

Higher-order soft and virtual corrections to H^+H^- production

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(in collaboration with Alberto Tonerò)

- $N^3\text{LO}$ soft corrections for colorless final states
- H^+H^- cross sections
- Charged-Higgs rapidity distributions



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Soft-gluon corrections for H^+H^- in 1PI kinematics

Analytical results are applicable to all processes with colorless final states

partonic processes $q(p_a) + \bar{q}(p_b) \rightarrow H^+(p_1) + H^-(p_2)$

define $s = (p_a + p_b)^2$, $t_1 = (p_a - p_1)^2 - m^2$, $u_1 = (p_b - p_1)^2 - m^2$ with $m = m_{H^+} = m_{H^-}$

In single-particle-inclusive (1PI) kinematics we define the threshold variable

$$s_4 = s + t_1 + u_1 = (p_2 + p_g)^2 - m^2$$

where extra gluon with momentum p_g is emitted

At partonic threshold $p_g \rightarrow 0$ and thus $s_4 \rightarrow 0$

Soft corrections $\left[\frac{\ln^k(s_4/m^2)}{s_4} \right]_+$ with $k \leq 2n - 1$ for the order α_s^n corrections

Resum these soft corrections for the double-differential cross section

Finite-order expansions \rightarrow no prescription needed or used
(this avoids underestimating the size of the corrections)

Approximate NNLO (aNNLO) and approximate N³LO (aN³LO) predictions
for cross sections and differential distributions

Soft-gluon Resummation

$$d\sigma_{pp \rightarrow H^+ H^-} = \sum_{q, \bar{q}} \int dx_a dx_b \phi_{q/p}(x_a, \mu_F) \phi_{\bar{q}/p}(x_b, \mu_F) d\hat{\sigma}_{q\bar{q} \rightarrow H^+ H^-}(s_4, \mu_F)$$

take Laplace transforms $d\tilde{\sigma}_{q\bar{q} \rightarrow H^+ H^-}(N) = \int (ds_4/s) e^{-Ns_4/s} d\hat{\sigma}_{q\bar{q} \rightarrow H^+ H^-}(s_4)$

$$\text{Then } d\tilde{\sigma}_{q\bar{q} \rightarrow H^+ H^-}(N) = \tilde{\phi}_{q/q}(N_q, \mu_F) \tilde{\phi}_{\bar{q}/\bar{q}}(N_{\bar{q}}, \mu_F) d\tilde{\sigma}_{q\bar{q} \rightarrow H^+ H^-}(N, \mu_F)$$

Refactorization of the cross section

$$d\sigma_{q\bar{q} \rightarrow H^+ H^-}(N) = \tilde{\psi}_q(N_q, \mu_F) \tilde{\psi}_{\bar{q}}(N_{\bar{q}}, \mu_F) H_{q\bar{q} \rightarrow H^+ H^-} \tilde{S}_{q\bar{q} \rightarrow H^+ H^-} \left(\frac{\sqrt{s}}{N\mu_F} \right)$$

where $H_{q\bar{q} \rightarrow H^+ H^-}$ is hard function and $S_{q\bar{q} \rightarrow H^+ H^-}$ is soft function

$$\text{Thus } d\tilde{\sigma}_{q\bar{q} \rightarrow H^+ H^-}(N) = \frac{\tilde{\psi}_{q/q}(N_q, \mu_F) \tilde{\psi}_{\bar{q}/\bar{q}}(N_{\bar{q}}, \mu_F)}{\tilde{\phi}_{q/q}(N_q, \mu_F) \tilde{\phi}_{\bar{q}/\bar{q}}(N_{\bar{q}}, \mu_F)} H_{q\bar{q} \rightarrow H^+ H^-} \tilde{S}_{q\bar{q} \rightarrow H^+ H^-} \left(\frac{\sqrt{s}}{N\mu_F} \right)$$

