Abelian Decomposition of QCD —Two Types of Gluons—

Y. M. Cho

School of Physics and Astronomy, Seoul National University, Seoul 08826
and
Center for Quantum Space-time, Sogang University, Seoul 04107
KOREA

August 13, 2024

Millennium Problem: Color Confinement in QCD

- An outstanding problem in QCD is the confinement of color. Two popular conjectures to resolve the problem are the monopole condensation and the Abelian dominance.
- To prove the monopole condensation we have to separate the monopole part from QCD. How can we do that?
- To prove the Abelian dominance, we have to know what is the Abelian part of QCD. Can we separate the Abelian part gauge independently?

More Questions

- Proton is made of three quarks, but obviously it must have the binding gluon to bind them. However, the quark model tells that it has no valence gluon. If so, what is the binding gluon and valence gluon?

- Group theoretically two of the eight gluons are color neutral. Can we separate them from the six colored gluons? If so, how?

- What is the difference between QED and QCD which allows QCD the color confinement?

History

- In 1974 Nambu and Mandelstam conjectured the monopole condensation as the confinement mechanism.
- In 1977 Savvidy calculated the SU(2) QCD effective action and obtained the Savvidy vacuum.
- In 1980 the Abelian decomposition of QCD was proposed. In 1981 't Hooft conjectured the Abelian dominance, which was proved in 1990.
- The lattice QCD was able to obtain the linear confining potential numerically, but unable to tell what is the confinement mechanism.

Contents

Abelian Decomposition: Two Types of Gluons

A. Abelian decomposition of SU(2) QCD

- Let $(\hat{n}_1, \hat{n}_2, \hat{n}_3 = \hat{n})$ be an orthonormal basis and $\hat{n}$ be the Abelian direction. With the Abelian projection we have the restricted potential $\hat{A}_\mu$,

$$D_\mu \hat{n} = \partial_\mu \hat{n} + g\vec{A}_\mu \times \hat{n} = 0,$$

$$\vec{A}_\mu \to \hat{A}_\mu = A_\mu \hat{n} - \frac{1}{g}\hat{n} \times \partial_\mu \hat{n} = \tilde{A}_\mu + \tilde{C}_\mu,$$

$$\tilde{A}_\mu = A_\mu \hat{n}, \quad \tilde{C}_\mu = -\frac{1}{g}\hat{n} \times \partial_\mu \hat{n}, \quad A_\mu = \hat{n} \cdot \vec{A}_\mu.$$

$\hat{A}_\mu$ has a dual structure, made of the non-topological Maxwellian $\tilde{A}_\mu$ which describes the color neutral binding gluon (the neuron) and the topological Diracian $\tilde{C}_\mu$ which describes the non-Abelian monopole.

- With this we have the gauge independent Abelian decomposition

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu, \quad (\hat{n} \cdot \vec{X}_\mu = 0).$$
$$\vec{F}_{\mu\nu} = \hat{F}_{\mu\nu} + \hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu + g \vec{X}_\mu \times \vec{X}_\nu,$$
$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + g \hat{A}_\mu \times \hat{A}_\nu = (F_{\mu\nu} + H_{\mu\nu})\hat{n},$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu,$$
$$C_\mu = -\frac{1}{g}\hat{n}_1 \cdot \partial_\mu \hat{n}_2.$$

1. $\hat{A}_\mu$ is Abelian, but has the full non-Abelian gauge degrees of freedom.
2. $\vec{X}_\mu$ transforms gauge covariantly, and describes the colored valence gluon (the chromon).

Two Types of Gluons!

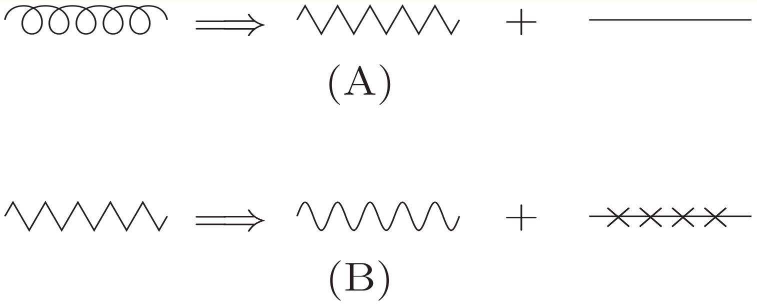


Figure: The gauge independent Abelian decomposition of QCD potential. (A) decomposes it to the restricted part and the chromon, and (B) decomposes the restricted part to the neuron and monopole.

B. RCD and QCD

- Define the restricted QCD (RCD) with $\hat{A}_\mu$,

$$\mathcal{L}_{RCD} = -\frac{1}{4}\hat{F}_{\mu\nu}{}^2 = -\frac{1}{4}(F_{\mu\nu} + H_{\mu\nu})^2$$
$$= -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2g}F_{\mu\nu}\hat{n}\cdot(\partial_\mu\hat{n}\times\partial_\nu\hat{n}) - \frac{1}{4g^2}(\partial_\mu\hat{n}\times\partial_\nu\hat{n})^2.$$

1. QCD has the Abelian core RCD which describes the Abelian sub-dynamics of QCD.

2. RCD is different from QED. It has the full color gauge symmetry and contains the monopole potential which is responsible for the confinement.

"Non-Abelian" theory of monopole

- Adding the valence gluon we recover QCD in the extended form

$$\mathcal{L}_{QCD} = -\frac{1}{4}\vec{F}_{\mu\nu}^2 = -\frac{1}{4}\hat{F}_{\mu\nu}^2$$

$$-\frac{1}{4}(\hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu)^2 - \frac{g}{2}\hat{F}_{\mu\nu} \cdot (\vec{X}_\mu \times \vec{X}_\nu) - \frac{g^2}{4}(\vec{X}_\mu \times \vec{X}_\nu)^2.$$

1. QCD can be interpreted as RCD made of the neuron which has the chromon as colored source. So the neuron plays the role of the binding gluon but the chromon becomes the constituent gluon.

