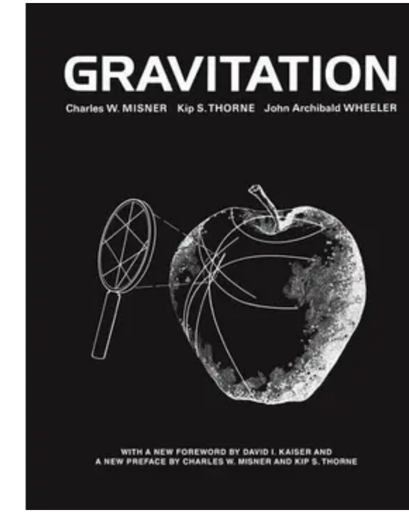
EPFL

Weyl-invariant Einstein-Cartan gravity: unifying the strong CP and hierarchy puzzles

Mikhail Shaposhnikov
Based on work with
Georgios Karananas and Sebastian Zell
arXiv: 2406.11956

Outline

- Gravity from particle physics: Einstein-Cartan
- Weyl symmetry/conformal symmetry
- EC Weyl invariant theory
- Solution of the strong CP problem
- Extra benefits of EC Weyl invariant theory: Higgs mass and HNL masses
- Euclidean continuation
- Conclusions



What is gravity?

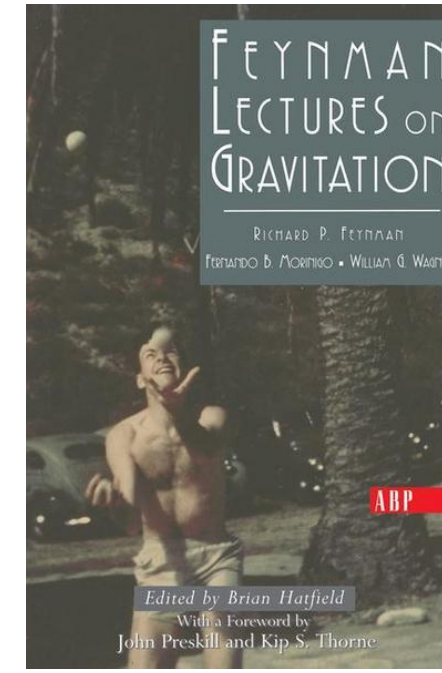
Einstein ‘1915: gravity is not an ordinary force, but rather a property of spacetime geometry: geometrodynamics.

Historical approach: start with the principle of equivalence, and develop the geometrical viewpoint.

Arguments in favour: solid experimental evidence at astronomical and human size distances, mathematical beauty,…

- Unified theory: geometrizing Electrodynamics?
- Unified theory: geometrizing other forces and all elementary particles?

What is gravity?



Feynman '1962: nothing special, just like electrodynamics, but associated with "spin" 2 massless particle - graviton instead of "spin" 1 massless photon. Geometrodynamics is an effective low energy theory.

Origin: the success of quantum mechanics and quantum field theory (QFT) in describing interactions of elementary particles.

Arguments in favour: Unitary Lorentz-invariant QFT:

Photons and Gravitons in *S*-Matrix Theory: Derivation of Charge Conservation and Equality of Gravitational and Inertial Mass*

STEVEN WEINBERG†
Physics Department, University of California, Berkeley, California
(Received 13 April 1964)

We give a purely *S*-matrix-theoretic proof of the conservation of charge (defined by the strength of soft photon interactions) and the equality of gravitational and inertial mass. Our only assumptions are the Lorentz invariance and pole structure of the *S* matrix, and the zero mass and spins 1 and 2 of the photon and graviton. We also prove that Lorentz invariance alone requires the *S* matrix for emission of a massless particle of arbitrary integer spin to satisfy a "mass-shell gauge invariance" condition, and we explain why there are no macroscopic fields corresponding to particles of spin 3 or higher.

