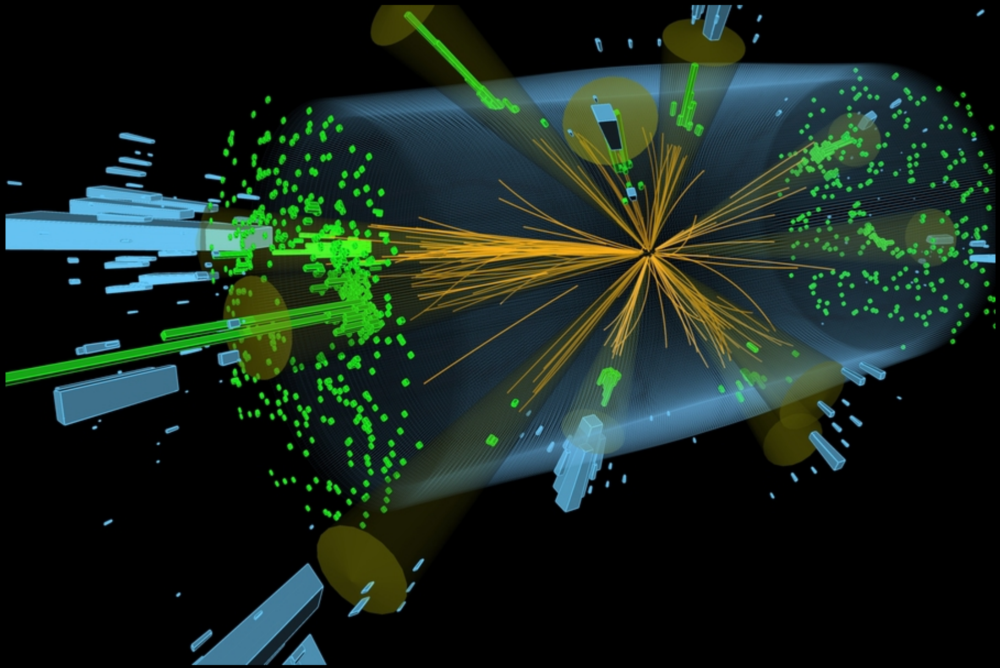
Theoretical Prospects for Jet Substructure

ALICE Meeting at Wright Lab - Yale University

Bianka Meçaj - Yale University

Jets are emergent phenomena in QCD

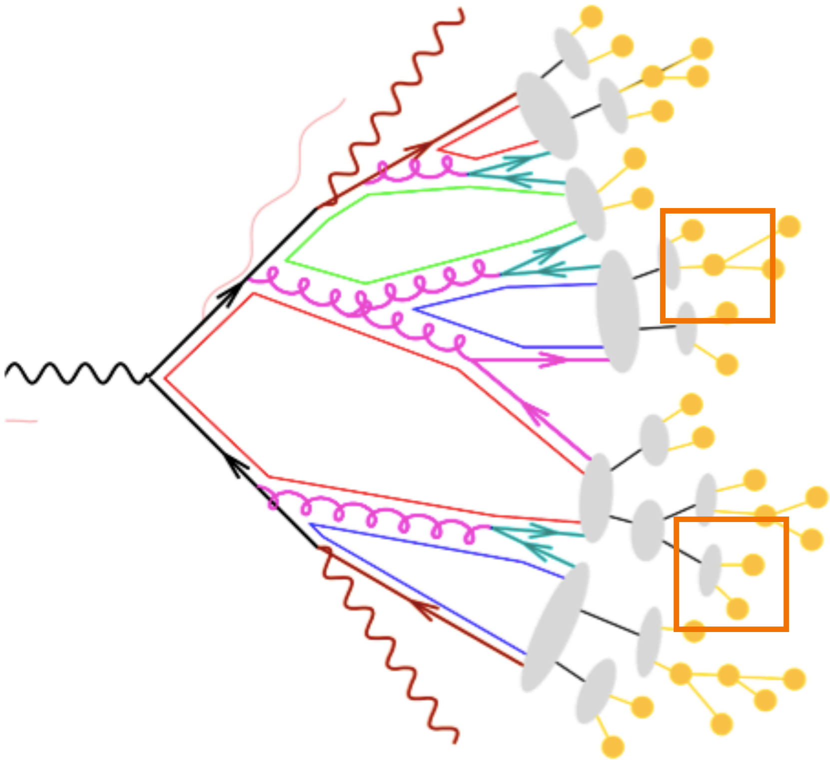
Almost every LHC event contains jets



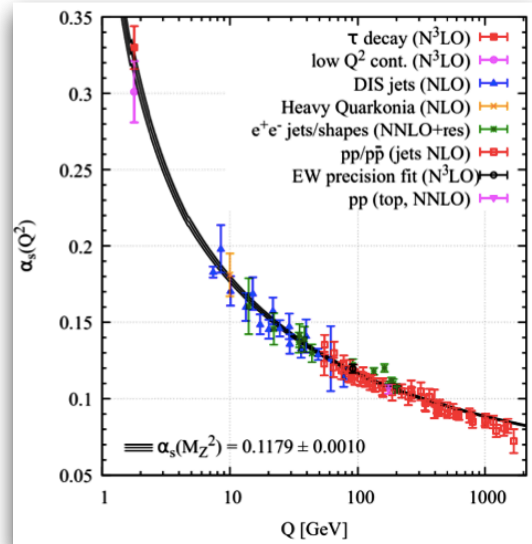
Jets are reconstructed using jet algorithms (anti-k_T)

Cacciari, Salam 2006
Salam, Soyez 2007

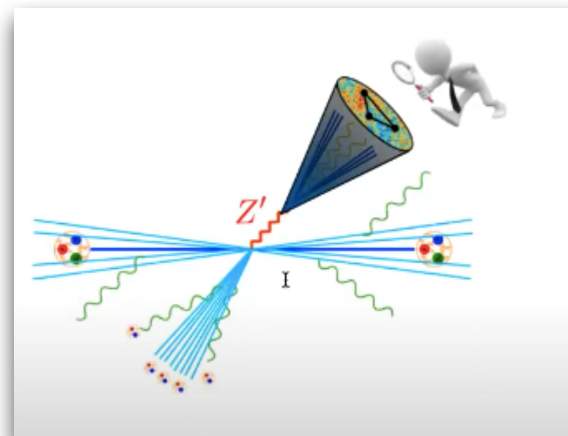
How can we learn the most about underlying physics from the reconstructed jets?



QCD precision studies



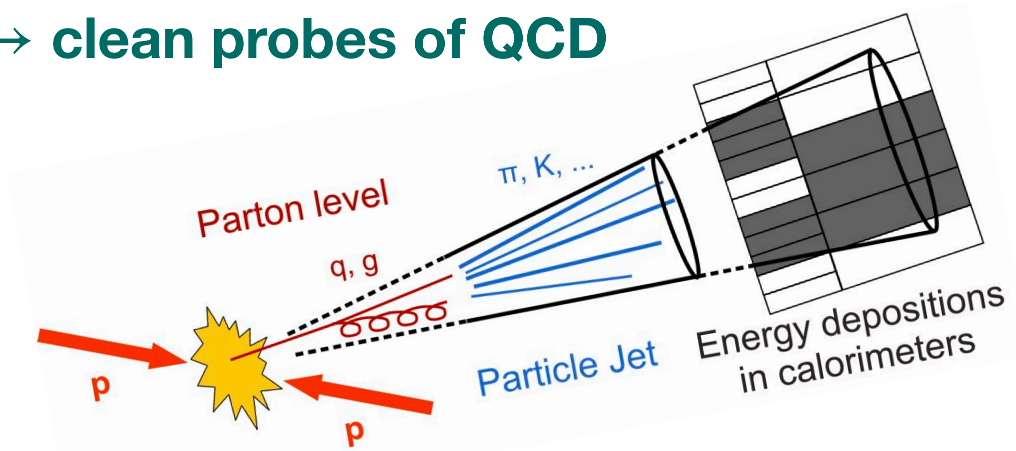
New Physics searches



Jet substructure

✦ Study the internal structure of a jet →theoretical analysis and measurements of kinematic properties

✦ Underlying Physics and intrinsic properties are imprinted in jet substructure → clean probes of QCD



Robust Jet Substructure Observables!

Event Shapes

- **Weighted cross-sections: distribution of outgoing particles/charges**

$$\sigma_\omega(q) = \sum_X (2\pi)^4 \delta^{(4)}\left(q - k_X\right) \omega(X) \, |\langle X | O(0) | 0 \rangle|^2$$

Local operator that creates the state $|X\rangle$ with momentum k_X

Event Shapes

- **Weighted cross-sections: distribution of outgoing particles/charges**

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Weight factor depends on the measurement

Local operator that creates the state $|X\rangle$ with momentum k_X

Event Shapes

- **Weighted cross-sections: distribution of outgoing particles/charges**

$$\sigma_\omega(q) = \sum_X (2\pi)^4 \delta^{(4)}\left(q - k_X\right) E_{\{X\}} \left|\langle X | O(0) | 0 \rangle\right|^2$$

Weight factor depends on the measurement

For $\omega(X)$ weighted energy this expression gives the distribution of energy inside the jet.

