Strong dynamics in Cosmology:

Cosmological Constant problem,
Dark Energy, Dark Matter, strong CP problem,
neutrino mass, and… gravitational radiowaves!

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Strongly coupled Yang Mills dynamics: an outline

- Important physical examples of gauge fields are realised in Nature (QCD and electroweak interactions)
- Non-perturbative QCD phenomena are far from being understood (e.g. quark confinement, mass gap, QCD phase transitions, hot/dense QCD phenomena etc)
- Non-abelian gauge (Yang-Mills) fields are present in most of UV completions of the Standard Model (e.g. GUTs, string/EDs compactifications etc)
- Confining dark Yang-Mills sectors are often considered as a possible source of Dark Matter in the Universe (e.g. dark glueballs)
- Relic abundance of dark glueballs, ubiquitous in string theory, overcloses the Universe for confining sectors with critical temperature above the eV-scale (a big problem for phenomenology!)
- We focus on the case of pure gluons

  $\Rightarrow$ confinement-deconfinement phase transition

Vacuum in Quantum Physics vs in Cosmology

Vacuum energy

"…the worst theoretical prediction in the history of physics" (Hobson 2006)

in Quantum Physics		in Cosmology
$\epsilon_{vac} \sim 10^{-2}\,\mathrm{GeV}^4$	**Topological QCD vacuum** **unique strongly-coupled subsystem!**	$\Lambda_{\rm cosm} \sim 10^{-47}\,\mathrm{GeV}^4$
$\sim 10^8\,\mathrm{GeV}^4$	**Higgs condensate**	"Old" CC problem: Why such small and positive? "New" CC problem: Why non-zeroth and exists at all?

Vacuum in Quantum Physics has incredibly wrong energy scale!

Quantum-topological (chromomagnetic) vacuum in QCD

$$\varepsilon_{vac(top)} = -\frac{9}{32}\langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x) : |0\rangle + \frac{1}{4}\left(\langle 0| : m_u \bar{u}u : |0\rangle + \langle 0| : m_d \bar{d}d : |0\rangle + \langle 0| : m_s \bar{s}s : |0\rangle\right)$$
$$\simeq -(5 \pm 1) \times 10^9\ \mathrm{MeV}^4.$$

Two possible approaches to this problem:

- **Let's forget about the "bare" vacuum (DE: "phantom", "quintessence", "ghost"… etc)**
  Zero vacuum density in the Minkowski limit, by (Casimir-like) definition, then (Zhitnitsky et al)

$$\Lambda_{\rm cosm} \equiv \epsilon_{\rm FLRW} - \epsilon_{\rm Mink}$$

simply imposing a cancellation of the "bare" vacuum by hands!!

- **Let's look closer at the vacuum state — why/how does it become "invisible" to gravity?**

An illustration: topological vs collective contributions

non-perturbative QCD vacuum

Quantum-topological (instanton) fluctuations

instantons/dyons carrying chromomagnetic and chromoelectric charges

$$\varepsilon_{vac(top)} < 0$$

Quantum-wave (hadronic) fluctuations

have quantum numbers of light hadrons

$$m_h \leq l^{-1}_{g(min)}$$

$$\varepsilon_{vac(h)} > 0$$

exist at the same space-time scales

$$l_{g(min)} < l_g < l_{g(max)},$$

$$l_{g(min)} \simeq (1500\ \text{MeV})^{-1}, \qquad l_{g(max)} \simeq (500\ \text{MeV})^{-1}$$

Can they mutually cancel each other? In principle, YES!

Taking into account ONLY metastable hadrons

$$B = \{N,\ \Lambda,\ \Sigma,\ \Xi\}$$

$$M = \{\pi,\ K,\ \eta,\ \eta'\}$$

$$\varepsilon_{vac(h)} = \frac{1}{32\pi^2}\left(2\sum_B (2J_B+1)m_B^4 \ln\frac{\mu}{m_B} - \sum_M (2J_M+1)m_M^4 \ln\frac{\mu}{m_M}\right)$$

$$\mu \simeq l^{-1}_{g(min)}$$

$$\varepsilon_{vac(top)} + \varepsilon_{vac(h)} = 0 \ \text{ for } \ \mu = 1.22\ \text{GeV}$$

!!!

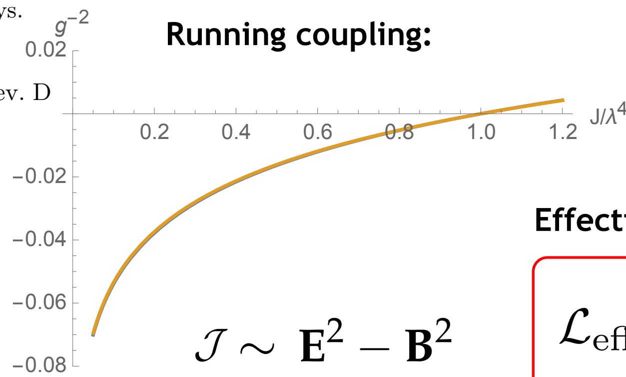
Effective YM action and gluon vacuum

At least, for SU(2) gauge symmetry,
the all-loop and one-loop effective Lagrangians
are practically indistinguishable (by FRG approach)

P. Dona, A. Marciano, Y. Zhang and C. Antolini, Phys. Rev. D **93** (2016) no.4, 043012.

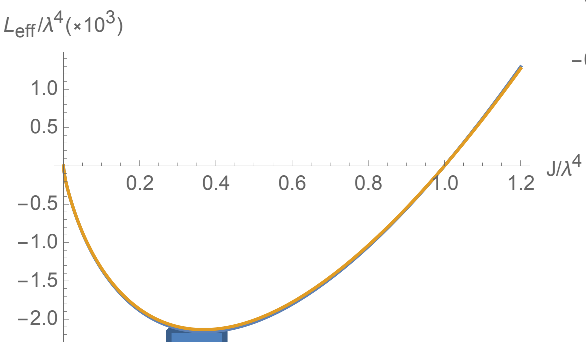
A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D **83** (2011) 045014 [Phys. Rev. D **83** (2011) 069903].

Running coupling:



$$\mathcal{J} \sim \mathbf{E}^2 - \mathbf{B}^2$$

Effective Lagrangian:



gluon condensate - which CM or CE, or both?

Discovery of CM condensate:

G. K. Savvidy, Phys. Lett. **71B**, 133 (1977)

G. Savvidy, Eur. Phys. J. C **80** (2020) 165

NOTE however, that the RG equation itself

$$\frac{d\ln|\bar{g}^2|}{d\ln|\mathcal{J}|/\mu_0^4} = \frac{\beta(\bar{g}^2)}{2}$$

appears to be invariant under

$$\mathcal{J} \longleftrightarrow -\mathcal{J}$$
$$\bar{g}^2 = \bar{g}^2(|\mathcal{J}|)$$

Is this RGE symmetry an accident, or smth deeper?

