The Art of Counting: where to cut-off SMEFT and HEFT in Higgs Pair Production

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based on arXiv: 24xx.xxxxx with Ilaria Brivio and Ramona Gröber

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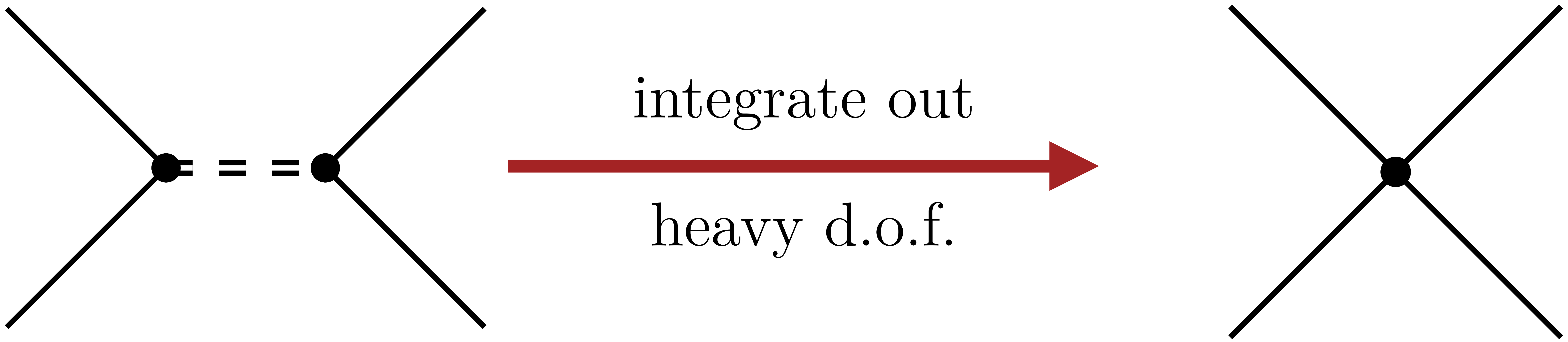
Why using Effective Field Theories (EFTs)?

UV completion with heavy degrees of freedom

Effective Field Theories
$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{dim. } 4} + \mathcal{L}_{\text{dim.} > 4}$



EFTs hide the effect of new Physics, i.e. parameterization of the **unknown**!

How to construct the effective part **without** knowing the UV completion?

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Bottom-up construction of effective operators

Particle Content

- light degrees of freedom
- in the following: SM particles

Symmetries

- Lorentz invariance
- gauge invariance

Power counting

- heavy suppression scales
- expansion in loops/couplings

These 3 ingredients and removing redundancies allows us to construct operator basis!

Focus of this talk: Standard Model EFT (**SMEFT**) and Higgs EFT (**HEFT**)

SMEFT vs. HEFT

SMEFT

SM Particles

physical Higgs is part of doublet under $SU(2)_L$

$SU(3)_c \times SU(2)_L \times U(1)_Y$

Higgs doublet gives rise to correlations:

$$H^n \sim (v+h)^n \xrightarrow[|H|^2 G^a_{\mu\nu} G^{a\mu\nu}]{} C_{HG}(v+h)^2 G^a_{\mu\nu} G^{a\mu\nu}$$

HEFT

treat physical Higgs and GBs separately

SM Particles

$SU(3)_c \times U(1)_{\mathrm{em}}$

two **different** building blocks:

$$U(\boldsymbol{\pi}) = \exp\left(i\,\frac{\pi^a T^a}{v}\right) \qquad \mathcal{F}_i(h) = \sum_n c_{in}\left(\frac{h}{v}\right)^n$$