Renormalization group evolution \rightarrow resummation

$$d\tilde{\sigma}_{q\bar{q} \rightarrow H^+ H^-}^{\text{resum}}(N) = \exp \left[E_q(N_q) + E_{\bar{q}}(N_{\bar{q}}) \right] \exp \left[2 \int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu} \left(\gamma_{q/q}(N_q, \alpha_s(\mu)) + \gamma_{\bar{q}/\bar{q}}(N_{\bar{q}}, \alpha_s(\mu)) \right) \right]$$

$$\times H_{q\bar{q} \rightarrow H^+ H^-} \left(\alpha_s(\sqrt{s}) \right) \tilde{S}_{q\bar{q} \rightarrow H^+ H^-} \left(\alpha_s \left(\frac{\sqrt{s}}{N} \right) \right)$$

$$\text{where } E_q(N_q) = \int_0^1 dz \frac{z^{N_q-1} - 1}{1-z} \left\{ \int_1^{(1-z)^2} \frac{d\lambda}{\lambda} A_q(\alpha_s(\lambda s)) + D_q \left[\alpha_s((1-z)^2 s) \right] \right\}$$

NLO soft+virtual corrections

$$\frac{d^2 \hat{\sigma}_{q\bar{q} \rightarrow H^+ H^-}^{(1)}}{dt_1 du_1} = F_{q\bar{q} \rightarrow H^+ H^-}^B \frac{\alpha_s(\mu_R)}{\pi} \{c_3 \mathcal{D}_1(s_4) + c_2 \mathcal{D}_0(s_4) + c_1 \delta(s_4)\}$$

where

$$\mathcal{D}_k(s_4) = \left[\frac{\ln^k(s_4/m^2)}{s_4} \right]_+,$$

$$c_3 = 4C_F \quad \text{and} \quad c_2 = -2C_F \ln\left(\frac{t_1 u_1}{m^4}\right) - 2C_F \ln\left(\frac{\mu_F^2}{s}\right),$$

$$c_1 = C_F \ln^2\left(\frac{-t_1}{m^2}\right) + C_F \ln^2\left(\frac{-u_1}{m^2}\right) + \left[C_F \ln\left(\frac{t_1 u_1}{m^4}\right) - 2\gamma_q^{(1)} \right] \ln\left(\frac{\mu_F^2}{s}\right) + V_1,$$

$$\text{and} \quad V_1 = 2C_F(-2 + \zeta_2)$$

NNLO soft+virtual corrections

$$\begin{aligned}
\frac{d^2 \hat{\sigma}_{q\bar{q} \rightarrow H^+ H^-}^{(2)}}{dt_1 du_1} &= F_{q\bar{q} \rightarrow H^+ H^-}^B \frac{\alpha_s^2(\mu_R)}{\pi^2} \left\{ \frac{1}{2} c_3^2 \mathcal{D}_3(s_4) + \left[\frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 \right] \mathcal{D}_2(s_4) \right. \\
&+ \left[c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} c_2 - \beta_0 C_F \ln \left(\frac{\mu_F^2}{\mu_R^2} \right) + 4A_q^{(2)} \right] \mathcal{D}_1(s_4) \\
&+ \left[c_2 c_1 - \zeta_2 c_3 c_2 + \zeta_3 c_3^2 + \frac{\beta_0}{4} c_2 \ln \left(\frac{\mu_R^2}{s} \right) \right. \\
&\quad \left. - \frac{\beta_0}{2} C_F \ln^2 \left(\frac{-t_1}{m^2} \right) - \frac{\beta_0}{2} C_F \ln^2 \left(\frac{-u_1}{m^2} \right) - 2A_q^{(2)} \ln \left(\frac{t_1 u_1}{m^4} \right) + 2D_q^{(2)} \right. \\
&\quad \left. + \frac{\beta_0}{4} C_F \ln^2 \left(\frac{\mu_F^2}{s} \right) - 2A_q^{(2)} \ln \left(\frac{\mu_F^2}{s} \right) \right] \mathcal{D}_0(s_4) \\
&+ \left[V_2 + \frac{1}{2} (c_1^2 - V_1^2) - \frac{\zeta_2}{2} c_2^2 + \zeta_3 c_3 c_2 + \frac{\beta_0}{6} C_F \left(\ln^3 \left(\frac{-t_1}{m^2} \right) + \ln^3 \left(\frac{-u_1}{m^2} \right) \right) \right. \\
&\quad + \left(\frac{\beta_0}{4} C_F + A_q^{(2)} \right) \left(\ln^2 \left(\frac{-t_1}{m^2} \right) + \ln^2 \left(\frac{-u_1}{m^2} \right) \right) + \frac{\beta_0}{4} c_1 \ln \left(\frac{\mu_R^2}{s} \right) - 2\gamma_q^{(2)} \ln \left(\frac{\mu_F^2}{s} \right) \\
&\quad \left. + A_q^{(2)} \ln \left(\frac{t_1 u_1}{m^4} \right) \ln \left(\frac{\mu_F^2}{s} \right) + \frac{\beta_0}{8} \left(2\gamma_q^{(1)} - C_F \ln \left(\frac{t_1 u_1}{m^4} \right) \right) \ln^2 \left(\frac{\mu_F^2}{s} \right) \right] \delta(s_4) \left. \right\}
\end{aligned}$$