H. Pagels and E. Tomboulis, Nucl. Phys. B **143**, 485 (1978).

Classical YM Lagrangian:

$$\mathcal{L}_{\rm cl} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_{\rm YM}\, f^{abc} A^b_\mu A^c_\nu$$

$$\mathcal{A}^a_\mu \equiv g_{\rm YM} A^a_\mu$$

Effective YM Lagrangian:

$$\mathcal{F}^a_{\mu\nu} \equiv g_{\rm YM} F^a_{\mu\nu}$$

$$\mathcal{L}_{\rm eff} = \frac{\mathcal{J}}{4\bar{g}^2(\mathcal{J})}, \qquad \mathcal{J} = -\mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu}_a$$

The energy-momentum tensor:

$$T^\nu_\mu = \frac{1}{\bar{g}^2}\left[\frac{\beta(\bar{g}^2)}{2} - 1\right]\left(\mathcal{F}^a_{\mu\lambda}\mathcal{F}^{\nu\lambda}_a + \frac{1}{4}\delta^\nu_\mu \mathcal{J}\right) - \delta^\nu_\mu \frac{\beta(\bar{g}^2)}{8\bar{g}^2}\mathcal{J}$$

Equations of motion:

$$\vec{\mathcal{D}}^{ab}_\nu\left[\frac{\mathcal{F}^{\mu\nu}_b}{\bar{g}^2}\left(1 - \frac{\beta(\bar{g}^2)}{2}\right)\right] = 0,$$
$$\vec{\mathcal{D}}^{ab}_\nu \equiv \left(\delta^{ab}\vec{\partial}_\nu - f^{abc}\mathcal{A}^c_\nu\right),$$

trace anomaly:

$$T^\mu_\mu = -\frac{\beta(\bar{g}^2)}{2\bar{g}^2}\mathcal{J}$$

Heterogenous quantum ground state: two-scale vacuum

The running coupling at one-loop

$$\mathcal{L}_{\rm eff}^{(1)} = \frac{bN}{384\pi^2}\mathcal{J}\ln\left(\frac{|\mathcal{J}|}{\lambda_\pm^4}\right) \qquad b = 11$$

$$\bar{g}_1^2(\mathcal{J}) = \frac{\bar{g}_1^2(\mu_0^4)}{1+\frac{bN}{96\pi^2}\bar{g}_1^2(\mu_0^4)\ln(|\mathcal{J}|/\mu_0^4)} = \frac{96\pi^2}{bN\ln(|\mathcal{J}|/\lambda_\pm^4)}$$

$$\beta_{(1)} = -\frac{bN}{48\pi^2}\,\bar{g}^2_{(1)}$$

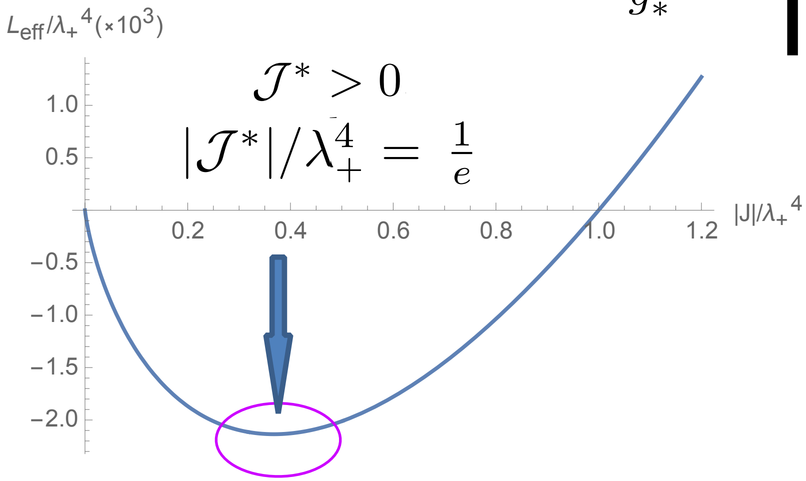
with two energy scales

$$\lambda_\pm^4 \equiv |\mathcal{J}^*|\exp\left[\mp\frac{96\pi^2}{bN|\bar{g}_1^2(\mathcal{J}^*)|}\right] \qquad |\mathcal{J}^*| = \lambda_+^2\lambda_-^2$$

CE vacuum: $\beta(\bar{g}_*^2) = 2$

e.o.m. is automatically satisfied!

Trace anomaly: $T^\mu_{\mu,\rm CE} = -\frac{1}{\bar{g}_*^2}\mathcal{J}^*$



Cosmological CE attractor

Mirror symmetry of the ground state

One-loop:

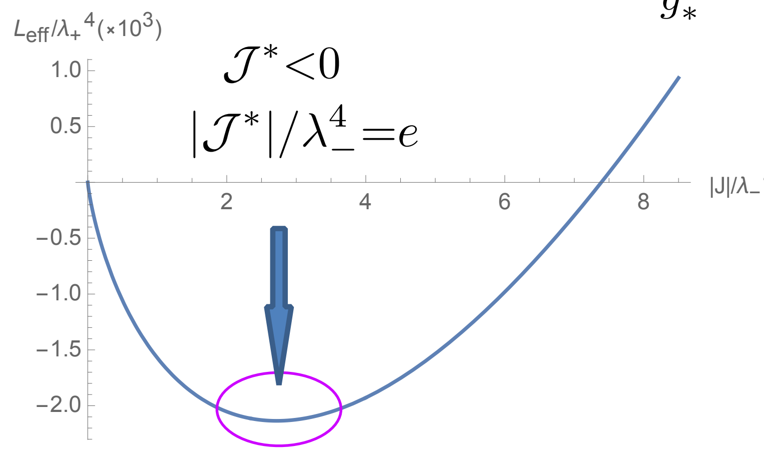
$$\lambda_+^2/\lambda_-^2 = e$$

CM vacuum: $\beta(\bar{g}_*^2) = -2$

Reduces to the standard YM e.o.m. discussed in e.g. in instanton theory

$$\overrightarrow{\mathcal{D}}_\nu^{ab}\left[\frac{\mathcal{F}_b^{\mu\nu}}{\bar{g}^2}\right] = 0, \qquad \bar{g}^2 \simeq \bar{g}_*^2$$

Trace anomaly: $T^\mu_{\mu,\rm CM} = +\frac{1}{\bar{g}_*^2}\mathcal{J}^*$



Cosmological CM attractor

Real-time evolution of the gluon condensate

$$\mathcal{J} = \frac{2}{\sqrt{-g}} \sum_a (\mathbf{E}_a \cdot \mathbf{E}_a - \mathbf{B}_a \cdot \mathbf{B}_a) \equiv \frac{2}{\sqrt{-g}} (\mathbf{E}^2 - \mathbf{B}^2)$$

FLRW metric in conformal time:

$$g \equiv \det(g_{\mu\nu}), \; g_{\mu\nu} = a(\eta)^2 \mathrm{diag}(1, -1, -1, -1)$$

$$\sqrt{-g} = a^4(\eta)\,, \qquad t = \int a(\eta) d\eta$$

- **Basic qualitative features on the non-perturbative YM action are noticed already at one loop**

Einstein-YM equations of motion for the effective YM theory:

$$\frac{1}{\varkappa}\left(R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R\right) = \bar{\epsilon}\delta_\mu^\nu + \frac{b}{32\pi^2}\frac{1}{\sqrt{-g}}\left[\left(-\mathcal{F}^a_{\mu\lambda}\mathcal{F}_a^{\nu\lambda} + \frac{1}{4}\delta_\mu^\nu \mathcal{F}^a_{\sigma\lambda}\mathcal{F}_a^{\sigma\lambda}\right)\ln\frac{e|\mathcal{F}^a_{\alpha\beta}\mathcal{F}_a^{\alpha\beta}|}{\sqrt{-g}\,\lambda^4} - \frac{1}{4}\delta_\mu^\nu\, \mathcal{F}^a_{\sigma\lambda}\mathcal{F}_a^{\sigma\lambda}\right],$$

$$\left(\frac{\delta^{ab}}{\sqrt{-g}}\overrightarrow{\partial}_\nu\sqrt{-g} - f^{abc}\mathcal{A}_\nu^c\right)\left(\frac{\mathcal{F}_b^{\mu\nu}}{\sqrt{-g}}\ln\frac{e|\mathcal{F}^a_{\alpha\beta}\mathcal{F}_a^{\alpha\beta}|}{\sqrt{-g}\,\lambda^4}\right) = 0$$

temporal (Hamilton) gauge $\quad A_0^a = 0 \qquad e_i^a A_k^a \equiv A_{ik} \qquad e_i^a e_k^a = \delta_{ik} \qquad e_i^a e_i^b = \delta_{ab}$

due to local SU(2) ~ SO(3) isomorphism $\quad A_{ik}(t,\vec{x}) = \delta_{ik}U(t) + \widetilde{A}_{ik}(t,\vec{x})$

The resulting equations:

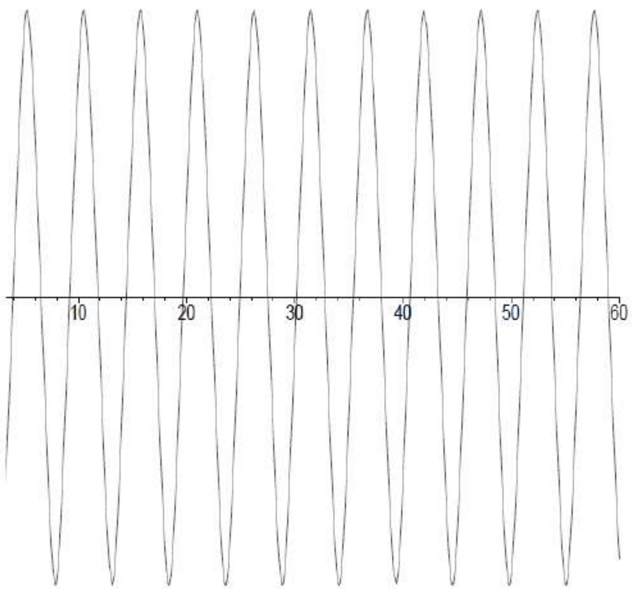
$$\frac{6}{\varkappa}\frac{a''}{a^3} = 4\bar{\epsilon} + T_\mu^{\mu,\mathrm{U}}\,, \qquad T_\mu^{\mu,\mathrm{U}} = \frac{3b}{16\pi^2 a^4}\left[(U')^2 - \frac{1}{4}U^4\right],$$

$$\frac{\partial}{\partial\eta}\left(U'\,\ln\frac{6e\left|(U')^2 - \frac{1}{4}U^4\right|}{a^4\lambda^4}\right) + \frac{1}{2}U^3\,\ln\frac{6e\left|(U')^2 - \frac{1}{4}U^4\right|}{a^4\lambda^4} = 0$$

Gluon condensate on non-stationary (FLRW) background

$$Q \equiv \frac{32}{11}\pi^2 e(\xi\Lambda_{\rm QCD})^{-4} T^\mu_\mu[U]$$
$$= 6e\left[(U')^2 - \frac{1}{4}U^4\right] a^{-4}(\xi\Lambda_{\rm QCD})^{-4}$$

Classical YM condensate



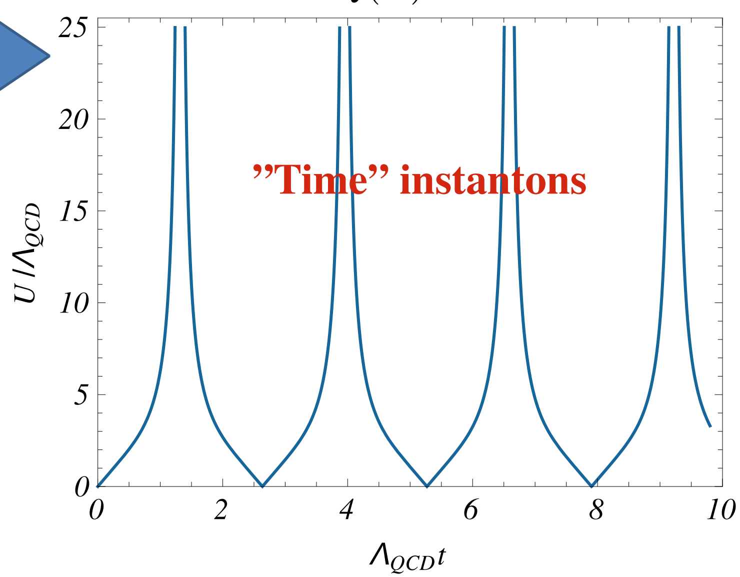
Exact partial solution:

$|Q| = 1$

Quantum corrections

Quantum YM vacuum

$Q(U) = 1$



"Radiation" medium

$\epsilon_{\rm YM} \propto 1/a^4$

Unstable solution!

QCD vacuum:
a ferromagnetic undergoing spontaneous magnetisation (Pagels&Tomboulis)

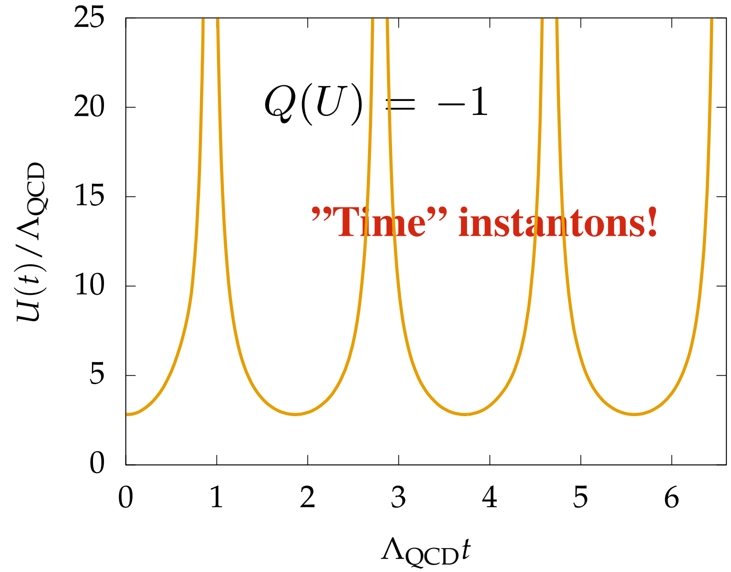
Asymptotic (attractor) solution

$\epsilon_{\rm CE} \to +{\rm const} \quad t \to \infty$

Stable solution!

- **In fact, both chromoelectric and chromomagnetic condensates are stable on non-stationary (FLRW) background of expanding Universe**

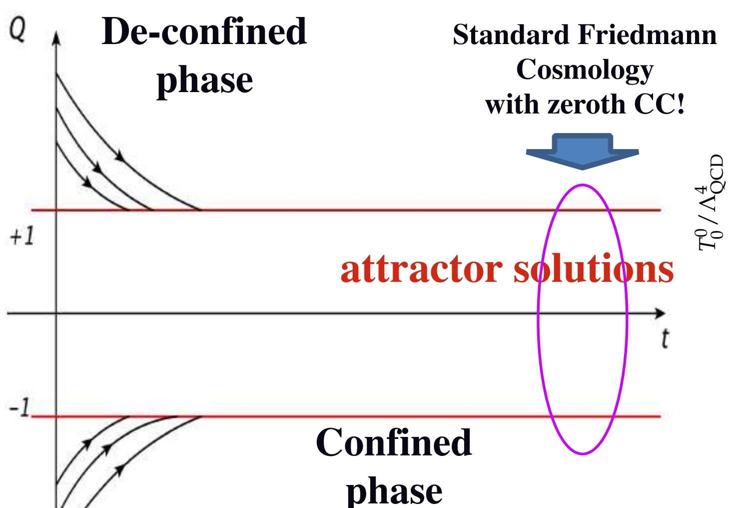
Infrared restoration of conformal invariance



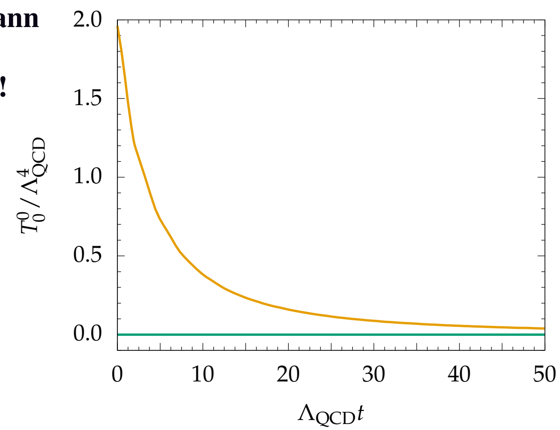
$$\epsilon_{\rm vac} \equiv \frac{1}{4}\langle T^{\mu}_{\mu}\rangle_{\rm vac} = \mp \mathcal{L}_{\rm eff}(\mathcal{J}^*)$$

$$\epsilon^{\rm CE}_{\rm vac}\Big|_{\mathcal{J}^*>0} + \epsilon^{\rm CM}_{\rm vac}\Big|_{\mathcal{J}^*<0} \equiv 0$$