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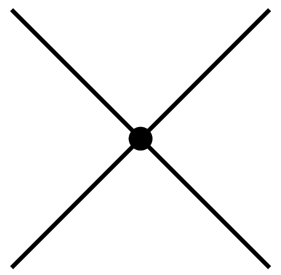
From operators to cross-sections

general procedure for perturbative QFTs:

set of operators $\{\mathcal{O}_i\}$ → vertices



→ matrix elements $\mathcal{M}$ → cross-sections $\sigma = \frac{1}{\Phi}\int d\Pi\,|\mathcal{M}|^2$

result: cross-section as expansion in one or multiple small parameters $\{\delta_i\}$

generic SM example with loop/coupling expansion:

$$\sigma = \sigma_0 \left(1 + A_1 \times \frac{\alpha_s}{4\pi} + A_2 \times \left(\frac{\alpha_s}{4\pi}\right)^2 + \dots\right)$$

truncation error

Scaling of general interactions

Investigate how powers of suppression scales and loop orders propagate in calculation!

keep track of mass dimensions and powers of $\hbar \longleftrightarrow 1/(4\pi)^2$ starting from the **action**:

$$S \sim \hbar \xrightarrow[\text{kinetic terms}]{\mathcal{L} \sim \hbar m^4} \phi \sim m\sqrt{\hbar},\ \psi \sim m^{3/2}\sqrt{\hbar} \xrightarrow[\text{interactions}]{} \mathcal{L} \sim g^P_{FS}\, \partial^P \phi^S \psi^F$$

use this most general operator to find scaling of **couplings/Wilson coefficients**:

$$g^P_{FS} = \hat{g}^P_{FS}\, m^{4-P-S-3F/2}\, \hbar^{1-(F+S)/2}$$

[Manohar, Georgi 1984]
[Jenkins et al. 2013]
[Buchalla et al. 2013, 2016]
[Gavela et al. 2016]

Examples:

gauge couplings: $g = \hat{g}\,(4\pi)$

Yukawa couplings: $y = \hat{y}\,(4\pi)$

scalar quartic couplings: $\lambda = \hat{\lambda}\,(4\pi)^2$

SMEFT example: $C_{HG}\,|H|^2 G^a_{\mu\nu} G^{a\mu\nu} \longrightarrow C_{HG} = \hat{C}_{HG}\,\dfrac{(4\pi)^2}{\Lambda^2}$

Scaling of matrix elements

$$\mathcal{M} \sim p^{F/2+P}\Lambda^{N_\Lambda}(4\pi)^{N_{4\pi}}$$

- $F/2$: polarizations of external fermions
- $P = 4L - 2I_S - I_F + \sum_v P_v$
- $N_\Lambda = \sum_v N_{\Lambda,v}$
- $N_{4\pi} = -2L + \sum_v N_{4\pi,v}$

employing the graph theory identity $L = I - V + 1$, one may also write:

$$N_{4\pi} = S + F - 2 \qquad N_\Lambda = 4 - P - S - \frac{3}{2}F$$

S – external scalars, F – external fermions

Scaling of cross-sections

$2 \to n$ process

$$\sigma = \frac{1}{\Phi} \int d\Pi \, \mathcal{M}_1 \mathcal{M}_2^*$$

$$\Phi = 2\sqrt{\lambda(s, m_a^2, m_b^2)}\,(2\pi)^{3n-4} \sim p^2 (4\pi)^{3n-4}$$

$$d\Pi = \prod_i^n \frac{d^3 p_i}{2E_i} \delta^{(4)}\left(\sum p\right) \sim p^{2n-4} (4\pi)^{n-1}$$

combining matrix elements, flux factor, and phase space:

$$\sigma \sim p^{-2} (4\pi)^3 \, (p/\Lambda)^{-(\alpha_\Lambda + \alpha_v)} \; (4\pi v/\Lambda)^{\alpha_v} \; (4\pi)^{-(\alpha_{4\pi}^g + \alpha_v)}$$

[Gavela et al. 2016]
[Buchalla et al. 2016]

