Towards HH at NNLO QCD: the n_h^2 Contribution

Marco Vitti (Karlsruhe Institute of Technology, TTP and IAP)

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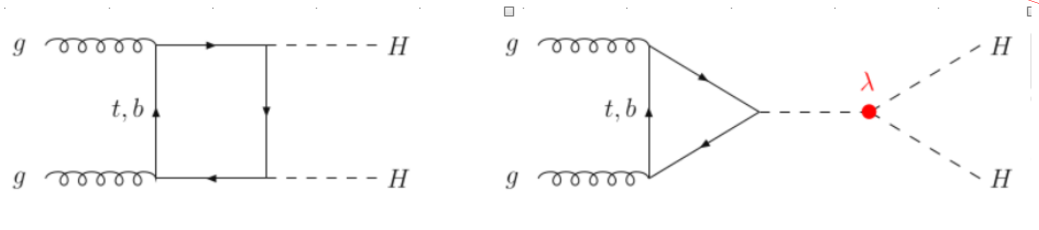
Work in collaboration with **J. Davies, K. Schönwald, M. Steinhauser**

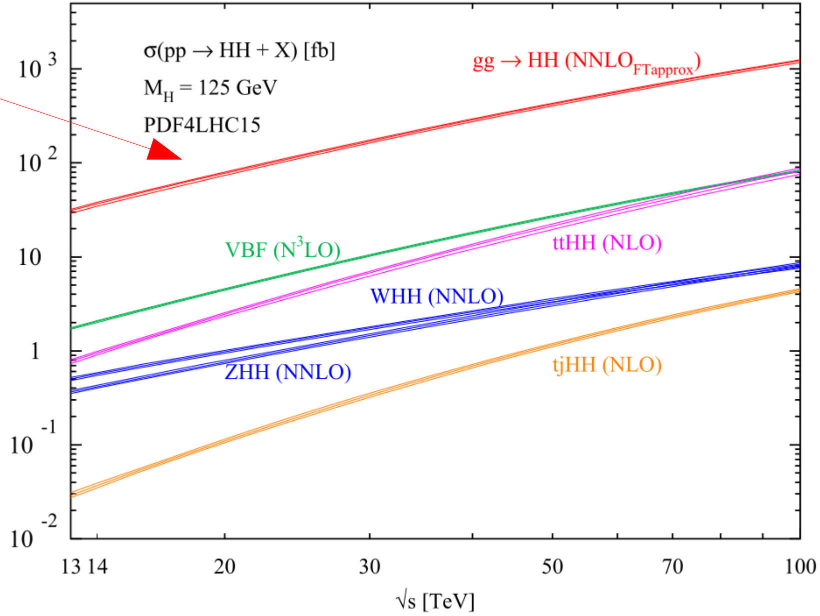
HH Production @LHC

$$V(h) = \frac{m_H^2}{2} h^2 + \lambda_3 v h^3 + \frac{\lambda_4}{4} h^4$$

$$\lambda_4^{SM} = \lambda_3^{SM} = \lambda = m_H^2/(2v^2)$$

- Best chance to measure λ_3
- Gluon-initiated channel dominant

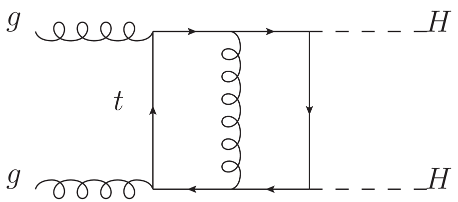




[Di Micco et al. - 1910.00012]

- LO computed in [Glover, van der Bij (‘88); Plehn et al. (96)]
- In the SM, destructive interference between triangle (signal) and box (background)
- Accurate higher-order predictions required for **both**

NLO QCD corrections for HH



Full top-mass dependence obtained via

- Numerical evaluation

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke - 1604.06447,
Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke - 1608.04798;
Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher - 1811.05692]

Multi-scale $(s, t, m_{H,} m_t)$
two-loop box integrals
No exact analytic results available

- Analytic approximations

Large Mass Expansion [Dawson, Dittmaier, Spira '98]

pT expansion [Bonciani, Degrassi, Giardino, Gröber - 1806.11564]

High-Energy expansion [Davies, Mishima, Steinhauser, Wellmann - 1811.05489]

Small-mass expansion [Wang, Wang, Xu, Xu, Yang - 2010.15649]

Full phase space covered in [Bellafronte, Degrassi, Giardino, Gröber, MV – 2202.12157;
Davies, Mishima, Schönwald, Matthias Steinhauser - 2302.01356]