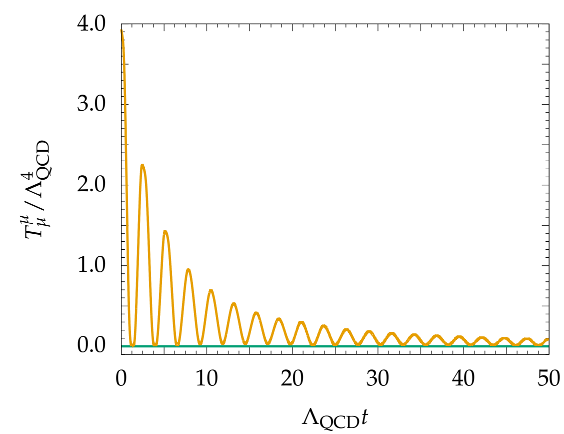
Exact compensation of CM and CE vacua as soon as the cosmological attractor is achieved!



CE energy density



CE EMT trace

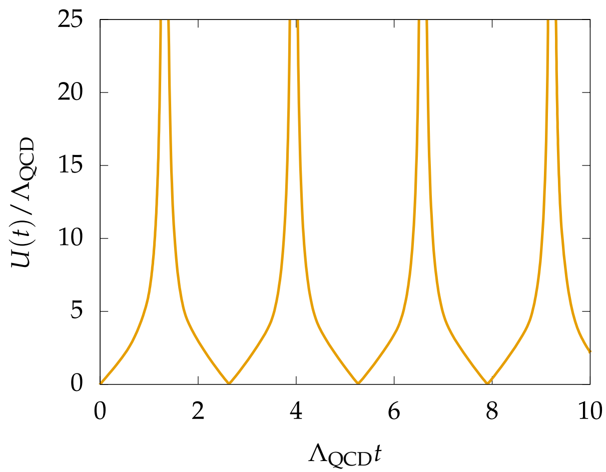


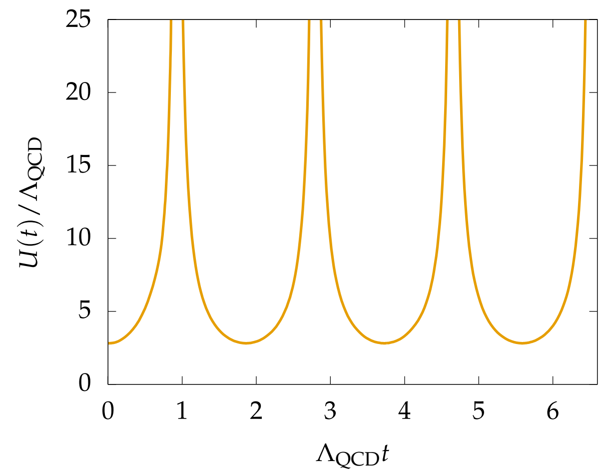
System with very unusual dynamical properties!

Addazi, A.; Marcianò, A.; Pasechnik, R.; Prokhorov, G. Mirror Symmetry of quantum Yang-Mills vacua and cosmological implications. *Eur. Phys. J. C* **2019**, *79*, 251, [arXiv:hep-th/1804.09826].

QCD “time crystal”

- **The emergence of spikes localised in time at a characteristic QCD time lapse $\Delta t \simeq \Lambda_{\rm QCD}^{-1}$ and extended in 3-space dimensions reveals the presence of an order state of space-like soliton/domain wall solutions (chronons)**





- **A time-ordered classical solution spontaneously breaking time translational invariance down to a discrete time shift symmetry $T_n \,:\, t \to t + n\Lambda_{\rm QCD}^{-1}$ is known as the “time crystal” first discovered by Wilczek in the context of superconductors and superfluids in** F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012)

- **The kink (anti-kink) profile localised in time corresponds to a space-like domain wall**

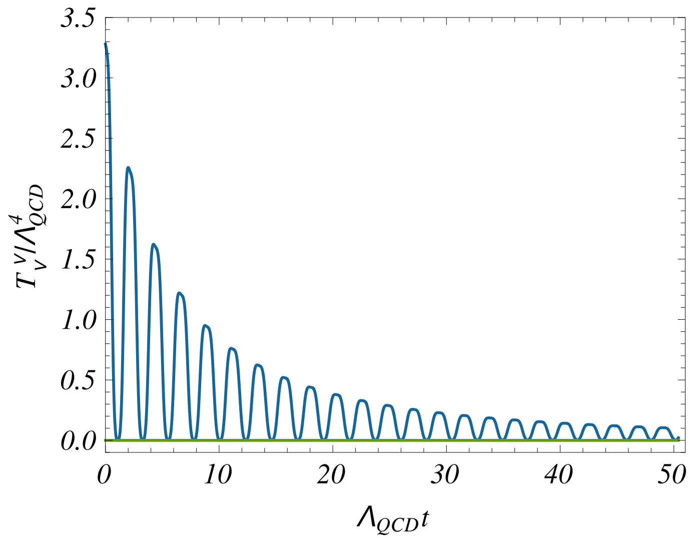
$$U(\eta) \simeq \frac{v}{\sqrt{2}} \tanh^{-1}[\frac{v}{\sqrt{2}}(\eta - \eta_0)] \qquad v \simeq \Lambda_{\rm QCD}$$

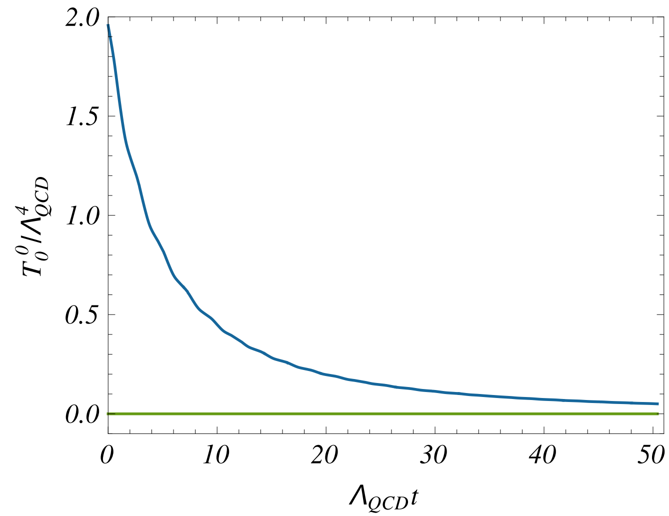
- **As the T-invariance is broken, a massless moduli field $\eta_0(x,y,z)$ localised on the domain wall world sheet x, y, z arises and corresponds to a Nambu-Goldstone boson**

Gravitational radio-waves from QCD relaxation

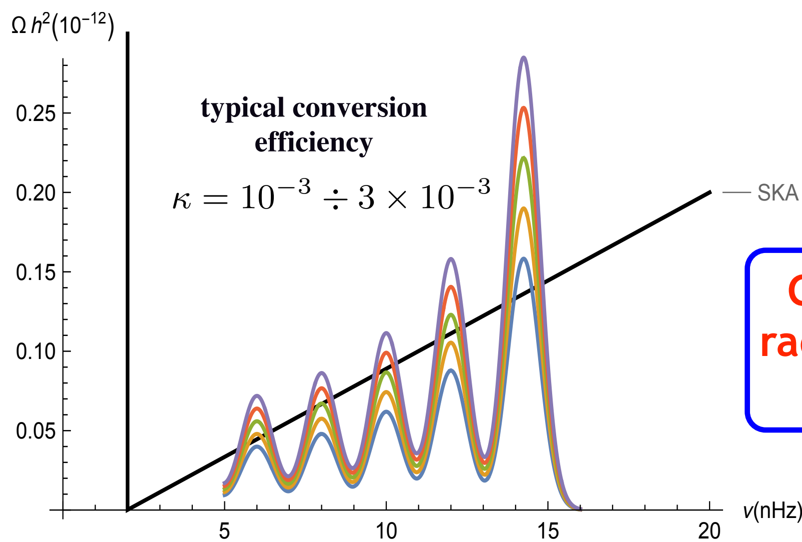
The pressure kinks get efficiently transmitted to the primordial plasma inducing shock sound waves and turbulence in it

A. Addazi, A. Marcianò, RP,
CPC 43 (2019) 6, 065101
arXiv: 1812.07376





In the domain of NANOGrav sensitivity!