Gravity as a gauge theory

Existence of electromagnetic field - U(1) global invariance of fermion Lagrangian promoted to be local

Gluons, W^+, W^- Z and γ of the Standard Model - SU(3)xSU(2)xU(1) global invariance of SM fermion Lagrangian promoted to be local

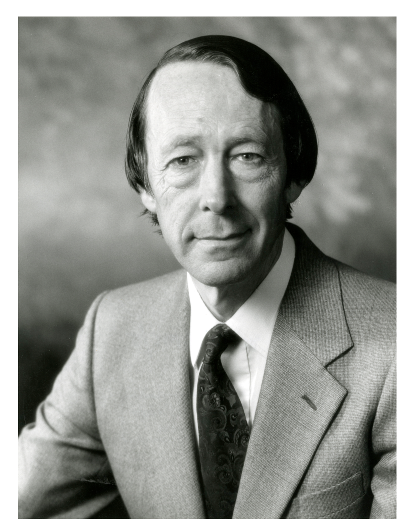
Existence of gravitational field - Poincare invariance of SM fermion Lagrangian promoted to be local?

Gauging of the Poincare group: Utiyama (1956), Kibble (1961) , Sciama (1962,1964).

VOLUME 101, NUMBER 5

Invariant Theoretical Interpretation of Interaction

RYOYU UTIYAMA*
Institute for Advanced Study, Princeton, New Jersey
(Received July 7, 1955)



Gravity as a gauge theory

Basic gauge fields

- e_μ^A - tetrad one-form (frame field, translations), A=0,1,2,3. 4x4=16 fields
- ω_μ^{AB} - spin connection one form (gauge field of the local Lorentz group, 4x3/2x4=24 fields).

Field strength, curvature:

$$F_{\mu\nu}^{AB} = \partial_\mu \omega_\nu^{AB} - \partial_\nu \omega_\mu^{AB} + \omega_{\mu C}^{A} \omega_\nu^{CB} - \omega_{\nu C}^{A} \omega_\mu^{CB}$$

Field strength, torsion:

$$T_{\mu\nu}^{A} = \partial_\mu e_\nu^{A} - \partial_\nu e_\mu^{A} + \omega_{\mu B}^{A} e_\nu^{B} - \omega_{\nu B}^{A} e_\mu^{B}$$

Gravity as a gauge theory

Lowest order invariants:

$$F \equiv \frac{1}{8\sqrt{g}} \epsilon_{ABCD} \epsilon^{\mu\nu\rho\sigma} F^{AB}_{\mu\nu} e^C_\rho e^D_\sigma$$

$$\tilde{F} \equiv \frac{1}{\sqrt{g}} \epsilon^{\mu\nu\rho\sigma} e_{\rho C} e_{\sigma D} F^{CD}_{\mu\nu}$$

Irreducible components of the torsion: $T_{\mu\nu\rho} = e_{\mu A} T^A_{\nu\rho}\,,$

$$v_\mu = T^\nu{}_{\mu\nu}\ , \quad a_\mu = \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma}\ , \quad \tau_{\mu\nu\rho} = \frac{2}{3}\left(T_{\mu\nu\rho} - v_{[\nu} g_{\rho]\mu} - T_{[\nu\rho]\mu} \right)$$

Gravity as a gauge theory

Pure gauge action of lowest dimensionality, M_P is the Planck mass:

$$M_P^2 \left[aF + b\tilde{F} + cv_\mu v^\mu + da_\mu a^\mu + ev_\mu v^\mu + f\tau^{\mu\nu\rho}\tau_{\mu\nu\rho} \right]$$

Equivalent to Einstein metric theory, only 2 out of 40 degrees of freedom are physical and propagating. They correspond to the graviton.

Adding Weyl symmetry

Weyl symmetry changes locally the scale of rulers, theory does not have any intrinsic mass parameters

Why having Weyl symmetry is interesting?

- Flat space limit - conformal symmetry
    - Symmetry of free Maxwell equations
    - Symmetry of the Standard Model without Higgs mass
    - Conformal field theories (CFT) are special

Applications of CFTs

- Conformal symmetry restricts the form of different correlation functions.

- CFTs are an indispensable tool to address the behaviour of many systems in the vicinity of the critical points associated with phase transitions.