Theoretical Uncertainties at NLO QCD

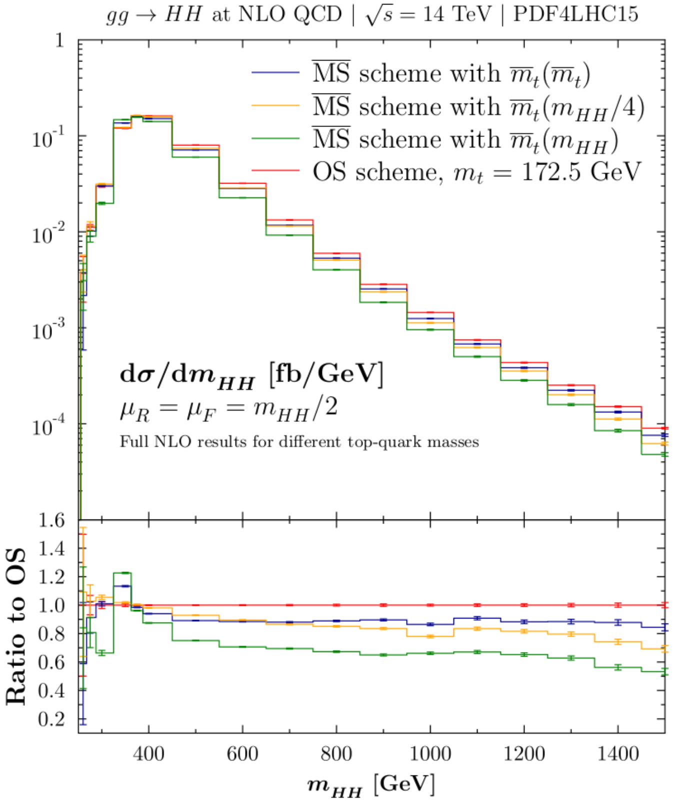
- Scale uncertainties reduced to O(15%)

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$
On-Shell	$18.22^{29.5\%}_{-21.3\%}$	-	$30.93^{13.7\%}_{-12.7\%}$	-

[Bagnaschi, Degrassi, Gröber - 2309.10525]

(Reduced to ~3% when approximate N3LO corrections are included)

- Uncertainty of ~20% due to choice of renormalization scheme and scale for the top mass
- Top mass effects must be retained at NNLO to reduce top-mass uncertainty



[Baglio et al. - 2003.03227]

Analytic approximations for NNLO QCD

Exploit **hierarchies** of masses/kinematic invariants

Pros: simplified integral structures; can change parameters and evaluate easily

Cons: proliferation of integrals; restricted to specific phase-space regions

$m_t \to \infty$ limit (N3LO)

[De Florian, Mazzitelli 1305.5206 and 1309.6594; . Grigo, Melnikov, Steinhauser – 1408.2422;
Chen, Li, Shao, Wang – 1909.06808 and 1912.13001;]

Finite $1/m_t$ effects (LME) (restricted to $s < 4m_t^2$)

[Grigo, Hoff, Steinhauser – 1508.00909;
Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli - 1803.02463;
Davies, Steinhauser – 1909.01361; Davies, Herren, Mishima, Steinhauser 2110.03697]

Forward Expansions?

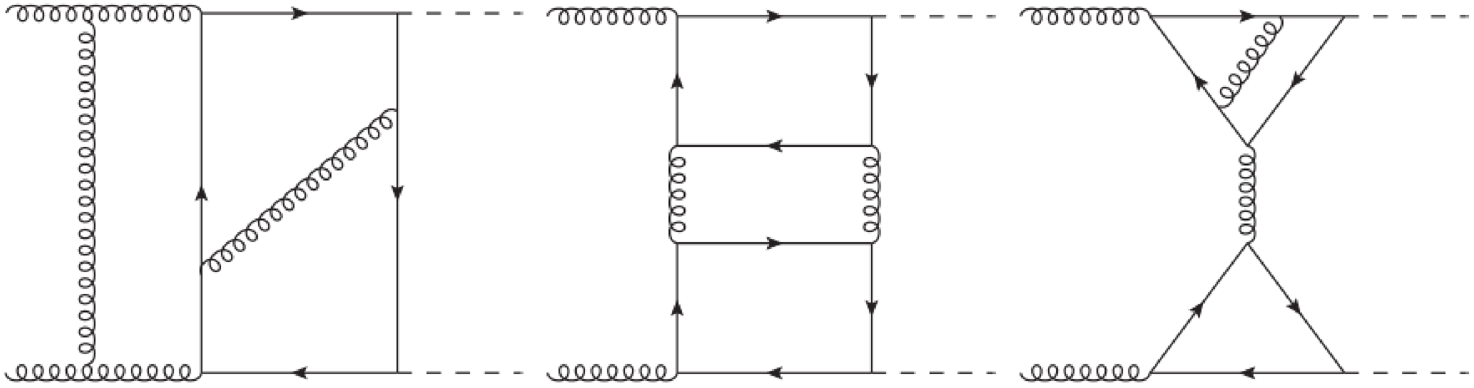
Cover ~95% of hadronic cross section at NLO QCD
Taylor expansions

- pT expansion $m_H^2, p_T^2 \ll 4m_t^2, \hat{s}$
  [Bonciani, Degrassi, Giardino, Gröber - 1806.11564]
- *t → 0* expansion $m_H^2, \hat{t} \ll 4m_t^2, \hat{s}$
  [Davies, Mishima, Schönwald, Steinhauser - 2302.01356]

Going to NNLO QCD...

