beta_2)} \sim \sum_{\text{color spin}} \mathcal{M}_{\alpha_2\beta_2}^* \mathcal{M}_{\alpha_1\beta_1}$$

4x4 matrix

SM:



Mixed state of qq and gg initiated channels,  
weighted by the luminosity functions

$$R(\hat{s}, \mathbf{k}) = \sum_I L^I(\hat{s}) R^I(\hat{s}, \mathbf{k})$$

# Spin production density matrix

Fano decomposition: (spanned by tensor prod. of Pauli and Identity)

$$R = \tilde{A} \mathbf{1}_2 \otimes \mathbf{1}_2 + \tilde{B}_i^+ \sigma^i \otimes \mathbf{1}_2 + \tilde{B}_i^- \mathbf{1}_2 \otimes \sigma^i + \tilde{C}_{ij} \sigma^i \otimes \sigma^j.$$

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**Cross-section:**  $\frac{d\sigma}{d\Omega d\hat{s}} = \frac{\alpha_s^2 \beta}{\hat{s}^2} \tilde{A}(\hat{s}, \mathbf{k})$

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**Degree of top and anti-top polarisation** (zero for P-inv interactions)

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**Spin-correlations**

NB: Spin-Correlations  $\supseteq$  Entanglement

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NB: Spin-Correlations  $\supseteq$  Entanglement

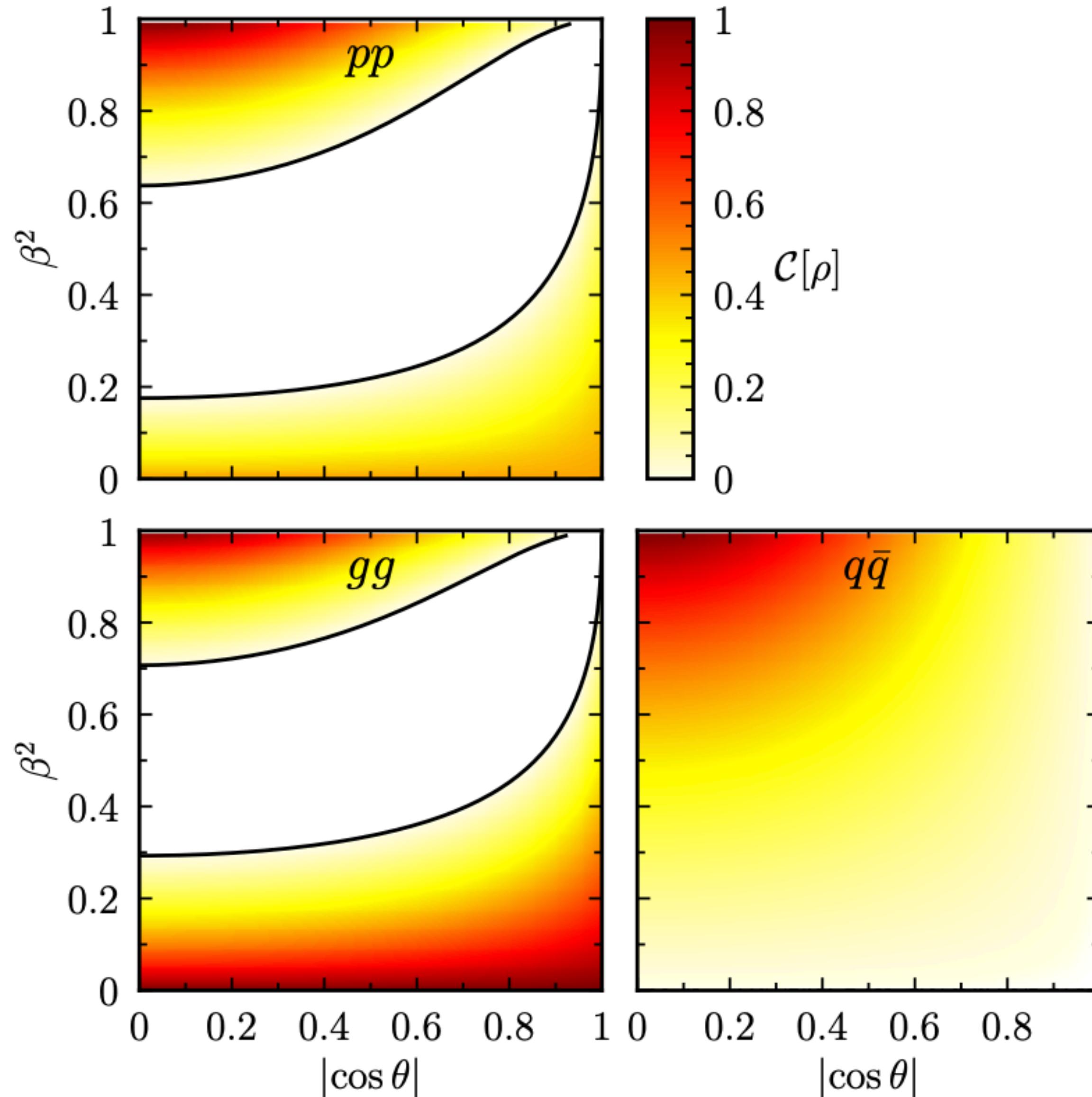
Normalize the state as

$$\rho = R / \text{tr}(R)$$

$$\rho = \frac{\mathbb{1}_2 \otimes \mathbb{1}_2 + B_i^+ \sigma^i \otimes \mathbb{1}_2 + B_i^- \mathbb{1}_2 \otimes \sigma^i + C_{ij} \sigma^i \otimes \sigma^j}{4}.$$

# What's the story for the SM?

[Afik and de Nova, 21']



Two-to-two scattering: 2 d.o.f  
velocity and angle of the top

$$\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}$$

$$\cos \theta$$

White regions: zero-entanglement

Maximal entanglement points/regions

- At threshold:  $\beta^2 = 0, \forall \theta$
- high-E:  $\beta^2 \rightarrow 1, \cos \theta = 0$

# Experimental detection (Quantum tomography)

Top quark cannot be measured directly

Spin information is directly transmitted to decay products  $t \rightarrow W^+ + b$

Lepton direction is totally correlated to the top spin

$(W^+ \rightarrow \ell^+ + \nu \text{ or } \bar{d} + u)$

D-coefficient:  $D = \frac{\text{tr}[\mathbf{C}]}{3}$

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \varphi} = \frac{1}{2} (1 - D \cos \varphi)$$

$\varphi$  angle between leptons in  
each parent direction rest frames

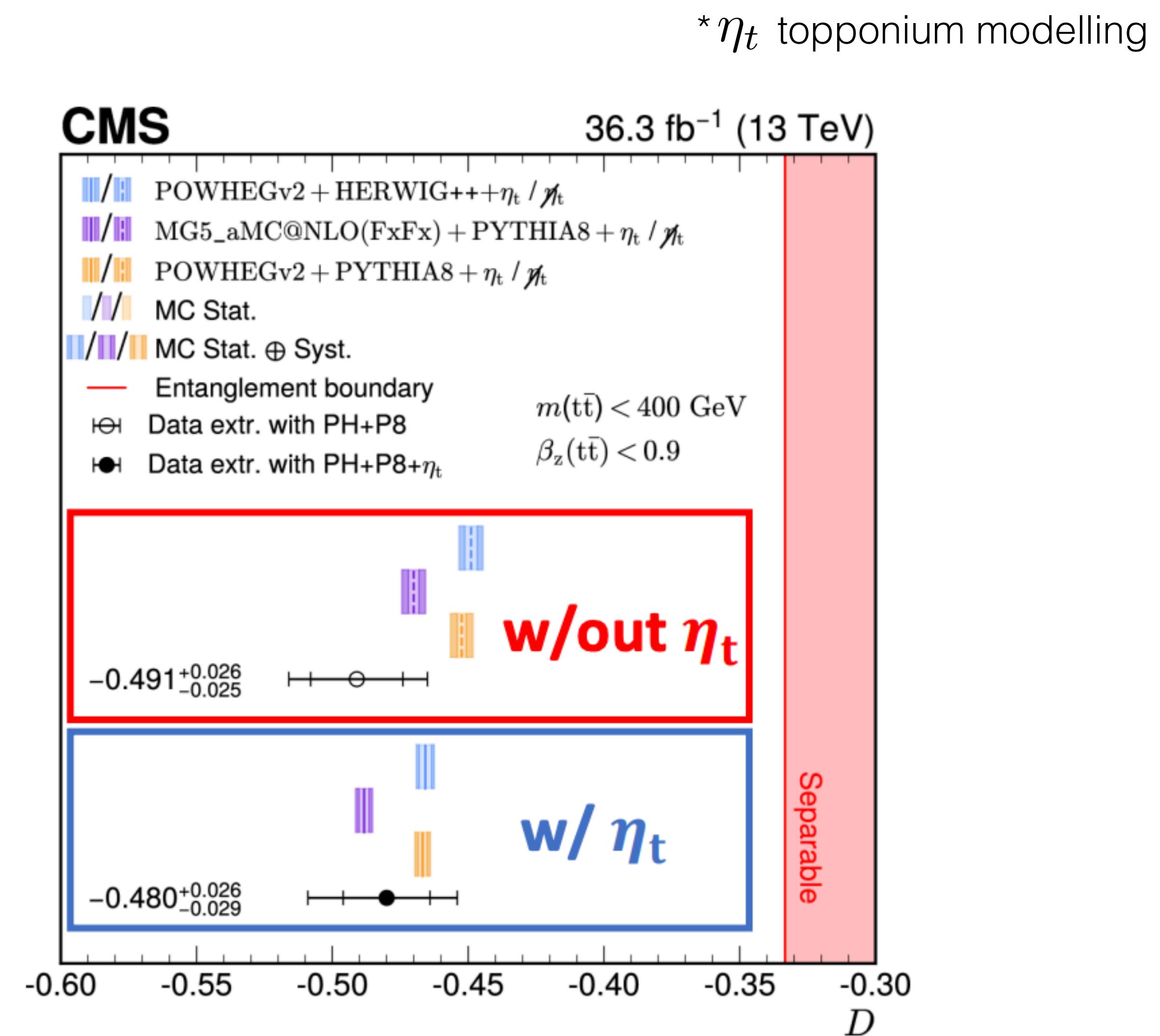
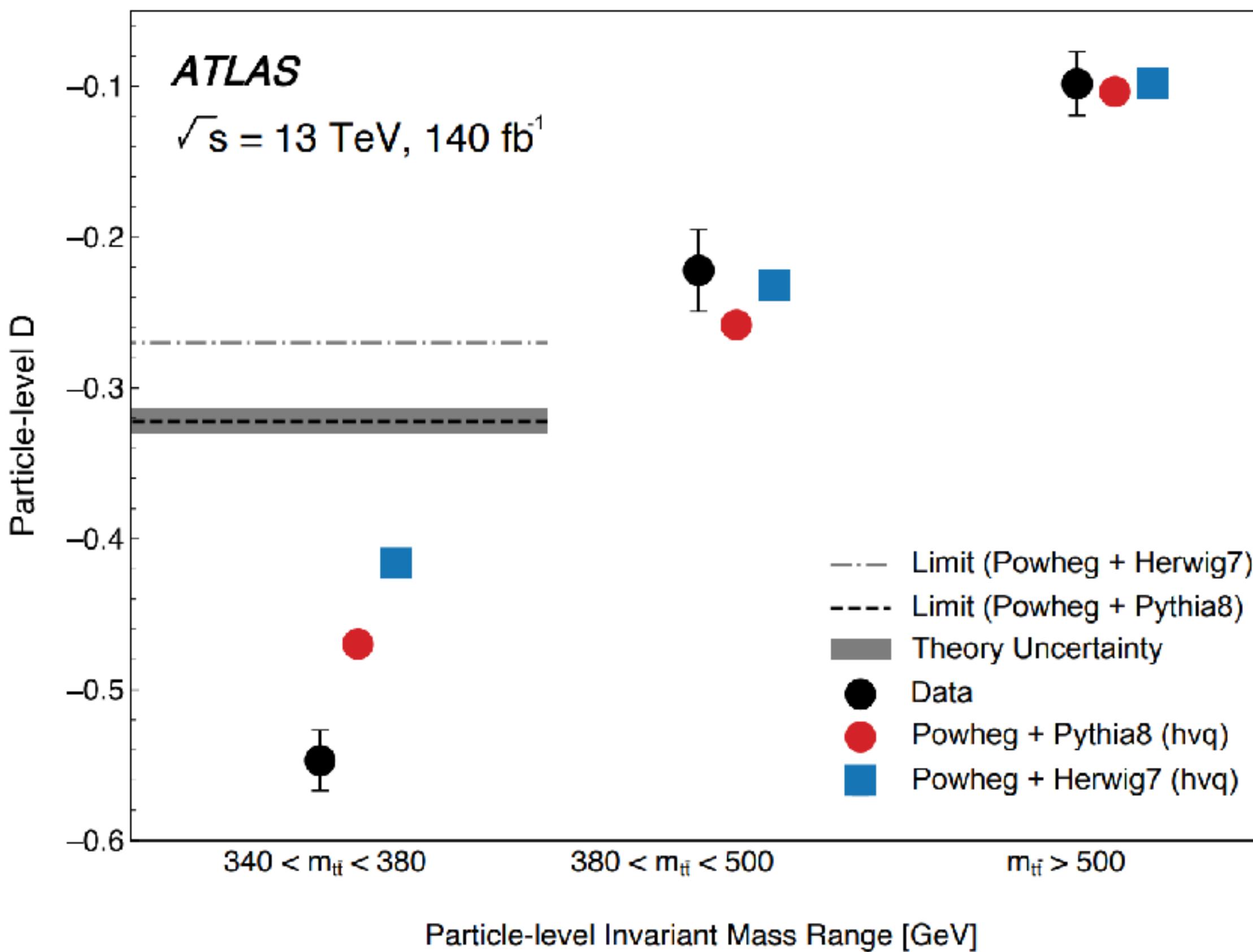
$$C[\rho] = \max(1 - 3D, 0)/2$$



entangled if  $D < -1/3$

# ATLAS and CMS measurement

At threshold  $\beta^2 \sim 0$



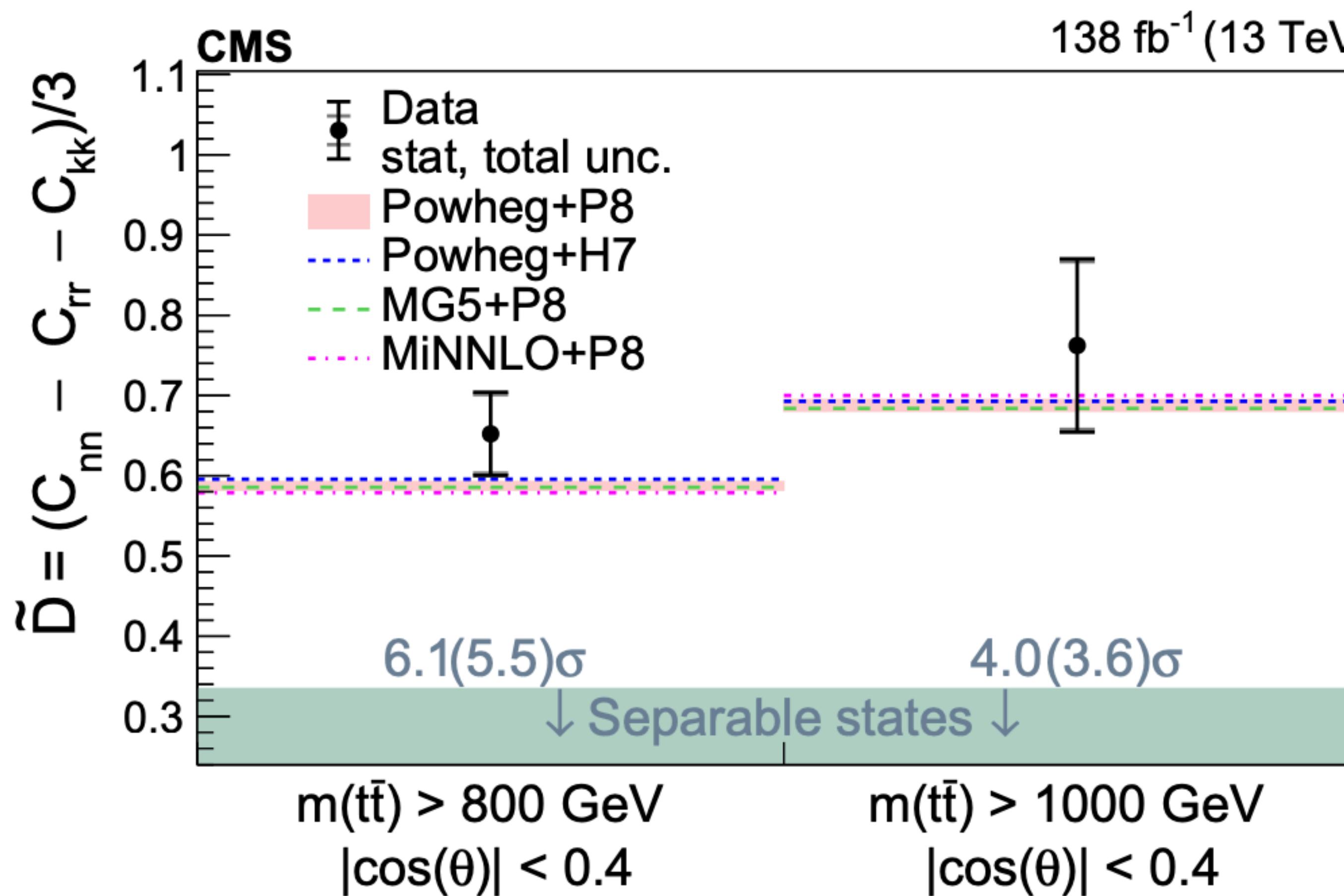
[Nature 633, 542-547 (2024)]

[Rep. Prog. Phys. 86 (2024) 117801]

$D < -1/3 \longleftrightarrow \text{entangled}$

# Also observed at high invariant mass by CMS

maximal at  $\beta^2 \rightarrow 1, \cos \theta = 0$

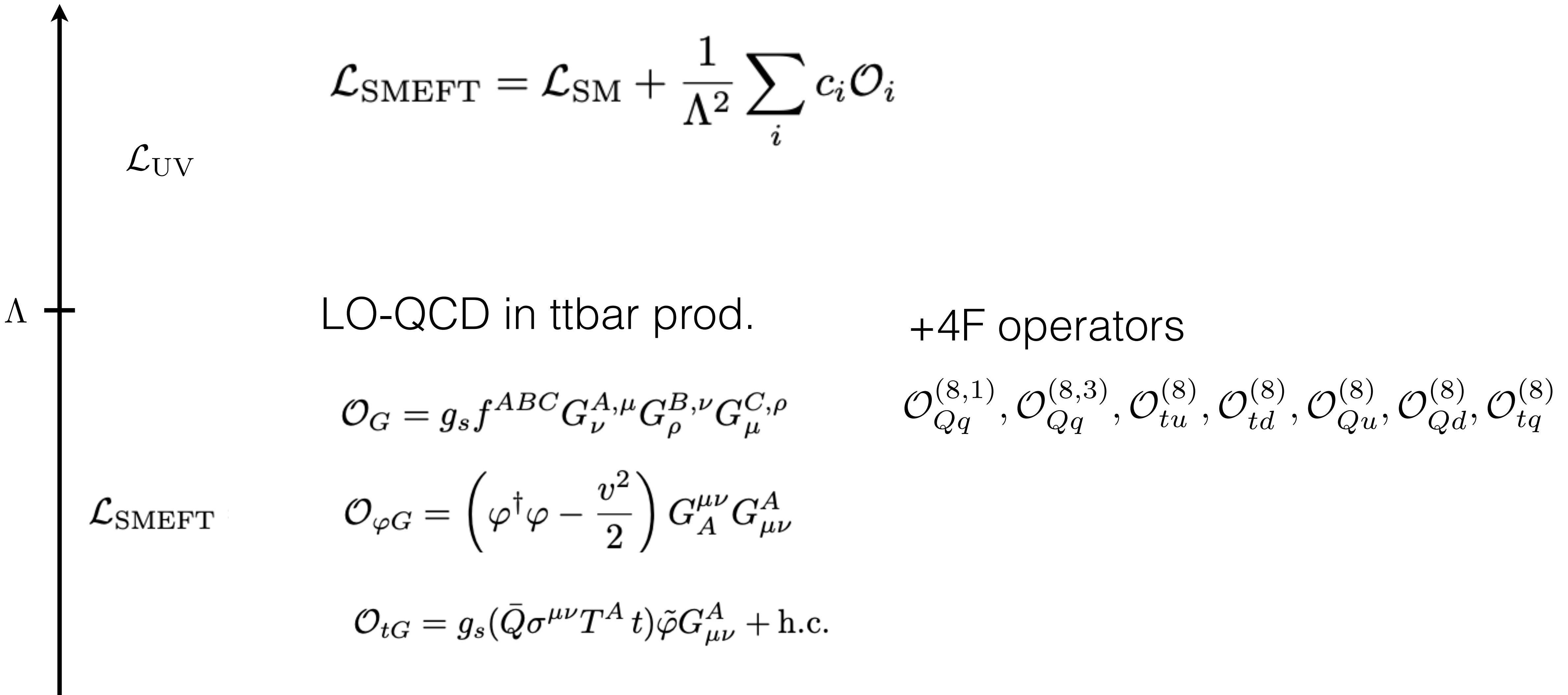


[Rep. Prog. Phys. 86 (2024) 117801]