where $V_2 = C_F^2 \left(\frac{511}{64} - \frac{35}{8} \zeta_2 - \frac{15}{4} \zeta_3 + \frac{\zeta_2^2}{10} \right) + C_F C_A \left(-\frac{1535}{192} + \frac{37}{9} \zeta_2 + \frac{7}{4} \zeta_3 - \frac{3}{20} \zeta_2^2 \right) + C_F n_f \left(\frac{127}{96} - \frac{7}{9} \zeta_2 + \frac{\zeta_3}{2} \right)$

N³LO soft-gluon corrections

$$\begin{aligned}
\frac{d^2 \hat{\sigma}^{(3)}}{dt_1 du_1} &= F_{q\bar{q} \rightarrow H^+ H^-}^B \frac{\alpha_s^3(\mu_R)}{\pi^3} \left\{ \frac{1}{8} c_3^3 \mathcal{D}_5(s_4) + \left[\frac{5}{8} c_3^2 c_2 - \frac{5}{24} c_3^2 \beta_0 \right] \mathcal{D}_4(s_4) \right. \\
&+ \left[c_3 c_2^2 + \frac{1}{2} c_3^2 c_1 - \zeta_2 c_3^3 + \frac{\beta_0^2}{12} c_3 - \frac{5}{6} \beta_0 c_3 c_2 - \beta_0 C_F c_3 \ln \left(\frac{\mu_F^2}{\mu_R^2} \right) + 4c_3 A_q^{(2)} \right] \mathcal{D}_3(s_4) \\
&+ \left[\frac{3}{2} c_3 c_2 c_1 + \frac{1}{2} c_2^3 - 3\zeta_2 c_3^2 c_2 + \frac{5}{2} \zeta_3 c_3^3 - \frac{\beta_0}{4} c_3 c_1 + \frac{9}{8} \beta_0 \zeta_2 c_3^2 \right. \\
&\quad \left. + (3c_2 - \beta_0) \left(-\frac{\beta_0}{4} c_2 - \frac{\beta_0}{2} C_F \ln \left(\frac{\mu_F^2}{\mu_R^2} \right) + 2A_q^{(2)} \right) - C_F \frac{\beta_1}{4} - \frac{3}{2} c_3 X_1 \right] \mathcal{D}_2(s_4) \\
&+ \left[\frac{1}{2} c_3 c_1^2 + c_2^2 c_1 - \zeta_2 c_3^2 c_1 - \frac{5}{2} \zeta_2 c_3 c_2^2 + 5\zeta_3 c_3^2 c_2 + \frac{5}{4} \zeta_2^2 c_3^3 - \frac{15}{4} \zeta_4 c_3^3 \right. \\
&\quad \left. - \frac{\beta_0^2}{4} \zeta_2 c_3 - \frac{5}{3} \beta_0 \zeta_3 c_3^2 + \beta_0 \zeta_2 c_3 c_2 + (2c_1 - 5\zeta_2 c_3) \left(-\frac{\beta_0}{4} c_2 - \frac{\beta_0}{2} C_F \ln \left(\frac{\mu_F^2}{\mu_R^2} \right) + 2A_q^{(2)} \right) \right. \\
&\quad \left. + (\beta_0 - 2c_2) X_1 + c_3 X_0 + 4A_q^{(3)} + C_F \frac{\beta_0^2}{4} \ln^2 \left(\frac{\mu_F^2}{\mu_R^2} \right) - 2\beta_0 A_q^{(2)} \ln \left(\frac{\mu_F^2}{\mu_R^2} \right) \right. \\
&\quad \left. + C_F \frac{\beta_1}{4} \ln \left(\frac{\mu_R^2}{s} \right) + C_F \frac{\beta_1}{4} \ln \left(\frac{t_1 u_1}{m^4} \right) \right] \mathcal{D}_1(s_4)
\end{aligned}$$