$E_{\{X\}}$ are different permutations for all $\{X\}$ final states

Local operator that creates the state $|X\rangle$ with momentum k_X

Event Shapes

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Weight factor depends on the measurement

Local operator that creates the state $|X\rangle$

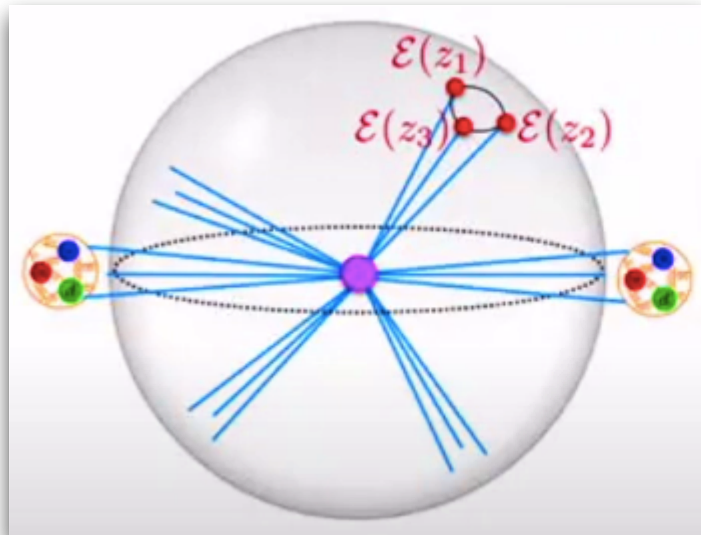
$E_{\{X\}}$ are different permutations for all $\{X\}$ final states

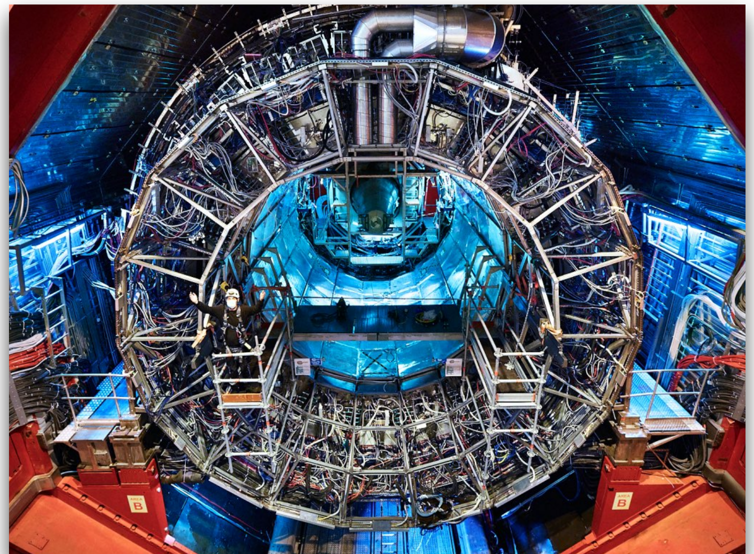
For $\omega(X)$ weighted energy this expression gives the distribution of energy inside the jet.

- **Reformulate such event shape distributions with correlation functions!**

Energy Correlators

Energy Correlators describe the calorimeter cells at infinity on the celestial sphere





$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\ldots\varepsilon(\vec{n}_n) \mid \Psi \rangle$$

Energy Flow Inside the Jet

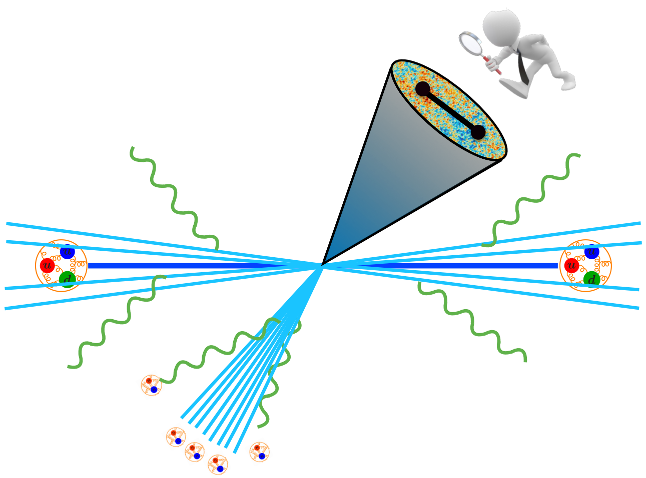
- Distribution of energy inside the jet is described by correlation functions of the energy flow operators ⇒ Energy Correlators.

[Basham, Brown, Ellis, Love]

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\ldots\varepsilon(\vec{n}_n) \mid \Psi \rangle$$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\; r^2 n^i T_{0i}(t, r\vec{n})$$

Defined from first principles in QFT!

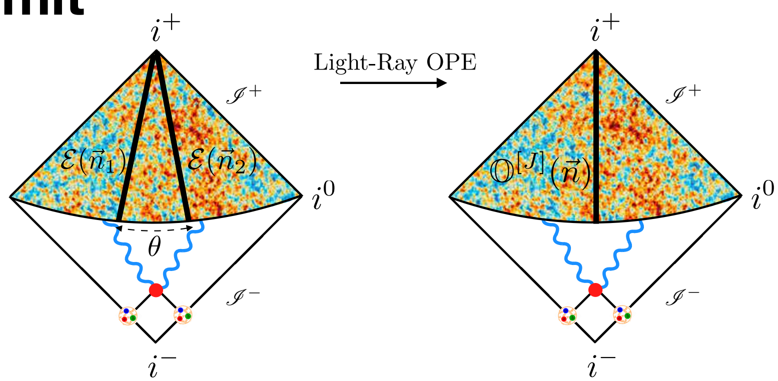


Any physics dynamics will be imprinted in the energy distributions inside the jet.

Scaling Behavior

Energy correlators inside high energy jets at the LHC

⇒small angle limit



Corresponds to $\omega(X) = E_i E_j$

$$\sigma_\omega(q) = \sum_X (2\pi)^4 \delta^{(4)}\left(q - k_X\right) \omega(X) \left| \langle X | O(0) | 0 \rangle \right|^2$$

- **Energy correlators admit an OPE:**

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$

[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

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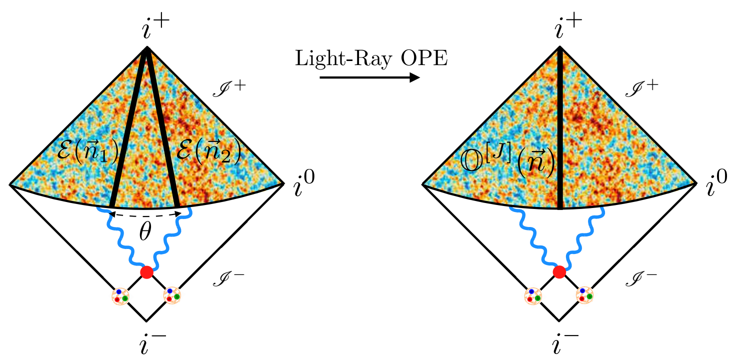

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⇒ Use LHC jets to test the leading QCD operators in this expansion

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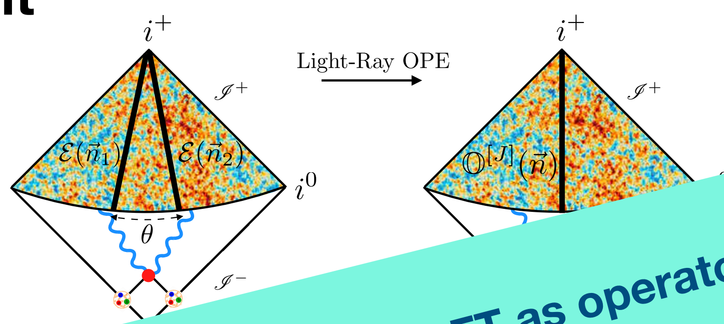

Universal scaling behavior in QFT as operators are brought together!

- **Energy**

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi\rangle \sim \sum \theta^{\gamma_i}\mathcal{O}_i(\vec{n}_1)$$

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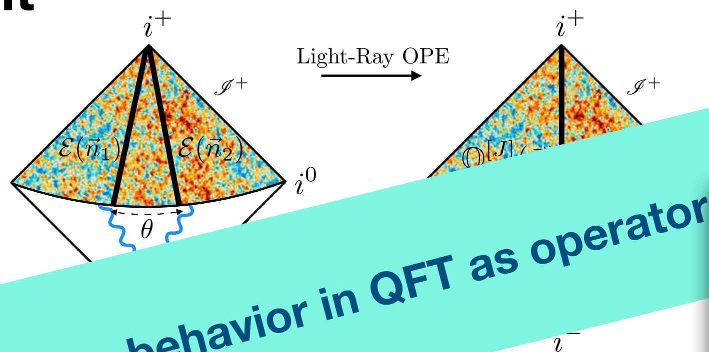
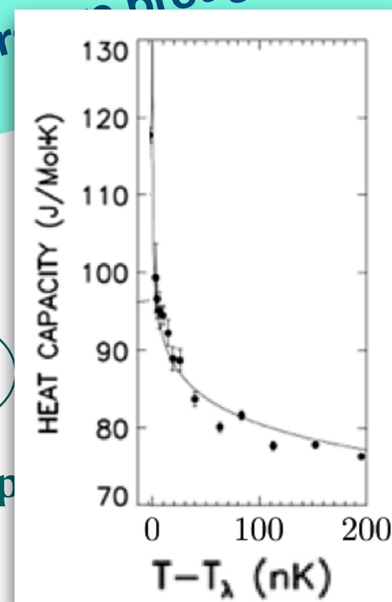


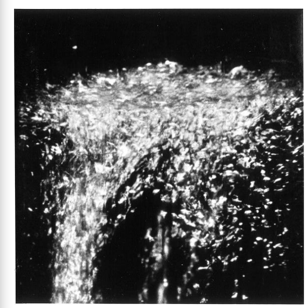
Universal scaling behavior in QFT as operator... e brought together!