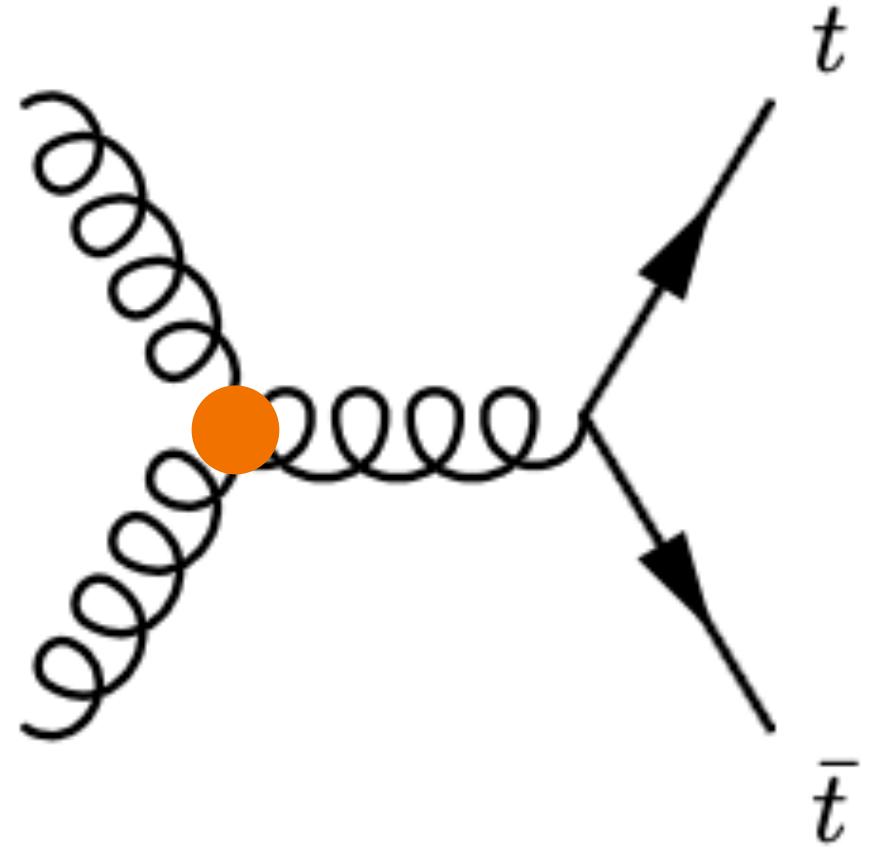
**SMEFT**

# SM Effective Field Theory

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 10']  
[Aguilar-Saavedra et. al, 18']  
[SMEFTatNLO: Degrande et. al, 20']



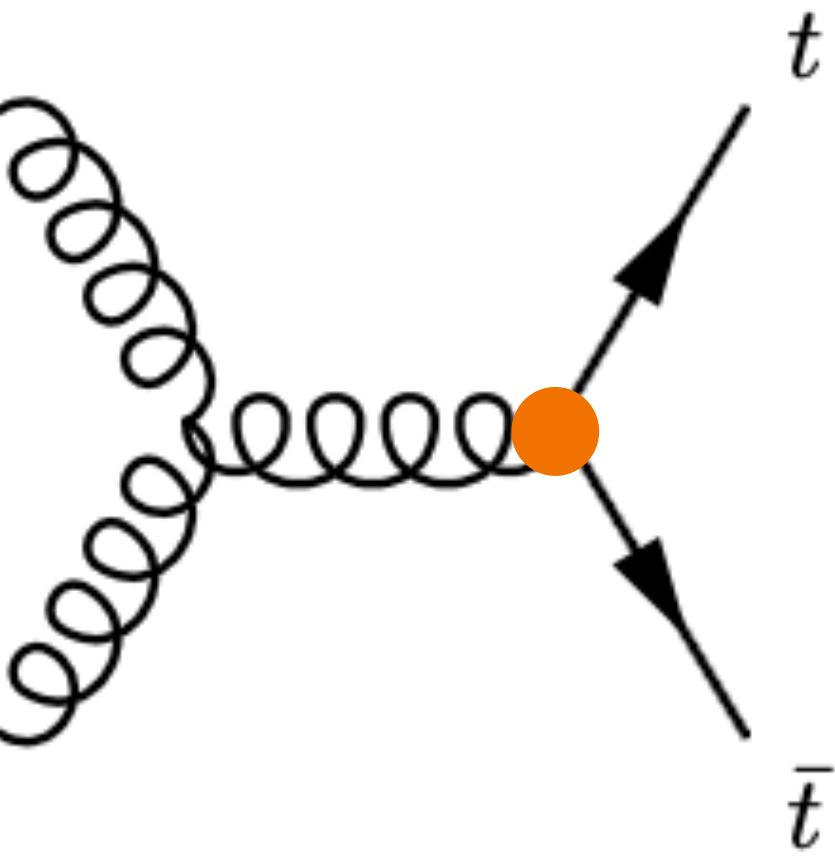
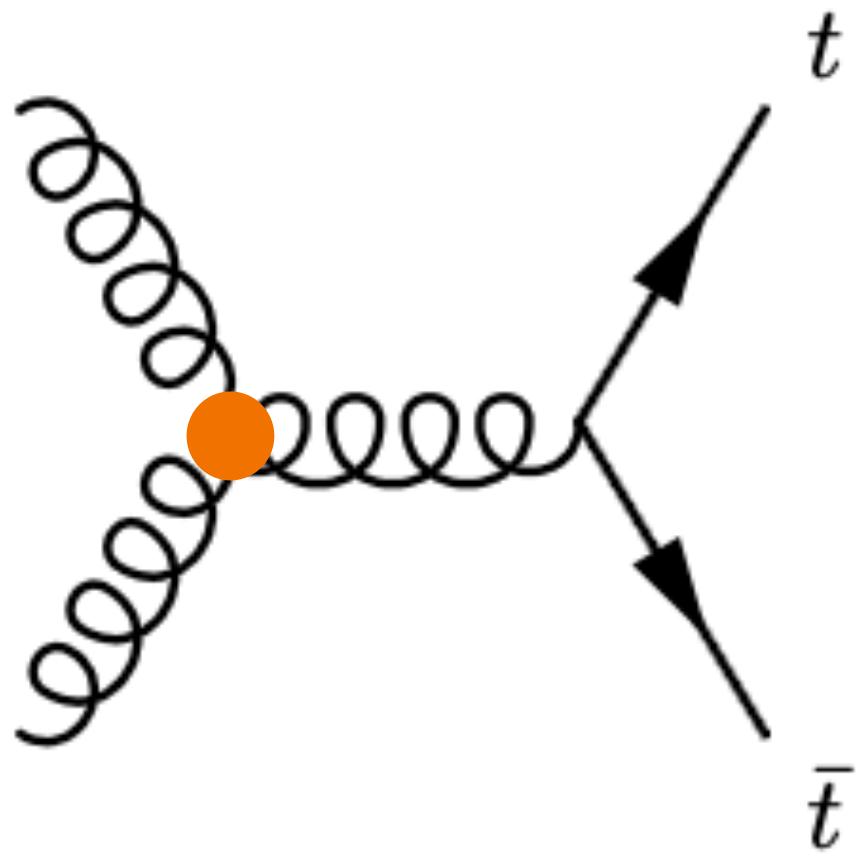
# Top pair within SMEFT



SM:  $-\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu}$

EFT:  $f^{ABC} G_{\nu}^{A,\mu} G_{\rho}^{B,\nu} G_{\mu}^{C,\rho}$

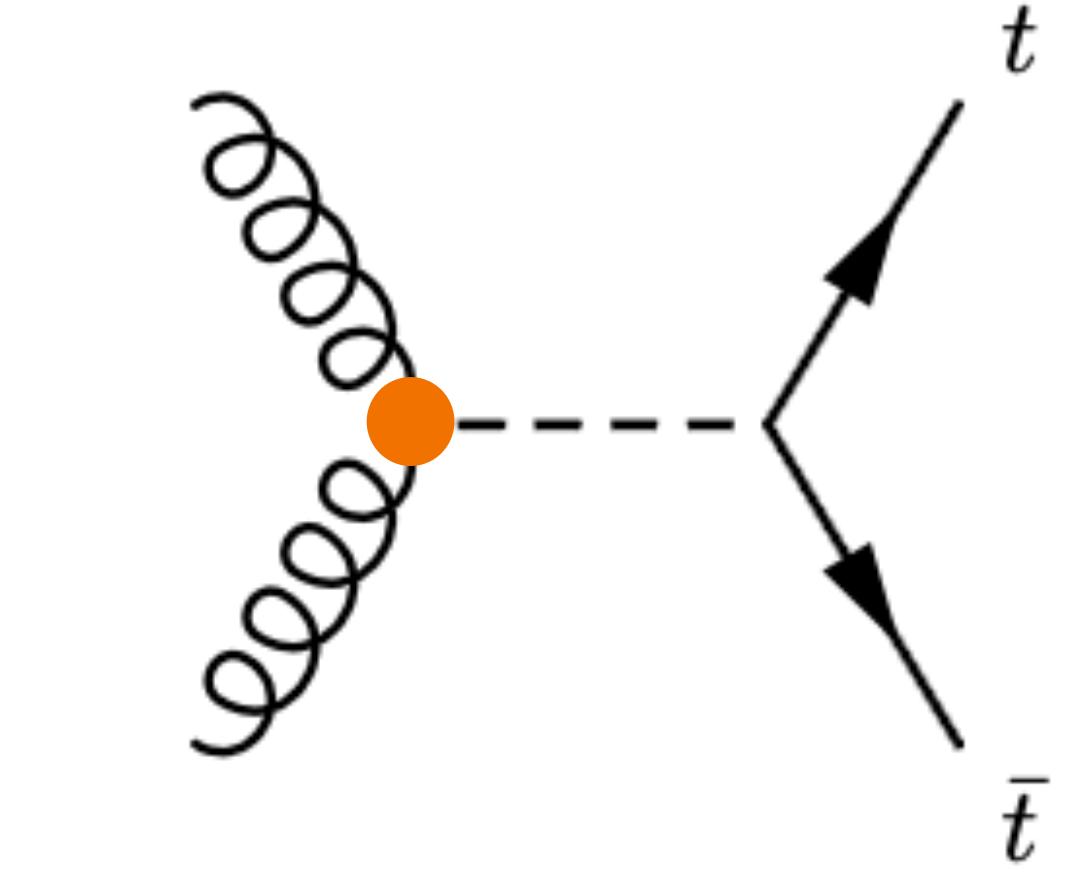
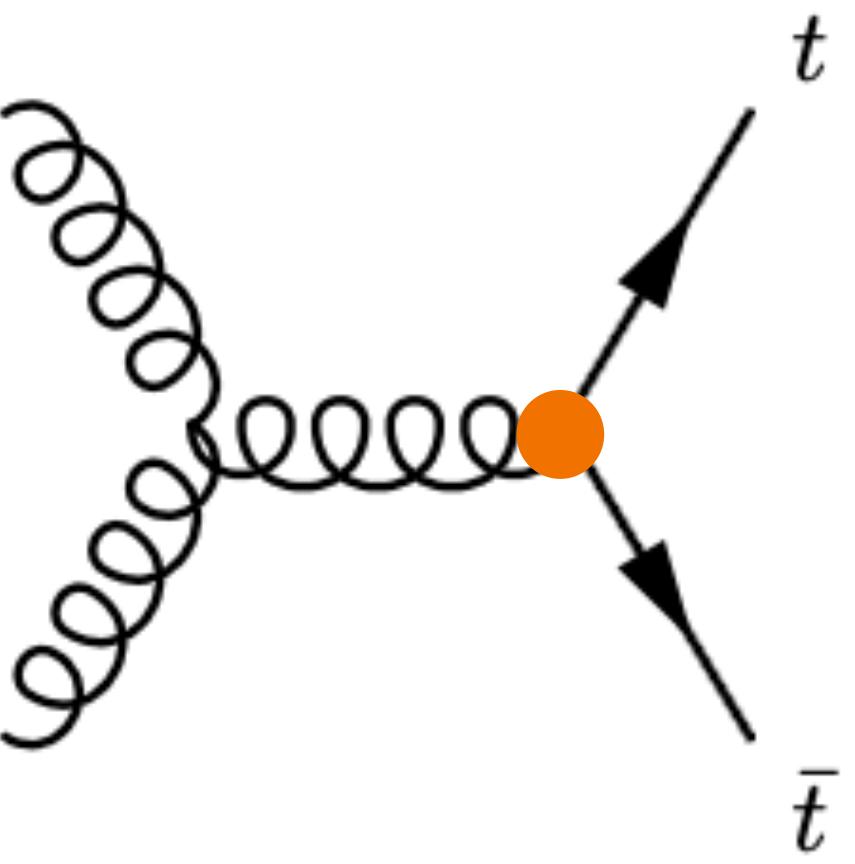
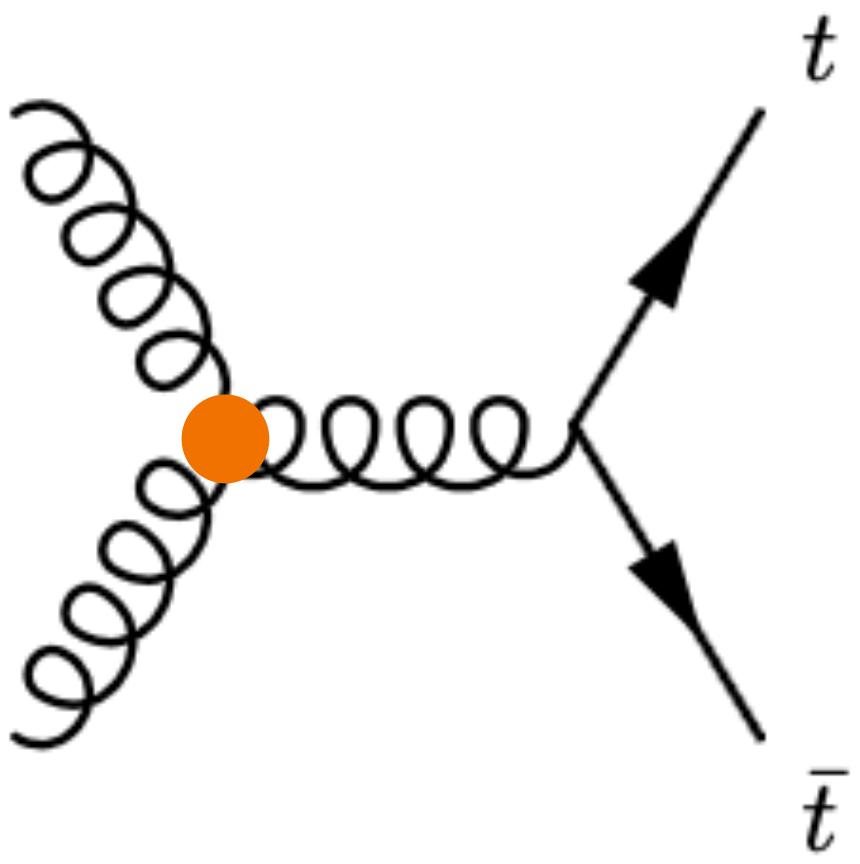
# Top pair within SMEFT



SM:  $\bar{t}\gamma^\mu t$

EFT:  $\bar{t}\sigma^{\mu\nu}t$

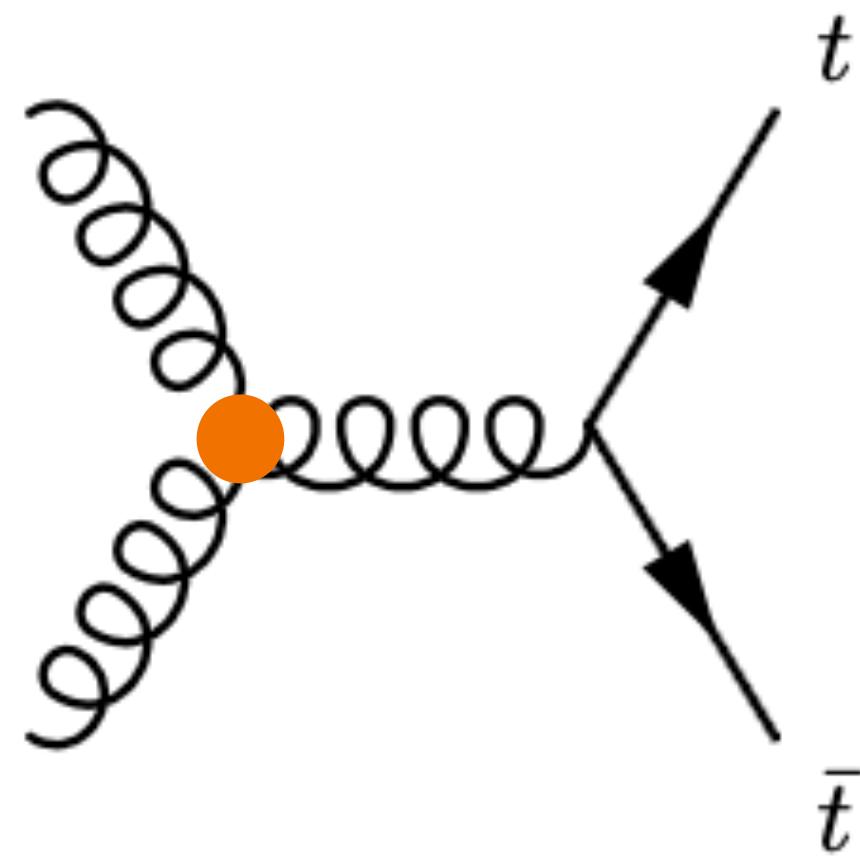
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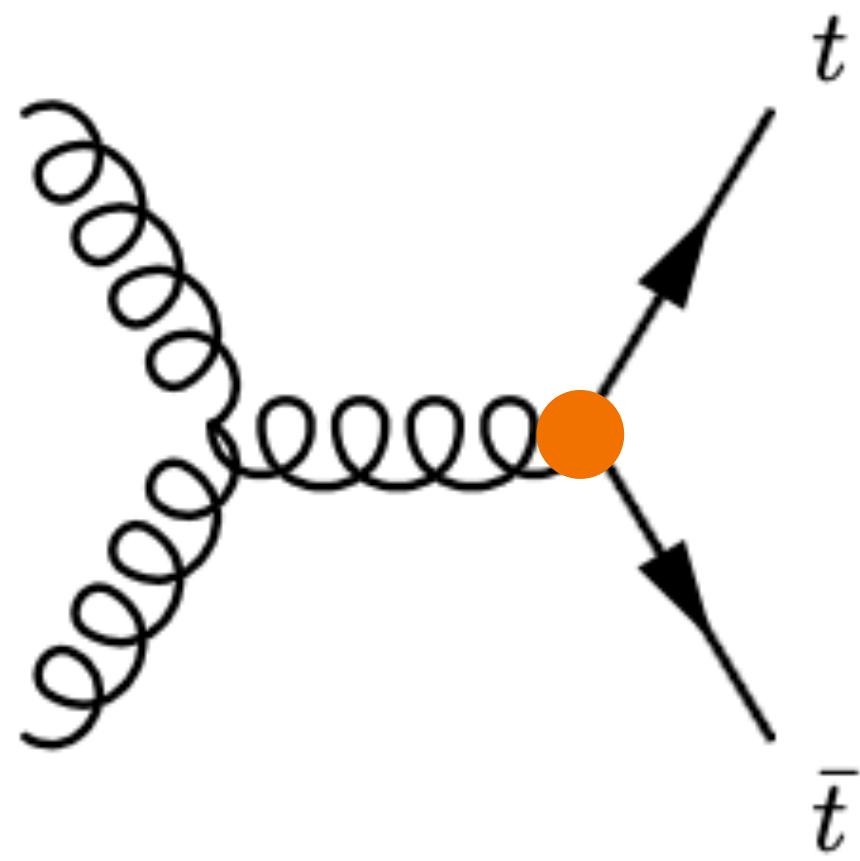
~~SM~~

EFT:  $\varphi G_A^{\mu\nu} G_{\mu\nu}^A$

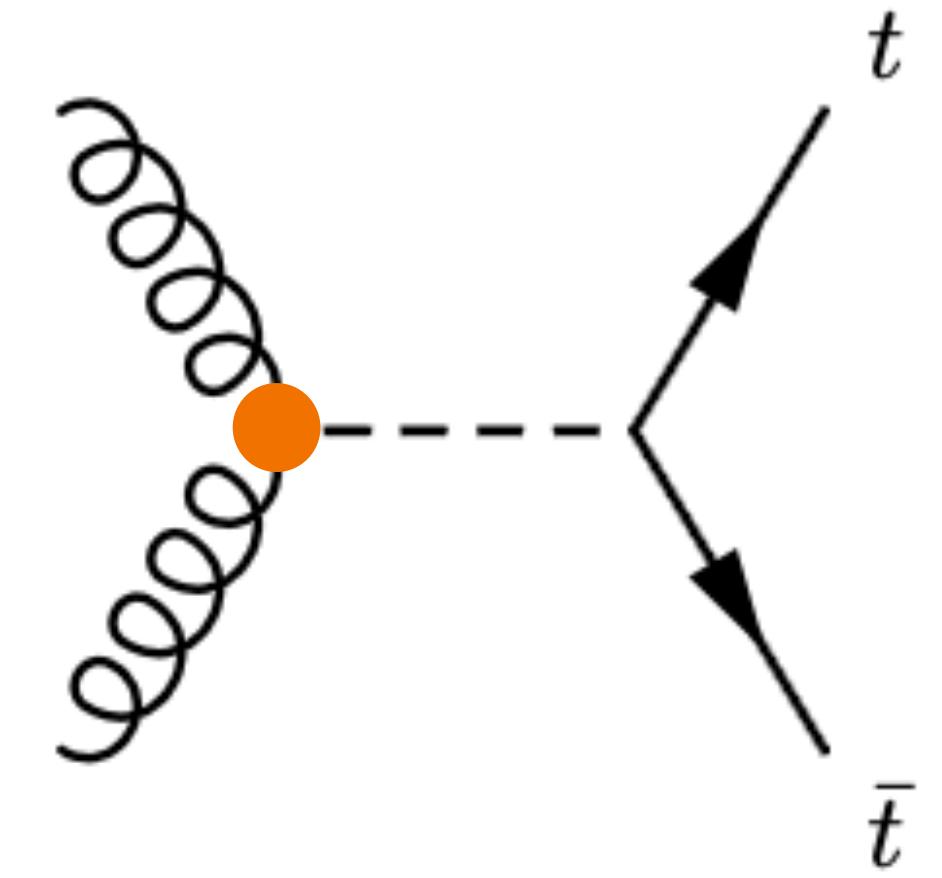
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$$\mathcal{O}_G = g_s f^{ABC} G_\nu^{A,\mu} G_\rho^{B,\nu} G_\mu^{C,\rho}$$

