Explorations in Metric-Affine Quadratic Gravity

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The present framework of modern cosmology consists of classical *General Relativity (GR)* as a theory of gravitation and *Quantum Field Theory (QFT)* as the theory of matter. A common working assumption is that the quantum aspects of gravitation can be ignored for energies below the Planck energy of $10^{19} GeV$ and, therefore, gravity can be treated classically. In contrast, the full quantum character of particle interactions is considered within QFT. The quantum interactions of the matter fields coupled to the classical gravitational field introduce modifications to the standard GR action with cosmological implications. Such are non-minimal couplings of the inflaton field or higher power terms of the Ricci curvature in models of cosmological inflation. The *Metric-Affine* formulation of gravity, where the metric and the connection are independent variables, although equivalent to the standard (*metric*) GR in the case of the Einstein-Hilbert action, leads to different predictions when the above corrections are included.

To be discussed

1) a metric-affine model with quadratic scalar curvature terms and its inflationary behaviour

2) a metric-affine model with quadratic scalar curvature terms with derivative couplings and its inflationary behaviour

Metric Versus Metric-Affine Formulation of Gravity

The **General Relativity Principle** states that all laws of physics should be invariant under general coordinate transformations. To implement such a principle we need to introduce a metric $g_{\mu\nu}$, which has to transform as

$$g'_{\alpha\beta}(x') = \left(\frac{\partial x^{\mu}}{\partial x'^{\alpha}}\right)\left(\frac{\partial x^{\nu}}{\partial x'^{\beta}}\right) g_{\mu\nu}(x) \,, \qquad (1)$$

as well as a **Connection** $\Gamma^{\rho}_{\mu\nu}$ in order to define covariant derivatives of tensors. In the standard **metric formulation** of gravity the connection is not an independent quantity but it is given by the **Levi-Civita relation** as

$$\Gamma^{\rho}_{\mu\nu}(g) \;=\; \frac{1}{2} g^{\rho\sigma}\left(\partial_{\mu} g_{\rho\nu} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu}\right) \,. \qquad (2)$$

In contrast, in the so-called **Metric-Affine** theories of gravity the connection is an **independent variable** not related to the metric through (2). Note that $D_{\mu} g_{\nu\rho}\big|_{LC} = 0$ (**metricity**), while $D_{\mu} g_{\nu\rho} \neq 0$ in general for a metric-affine theory.

Torsion and Non-Metricity

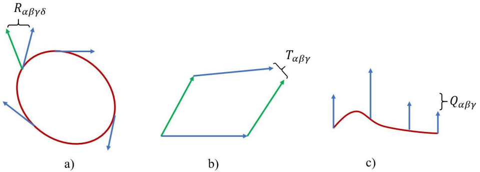
The **Curvature (Riemann tensor)** of a metric-affine theory is defined as

$$\mathcal{R}_{\mu\nu}{}^{\rho}{}_{\sigma} = \partial_\mu \Gamma^{\ \rho}_{\nu\sigma} - \partial_\nu \Gamma^{\ \rho}_{\mu\sigma} + \Gamma^{\ \rho}_{\mu\lambda}\Gamma^{\ \lambda}_{\nu\sigma} - \Gamma^{\ \rho}_{\nu\lambda}\Gamma^{\ \lambda}_{\mu\sigma} \,. \tag{3}$$

A general $\Gamma^{\rho}_{\mu\nu}$ implies non-zero **Torsion** and **Non-Metricity**

$$\begin{aligned} T^{\alpha}_{\beta\gamma} &\equiv \Gamma^{\ \alpha}_{\beta\gamma} - \Gamma^{\ \alpha}_{\gamma\beta} \qquad (\textbf{Torsion}) \\ Q_{\mu\alpha\beta} &= \nabla_\mu g_{\alpha\beta} \qquad (\textbf{Non-Metricity}) \end{aligned} \tag{4}$$

Curvature measures the change of a vector under rotation after parallel transport in a closed loop. Torsion measures the non-closure of parallelograms formed from parallel-transported vectors. Non-metricity measures the change of the length of parallel-transported vectors.



Formulation of Gravity	$\mathcal{R}_{\mu\nu\rho\sigma}$	$T_{\mu\nu\rho}$	$Q_{\mu\nu\rho}$
Metric-affine			
Einstein-Cartan			0
Weyl		0	
Metric		0	0
Generic Teleparallel	0		
Metric Teleparallel	0		0

Symmetry	Metric	Einstein-Cartan	Metric-affine
$R_{\mu\nu[\rho\sigma]}$	yes	yes	yes
$R_{[\mu\nu]\rho\sigma}$	yes	yes	no
$R_{(\mu\nu)(\rho\sigma)}$	yes	no	no

The Distortion Tensor

The difference between the independent connection of a metric-affine theory and the corresponding Levi-Civita one is a tensor called ***the Distrortion tensor***

$$C_{\mu\ \nu}^{\ \rho} = \Gamma_{\mu\ \nu}^{\ \rho} - \Gamma_{\mu\ \nu}^{\ \rho}(g)\,. \tag{5}$$

The distortion tensor vanishes for metric theories.

The curvature tensor can be written in terms of the distortion as

$$\mathcal{R}^{\alpha}_{\ \beta\gamma\delta} = R^{\alpha}_{\ \beta\gamma\delta}(g) + \nabla_{\gamma} C_{\delta\ \beta}^{\ \alpha} - \nabla_{\delta} C_{\gamma\ \beta}^{\ \alpha} + C_{\gamma\ \lambda}^{\ \alpha} C_{\delta\ \beta}^{\ \lambda} - C_{\delta\ \lambda}^{\ \alpha} C_{\gamma\ \beta}^{\ \lambda} \tag{6}$$

where $R^{\alpha}_{\ \beta\gamma\delta}(g)$ is the standard metric Riemann tensor and ∇ is the standard metric covariant derivative in terms of the Levi-Civita connection. Note that the only symmetry is $\mathcal{R}^{\alpha}_{\ \beta\gamma\delta} = -\mathcal{R}^{\alpha}_{\ \beta\delta\gamma}$.