- **Energ... an OPE:**

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$

⇒ **Use LHC jets to test the leading QCD op... expansion**

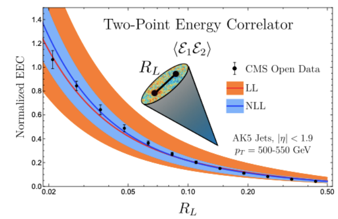


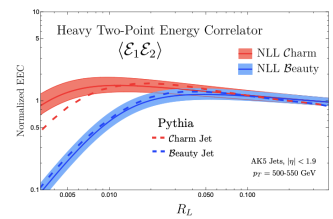


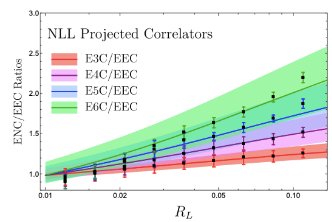
Energy correlators for jet substructure at LHC

Outline

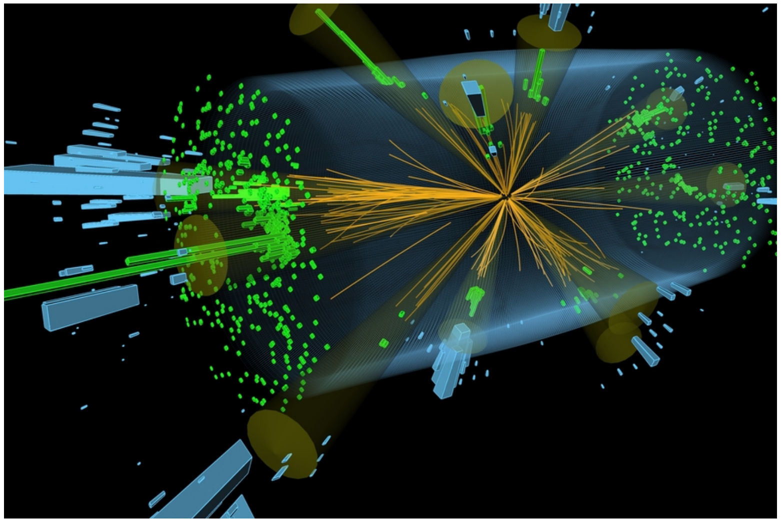
- Massless Energy Correlators
- Massive Energy Correlators
- Higher Point Correlators



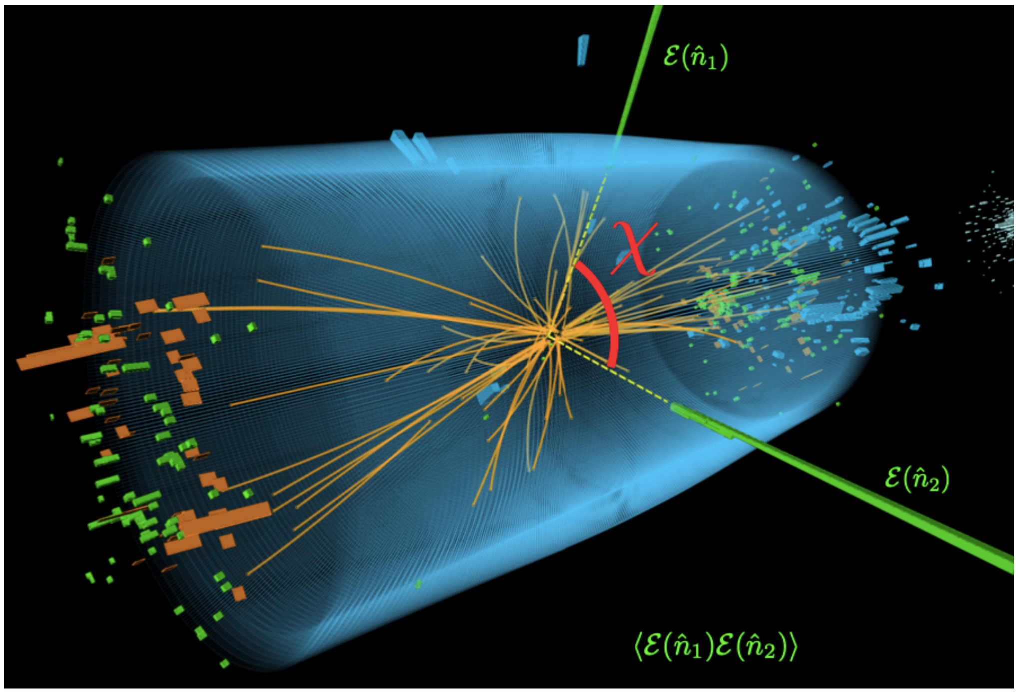




Energy Correlators at the LHC
(Light quarks)



Energy Correlator as an observable

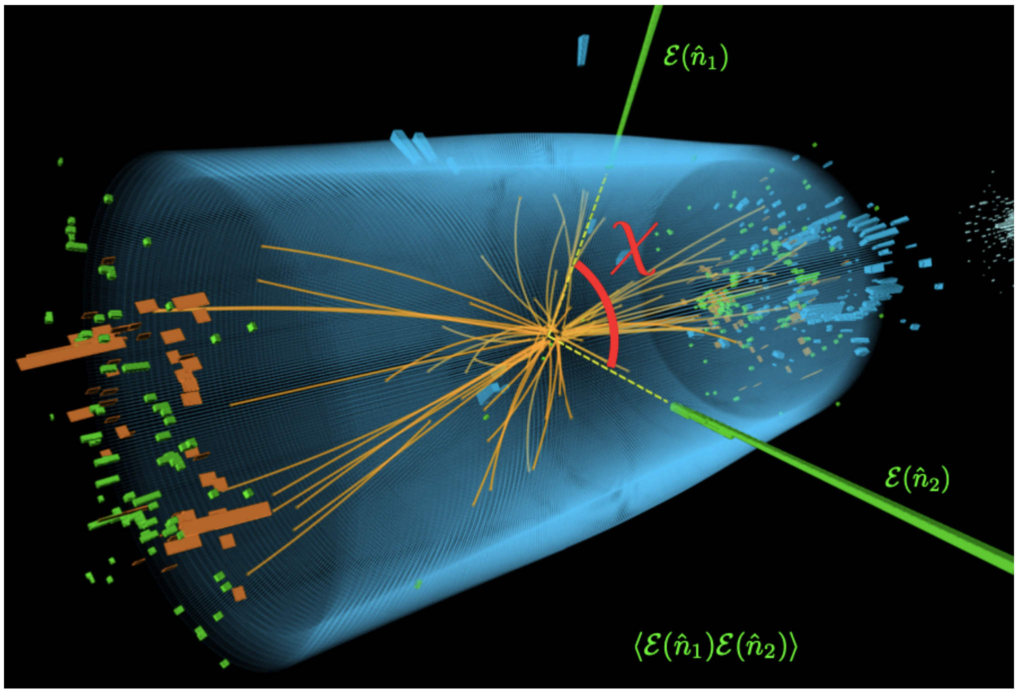


Energy weighted cross sections

$$\text{EEC} = \sum_{i,j} d\sigma \frac{2E_iE_j}{Q^2\sigma_{\text{tot}}} \delta(\cos\theta_{ij} - \cos\chi)$$

Studied first in e^+e^- by [Basham, Brown, Ellis, Love]

Energy Correlator as an observable

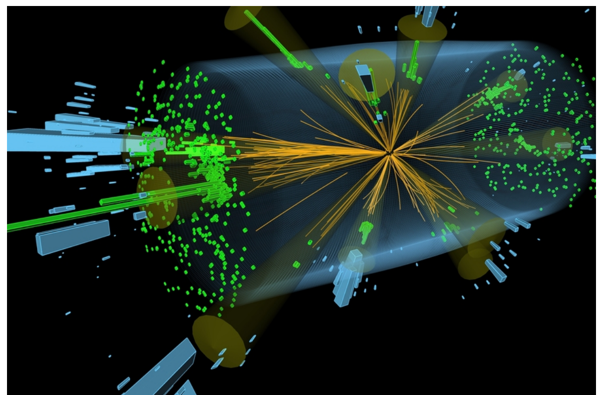


$$\text{EEC} = \sum_{i,j} d\sigma \frac{2E_i E_j}{Q^2 \sigma_{\text{tot}}} \delta(\cos\theta_{ij} - \cos\chi)$$

- **Generally one can study EEC for any angle χ**
- **Most interesting phenomenological case: $\chi \to 0$ and $\chi \to \pi$**
- **Here we study $\chi \to 0$ case.**

Energy Correlators at the LHC

Factorization Formula



[Lee, BM, Moult]

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}z} = \sum_i \mathscr{H}_i \left(p_T, \eta, \mu\right) \otimes \int_0^1 dx\, x^N \mathscr{J}_{ij}(z, x, p_T R, \mu)\, J_j^{[N]}(z, x, \mu)$$

