Jet Bundle Geometry of Scalar EFTs

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Outline

- Motivation for using geometry
- Overview of other geometric approaches
- Introduction to fibre and jet bundles
- Amplitudes and field redefinitions
- Outlook and conclusions

Physics Beyond the Standard Model

- The shortcomings of the standard model indicate the presence of new undiscovered physical theories
- EFTs serve as a primary method of probing new physics using the standard model as a base
- SMEFT and HEFT emerged as the main EFT extensions of the standard model

SMEFT vs. HEFT

- In the standard model effective field theory (SMEFT) the Higgs boson is embedded in an SU(2) doublet

- In the Higgs effective field theory (HEFT) the Higgs boson is introduced as a singlet

- In terms of expansions we typically say:
  - SMEFT: Expansion around the EW-symmetric point
  - HEFT: Expansion around the EW-vacuum

R. Alonso, E. Jenkins, A. Manohar., arXiv:1605.03602

SMEFT vs. HEFT: Motivation for Geometry

- While mapping SMEFT to HEFT is well defined, the map from HEFT to SMEFT is plagued with singularities in the scalar sector.

R. Gomez-Ambrosio, et al., arXiv:2204.01763

- Singularities may be non-physical → S-Matrix invariant under the map → S-Matrix could be expressed via geometric invariants

R. Alonso, E. Jenkins, A. Manohar., arXiv:1511.00724

- If any of the singularities introduced by the mapping are physical, then the HEFT theory cannot be expressed in terms of SMEFT

T. Cohen, et al, arXiv:2008.08597

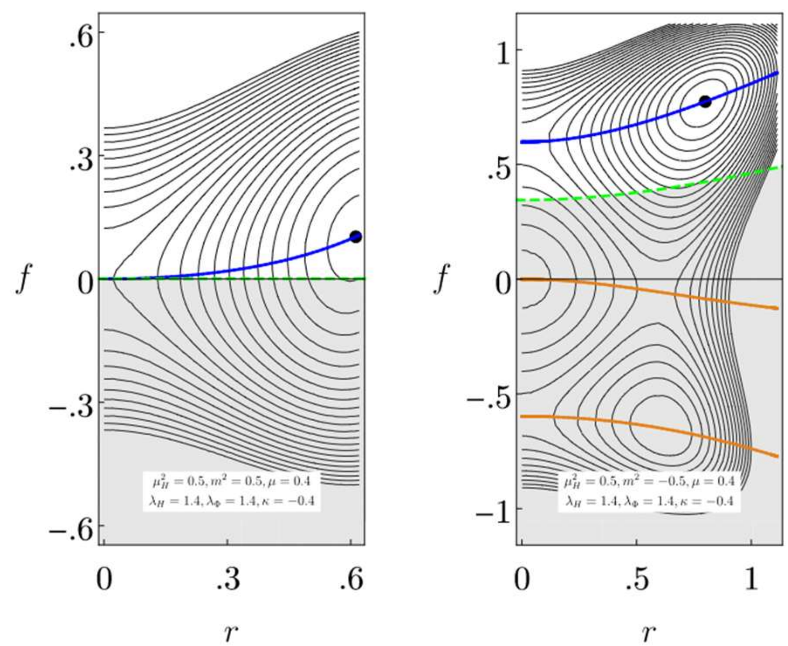
SMEFT vs HEFT: Physical Singularities

- A phenomenological difference between the two extensions would likely emerge in such a scenario

T.Cohen, et al., arXiv:2108.03240

- Loryons serve as a simple example of such a situation. By acquiring more than ½ their mass from EW symmetry breaking they require HEFT

R. Alonso, E. Jenkins, A. Manohar, arXiv:1602.00706



T. Cohen, et al, arXiv:2008.08597

Geometric formalism of EFTs

- Treat fields as if they are coordinates on a manifold M equipped with a Riemannian metric g

- Two derivative term in the Lagrangian comes from the metric

$$L = \frac{1}{2} g_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi) + O(\partial^4)$$