2. This justifies us to replace the quark and gluon model by the quark and chromon model.

"Extended QCD (ECD)"

- This puts QCD to the background field formalism which has two gauge symmetries, the classical (active) gauge symmetry

$$\delta\hat{n} = -\alpha \times \hat{n}, \quad \delta\hat{A}_\mu = \frac{1}{g}\hat{D}_\mu\vec{\alpha}, \quad \delta\vec{X}_\mu = -\vec{\alpha} \times \vec{X}_\mu,$$

and the quantum (passive) gauge symmetry

$$\delta\hat{n} = 0, \quad \delta\hat{A}_\mu = \frac{1}{g}(\hat{n} \cdot D_\mu\vec{\alpha})\hat{n}, \quad \delta\vec{X}_\mu = \frac{1}{g}\hat{n} \times (D_\mu\vec{\alpha} \times \hat{n}).$$

- This provides us an ideal platform to calculate the QCD effective action and prove the monopole condensation.

ICAL REVIEW D VOLUME 21, NUMBER 4 15 FEBRUARY 1980

Restricted gauge theory

Y. M. Cho
Max-Planck-Institut für Physik und Astrophysik, München, Federal Republic of Germany
(Received 27 April 1979)

A restricted gauge theory is obtained as a self-consistent subset of a non-Abelian gauge theory by imposing an extra magnetic symmetry to the gauge symmetry. The theory describes the dual dynamics between the color isocharges (i.e., the electric charges) and the topological charges (i.e., the magnetic charges) of the non-Abelian symmetry, and contains two potentials, the electric and the magnetic potentials, in a dual-symmetric way. The topological charge is identified as the dual of the Noether charge of the magnetic symmetry of the theory. A possible role of the restricted chromodynamics for quark confinement in quantum chromodynamics is speculated.

I. INTRODUCTION

Since Dirac[1] introduced his magnetic monopole ... of Abelian gauge theory, at- ... truct a local field ...

of the group G when compared to the canonical (i.e., the unrestricted) gauge theory that does not have any additional symmetry. As we will see in detail in the following, owing to the magnetic sym metry one can choose a particular gauge (the ma netic gauge) in which the nonvanishing componen ... restricted gauge field become only those ... H that is uniquely determin ... one can form

C. SU(3) QCD

- Since the SU(3) QCD has two Abelian directions, the Abelian projection is given by two magnetic symmetries,

$$D_\mu \hat{n} = 0, \qquad D_\mu \hat{n}' = 0, \qquad (\hat{n}^2 = \hat{n}'^2 = 1)$$

where $\hat{n}$ and $\hat{n}' = \hat{n} * \hat{n}$ are λ_3-like and λ_8-like octet unit vectors.

- With this we have the following Abelian projection,

$$\vec{A}_\mu \rightarrow \hat{A}_\mu, \quad \hat{A}_\mu = A_\mu \hat{n} + A'_\mu \hat{n}' - \frac{1}{g}(\hat{n} \times \partial_\mu \hat{n} + \hat{n}' \times \partial_\mu \hat{n}')$$

$$A_\mu = \hat{n} \cdot \vec{A}_\mu, \quad A'_\mu = \hat{n}' \cdot \vec{A}_\mu, \quad \hat{n} \cdot \vec{X}_\mu = \hat{n}' \cdot \vec{X}_\mu = 0.$$

- $\hat{A}_\mu$ can be expressed by the three neurons of SU(2) subgroups in Weyl symmetric form

$$\hat{A}_\mu = \sum_p \frac{2}{3}\hat{A}^p_\mu, \quad (p=1,2,3),$$

$$\hat{A}^p_\mu = A^p_\mu\ \hat{n}^p - \frac{1}{g}\hat{n}^p \times \partial_\mu \hat{n}^p = \tilde{A}^p_\mu + \tilde{C}^p_\mu,$$

$$A^1_\mu = A_\mu, \ \ A^2_\mu = -\frac{1}{2}A_\mu + \frac{\sqrt{3}}{2}A'_\mu, \ \ A^3_\mu = -\frac{1}{2}A_\mu - \frac{\sqrt{3}}{2}A'_\mu,$$

$$\hat{n}^1 = \hat{n}, \quad \hat{n}^2 = -\frac{1}{2}\hat{n} + \frac{\sqrt{3}}{2}\hat{n}', \quad \hat{n}^3 = -\frac{1}{2}\hat{n} - \frac{\sqrt{3}}{2}\hat{n}'.$$

- With this we have the Abelian decomposition of SU(3) QCD,

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu = \sum_p (\frac{2}{3}\hat{A}^p_\mu + \vec{W}^p_\mu), \quad \vec{X}_\mu = \sum_p \vec{W}^p_\mu,$$
$$\vec{W}^1_\mu = X^1_\mu \hat{n}_1 + X^2_\mu \hat{n}_2,\ \vec{W}^2_\mu = X^6_\mu \hat{n}_6 + X^7_\mu \hat{n}_7,\ \vec{W}^3_\mu = X^4_\mu \hat{n}_4 + X^5_\mu \hat{n}_5.$$

- $\vec{W}^p_\mu$ can be expressed by red, blue, and green chromons of SU(2) subgroups (R_μ, B_μ, G_μ),

$$R_\mu = \frac{X^1_\mu + iX^2_\mu}{\sqrt{2}},\ B_\mu = \frac{X^6_\mu + iX^7_\mu}{\sqrt{2}},\ G_\mu = \frac{X^4_\mu + iX^5_\mu}{\sqrt{2}}.$$

But unlike $\hat{A}^p_\mu$, they are mutually independent.