- CFTs describe the limiting behaviour of different quantum field theories deeply in the ultraviolet (UV) and/or infrared (IR) domains of energy.

Applications of CFTs

CFTs may be relevant for the solution of the most puzzling fine-tuning issues of fundamental particle physics

- CFTs are theories without infinities.

- The Lagrangian of the Standard Model is invariant under the full conformal group (at the classical level) if the mass of the Higgs boson is put to zero. Perhaps, the observed smallness of the Fermi scale in comparison with the Planck scale is a consequence of this,
125 GeV $\simeq M_H \ll M_P \sim 10^{19}$ GeV?

- The energy of the ground state in CFTs is equal to zero. Perhaps, this is relevant for the explanation of the amazing smallness of the cosmological constant?

Weyl symmetry

What is Weyl symmetry in EC gravity? Local change of the rulers,

$$e_\mu^A \to \Omega(x) e_\mu^A, \quad \omega_\mu^{AB} \to \omega_\mu^{AB},$$

$$a_\mu \to a_\mu, \;\; v_\mu \to v_\mu + 3\Omega\partial_\mu\Omega^{-1}, \;\; \tau_{\mu\nu\rho} \to \Omega^2 \tau_{\mu\nu\rho}$$

Torsion vector is an Abelian gauge field of the Weyl symmetry. It can be used for construction of Weyl-invariant actions.

Weyl-invariant EC gravity

The most general Weyl-invariant action with at most two derivatives contains 10 different structures constructed from $F_{\mu\nu}^{AB}$ and e_μ^A (dimensionful couplings are not admitted).

In general, it contains many propagating degrees of freedom and is pathological because of ghosts.

The only structure which is ghost free for any parameter choice is

$$aF^2 + b\tilde{F}^2 + cF\tilde{F}$$

The spectrum of the theory contains graviton and a scalar degree of freedom.

Weyl-invariant EC gravity

Demonstration for c=0, f and $\tilde{f}$ are Lorentz group gauge couplings

$$S_{\rm gr} = \int d^4x \,\det(e) \left[\frac{1}{f^2} F^2 + \frac{1}{\tilde{f}^2} \tilde{F}^2 \right], \quad \text{metric } (-,+,+,+).$$

Field strengths in terms of torsion and torsion-free curvature and covariant derivatives:

$$F = \frac{R}{2} + v^{\mu}_{;\mu} - \frac{1}{3} v_\mu v^\mu + \frac{1}{48} a_\mu a^\mu + \frac{1}{4} \tau_{\mu\nu\rho} \tau^{\mu\nu\rho}$$

$$\tilde{F} = -\, a^{\mu}_{;\mu} + \frac{2}{3} a_\mu v^\mu - \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \tau_{\lambda\mu\nu} \tau^{\lambda}_{\rho\sigma}$$

Weyl-invariant EC gravity

Introduce auxiliary fields χ (dilaton) and ϕ (ALP- axion like particle),

$$S_{\text{gr}} = \int d^4x\sqrt{g}\left[\chi^2 F + M_P^2\phi\tilde{F} - \frac{f^2\chi^4}{4} - \frac{\tilde{f}^2 M_P^4\phi^2}{4}\right]$$

Use the Weyl invariance and replace $\chi = \dfrac{M_P}{\sqrt{2}}$,

$$S_{\text{gr}} = M_P^2\int d^4x\sqrt{g}\left[\frac{F}{2} + \phi\tilde{F} - \frac{\tilde{f}^2 M_P^2\phi^2}{4} - \frac{f^2 M_P^2}{16}\right]$$

Weyl-invariant EC gravity

Write equations of motion for torsion and solve them (torsion is non-dynamical). Result: gravity in metric formulation, containing propagating graviton and a scalar particle.

Features:

- The gauge coupling f determines the vacuum energy, $\epsilon_{vac} \propto f^2 M_P^4$

- The gauge coupling $\tilde{f}$ determines the scalar mass, $m_a^2 \propto \tilde{f}^2 M_P^2$

Very small gauge couplings are needed for theory to be viable.