$$\begin{aligned}
& + \left[\frac{1}{2} c_2 c_1^2 + 3\zeta_5 c_3^3 - \frac{15}{4} \zeta_4 c_3^2 c_2 - 2\zeta_2 \zeta_3 c_3^3 + \zeta_3 c_3^2 c_1 + 2\zeta_3 c_3 c_2^2 + \frac{5}{4} \zeta_2^2 c_3^2 c_2 - \zeta_2 c_3 c_2 c_1 - \frac{\zeta_2}{2} c_2^3 \right. \\
& \quad + \frac{\beta_0}{12} c_3 \left(15\zeta_4 c_3 - 8\zeta_3 c_2 - 6\zeta_2^2 c_3 + 3\zeta_2 c_1 \right) \\
& \quad + (4\zeta_3 c_3 - 3\zeta_2 c_2) \left(-\frac{\beta_0}{4} c_2 - \frac{\beta_0}{2} C_F \ln \left(\frac{\mu_F^2}{\mu_R^2} \right) + 2A_q^{(2)} \right) \\
& \quad + (\zeta_2 c_3 - c_1) X_1 + c_2 X_0 - \frac{\beta_0^2}{4} C_F \left(\ln^2 \left(\frac{-t_1}{m^2} \right) + \ln^2 \left(\frac{-u_1}{m^2} \right) \right) \ln \left(\frac{\mu_R^2}{s} \right) \\
& \quad + \frac{\beta_0^2}{16} c_2 \ln^2 \left(\frac{\mu_R^2}{s} \right) + \frac{\beta_0^2}{8} C_F \ln \left(\frac{\mu_F^2}{s} \right) \ln^2 \left(\frac{\mu_R^2}{s} \right) \\
& \quad \left. - \frac{\beta_1}{8} C_F \left(\ln^2 \left(\frac{-t_1}{m^2} \right) + \ln^2 \left(\frac{-u_1}{m^2} \right) \right) + Y_0 \right\} \mathcal{D}_0(s_4)
\end{aligned}$$

where

$$\begin{aligned}
X_1 & = \frac{\beta_0}{4} \zeta_2 c_3 - \frac{\beta_0}{4} c_2 \ln \left(\frac{\mu_R^2}{s} \right) + \frac{\beta_0}{2} C_F \ln^2 \left(\frac{-t_1}{m^2} \right) + \frac{\beta_0}{2} C_F \ln^2 \left(\frac{-u_1}{m^2} \right) \\
& \quad + 2A_q^{(2)} \ln \left(\frac{t_1 u_1}{m^4} \right) - 2D_q^{(2)} - \frac{\beta_0}{4} C_F \ln^2 \left(\frac{\mu_F^2}{s} \right) + 2A_q^{(2)} \ln \left(\frac{\mu_F^2}{s} \right),
\end{aligned}$$

$$\begin{aligned}
X_0 = & V_2 - \frac{1}{2}V_1^2 - \frac{1}{4}\zeta_2^2 c_3^2 + \frac{3}{4}\zeta_4 c_3^2 + \frac{\beta_0}{4}c_1 \ln\left(\frac{\mu_R^2}{s}\right) + \frac{\beta_0}{6}\zeta_3 c_3 - \frac{\beta_0}{4}\zeta_2 c_2 - \frac{\beta_0}{2}\zeta_2 C_F \ln\left(\frac{\mu_F^2}{\mu_R^2}\right) \\
& + 2A_q^{(2)}\zeta_2 - 2\gamma_q^{(2)} \ln\left(\frac{\mu_F^2}{s}\right) + \frac{\beta_0}{8}\left[2\gamma_q^{(1)} - C_F \ln\left(\frac{t_1 u_1}{m^4}\right)\right] \ln^2\left(\frac{\mu_F^2}{s}\right) \\
& + A_q^{(2)} \ln\left(\frac{t_1 u_1}{m^4}\right) \ln\left(\frac{\mu_F^2}{s}\right) + \frac{\beta_0}{6}C_F \ln^3\left(\frac{-t_1}{m^2}\right) + \frac{\beta_0}{6}C_F \ln^3\left(\frac{-u_1}{m^2}\right) \\
& + \left(C_F \frac{\beta_0}{4} + A_q^{(2)}\right) \left[\ln^2\left(\frac{-t_1}{m^2}\right) + \ln^2\left(\frac{-u_1}{m^2}\right)\right],
\end{aligned}$$