- From this we have the Weyl symmetric SU(3) RCD and QCD

$$\mathcal{L}_{RCD} = -\sum_p \frac{1}{6}(\hat{F}^p_{\mu\nu})^2,$$

$$\mathcal{L}_{QCD} = -\frac{1}{4}\vec{F}^2_{\mu\nu} = -\sum_p \Big\{\frac{1}{6}(\hat{F}^p_{\mu\nu})^2 + \frac{1}{4}(\hat{D}^p_\mu \vec{W}^p_\nu - \hat{D}^p_\nu \vec{W}^p_\mu)^2$$

$$+\frac{g}{2}\hat{F}^p_{\mu\nu}\cdot(\vec{W}^p_\mu \times \vec{W}^p_\nu)\Big\} - \sum_{p,q} \frac{g^2}{4}(\vec{W}^p_\mu \times \vec{W}^q_\mu)^2$$

$$-\sum_{p,q,r} \frac{g}{2}(\hat{D}^p_\mu \vec{W}^p_\nu - \hat{D}^p_\nu \vec{W}^p_\mu)\cdot(\vec{W}^q_\mu \times \vec{W}^r_\mu)$$

$$-\sum_{p\neq q} \frac{g^2}{4}\big[(\vec{W}^p_\mu \times \vec{W}^q_\nu)\cdot(\vec{W}^q_\mu \times \vec{W}^p_\nu) + (\vec{W}^p_\mu \times \vec{W}^p_\nu)\cdot(\vec{W}^q_\mu \times \vec{W}^q_\nu)\big].$$

- The Abelian decomposition does not change QCD, but reveals important hidden structures of QCD.

  1. It decomposes the QCD Feynman diagrams in such a way that the color conservation is explicit.

  2. It tells that the chromon (being colored) can not play any role in confinement, because the prisoner can not be the jailer (the confiner).

  3. It replaces the quark and gluon model of hadrons to the quark and chromon model.

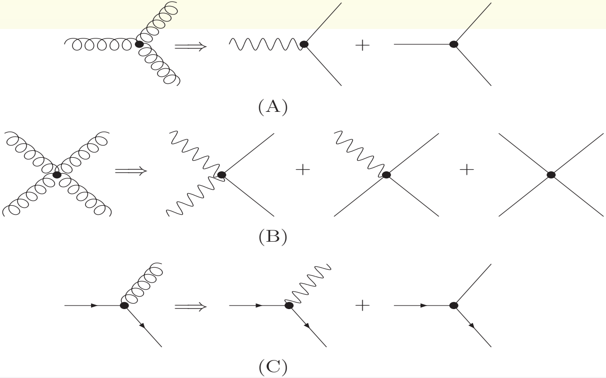


Figure: The Abelian decomposition of Feynman diagrams in SU(3) QCD. Notice that the monopole does not appear in the diagram because it describes a topological degree.

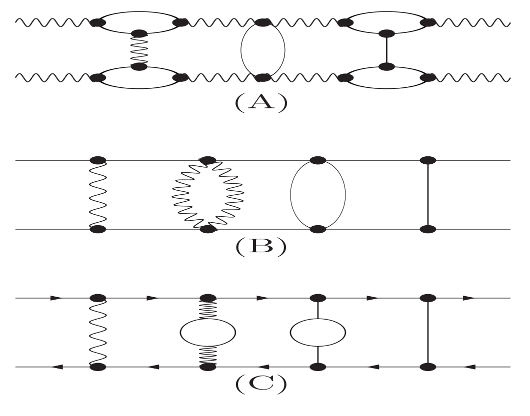


Figure: The possible Feynman diagrams of the neuron and chromon bindings. Two neuron binding is shown in (A), two chromon binding is shown in (B). In comparison the quark-antiquark binding is shown in (C).

D. Abelian Dominance versus Monopole Dominance

- The Abelian decomposition allows us to prove the Abelian dominance regorously, that the restricted potential generates the confining force in the Wilson loop integral.
- Implementing the Abelian decomposition on lattice, we can prove the monopole dominance.
- But this does not tell how the monopole confines the color.

PHYSICAL REVIEW D, VOLUME 62, 074009

Abelian dominance in Wilson loops

Y. M. Cho*

Asia Pacific Center for Theoretical Physics,
and Department of Physics, College of Natural Sciences, Seoul National University, Seoul 151-742 Korea

(Received 23 December 1999; published 6 September 2000)

It has been conjectured that the Abelian projection of QCD is responsible for the confinement of color. Using a gauge independent definition of the Abelian projection which does *not* employ any gauge fixing, we provide strong evidence for Abelian dominance in the Wilson loop integral. In specific we prove that the gauge potential which contributes to the Wilson loop integral is precisely the one restricted by the Abelian projection.

PACS number(s): 12.38.Aw, 11.15.Ha, 11.15.Tk, 12.38.Lg

The confinement problem in QCD is perhaps one of the
ost difficult problems in theoretical physics. It has long
een argued that the monopole condensation could provide
e confinement of color through a dual Meissner effect
2]. More explicitly it has been conjectured that the re-
cted part of QCD which comes from the "Abelian pro-
ion" of the theory to its maximal Abelian subgroup is
onsible for the dynamics of the dual Meissner effect

dynamical mechanism for the confinement of color in QCD.

Consider a non-Abelian gauge theory of a given gauge group G:

$$\mathcal{L} = -\frac{1}{4}\vec{F}^2_{\mu\nu},$$

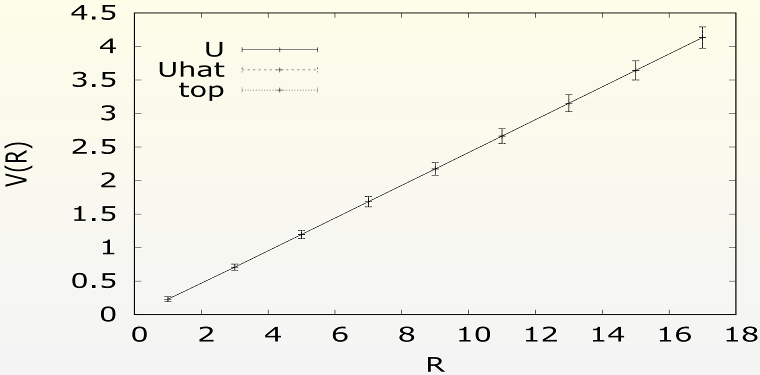


Figure: The Abelian dominance versus the monopole dominance in the lattice calculation. Here U, Uhat, top represent the full, restricted, and monopole potentials.

A. Savvidy Action of SU(2) QCD: A Review

- Savvidy first calculated the one-loop effective action of SU(2) QCD and obtained the Savvidy vacuum, integrating out gluons in a constant magnetic background.