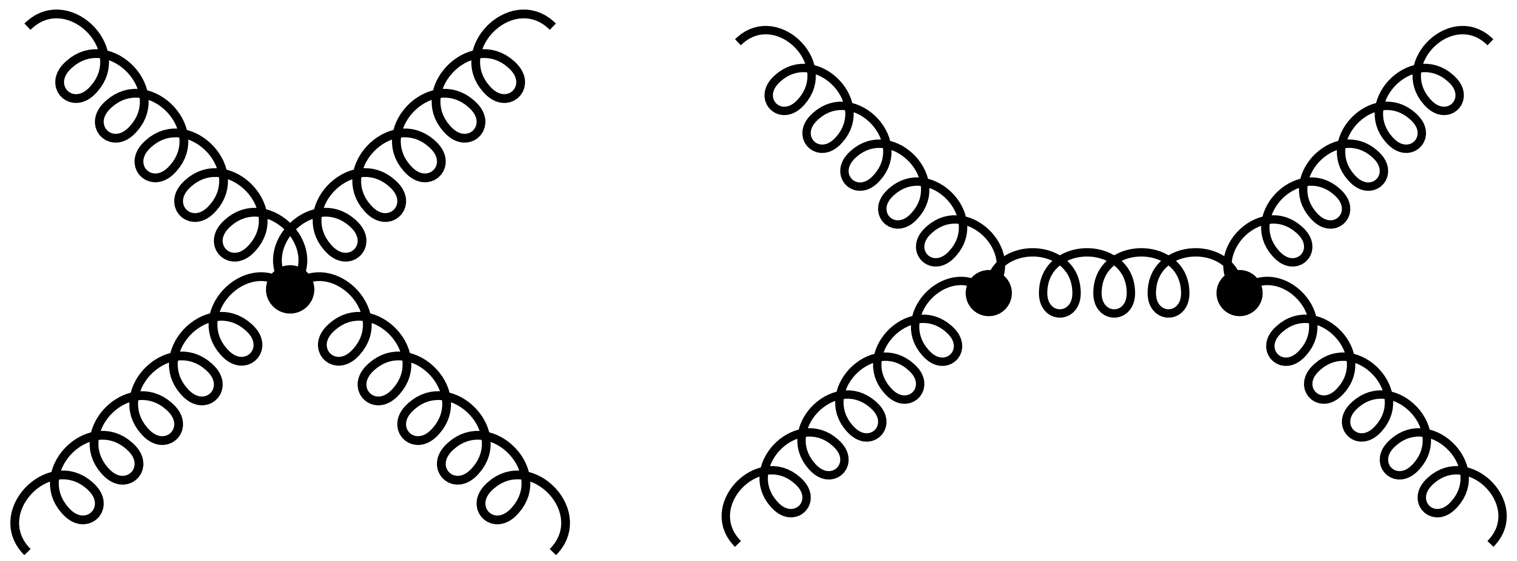
α_Λ – powers of Λ, α_v – powers of v, $\alpha_{4\pi}^g$ – 4π's from couplings/Wilson coefficients/v

A renormalizable example

$AA \to AA$ in QCD with a coloured fermion:

$$\mathcal{L} = -\frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \bar{\psi} i \not{D} \psi \sim g\,\partial A^3, g^2 A^4, g\psi^2 A$$