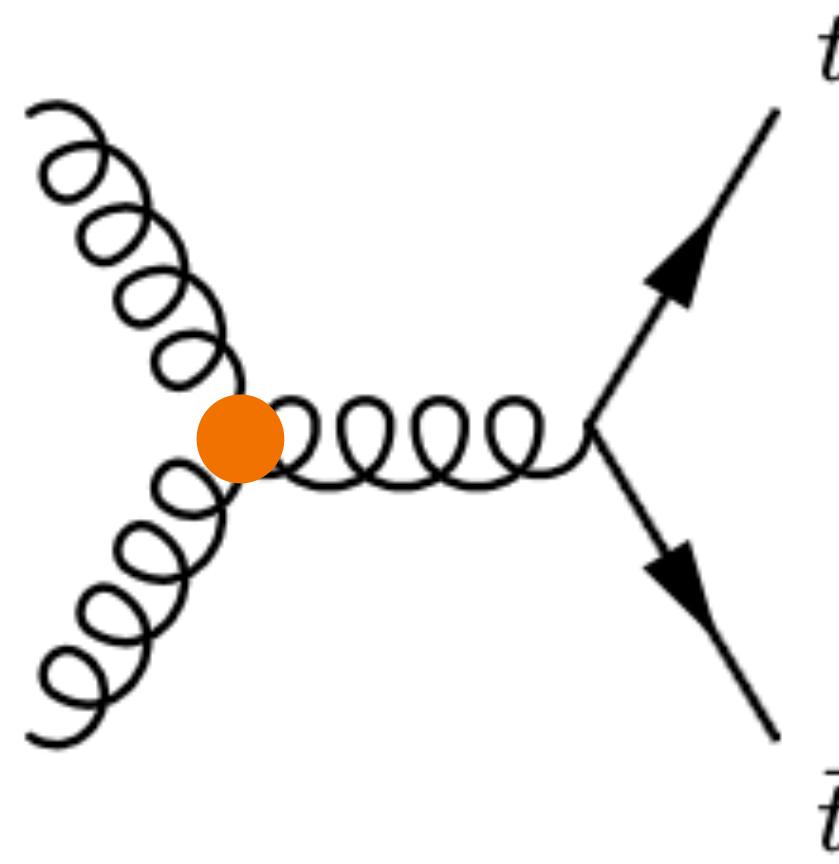


$$\mathcal{O}_{tG} = g_s (\bar{Q} \sigma^{\mu\nu} T^A t) \tilde{\varphi} G_{\mu\nu}^A + \text{h.c.}$$

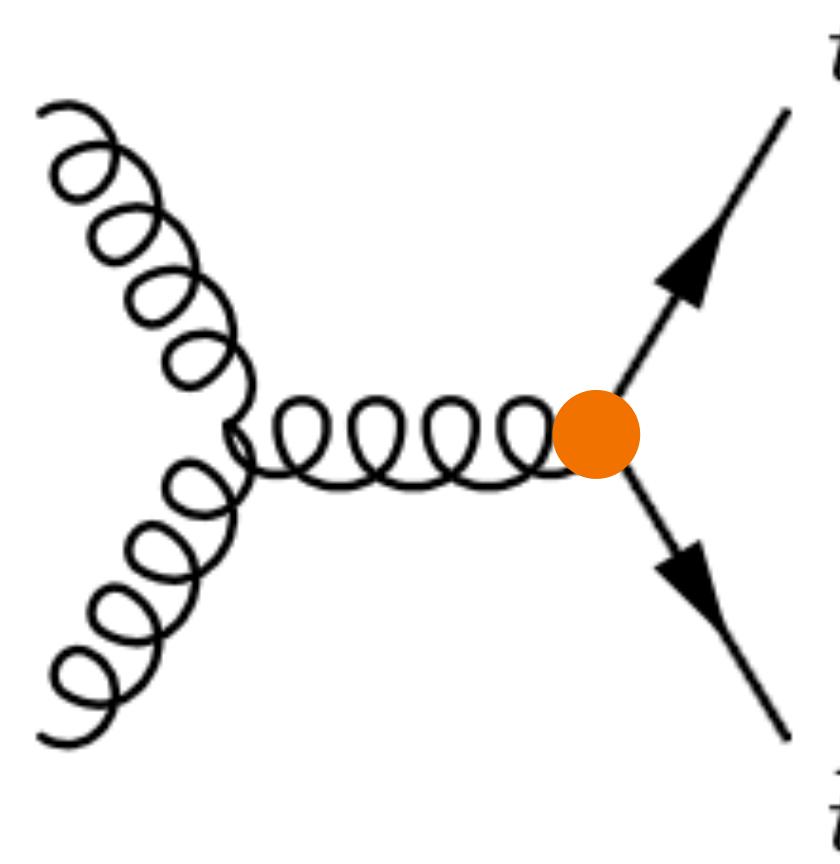


$$\mathcal{O}_{\varphi G} = \left( \varphi^\dagger \varphi - \frac{v^2}{2} \right) G_A^{\mu\nu} G_{\mu\nu}^A$$

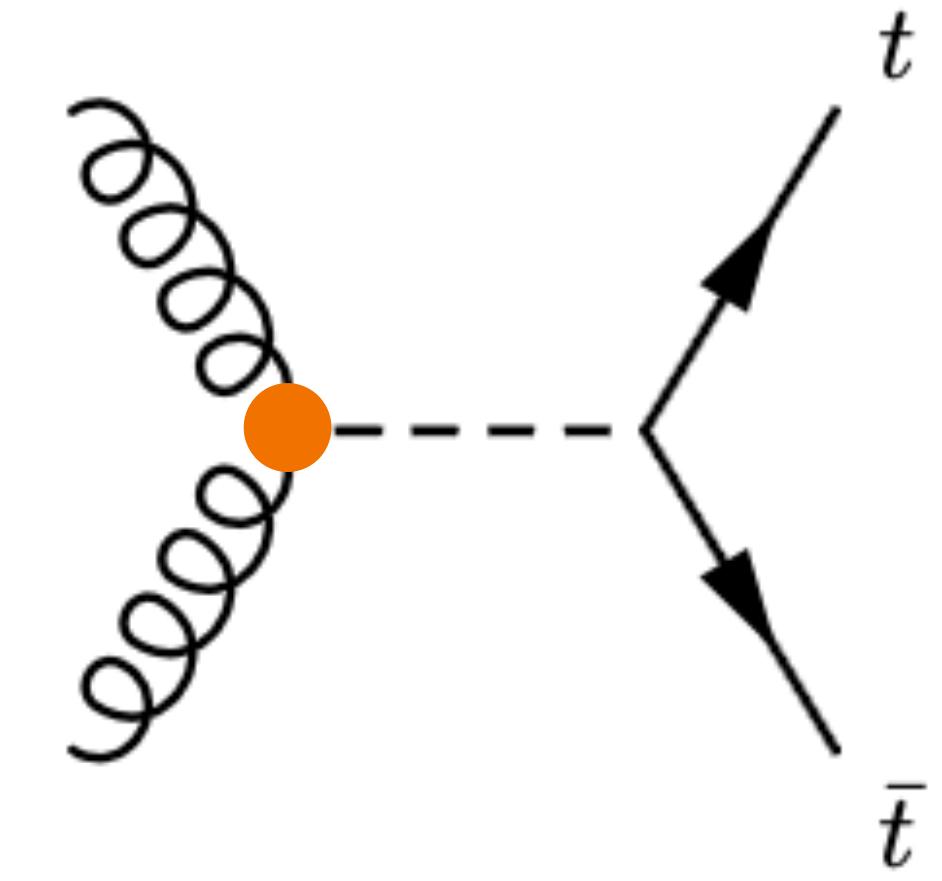
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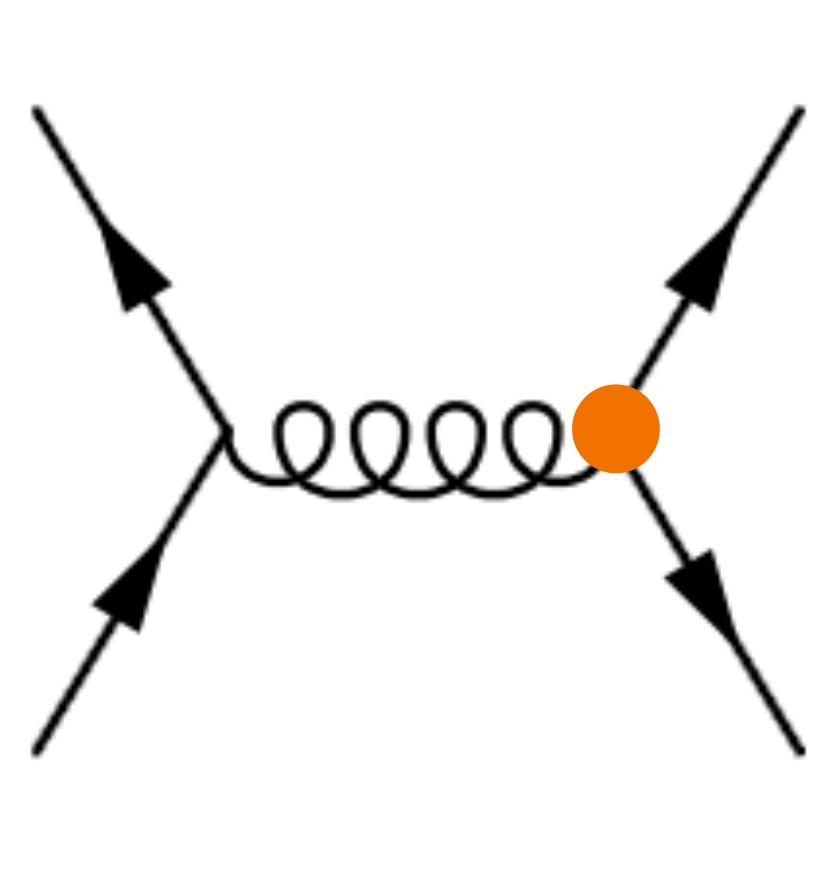
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$$\mathcal{O}_{tG} = g_s (\bar{Q} \sigma^{\mu\nu} T^A t) \tilde{\varphi} G_{\mu\nu}^A + \text{h.c.}$$

Four-fermion op.

# SMEFT averaged concurrence

[Aoude, Madge,  
Maltoni, Mantani, 22']

Average over the solid angle

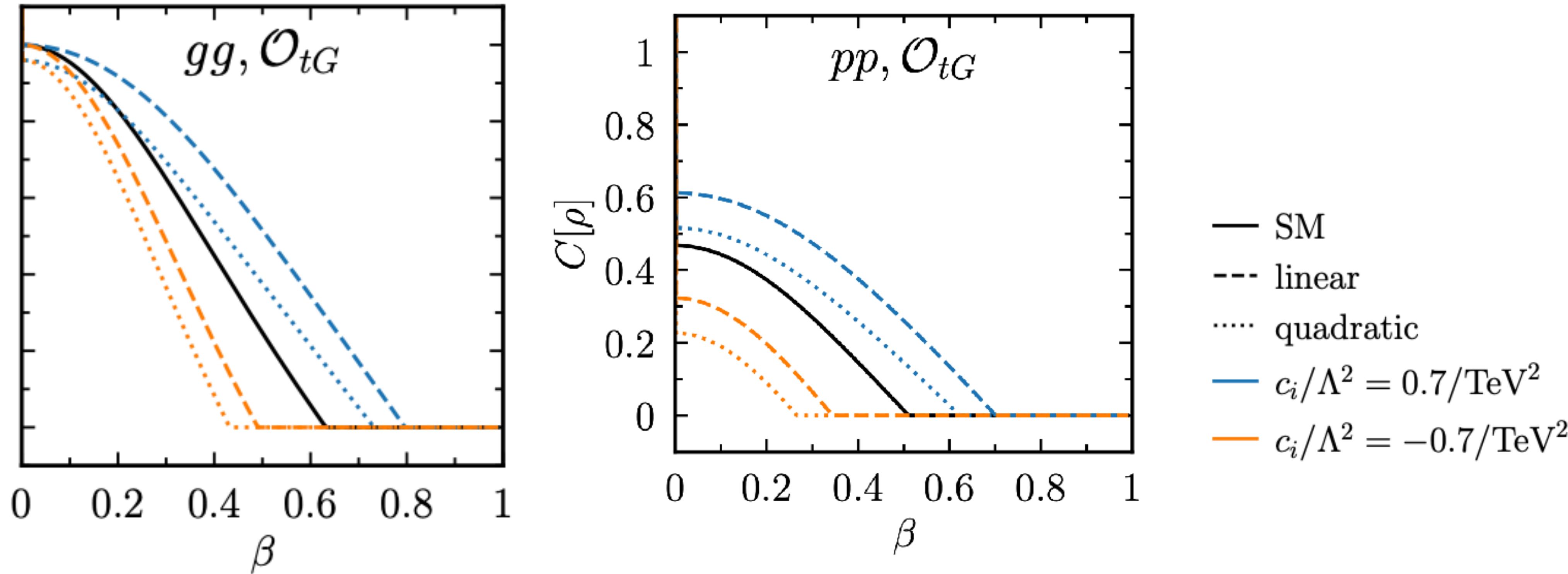
$$\bar{R} = (4\pi)^{-1} \int d\Omega R(\hat{s}, \mathbf{k}) \longrightarrow \text{Only function of energy (top velocity) } \beta$$

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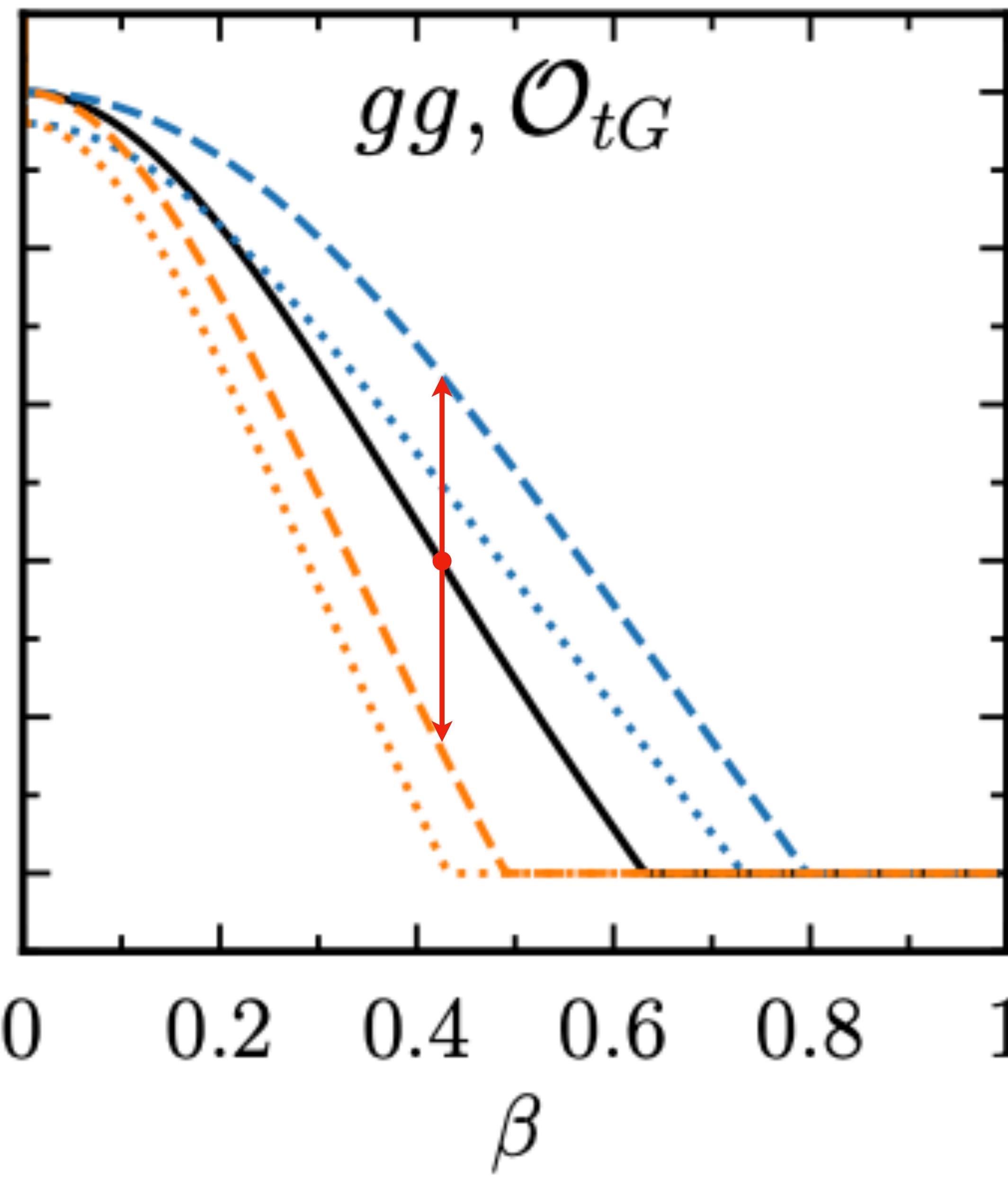
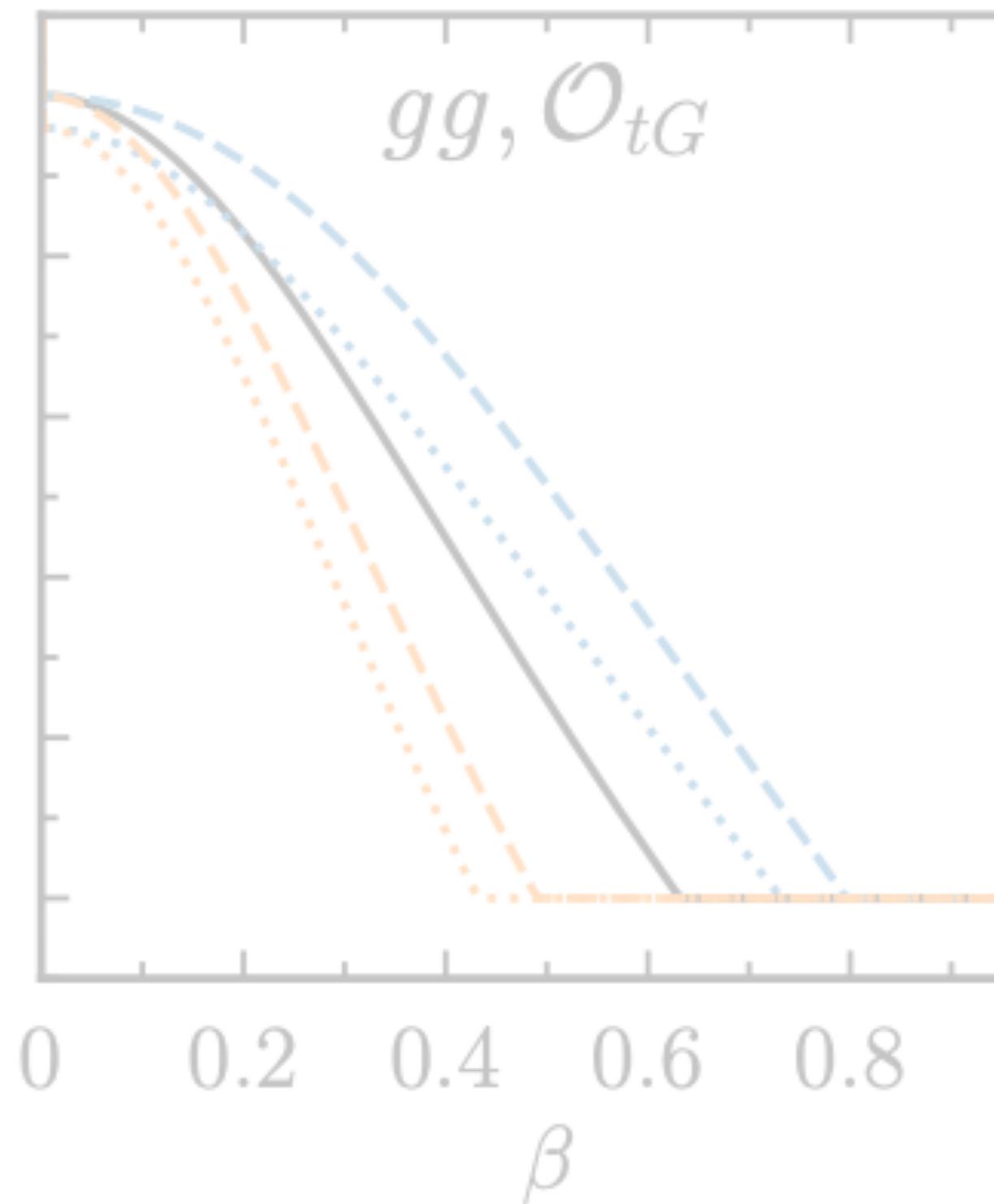
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Average over the so