It is possible to express the distortion $C_{\mu}{}^{\rho}{}_{\nu}$ (or $\Gamma_{\mu}{}^{\rho}{}_{\nu}$) in terms of the torsion and the non-metricity

$$C_{\mu}{}^{\rho}{}_{\nu} = \frac{1}{2}\left(Q^{\rho}{}_{\mu\nu} - Q_{\nu}{}^{\rho}{}_{\mu} - Q_{\mu}{}^{\rho}{}_{\nu} + T^{\rho}{}_{\mu\nu} + T_{\nu}{}^{\rho}{}_{\mu} + T_{\mu}{}^{\rho}{}_{\nu}\right) \tag{7}$$

or

$$\begin{aligned} C_{\mu}{}^{\rho}{}_{\nu} \; &= \; \delta^{\rho}_{\mu}\left[-\tfrac{1}{3}T_{\nu} + \tfrac{1}{36}\left(2\hat{Q}_{\nu} - 5Q_{\nu}\right)\right] + \tfrac{1}{36}\delta^{\rho}_{\nu}\left[2\hat{Q}_{\mu} - 5Q_{\mu}\right] \\ &+ g_{\mu\nu}\left[\tfrac{1}{3}T^{\rho} + \tfrac{1}{36}\left(7Q^{\rho} - 10\hat{Q}^{\rho}\right)\right] + \tfrac{1}{12}\epsilon^{\rho}{}_{\mu\nu\sigma}\hat{T}^{\sigma} + \tau_{\mu}{}^{\rho}{}_{\nu} \end{aligned} \tag{8}$$

This expression gives the distortion in terms of the four vectors

$$T_{\mu} = T^{\alpha}{}_{\mu\alpha}, \quad \hat{T}_{\mu} = \epsilon_{\mu}{}^{\alpha\beta\gamma}T_{\alpha\beta\gamma}, \quad Q_{\mu} = Q_{\mu}{}^{\alpha}{}_{\alpha}, \quad \hat{Q}_{\mu} = Q^{\alpha}{}_{\alpha\mu} \tag{9}$$

and a purely tensorial part

$$\tau_{\mu\rho\nu} = \frac{1}{2}\left(t_{\rho\mu\nu} + t_{\mu\rho\nu} + t_{\nu\rho\mu} + q_{\rho\mu\nu} - q_{\mu\rho\nu} - q_{\nu\rho\mu}\right) \tag{10}$$

$(q_{\alpha\beta\gamma} = q_{\alpha\gamma\beta},\; t_{\alpha\beta\gamma} = -t_{\alpha\gamma\beta},\;\; q^{\alpha\beta}{}_{\beta} = q^{\beta\alpha}{}_{\beta} = t^{\alpha\beta}{}_{\beta} = t^{\beta\alpha}{}_{\beta} = 0)$

"Equivalent" metric theory

The metric-affine version of the Einstein-Hilbert action can be written in terms of the distortion as

$$\int d^4x\sqrt{-g}\mathcal{R} = \int d^4x\sqrt{-g}\left\{R(g)+\nabla_\rho C_\nu{}^{\rho\nu}-\nabla_\nu C_\rho{}^{\rho\nu}+C_\rho{}^\rho{}_\lambda C_\nu{}^{\lambda\nu}-C_\nu{}^\rho{}_\lambda C_\rho{}^{\lambda\nu}\right\} \tag{11}$$

Variation with respect to the distortion gives a linear algebraic equation with a trivial solution that reduces the action into the standard metric GR form. Therefore, The metric-affine Einstein-Hilbert action is entirely equivalent to standard GR. Nevertheless, this is not so if the action includes quadratic terms of the curvature

$$\int d^4x\sqrt{-g}\left\{\mathcal{R}+f^{\mu\mu'\nu\nu'\sigma\sigma'}_{\rho\rho'}\mathcal{R}_{\mu\nu}{}^\rho{}_\sigma\mathcal{R}_{\mu'\nu'}{}^{\rho'}{}_{\sigma'}\right\} \tag{12}$$

Introducing the distortion, we obtain

$$\int d^4x\sqrt{-g}\left\{R(g)+f^{\mu\mu'\nu\nu'\sigma\sigma'}_{\rho\rho'}R_{\mu\nu}{}^\rho{}_\sigma(g)R_{\mu'\nu'}{}^{\rho'}{}_{\sigma'}(g)+\Delta(g,\,C)\right\} \tag{13}$$

where the Δ-term contains up to quartic distortion terms and upon variation the resulting equation is dynamical corresponding to the extra dynamical degrees of freedom of the connection.

Non-Minimal Coupling to Scalars

One can derive the **metric-equivalent** of any metric-affine theory based on an action, where gravity couples to a scalar field,

$$\mathcal{S} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2}\Omega^2(\phi)\mathcal{R} + \mathcal{L}(\phi, g_{\mu\nu}, \partial_\mu\phi) \right\} . \qquad (14)$$

Note that any $F(\mathcal{R})$ theory can also be set in this form. Indeed the action $\mathcal{S} = \frac{1}{2}\int d^4x \sqrt{-g}\, F(\mathcal{R})$, corresponding to the metric-affine formulation of $f(R)$ theories studied in the standard metric formulation. The action can be set in the form

$$\mathcal{S} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2}F'(\chi)\,\mathcal{R} - V(\chi) \right\} \text{ where } V(\chi) = \frac{1}{2}\left(\chi F'(\chi) - F(\chi)\right) , \qquad (15)$$

in terms of the auxiliary scalar χ.

Substituting the expression of $\mathcal{R}$ in terms of the distortion, we obtain

$$\mathcal{S} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2}\Omega^2(\phi)R(g) + \frac{1}{2}\Omega^2(\phi)\left(D_\mu C_\nu{}^{\mu\nu} - D_\nu C_\mu{}^{\mu\nu} \right.\right.$$

$$\left.\left. + C_\mu{}^\mu{}_\lambda C_\nu{}^{\lambda\nu} - C_\nu{}^\mu{}_\lambda C_\mu{}^{\lambda\nu}\right) + \mathcal{L}(\phi, g_{\mu\nu}, \partial_\mu\phi) \right\} , \qquad (16)$$

Variation with respect to the distortion gives an algebraic equation with a solution for it (up to terms $U_\mu\, g_{\nu\rho}$ of an arbitrary vector U_μ)

$$C_{\mu\nu\rho} = \frac{1}{2}\left(g_{\mu\nu}\partial_\rho \ln\Omega^2 - g_{\mu\rho}\partial_\nu \ln\Omega^2\right) \tag{17}$$

Note that this corresponds to $Q = \hat{Q} = 0$ and $\tau = 0$. Substituting C back into the action we obtain

$$\mathcal{S} = \int d^4x\,\sqrt{-g}\left\{\frac{1}{2}\Omega^2(\phi)R(g) + \frac{3}{4}\frac{(\nabla\Omega^2)^2}{\Omega^2} + \mathcal{L}(\phi, g_{\mu\nu}, \partial_\mu\phi)\right\}\,. \tag{18}$$

This is a metric theory and the appearing connection is the Levi-Civita one. Note that the extra term has the form of the extra kinetic term that appears when we Weyl-rescale the metric theory to the Einstein frame, albeit *with an opposite sign*. The inequivalence of the two formulations rests on this term. Going to the Einstein frame we obtain standard GR without any additional scalar dof apart from a modified matter Lagrangian.