Can we use the forward expansion for higher orders?

- Classes of three loop diagrams



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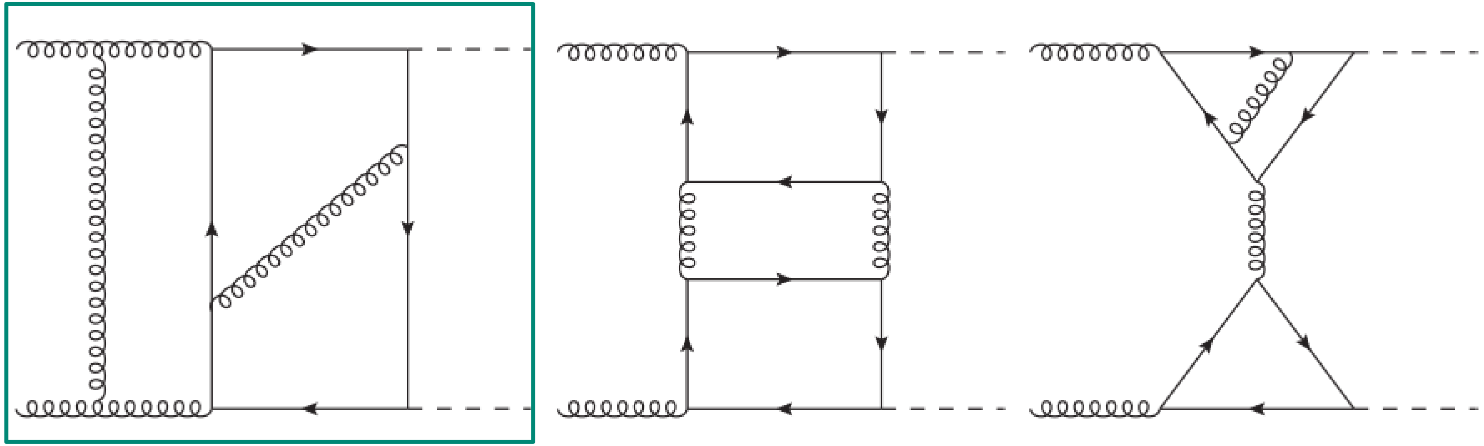
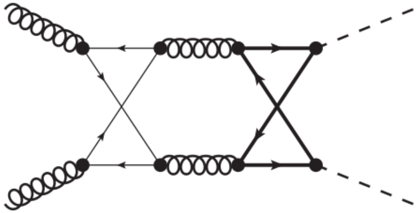


Conceptually yes

Practical implementation promising
for the *t*→*0* expansion $\{t^0, m_H^0\}$

[Davies, Schönwald, Steinhauser 2307.04796]



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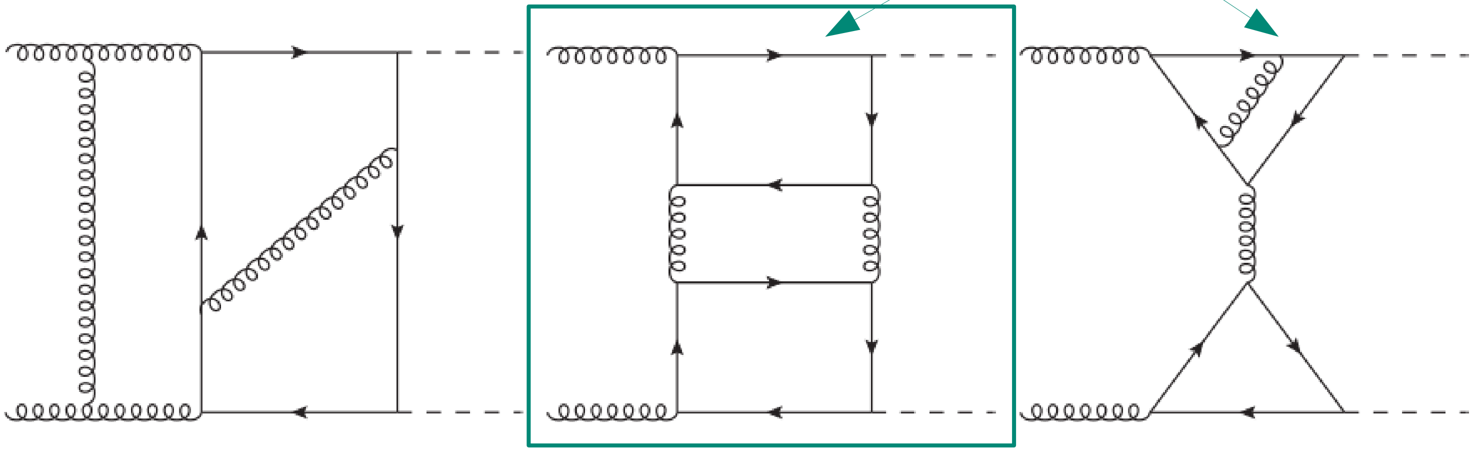
n_h^2 contribution



More involved due to t-channel cuts
through massless lines
$\Rightarrow$ A Taylor expansion is not sufficient

Going to NNLO QCD...