Adding the Standard Model

- For fermions,

$$S_{\rm f} = \int d^4x\sqrt{g}\left[\frac{i}{2}\overline{\Psi}\gamma^\mu D_\mu\Psi + \text{h.c.} + \left(\zeta_V^a V_\mu + z_A^a A_\mu\right) a^\mu\right]$$

vector and axial currents

$$V_\mu = \bar{\Psi}\gamma_\mu\Psi\ , \quad A_\mu = \bar{\Psi}\gamma_5\gamma_\mu\Psi$$

covariant derivative

$$D_\mu = \mathscr{D}_\mu + \frac{1}{8}\omega_\mu{}^{AB}\left(\gamma_A\gamma_B - \gamma_B\gamma_A\right),$$

$\mathscr{D}_\mu$ contains SM gauge fields

Adding the Standard Model

- For the Higgs field, use the Weyl covariant derivative with the torsion vector $D^W_\mu h = \partial_\mu h + \frac{v_\mu}{3} h,$ and add all sorts of non-minimal couplings

$$S_{\text{Higgs}} = \frac{1}{2}\int d^4x\sqrt{g}\Bigg[\xi_h h^2 F + \zeta_h h^2 \tilde{F} + c_{aa} h^2 a_\mu a^\mu + c_{\tau\tau} h^2 \tau^2_{\mu\nu\rho}$$

$$+\tilde{c}_{\tau\tau} h^2 \epsilon^{\mu\nu\rho\sigma}\tau_{\lambda\mu\nu}\tau^{\lambda}{}_{\rho\sigma} - \left(D^W_\mu h\right)^2 - \frac{\lambda h^4}{2}\Bigg]$$

To get a theory in metric formulation, integrate out torsion.

Strong CP-problem

Low energy effective action for ALP, related to ϕ as $a = 2\sqrt{6}M_P\phi$

$$S \approx -\frac{1}{2}\int d^4x\sqrt{g}\left[(\partial_\mu a)^2 + \frac{\tilde{f}^2 M_P^2}{48}a^2 - 4\sqrt{6}\zeta_A^a \frac{a}{M_P} A^\mu_{;\mu}\right]$$

$A^\mu_{;\mu}$ is the divergence of the axial current, containing SU(3) topological charge $\propto G\tilde{G}$. Gravitationally induced ALP mass:

$$m_{a,\mathrm{grav}} = \frac{\tilde{f}M_P}{4\sqrt{3}}$$

If it is smaller than the QCD induced mass, the strong CP problem is solved. Requirement:

$$\frac{\tilde{f}}{\zeta_A^a} \lesssim 10^{-5}\frac{m_\pi f_\pi}{M_P^2}\frac{\sqrt{m_u m_d}}{m_u + m_d} \sim \mathcal{O}\left(10^{-43}\right)$$

To compare with the known axion solutions

QCD without axion:

- One “unnatural” number, $\theta \lesssim 10^{-10}$

QCD with axion:

- 6 new degrees of freedom (KSVZ - one complex scalar field and a new massive quark, DFSZ - two complex scalar fields, one is the doublet with respect to the SU(2) weak isospin and another is a singlet).

- Two “unnatural” numbers:

- Ratio of EW scale and PQ scale: $\left(v_{EW}/F_{PQ}\right)^2 \lesssim 10^{-14}$

- Quality of PQ symmetry: $\left(m_{PQ\ breaking}/F_{PQ}\right)^2 \lesssim 10^{-50}$

- What is the need for the PQ symmetry, besides the solution of the strong CP problem?