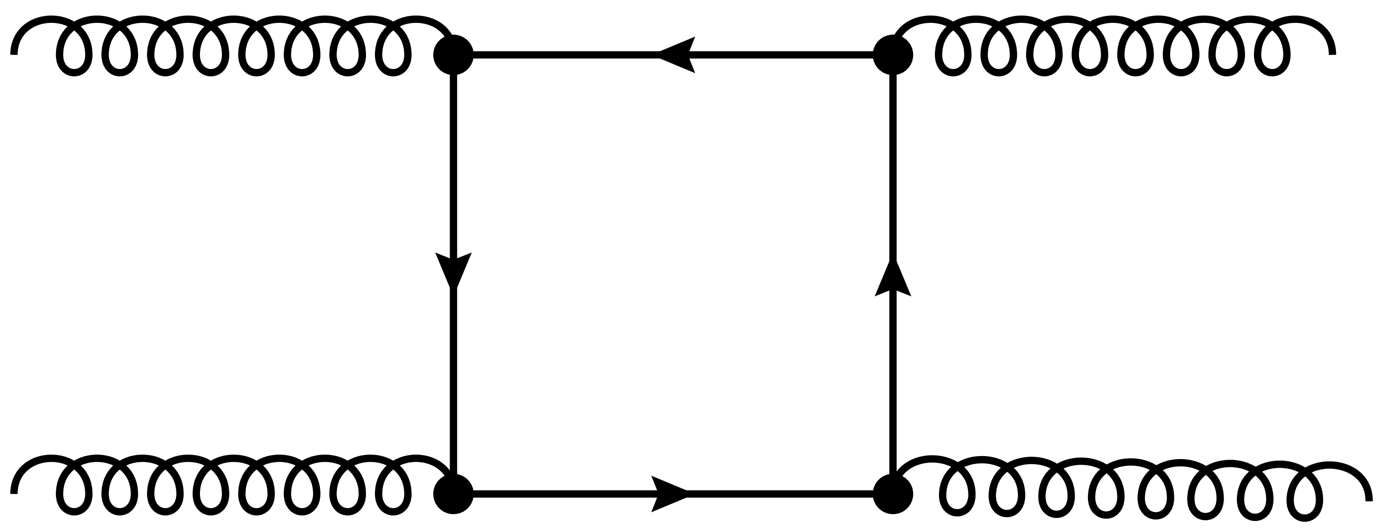
tree:



$\xrightarrow[g = \hat{g}\,(4\pi)]{|\mathcal{M}|^2 \sim g^4}$ $\alpha_\Lambda = 0$, $\alpha_v = 0$, $\alpha^g_{4\pi} = 4$ $\longrightarrow$ $p^{-2}(4\pi)^3 \left(\frac{g}{4\pi}\right)^4$

loops $\longleftrightarrow$ couplings

one-loop:



$\xrightarrow[g = \hat{g}\,(4\pi)]{|\mathcal{M}|^2 \sim g^8}$ $\alpha_\Lambda = 0$, $\alpha_v = 0$, $\alpha^g_{4\pi} = 8$ $\longrightarrow$ $p^{-2}(4\pi)^3 \left(\frac{g}{4\pi}\right)^8$

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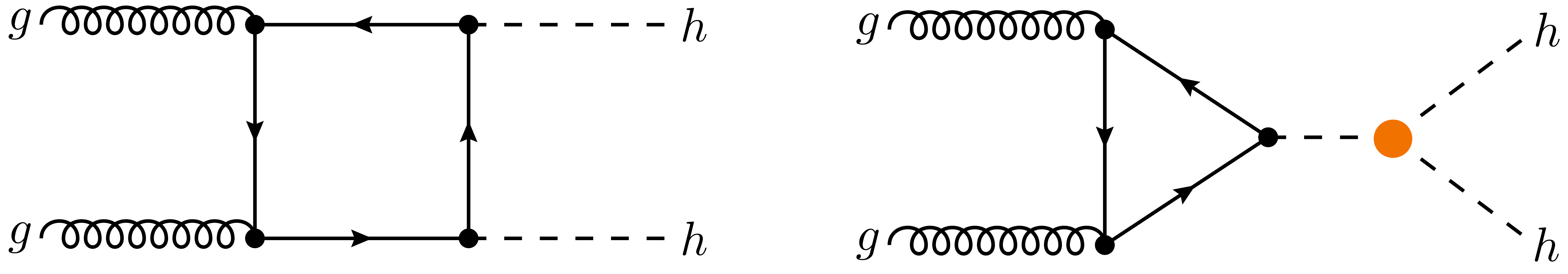
Di-Higgs production - Motivation