GW signal lies at the radio-astronomy pulsar timing scale

$10^{-9} \div 10^{-8}$ Hz

PTAs should be able to probe QCD relaxation through detection of primordial GW radio waves

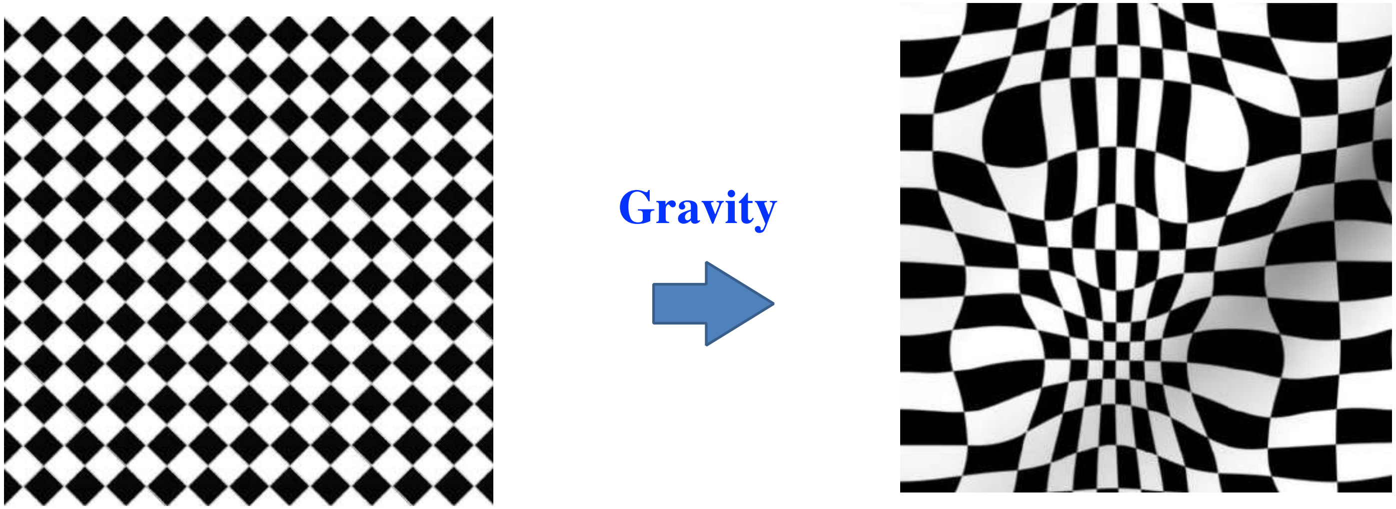
Breaking of Mirror symmetry and Cosmological Constant

Exact mirror symmetry of the YM ground state

Exact conformal invariance at macroscopic scales

Quantum Gravity in the quasi classical approximation

Mirror symmetry and conformal invariance breakdown at cosmological scales

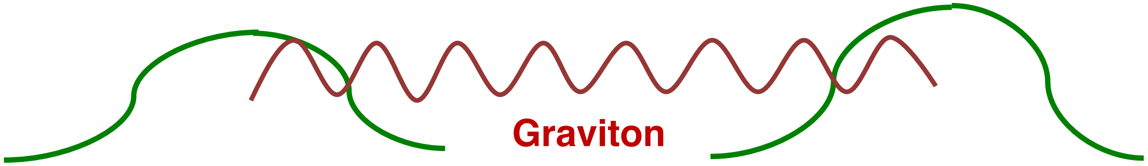


Pasechnik, R.; Beylin, V.; Vereshkov, G. Dark Energy from graviton-mediated interactions in the QCD vacuum. *JCAP* **2013**, *06*, 011, [arXiv:gr-qc/1302.6456].

Ya. Zeldovich (1967):

$$\Lambda \sim G m^6$$

A. Sakharov (1967):
extra terms describing an effect of graviton exchanges between *identical particles (bosons occupying the same quantum state) should appear in the right hand side of Einstein equations (averaged over quantum ensemble)*



$$\varepsilon_\Lambda \sim G \Lambda_{\rm QCD}^6$$

Quasiclassical gravity

Zeldovich-Sakharov scenario can be realized in the following consistent way:

Action $S = \int L d^4x, \quad L = -\frac{1}{2\varkappa}\sqrt{-\hat{g}}\hat{g}^{ik}\hat{R}_{ik} + L\left(\hat{g}^{ik}, \chi_A\right)$

Metric operator $\hat{g}^{ik}$

Macroscopic geometry (c-number part) g^{ik}

Quantum graviton field Φ_i^k

$\langle 0|\Phi_i^k|0\rangle = 0$

Independent variations over classical and quantum fields:

$$\delta \int L d^4x = -\frac{1}{2}\int d^4x \left(\sqrt{-g}\,\delta g^{ik}\hat{G}_{ik}\right)_{\Phi_i^k = \text{const}}$$
$$= -\frac{1}{2}\int d^4x \left(\sqrt{-g}\,\delta \Phi^{ik}\hat{G}_{ik}\right)_{g^{ik} = \text{const}}$$

Heisenberg state vector containing info about initial states of all fields exists!

Averaging over initial states

same operator eqns:

$$\hat{G}_i^k = \frac{1}{2}\left(\delta_l^k\delta_i^m + g^{km}g_{il}\right)\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{E}_m^l = 0,$$
$$\hat{E}_m^l = \frac{1}{\varkappa}\left(\hat{g}^{lp}\hat{R}_{pm} - \frac{1}{2}\delta_m^l\hat{g}^{pq}\hat{R}_{pq}\right) - \hat{g}^{lp}\hat{T}_{pm}\left(\hat{g}^{ik}, \chi_A\right)$$

e.o.m. for macroscopic geometry

$$\langle 0|\hat{G}_i^k|0\rangle = 0$$

e.o.m. for graviton field

$$\hat{G}_i^k - \langle 0|\hat{G}_i^k|0\rangle = 0$$

Λ-term calculation

We start from the Einstein equations for macroscopic geometry:

$$\frac{1}{\varkappa}\left(R_i^k - \frac{1}{2}\delta_i^k R\right) = \langle 0|\hat{T}_i^k|0\rangle \qquad \hat{T}_i^k = \hat{T}_{i(G)}^k + \frac{1}{2}\left(\delta_l^k\delta_i^m + g^{km}g_{il}\right)\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{lp}\hat{T}_{pm}\left(\hat{g}^{ik},\chi_A\right)$$

Trace: $R + 4\varkappa\Lambda = 0$

$$\Lambda = -\frac{b_{eff}}{32}\langle 0|\frac{\alpha_s}{\pi}\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{il}\hat{g}^{km}\hat{F}_{ik}^a\hat{F}_{lm}^a|0\rangle + \frac{1}{4}\langle 0|\hat{T}_{(G)}|0\rangle$$

Stress tensor in Riemann space is found from YM eqs:

$$\left(\delta^{ab}\frac{\partial}{\partial x^k} - g_s f^{abc}\hat{A}_k^c\right)\sqrt{-\hat{g}}\hat{g}^{il}\hat{g}^{km}\hat{F}_{lm}^b = 0$$

$$\hat{F}_{ik}^a = F_{ik}^a + \underbrace{\frac{1}{2}\psi F_{ik}^a - \psi_i^l F_{lk}^a - \psi_k^l F_{il}^a}_{} + O(\alpha_s G)$$

induce interactions of YM field with metric fluctuations

Equation for gravitons turns into:

$$\psi_{i,\,l}^{k,\,l} - \psi_{i,\,l}^{l,\,k} - \psi_{l,\,i}^{k,\,l} + \delta_i^k\psi_{l,\,m}^{m,\,l} = \frac{\varkappa b_{eff}\alpha_s}{\pi}\left(-F_{il}^a F_a^{kl} + \frac{1}{4}\delta_i^k F_{ml}^a F_a^{ml}\right)\ln\frac{L_g^{-1}}{\Lambda_{QCD}}$$

After exact cancellation of unperturbed part of EMT tensor we get:

$$\Lambda = -\frac{b_{eff}}{16}\ln\frac{L_g^{-1}}{e\Lambda_{QCD}}\langle 0|\frac{\alpha_s}{\pi}F_{il}^a F_a^{kl}\left(\psi_k^i - \frac{1}{4}\delta_k^i\psi\right)|0\rangle$$

linear in graviton field!