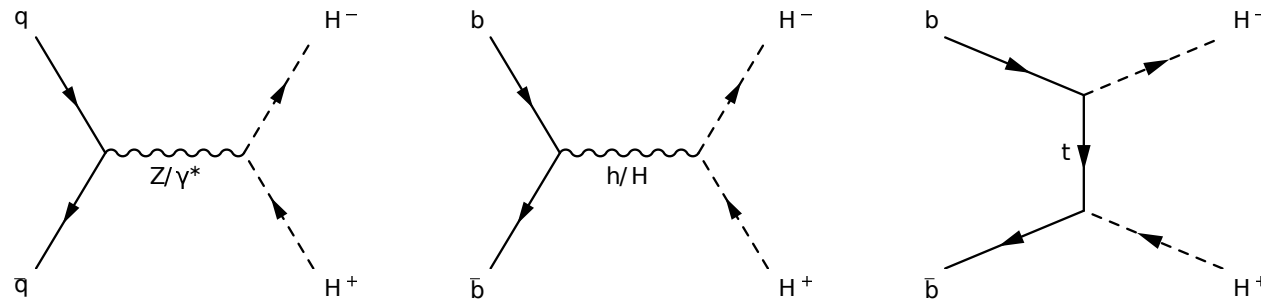
and

$$\begin{aligned}
Y_0 = & -C_F \frac{\beta_0^2}{24} \left[\ln^3\left(\frac{\mu_F^2}{\mu_R^2}\right) + \ln^3\left(\frac{\mu_R^2}{s}\right)\right] + \frac{1}{16} \left(C_F \beta_1 + 8\beta_0 A_q^{(2)}\right) \left[\ln^2\left(\frac{\mu_F^2}{\mu_R^2}\right) - \ln^2\left(\frac{\mu_R^2}{s}\right)\right] \\
& - \left(A_q^{(2)}\beta_0 + C_F \frac{\beta_1}{8}\right) \ln\left(\frac{t_1 u_1}{m^4}\right) \ln\left(\frac{\mu_R^2}{s}\right) + D_q^{(2)}\beta_0 \ln\left(\frac{\mu_R^2}{s}\right) - 2A_q^{(3)} \ln\left(\frac{\mu_F^2}{s}\right) + 2D_q^{(3)} \\
& - \frac{C_F}{6}\beta_0^2 \left[\ln^3\left(\frac{-t_1}{m^2}\right) + \ln^3\left(\frac{-u_1}{m^2}\right)\right] - \frac{\beta_0}{4} \left(C_F \beta_0 + 4A_q^{(2)}\right) \left[\ln^2\left(\frac{-t_1}{m^2}\right) + \ln^2\left(\frac{-u_1}{m^2}\right)\right] \\
& + \left(\beta_0 D_q^{(2)} - 2A_q^{(3)}\right) \ln\left(\frac{t_1 u_1}{m^4}\right)
\end{aligned}$$

$H^+ H^-$ processes in 2HDM

[for details see N. Kidonakis and A. Tonerio, JHEP 06, 138 (2024)]

partonic processes for charged-Higgs pair production
in type II Two-Higgs-Doublet Model