1) General theoretical point of view - probe of the Higgs sector



→ Higgs trilinear coupling and BSM searches

2) Experimental point of view

$\sigma_{\mathrm{SM}}^{\mathrm{NLO}} \sim \mathcal{O}(30)\,\mathrm{fb}$ at LHC →

- increase in luminosity
- identification of final states $(b\bar{b}b\bar{b}, b\bar{b}\tau^+\tau^-, b\bar{b}\gamma\gamma)$

3) EFT point of view

- EFTs give rise to tree-level diagrams (scaling EFT vs. SM)
- different treatment of Higgs in SMEFT vs. HEFT

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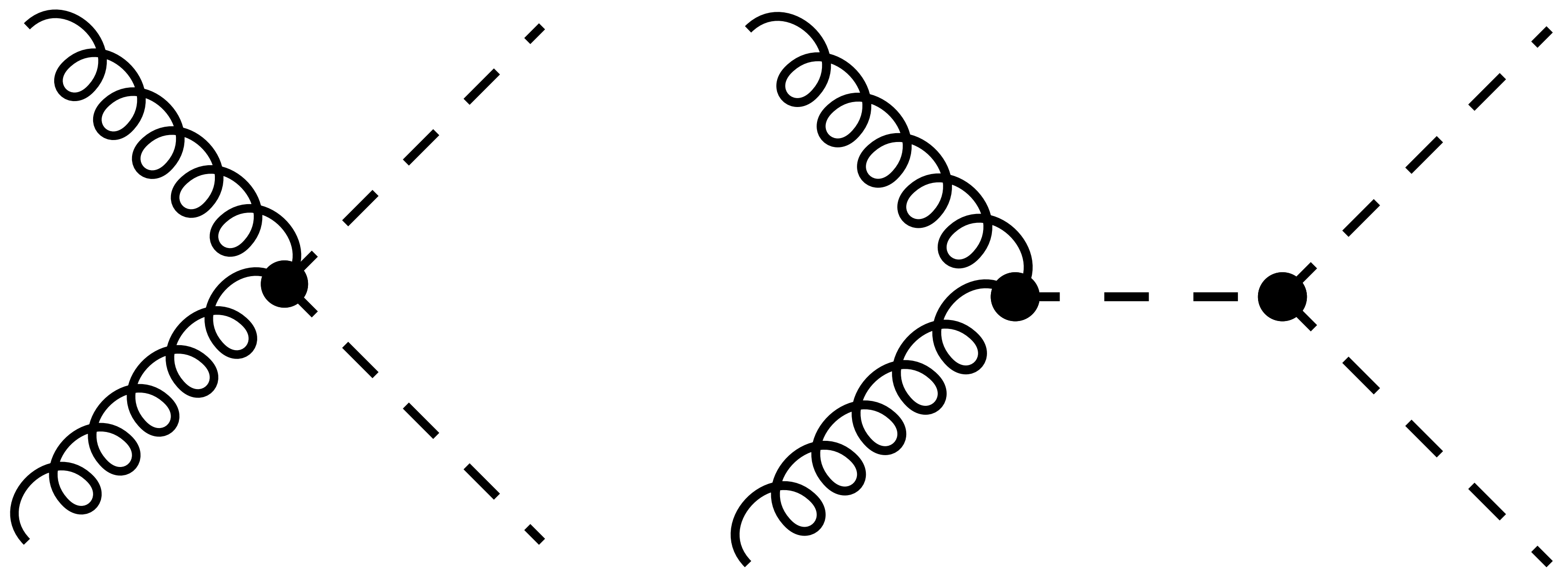
Di-Higgs production in SMEFT

[Heinrich, Lang 2022, 2023, 2024]
[Heinrich et al. 2022]

tree-level contribution at dimension 6 from $\frac{C_{HG}}{\Lambda^2} |H|^2 G^a_{\mu\nu} G^{a\mu\nu}$



$$|\mathcal{M}|^2 \sim \left\{ \left(\frac{C_{HG}}{\Lambda^2}\right)^2, \left(\frac{C_{HG}}{\Lambda^2}\lambda v^2\right)^2 \right\} \longrightarrow \begin{aligned} \alpha_\Lambda &= \{-4, -4\} \\ \alpha_v &= \{0, 4\} \\ \alpha^g_{4\pi} &= \{4, 4\} \end{aligned}$$