Λ-term calculation

Fock gauge: $\psi^{k}_{i;\,k} = 0$

Metric fluctuations are induced by QCD vacuum fluctuations!

Exact solution of graviton equation:

$$\psi_i^k(x) = \varkappa b_{eff} \ln\frac{L_g^{-1}}{\Lambda_{QCD}} \int d^4x' \underbrace{\mathcal{G}(x-x')}_{} \times \left(\frac{\alpha_s}{\pi} F^a_{il}(x')F^{kl}_a(x') - \delta_i^k \frac{\alpha_s}{4\pi} F^a_{ml}(x')F^{ml}_a(x')\right)$$

Green function: $\mathcal{G}_{;\,l}^{;\,l} = -\delta(x-x')$

After explicit calculation of averages, we get

$$\Lambda = -\pi G \langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}F^{ik}_a : |0\rangle^2 \times \left(\frac{b_{eff}}{8}\right)^2 \ln\frac{L_g^{-1}}{e\Lambda_{QCD}} \ln\frac{L_g^{-1}}{\Lambda_{QCD}} \int d^4y \mathcal{G}(y) D^2(y) =$$
$$= (1 \pm 0.5) \times 10^{-29} \Delta \text{ MeV}^4.$$

where

$$\langle 0| \frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x')|0\rangle = \langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(0)F^{ik}_a(0) : |0\rangle D(x-x'),$$

$$\Delta = -\frac{1}{L_g^2}\int d^4y \mathcal{G}(y) D^2(y)$$

$$D(x-x') = D_{top}(x-x') - D_h(x-x'), \qquad D(0) = 0.$$

must be established in a dynamical theory of NPT QCD vacuum!

It is expected to be generated by chiral symmetry breaking

In terms of known NPT QCD parameters

$$1/L_{top} \sim 1/L_h \sim 1/L_g,$$

$$|1/L_{top} - 1/L_h| \sim m_u + m_d + m_s$$

$$\Delta = k \cdot \frac{(m_u + m_d + m_s)^2 L_g^2}{(2\pi)^4} \sim 3 \cdot 10^{-6}$$

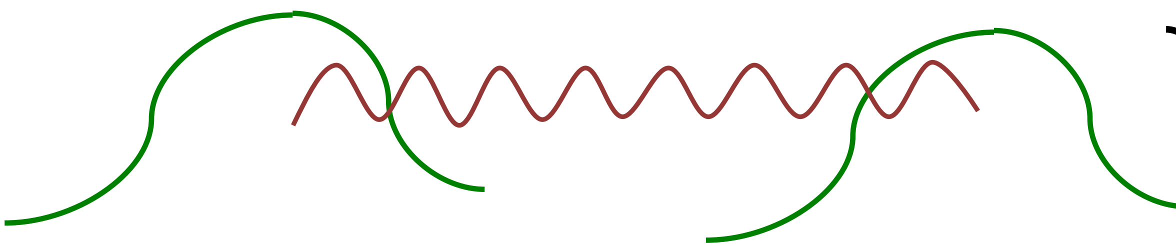
!!!

Observable Λ-term from QCD?

"zeroth" order in QG

Must cancel exactly due to QCD confinement
(only a new symmetry of the ground state can do that!)

"first" order in QG



virtual (strong) NPT fluctuations of quark and gluon fields dynamically induce metric fluctuations (gravitons)!

cancel NOT exactly!
(due the chiral SB in QCD)

Observable Λ-term!

$$\varepsilon_\Lambda \sim G\Lambda^6_{\rm QCD}$$

$$\Lambda = \frac{m_\pi^6}{(2\pi)^4 M_{Pl}^2} \simeq 2.98 \times 10^{-35}\ {\rm MeV}^4 \qquad m_\pi \simeq 138\ {\rm MeV}$$

$$\Lambda_{\rm exp} = (3.0 \pm 0.7) \times 10^{-35}\ {\rm MeV}^4$$

Only NPT QCD vacuum fluctuations coupled to Gravity at lowest (hadron) scales of Particle Physics gives rise to Λ-term if UV terms are canceled

Including temperature: CM field as a glueball

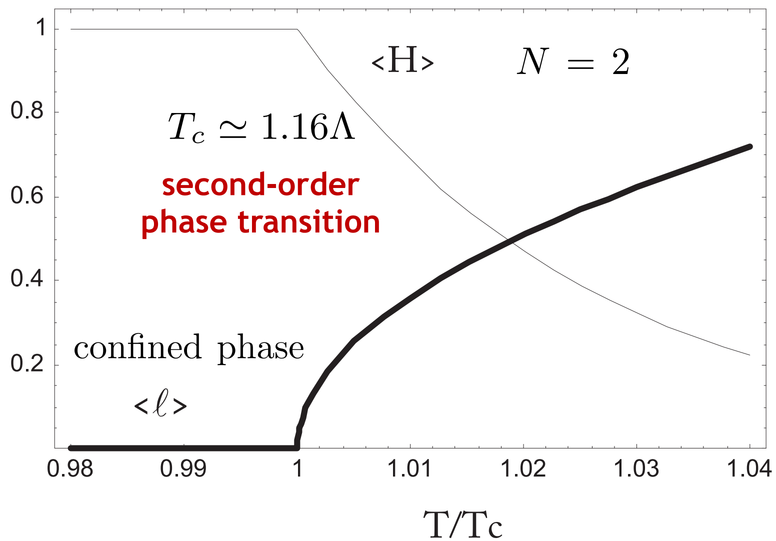
Carenza, RP, Salinas, Wang, Phys. Rev. Lett. 129 (2022) no.26, 26

- **Polyakov loop operator charged under the center of SU(N):**

$$\ell(x) = \frac{1}{N}\mathrm{Tr}[\mathbf{L}] \equiv \frac{1}{N}\mathrm{Tr}\left[\mathcal{P}\exp\left[i\,g\int_0^{1/T} A_0(\tau,\mathbf{x})d\tau\right]\right]$$

$$\ell \;\rightarrow\; z\ell \qquad z\in\mathbb{Z}_N \qquad N\geq 2$$

- **Polyakov loop VEV is an order parameter of confinement phase transition:**



F. Sannino, *Polyakov loops versus hadronic states*, *Phys. Rev. D* **66** (2002) 034013 [hep-ph/0204174].

$$\mathcal{L} = \frac{c}{2}\frac{\partial_\mu\mathcal{H}\partial^\mu\mathcal{H}}{\mathcal{H}^{3/2}} - V[\mathcal{H},\ell]$$

$$V[\mathcal{H},\ell] = \frac{\mathcal{H}}{2}\ln\left[\frac{\mathcal{H}}{\Lambda^4}\right] + T^4\mathcal{V}[\ell] + \mathcal{H}\mathcal{P}[\ell] + V_T[\mathcal{H}]$$

$$c = \frac{1}{2\sqrt{e}}\left(\frac{\Lambda}{m_{\rm gb}}\right)^2$$

PLM well captures essential thermodynamical observables predicted by lattice simulations