Hard function: includes pdfs

Matching coefficient, jet algorithm

Energy correlator jet function

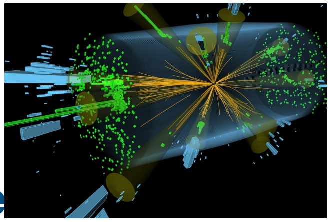
Can calculate any higher point correlator at the LHC

$$z = \frac{1 - \cos\theta_{ij}}{2}$$

$$x = \frac{2E_i}{Q}$$

Two-point energy correlator

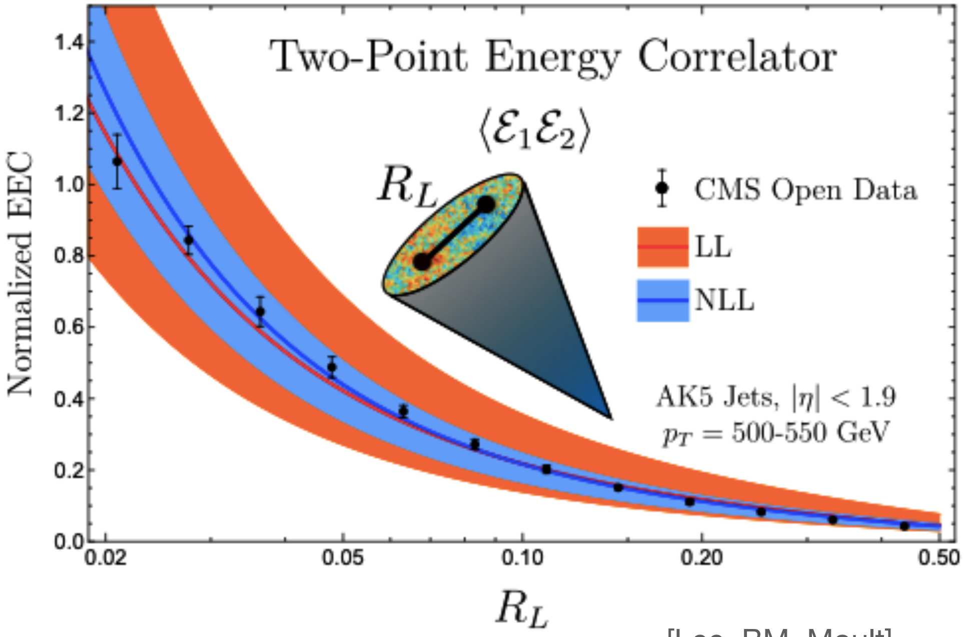
The simplest jet substructure observable



- **The complicated LHC environment is described by a simple observable**
- **Probe the OPE structure of $\langle \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\rangle$**

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$

- **A jet substructure observable that can test quantum scaling behavior of operators.**

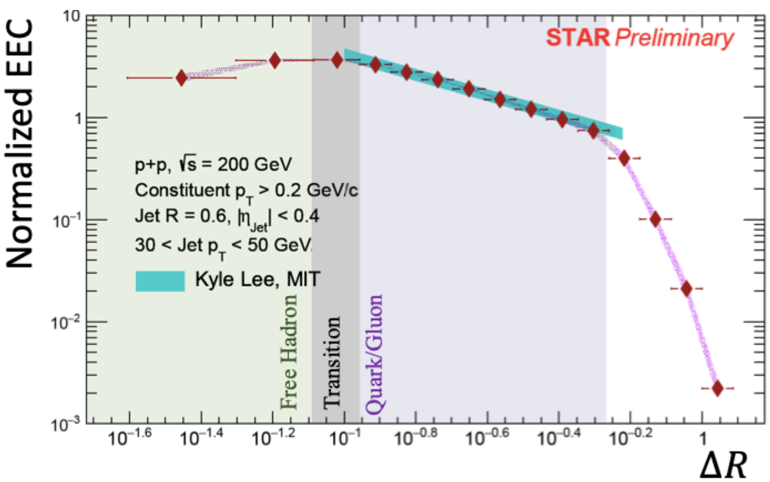


[Lee, BM, Moult]

Experimental results

Talk by N.Sahoo and A.Tamis at
HARD PROBES-March 2023

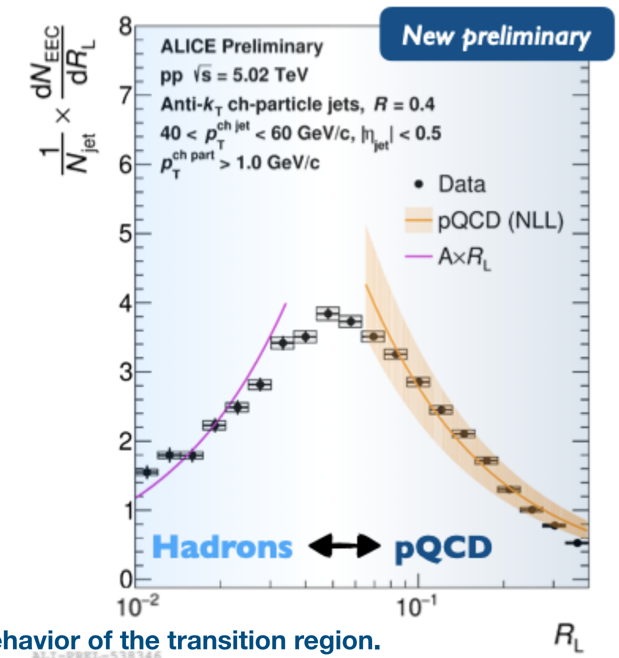
- STAR collaboration $\sqrt{s} = 200GeV$



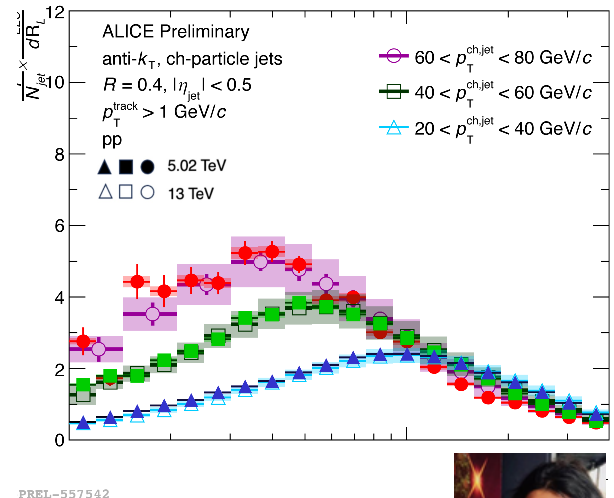
$$\text{Normalized EEC} = \frac{1}{\sum_{\text{jets}} \sum_{i \neq j} \frac{E_i E_j}{p^2_{\text{T,jet}}}} \frac{d(\sum_{\text{jets}} \sum_{i \neq j} \frac{E_i E_j}{p^2_{\text{T,jet}}})}{d\,(\Delta R)}$$

Direct observation of the transition from free hadrons to quarks/gluons at a universal scaling!

- ALICE collaboration $\sqrt{s} = 5TeV,\ 20GeV,\ 40GeV,\ 60GeV$



Preliminary results from Ananya Rai



Universal behavior of the transition region.

Talk by J.Mulligan and R.Cruz-Torres
at HARD PROBES-March 2023

Experimental results

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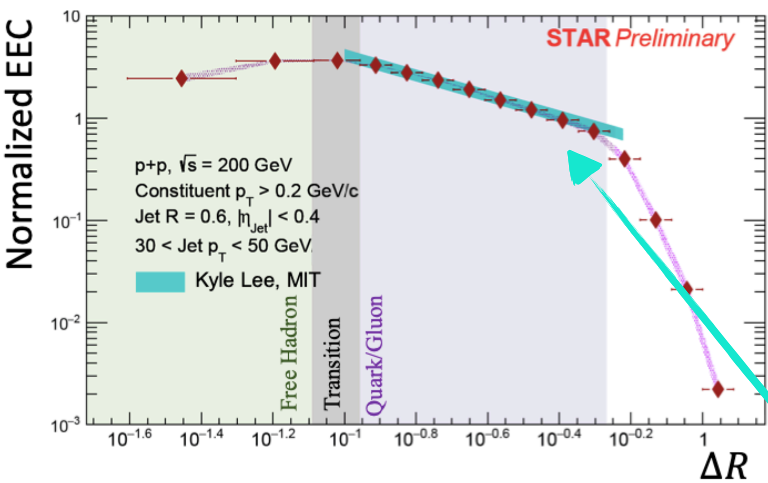

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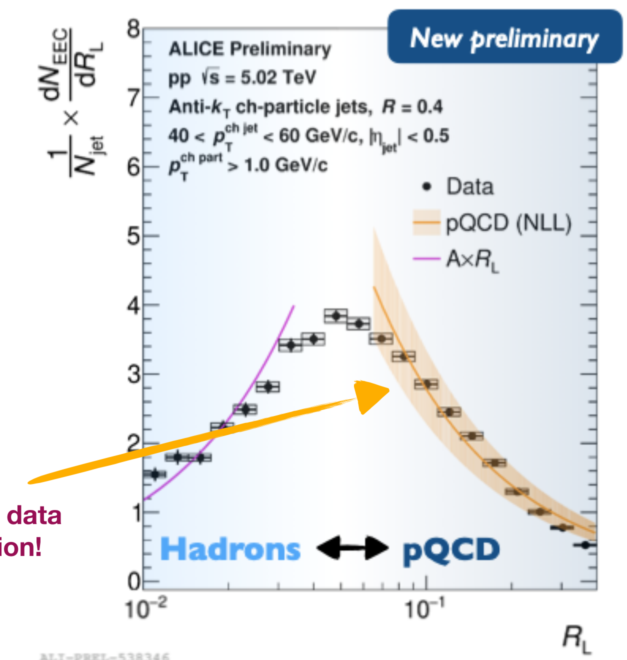
Excellent agreement of data with our NLL calculation!