- Physical amplitudes related to tensor on the manifold M

$$\overline{V}_{;(\alpha_1\alpha_2\alpha_3\alpha_4)} + \frac{2}{3}\left(s_{12}\overline{R}_{\alpha_1(\alpha_3\alpha_4)\alpha_2} + s_{13}\overline{R}_{\alpha_1(\alpha_2\alpha_4)\alpha_3} + s_{14}\overline{R}_{\alpha_1(\alpha_2\alpha_3)\alpha_4}\right)$$

Two-to-two scattering. T.Cohen, et al., arXiv:2108.03240

Standard Geometric Formalism: Mathematical Details

Fields as coordinates on a manifold M

Introduce a (pseudo-)Riemannian metric on M

$$u^i \in M$$

$$g = g_{ij}(u) du^i \otimes du^j : TM \times TM \to \mathbb{R}$$

Pullback to space-time along a map

Coordinates pullback to fields

One-forms pullback to derivatives

Two derivative term provided by geometry

$$\phi : \Sigma \to M$$

$$(\phi^* u^i)(x) = u^i(\phi(x)) = \phi^i(x)$$

$$(\phi^* du^i)(x) = d(u^i(\phi(x)) = d\phi^i(x) = \partial_\mu \phi^i(x) dx^\mu$$

$$\phi^* g = g_{ij}(\phi(x)) \partial_\mu \phi^i \partial_\nu \phi^j dx^\mu \otimes dx^\nu$$

Potential must be added in by hand

Formalism limited to two derivative terms

$$V(\phi(x)) \notin \phi^* g$$

$$ddu = 0 \implies \partial_\mu \partial_\nu \phi^i \notin \phi^* g$$

Fibre Bundles

- Aim to obtain the entire Lagrangian from geometry

$$L = \frac{1}{2}\langle \eta^{-1}, \phi^* \, g \, \rangle$$

- Replace manifold M where fields are coordinates with a manifold E with both space time coordinates x^μ and "field coordinates" u^i

- Introduce a map from space-time to the manifold; $\pi: \Sigma \to E$

- Map has local inverses ϕ called sections which become our fields

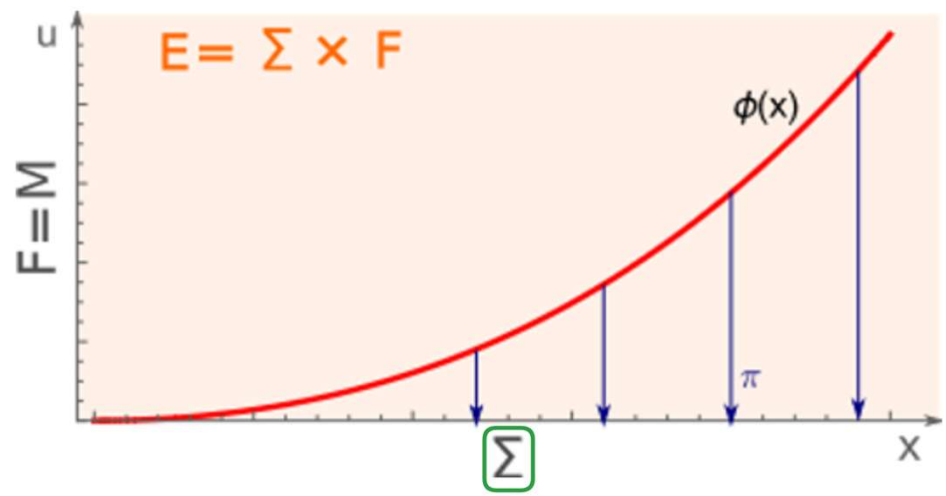
- Field redefinitions become bundle morphisms

Fibre Bundles

A bundle consists of a:

- Total space
- Base Space
- Surjective Map

Locally the bundle looks like a direct product of the base space and the fibres



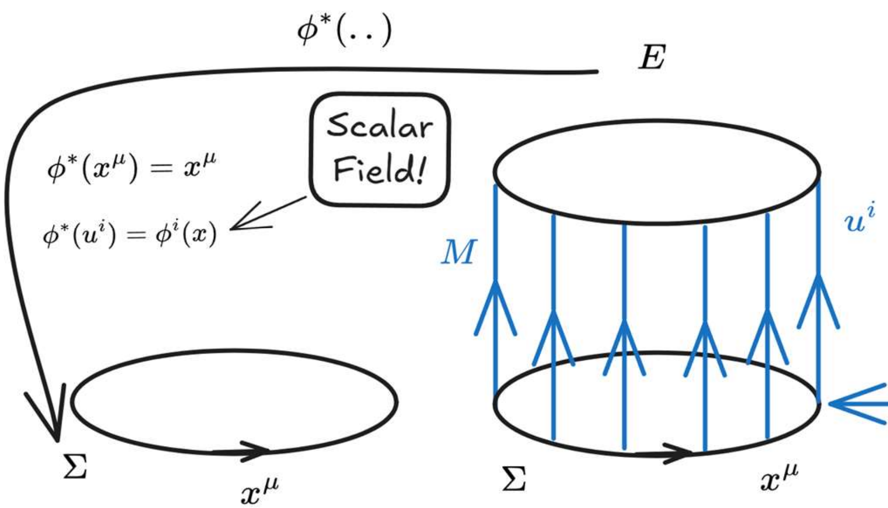
At every point in the bundle local inverses to the surjective map exist. The maps are called sections

Fibre Bundles Geometry

Locally a fibre bundle admits fibred coordinates ⟷ $\forall p \in E, \quad \exists E_p \quad s.t \quad (x^\mu, u^i) \in E_p$

The coordinates naturally split into fibre and base coordinates

$$E_p = (M \times \Sigma)_p$$
$$u_i \in M, x^\mu \in \Sigma$$



A metric on E consists of four blocks

$$\begin{pmatrix} g_{\mu\nu} & g_{\mu i} \\ g_{\mu i} & g_{ij} \end{pmatrix}$$

Poincaré invariance implies the off-diagonal blocks vanish →

$$\begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & g_{ij} \end{pmatrix}$$