(no interference for sake of brevity)

$$\sigma \sim p^{-2}(4\pi)^3 \left\{ \left(\frac{p}{\Lambda}\right)^4, \left(\frac{4\pi v}{\Lambda}\right)^4 \left(\frac{\lambda}{(4\pi)^2}\right)^2 \right\} \frac{C^2_{HG}}{(4\pi)^4} \longrightarrow$$

p can also be a light mass!
$p \sim m \sim gv \sim (4\pi)\hat{g}v$

comparison to SM: $\frac{\sigma^{\text{SMEFT}}_{\mathcal{O}_{HG}}}{\sigma^{\text{SM}}} \sim \frac{(p/\Lambda)^4}{(4\pi)^{-4}} \longrightarrow$ two **orthogonal** expansions

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A comment on chiral dimensions

[Buchalla et al. 2012, 2013, 2015, 2016]
[Gavela et al. 2016]

HEFT is a mixture between SMEFT (fermion/gauge sector) and χPT (scalar sector)

idea: base counting on loop orders introducing **chiral dimensions** $d_\chi = N_{4\pi} + N_\Lambda$

⟶ e.g. for renormalizable couplings: $d_\chi(g) = d_\chi(y) = 1,\; d_\chi(\lambda) = 2$

One can always define another counting parameter as a linear combination!

But what does it mean to truncate cross-sections in this parameter?

$$\sigma \sim p^{-2}(4\pi)^3\,(p/\Lambda)^{2n}(p/\Lambda)^{-(\alpha_\chi+\alpha_\chi^g)}(4\pi v/\Lambda)^{\alpha_v}(4\pi)^{-\alpha_\chi^g}$$

Summary and Outlook

- starting from an operator (renormalizable/EFT), we can directly give an estimate of the cross-section scaling for a certain process

  ⟶ validations for SM and SMEFT (tree/loop/interference)

- counting/truncation in HEFT:
  - mixture between SMEFT and χPT
  - chiral dimensions
  - different treatment of the vev $\mathcal{F}_i(h) = \sum_n c_{in} \left(\frac{h}{v}\right)^n$

- perform consistent calculations in HEFT and application to Di-Higgs production

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Backup

Naive Dimension Analysis (NDA) formula

$$g^P_{FS} = \hat{g}^P_{FS}\, m^{4-P-S-3F/2}\, \hbar^{1-(F+S)/2}$$

$$m \longleftrightarrow \Lambda$$

$$\hbar \longleftrightarrow 1/(4\pi)^2$$

$$\hat{g}^P_{FS}\, \frac{\Lambda^4}{(4\pi)^2} \left(\frac{\partial_\mu}{\Lambda}\right)^P \left(\frac{4\pi}{\Lambda}\phi\right)^S \left(\frac{4\pi}{\Lambda^{3/2}}\psi\right)^F$$

compare to normalization of vev on slide 8

[Manohar, Georgi 1984] [Jenkins et al. 2013] [Gavela et al. 2016]

HEFT Lagrangian

$$\mathbf{V}_\mu = (D_\mu \mathbf{U})\mathbf{U}^\dagger$$

$$\begin{aligned}\mathcal{L}_2 = &-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{v^2}{4}\mathrm{Tr}\left(\mathbf{V}_\mu \mathbf{V}^\mu\right)\mathcal{F}_C(h) - V(h) \\ &+ i\overline{Q}_L \not{D} Q_L + i\overline{Q}_R \not{D} Q_R + i\overline{L}_L \not{D} L_L + i\overline{L}_R \not{D} L_R \\ &- \frac{v}{\sqrt{2}}\left(\overline{Q}_L \mathbf{U} \mathcal{Y}_Q(h) Q_R + \mathrm{h.c.}\right) - \frac{v}{\sqrt{2}}\left(\overline{L}_L \mathbf{U} \mathcal{Y}_L(h) L_R + \mathrm{h.c.}\right)\end{aligned}$$