Extra benefit: Higgs boson mass

The tree value of the Higgs mass is very small,

$$m^2_{\mathrm{h,grav}} = -\frac{f^2 \xi_h M_P^2}{4}$$

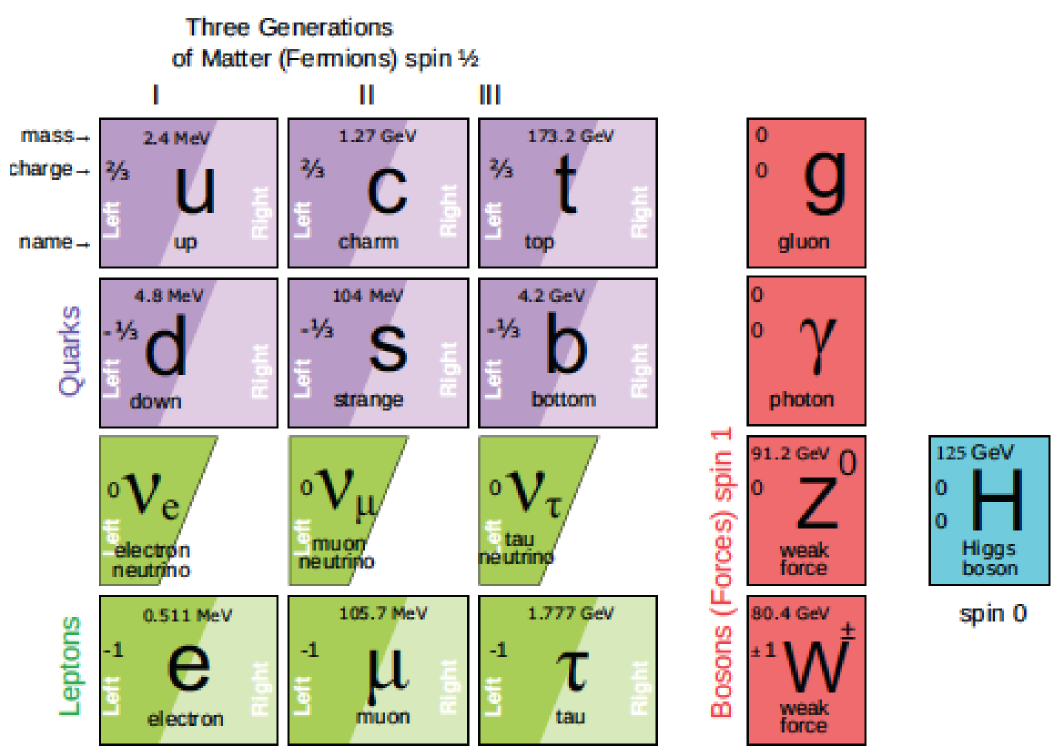
since it is proportional to the Lorentz gauge coupling related to the cosmological constant. So, the Higgs mass is computable in terms of other parameters. Non-perturbative scalar-gravitational instantons can give

$$m_h \propto e^{-S} M_P \,,$$

where S is the instanton action, it may be $S \gg 1$ (Sherin, MS'2018, Shkerin, MS, Zell'2021)