Lagrangian from Metric

- After Poincare invariance

$$g = g_{\mu\nu} dx^{\mu} \otimes dx^{\nu} + g_{ij} du^{i} \otimes du^{j}$$

$\phi^{*}(\ldots)$ $\phi^{*}(\ldots)$

$$g_{\mu\nu}\eta^{\mu\nu} = -V(\phi) \subset L$$

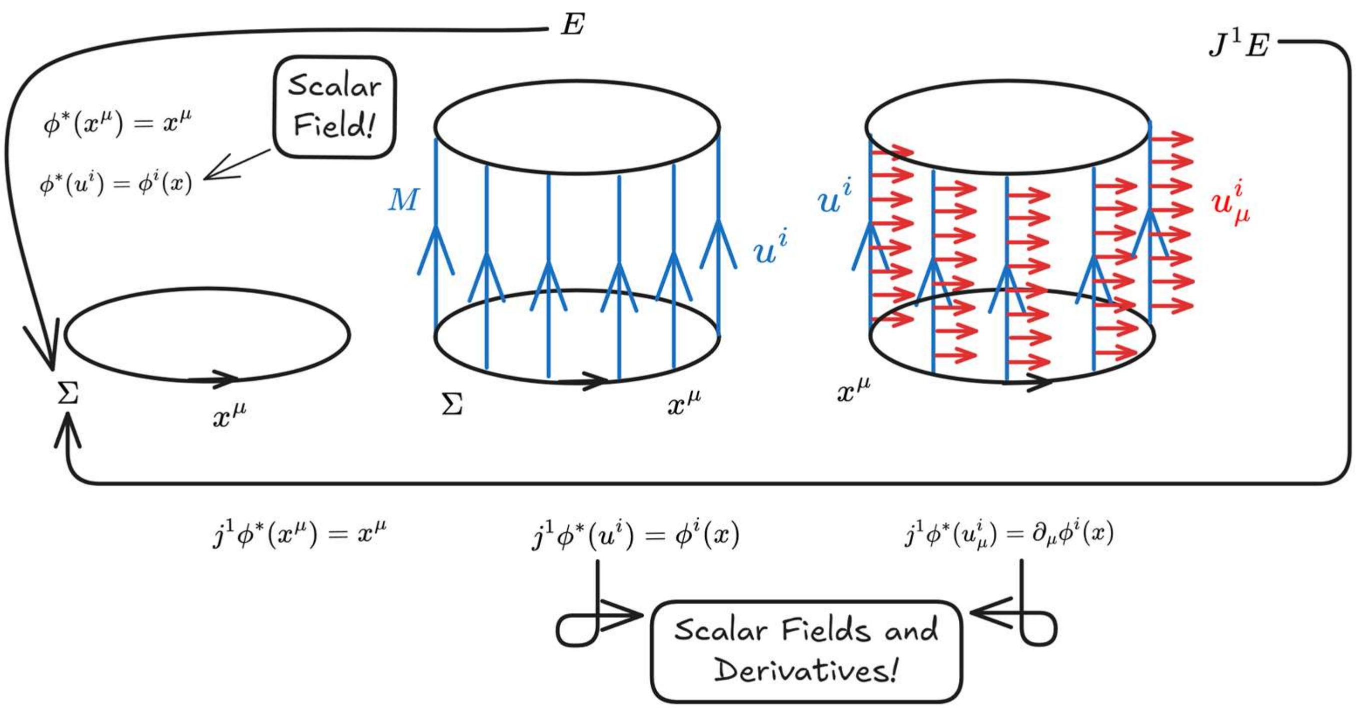
$$g_{ij}\eta^{\mu\nu}\partial_{\mu}\phi^{i}\partial_{\nu}\phi^{j} \subset L$$

- Scalar potential is now geometric!

Jet Bundles

- Given a smooth fibre bundle (E, Σ, π) we can extend it to create a jet bundle
- At each point $p \in E$ we define equivalence classes $j_p^1\phi$. The set of all equivalence classes is a smooth manifold J^1E
- The manifolds Σ, J^1E create the first jet bundle (Σ, J^1E, π_1)

Jet Bundle Geometry



Equivalence Classes

$$\phi, \psi \in j^1_p\phi$$

1. $\phi(p) = \psi(p)$

2. $\left.\dfrac{\partial \phi \circ u^i}{\partial x^\mu}\right|_p = \left.\dfrac{\partial \psi \circ u^i}{\partial x^\mu}\right|_p$

Geometry on Jet Bundle

- The first jet bundle introduces an additional coordinate u^i_μ thus the metric becomes

$$g = \begin{pmatrix} dx^\mu & du^i & du^i_\mu \end{pmatrix} \begin{pmatrix} g_{\mu\nu} & g_{\mu i} & g_{\mu i}^{\ \ \nu} \\ g_{\mu i} & g_{ij} & g_{ij}^{\ \ \nu} \\ g_{\mu i}^{\ \ \nu} & g_{ij}^{\ \ \nu} & g_{ij}^{\mu\nu} \end{pmatrix} \begin{pmatrix} dx^\nu \\ du^j \\ du^j_\nu \end{pmatrix}$$