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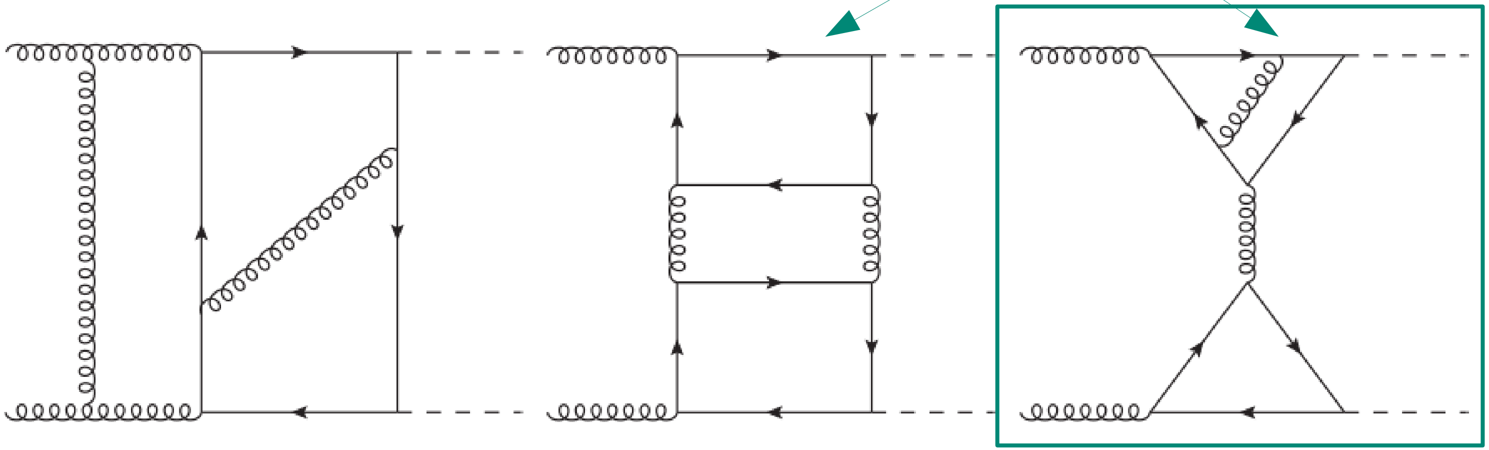
n_h^2 contribution

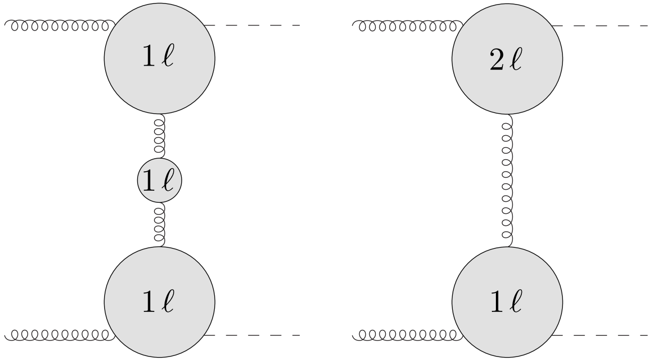


Start by studying the 1PR piece

1PR Contribution to *gg*→*HH* @ 3 Loops

[Davies, Schönwald, Steinhauser, MV - 2405.20372]

$$\mathcal{M}^{ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\mathcal{M}^{\mu\nu,ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\delta^{ab}X_0 s\left(F_1 A_1^{\mu\nu} + F_2 A_2^{\mu\nu}\right)$$



Goal: compute $F_1^{(3\ell,\ 1\mathrm{PR})}$ $\quad F_2^{(3\ell,\ 1\mathrm{PR})}$

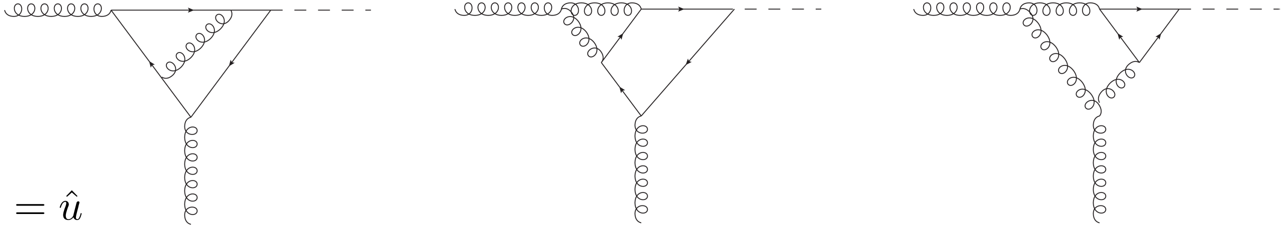
Approach: construct the *gg*→*HH* form factors from the 1PI gg*H subamplitudes

$$\mathcal{V}^{\alpha\beta}(q_s,q_2) = F_a\ g^{\alpha\beta}(q_s\cdot q_2) + F_b\ q_s^\alpha q_2^\beta + F_c\ q_2^\alpha q_s^\beta + F_d\ q_s^\alpha q_s^\beta + F_e\ q_2^\alpha q_2^\beta$$