$$\bar{R} = (4\pi)^{-1} \int$$



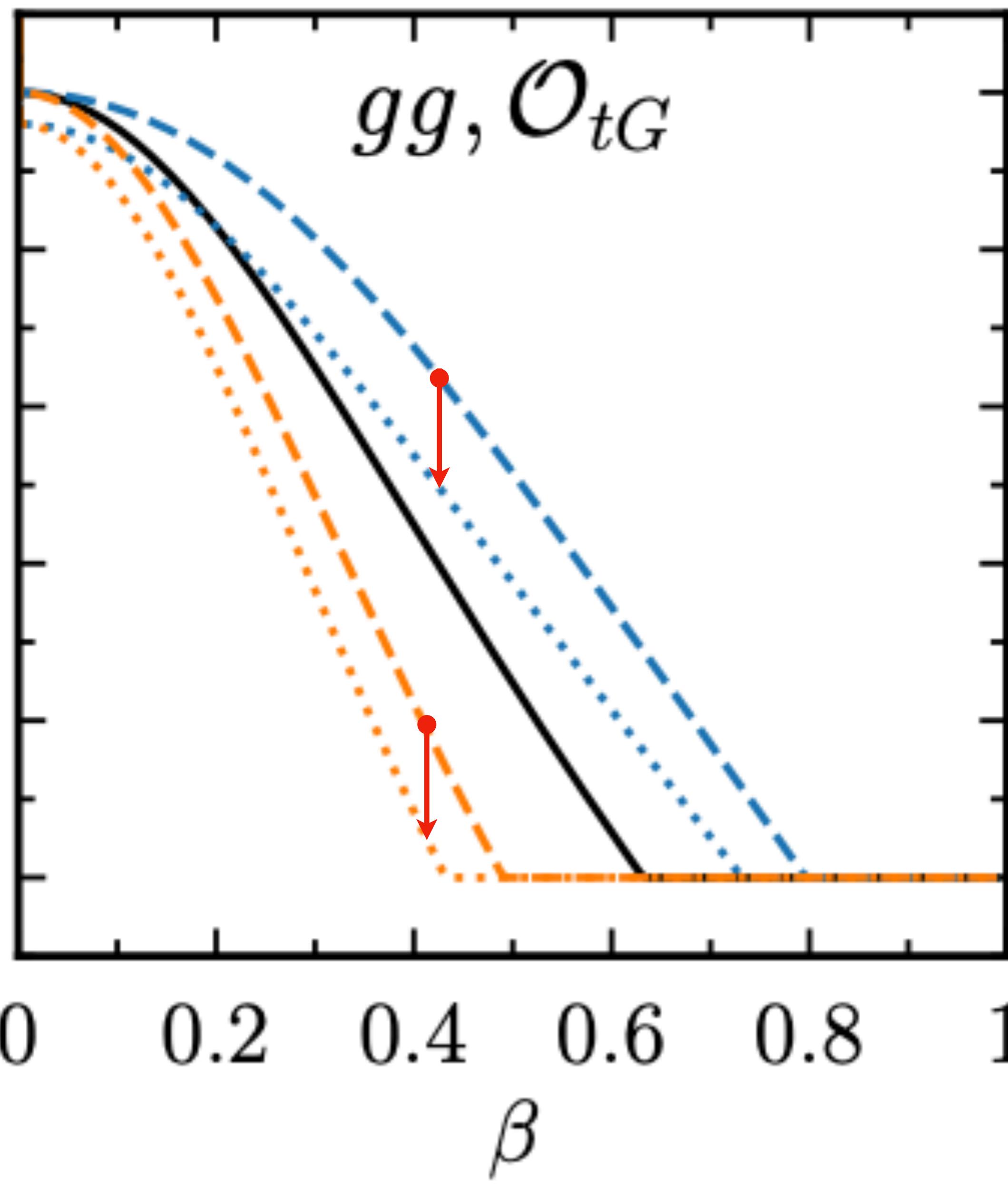
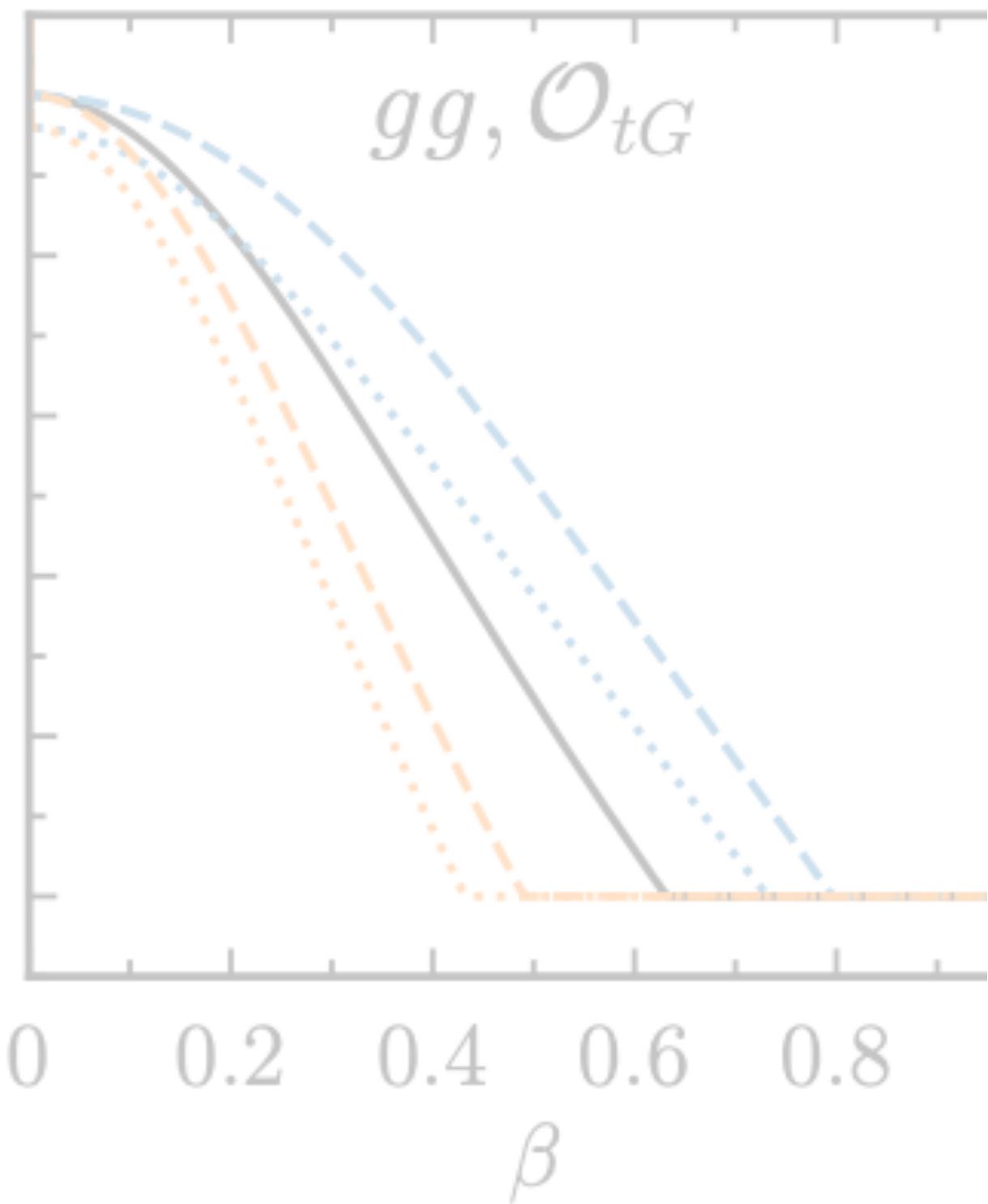
linear depends  
on the sign

- SM
- - - linear
- .... quadratic
- $c_i/\Lambda^2 = 0.7/\text{TeV}^2$
- $c_i/\Lambda^2 = -0.7/\text{TeV}^2$

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Average over the so

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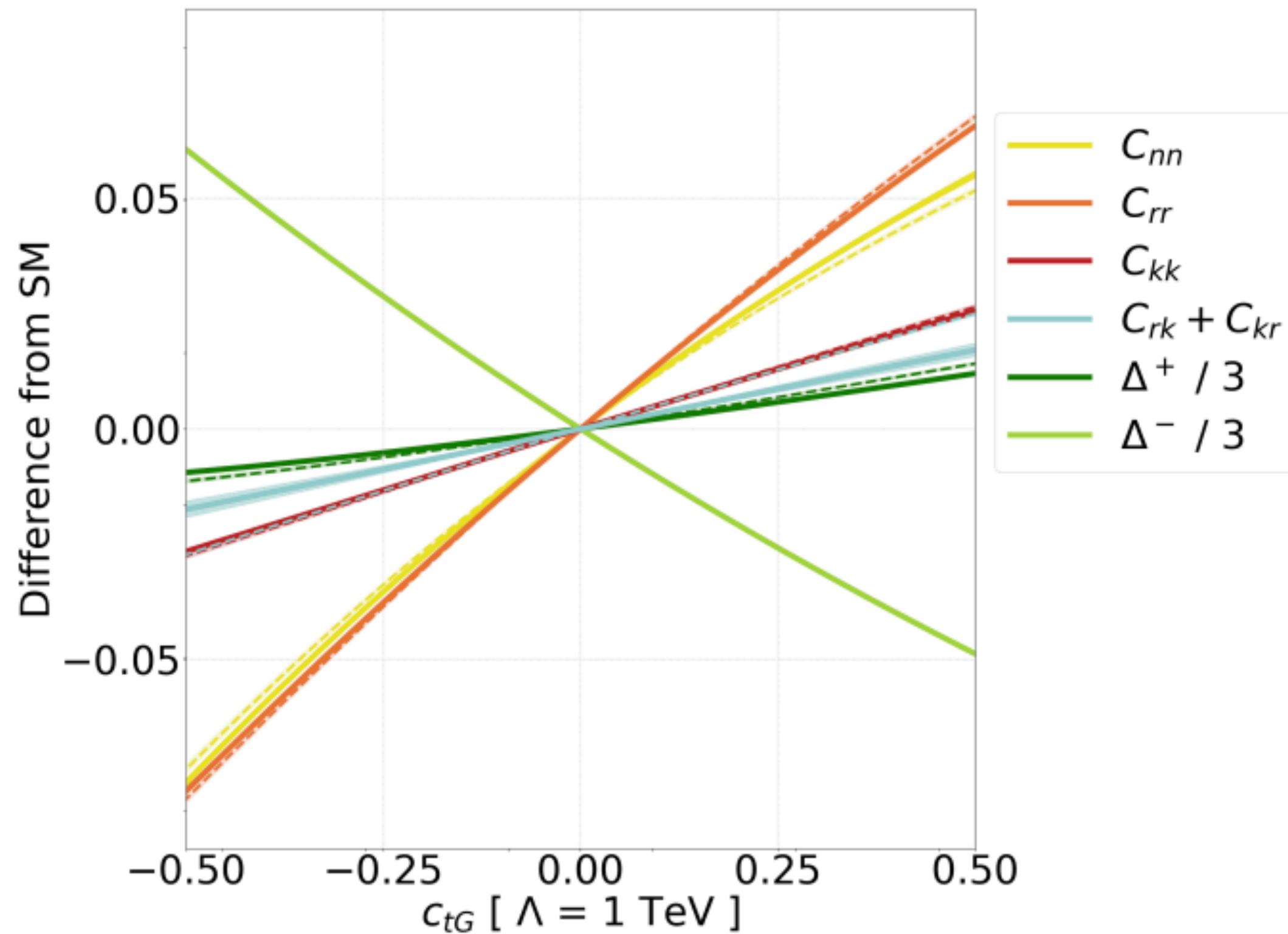
quad  
lowers the ent.

- SM
- - - linear
- .... quadratic
- $c_i/\Lambda^2 = 0.7/\text{TeV}^2$
- $c_i/\Lambda^2 = -0.7/\text{TeV}^2$

# NLO QCD SMEFT and resonant new physics

[Severi, Vryonidou 22']

$O_{tG}$



(a)

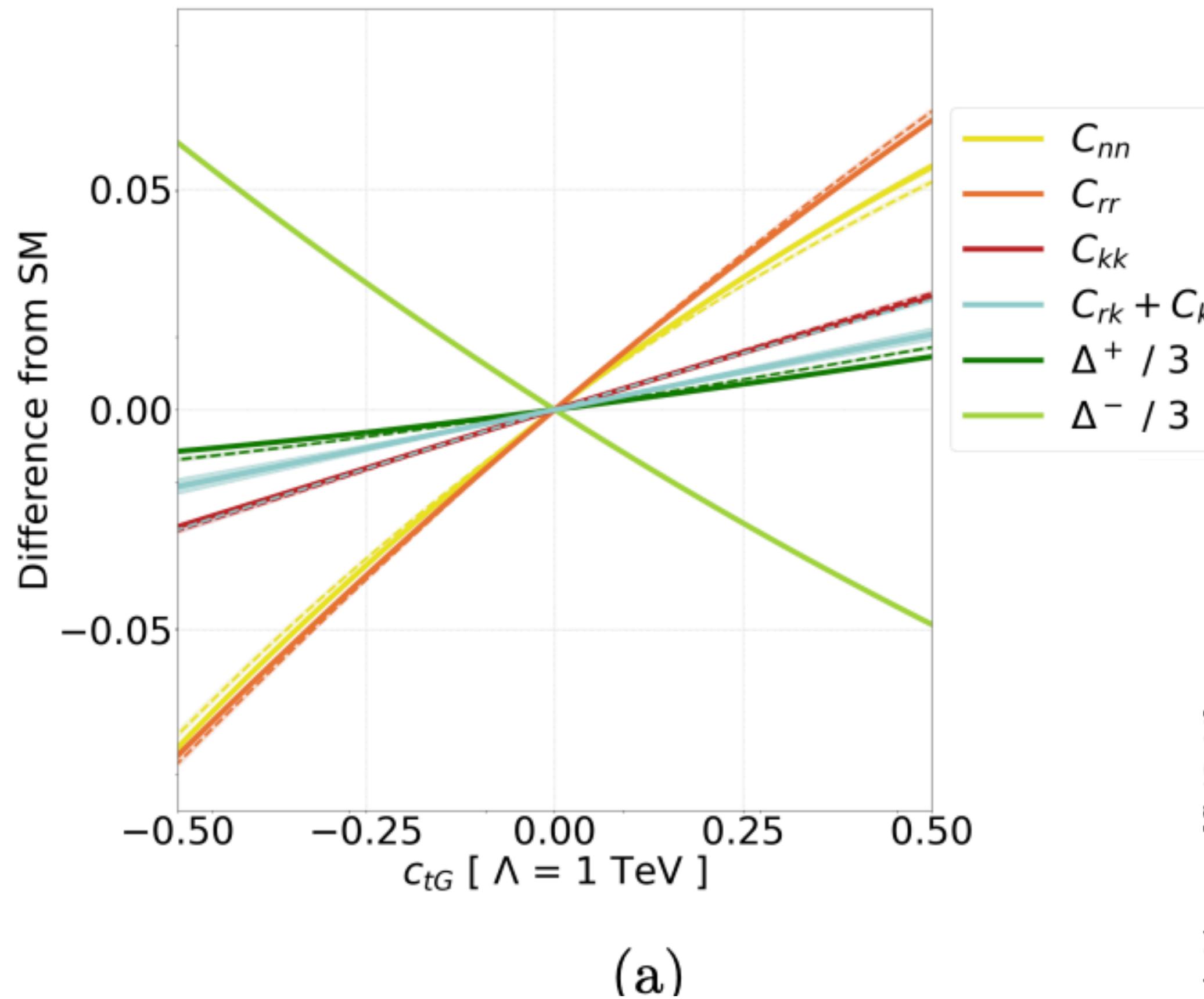
$$D \equiv (\Delta^- + 1)/3.$$

LO (dashed)  
NLO (solid)

# NLO QCD SMEFT and resonant new physics

[Severi, Vryonidou 22']

$O_{tG}$

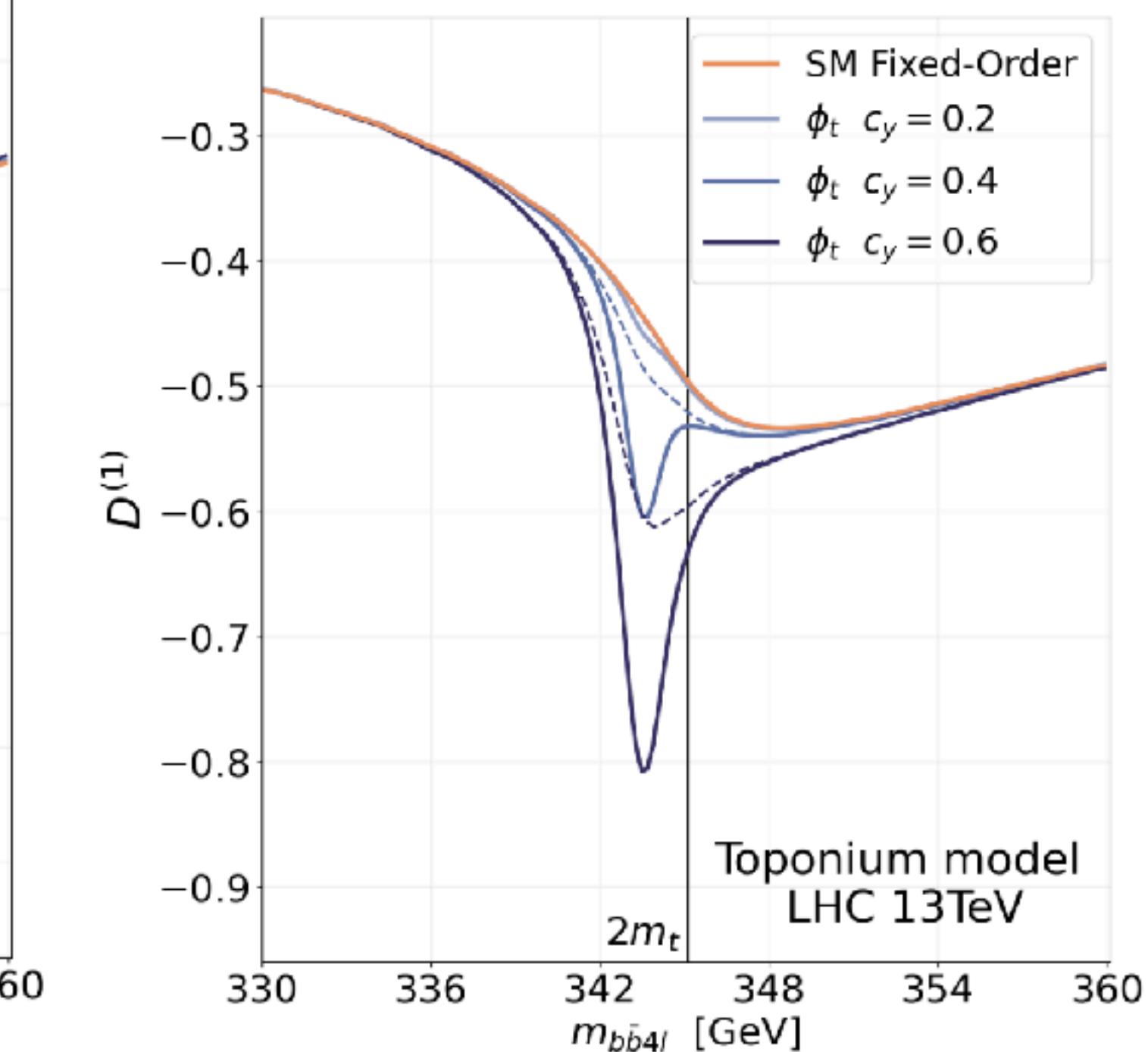
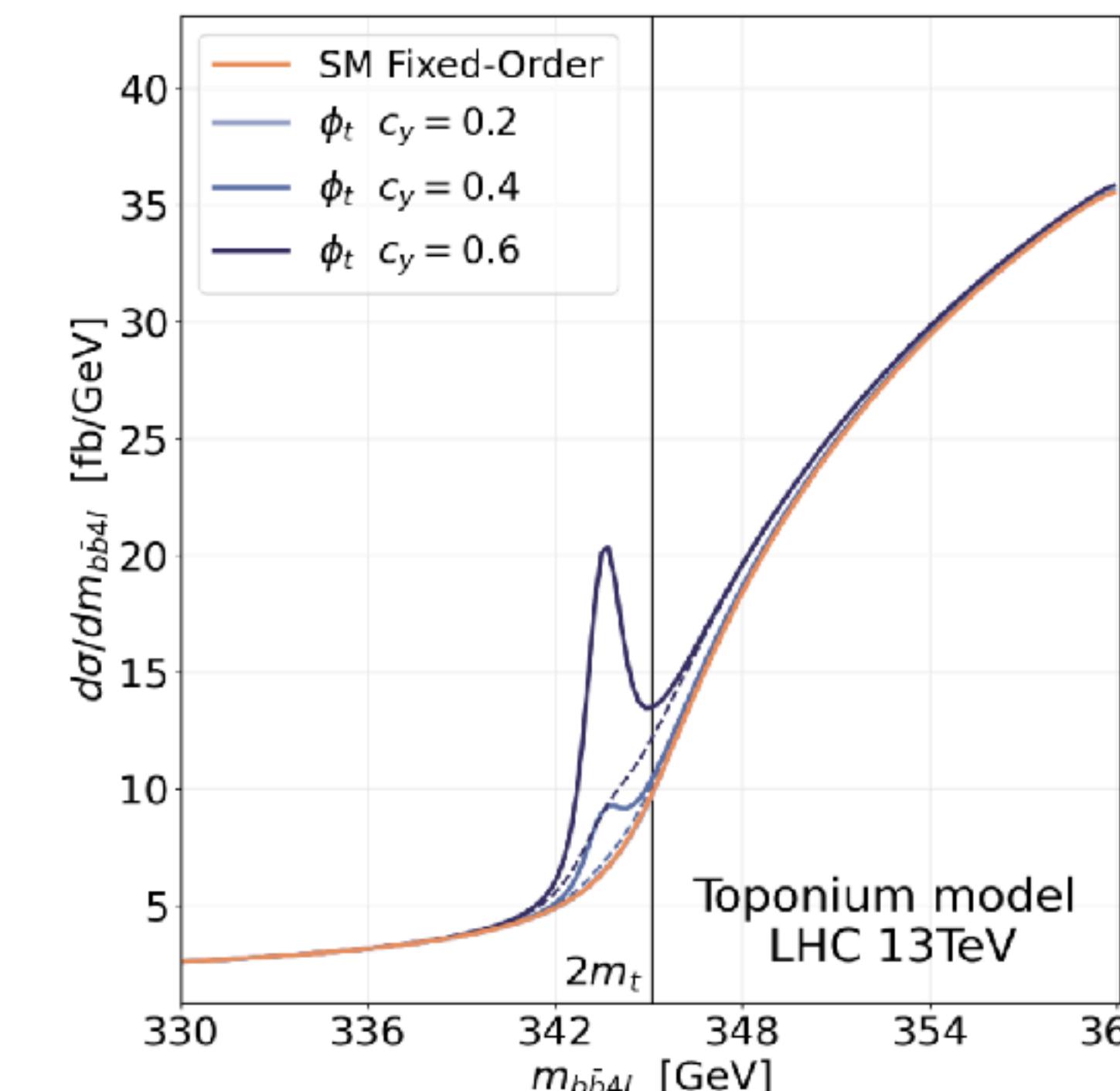
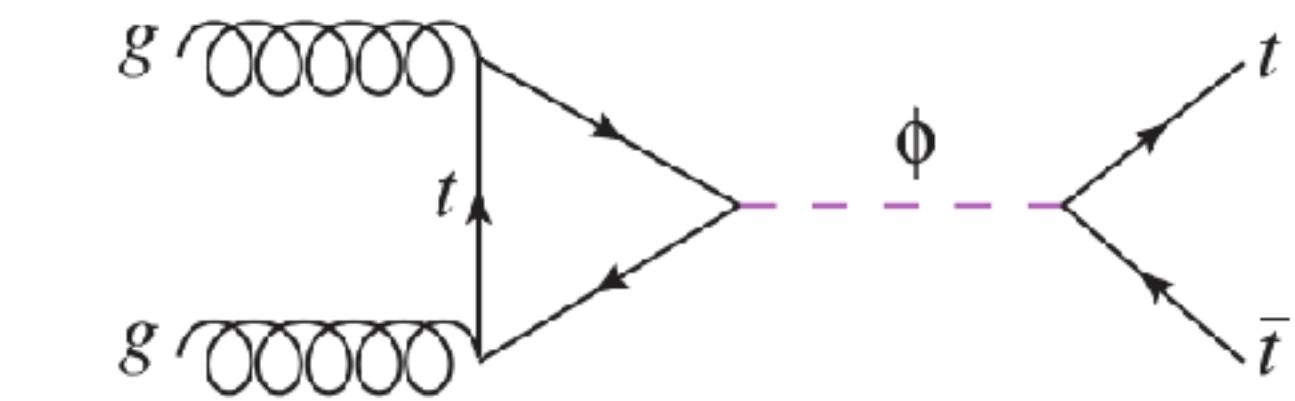