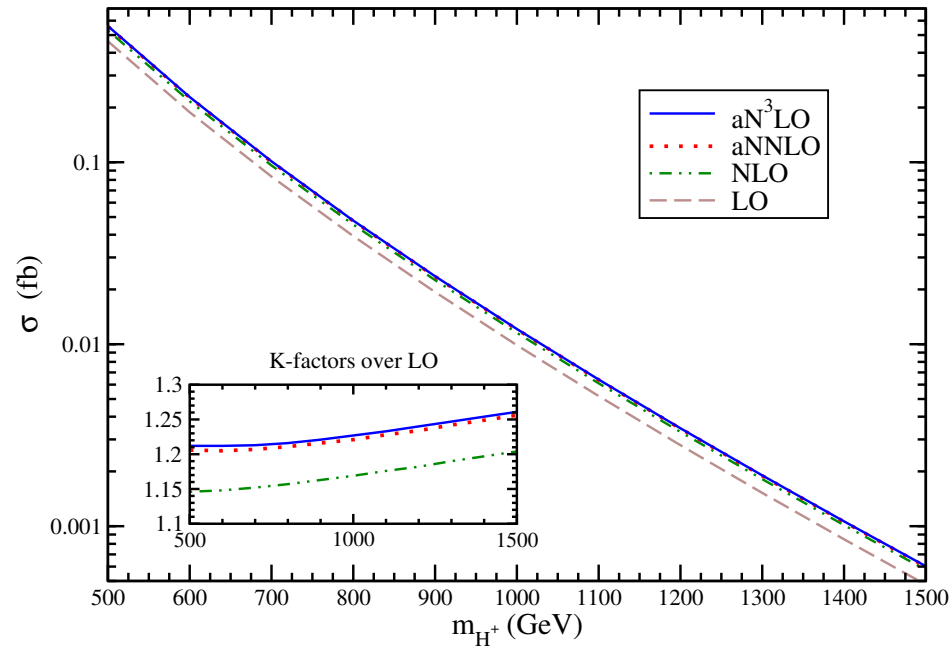
we consider Drell-Yan type production of $H^+ H^-$ (left diagram),
which is dominant at low $\tan \beta$

H⁺H⁻ cross sections at 13 TeV

pp → H⁺H⁻ cross section via quark-antiquark annihilation at 13 TeV

<i>m_H</i> (GeV)	σ LO (fb)	σ NLO (fb)	σ aNNLO (fb)	σ aN ³ LO (fb)
600	0.188 ^{+0.016+0.006} _{-0.014-0.006}	0.216 ^{+0.005+0.007} _{-0.006-0.007}	0.227 ^{+0.003+0.007} _{-0.001-0.007}	0.228 ^{+0.003+0.007} _{-0.002-0.007}
800	0.0393 ^{+0.0040+0.0017} _{-0.0034-0.0015}	0.0454 ^{+0.0011+0.0019} _{-0.0013-0.0018}	0.0476 ^{+0.0004+0.0020} _{-0.0002-0.0019}	0.0478 ^{+0.0006+0.0020} _{-0.0002-0.0019}
1000	0.00988 ^{+0.00114+0.00052} _{-0.00096-0.00049}	0.01155 ^{+0.00030+0.00060} _{-0.00038-0.00057}	0.01207 ^{+0.00008+0.00063} _{-0.00010-0.00060}	0.01212 ^{+0.00009+0.00063} _{-0.00003-0.00060}
1200	0.00279 ^{+0.00035+0.00019} _{-0.00030-0.00018}	0.00329 ^{+0.00009+0.00022} _{-0.00012-0.00021}	0.00344 ^{+0.00002+0.00023} _{-0.00004-0.00022}	0.00345 ^{+0.00001+0.00023} _{-0.00001-0.00022}