$q_2^2 = 0, q_s^2 \neq 0$

$m_H \neq 0$

$q_s^2 = \hat{t}$ or $q_s^2 = \hat{u}$



Outline of Calculation

1. Generation of diagrams with qgraf [Nogueira, ‘93]

2. Manipulation with tapir [Gerlach, Herren, Lang - 2201.05618], q2e/exp [Harlander, Seidensticker Steinhauser – ‘97], FORM [Ruijl, Ueda, Vermaseren - 1707.06453]

3. IBP reduction (KIRA [Klappert, Lange, Maierhöfer, Usovitsch – 2008.06494])

4. Perform asymptotic expansions in two limits

$$m_H^2 \ll q_s^2, m_t^2$$

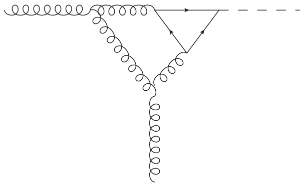
Same MIs from $t \to 0$ expansion at NLO QCD
Evaluated using “expand and match” approach

[Fael, Lange, Schönwald, Steinhauser – 2106.05296; 2202.05276]

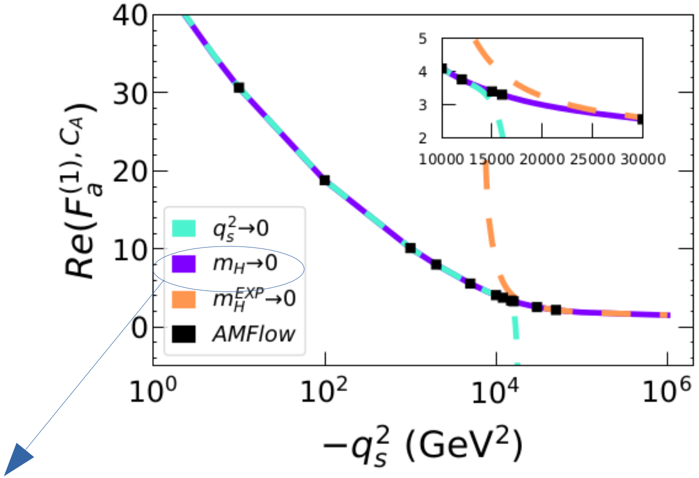
$$q_s^2 \ll m_H^2, m_t^2$$

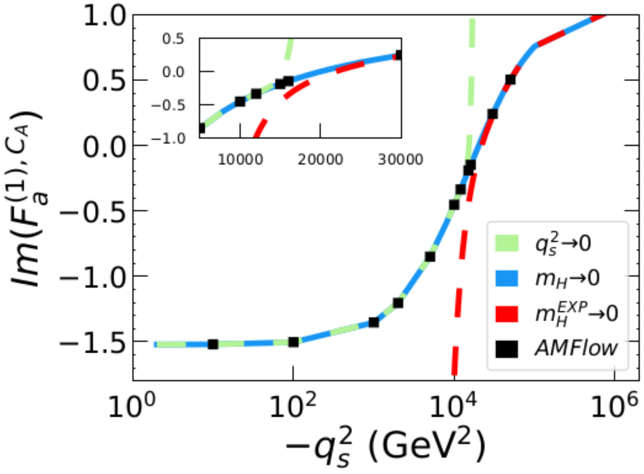
Analytic evaluation in terms of HPLs

*gg*H* Form Factors



$$\mathcal{V}^{\alpha\beta}(q_s, q_2) = \boxed{F_a}\, g^{\alpha\beta}(q_s \cdot q_2) + F_b\; q_s^\alpha q_2^\beta + F_c\; q_2^\alpha q_s^\beta + F_d\; q_s^\alpha q_s^\beta + F_e\; q_2^\alpha q_2^\beta$$