$$D \equiv (\Delta^- + 1)/3.$$

LO (dashed)  
NLO (solid)

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{2} \phi (\partial^2 + M_\phi^2) \phi + c_y \frac{y_t}{\sqrt{2}} \phi \bar{t} (\cos \alpha + i \gamma^5 \sin \alpha) t.$$

[Maltoni, Severi, Tentori, Vryonidou 24']



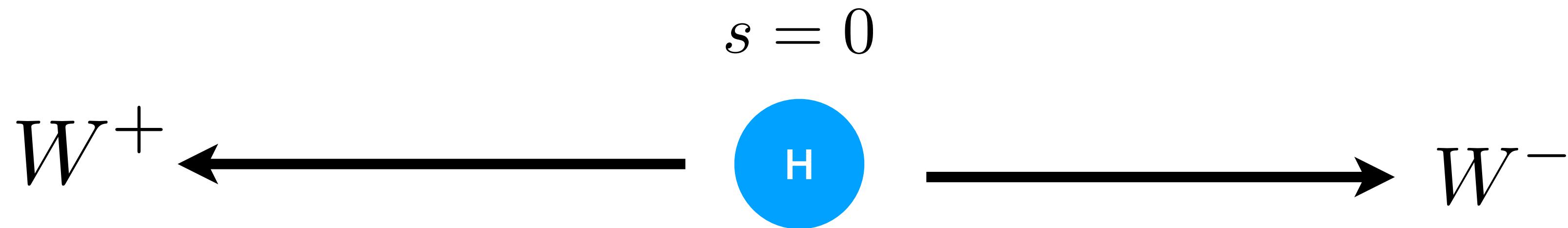
# Many many questions

- Threshold bound state effects (topponium)
- Better observables ?
- Can we test beyond QM ?
- Can we test Bell inequalities ?
- What happens at higher-orders? NNLO QCD? EW?
- Actually, Bell inequalities were proposed a while back  
[Abel, Dittmar, Dreiner '92]  
[Dittmar, Dreiner '96]
- and many more...

**but...**

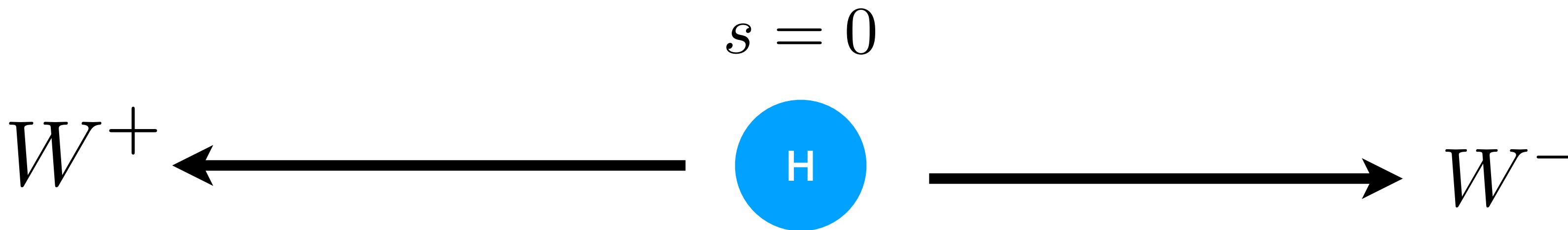
# We are in the Higgs 2024 conference.

Entanglement in Higgs decay?



# We are in the Higgs 2024 conference.

Entanglement in Higgs decay?

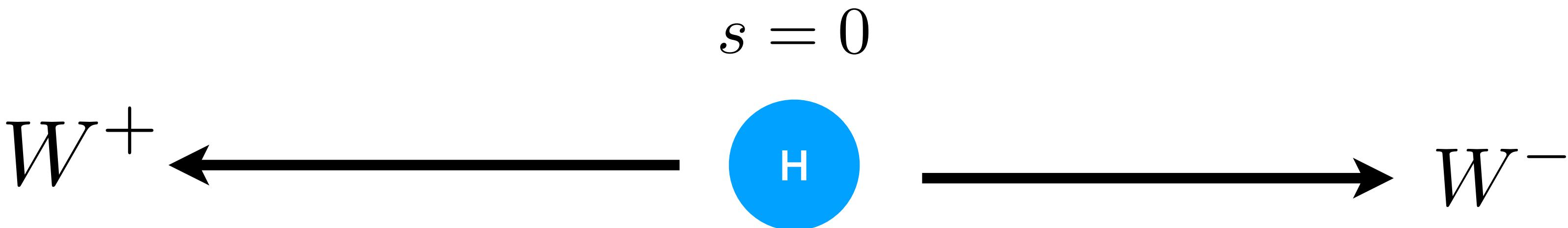


$W$  acts as its own polarimeter: It decays along the axis of the emitted lepton

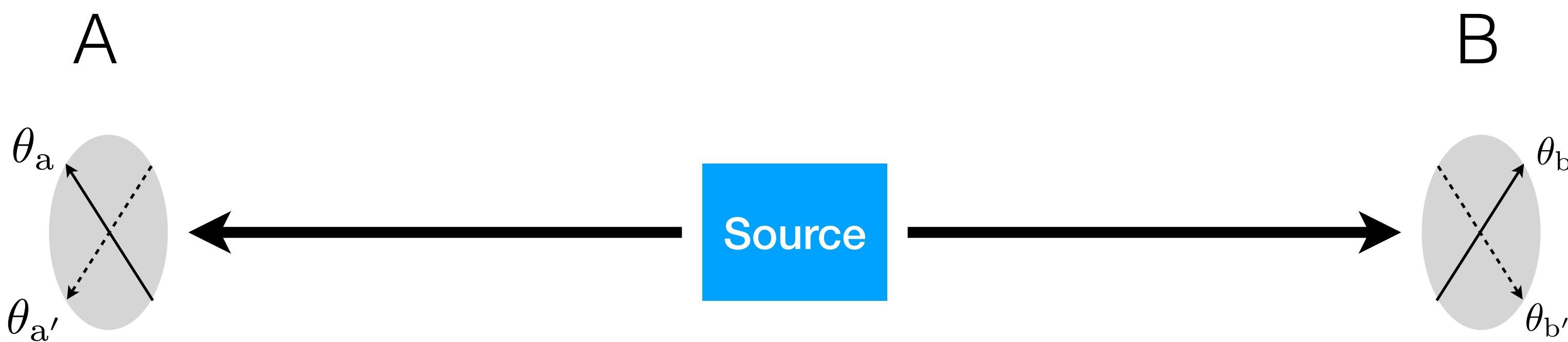


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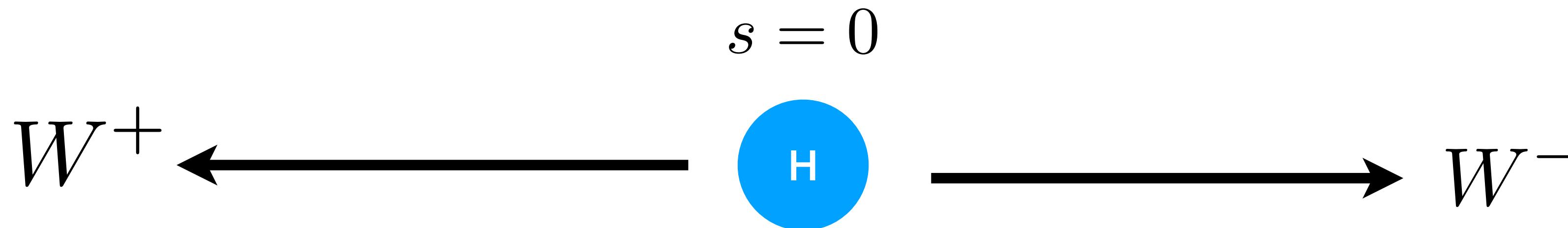


Bell-type experiment!

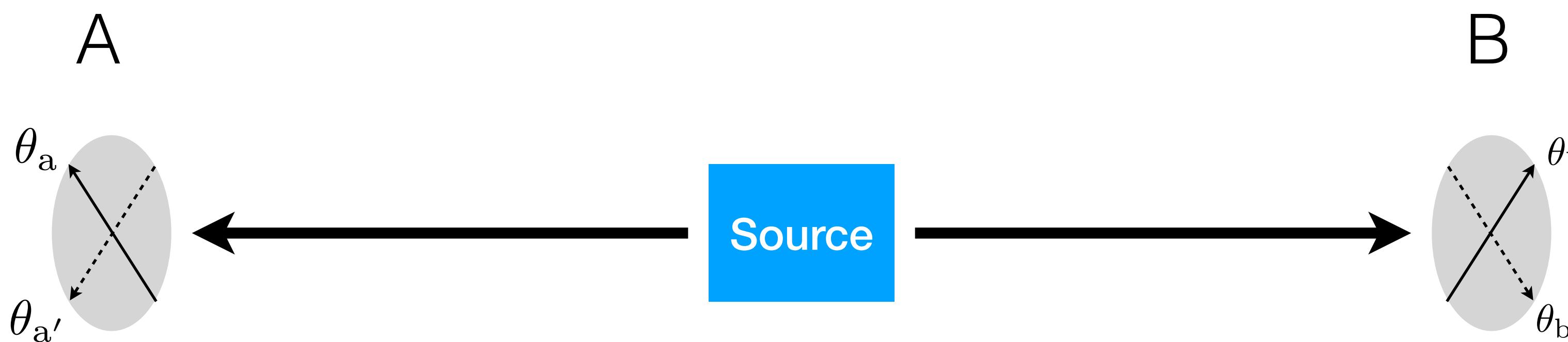


# We are in the Higgs 2024 conference.

Entanglement in Higgs decay?



Bell-type experiment!



But the  $W$  has 3 polarisations. It is not a qubit anymore: **Qutrit!**

# Qutrits

[Ashby-Pickering, Barr, Wierzchuca, '22]

Massive spin-1 boson (W,Z):  $|V\rangle = c_+|V_+\rangle + c_0|V_L\rangle + c_-|V_-\rangle = \begin{pmatrix} c_+ \\ c_0 \\ c_- \end{pmatrix}$

Single Qutrit:  
(3x3)  $\rho = \frac{1}{3}\mathbb{I} + \sum_{i=1}^8 a_i \lambda_i$   $a_i$  : 8 real parameters

Two Qutrits  
(9x9)  $\rho = \frac{1}{9}\mathbb{I} \otimes \mathbb{I} + \frac{1}{3} \sum_{i=1}^8 a_i \lambda_i \otimes \mathbb{I} + \frac{1}{3} \sum_{j=1}^8 b_j \mathbb{I} \otimes \lambda_j + \sum_{i=1}^8 \sum_{j=1}^8 c_{ij} \lambda_i \otimes \lambda_j$

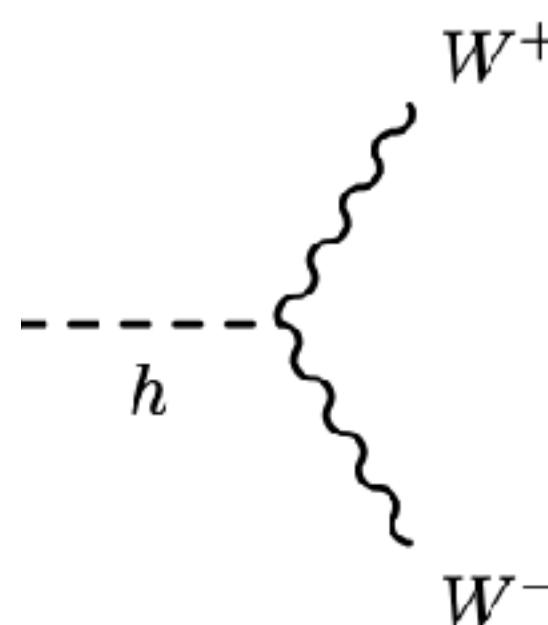
$a_i, b_j, c_{ij}$  8+8+64 real parameters

$\lambda_i$  are Gell-Mann matrices

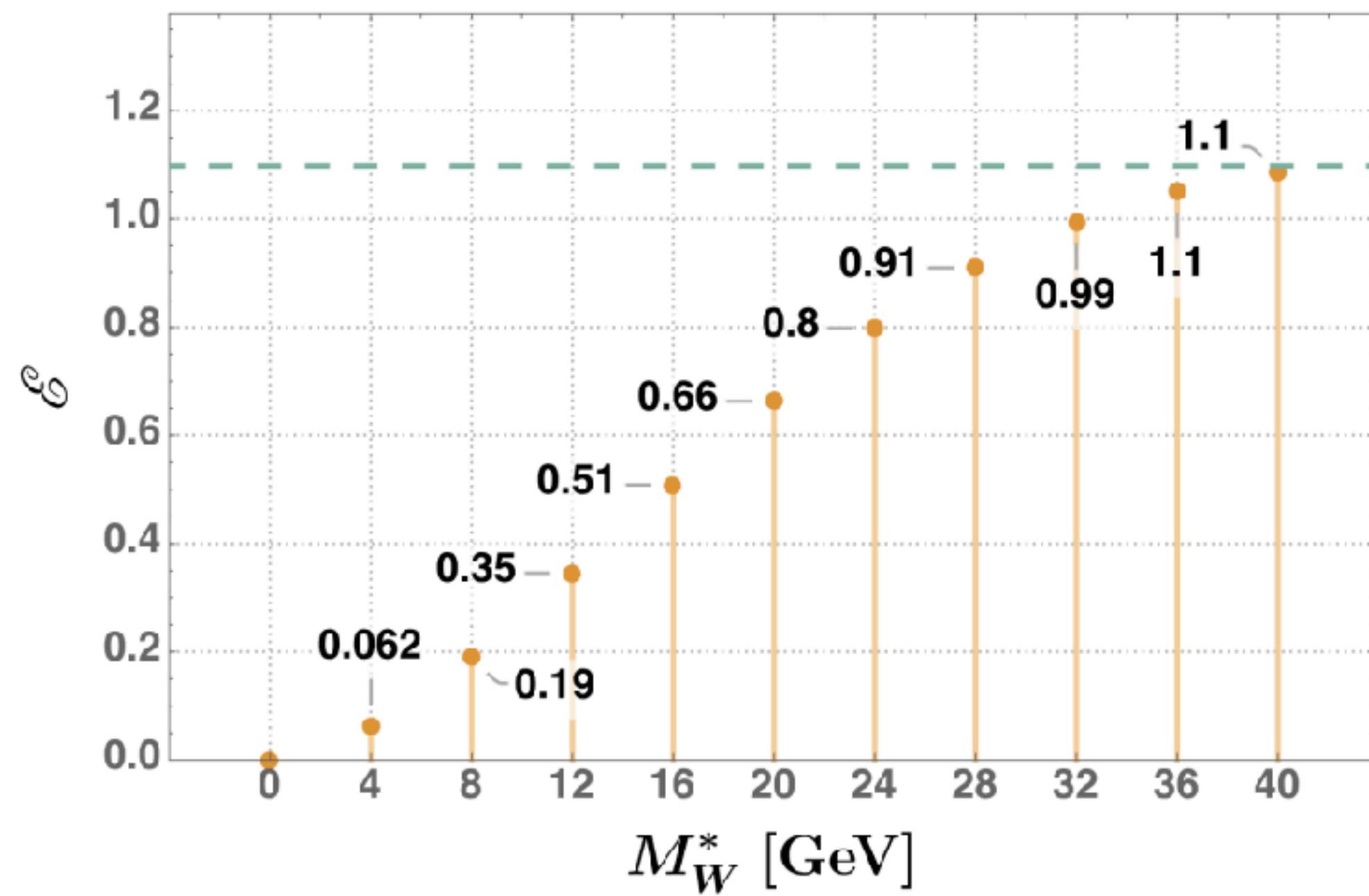
# Entanglement in Higgs decays

[Fabbrichesi, Floreanini, Gabrielli, Mazola '23]

At tree level:



$$\rho_{h \rightarrow WW} = 2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{44} & 0 & \rho_{16} & 0 & \rho_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{16} & 0 & 2\rho_{33} & 0 & \rho_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{44} & 0 & \rho_{16} & 0 & \rho_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



Pure state!