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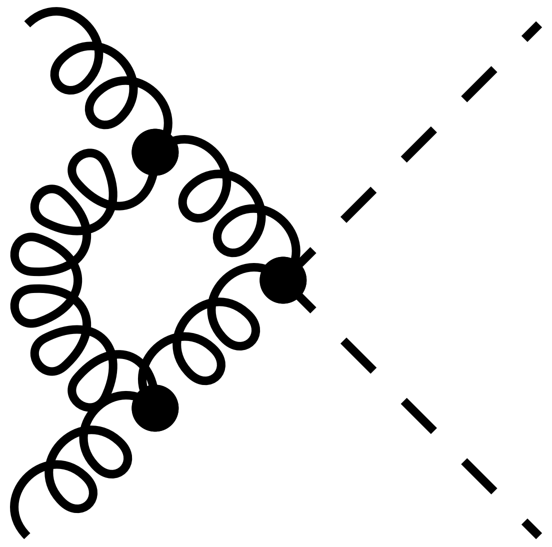
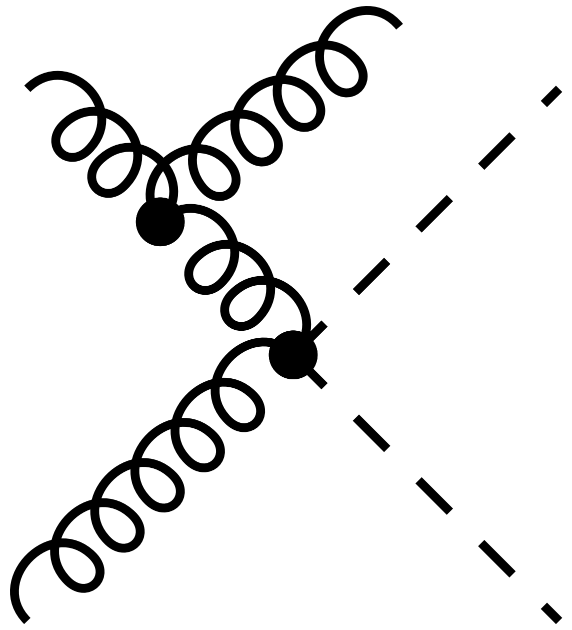
A real emission example

consider again $\frac{C_{HG}}{\Lambda^2}|H|^2 G^a_{\mu\nu} G^{a\mu\nu}$



$\longrightarrow \quad \mathcal{M}_{\text{loop}} \sim g^2 \frac{C_{HG}}{\Lambda^2}$



$\longrightarrow \quad \mathcal{M}_{\text{real}} \sim g \frac{C_{HG}}{\Lambda^2}$

$$\mathcal{M}_{\text{tree}}\,\mathcal{M}^*_{\text{loop}} \sim |\mathcal{M}_{\text{real}}|^2 \sim g^2 \left(\frac{C_{HG}}{\Lambda^2}\right)^2$$

$\longrightarrow$ same cross-section scaling!