Quadratic Scalar Curvature terms

There are only two scalars, linear in the Riemann tensor, defined as

$$\begin{cases} \mathcal{R} = \mathcal{R}^{\mu\nu}{}_{\mu\nu} = R(g) + \nabla_\rho C_\nu{}^{\rho\nu} - \nabla_\nu C_\rho{}^{\rho\nu} + C_\rho{}^\rho{}_\lambda C_\nu{}^{\lambda\nu} - C_\nu{}^\rho{}_\lambda C_\rho{}^{\lambda\nu} & (\textbf{Ricci scalar}) \\ \tilde{\mathcal{R}} = (-g)^{-1/2}\epsilon^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma} = 2(-g)^{-1/2}\epsilon^{\mu\nu\rho\sigma}\left(\nabla_\mu C_{\nu\rho\sigma} + C_{\mu\rho\lambda}C_\nu{}^\lambda{}_\sigma\right) & (\textbf{Holst invariant}) \end{cases} \tag{19}$$

Consider the following metric-affine generalization of the Starobinsky model

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\alpha\mathcal{R} + \frac{1}{2}\beta\tilde{\mathcal{R}} + \frac{1}{4}\gamma\mathcal{R}^2 + \frac{1}{4}\delta\tilde{\mathcal{R}}^2\right\}, \tag{20}$$

where $\mathcal{R}$ is the Ricci scalar curvature and $\tilde{\mathcal{R}}$ is the Holst invariant. This is a general quadratic action of these scalars. In what follows we shall use Planck-mass units taking $\alpha = 1$. An equivalent way to express the action is in terms of the auxiliary scalars χ and ζ as

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}(1+\gamma\chi)\mathcal{R} + \frac{1}{2}(\beta+\delta\zeta)\tilde{\mathcal{R}} - \frac{1}{4}\left(\gamma\chi^2 + \delta\zeta^2\right)\right\}. \tag{21}$$

Next, we may use the expressions of $\mathcal{R}$ and $\tilde{\mathcal{R}}$ in terms of the Distortion C, given in (19), and obtain

$$\mathcal{S} = \int d^4x\sqrt{-g}\,\{\tfrac{1}{2}(1+\gamma\chi)R$$

$$+\tfrac{1}{2}(1+\gamma\chi)\left(D_\mu C_\nu{}^{\mu\nu} - D_\nu C_\mu{}^{\mu\nu} + C_\mu{}^\mu{}_\lambda C_\nu{}^{\lambda\nu} - C_\nu{}^\mu{}_\lambda C_\mu{}^{\lambda\nu}\right) \quad (22)$$

$$+(\beta+\delta\zeta)(-g)^{-1/2}\epsilon^{\mu\nu\rho\sigma}\left(D_\mu C_{\nu\rho\sigma} + C_{\mu\rho\lambda}C_\nu{}^\lambda{}_\sigma\right) - \tfrac{1}{4}\left(\gamma\chi^2+\zeta^2\right)\}$$

where $R = R(g)$ and the covariant derivatives are with respect to $\Gamma^\lambda_{\mu\nu}\big|_{LC}$. Variation with respect to $C_\alpha{}^{\beta\gamma}$ gives an algebraic equation with a solution

$$C_{\mu\nu\rho} = \frac{g_{\mu\nu}}{2\Delta}\left(\Omega^2\partial_\rho\Omega^2 + 4\overline{\Omega}^2\partial_\rho\overline{\Omega}^2\right) - \frac{g_{\mu\rho}}{2\Delta}\left(\Omega^2\partial_\nu\Omega^2 + 4\overline{\Omega}^2\partial_\nu\overline{\Omega}^2\right)$$

$$+\frac{\epsilon_{\mu\nu\rho\sigma}}{\Delta\sqrt{-g}}\left(\Omega^2\partial^\sigma\overline{\Omega}^2 - \overline{\Omega}^2\partial^\sigma\Omega^2\right), \quad (23)$$

where

$$\Omega^2 \equiv 1+\gamma\chi, \quad \overline{\Omega}^2 = \beta+\delta\zeta \quad (24)$$

and $\Delta \equiv \Omega^4 + 4\overline{\Omega}^4$. Note that this corresponds to $Q = \hat{Q} = \tau = 0$.

Substituting C back into the action, we obtain the corresponding metric action

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\Omega^2 R(g) + \frac{3}{4}\frac{(\nabla\Omega^2)^2}{\Omega^2} - \frac{3}{\Omega^2\Delta}\left(\Omega^2\nabla\overline{\Omega}^2 - \overline{\Omega}^2\nabla\Omega^2\right)^2 \right.$$
$$\left. -\frac{1}{4\gamma}(\Omega^2-1)^2 - \frac{1}{4\delta}(\overline{\Omega}^2-\beta)^2\right\} \quad (25)$$

The Weyl rescaling $g_{\mu\nu} = \Omega^{-2}\bar{g}_{\mu\nu}$ takes us to the Einstein frame. The action is

$$\mathcal{S} = \int d^4x\sqrt{-\bar{g}}\left\{\frac{1}{2}\bar{R}(\bar{g}) - \frac{3}{\Omega^4\Delta}\left(\Omega^2\bar{\nabla}\overline{\Omega}^2 - \overline{\Omega}^2\bar{\nabla}\Omega^2\right)^2 \right.$$
$$\left. -\frac{1}{\Omega^4}\left(\frac{1}{\gamma}(\Omega^2-1)^2 + \frac{1}{\delta}(\overline{\Omega}^2-\beta)^2\right)\right\} \quad (26)$$

Introducing the field $\sigma \equiv \overline{\Omega}^2/2\Omega^2$ the scalar part of the Lagrangian becomes

$$\mathcal{L} = -\frac{12(\bar{\nabla}\sigma)^2}{(1+16\sigma^2)} - \frac{1}{4}\left(\frac{1}{\gamma}(\Omega^{-2}-1)^2 + \frac{1}{\delta}(2\sigma-\beta\Omega^{-2})^2\right) . \quad (27)$$

Variation with respect to the non-dynamical Ω^2 gives

$$\frac{\delta \mathcal{L}}{\delta \Omega^2} = 0 \Longrightarrow \Omega^{-2} = \frac{\delta + 2\beta\gamma\sigma}{\delta + \beta^2\gamma} \Longrightarrow \mathcal{L} = -\frac{12(\bar{\nabla}\sigma)^2}{(1+16\sigma^2)} - \frac{1}{4}\frac{(2\sigma - \beta)^2}{(\delta + \beta^2\gamma)} . \tag{28}$$

The theory can be expressed in terms of a canonical scalar s defined by

$$\sigma = \frac{1}{4}\sinh(\sqrt{2/3}\, s) \tag{29}$$

as

$$\mathcal{L} = -\frac{1}{2}(\nabla s)^2 - \frac{1}{16}\frac{\left(\sinh(\sqrt{2/3}\, s) - 2\beta\right)^2}{(\delta + \beta^2\gamma)} . \tag{30}$$

At least one of γ and δ has to be included in order to generate the additional pseudoscalar degree of freedom represented by σ. The inflationary behaviour of this model has been studied by G.Pardisi and A.Salvio (2022). Note that the parameters γ and δ, associated with $\mathcal{R}^2$ and $\tilde{\mathcal{R}}^2$, can only have a secondary role in a possible inflationary behaviour, which would be controlled by β.