$$\rho^2 = \rho = |\Psi\rangle\langle\Psi|$$

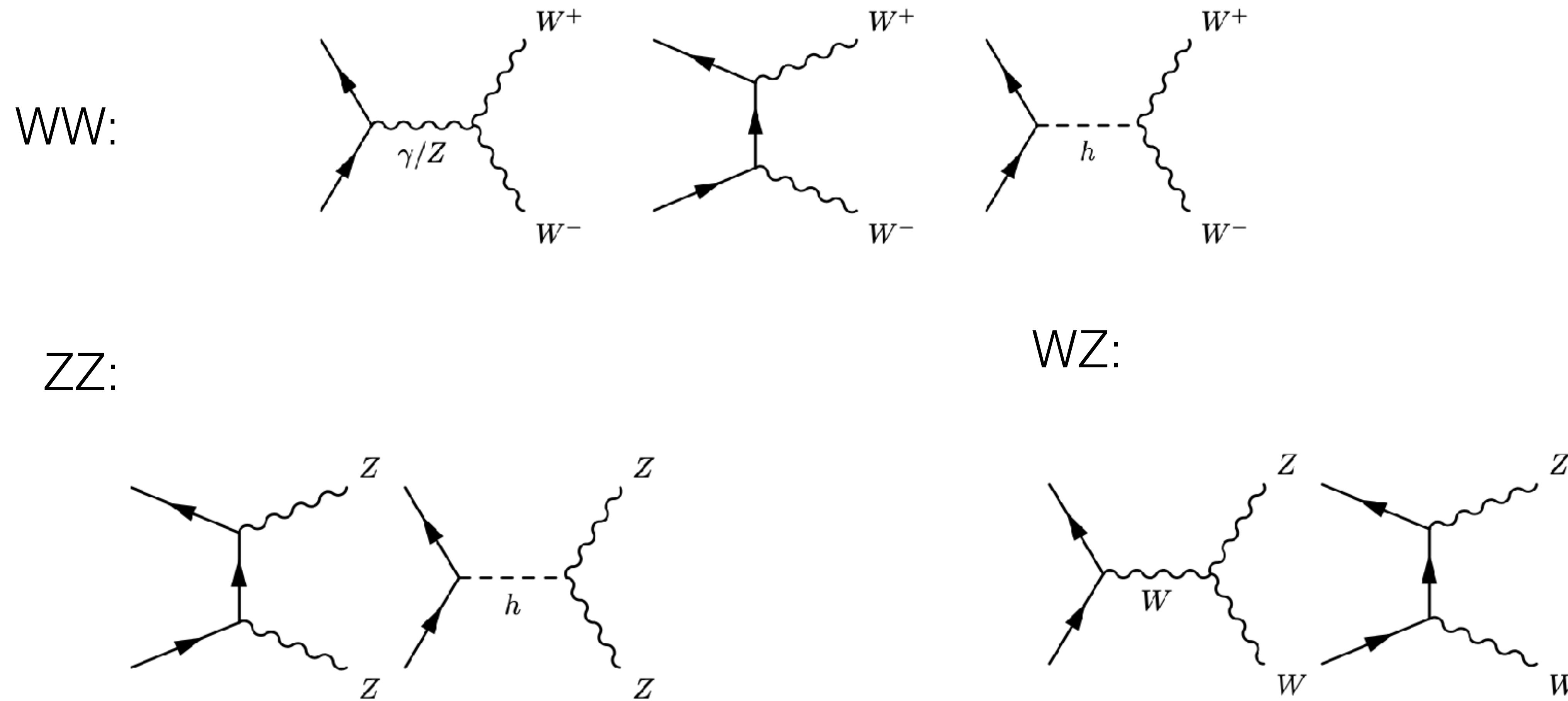
$$|\Psi\rangle = \frac{1}{\sqrt{2 + \kappa^2}} \{ | \uparrow\uparrow \rangle - \kappa | 00 \rangle + | \downarrow\downarrow \rangle \}$$

See also Bell inequalities

[Fabbri, Howarth, Maurin, 24']

[Horodecki, Horodecki, Horodecki, Horodecki, 09']  
[Ashby-Pickering, Barr, Wierzchucha, '22]

# EW boson production at colliders



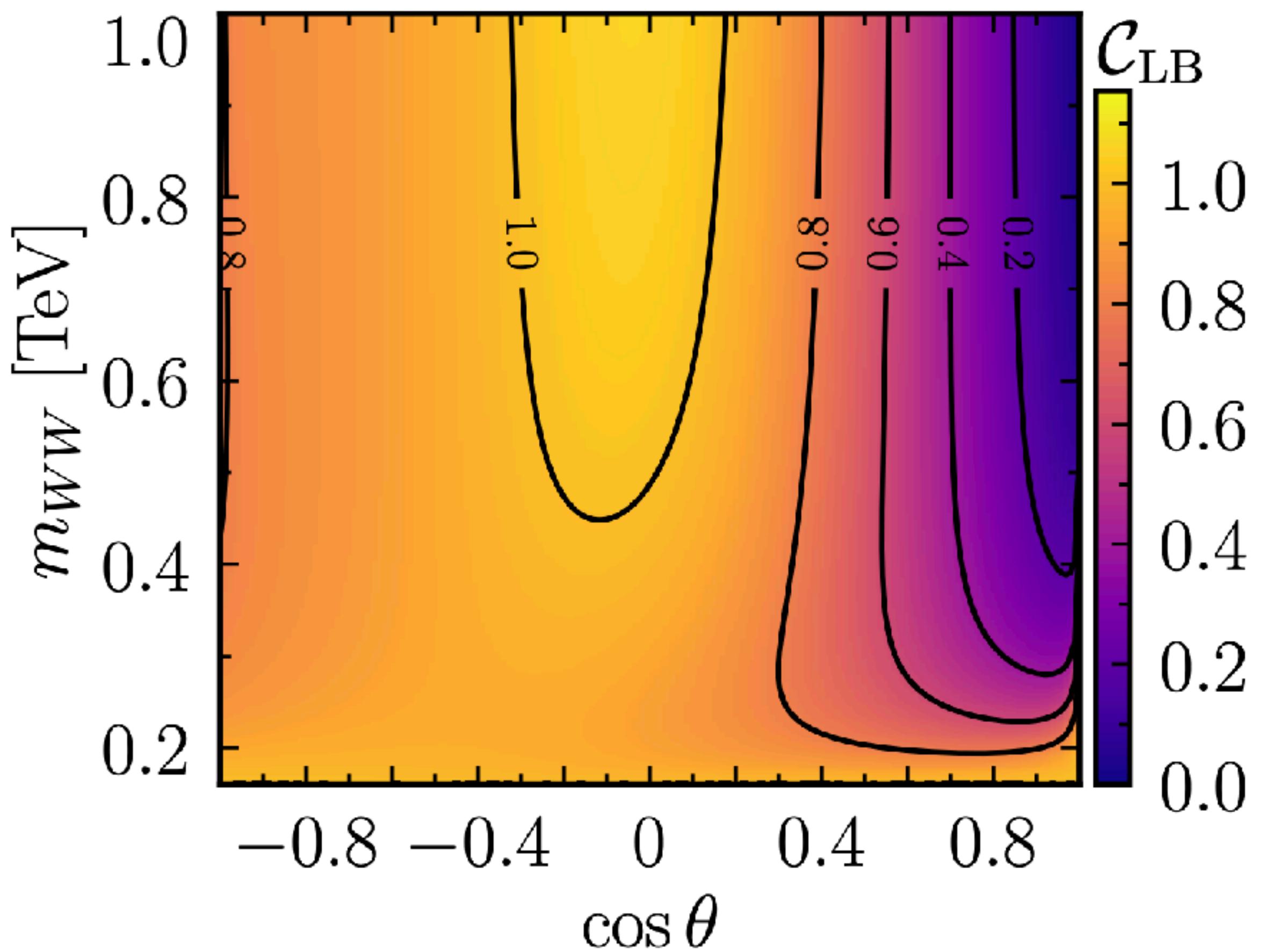
# What's the story for the Standard Model?

[Ashby-Pickering, Barr, Wierzchuca, '22]

$$e^+ e^- \rightarrow W^+ W^-$$

[Aoude, Madge, Maltoni, Mantani, 23']

Lower bound:



No symmetry around  $\theta = \pi/2$  as in ttbar

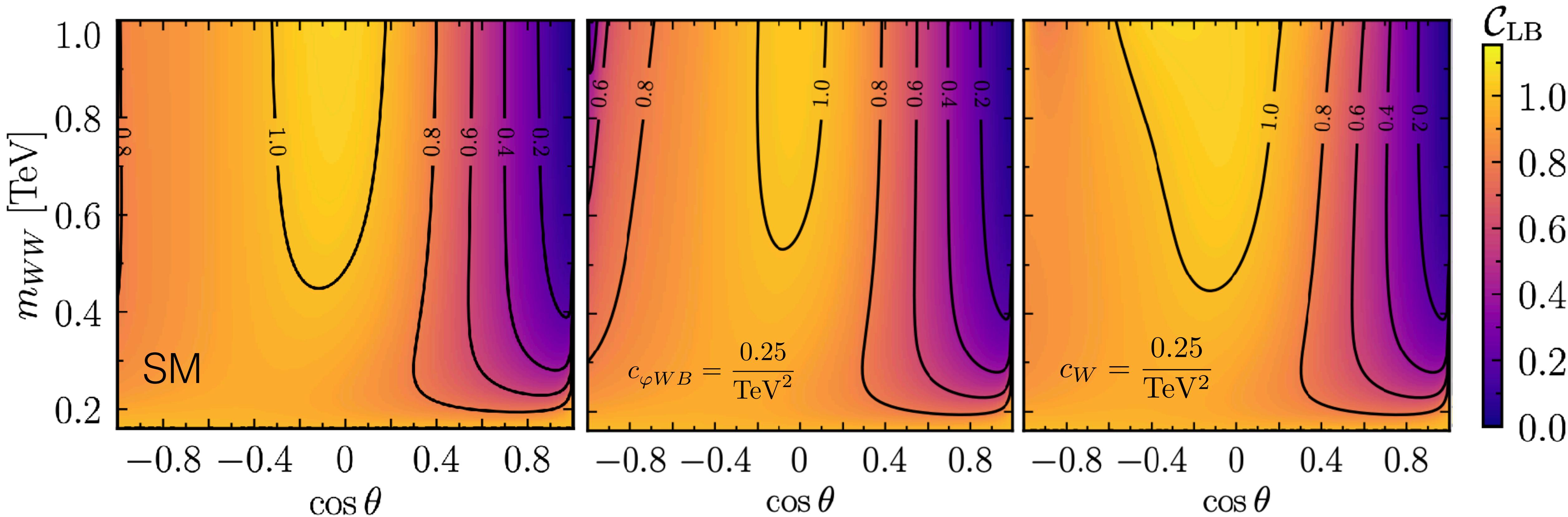
Entanglement is mostly present across the phase space

Zero entanglement at  $\theta = 0$

High entanglement at central HE region

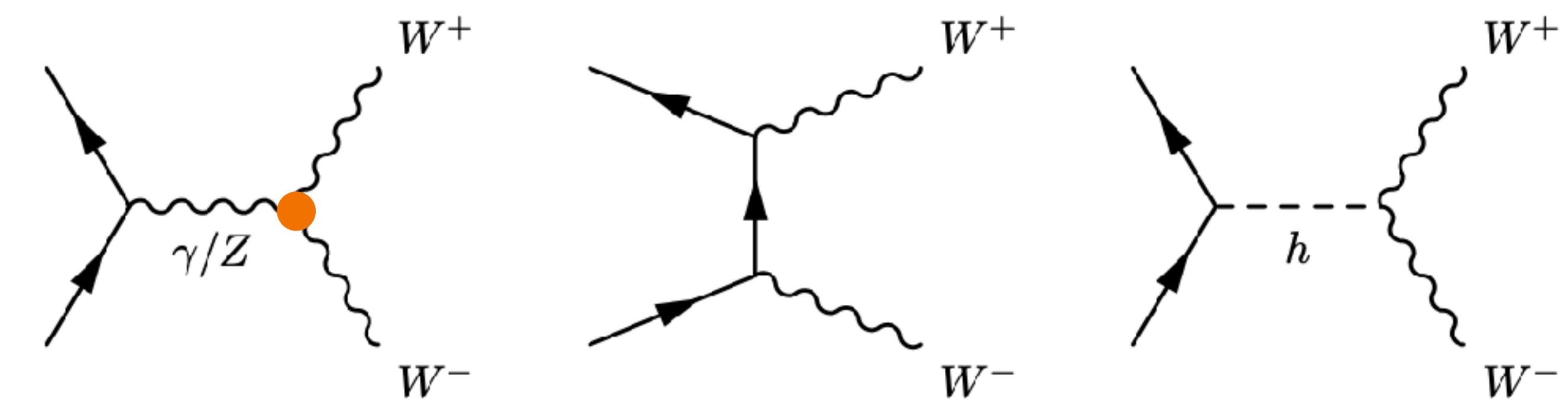
# SMEFT entanglement deviations

$e^+e^- \rightarrow W^+W^-$



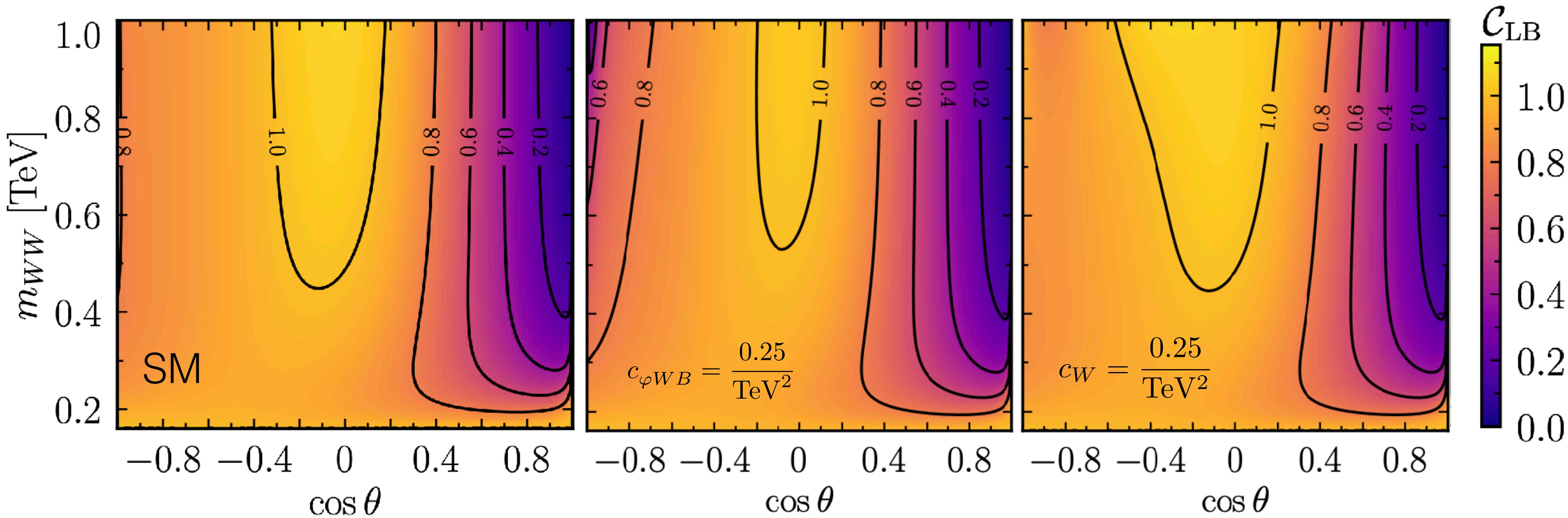
$\mathcal{O}_{\varphi WB}$  changes TGC coupling

$\mathcal{O}_W$  new Lorentz structure



# SMEFT entanglement deviations

$e^+e^- \rightarrow W^+W^-$

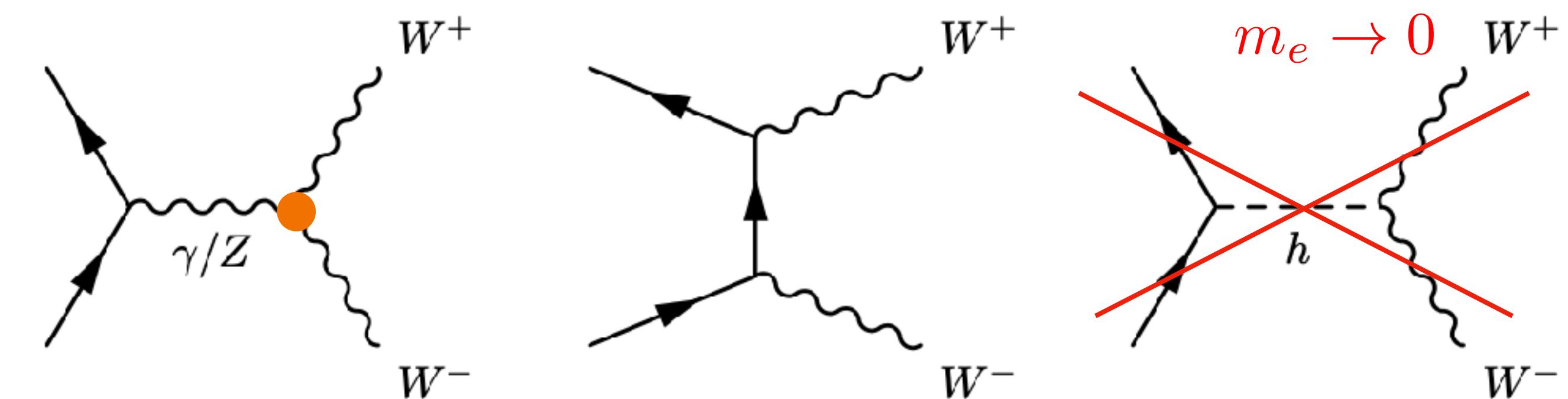


$\mathcal{O}_{\varphi WB}$  changes TGC coupling

- more ent. in central and backward HE

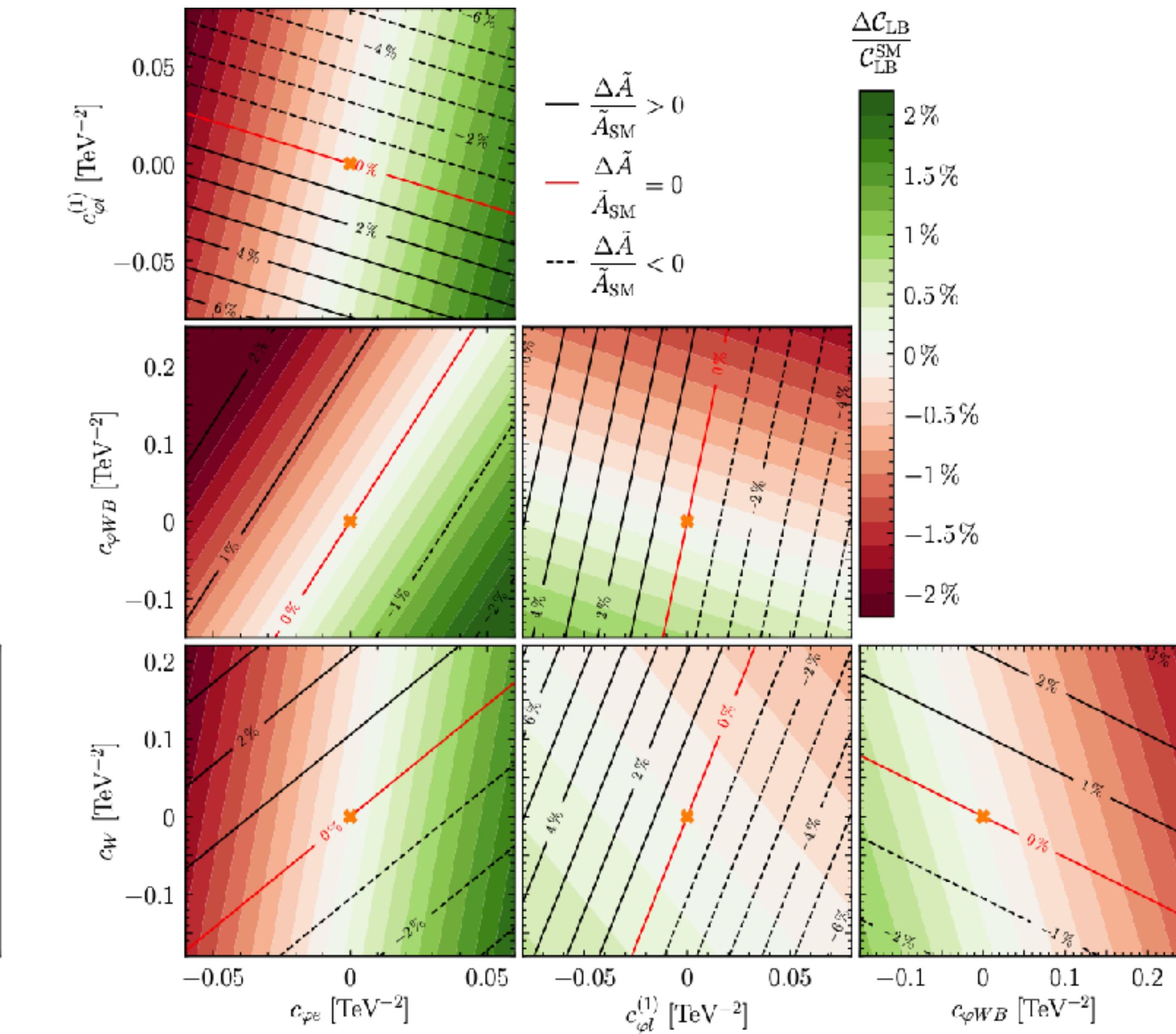
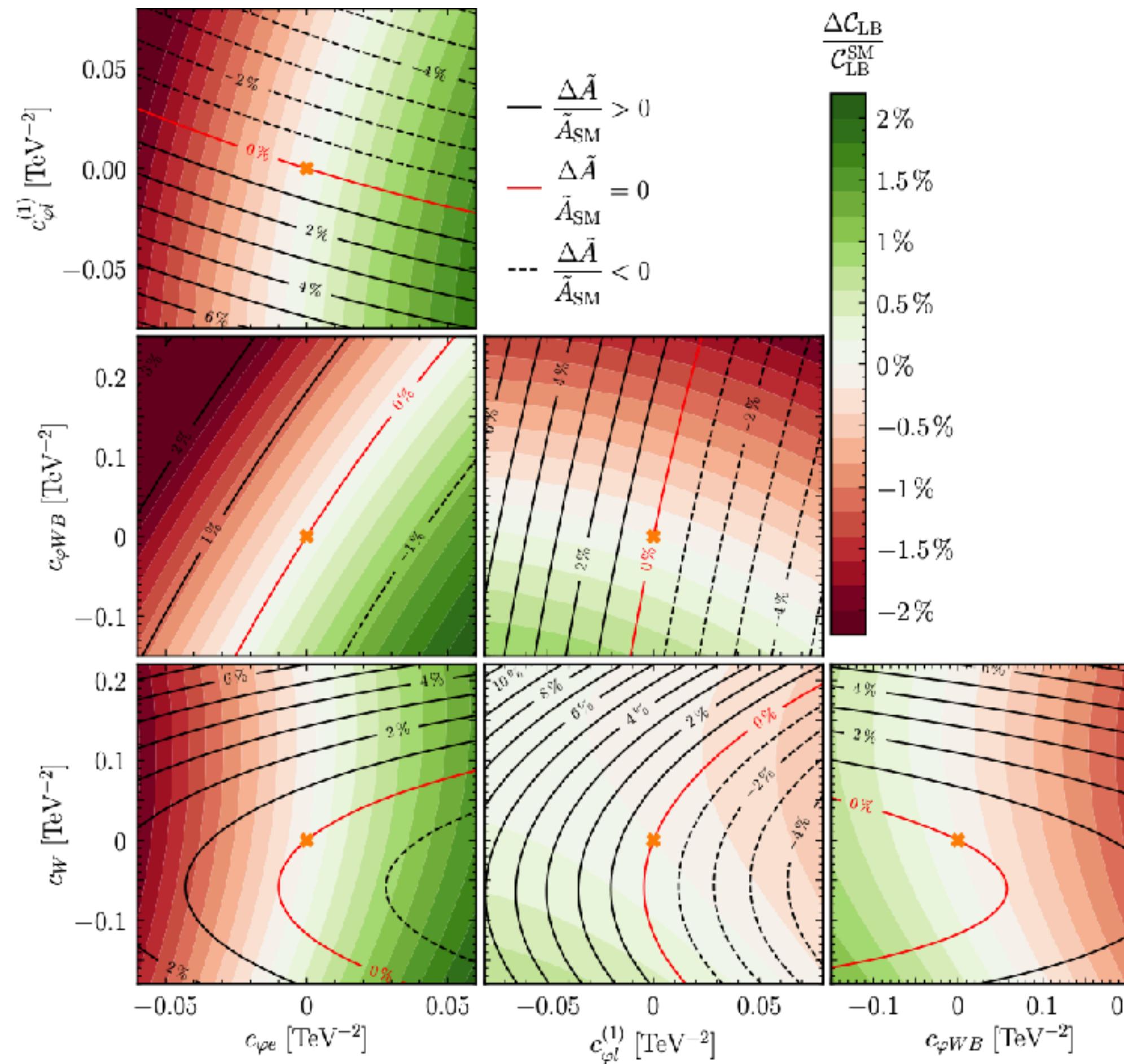
$\mathcal{O}_W$  new Lorentz structure