p p → H⁺H⁻ cross section $\sqrt{s}=13$ TeV
MSHT20 aN³LO pdf $\mu=m_{H^+}$

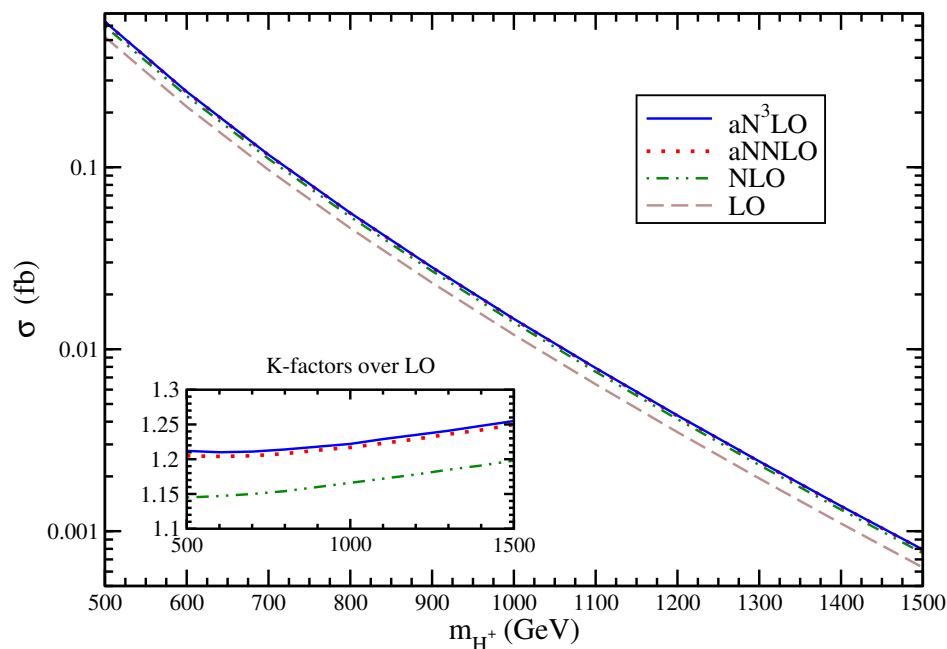


H⁺H⁻ cross sections at 13.6 TeV

pp → H⁺H⁻ cross section via quark-antiquark annihilation at 13.6 TeV

<i>m_H</i> (GeV)	σ LO (fb)	σ NLO (fb)	σ aNNLO (fb)	σ aN ³ LO (fb)
600	0.215 ^{+0.017+0.007} -0.015-0.007	0.247 ^{+0.006+0.008} -0.006-0.007	0.259 ^{+0.003+0.008} -0.002-0.008	0.260 ^{+0.004+0.008} -0.002-0.008
800	0.0463 ^{+0.0046+0.0018} -0.0039-0.0017	0.0534 ^{+0.0012+0.0021} -0.0016-0.0020	0.0559 ^{+0.0005+0.0022} -0.0003-0.0021	0.0562 ^{+0.0007+0.0022} -0.0003-0.0021
1000	0.0120 ^{+0.0014+0.0006} -0.0011-0.0006	0.0140 ^{+0.0004+0.0007} -0.0005-0.0007	0.0146 ^{+0.0001+0.0007} -0.0001-0.0007	0.0147 ^{+0.0001+0.0007} -0.0001-0.0007
1200	0.00350 ^{+0.00043+0.00023} -0.00036-0.00022	0.00413 ^{+0.00011+0.00026} -0.00015-0.00025	0.00431 ^{+0.00003+0.00027} -0.00004-0.00026	0.00432 ^{+0.00002+0.00027} -0.00001-0.00026

p p → H⁺H⁻ cross section √S=13.6 TeV
MSHT20 aN³LO pdf μ=m_{H⁺}



cross section with only NLO soft-gluon corrections is $\sim 1\%$ or less different from exact NLO \rightarrow the soft-gluon corrections are dominant

Relative contributions of the different logarithmic powers

consider $m_H = 600$ GeV and an energy of 13.6 TeV and set $\mu_F = \mu_R = m_H$

contributions to the NNLO corrections

\mathcal{D}_3 terms: 0.0134 fb

$\mathcal{D}_3 + \mathcal{D}_2$ terms: 0.0189 fb

$\mathcal{D}_3 + \mathcal{D}_2 + \mathcal{D}_1$ terms: 0.0150 fb

$\mathcal{D}_3 + \mathcal{D}_2 + \mathcal{D}_1 + \mathcal{D}_0$ terms: 0.0118 fb

further add the NNLO $\delta(s_4)$ terms, then full NNLO S+V correction is 0.0123 fb