$\mathcal{R}$	GR
$\mathcal{R}+\tilde{\mathcal{R}}$	GR
$\mathcal{R}+\mathcal{R}^2$	GR
$\mathcal{R}+\tilde{\mathcal{R}}^2$	σ, No Inflation
$\mathcal{R}+\tilde{\mathcal{R}}+\mathcal{R}^2$	σ, Inflation possible
$\mathcal{R}+\tilde{\mathcal{R}}+\tilde{\mathcal{R}}^2$	σ, Inflation possible
$\mathcal{R}+\tilde{\mathcal{R}}^2+\mathcal{R}^2$	σ, No Inflation
$\mathcal{R}+\tilde{\mathcal{R}}+\mathcal{R}^2+\tilde{\mathcal{R}}^2$	σ, Inflation possible

I.Antoniadis, A.Karam, A.Lykkas, KT (2018)

I.Gialamas, KT (2023)

G.Pradisi, A.Salvio (2022)

Coupling to a Fundamental Scalar

We consider a scalar ϕ coupled to quadratic metric-affine gravity non-minimally. The action is

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}f(\phi)\mathcal{R} + \frac{1}{2}h(\phi)\tilde{\mathcal{R}} + \frac{\gamma}{4}\mathcal{R}^2 + \frac{\delta}{4}\tilde{\mathcal{R}}^2 + \mathcal{L}_\phi\right\}, \tag{31}$$

with

$$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\,. \tag{32}$$

Introducing the auxiliaries χ and ζ, we arrive at

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}(\gamma\chi + f(\phi))\mathcal{R} + \frac{1}{2}(\delta\zeta + h(\phi))\tilde{\mathcal{R}} - \frac{1}{4}\left(\gamma\chi^2 + \delta\zeta^2\right) + \mathcal{L}_\phi\right\} \tag{33}$$

or, introducing

$$\Omega^2 = \gamma\chi + f(\phi), \quad \overline{\Omega}^2 = \delta\zeta + h(\phi)\,, \tag{34}$$

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\Omega^2\mathcal{R} + \frac{1}{2}\overline{\Omega}^2\tilde{\mathcal{R}} - \frac{1}{4}\left(\frac{1}{\gamma}(\Omega^2 - f(\phi))^2 + \frac{1}{\delta}(\overline{\Omega}^2 - h(\phi))^2\right) + \mathcal{L}_\phi\right\} \tag{35}$$

The Corresponding Metric Theory

Rewriting the action in terms of the Distortion and solving for it we arrive at the action of the corresponding metric theory in the Jordan frame

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\Omega^2 R(g) + \frac{3}{4}\frac{(\nabla\Omega^2)^2}{\Omega^2} - \frac{3}{\Omega^2\Delta}\left(\Omega^2\nabla\overline{\Omega}^2 - \overline{\Omega}^2\nabla\Omega^2\right)^2 \right.$$
$$\left. -\frac{1}{4}\left(\frac{1}{\gamma}(\Omega^2 - f(\phi))^2 + \frac{1}{\delta}(\overline{\Omega}^2 - h(\phi))^2\right) + \mathcal{L}_\phi\right\} \qquad (36)$$

The Weyl rescaling $g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu}$ takes the action into the Einstein frame

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}R(g) - \frac{3}{\Omega^4\Delta}\left(\Omega^2\nabla\overline{\Omega}^2 - \overline{\Omega}^2\nabla\Omega^2\right)^2 \right.$$
$$\left. -\frac{1}{4\Omega^4}\left(\frac{1}{\gamma}(\Omega^2 - f(\phi))^2 + \frac{1}{\delta}(\overline{\Omega}^2 - h(\phi))^2\right) - \frac{1}{2}\frac{(\nabla\phi)^2}{\Omega^2} - \frac{V(\phi)}{\Omega^4}\right\} \qquad (37)$$

Introducing the field $\sigma = \frac{\overline{\Omega}^2}{2\Omega^2}$, we get the action in the form

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}R - \frac{12(\nabla\sigma)^2}{(1+16\sigma^2)} - \frac{1}{2}\frac{(\nabla\phi)^2}{\Omega^2} - \frac{\sigma^2}{\delta}\right.$$

$$\left. -\frac{1}{4\gamma\Omega^4}\left(f(\phi) - \Omega^2\right)^2 - \frac{h(\phi)}{4\delta\Omega^4}\left(h(\phi) - 4\sigma\Omega^2\right) - \frac{V(\phi)}{\Omega^4}\right\} \quad (38)$$

Note that no kinetic term for Ω^2 appears. Solving for it we obtain

$$\frac{\delta\mathcal{S}}{\delta\Omega^2} = 0 \implies \Omega^2 = \frac{f(\phi)^2 + 4\gamma V(\phi) + \gamma h^2(\phi)/\delta}{f(\phi) + 2\gamma\sigma h(\phi)/\delta - \gamma(\nabla\phi)^2} \quad (39)$$

Substituting Ω^2 into the action we get it in the form

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{2}K_\phi(\phi,\sigma)(\nabla\phi)^2 + \frac{1}{4}L_\phi(\phi)(\nabla\phi)^4 - \frac{1}{2}K_\sigma(\sigma)(\nabla\sigma)^2 - U(\phi,\sigma)\right\} \tag{40}$$

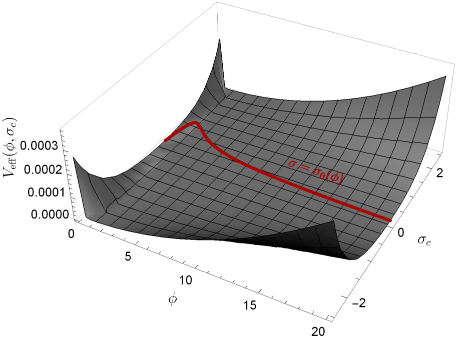
where

$$\begin{cases} K_\phi(\phi,\sigma) = \frac{f(\phi)+2\gamma\sigma h(\phi)/\delta}{\gamma h^2(\phi)/\delta+f^2(\phi)+4\gamma V(\phi)} \\ L_\phi = \frac{\gamma}{\gamma h^2(\phi)/\delta+f^2(\phi)+4\gamma V(\phi)} \\ K_\sigma(\sigma) = \frac{24}{1+16\sigma^2} \\ U(\phi,\sigma) = \frac{V(\phi)}{f^2(\phi)+4\gamma V(\phi)} + \frac{1}{\delta}\left(\frac{f^2(\phi)+4\gamma V(\phi)}{\gamma h^2(\phi)/\delta+f^2(\phi)+4\gamma V(\phi)}\right)(\sigma-\sigma_0(\phi))^2 \end{cases} \tag{41}$$

where

$$\sigma_0(\phi) = \frac{h(\phi)f(\phi)/2}{f^2(\phi)+4\gamma V(\phi)}\,. \tag{42}$$

Note that the potential is positive-definite with a minimum line along $\sigma = \sigma_0(\phi)$.