- small effect



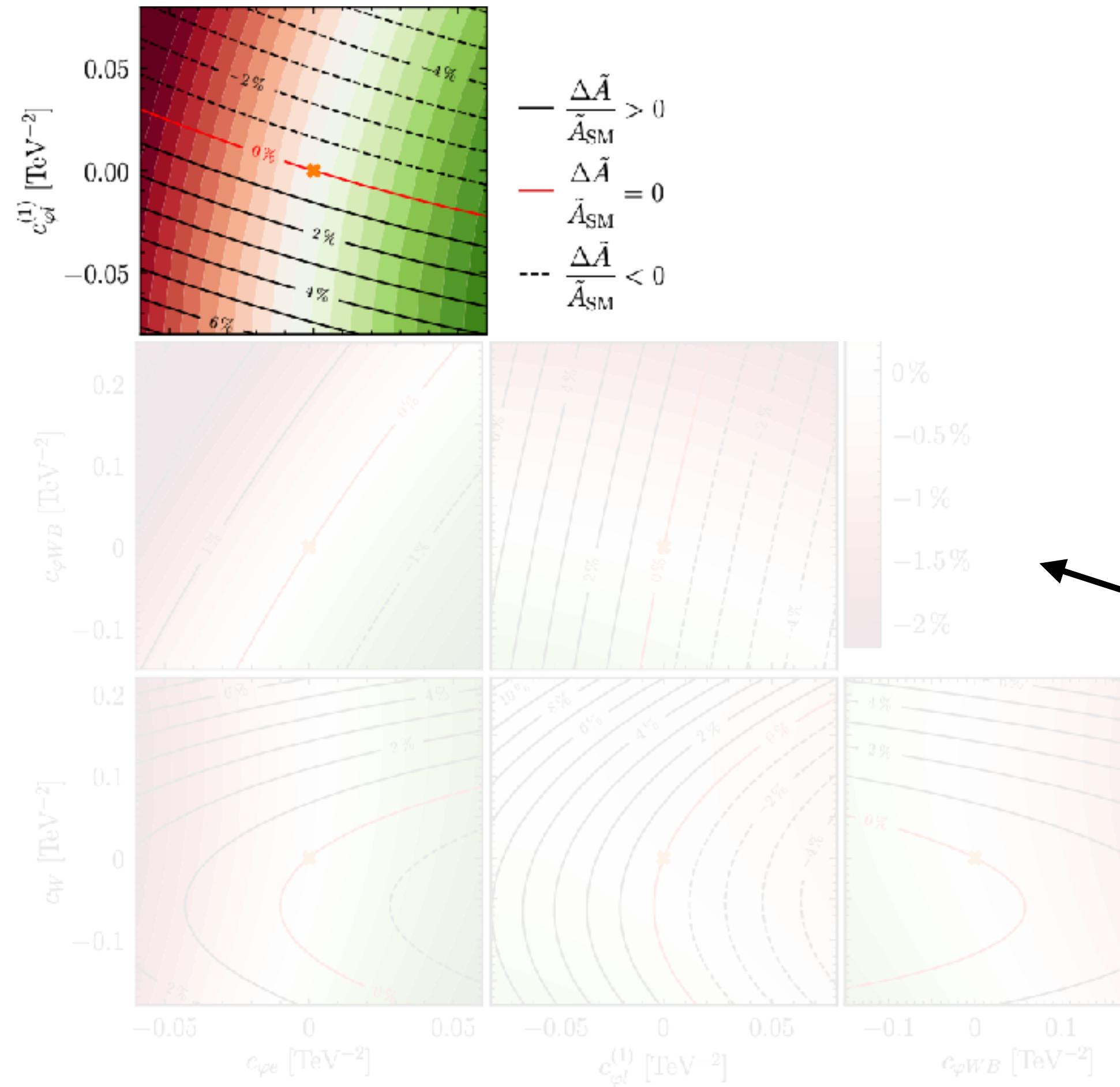
# SMEFT entanglement deviations: Central region

$$m_{WW} = 500 \text{ GeV} \quad \cos \theta = 0$$

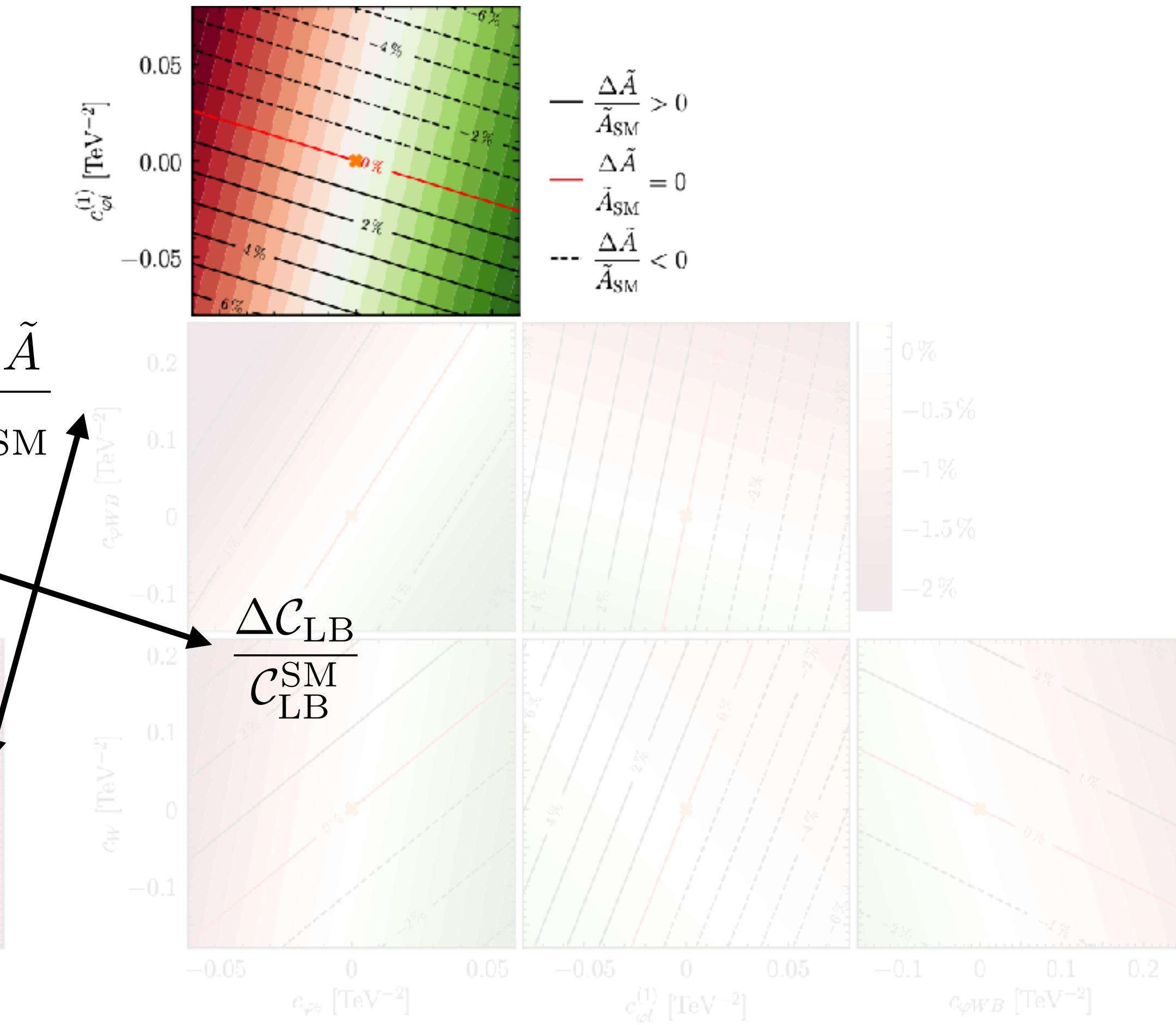


# SMEFT entanglement deviations: Central region

$$m_{WW} = 500 \text{ GeV} \quad \cos \theta = 0$$



linear+squared dim-6



Linear dim-6

# Growing subfield: many new directions

\*not a complete list!

## Symmetries from Entanglement

[Beane, Kaplan, Klco, Savage '18]

[Beane, Farrell '21]

[Low, Mehen 21'] [Liu, Low, Mehen, '21]

[Carena, Low, Wagner, Xiao, '24]

[Thaler, Trifinopoulos, '24]

## Post decay

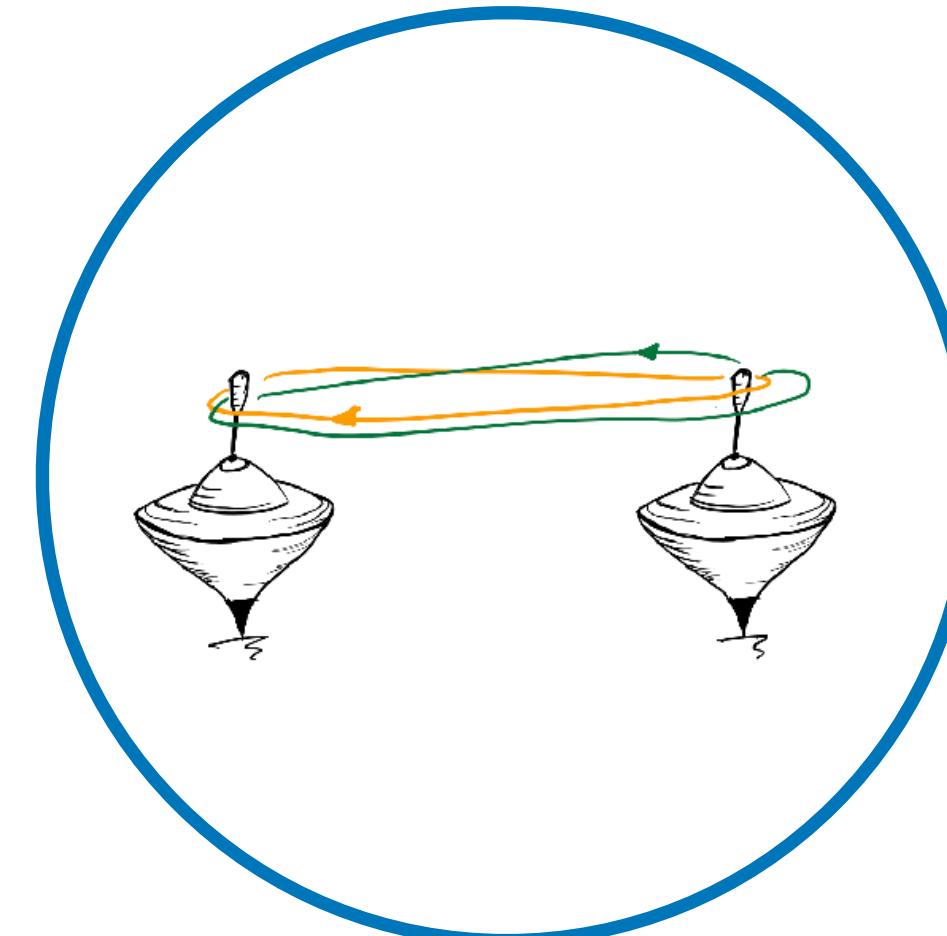
[J. A. Aguilar-Saavedra 24']

## Multipartite entanglement

[Sakurai, Spannowsky 23']

[Bernal, Casas, Moreno 23']

[Bernal 24']



## Entanglement in flavour space/ Momenta space

[Cheung, He, Sivaramakrishnan 24']

[Aoude, Elor, Remen, Sumensari, 24']

[Low, Yin 24']

[Duaso Pueyo, Goodhew, McCulloch, Pajer 24']

## Beyond Quantum Mechanics

[Eckestein]

## Bell Inequalities

[many refs!]

## Quantum Simulations in HEP

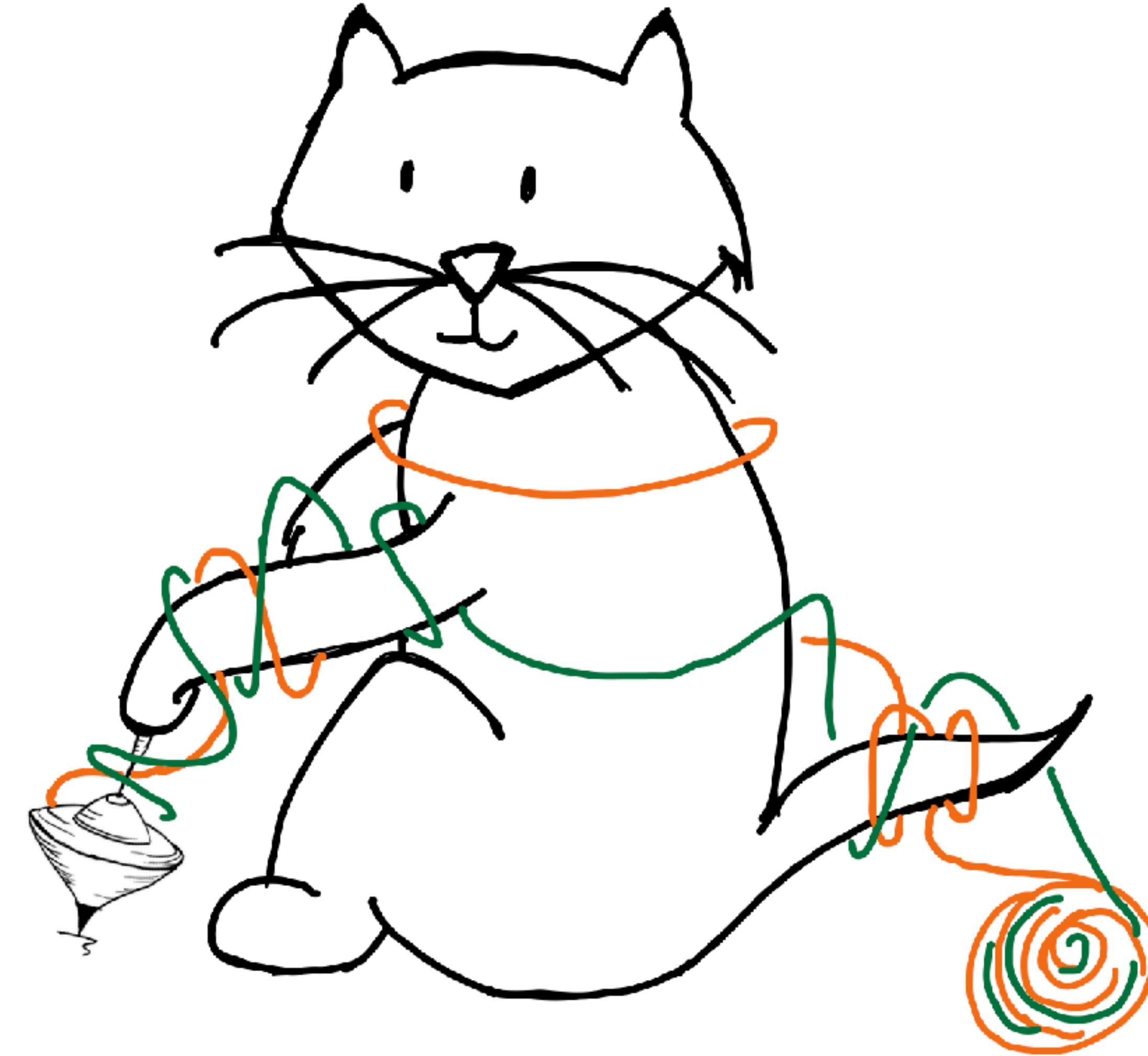
[Gustafson, Prestel, Spannowky, Williams, '22]

[many refs!]