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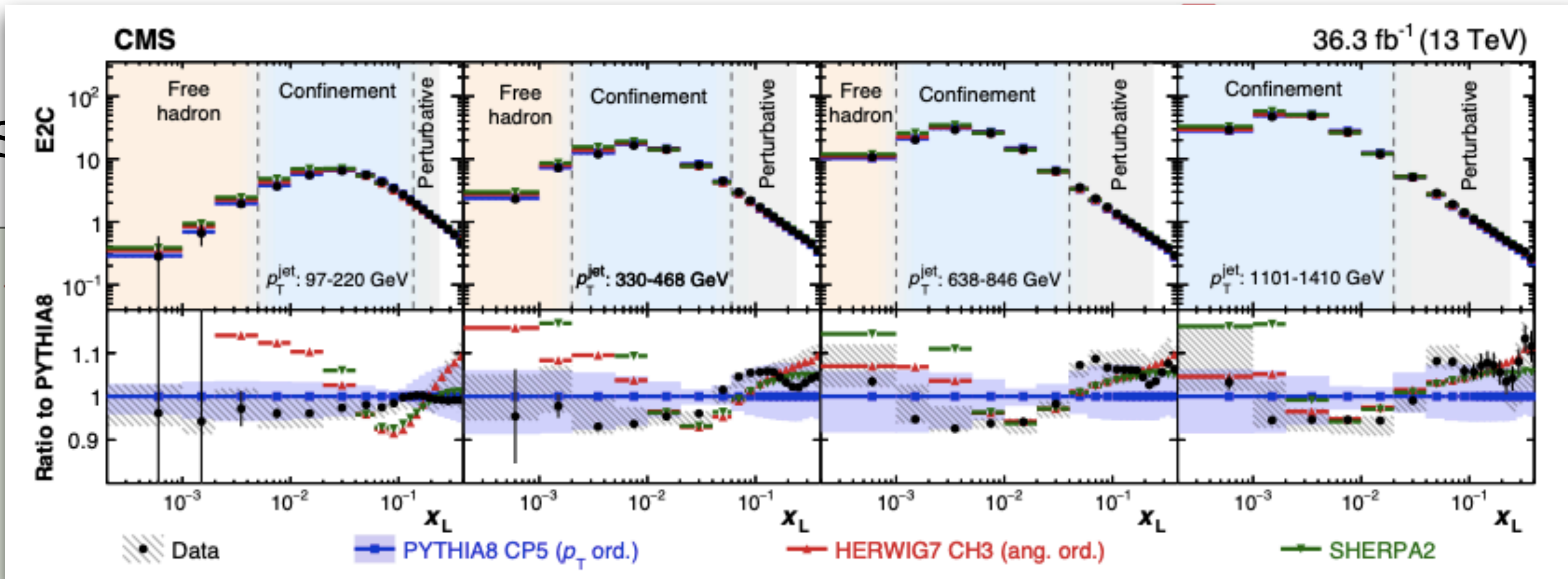

an and R.Cruz-Torres
OBES-March 2023

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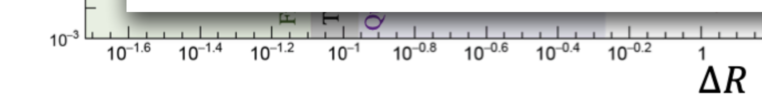

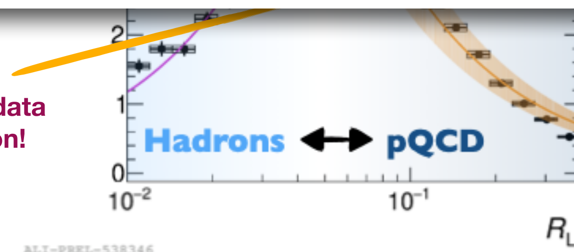

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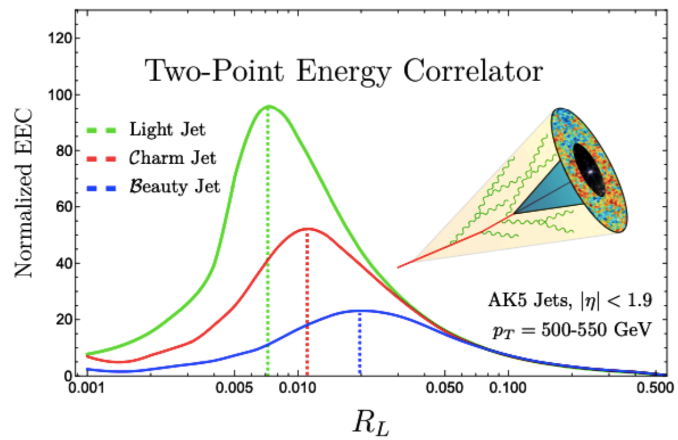
Direct observation of the transition from free hadrons to quarks/gluons at a universal scaling!

Universal behavior of the transition region.

Beautiful and Charming Energy Correlators



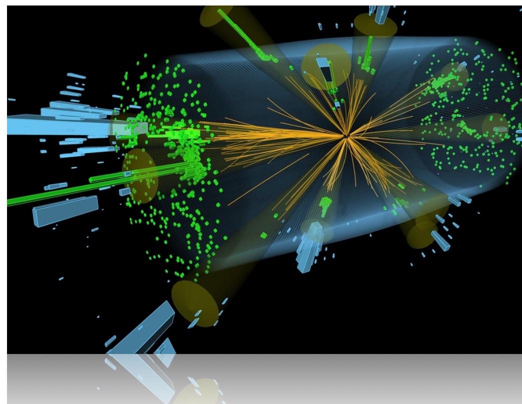
Application on Heavy Jets

Introduce an additional scale

- At the LHC energies there is access to the transition phase from massless to massive behaviour $\Rightarrow$ more complexity
- Also very interesting!
  - Can probe intrinsic mass effects of quarks before confinement into hadrons

Factorization theorem

Can compute any higher point correlators on massive quarks at LHC at NLL



Hard function (NNLO)

$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

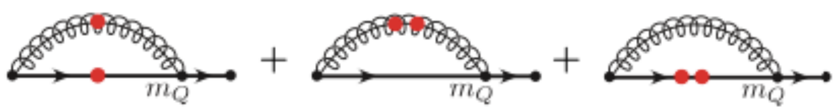
[Craft, Lee, BM, Moult]

[Czakon, Generet, Mitov, Poncelet; 2021]

$\mu_H \sim p_T$

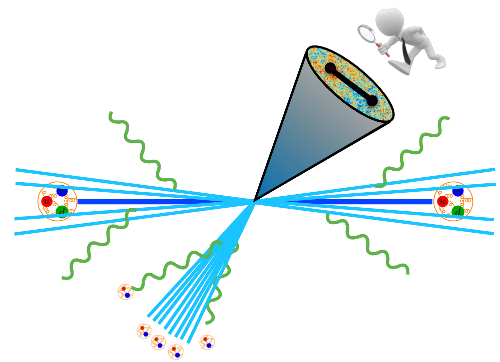
$\mu_J \sim p_T R$

Massive Energy Correlator Jet Function (NLO)



Heavy quark jet function

Result



$$J_Q^{EEC}(z,M,\mu) = \delta(z)\left(1+\frac{\alpha_s C_F}{4\pi}\left[-\left(\gamma_{qq}^{(0)}(3)+\gamma_{gq}^{(0)}(3)\right)\left(\frac{1}{\epsilon_{\mathrm{UV}}}+\ln\frac{\mu^2}{M^2}\right)-\frac{19}{6}\right]\right)$$

$$+\frac{\alpha_s C_F}{\pi}\frac{1}{z}\left[\frac{3}{4}-\frac{5}{2}\delta^2-\frac{\delta^4}{1+\delta^2}+3\delta^3\arctan\left(\frac{1}{\delta}\right)+\frac{1}{2}\delta^2\left(1-\delta^2\right)\ln\frac{\delta^2}{1+\delta^2}\right]$$

The mass should not affect the UV behavior of the jet function.
This can be seen from comparing the UV poles with the light quark jet function.

$$J_q^{EEC} = \delta(z)+\frac{\alpha_s C_F}{4\pi}\left[\delta(z)\left(-\left(\gamma_{qq}^{(0)}(3)+\gamma_{gq}^{(0)}(3)\right)\frac{1}{\epsilon_{\mathrm{UV}}}-\frac{37}{3}\right)+3\frac{Q^2}{\mu^2}\mathscr{L}_0\left(\frac{Q^2}{\mu^2}z\right)\right]$$

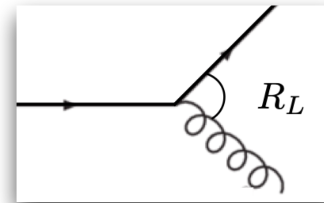
[Craft, Lee, BM, Moult]

Massive jets

Massive Energy Correlator Jet Function

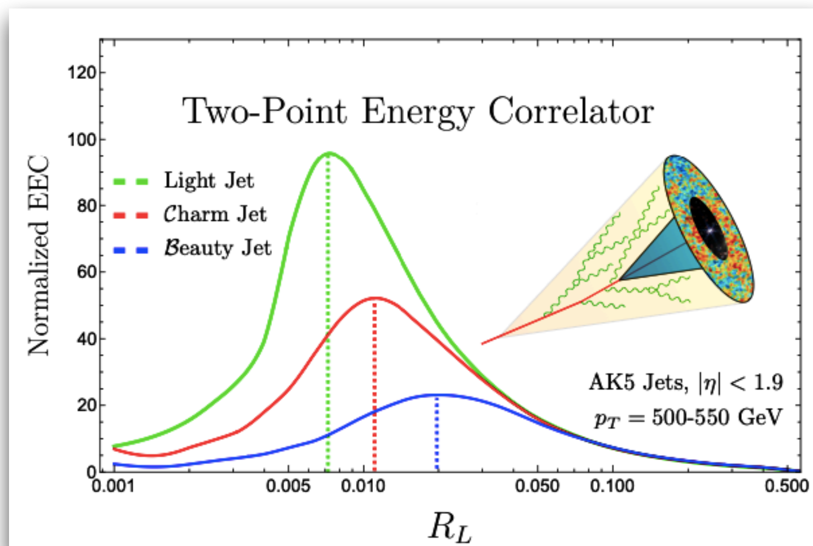
$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

Hard function



Virtuality $\sim p_T R_L + m_Q^2$

- Formation time changes with the mass of the quark.
- Can clearly see this from the two-point EEC.