3D plot of $U(\phi, \sigma)$ for $f(\phi) = 1 + \xi\phi^2$, $h(\phi) = \bar{\xi}\phi + \bar{\xi}'\phi^3$ and $V(\phi) = \frac{\lambda}{4}\phi^4$

Inflation in the $\mathcal{R}^2$, $\tilde{\mathcal{R}}^2$, ϕ model

Addopting the following leading terms of $f(\phi)$ and $h(\phi)$, namely

$$f(\phi) = 1 + \xi\phi^2, \quad h(\phi) = \bar{\xi}\phi + \bar{\xi}'\phi^3 . \tag{43}$$

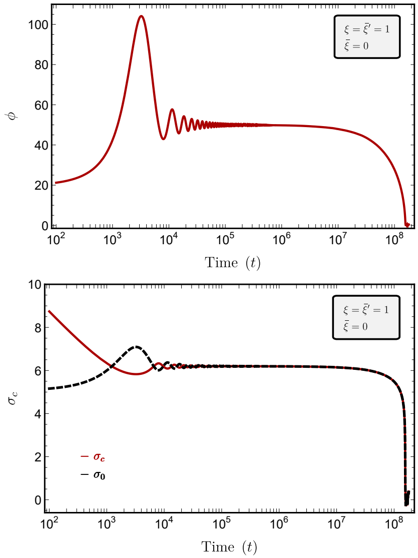
Note that $h(\phi)$ is chosen this way to counteract the parity-odd coupling $h(\phi)\tilde{\mathcal{R}}$. We also replace σ with the canonical field

$$\sigma_c = 2\sqrt{6}\int \frac{d\sigma}{\sqrt{1+16\sigma^2}} \implies \sigma = \frac{1}{4}\sinh\left(\sqrt{\frac{2}{3}}\,\sigma_c\right) . \tag{44}$$

In an FRW background the equations of motion read

$$\begin{aligned}
&(K_\phi + 3L_\phi\dot{\phi}^2)\ddot{\phi} + 3H(K_\phi + L_\phi\dot{\phi}^2)\dot{\phi} + \dot{\phi}\dot{\sigma}_c\frac{\partial K_\phi}{\partial\sigma_c} + \left(\frac{1}{2}\frac{\partial K_\phi}{\partial\phi} + \frac{3}{4}\frac{\partial L_\phi}{\partial\phi}\dot{\phi}^2\right)\dot{\phi}^2 + \frac{\partial U}{\partial\phi} = 0\\
&\ddot{\sigma}_c + 3H\dot{\sigma}_c - \frac{1}{2}\frac{\partial K_\phi}{\partial\sigma_c}\dot{\phi}^2 + \frac{\partial U}{\partial\sigma_c} = 0\\
&H^2 = \frac{1}{3}\rho\,, \qquad \rho = \frac{1}{2}K_\phi\dot{\phi}^2 + \frac{3}{4}L_\phi\dot{\phi}^4 + \frac{1}{2}\dot{\sigma}_c^2 + U\\
&\dot{H} = -\frac{1}{2}(\rho + p)\,, \qquad p = \frac{1}{2}K_\phi\dot{\phi}^2 + \frac{1}{4}L_\phi\dot{\phi}^4 + \frac{1}{2}\dot{\sigma}_c^2 - U
\end{aligned} \tag{45}$$

Solving numerically the equations of motion (with $V = \lambda\phi^4/4$) we obtain the plots



showing that very soon the system falls along the minimum line $\sigma_0(\phi)$.

Therefore, for the inflationary period it would be sufficient to study the *single-field problem* described by $\phi, \sigma_0(\phi)$

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{2}\bar{K}(\phi)(\nabla\phi)^2 + \frac{1}{4}L(\phi)(\nabla\phi)^4 - U(\phi)\right\}, \tag{46}$$

where

$$\bar{K}(\phi) = \frac{f(\phi)}{f^2(\phi)+4\gamma V(\phi)} + \left(\frac{12}{1+\frac{4g(\phi)^2 f^2(\phi)}{[f^2(\phi)+4\gamma V(\phi)]^2}}\right)\Bigg\{ \frac{g'(\phi)f(\phi)+g(\phi)f'(\phi)}{f^2(\phi)+4\gamma V(\phi)} - \frac{g(\phi)f(\phi)}{[f^2(\phi)+4\gamma V(\phi)]^2}\left(2f'(\phi)f(\phi) + 4\gamma V'(\phi)\right) \Bigg\}^2,$$

$$L(\phi) = \frac{\gamma}{\gamma g^2(\phi)/\delta + f^2(\phi) + 4\gamma V(\phi)},$$

$$U(\phi) = \frac{V(\phi)}{f^2(\phi)+4\gamma V(\phi)}. \tag{47}$$

Note that both kinetic functions $\bar{K}(\phi)$ and $L(\phi)$ are positive definite.

Considering an FRW background, we obtain the set of equations

$$H^2 = \frac{\rho}{3} \quad \text{and} \quad \dot{H} = -\frac{1}{2}(\rho + p) \ , \tag{48}$$

where the energy density and pressure are given by

$$\rho = \frac{1}{2}\bar{K}(\phi)\dot{\phi}^2 + \frac{3}{4}L(\phi)\dot{\phi}^4 + U(\phi) \quad \text{and} \quad p = \frac{1}{2}\bar{K}(\phi)\dot{\phi}^2 + \frac{1}{4}L(\phi)\dot{\phi}^4 - U(\phi) \, . \tag{49}$$

In the analysis of inflationary observables we have focused on $f(\phi) = 1 + \xi\phi^2$, $g(\phi) = \bar{\xi}\phi + \bar{\xi}'\phi^3$ and $V(\phi) = \lambda\phi^4/4$.
Due to the $\dot{\phi}^4$ terms the speed of sound deviates from unity

$$c_s^2 = \frac{1 + L(\phi)\dot{\phi}^2/\bar{K}(\phi)}{1 + 3L(\phi)\dot{\phi}^2/\bar{K}(\phi)} \, . \tag{50}$$

Nevertheless, the deviation from unity turns out to be quite small.