virtual contribution to the NNLO soft-plus-virtual corrections is small

the soft-gluon corrections alone are dominant

contributions to N³LO corrections

\mathcal{D}_5 terms: 0.00993 fb

$\mathcal{D}_5 + \mathcal{D}_4$ terms: 0.0176 fb

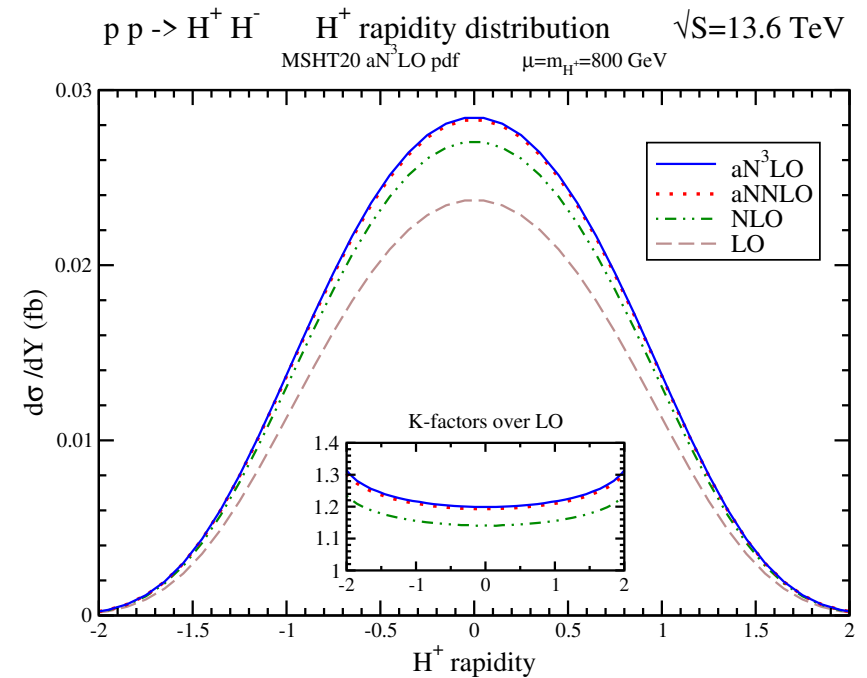
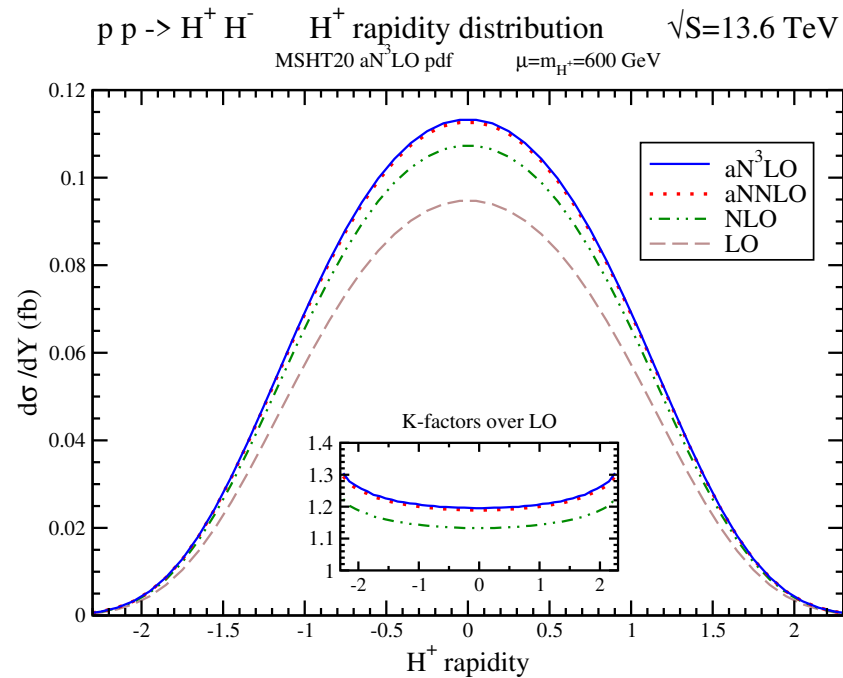
$\mathcal{D}_5 + \mathcal{D}_4 + \mathcal{D}_3$ terms: 0.0155 fb

$\mathcal{D}_5 + \mathcal{D}_4 + \mathcal{D}_3 + \mathcal{D}_2$ terms: 0.00898 fb

$\mathcal{D}_5 + \mathcal{D}_4 + \mathcal{D}_3 + \mathcal{D}_2 + \mathcal{D}_1$ terms: 0.00200 fb

$\mathcal{D}_5 + \mathcal{D}_4 + \mathcal{D}_3 + \mathcal{D}_2 + \mathcal{D}_1 + \mathcal{D}_0$ terms: 0.00133 fb

H^+ rapidity distributions at 13.6 TeV



K -factors increase at larger rapidities

Summary

- $H^+ H^-$ production
- soft-gluon corrections through N³LO in single-particle-inclusive kinematics
- results for total cross sections and rapidity distributions
- higher-order corrections further enhance and improve the theoretical predictions
- analytical results for resummation and N³LO expansions applicable to other processes with colorless final states