- The coordinates are independent on the manifold J^1E

Lagrangian from Jet Bundle Metric

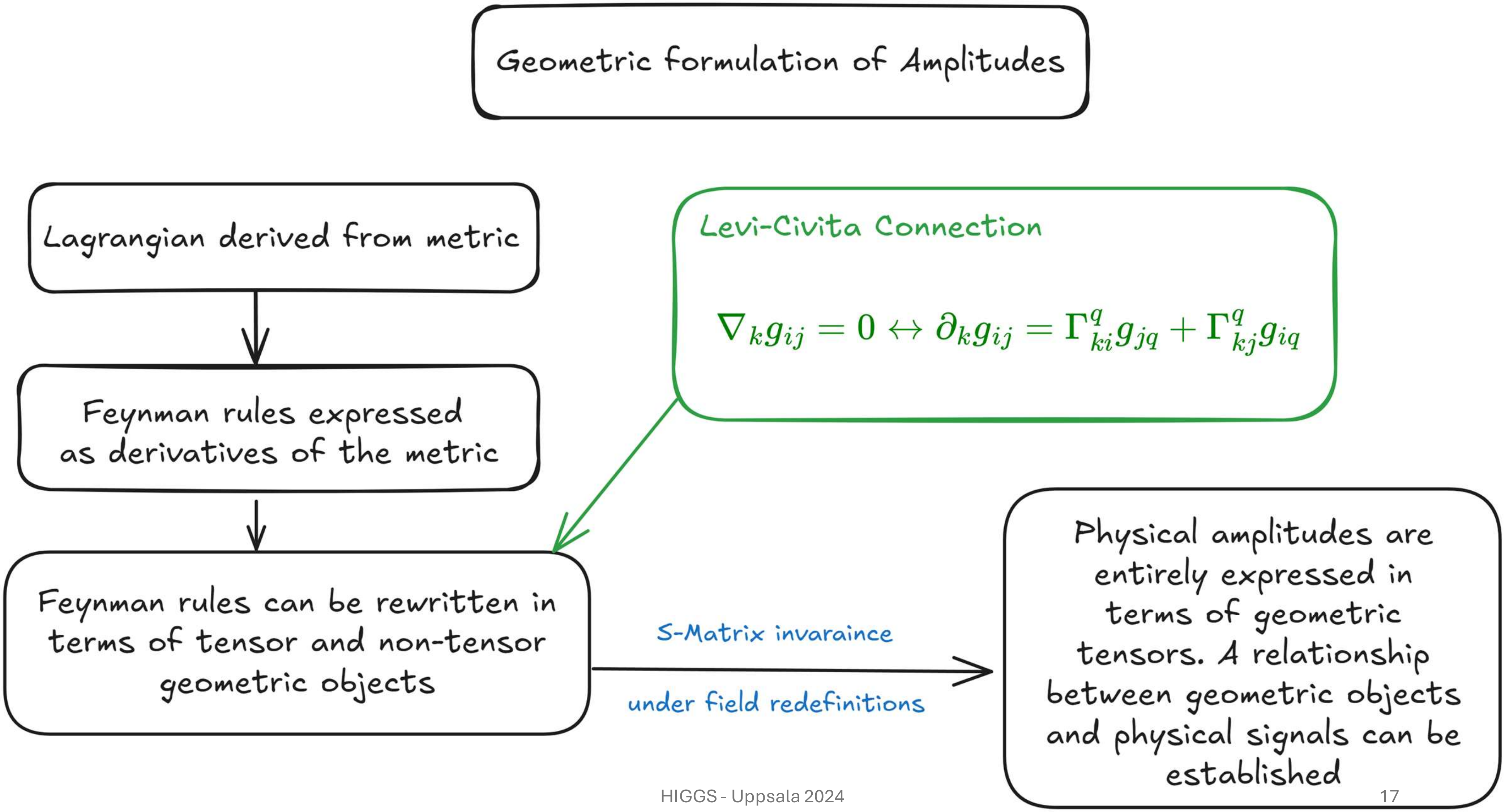
$$g = g_{\mu\nu} dx^\mu \otimes dx^\nu + g_{ij} du^i \otimes du^j + g_{ij}^{\mu\nu} du_\mu^i \otimes du_\nu^j$$

$$+ g_{i\mu} du^i \otimes dx^\mu + g_{ij}^{\mu} du_\mu^i \otimes du^j + g_{i\nu}^{\mu} du_\mu^i \otimes dx^\nu$$

$$g_{ij}\eta^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j \subset L$$

$$g_{\mu\nu}\eta^{\mu\nu} = -V(\phi) + \cdots \subset L$$

$$g_{ij}^{\mu\nu}\eta^{\rho\sigma}\partial_\rho\partial_\mu\phi^i\partial_\sigma\partial_\nu\phi^j \subset L$$