Thermal evolution of the glueball-gluon system

Carenza, RP, Salinas, Wang, Phys. Rev. Lett. 129 (2022) no.26, 26

- **Introducing canonically normalised field** $\mathcal{H} = 2^{-8} c^{-2} \phi^4$

  the effective Lagrangian reads:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V[\phi, \ell] \,,$$

$$V[\phi,\ell] = \frac{\phi^4}{2^8 c^2}\left[2\ln\left(\frac{\phi}{\Lambda}\right) - 4\ln 2 - \ln c\right] + $$

$$+ \frac{\phi^4}{2^8 c^2}\mathcal{P}[\ell] + T^4 \mathcal{V}[\ell] \,,$$

$$\mathcal{P}[\ell] = c_1 |\ell|^2 \,,$$

$$\mathcal{V}[\ell] = -\frac{b_2(T)}{2}|\ell|^2 + b_4|\ell|^4 - b_3(\ell^3 + (\ell^*)^3) \,,$$

$$b_2(T) = \sum_{i=0}^{4} a_i \left(\frac{T_c}{T}\right)^i \,,$$

- **Fits to lattice results for observables provide:**

a_0	a_1	a_2	a_3	a_4	b_3	b_4
3.72	−5.73	8.49	−9.29	0.27	2.40	4.53

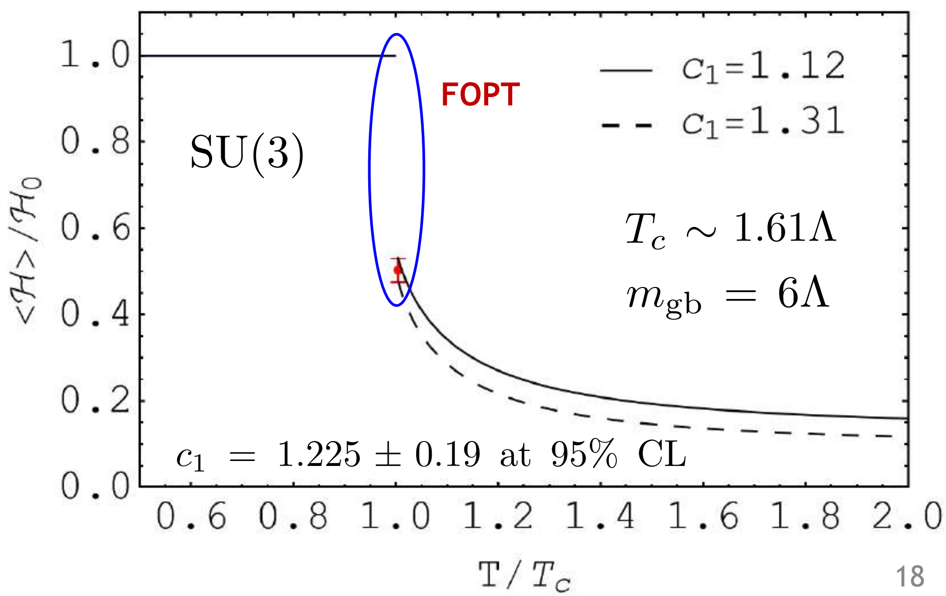
Huang, Reichert, Sannino and Wang, PRD 104 (2021) 035005

- **Integrating out the Polyakov loop in the high-T phase provides**

$$V[\phi, T] = V[\phi, \ell(\phi, T)]$$

matching the size of discontinuity to lattice:

M. D'Elia, A. Di Giacomo and E. Meggiolaro, *Gauge invariant field strength correlators in pure Yang-Mills and full QCD at finite temperature*, *Phys. Rev. D* **67** (2003) 114504 [hep-lat/0205018].



Cosmological evolution of the glueball field

- Since quantum effects are embedded into the effective Lagrangian, the evolution can be treated as if it were classical

- The glueball field is considered homogeneous and evolves in expanding FLRW Universe, with the Klein-Gordon e.o.m.

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V[\phi, T] = 0$$

- If there are no interactions with the SM, the dark sector is colder than the SM thermal bath, with the visible-to-dark sector temperature ratio ξ_T

- Time variable is found in terms of the photon temperature:

$$t = \frac{1}{2}\sqrt{\frac{45}{4\pi^3 g_{*,\rho}(T_\gamma)}}\frac{m_P}{T_\gamma^2} \qquad T_\gamma = \xi_T T$$

- E.o.m. in terms of the dark sector temperature:

$$\boxed{\frac{4\pi^3 g_{*,\rho}}{45 m_P^2}\xi_T^4 T^6 \frac{d^2\phi}{dT^2} + \frac{2\pi^3}{45 m_P^2}\frac{dg_{*,\rho}}{dT}\xi_T^4 T^6 \frac{d\phi}{dT} + \partial_\phi V[\phi, T] = 0\,,} \qquad g_{*,\rho} = 100$$

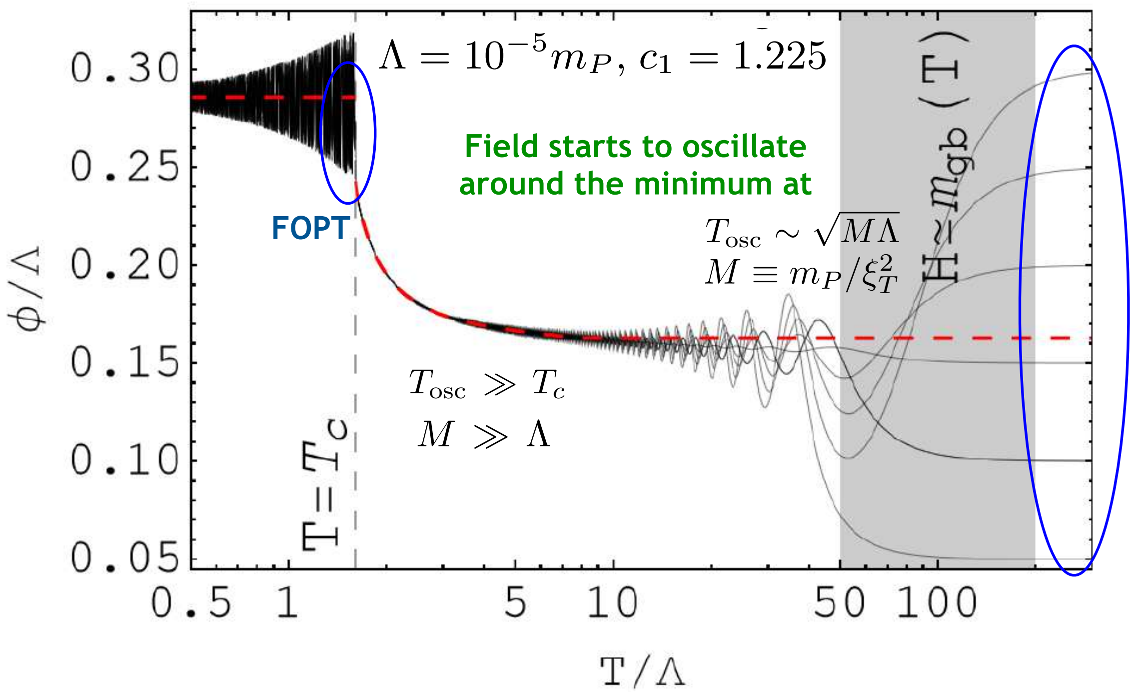
encodes non-perturbative dynamics of the glueball field!