- But the separation of the classical and quantum parts was ad hoc. More seriously, it had two critical defects:

  1. The Savvidy vacuum was unstable.
  2. It was not gauge invariant nor parity conserving.

Savvidy-Nielsen-Olesen Instability

Volume 71B, number 1

PHYSICS LETTERS

7 November 1977

INFRARED INSTABILITY OF THE VACUUM STATE OF GAUGE THEORIES AND ASYMPTOTIC FREEDOM

G.K. SAVVIDY

Yerevan Physics Institute

Yerevan, Armenian S.S.R., USSR

Received 12 April 1977

The radiative corrections to the Yang-Mills massless theory are found to lead to an instability of the vacuum state. This fact is in full compliance with the asymptotic freedom of gauge theories and is due to infrared singularities.

Gross, Wilczek and Politzer have found that non-Abelian gauge theories are "asymptotically free" [1]. Physical application of this observation encounters … singularities and quark confine-

The gauge invariance implies that the $\bar{\mathcal{L}}$ function depends only on invariants $\mathcal{F} = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$ and $\mathcal{G} = \frac{1}{4} G^a_{\mu\nu} G^{*a}_{\mu\nu}$ but does not depend on covariant derivatives G^a… The $\widetilde{\mathcal{L}}$ function depends on invariant

B. Gauge Invariant Effective Action of SU(2) QCD

- To remove these defects, we need the followings.

1. Treat RCD as the classical part and choose the monopole background

$$\hat{F}^{(b)}_{\mu\nu} = \bar{H}_{\mu\nu}\hat{n}, \quad \bar{H}_{\mu\nu} = H\delta^1_{[\mu}\delta^2_{\nu]}.$$

2. Integrate out the chromon pair $\vec{X}_\mu$ and $\vec{X}^{(c)}_\mu$ simultaneously on the same footing.

- Adopting the quantum gauge condition $\bar{D}_\mu \vec{X}_\mu = 0$ we have

$$\exp\left[iS_{eff}(\hat{A}_\mu)\right] \simeq \int \mathcal{D}\vec{X}_\mu \mathcal{D}\vec{X}_\mu^{(c)} \mathcal{D}\vec{c}\mathcal{D}\vec{c}^*$$
$$\exp\Big\{ -i\int \big[\frac{1}{4}\hat{F}_{\mu\nu}^2 + \frac{1}{4}(\hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu)^2 + \frac{g}{2}\hat{F}_{\mu\nu}\cdot(\vec{X}_\mu \times \vec{X}_\nu)$$
$$+\vec{c}^* \bar{D}_\mu D_\mu \vec{c} + \frac{1}{2\xi}(\bar{D}_\mu \vec{X}_\mu)^2\big] d^4x \Big\},$$

where $\vec{c}$ and $\vec{c}^*$ are the ghost fields.

- Under the color reflection the eigenvalues of the chromon functional determinant change to

$$2gH(n+\frac{1}{2}\mp S_3)+k^2 \rightarrow 2gH(n+\frac{1}{2}\pm S_3)+k^2.$$

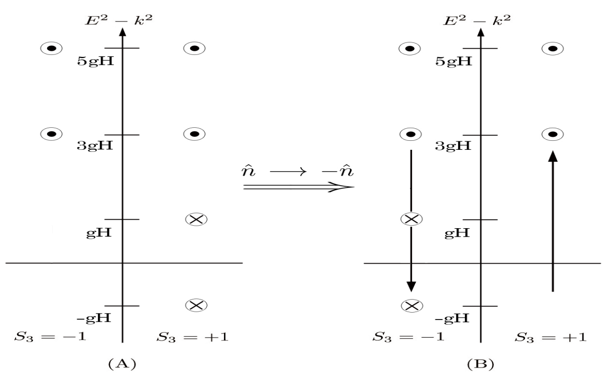


Figure: The gauge invariant eigenvalues of the chromon functional determinant. Notice that the C-projection excludes the lowest two eigenmodes, in particular the tachyonic modes.

- So we must make the C-projection to exclude the lowest two eigenmodes, in particular the tachyonic mode. With this we have

$$\ln \mathrm{Det}^{1/2} K = \ln \mathrm{Det} \left[(-\hat{D}^2 + 2gH)(-\hat{D}^2 + 2gH)\right],$$

$$\Delta\mathcal{L} = \frac{1}{16\pi^2}\int_0^\infty \frac{dt}{t^{3-\epsilon}} \frac{gHt/\mu^2}{\sinh(gHt/\mu^2)} \times \Big[\exp(-2gHt/\mu^2) + \exp(-2gHt/\mu^2) - 1\Big].$$

- Just like the GSO-projection which removes tachyons in string theory, the C-projection removes the tachyonic modes and restores the stability of the monopole condensation in QCD.

No Infra-red Divergence!

Perturbative Calculation of Stability: Schwinger's Method

- The imaginary part of the SNO effective potential is of the order of g^2, and there is a well-known Schwinger's method tested in QED to calculate the effective action perturbatively to this order.

- Using the Schwinger's method we have (up to the order g^2)

$$\Delta S = -\frac{g^2}{4\pi^2} \int d^4p G_{\mu\nu}(p) G_{\mu\nu}(-p) \Big[\int_0^1 dv \frac{v^2(1 - v^2/12)}{1 - v^2} + C \Big],$$

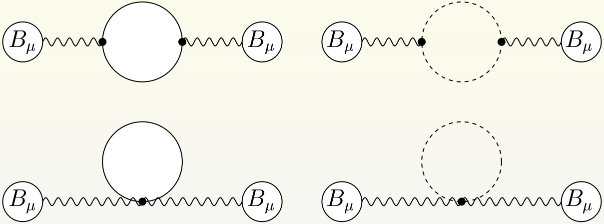
where C is a regularization-dependent constant. Notice that the imaginary part of ΔS can only come from the pole at $v = 1$.

- Perform the integral and find

$$\text{Im}\ \mathcal{L}_{eff} = \begin{cases} 0, & E = 0 \\ -\dfrac{11E^2}{96\pi}. & H = 0 \end{cases}$$

This confirms that the monopole condensation is indeed stable.

- One can also calculate the imaginary part directly with Feynman diagrams. There are four Feynman diagrams which contribute to the effective action to this order, and a straightforward calculation reproduces the same result.