Di-Higgs production in the SM

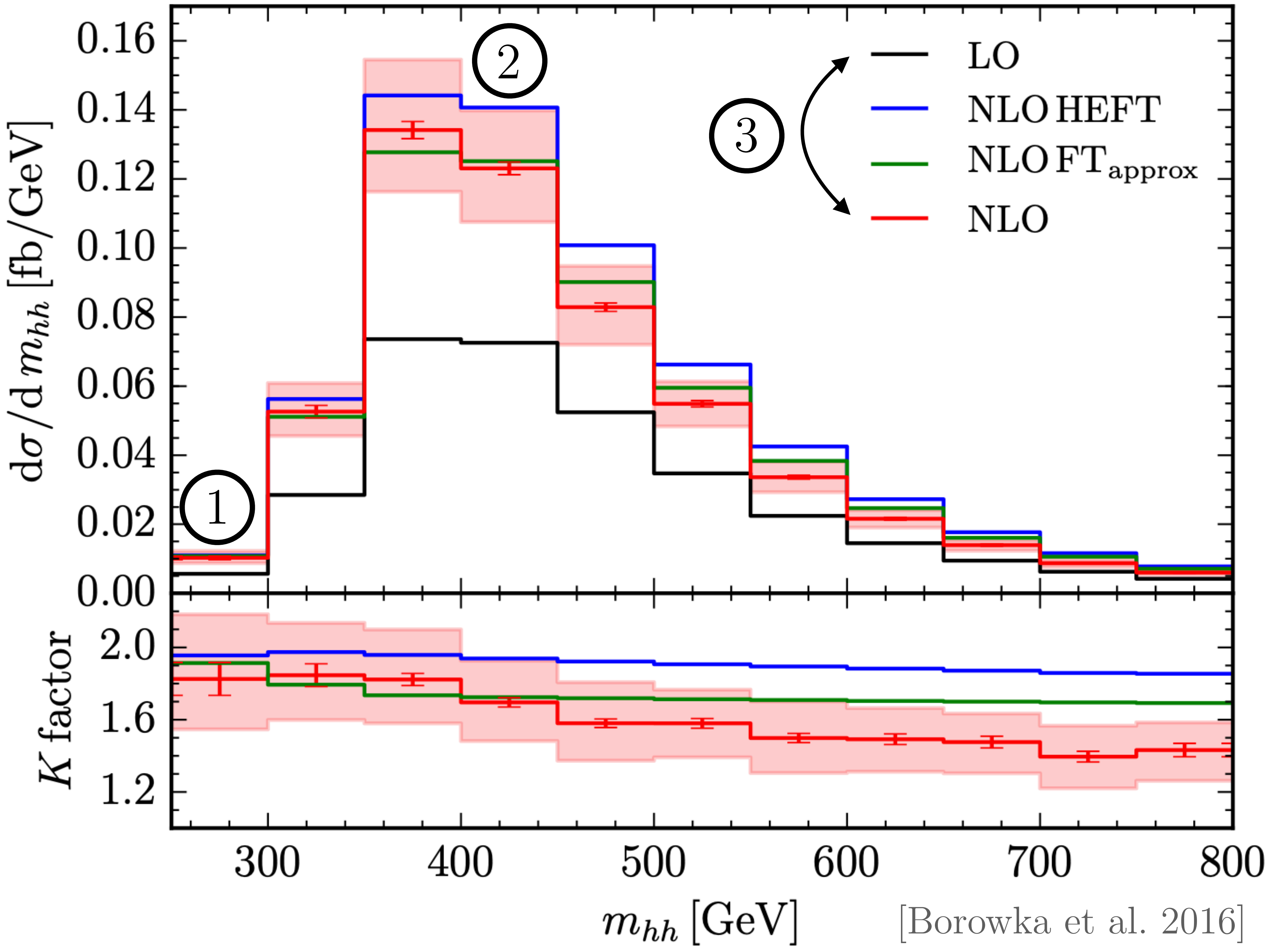
general amplitude decomposition: $\mathcal{M}(g_a g_b \to hh) = \delta^{ab} \left(\mathcal{M}_1 \, A_1^{\mu\nu} + \mathcal{M}_2 \, A_2^{\mu\nu} \right) \epsilon_\mu \epsilon_\nu$

<u>Observations</u>

① approx. cancellation at threshold (exact in HTL)

② peak at $m_{hh} \simeq 2m_t$

③ NLO QCD important $\sigma_{\text{NLO}} \sim 2\sigma_{\text{LO}} \sim \mathcal{O}(30)\,\text{fb}$



[Borowka et al. 2016]