Geometric formulation of Amplitudes
Lagrangian derived from metric
Feynman rules expressed as derivatives of the metric
Feynman rules can be rewritten in terms of tensor and non-tensor geometric objects
Levi-Civita Connection
$\nabla_k g_{ij} = 0 \leftrightarrow \partial_k g_{ij} = \Gamma^q_{ki} g_{jq} + \Gamma^q_{kj} g_{iq}$
S-Matrix invaraince
under field redefinitions
Physical amplitudes are entirely expressed in terms of geometric tensors. A relationship between geometric objects and physical signals can be established

Three Point Amplitude on 0-Jet Bundle

- Accounting for the symmetry factors the three-point amplitude is given by

$$i\left(\frac{1}{6}(\overline{\nabla_{a_3}R^{\mu}_{a_1\mu a_2}}+\overline{\nabla_{a_2}R^{\mu}_{a_1\mu a_3}}+\overline{\nabla_{a_1}R^{\mu}_{a_2\mu a_3}})\right.$$
$$\left.+(p_1^2-m_1^2)\overline{\Gamma_{a_1a_2a_3}}+(p_2^2-m_2^2)\overline{\Gamma_{a_2a_1a_3}}+(p_3^2-m_3^2)\overline{\Gamma_{a_3a_2a_1}}\right)$$

- On-shell only the tensorial piece survives

- Agrees with results from previous formalisms

$$\overline{V}_{,\alpha\beta\gamma}-m_\alpha^2\overline{\Gamma}_{\alpha\beta\gamma}-m_\beta^2\overline{\Gamma}_{\beta\gamma\alpha}-m_\gamma^2\overline{\Gamma}_{\gamma\alpha\beta}$$

[T. Cohen, N. Craig, X. Lu and D. Sutherland, arXiv:2108.03240]

Diffeomorphisms and Tensors

- Under a general diffeomorphism f the Riemann tensor is not invariant

$$R_{IJKL}(x)dx^I dx^J dx^K dx^L \to R_{IJKL}(f(x))\frac{\partial(f\circ x^I)}{\partial x^A}\frac{\partial(f\circ x^J)}{\partial x^B}\frac{\partial(f\circ x^K)}{\partial x^C}\frac{\partial(f\circ x^L)}{\partial x^D}dx^A dx^B dx^C dx^D$$

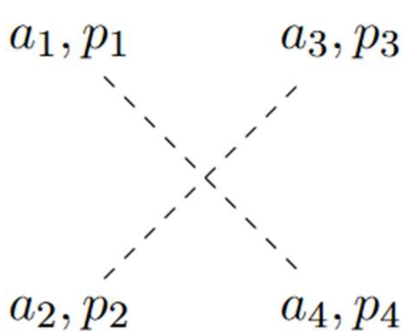
- A diffeomorphism of the form $u \to f(u) = u + c_n u^n$ with $n \geq 2$ is special since at the point $u = 0$ we have

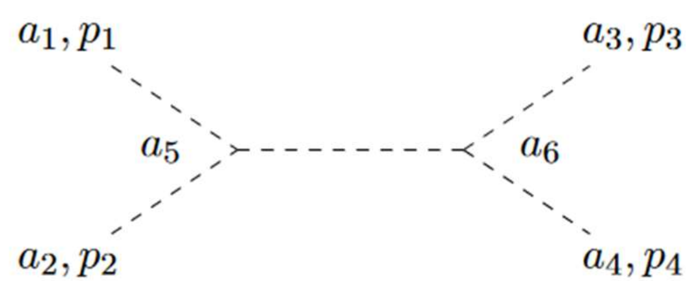
$$\lim_{u\to 0} f(u) = \lim_{u\to 0} u = 0 \qquad \lim_{u\to 0}\frac{\partial(f\circ u^i)}{\partial u^j} = \delta^i_j$$