# Conclusions

- Entanglement (Quantum Information) in High Energy Physics is cool!
- SM in ttbar has high entanglement at threshold and high-pT
- SMEFT typically decreases the amount of entanglement
- Recent experimental results appeared this year!
- Synergies between HEP and QIS
- Useful to probe/constrain new physics
- New interpretation for a spin correlations subset

**Tack!**



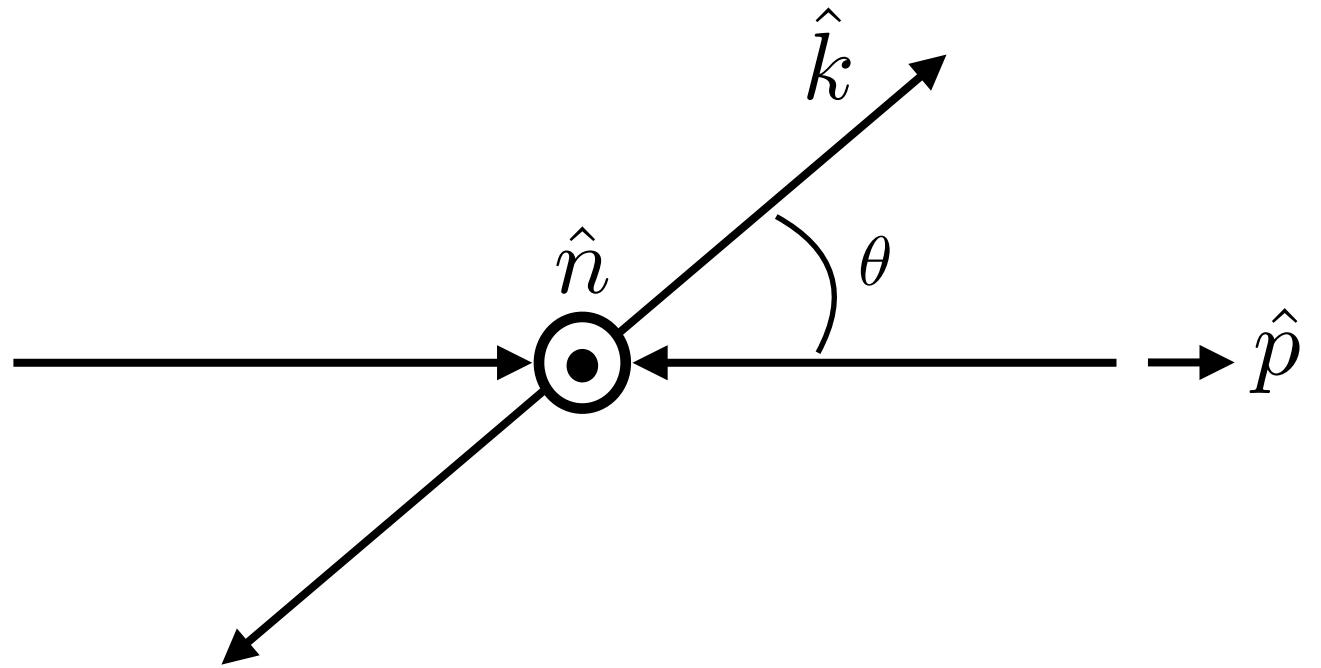
THE UNIVERSITY  
of EDINBURGH



# Density matrix and helicity-basis

Helicity basis:

$$\{\mathbf{k}, \mathbf{n}, \mathbf{r}\} : \mathbf{r} = \frac{(\mathbf{p} - z\mathbf{k})}{\sqrt{1 - z^2}}, \quad \mathbf{n} = \mathbf{k} \times \mathbf{r},$$



To expand in this basis, e.g.

$$C_{nn} = \text{tr}[C_{ij} \mathbf{n} \otimes \mathbf{n}]$$

Phase-space parametrized by:  $\beta^2 = (1 - 4m_t^2/\hat{s})$  and  $z = \cos \theta$

# SMEFT R-matrix

$$R^{\text{SM}} \sim \mathcal{M}^{\text{SM}} \mathcal{M}^{\dagger \text{SM}}$$

$$R^{\text{EFT},(1)} \sim \mathcal{M}^{\text{SM}} \mathcal{M}^{\dagger \text{(d6)}}$$

$$R^{\text{EFT},(2)} \sim \mathcal{M}^{\text{(d6)}} \mathcal{M}^{\dagger \text{(d6)}}$$

The R-matrix coefficients  $X = X^{(0)} + \frac{1}{\Lambda^2}X^{(1)} + \frac{1}{\Lambda^4}X^{(2)}$  where

$$X = \tilde{A}, \tilde{C}_{ij} \text{ and } \tilde{B}_i^\pm$$

At  $\mathcal{O}(\Lambda^{-2})$

$$\tilde{C}_{nn}^{gg,(1)} = \frac{g_s^2}{\Lambda^2} \frac{1}{1 - \beta^2 z^2} \left[ \frac{-7g_s^2 v m_t}{12\sqrt{2}} c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2 - (1 - \beta^2)m_h^2} c_{\varphi G} + \frac{9g_s^2 \beta^2 m_t^2 z^2}{8} c_G \right]$$

# SMEFT entanglement: gg-initiated

[Aoude, Madge,  
Maltoni, Mantani, 22']

only  $\mathcal{O}_{tG}, \mathcal{O}_G, \mathcal{O}_{\varphi G}$  contributes

$$\rho = \frac{R^{\text{SM}} + R^{\text{EFT}}}{\text{tr}(R^{\text{SM}}) + \text{tr}(R^{\text{EFT}})}$$

gg-initiated at threshold  $\beta^2 = 0$

- linear interference exactly cancel, maximally entangled state unchanged
- quadratics vanish for  $\mathcal{O}_{\varphi G}$  and decreases for  $\mathcal{O}_{tG}, \mathcal{O}_G$

gg-initiated at high-E:  $\beta^2 \rightarrow 1$  : EFT not valid but  $m_t^2 \ll \hat{s} \ll \Lambda^2$

- linear interference: sign dependent
- quadratics always decreases

# SMEFT entanglement: qq-initiated

[Aoude, Madge,  
Maltoni, Mantani, 22']

only  $\mathcal{O}_{tG}$  and 4F contributes

$$\rho = \frac{R^{\text{SM}} + R^{\text{EFT}}}{\text{tr}(R^{\text{SM}}) + \text{tr}(R^{\text{EFT}})}$$

qq-initiated at threshold  $\beta^2 = 0$

- no contributions for linear and quad

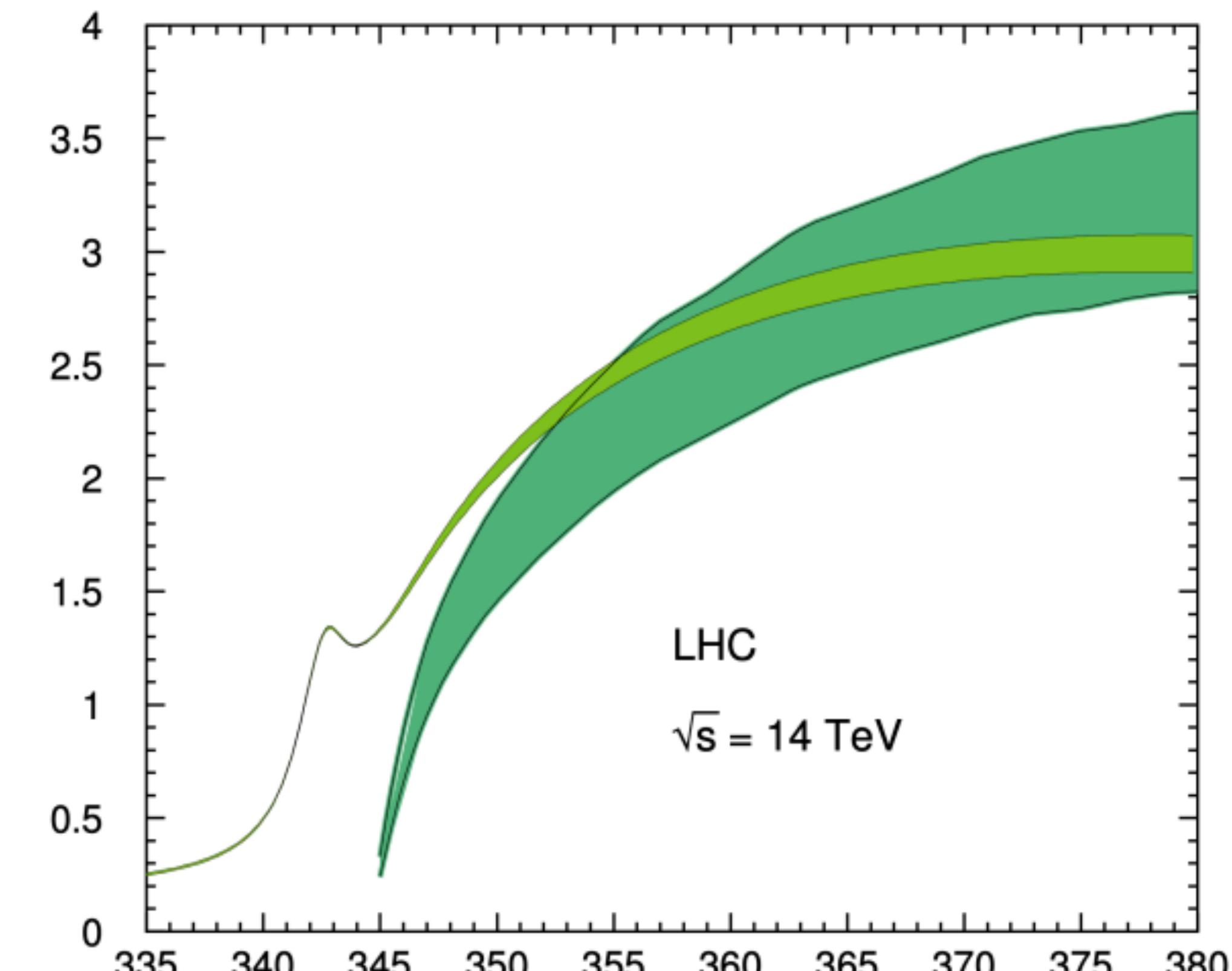
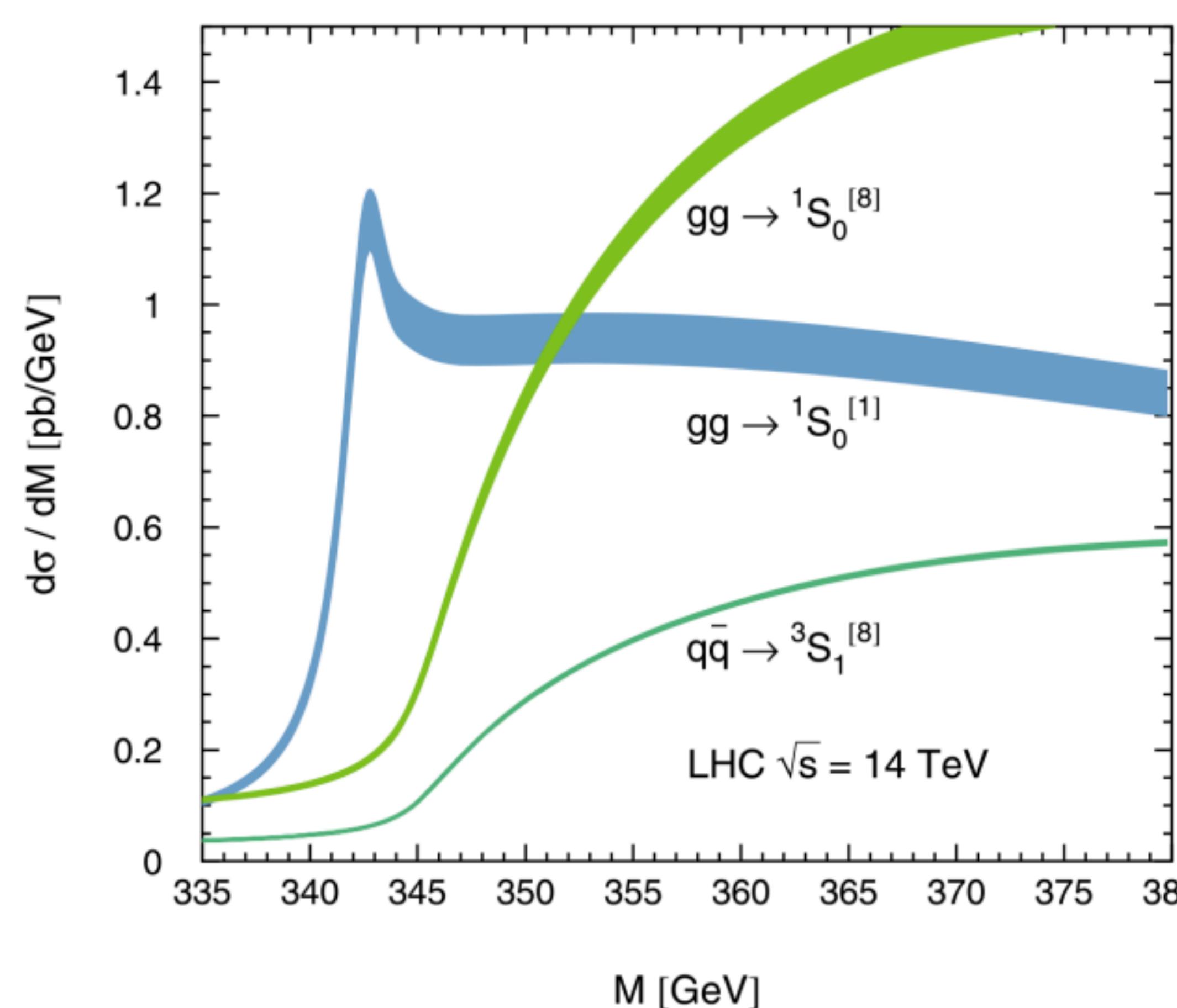
qq-initiated at high-E:  $m_t^2 \ll \hat{s} \ll \Lambda^2$

- sign dependent for linear and quadratics always decreases

everything gets more involved for pp

# Threshold effects (“topponium”)

[Kiyo, Kuhn, Moch, Steinhouse, Uwer, 09’]



# Concurrence Bounds for qutrits

However, we can define lower and upper bounds  $\mathcal{C}_{\text{LB}} \leq \mathcal{C}(\rho) \leq \mathcal{C}_{\text{UB}}$ ,

Lower:  $(\mathcal{C}(\rho))^2 \geq 2 \max(0, \text{Tr}[\rho^2] - \text{Tr}[\rho_A^2], \text{Tr}[\rho^2] - \text{Tr}[\rho_B^2]) \equiv \mathcal{C}_{\text{LB}}^2,$

Upper:  $(\mathcal{C}(\rho))^2 \leq 2 \min(1 - \text{Tr}[\rho_A^2], 1 - \text{Tr}[\rho_B^2]) \equiv \mathcal{C}_{\text{UB}}^2,$

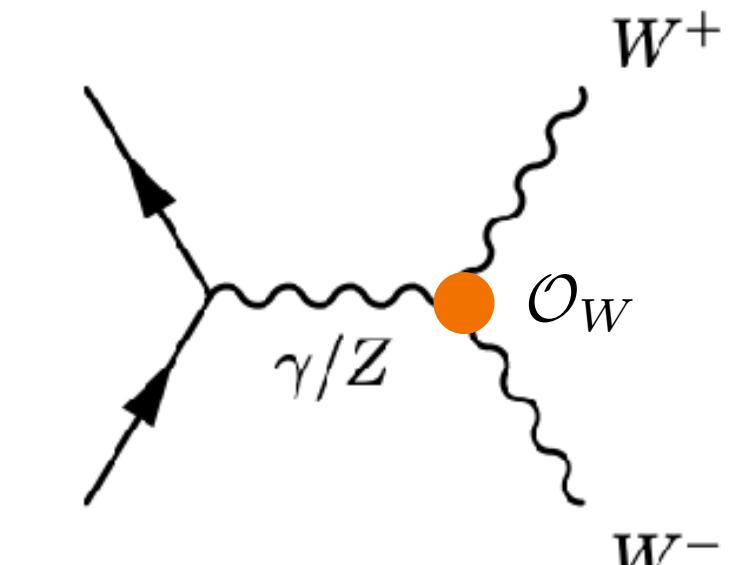
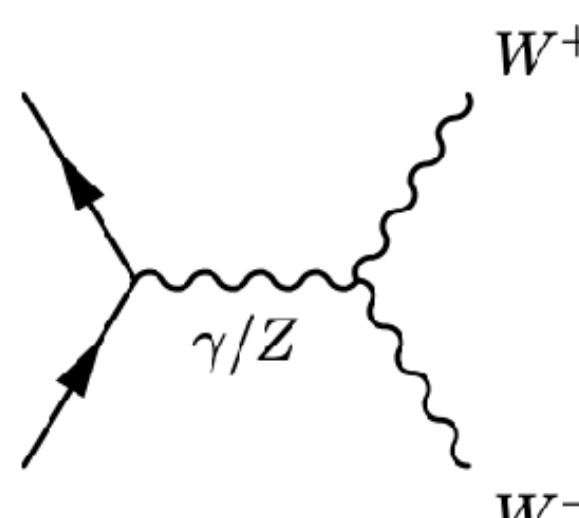
For a pure state:  $P(\rho) = \text{tr}\rho^2 = 1 \quad \longrightarrow \quad \mathcal{C}_{\text{LB}}(\rho) = \mathcal{C}(\rho) = \mathcal{C}_{\text{UB}}(\rho)$

# Perturbative Unitarity and Entanglement

The density matrix (and angular observables) are sensitive to new directions

$$e^+ e^- \rightarrow W^+ W^-$$

| $(\lambda_1 \lambda_2   \alpha \beta)$ | SM  | EFT $\Lambda^{-2} : c_{WWW}$                                      |
|--|---|---|
| $+ - 00$                               | $-2\sqrt{2}G_F m_Z^2 \sin \theta$                             | -   |
| $+ - - +$                              | $2\sqrt{2}G_F m_W^2 \sin \theta$                              | -   |
| $+ - +-$                               | $-\frac{1}{\sqrt{2}}G_F m_W^2 \sin^3 \theta \csc^4(\theta/2)$ | -   |
| $+ - \pm \pm$                          | -   | $3 \cdot 2^{1/4} \sqrt{G_F} m_W \sin \theta (4m_W^2 x^2 - m_Z^2)$ |
| $+ - 0\pm$                             | -   | $-3 \cdot 2^{3/4} \sqrt{G_F} m_W^3 (\pm 1 + \cos \theta) x$       |
| $+ - \pm 0$                            | -   | $-3 \cdot 2^{3/4} \sqrt{G_F} m_W^3 (\mp 1 + \cos \theta) x$       |
| $- + 00$                               | $2\sqrt{2}G_F(m_Z^2 - m_W^2) \sin \theta$                     | -   |
| $- + \pm \pm$                          | -   | $6 \cdot 2^{1/4} \sqrt{G_F} m_W(m_Z^2 - m_W^2) \sin \theta$       |

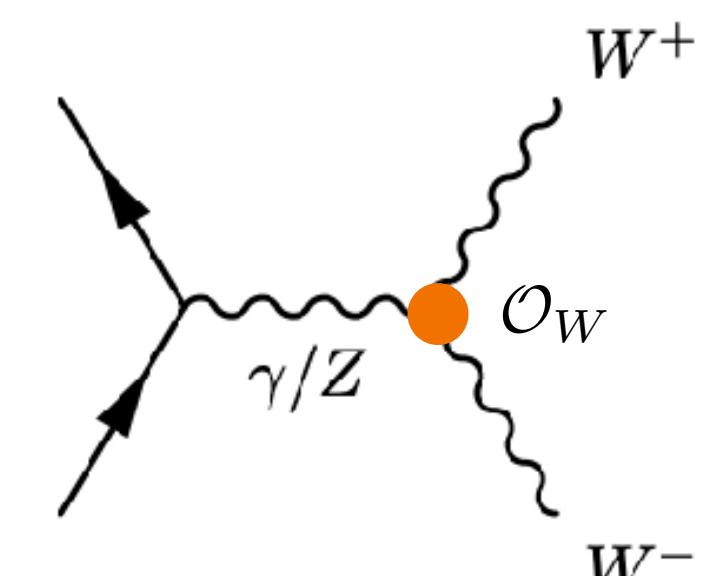


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| $(\lambda_1 \lambda_2   \alpha \beta)$ | SM  | EFT $\Lambda^{-2} : c_{WWW}$  |
|--|---|---|
| + - 00                                 | $-2\sqrt{2}G_F m_Z^2 \sin \theta$                             | $\longleftrightarrow$   |
| + - -+                                 | $2\sqrt{2}G_F m_W^2 \sin \theta$                              | $\longleftrightarrow$   |
| + - +-                                 | $-\frac{1}{\sqrt{2}}G_F m_W^2 \sin^3 \theta \csc^4(\theta/2)$ | $\longleftrightarrow$   |
| + - ±±                                 | -   | $\longleftrightarrow 3 \cdot 2^{1/4} \sqrt{G_F} m_W \sin \theta (4m_W^2 x^2 - m_Z^2)$ |
| + - 0±                                 | -   | $\longleftrightarrow -3 \cdot 2^{3/4} \sqrt{G_F} m_W^3 (\pm 1 + \cos \theta) x$       |
| + - ±0                                 | -   | $\longleftrightarrow -3 \cdot 2^{3/4} \sqrt{G_F} m_W^3 (\mp 1 + \cos \theta) x$       |
| <hr/>                                  |   |   |
| - + 00                                 | $2\sqrt{2}G_F(m_Z^2 - m_W^2) \sin \theta$                     | $\longleftrightarrow$   |
| - + ±±                                 | -   | $\longleftrightarrow 6 \cdot 2^{1/4} \sqrt{G_F} m_W(m_Z^2 - m_W^2) \sin \theta$       |
| <hr/>                                  |   |   |



No interference!



Cross-section

$$\tilde{A}(O_W) \sim 0$$

# Perturbative Unitarity and Entanglement

| $(\lambda_1 \lambda_2   \alpha \beta)$ | SM  | EFT $\Lambda^{-2} : c_{WWW}$                                      |
|--|---|---|
| + - 00                                 | $-2\sqrt{2}G_F m_Z^2 \sin \theta$                             | -   |
| + - - +                                | $2\sqrt{2}G_F m_W^2 \sin \theta$                              | -   |
| + - +-                                 | $-\frac{1}{\sqrt{2}}G_F m_W^2 \sin^3 \theta \csc^4(\theta/2)$ | -   |
| + - ±±                                 | -   | $3 \cdot 2^{1/4} \sqrt{G_F} m_W \sin \theta (4m_W^2 x^2 - m_Z^2)$ |
| + - 0±                                 | -   | $-3 \cdot 2^{3/4} \sqrt{G_F} m_W^3 (\pm 1 + \cos \theta) x$       |
| + - ±0                                 | -   | $-3 \cdot 2^{3/4} \sqrt{G_F} m_W^3 (\mp 1 + \cos \theta) x$       |
| - + 00                                 | $2\sqrt{2}G_F(m_Z^2 - m_W^2) \sin \theta$                     | -   |
| - + ±±                                 | -   | $6 \cdot 2^{1/4} \sqrt{G_F} m_W(m_Z^2 - m_W^2) \sin \theta$       |

$$\rho = \begin{bmatrix} \mathcal{M}_{++} & \mathcal{M}_{++}^* & & \\ \mathcal{M}_{+-} & \mathcal{M}_{++}^* & \cdots & \\ \mathcal{M}_{+-}^* & \mathcal{M}_{+-} & \cdots & \\ \vdots & \ddots & & \end{bmatrix}$$

The spin-density matrix has different helicity products

$$\tilde{a}_1(\mathcal{O}_W) \simeq \tilde{b}_1(\mathcal{O}_W) \simeq \bar{c}_W 2^{5/4} x \cos^4(\theta/2)(\cos \theta + 3) \csc \theta,$$

Entanglement is sensitive to off-diagonal contractions

$$\tilde{c}_{13} \simeq 3 \bar{c}_W \cdot 2^{3/4} \cos^2(\theta/2)(3 \cos \theta + 1) \cot(\theta/2) x$$

Recovers the energy growth!