- Figure: Feynman diagrams that contribute to the effective action at g^2 order. Solid and dotted loops represent the chromon and ghost loops, and B_μ are the background field.

C. Monopole Condensation and Asymptotic Freedom

- With the C-projection we have the effective potential which has the non-trivoal minimum

$$V = \frac{H^2}{2}\Big[1 + \frac{11g^2}{24\pi^2}\big(\ln\frac{gH}{\mu^2} - c\big)\Big].$$

- Define the running coupling $\bar{g}$ by $\frac{\partial^2 V}{\partial H^2}\Big|_{H=\bar{\mu}^2} = \frac{g^2}{\bar{g}^2}$ and find

$$\frac{1}{\bar{g}^2} = \frac{1}{g^2} + \frac{11}{24\pi^2}\big(\ln\frac{\bar{\mu}^2}{\mu^2} - c + \frac{3}{2}\big), \quad \beta(\bar{\mu}) = \bar{\mu}\frac{\partial\bar{g}}{\partial\bar{\mu}} = -\frac{11\bar{g}^3}{24\pi^2}.$$

Asymptotic (Ultra-violet) Freedom

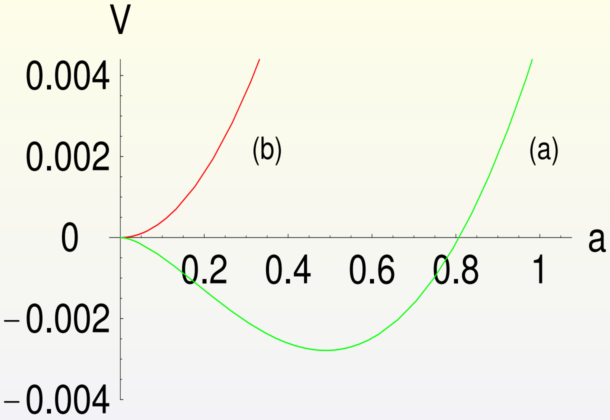


Figure: The one-loop effective potential of SU(2) QCD. Here (a) and (b) represent the effective potential and the classical potential.

- In general for arbitray constant electromagnetic background $\bar{H}_{\mu\nu}$ we find

$$\mathcal{L}_{eff} = \begin{cases} -\dfrac{H^2}{2} - \dfrac{11g^2H^2}{48\pi^2}\Big(\ln\dfrac{gH}{\mu^2} - c\Big), & E = 0 \\ -\dfrac{E^2}{2} + \dfrac{11g^2E^2}{48\pi^2}\Big(\ln\dfrac{gE}{\mu^2} - c\Big) - i\dfrac{11g^2E^2}{96\pi}, & H = 0 \end{cases}$$

$$c = 1 - \ln\, 2 - \frac{24}{11}\zeta'(-1, \frac{3}{2}) = 0.94556...$$

- The negative imaginary part in the chromo-electric background tells that the chromo-electric field annihilates the chromon pairs. This is the origin of the asymptotic freedom.

D. Effective Potential of SU(3) QCD

- With the Weyl symmetry of SU(3) QCD we have

$$\exp\left[iS_{eff}(\hat{A}_\mu)\right] \simeq \sum_p \int \mathcal{D}\vec{W}_\mu^p \mathcal{D}\vec{W}_\mu^{(c)p} \mathcal{D}\vec{c}^p \mathcal{D}\vec{c}^{*p}$$

$$\exp\Big\{-i\int\Big[\frac{1}{6}(\hat{F}_{\mu\nu}^p)^2 + \frac{1}{4}(\hat{D}_\mu^p \vec{W}_\nu^p - \hat{D}_\nu^p \vec{W}_\mu^p)^2 + \frac{g}{2}\hat{F}_{\mu\nu}^p \cdot (\vec{W}_\mu^p \times \vec{W}_\nu^p)$$

$$-\vec{c}^{*p}\bar{D}_\mu^p D_\mu^p \vec{c}^p - \frac{1}{2\xi}(\bar{D}_\mu^p \vec{W}_\mu^p)^2\Big] d^4x\Big\},$$

at one-loop level. This allows us to calculate the effective action of SU(3) QCD from that of SU(2) QCD.

- So we have the Weyl symmetric SU(3) QCD effective Lagrangian

$$\mathcal{L}_{eff} = \begin{cases} -\sum_p \Big(\frac{H_p^2}{3} + \frac{11g^2 H_p^2}{48\pi^2}(\ln \frac{gH_p}{\mu^2} - c)\Big), & (E_p = 0) \\ \sum_p \Big(\frac{E_p^2}{3} + \frac{11g^2 E_p^2}{48\pi^2}(\ln \frac{gE_p}{\mu^2} - c) \\ \qquad - i\frac{11g^2}{96\pi} E_p^2\Big). & (H_p = 0) \end{cases}$$

- This assures that the essential features of SU(2) QCD remains the same. In particular, this tells that the chromo-electric field makes the pair annihilation of chromon.

- The effective potential for the monopole background is given by

$$V = \frac{3}{4}\sum_p H_p^2 + \frac{11g^2}{48\pi^2}\sum_p H_p^2 \ln\left(\frac{gH_p}{\mu^2} - c\right).$$

- Define the renormalized coupling $\bar{g}$ by

$$\forall_p \quad \frac{\partial^2 V}{\partial H_p^2}\Big|_{H_1=H_2=H_3=\bar{\mu}^2} = \frac{g^2}{\bar{g}^2},$$

and find

$$\frac{1}{\bar{g}^2} = \frac{1}{g^2} + \frac{11}{16\pi^2}\left(\ln\frac{\bar{\mu}^2}{\mu^2} - c + \frac{5}{4}\right), \quad \beta(\bar{\mu}) = \bar{\mu}\frac{\partial \bar{g}}{\partial \bar{\mu}} = -\frac{11\bar{g}^3}{16\pi^2}.$$

Asymptotic Freedom!