Cosmological evolution of the glueball field

- After the phase transition, we assume that the energy stored in the glueball fields gives rise to the Dark Matter relic density (no further decays implied)

- Due to the interaction term, dark glueballs are formed from dark gluons populating the Universe in the deconfined regime

- Higher-order non-linear interaction terms among glueballs are important for large amplitudes of glueball field oscillations around the minimum (particularly relevant for phase transition)



In early times in deconfined regime, for different initial conditions the field evolution is dominated by Hubble friction (slow evolution)

Oscillations have long time to decay regardless of the initial condition (field follows the minimum)

FOPT washes out any dependence on initial conditions

Glueball relic density

Carenza, RP, Salinas, Wang, Phys. Rev. Lett. 129 (2022) no.26, 26

- In the confined phase, due to large oscillations, annihilation of n glueballs into m<n glueballs is possible due to (n+m)-interaction term

- As the glueball number density decreases, only 3->2 processes remain efficient and determine the relic abundance when $\Gamma_{3\to 2} < H$

- The evolution of the glueball field is that of a dumped harmonic oscillator in a non-linear potential, with oscillations about the minimum $\phi_{\rm min} \approx 0.28\Lambda$

- Energy stored in those oscillations gives rise to the relic DM abundance:

$$\Omega h^2 = \rho/\rho_c \qquad \rho_c = 1.05 \times 10^4\,\mathrm{eV\,cm^{-3}}$$

$$\rho = \frac{2\pi^3}{45} g_{*,\rho}(T)\frac{T^6}{M^2}\left(\frac{d\phi}{dT}\right)^2 + V[\phi] \;\sim T^3$$

after decoupling of 3->2 transitions, the density scales as that of Cold Dark Matter

- Below freeze-out temperature, the relic abundance

$$0.12\zeta_T^{-3}\frac{\Lambda}{137.9\,\mathrm{eV}} \lesssim \Omega h^2 \lesssim 0.12\zeta_T^{-3}\frac{\Lambda}{82.7\,\mathrm{eV}}$$

$$\Lambda \lesssim 0.1\,M$$

$$1.035 < c_1 < 1.415$$

Comparison with early studies

- We confirm the existence of the glueball overabundance problem for high-scale confinement previously found in the literature due to the linear scaling

$$\Omega h^2 \sim \Lambda$$

- Our prediction is an order of magnitude smaller than the existing glueball abundance results in the literature

$$\Omega h^2 \quad \sim \quad 0.12\, \zeta_T^{-3} \Lambda / 5.45\, \mathrm{eV}$$

E. D. Carlson, M. E. Machacek and L. J. Hall, *Self-interacting dark matter*, *Astrophys. J.* **398** (1992) 43.

Two main differences

the energy density of dark gluons for temperatures right above the critical one strongly deviates (reduced by a factor ~ 50) from that of an ideal gas, in agreement with lattice results

glueballs do not redshift as CDM immediately after the phase transition but dilute slower than dust, going through a phase with equation of state

$$-1 \lesssim p/\rho \lesssim 0$$

- Thermal corrections to the glueball potential are expected to increase the glueball relic density by up to 80%, due to displacing the high-temperature minimum of a ~10% farther from the low-temperature minimum (in progress)

- Contribution of dark gluons to the effective number of relativistic species, is constrained to be

$$\Delta N_{\mathrm{eff}} < 0.35 \qquad \zeta_T \gtrsim 2$$

is enough to avoid this constraint

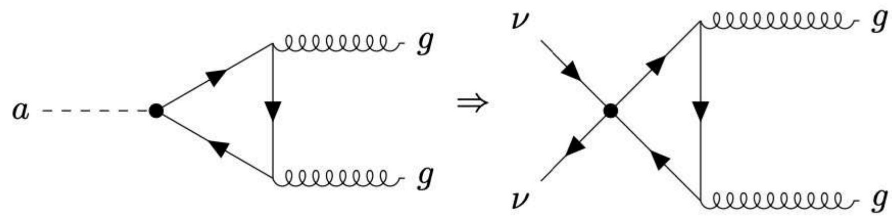
Composite QCD axion and neutrino mass

A. Addazi, A. Marcianò, RP, K. Zeng, Phys.Dark Univ. 36 (2022) 101007

- **Ginzburg, Zharkov 1967**: neutrinos can be non-relativistically produced in a superfluid phase, related to the neutrino mass gap and Dark Energy

- We **postulate** that neutrinos have a Peccei-Quinn (PQ) scale suppressed **portal with the strongly-coupled QCD sector** through quantum anomalies:

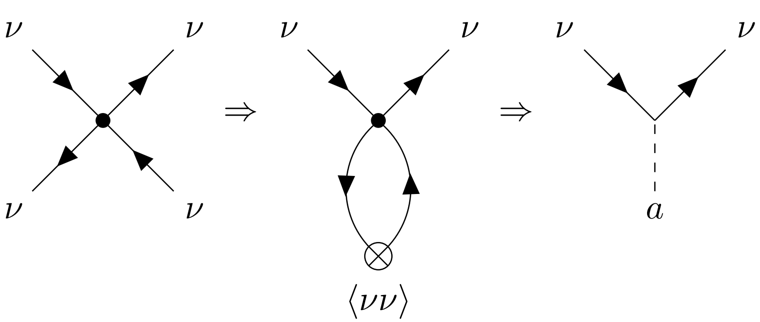
$$\frac{1}{f_{\rm PQ} f_a^2}(\nu^T \mathcal{C}^{-1}\gamma_5 \nu)\, G\widetilde{G}$$



- The **QCD axion** is then identified with **the composite pair of neutrinos**:

$$\mathcal{L} = \partial\Phi^\dagger\partial\Phi - V(\Phi)\,, \quad V(\Phi) = (|\Phi|^2 - f_{\rm PQ}^2)^2\,, \quad \Phi = \rho\, e^{\imath \frac{a}{f_{\rm PQ}}}\,, \qquad a = \frac{1}{f_a^2}\nu^T \mathcal{C}^{-1}\gamma_5\nu$$

- A four-fermion interaction responsible **for condensation of neutrino pairs and for generation of the neutrino mass scale in the SM**:



Neutrinos can be produced in pairs as cold during the QCD phase transition and condense as non-relativistic Cooper pairs via the misalignment mechanism

- **Neutrino condensate** can naturally account for **the right amount of Dark Matter**

Concluding remarks

- Local loss of continuous time-translational invariance leads to "time crystal"-type configurations in the QCD vacuum

- Nielsen-Olsen proof of instability of CE condensate on a rigid Minkowski in NOT in contradiction with our picture: we consider YM evolution on a dynamical (FLRW) spacetime while equilibrium is achieved only asymptotically.

- A possible decay of CE condensate into an anisotropic vacuum after a cosmological relaxation time would be exponentially suppressed and is practically never realised

- Even starting from an initial non-zero energy-density, the evolution of localised 3-space "pockets" of the CE and CM condensates trigger a mutual screening, flowing towards a zero-energy density attractor and accompanying by a formation of the domain walls corresponding to an asymptotic restoration of the Z2 (Mirror) symmetry and effectively protecting the "false" CE vacua pockets from further decay

- The vacua cancellation mechanism seems to naturally marry the existing confinement pictures related to a formation of a network of t'Hooft monopoles or chromovortices. In this approach, the scalar kink profile may correspond the J-invariant whose change may be related to the presence of monopole or vortex solutions localise inside the space-time domain walls. This implies the existence of space-time solitonic objects of a new type.

Concluding remarks

- Breaking of the Mirror symmetry by gravitational interactions induces non-vanishing leading order contribution to the QCD ground state energy compatible with the observed cosmological constant value that must be taken into account in any model of DE

- Pressure oscillations during the QCD relaxation epoch trigger multi-peaked primordial gravitational wave spectrum in the radio-frequency range that can be potentially probed by the SKA telescope

- We developed a new approach based upon the well-established thermal EFT and the existing lattice results to calculate the glueball CDM relic density incorporating confinement effects and non-perturbative self-interactions

- Cold neutrino pairs can be produced during the QCD transition and condense into axions through a possible four-fermion neutrino interaction and a coupling to the QCD anomaly enabling neutrino mass gap and Dark Matter generation