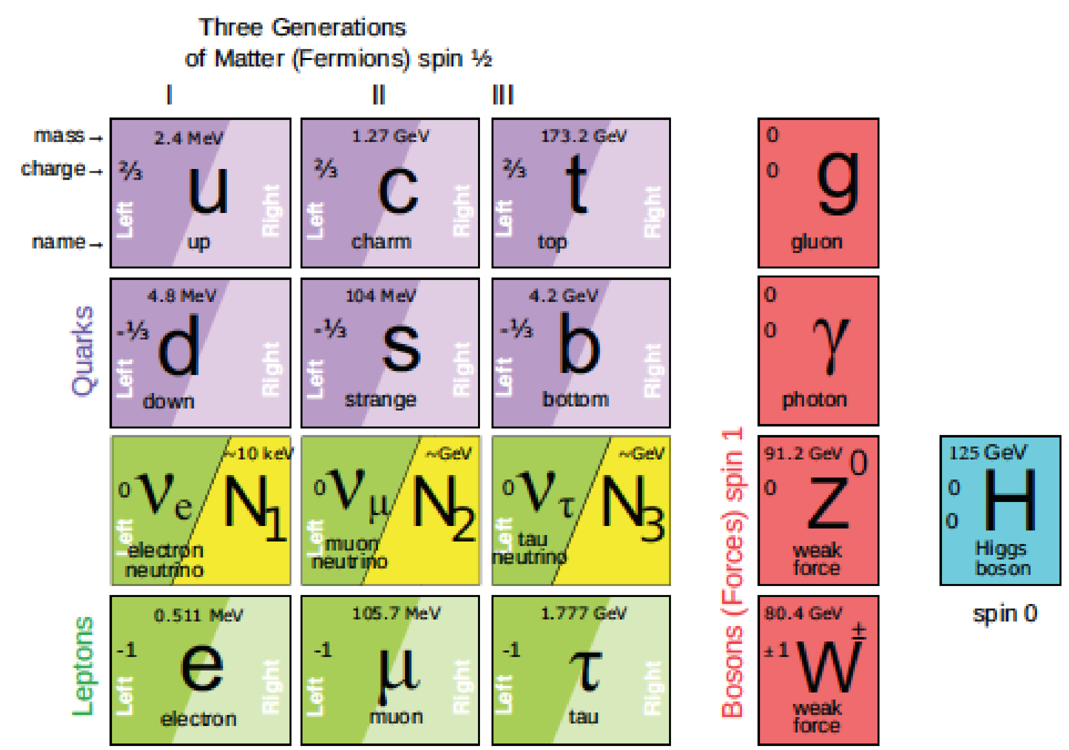
Extra benefit: HNL masses

νMSM- Neutrino Minimal Standard Model: 3 extra neutrinos $N_{1,2,3}$ can solve simultaneously three outstanding problems of the Standard Model. They can give masses to ordinary neutrinos, one of them can be a dark matter particle. They can also explain why the Universe contains more matter than antimatter.



Standard Model

⇒



νMSM

Extra benefit: HNL masses

In Weyl-invariant EC gravity the tree values of HNL masses are zero. They cannot be generated in any order of perturbation theory because of global B-L symmetry. But this means that they are computable.

Gravity leads to breaking of all global symmetries. Perhaps, HNL masses can be generated non-perturbatively. It is not clear what their values could be and how to make a computation.

Continuation to Euclidean space-time

EC gravity allows for transparent continuation into Euclidean space-time, what may help to define path integral

$$e^0_\mu \to -ie^0_\mu\,, \quad e^j_\mu \to e^j_\mu\,, \quad \omega^{0j}_\mu \to -i\omega^{0j}_\mu\,, \quad \omega^{jk}_\mu \to \omega^{jk}_\mu\,, \quad j,k = 1{,}2{,}3$$

Euclidean curvature and torsion:

$$F^{0j}_{\mu\nu} \to -iF^{0j}_{\mu\nu}\,, \quad F^{jk}_{\mu\nu} \to F^{jk}_{\mu\nu}\,, \quad T^0_{\mu\nu} \to -iT^0_{\mu\nu}\,, \quad T^j_{\mu\nu} \to T^j_{\mu\nu}$$

Euclidean action of Weyl-invariant EC gravity:

$$S^{\mathrm{E}}_{\mathrm{gr}} = -\int d^4x\,\det(e)\left[\frac{1}{f^2}F^2 + \frac{1}{\tilde{f}^2}\tilde{F}^2\right].$$

It is bounded from below for negative f^2 and $\tilde{f}^2$, the choice which does not seem to spoil other consideration presented above. Well defined Euclidean path integral? No problem with the conformal part of the metric in the standard metric gravity?

Conclusions

Weyl-invariant Einstein-Cartan gravity is an interesting theory with the following properties:

- It has one extra scalar degree of freedom - the ALP - in comparison with the SM (or νMSM).

- The cosmological constant, the tree value of the Higgs boson mass and the gravitationally induced ALP mass are all proportional to the gauge couplings of the Lorentz group, which are required to be very small (gravity is the weakest gauge force in Nature). Non-perturbative generation of the Higgs mass?

- The HNL masses are vanishing at the classical level. Non-perturbative generation of HNL masses?

- Euclidean action is bounded from below for specific choice of gauge coupling constants. No problem with conformal part of the metric?

Conclusions

Open problems, currently under investigation:

- Cosmological applications:
  - Higgs-axion inflation, isocurvature perturbations?
  - Axion behaviour in the early Universe, axion as dark matter?
- Quantum Weyl anomaly in EC gravity
- Theory is not renormalisable by standard methods