Inflationary Observables

Assuming the *slow-roll approximation* we have the *scalar and tensor power spectrum*

$$\mathcal{P}_\zeta(k) \approx \frac{U(\phi_*)}{24\pi^2\epsilon_U(\phi_*)}\left(\frac{k}{k_*}\right)^{n_s-1}, \quad \mathcal{P}_T \approx \frac{2U(\phi_*)}{3\pi^2}\left(\frac{k}{k_*}\right)^{n_t} \qquad (51)$$

The *scalar* and *tensor spectral indices* are

$$n_s = 1 + \frac{d\ln\mathcal{P}_\zeta}{d\ln k} \approx -6\epsilon_U + 2\eta_{\bar{U}} \quad end \quad n_t = \frac{d\ln\mathcal{P}_T}{d\ln k} \qquad (52)$$

where the *(potential) slow-roll parameters* are defined

$$\epsilon_U = \frac{1}{2\bar{K}}\left(\frac{U'}{U}\right)^2, \quad \eta_U = \frac{\left(\bar{K}^{-1/2}U'\right)'}{\bar{K}^{1/2}U} \qquad (53)$$

The *tensor-to-scalar ratio* is $r = \mathcal{P}_T/\mathcal{P}_\zeta \approx 16\epsilon_U$. Recent observations yield the constraints ($k_* = 0.05\, Mpc^{-1}$)

$$A_s = (2.10 \pm 0.03) \times 10^{-9}, \quad n_s = 0.9649 \pm 0.0042\,(1\sigma \text{ region}), \quad r < 0.03 \qquad (54)$$

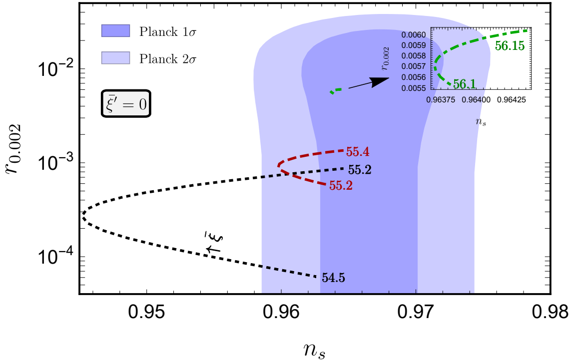


Figure: Predictions of the model using pivot scales 0.05 Mpc^{-1} for n_s and 0.002 Mpc^{-1} for r. Shaded regions are the allowed parameter regions at 68% and 95% confidence coming from the latest combination of Planck, BICEP/Keck and BAO data. The values of the parameters are $\bar{\xi}' = 0$ and $\gamma = 10^6$, while $\xi = 0.1$ (green dashed-dotted line), $\xi = 1$ (red dashed line) and $\xi = 10$ (black dotted line). The parameter $\bar{\xi}$ varies from 10^{-3} to 10^3 in each curve in a clockwise direction indicated by the arrow. The small numbers at the edges of the curves indicate the number of $e-$folds $N_{0.05}$, for the extreme values of the parameter $\bar{\xi}$.

Derivative Couplings

In the general metric-affine framework there are three non-zero contractions of the Riemann tensor given by

$$\mathcal{R}_{\mu\nu} = \mathcal{R}^{\rho}{}_{\mu\rho\nu}, \quad \hat{\mathcal{R}}^{\mu}{}_{\nu} = g^{\alpha\beta}\mathcal{R}^{\mu}{}_{\alpha\beta\nu}, \quad \mathcal{R}'_{\mu\nu} = \mathcal{R}^{\alpha}{}_{\alpha\mu\nu} \tag{55}$$

called the Ricci, co-Ricci, and homothetic curvature tensor, respectively. There is a single Ricci scalar determined through an additional contraction of either the Ricci tensor or the co-Ricci tensor, expressed as $\mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu} = -\hat{\mathcal{R}}^{\mu}{}_{\mu}$. In what follows we shall consider the following metric-affine action of a scalar field ϕ coupled non-minimally to the Ricci scalar as well as the Ricci tensors through derivative couplings:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}(f(\phi) + \alpha_1 X)\mathcal{R} - \frac{1}{2}K(\phi)X + \alpha_2\mathcal{R}^{\mu\nu}X_{\mu\nu} + \alpha_3\hat{\mathcal{R}}^{\mu\nu}X_{\mu\nu} + \frac{1}{4}\beta\mathcal{R}^2 - V(\phi)\right\} \tag{56}$$

where $X_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi$ and $X = g^{\mu\nu}X_{\mu\nu}$. There is no $\mathcal{R}'_{\mu\nu}$ coupling due to the antisymmetry of its indices. In the metric case there is no a_3 coupling since $\hat{\mathcal{R}}_{\mu\nu} = -R_{\mu\nu}$.

The functions $F(\phi)$, $K(\phi)$, and $V(\phi)$ represent nonminimal couplings, non-canonical kinetic terms, and the potential term of the scalar field, respectively. General quadratic terms of the Riemann and Ricci tensors are known to be associated with unphysical degrees of freedom in contrast to quadratic terms of the Ricci scalar with its reliable inflationary predictions. Introducing the auxiliary scalar field χ, the action takes the form

$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\left(\mathcal{F}(\phi,\chi) + \alpha_1 X\right)\mathcal{R} - \frac{1}{2}K(\phi)X + \alpha_2\mathcal{R}^{\mu\nu}X_{\mu\nu} + \alpha_3\hat{\mathcal{R}}^{\mu\nu}X_{\mu\nu} - U(\phi,\chi)\right\} \tag{57}$$

with

$$\mathcal{F}(\phi,\chi) = f(\phi) + \beta\chi \quad \textit{and} \quad U(\phi,\chi) = V(\phi) + \frac{1}{4}\beta\chi^2 \,. \tag{58}$$

In order to discuss inflationary dynamics it is necessary to go to the Einstein frame.

Disformal Transformations

While actions constructed solely from the Ricci scalar can be transformed via a Weyl rescaling, since our action (57) involves derivative couplings of the scalar field to the Ricci tensors, we need to employ a broader set of transformations, namely the *disformal transformations*. These transformations are defined as

$$g_{\mu\nu} = \gamma_1(\phi, \tilde{X})\tilde{g}_{\mu\nu} + \gamma_2(\phi, \tilde{X})X_{\mu\nu} \,, \tag{59}$$

where $\tilde{X} = \tilde{g}^{\mu\nu}X_{\mu\nu}$. The inverse transformation is

$$\tilde{g}_{\mu\nu} = \tilde{\gamma}_1(\phi, X)g_{\mu\nu} + \tilde{\gamma}_2(\phi, X)X_{\mu\nu} \,, \tag{60}$$

with $\tilde{\gamma}_1 = 1/\gamma_1$ and $\tilde{\gamma}_2 = -\gamma_2/\gamma_1$, while the determinants are related by the equation $g = \tilde{g}\gamma_1^3(\phi, \tilde{X})(\gamma_1(\phi, \tilde{X}) + \gamma_2(\phi, \tilde{X})\tilde{X})$. Using the relations for the inverse metrics $g^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ we obtain also that

$$X = \frac{\tilde{X}}{\gamma_1(\phi, \tilde{X}) + \gamma_2(\phi, \tilde{X})\tilde{X}} \qquad \text{and} \qquad \tilde{X} = \frac{X}{\tilde{\gamma}_1(\phi, X) + \tilde{\gamma}_2(\phi, X)X} \,. \tag{61}$$