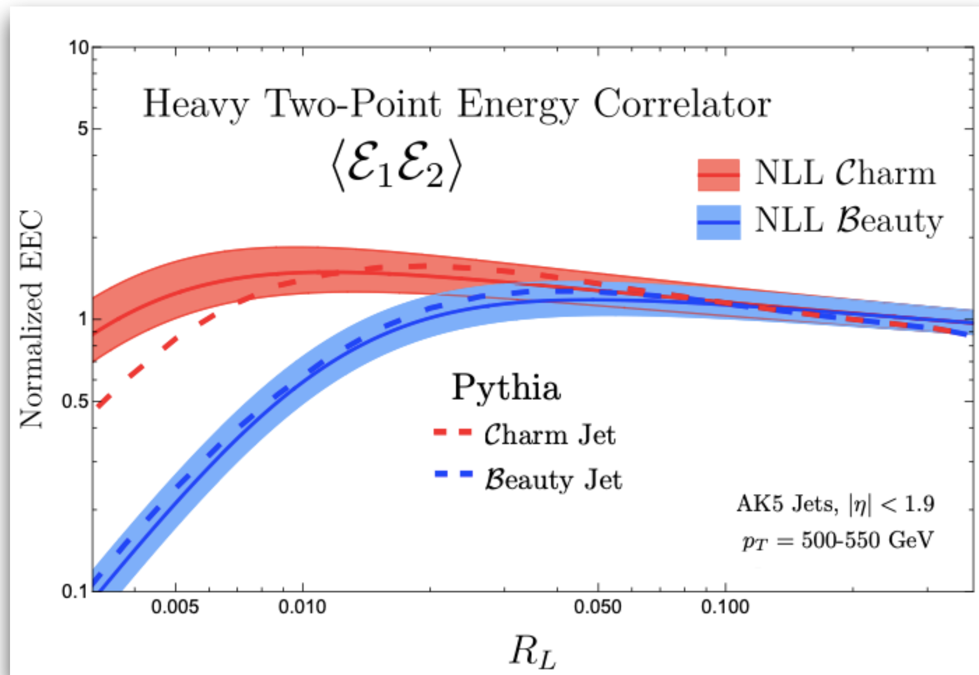


[Craft, Lee, BM, Moult]

Massive two point correlator

First massive jet substructure observable at NLL

- **Scaling behaviour identical to massless case for larger scales.**
- **A turn-over for** $R_L \to m_Q/p_T$
- **The change in the slope is perturbative effect contrary to massless jets:** $R_L \to \Lambda_{QCD}/p_T$
- **The turn-over region is of interest for improving heavy quark description in parton shower.**

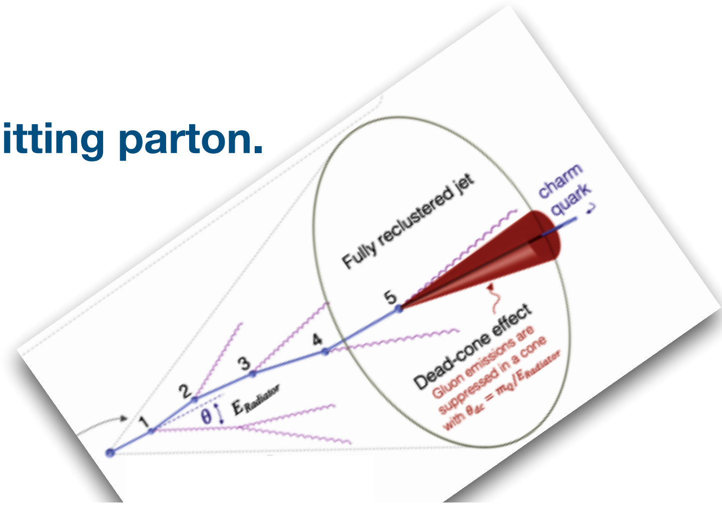


[Craft, Lee, BM, Moult]

Dead-cone effect in QCD

Fundamental phenomena

- **Parton-shower pattern depends on the mass of the emitting parton.**
- **Angular suppression** $\propto \frac{M}{E}$.



Observable used for the observation of the dead-cone effect in LHC data

$$R(\theta) = \frac{1}{N^{\mathrm{D^0\,jets}}} \frac{\mathrm{d}n^{\mathrm{D^0\,jets}}}{\mathrm{d}\ln(1/\theta)} \Big/ \frac{1}{N^{\mathrm{inclusive\,jets}}} \frac{\mathrm{d}n^{\mathrm{inclusive\,jets}}}{\mathrm{d}\ln(1/\theta)} \Bigg|_{k_\mathrm{T}, E_\mathrm{Radiator}}$$

- **Can we observe the dead-cone with EEC?**

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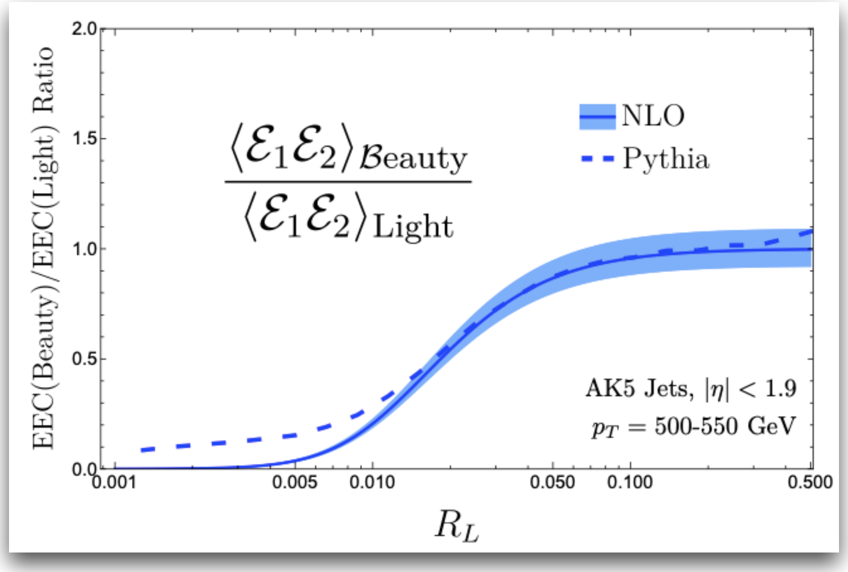
Direct observation of the dead-cone effect in quantum chromodynamics

ALICE Collaboration

Intrinsic mass effects

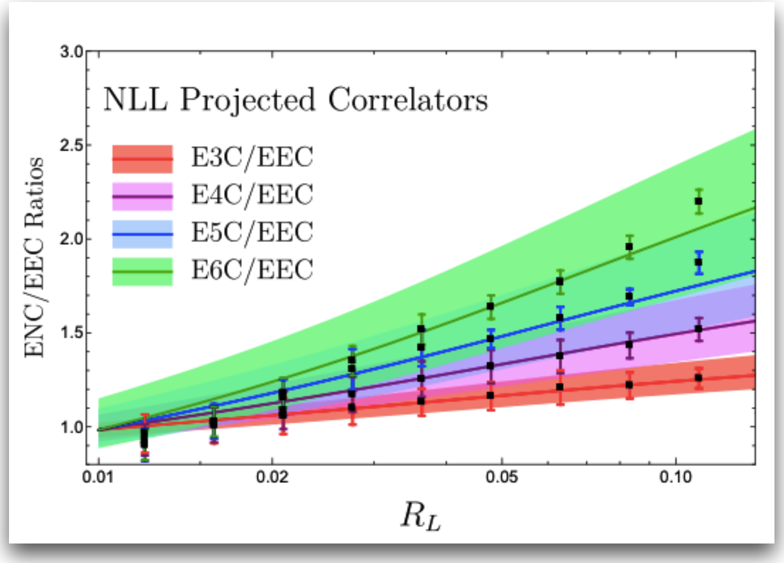
Dead-cone effect



[Craft, Lee, BM, Moult]

- **Ratios of the massive and massless EEC isolate mass (IR) effects.**
- **A transition region related to the quark mass: perturbatively calculable.**
- **Excellent agreement with MC.**
- **Small angle suppression can be interpreted as a dead-cone effect.**

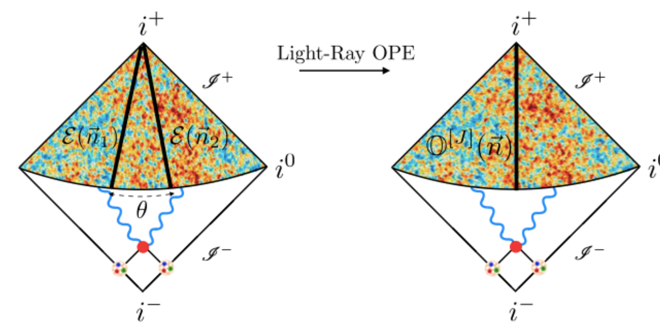
Higher point correlators



The light-ray OPE

- The leading scaling behavior at the LHC is described by the leading terms in the OPE: **twist two light-ray operators**.
- Light-ray OPE is a rigorous and convergent expansion in CFT.