- Tensors are invariant under such a transformation at the vacuum just like amplitudes

Two to Two Scattering

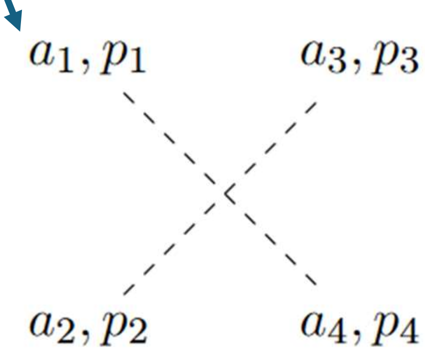
$$i\left(\frac{1}{48}\left(\overline{\nabla_{a_1}\nabla_{a_2}R^{\mu}_{a_3\mu a_4}} - \overline{R^{\mu}_{a_1\nu a_2}R^{\nu}_{a_3\mu a_4}} + \text{perms}(1234)\right) - \left(\frac{2}{3}s_{12}\overline{R_{a_1(a_3a_4)a_2}}\right.\right.$$
$$\left.\left. + \frac{1}{144}\left(\frac{\overline{g^{a_5a_6}}}{s_{12}-m_5^2}\left(\overline{\nabla_{a_5}R^{\mu}_{a_1\mu a_2}} + \text{perms}(125)\right)\left(\overline{\nabla_{a_6}R^{\mu}_{a_3\mu a_4}} + \text{perms}(346)\right)\right) + \text{cycs}(234)\right)\right)$$

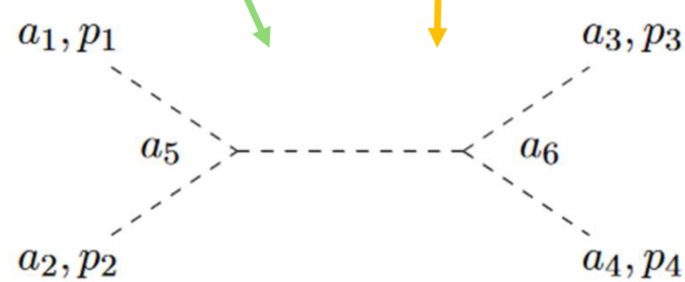




Two to Two Scattering

$$i\left(\frac{1}{48}\left(\overline{\nabla_{a_1}\nabla_{a_2}R^{\mu}_{a_3\mu a_4}}-\overline{R^{\mu}_{a_1\nu a_2}R^{\nu}_{a_3\mu a_4}}+\text{perms}(1234)\right)-\left(\frac{2}{3}s_{12}\overline{R_{a_1(a_3a_4)a_2}}\right.\right.$$
$$\left.\left.+\frac{1}{144}\left(\frac{\overline{g^{a_5a_6}}}{s_{12}-m_5^2}\left(\overline{\nabla_{a_5}R^{\mu}_{a_1\mu a_2}}+\text{perms}(125)\right)\left(\overline{\nabla_{a_6}R^{\mu}_{a_3\mu a_4}}+\text{perms}(346)\right)\right)+\text{cycs}(234)\right)\right)$$





Conclusion

- Jet Bundles provide a way to describe scalar EFTs geometrically to any order.
- Amplitudes are naturally related to geometric invariants.
- Geometric differences between theories can be related to physical observables.

Outlook

- Automation of computation of amplitudes in terms of geometric tensors – Coming soon!
- Introduction of gauge field to the bundle formulation – In progress
- Renormalization of scalar theories on the 0-jet bundle- In progress
- Computation of amplitudes on 1-jet bundle and beyond