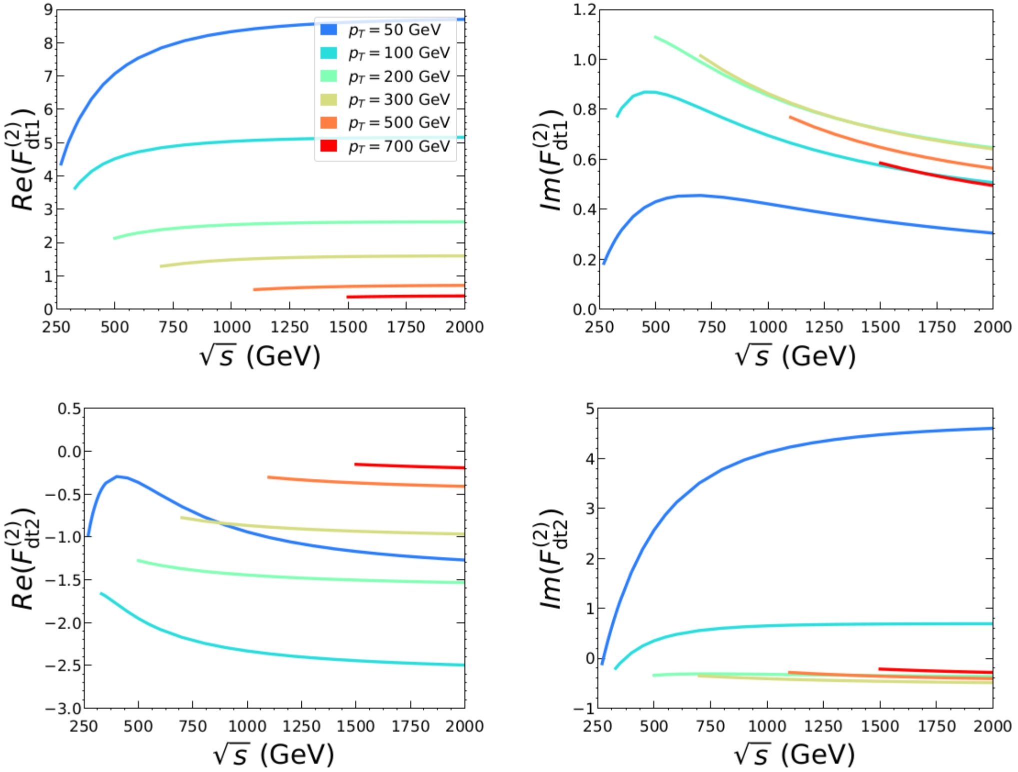


- Use expanded MIs but keep coefficients exact ($m_H \to 0$)
- Results checked with AMFlow [Liu, Ma - 2201.11669]
- Complete coverage of q_s^2 range

$gg \to HH$ Form Factors

$$\tilde{F}^{(2)}_{\mathrm{dt1}}(t) = F_a^{(0)}(t)\Bigg[F_a^{(1)}(t) + \frac{1}{2}\,F_a^{(0)}(t)\,\Pi_{gg}(t) - \frac{s\,\left(\epsilon\,(m_H^2 - 2p_T^2 + t) + 2p_T^2\right)}{(1-2\epsilon)(m_H^2 - s)t}\,F_d^{(1)}(t)\Bigg]$$

$$\tilde{F}^{(2)}_{\mathrm{dt2}}(t) = F_a^{(0)}(t)\Bigg[\frac{p_T^2}{t}\left(F_a^{(1)}(t) + \frac{1}{2}\,F_a^{(0)}(t)\,\Pi_{gg}(t)\right) - \frac{s\,\left(\epsilon\,(2p_T^2 - m_H^2 - t) + m_H^2 + t\right)}{(1-2\epsilon)(m_H^2 - s)t}\,F_d^{(1)}(t)\Bigg]$$



- Agreement with LME result of
  [Davies, Steinhauser - 1909.01361]
- Complete coverage of physical phase space for HH form factors

Conclusions

- Including NNLO QCD effects in $gg \to HH$ would allow control over scale and top-mass-scheme uncertainties
- An expansion in the forward-scattering limit is a promising way to obtain fast and flexible results *and* a wide coverage of the phase space
- At three loops, asymptotic expansions are necessary to account for the n_h^2 contribution, already in the reducible piece

Outlook

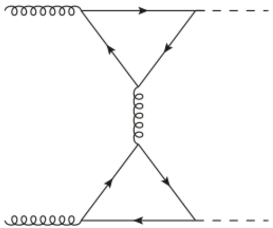
- Computation of the 1PI n_h^2 contribution (work in progress)
- Combination of all purely-virtual corrections
- Inclusion of real emission contributions
- Lots of challenges ahead...