We may replace the co-Ricci tensor with the average Ricci tensor, defined as $\overline{\mathcal{R}}_{\mu\nu} = (\mathcal{R}_{\mu\nu} + \widehat{\mathcal{R}}_{\mu\nu})/2$, which vanishes in the metric case. Omitting the tildes for brevity, we obtain

$$\begin{aligned}
\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\,\Big[& F_1(\phi,X,\chi)\tfrac{\mathcal{R}}{2} + F_2(\phi,X)\overline{\mathcal{R}}^{\mu\nu}X_{\mu\nu} + F_3(\phi,X,\chi)\mathcal{R}^{\mu\nu}X_{\mu\nu} \\
& -F_4(\phi,X)X - F_5(\phi,X,\chi)U(\phi,\chi)\Big] \\
F_1(\phi,X,\chi) &= (1+\gamma X)^{1/2}\left(\gamma_1\mathcal{F}(\phi,\chi) + \tfrac{\alpha_1 X}{1+\gamma X}\right)\,, \\
F_2(\phi,X) &= \alpha_3(1+\gamma X)^{-1/2} \\
F_3(\phi,X,\chi) &= \tfrac{1}{2}(1+\gamma X)^{-1/2}\left(-\gamma_2\mathcal{F}(\phi,\chi) + \tfrac{\alpha_2-\alpha_3-(\alpha_1+\alpha_3)\gamma X}{(1+\gamma X)}\right) \\
F_4(\phi,X) &= \tfrac{1}{2}\gamma_1(1+\gamma X)^{-1/2}K(\phi), \qquad F_5(\phi,X,\chi) = \gamma_1^2(1+\gamma X)^{1/2}\,,
\end{aligned} \tag{62}$$

General metric-affine theories have non-zero torsion $T_{\mu\lambda\nu} = 2\mathcal{C}_{[\mu|\lambda|\nu]}$ and non-metricity $Q_{\rho\mu\nu} = \nabla_\rho g_{\mu\nu} = -2\mathcal{C}_{(\mu\nu)\rho}$. We intend to focus on the Einstein-Cartan gravity case, where $Q_{\rho\mu\nu} = 0$, considering $\alpha_3 \to 0$.

Focusing on the $\alpha_3 = 0$ case (Einstein-Cartan), we need to solve the system of equations $F_1(\phi, X, \chi) = 1$ and $F_3(\phi, X, \chi) = 0$. We approximate the solutions by assuming that in the slow-roll approximation, the higher-order kinetic terms are negligible (i.e. $X \ll 1$), particularly during inflation as well as during reheating. An approximate solution under this assumption is

$$\gamma \simeq \alpha_2 - \frac{\alpha_2^2}{2}X + \frac{5\alpha_2^3}{8}X^2\,, \quad \gamma_1 \simeq \frac{1}{\mathcal{F}(\phi,\chi)}\left(1 - (\alpha_1 + \alpha_2/2)X + (\alpha_1\alpha_2 + 5\alpha_2^2/8)X^2\right)\,, \tag{63}$$

where we kept terms up to $\mathcal{O}(X^2)$. Substituting the solution back to the action we obtain

$$\begin{aligned}\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\Big[\tfrac{1}{2}R(g) - \tfrac{K(\phi)X}{2\mathcal{F}(\phi,\chi)}(1 - (\alpha_1 + \alpha_2)X) \\ - \tfrac{U(\phi,\chi)}{\mathcal{F}^2(\phi,\chi)}\left(1 - (2\alpha_1 + \alpha_2/2)X + (\alpha_1^2 + 2\alpha_1\alpha_2 + 5\alpha_2^2/8)X^2\right)\Big]\end{aligned} \tag{64}$$

Varying the action with respect to the auxiliary field χ, we obtain a solution $\chi(\phi, X)$, which is re-expanded in powers of X and then substituted back into the action. The resulting effective action will represent the final metric action of the scalar field ϕ, featuring modified potential and kinetic terms.

The χ-variation gives

$$\frac{\delta \mathcal{S}}{\delta \chi} = 0 \quad \Rightarrow \quad \chi = \frac{4V(\phi) + A(\phi)X + B(\phi)X^2}{f(\phi) + C(\phi)X + D(\phi)X^2}\,, \tag{65}$$

with

$$A(\phi) = K(\phi)f(\phi) - 4V(\phi)(2\alpha_1 + \alpha_2/2)\,, \tag{66a}$$

$$B(\phi) = 4V(\phi)(\alpha_1^2 + 2\alpha_1\alpha_2 + 5\alpha_2^2/8) - (\alpha_1 + \alpha_2)K(\phi)f(\phi)\,, \tag{66b}$$

$$C(\phi) = -\beta K(\phi) - f(\phi)(2\alpha_1 + \alpha_2/2)\,, \tag{66c}$$

$$D(\phi) = \beta(\alpha_1 + \alpha_2)K(\phi) + (\alpha_1^2 + 2\alpha_1\alpha_2 + 5\alpha_2^2/8)f(\phi)\,. \tag{66d}$$

Expanding in powers of X we obtain[1]

$$\chi \simeq \frac{4V(\phi)}{f(\phi)} + XK(\phi)\left(1 + 4\beta\frac{V(\phi)}{f^2(\phi)}\right) + X^2\frac{K(\phi)}{2f(\phi)}\left(1 + 4\beta\frac{V(\phi)}{f^2(\phi)}\right)\left((2\alpha_1 - \alpha_2)f(\phi) + 2\beta K(\phi)\right)\,. \tag{68}$$

[1] In the minimal case $f(\phi) = K(\phi) = 1$ the auxiliary field reads

$$\chi \simeq 4V(\phi) + (1 + 4\beta V(\phi))X + (1 + 4\beta V(\phi))(\beta + \alpha_1 - \alpha_2/2)X^2 + \mathcal{O}(X^3)\,. \tag{67}$$

Substituting back into the action and re-expanding in powers of X, we obtain

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}R(g) - \frac{1}{2}\bar{K}(\phi)X - \bar{U}(\phi) + \mathcal{O}(X^2)\right) , \qquad (69)$$

with

$$\bar{K}(\phi) = \frac{-\tilde{\alpha}V(\phi) + f(\phi)K(\phi)}{(f^2(\phi) + 4\beta V(\phi))} \qquad \text{and} \qquad \bar{U}(\phi) = \frac{V(\phi)}{f^2(\phi) + 4\beta V(\phi)} , \qquad (70)$$

where we have defined $4\alpha_1 + \alpha_2 = \tilde{\alpha}$.
Note that for large values of the potential $V(\phi) >>$, the effective potential tends to a constant $1/4\beta$. This the well known *Palatini-*$\mathcal{R}^2$ *plateau*, inducive of inflationary behaviour.