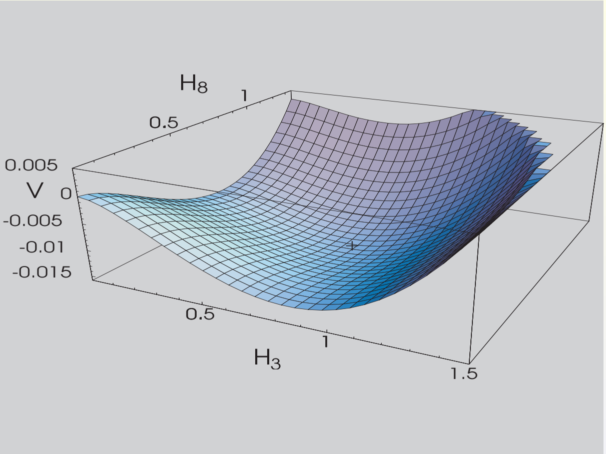


Figure: The effective potential with $\cos\theta = 0$, which has a unique minimum at $H = H' = H_0$ (or $H_1 = H_2 = H_3 = H_0$). Notice that H and H' are orthogonal in space.

Eur. Phys. J. C (2019) 79:498
https://doi.org/10.1140/epjc/s10052-019-7023-x

Regular Article - Theoretical Physics

THE EUROPEAN PHYSICAL JOURNAL C

Abelian decomposition and Weyl symmetric effective action of SU(3) QCD

Y. M. Cho[1,2,a], **Franklin H. Cho**[3]

[1] Center for Quantum Spacetime, Sogang University, Seoul 04107, Korea
[2] School of Physics and Astronomy, Seoul National University, Seoul 08826, Korea
[3] Department of Physics and Astronomy, Institute for Quantum Computing, University of Waterloo, Waterloo, ON N2L 3G1, Canada

Received: 7 May 2019 / Accepted: 1 June 2019

Abstract We show how to calculate the effective potential of SU(3) QCD which tells that the true minimum is given by the monopole condensation. To do this we make the gauge independent Weyl symmetric Abelian decomposition of the SU(3) QCD which decomposes the gluons to the color neutral neurons and the colored chromons. In the perturbative regime this decomposes the Feynman diagram in such a way that the conservation of color is explicit. Moreover, this shows the

ons which are destined to be confined. Since the confined prisoner can not be the confining agent (the jailer), only the Abelian part can play the role of the confiner.

In fact we can prove this Abelian dominance rigorously. Theoretically we can show that the contribution of the non-Abelian part in the area law of the Wilson loop integral is negligible [9]. Moreover, numerically we can confirm this in the lattice QCD [10–15].

A. Two Types of Gluon Jets

- The Abelian decomposition is not just a theoretical proposition. It can be verified by experiment.
- The neurons and chromons have different color quantum number and color factor, so that they must behave differently.
- This means that QCD must have two types of gluon jets, the neuron jet and the chromon jet.

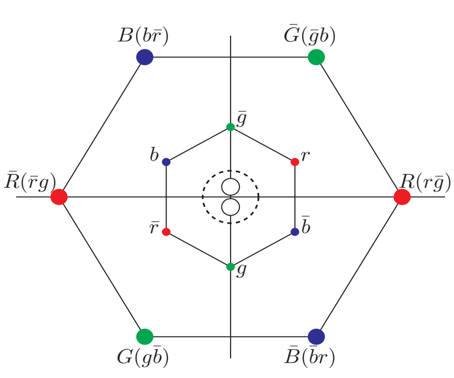


Figure: Separation of neurons and chromons from the gluon octet. The neurons are shown by two circles at the center, and the chromons are represented by the outer sextet. The inner sextet represents the quarks.

- Experimental confirmation of the gluon jet has assured that QCD is the right theory of strong interaction.
- Jets are produced in two steps, parton shower and hadronization of the hard partons (neuron, chromon, and quark). The hadronization is basically the same for all jets.
- The neuron jet and chromon jet have different parton showers and different color factors.

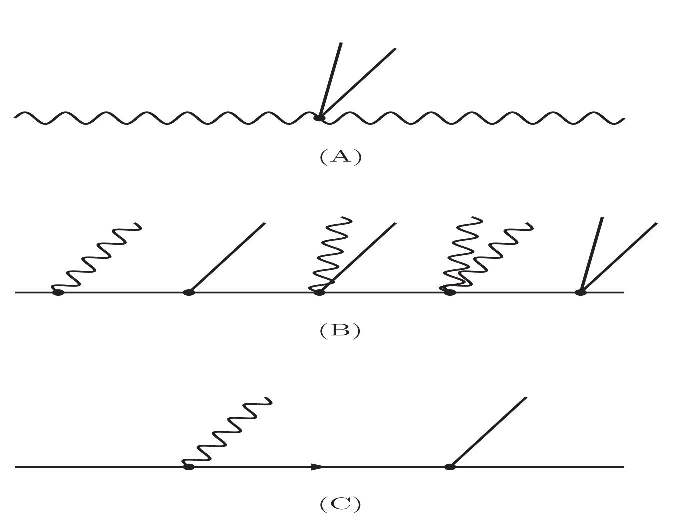


Figure: The parton shower of neuron, chromon, and quark. The chromon and quark showers are of $O(g)$, but the neuron shower is of $O(g^2)$. Moreover, the neuron has only one type of parton shower, but the chromon has five.

- The parton shower tells that the neuron jet has sharp jet shape and less particle multiplicity, but the chromon jet has broad jet shape and more particle multiplicity.
- So the particles from the neuron jets in general have more energy than those from the chromon jets.
- Moreover, the neuron jet has an ideal color dipole pattern, but the chromon jet has a distorted color dipole pattern.
- The fact that the color factors of quark, neuron, and chromon are given by $C_q : C_n : C_c \simeq 1 : 0.56 : 1.69$ supports these predictions.