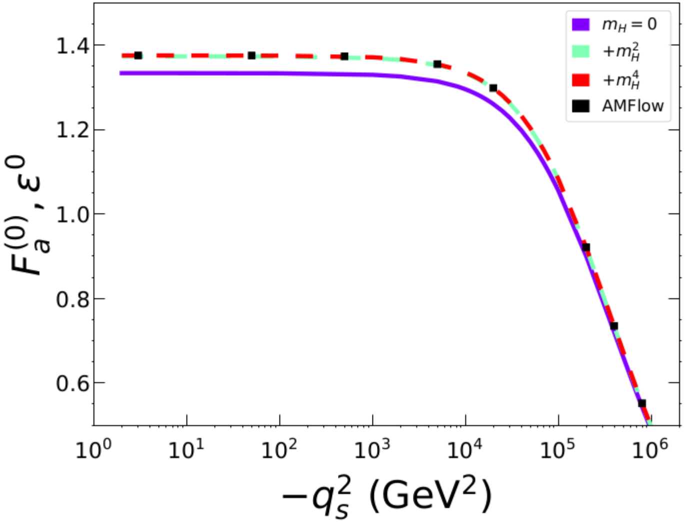
Karlsruhe Institute of Technology

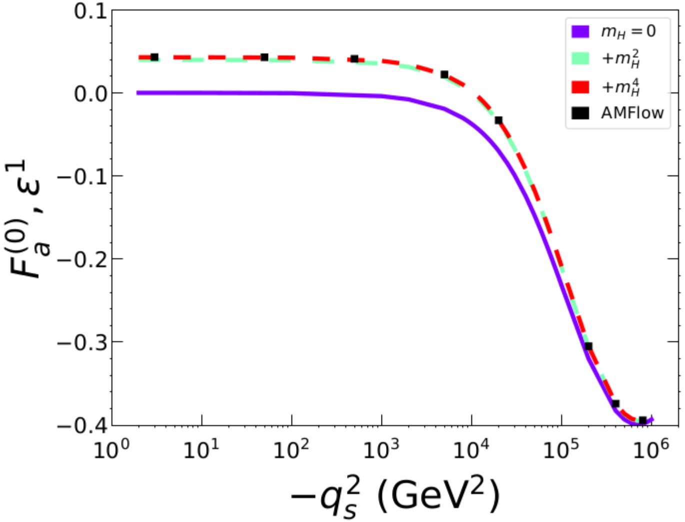
Thank you for your attention

Backup

LO Validation

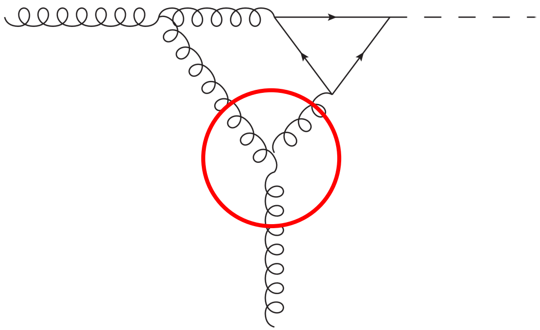




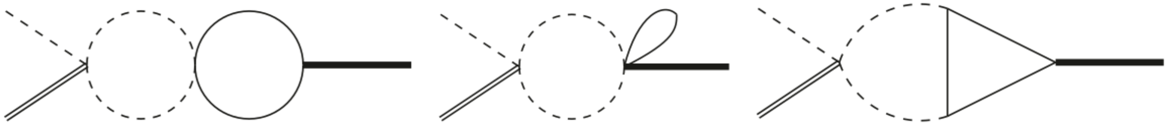


- Two-loop: results in agreement with [Degrassi, Giardino, Gröber – 1603.00385]
- NEW: inclusion of $O(\varepsilon^2)$ terms (renormalization and IR subtraction)