INFLATION IN THE DERIVATIVE-COUPLED MODEL

We shall simplify the analysis by considering the minimal case where $f(\phi) = K(\phi) = 1$. Then,

$$\bar{K} = \frac{1 - \tilde{\alpha} V}{1 + 4\beta V}, \quad \bar{U} = \frac{V}{1 + 4\beta V} . \tag{71}$$

A canonically normalized inflaton ϕ_c can be expressed as a function of ϕ through $d\phi_c = d\phi\sqrt{\bar{K}(\phi)}$.

We consider the simple case of the minimally coupled ($f(\phi) = 1$) **quadratic model** with a potential $V(\phi) = m^2\phi^2/2$, which can be treated analytically. For small $O(X^2)$ terms the first slow-roll parameters are

$$\epsilon_{\bar{U}} = \frac{4}{\phi^2(2 - \tilde{\alpha}m^2\phi^2)(1 + 2\beta m^2\phi^2)} , \quad \eta_{\bar{U}} = \frac{8\left(1 + \beta m^2\phi^2(3\tilde{\alpha}m^2\phi^2 - 4)\right)}{\phi^2(2 - \tilde{\alpha}m^2\phi^2)^2(1 + 2\beta m^2\phi^2)} \tag{72}$$

The number of $e-$folds, left to the end of on inflation are

$$N_\star = \frac{1}{16}(\phi_{end}^2 - \phi_\star^2)\left(\tilde{\alpha} m^2(\phi_{end}^2 + \phi_\star^2) - 4\right) \simeq \frac{1}{16}\phi_\star^2(4 - \tilde{\alpha} m^2 \phi_\star^2)\,, \tag{73}$$

where the second equality holds for $\phi_{end}^2 \ll \phi_\star^2$. The above equation has a solution

$$\phi_\star^2 = \frac{2 - \sqrt{\tilde{\alpha}^2 m^4 \phi_{end}^4 - 4\tilde{\alpha} m^2 \phi_{end}^2 + 4 - 16\tilde{\alpha} m^2 N_\star}}{\tilde{\alpha} m^2} \xrightarrow{\tilde{\alpha}\to 0} \phi_{\rm end}^2 + 4N_\star\,. \tag{74}$$

The field value at the end of inflation is defined by $\epsilon_{\bar{U}}(\phi_{\rm end}) = 1 \Rightarrow$

$$2\tilde{\alpha}\beta m^4 \phi_{\rm end}^6 + (\tilde{\alpha} m^2 - 4\beta m^2)\phi_{\rm end}^4 - 2\phi_{\rm end}^2 + 4 = 0\,. \tag{75}$$

We have seen numerically that the approximation $\phi_{\rm end}^2 \ll \phi_\star^2$ holds true and we safely omit the term ϕ_{end}. Under this approximation the field value at the horizon crossing is given by

$$\phi_\star^2 \simeq \frac{2 - 2\sqrt{1 - 4\tilde{\alpha} m^2 N_\star}}{\tilde{\alpha} m^2}\,. \tag{76}$$

Using $\phi_\star$, given above, A_s is written in terms of $N_\star$ as

$$A_s \simeq \frac{\sqrt{1-4\tilde{\alpha}m^2N_\star}\left(1-\sqrt{1-4\tilde{\alpha}m^2N_\star}\right)^2}{24\pi^2\tilde{\alpha}^2m^2}\,, \qquad (77)$$

which under this approximation does not depend on the parameter β. The above equation can be used to see the impact of $\tilde{\alpha}$ on m^2. For $N_\star \sim 50-60$ this becomes drastic for $|\tilde{\alpha}| \gtrsim 10^8$. Given that the parameter m represents the mass of the scalar field ϕ it is essential for it to remain sub-Planckian. Thus, we may derive an upper limit for the parameter $|\tilde{\alpha}|$ given by

$$|\tilde{\alpha}| \lesssim 4.3\times 10^{19}\left(\frac{N_\star}{55}\right)^3\,. \qquad (78)$$

The spectral index is

$$n_s \simeq 1-\frac{1+\sqrt{1-4\tilde{\alpha}m^2N_\star}-6\tilde{\alpha}m^2N_\star}{N_\star(1-4\tilde{\alpha}m^2N_\star)} \simeq \begin{cases} 1-\dfrac{2}{N_\star}\,, & \text{if } |\mathring{A}|/N_\star \ll 1 \\ 1-\dfrac{3}{2N_\star}\,, & \text{if } |\mathring{A}|/N_\star \gg 1\,. \end{cases} \qquad (79)$$

being β-independent to leading order. Therefore, for small $|\mathring{A}|$, this prediction aligns with that of the simple quadratic model of inflation.

Finally, the tensor-to-scalar ratio is given by

$$r \simeq \frac{16\tilde{\alpha}m^2}{\sqrt{1-4\tilde{\alpha}m^2N_\star}\left(\sqrt{1-4\tilde{\alpha}m^2N_\star}-1\right)\left[4\beta\left(\sqrt{1-4\tilde{\alpha}m^2N_\star}-1\right)-\tilde{\alpha}\right]}\,, \tag{80}$$

while its limiting cases are

$$r \simeq \begin{cases} \dfrac{8}{N_\star + 48\pi^2 A_s\beta}\,, & \text{if } |\mathring{A}|/N_\star \ll 1 \\ \dfrac{4}{N_\star + 24\pi^2 A_s\beta}\,, & \text{if } |\mathring{A}|/N_\star \gg 1\,. \end{cases} \tag{81}$$

Here the introduction of a substantial β parameter ($\beta \gtrsim 10^8$) becomes necessary in order to bring r within the observational limit ($\beta > 10^8$).

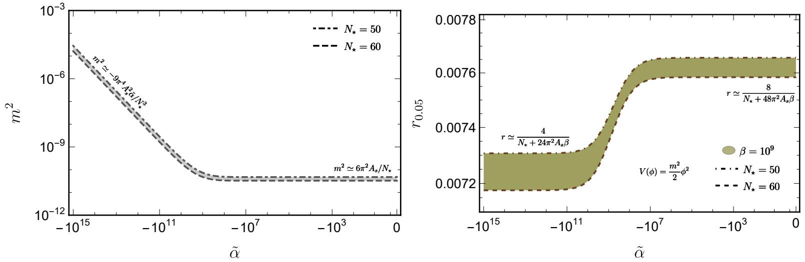


Figure: **Left:** The mass parameter m^2 as function of the parameter $\tilde{\alpha}$. **Right:**The tensor-to-scalar ratio as function of the parameter $\tilde{\alpha}$ for $\beta = 10^9$.

(I.Gialamas, T.Katsoulas and KT, JCAP 2024)

CONCLUSIONS

Metric-Affine Ricci and Holst-squared models coupled to scalars yield an extra dynamical pseudoscalar and exhibit one-field inflation for a large class of potentials.

Metric-Affine Ricci-squared models with derivative couplings of scalars to the Ricci tensors exhibit one-field inflation for a large class of potentials.