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle = \sum c_i \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$



$$\langle \varepsilon\left(\vec{n}_1\right) \varepsilon\left(\vec{n}_2\right) \cdots \varepsilon\left(\vec{n}_k\right) \rangle = \frac{1}{R_L^2} \left\{ f_q^{[k]}\left(u_i, v_i\right) \mathbb{O}_q^{[k+1]}\left(\vec{n}_1\right) + f_g^{[k]}\left(u_i, v_i\right) \mathbb{O}_g^{[k+1]}\left(\vec{n}_1\right) \right\} + \mathcal{O}\left(R_L^0\right)$$

$$u_i = \left(\frac{x_{i_1 i_2} x_{i_3 i_3}}{x_{i_1 i_3} x_{i_2 i_4}}\right)^2 \qquad v_i = \left(\frac{x_{i_1 i_2} x_{i_3 i_4}}{x_{i_1 i_4} x_{i_2 i_3}}\right)^2$$

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J} \bar{\psi} \gamma^+ \left(iD^+\right)^{J-1} \psi,$$

$$\vec{\mathbb{O}}^{[J]} = \left(\mathbb{O}_q^{[J]}, \mathbb{O}_g^{[J]}\right)^T = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{\mathcal{O}}^{[J]}(t, r\vec{n})$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J} F_a^{\mu+} \left(iD^+\right)^{J-2} F_a^{\mu+}$$

Leading twist light-ray OPE

Control scaling at leading power

- **Twist-2 operators in QCD are characterized by a spin J and transverse spin j=0,2.**
- **They can be transformed to a twist-2 light-ray operator vector parametrized by J**

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+ \left(iD^+\right)^{J-1}\psi,$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}\left(iD^+\right)^{J-2}F_a^{\mu+}$$

$$\mathcal{O}_{\bar{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}\left(iD^+\right)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

$$\xrightarrow{\lim_{r\to\infty} r^2 \int_0^\infty dt}$$

$$\vec{\mathbb{O}}^{[J]}(\vec{n}) = \begin{bmatrix} \mathbb{O}_q^{[J]}(\vec{n}) \\ \mathbb{O}_g^{[J]}(\vec{n}) \\ \mathbb{O}_{\bar{g},+}^{[J]}(\vec{n}) \\ \mathbb{O}_{\bar{g},-}^{[J]}(\vec{n}) \end{bmatrix}$$

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Unpolarized

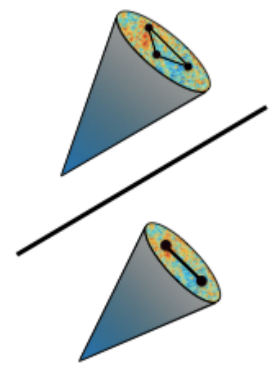
Polarized

Unpolarized Scaling

LHC scenario

- **Probe the unpolarized spin $j = 0$ operators**
- **The leading scaling behavior is determined by the anomalous dimension $\gamma(N+1)$ for an operator of spin $N+1$.**

→ can isolate the anomalous dimensions!

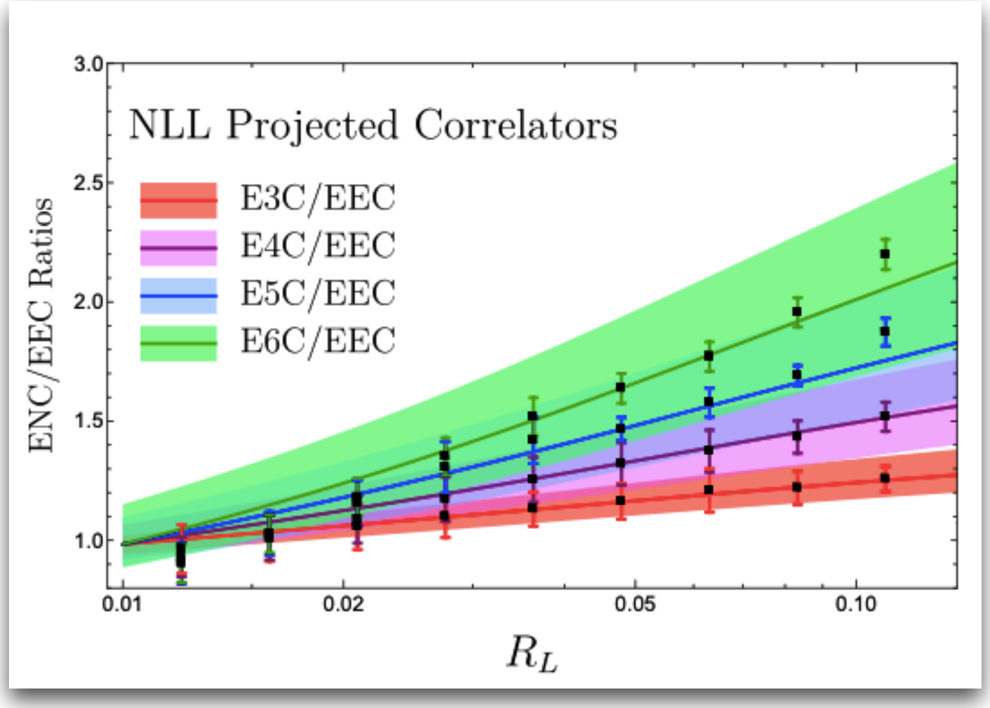


$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_{J-1} \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathbb{O}^{[J]} \rangle}{\langle \mathbb{O}^{[3]} \rangle}$$

The jet spectrum

Higher-point correlators

- **Asymptotic energy flux directly probes the spectrum of (twist-2) lightray operators at the quantum level**
- **Ratio of the higher-point correlators with the two-point isolates anomalous scaling!**
- **The anomalous scaling behavior depends on N (slope increases with N)**
- **First hand probe of the anomalous dimensions of QCD operators.**



[Lee, BM, Moult]

[Chen, Moult, Zhang, Zhu]

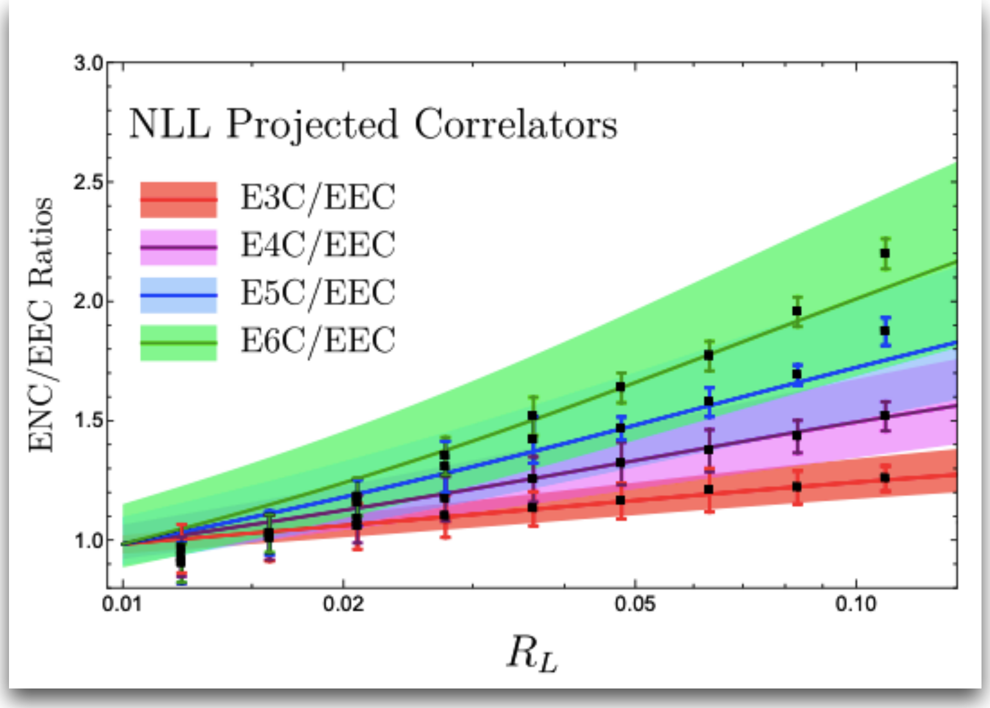
The jet spectrum

Higher-point correlators

- **Non-perturbative effects cancel in the ratio**
- **A clean measurement of strong coupling**

$$\theta^{\gamma} \to \exp\left(\frac{\hat{\gamma}}{2\beta_0} \ln \frac{\alpha_s(\theta Q)}{\alpha_s(Q)}\right)$$