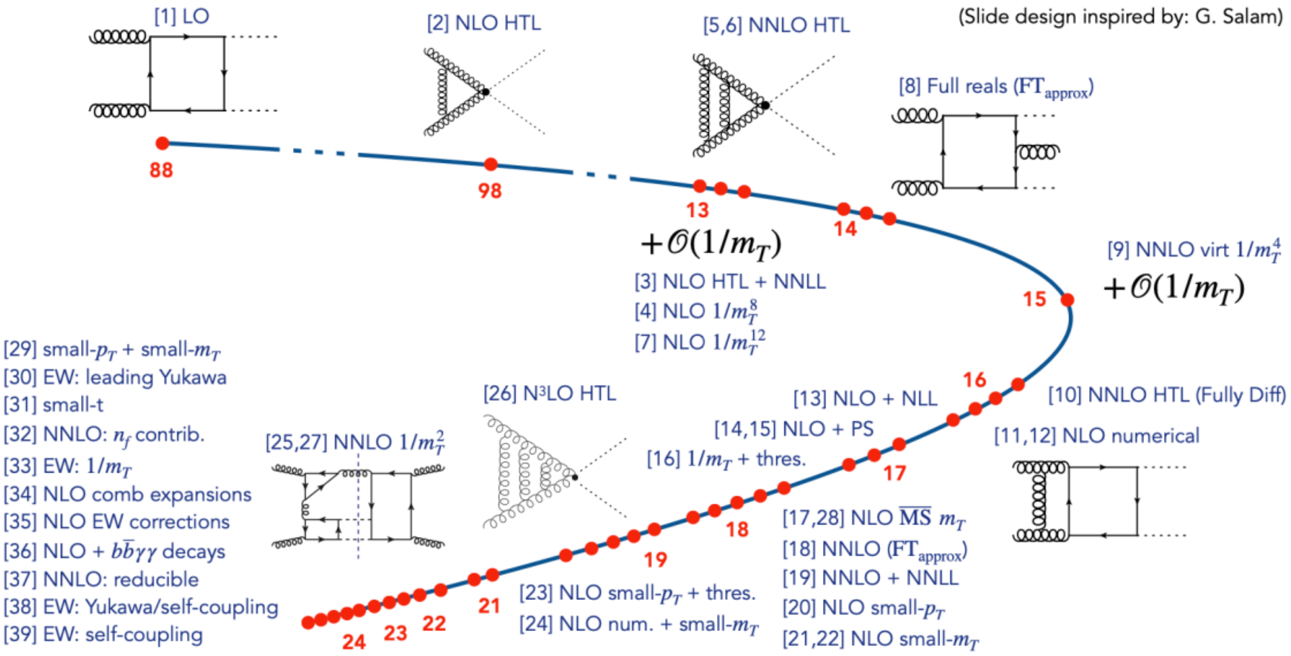
*gg*H* Form Factors



- A Taylor expansion of the two-loop integrals is not possible due to diagrams where the off-shell gluon couples to massless internal lines
- Three topologies require an asymptotic expansion



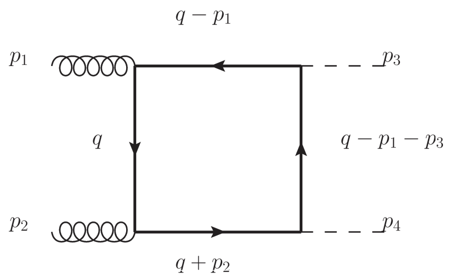
Overview



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23 [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, SPJ, Kerner, Stone, Vestner [39] Li, Si, Wang, Zhang, Zhao 24

[Credit: Stephen Jones]

pT Expansion - Details



- We assume the limit of a **forward kinematics**

$$(p_1 + p_3)^2 \to 0 \Leftrightarrow \hat{t} \to 0 \Rightarrow p_T \to 0$$

- Then Taylor-expand the form factors in the ratios

$$\frac{m_H^2}{\hat{s}}, \frac{p_T^2}{\hat{s}} \ll 1 \qquad \frac{p_T^2}{4m_t^2} \ll 1$$

Expansion at integrand level

- After tensor + IBP reduction the MIs depend on the ratio $\hat{s}/m_t^2$

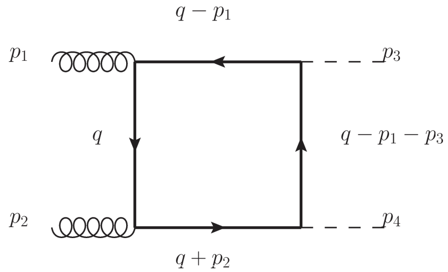
$\Rightarrow$ **single-scale integrals!**

$$I(\hat{s}, p_T^2, m_H^2, m_t^2) \to \mathrm{MI}(\hat{s}/mt^2)$$

- The MIs can be evaluated semi-analytically (e.g. “expand and match”)

[Fael, Lange, Schönwald, Steinhauser – 2106.05296; 2202.05276]

pT Expansion $m_H^2, p_T^2 \ll m_t^2, \hat{s}$



1) Consider a **one-loop** box integral

$$\int d^D q \, \frac{(q^2)^{n_1}(q\cdot p_1)^{n_2}(q\cdot p_2)^{n_3}(q\cdot p_3)^{n_4}}{(q^2-m_t^2)[(q+p_2)^2-m_t^2][(q-p_1-p_3)^2-m_t^2][(q-p_1)^2-m_t^2]}$$

2) Focus on the p3-dependent part; explicit transverse momentum (Sudakov)

$$p_3^\mu = -p_1^\mu - \frac{t'}{s'}(p_1-p_2)^\mu + r_\perp^\mu$$

$$\frac{t'}{s'} = -\frac{1}{2}\left\{1-\sqrt{1-2\frac{p_T^2+m_H^2}{s'}}\right\}$$

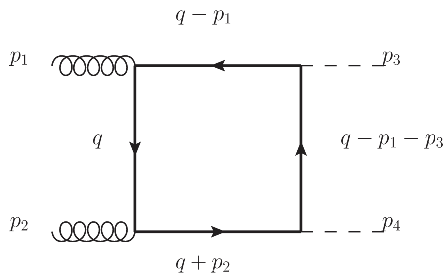
$$r_\perp^2 = -p_T^2$$

3) In the forward limit $p_3^\mu \simeq -p_1^\mu$

Taylor expansion $\rightarrow$ $$\int d^D q \, \frac{(q^2)^{n_1}(q\cdot p_1)^{n_2'}(q\cdot p_2)^{n_3'}(q\cdot r_\perp)^{n_4'}}{(q^2-m_t^2)^{l_1}[(q+p_2)^2-m_t^2][(q-p_1)^2-m_t^2]}$$

4) Tensor + IBP Reduction $\rightarrow$ Dependence on $r_\perp$ removed

pT Expansion $m_H^2, p_T^2 \ll m_t^2, \hat{s}$



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