PHYSICAL REVIEW D **107**, 054024 (2023)

Experimental verification of two types of gluon jets in QCD

Y. M. Cho,[1,2,*] Pengming Zhang,[3,†] and Li-Ping Zou[4,‡]

[1]*School of Physics and Astronomy, Seoul National University, Seoul 08826, Korea*
[2]*Center for Quantum Spacetime, Sogang University, Seoul 04107, Korea*
[3]*School of Physics and Astronomy, Sun Yat-Sen University, Zhuhai 519082, China*
[4]*Sino-French Institute of Nuclear Engineering and Technology, Sun Yat-Sen University, Zhuhai 519082, China*

(Received 25 December 2019; accepted 21 December 2022; published 17 March 2023)

The Abelian decomposition of QCD tells that there are two types of gluons, the color neutral neurons and colored chromons. We propose to confirm the Abelian decomposition testing the existence of two types of gluon jets experimentally. We predict that one quarter of the gluon jet is made of the neurons which has the color factor 3/4 and the sharpest jet radius and smallest charged particle multiplicity, while the three quarters of the gluon jet are made of the chromons with the color factor 9/4, which have the broadest jet radius (broader than the quark jet). Moreover, we argue that the neuron jet has a distinct color flow which forms an ideal color dipole, while the quark and chromon jets have distorted dipole pattern. To test the plausibility of this proposal, we suggest to analyze the gluon distribution against the jet shape (the sphericity) and/or particle multiplicity from the existing gluon jet events and look for two distinct peaks in distribution.

Abelian dec

- We can re-analize the existing ALEPH and CMS jet data, and confirm the existence of two types of gluon jets experimentally.

- Consider the old ALEPH gluon jet data coming from

$$e\bar{e} \to Z \to b\bar{b}g.$$

  The gluon jet has two anormalies, the one in energy fragmantation of particles and the other in particle multiplicity.

- These anormalies could be viewed as the evidences of the two types of gluon jets.

ALEPH 2-jet vs 3-jet events

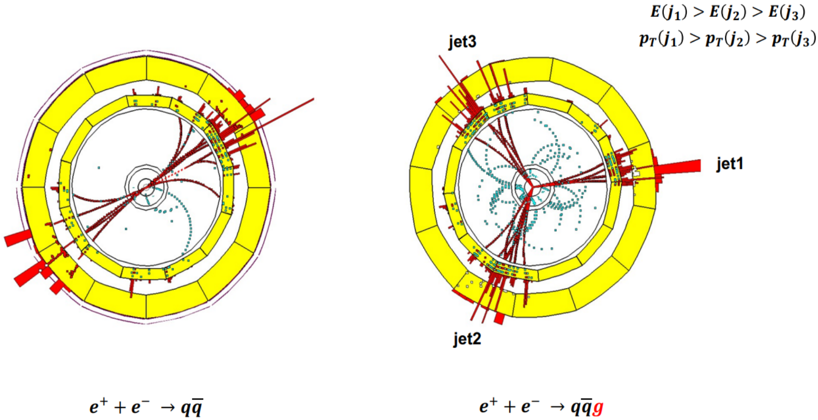


Figure: The ALEPH 3 jets event which confirmed the existence of quark and gluon in QCD.

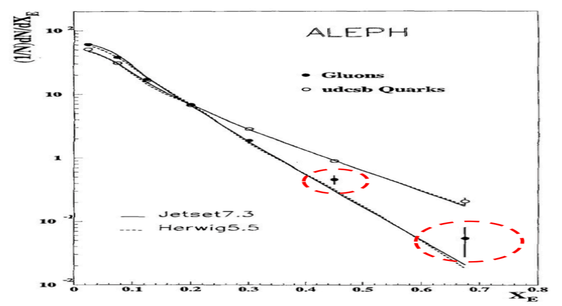


Figure: The energy fragmantation of the particles in the gluon jet in the ALEPH data which shows the excess of more energetic particles.

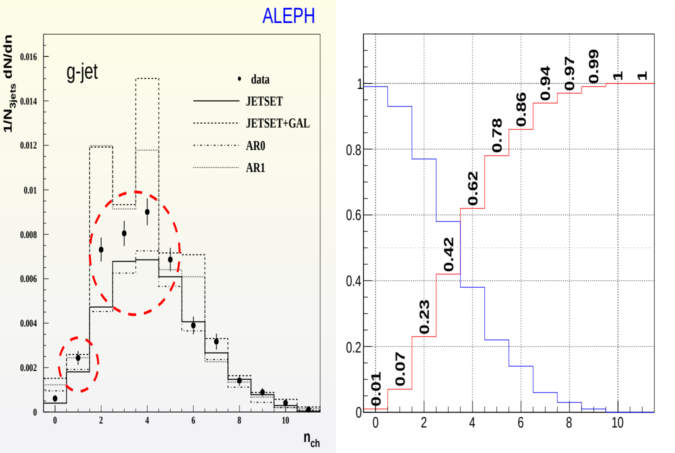


Figure: The asymmetry of the gluon distribution against the number of the charged particles at ALEPH, which shows 20% more jet events for less charged particle numbers.

- These anormalies are precisely what we have predicted based on the existence of the neuron jet.
- The two anormalies actually come from the same origin, the neuron jet. This means that the two anormalies must be correlated.
- Remarkably, we can confirm this correlation from the same ALEPH gluon jet data. This provides a crucial piece of infromation which strongly indicates the existence of the neuron jet.

Correlation!

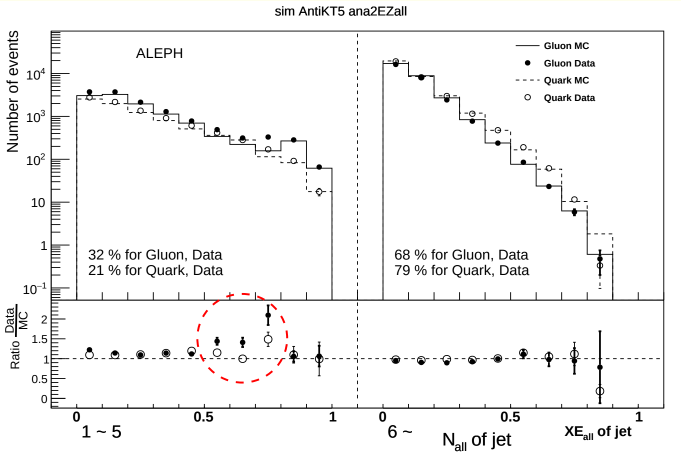


Figure: The correlation of the gluon jets between the charged particle multiplicity and the energy fragmantation of particles at ALEPH. This shows that 32 % of the events below $N = 6$ expain the anormaly in the energy fragmantation.