- **Can be observed at the high energies at the LHC at high precision**



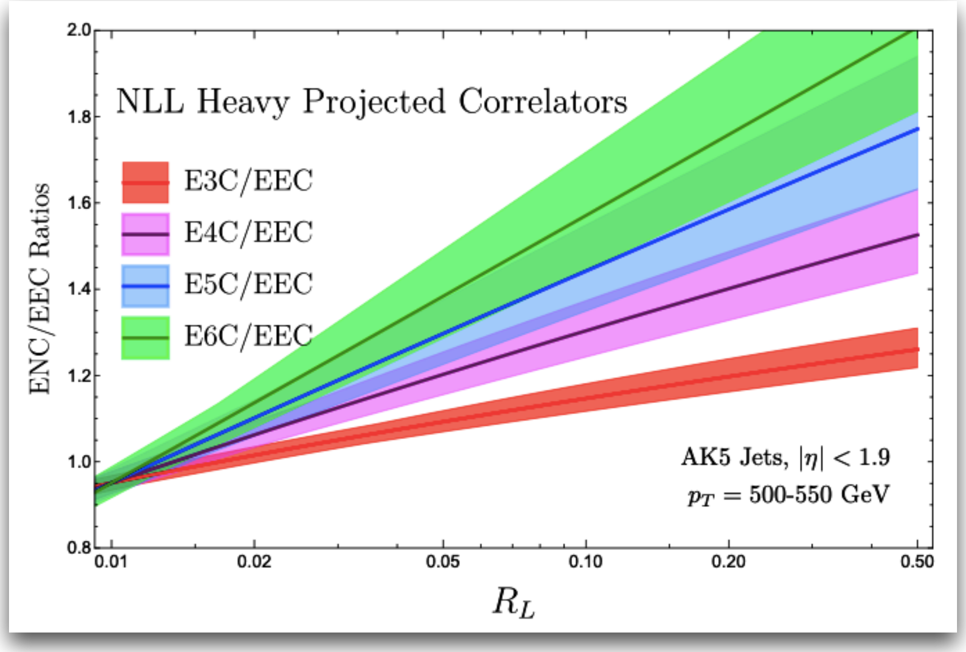
[Lee, BM, Moult]

[Chen, Moult, Zhang, Zhu]

Heavy Projected Energy Correlators

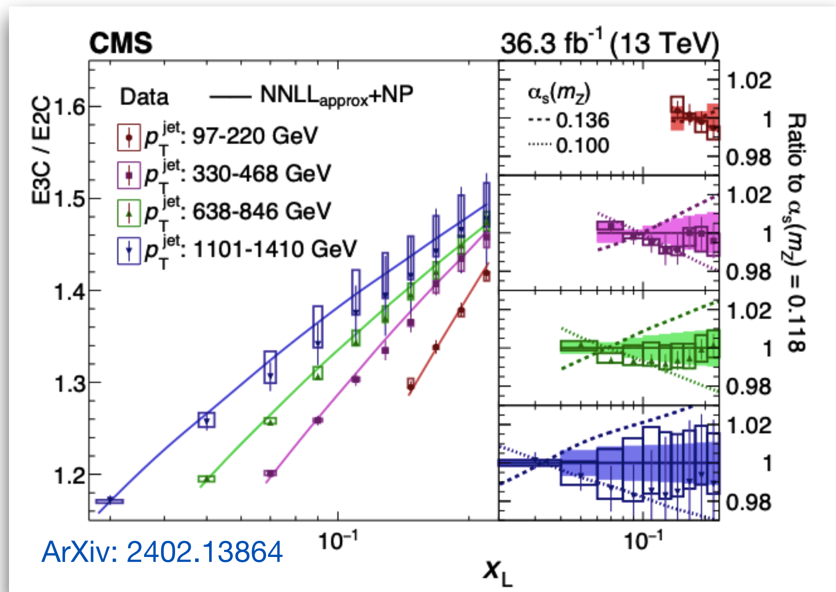
Resolve the UV scaling behaviour

- **Ratios of higher point correlators with the two point EEC are independent of IR effects, including quark mass.**
- **The exact behavior as the massless case.**
- **Non-trivial cross check of the factorization theorem!**
- **Anomalous dimensions should not be affected by the IR physics.**



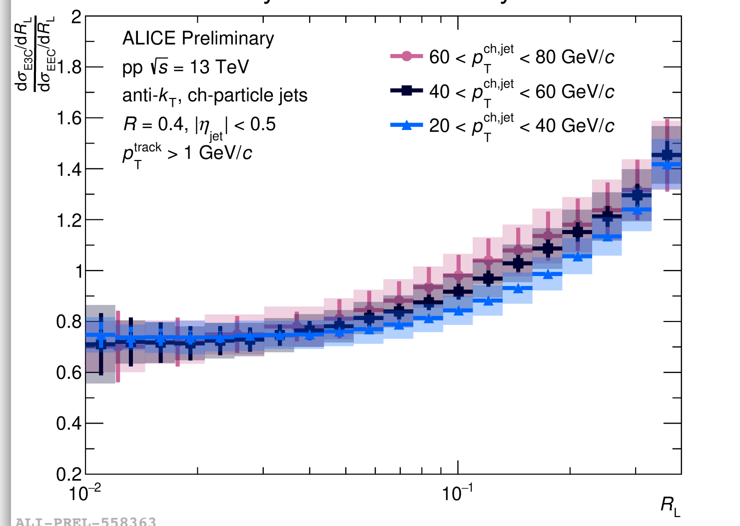
[Craft, Lee, BM, Moult]

Experimental Results

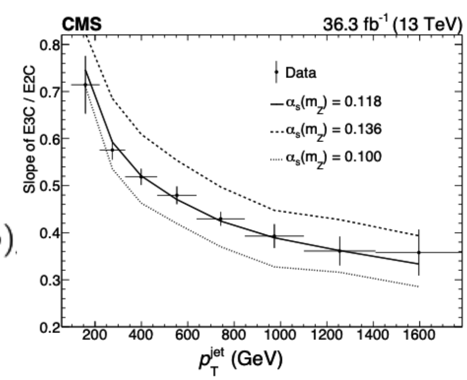


ArXiv: 2402.13864

Preliminary results from Ananya Rai



Extraction of the strong coupling

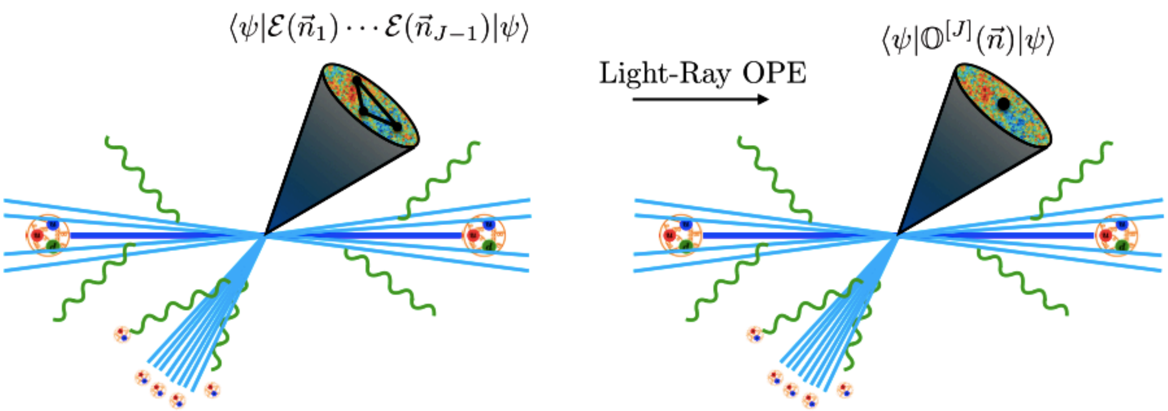


$$\alpha_S(m_Z) = 0.1229^{+0.0014}_{-0.0012}\,(\text{stat})^{+0.0030}_{-0.0033}\,(\text{theo})^{+0.0023}_{-0.0036}\,(\text{exp})$$

Jet substructure from first principles!

- Energy correlator is a jet substructure observable defined from first principles in QFT

$\Rightarrow$ No ambiguity between what is measured and the theory calculation.



- Formalism can be applied for any conserved charge for LHC processes.
- No jet grooming or pruning is needed to extract the final results, pure QFT calculation!
- Not sensitive to soft and wide angle radiations.

Applications of these results

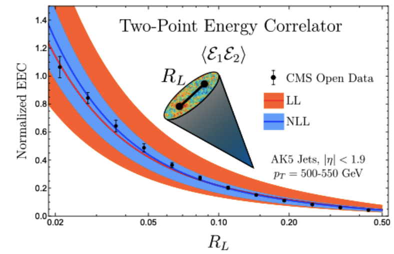
- **Precision measurements: α_s measurement**
- **Jet modeling in MC simulations: heavy flavours**
- **Precision in parton showers: “reference resummation” for testing DGLAP finite moments.**
- **Understand properties of the QGP: multi-scale problem too, global properties of plasma.**

[Andres, Dominguez, Kunnawalkam Elayawalli, Holguin, Marquet, Moult,…]

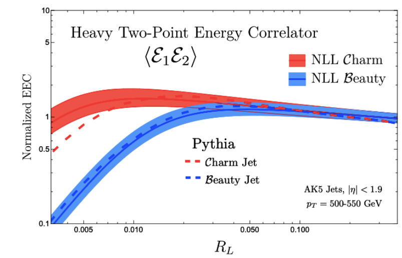
Conclusions

- **Factorization formula for calculating energy correlators for jet substructure at the LHC.**

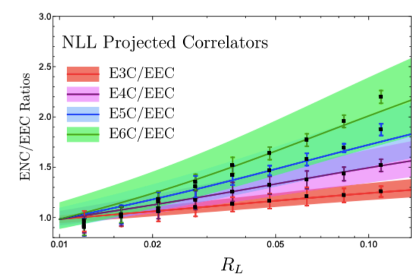
$$\frac{d\Sigma}{dp_T d\eta dz} = \sum_i \mathscr{H}_i \left(p_T z, \eta, \mu\right) \otimes \int_0^1 dx\, x^N \mathscr{J}_{ij}(z, x, p_T R, \mu)\, J_j^{[N]}(z, x, \mu)$$



- **Intrinsic mass effects of strongly interacting elementary particles.**



- **Higher-point correlators can be calculated for LHC and probe anomalous scaling dimension of QCD operators.**



Thank You!