- There are two more circumferential evidences of the neuron jet comes from unexpected quarters, the DELPHI gluon jet from $e\bar{e} \rightarrow Z \rightarrow b\bar{b}g$ and the gluon jets from the CMS heavy ion collision.

- The DELPHI separated the quark jets with purity about 90 % and the gluon enriched jets with purity about 70 %, and reported an excess of the neutral particles in the gluon jet with $E \leq 2$ GeV.

- DELPHI: "an indication that the gluon jet might have an additional hithertoo undetected fragmantation mode via a two-gluon system...as predicted by QCD".

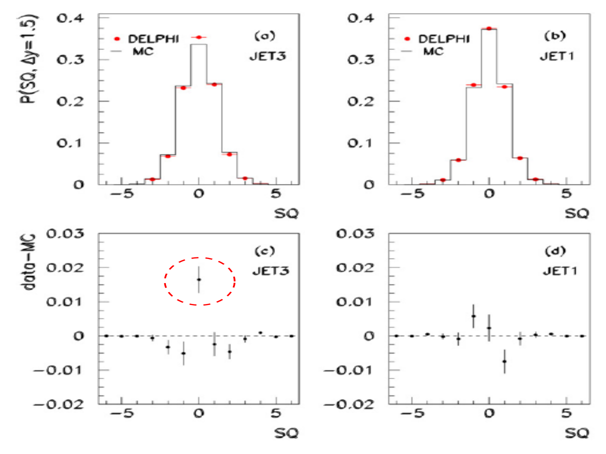


Figure: The DELPHI experiment which shows a clear excess of the neutral particles in gluon jet in (a) and (c), which is absent in the quark jet in (b) and (d).

- This, however, is precisely a characteristic feature of the neuron jet that we predicted, which could most likely produce the chromoballs made of the chromon-antichromon pair.

- So our prediction is not only in line with the DELPHI interpretation, but more importantly provides a theoretical justification of this interpretation.

- The gluon jets from the heavy ion collision are expected to undergo the "quenching" when they go through the quark-gluon plasma, so that there should be less gluon jets in heavy ion collosion than in p-p collision.

- Experimentally, however, the gluon jets in the CMS Pb-Pb collision looks almost identical to those coming from the p-p collision, so that there is very little quenching of the gluon jets in heavy ion collision. This puzzle could be explained by the existence of the neuron jet.

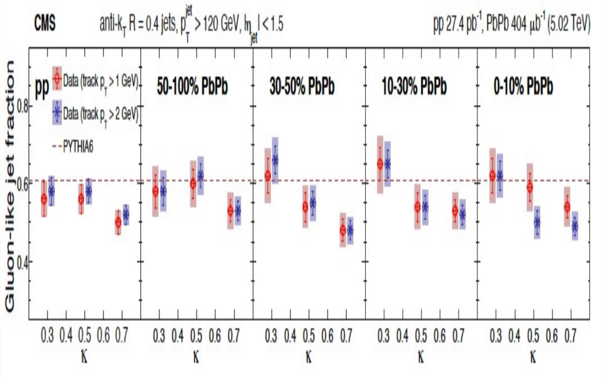


Figure: The CMS gluon jet data in Pb-Pb heavy ion collision compared to the gluon jets in p-p collision, which shows that the expected gluon quenching in Pb-Pb collision is hardly detectable.

No New Experiment!

B. Quark and Chromon Model: Chromoballs and Mixed States

- The Abelian decomposition replaces the quark and gluon model to the quark and chromon model. This provides a clear picture of glueballs and their mixing with quarkoniums.

- The model predicts the chromoballs made of chromons. But experimentally, there are not so many candidates of chromoballs.

- There are two reasons for this. Unlike the quarks the chromoballs have intrinsic instability, and often mix with quarkoniums. This makes the identification complicated.

- Nevertheless we can make a systematic mixing analysis of chromoball-quarkonium in 0^{++}, 2^{++}, and 0^{-+} sectors below 2 GeV.
- The result shows that $f_0(1500)$ in the 0^{++} sector, $f_2(1950)$ in the 2^{++} sector, and $\eta(1405)$ and $\eta(1475)$ in the 0^{-+} sector could be identified as predominantly the glueball states.
- The quark and chromon model also predicts the hybrid hadrons made of chromons and quarks, which could be verified experimentally.

C. Monoball: Vacuum Fluctuation of Monopole Condensation

- The monopole condensation could generate the quantum fluctuation. This suggests the existence of the monoball, the 0^{++} vacuum fluctuation mode. A possible candidate is $f_0(500)$.

- Unlike all other hadrons, it originates from the QCD vacuum. This makes the experimental verification of the monoball an important issue in QCD.

Summary

- The experimental discovery of the gluon jet and the proof of the asymptotic freedom was a big step for us to understand QCD.
- The experimental confirmation of two types of gluons predicted by the Abelian decomposition will be another giant step for QCD.
- There are five circumferential evidences of the existence of two types of gluons.
- If confirmed, this could be a most important discovery in QCD which could extend the horizon of our understanding of nature one step further.

- References

1. G. Savvidy, PLB 71, 133 (1977); N. Nielsen and P. Olesen, NPB 144, 485 (1978).
2. Y.M. Cho, PRD 21, 1080 (1980); PRL 46, 302 (1981); PRD 23, 2415 (1981); See also Y.S. Duan and M.L. Ge, SS 11, 1072 (1979).
3. Y.M. Cho, M. Walker, and D.G. Pak, JHEP 05, 073 (2004); Y.M. Cho and D.G. Pak, PLB 632, 745 (2006).
4. Y.M. Cho, Franklin H. Cho, and J. H. Yoon, PRD 87, 085025 (2013).
5. Pengming Zhang, Liping Zou, and Y.M. Cho, PRD 98, 096015 (2018).
6. Y.M. Cho and Franklin H. Cho, EPJC 79, 498 (2019).
7. Y.M. Cho, Pengming Zhang, and Liping Zou, PRD 107, 054024 (2023).
8. Y.M. Cho, arXiv:2401.08704 [hep-ph], to be published.