Caustics in Self-gravitating N-body and Large Scale Structure of Universe

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Caustics in self-gravitating N-body systems
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large scale structure of universe

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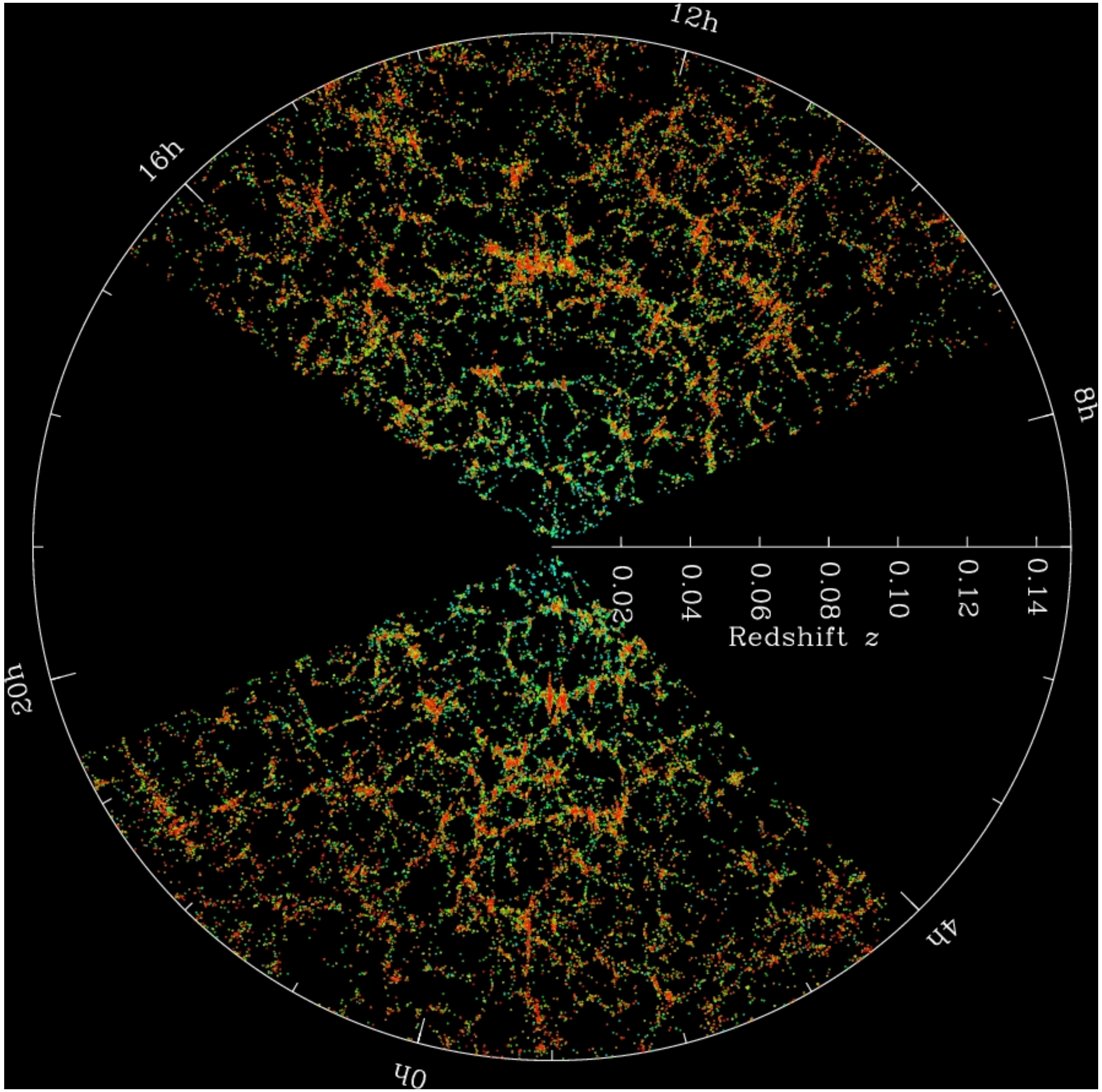
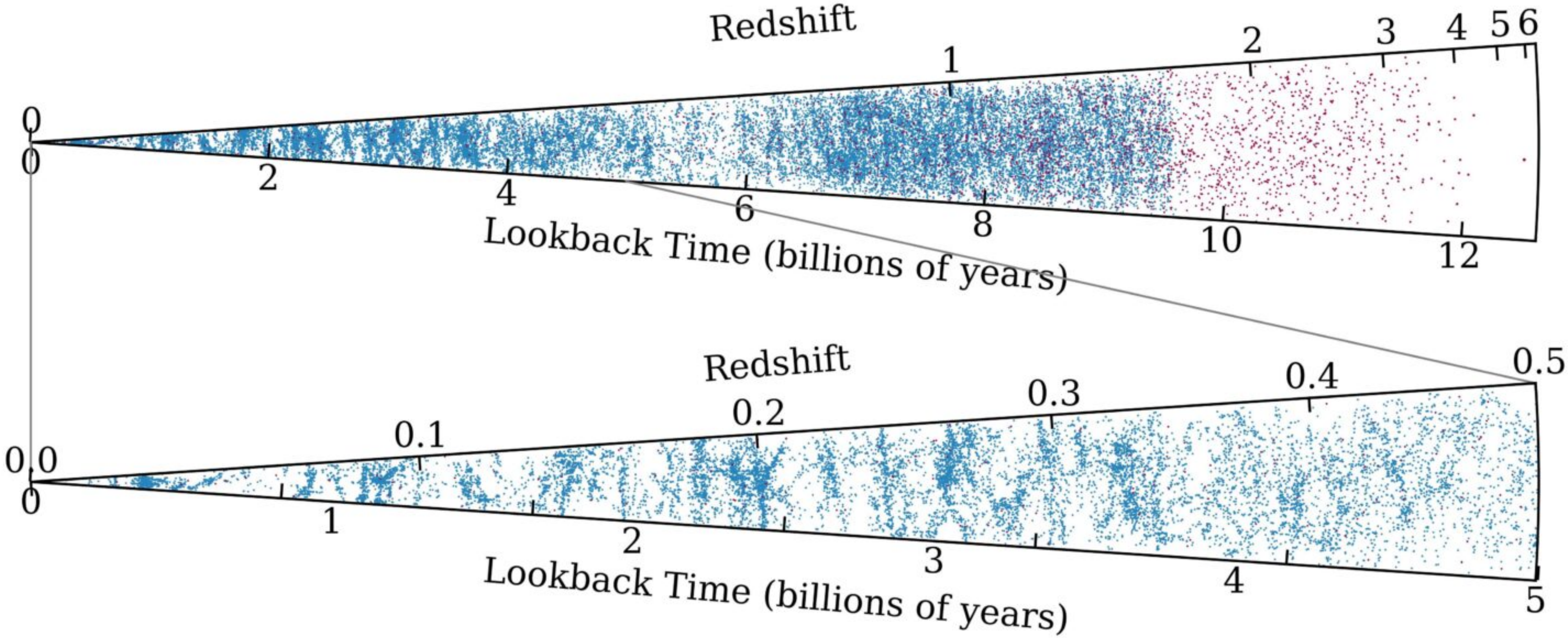


This figure shows galaxies discovered by the Sloan Digital Sky Survey (SDSS). Galaxy filaments forming the cosmic web consist of walls of gravitationally bound galactic superclusters that can be seen by eye. The figure shows galaxies up to around 2 billion light-years away (z=0.14). Figure Credit: M. Blanton and SDSS



The space-time distribution of galaxies as a function of redshift. This DESI data has the Earth on the left and looks back in time to the right. Every dot represents a galaxy (blue) or quasar (red). The upper wedge includes objects all the way back to about 12 billion years ago. The bottom wedge zooms in on the closer galaxies in more detail. The clumps, strands, and blank spots are real structures in the Universe showing how galaxies group together or leave voids on gigantic scales. Figure Credit: Eleanor Downing/ DESI collaboration

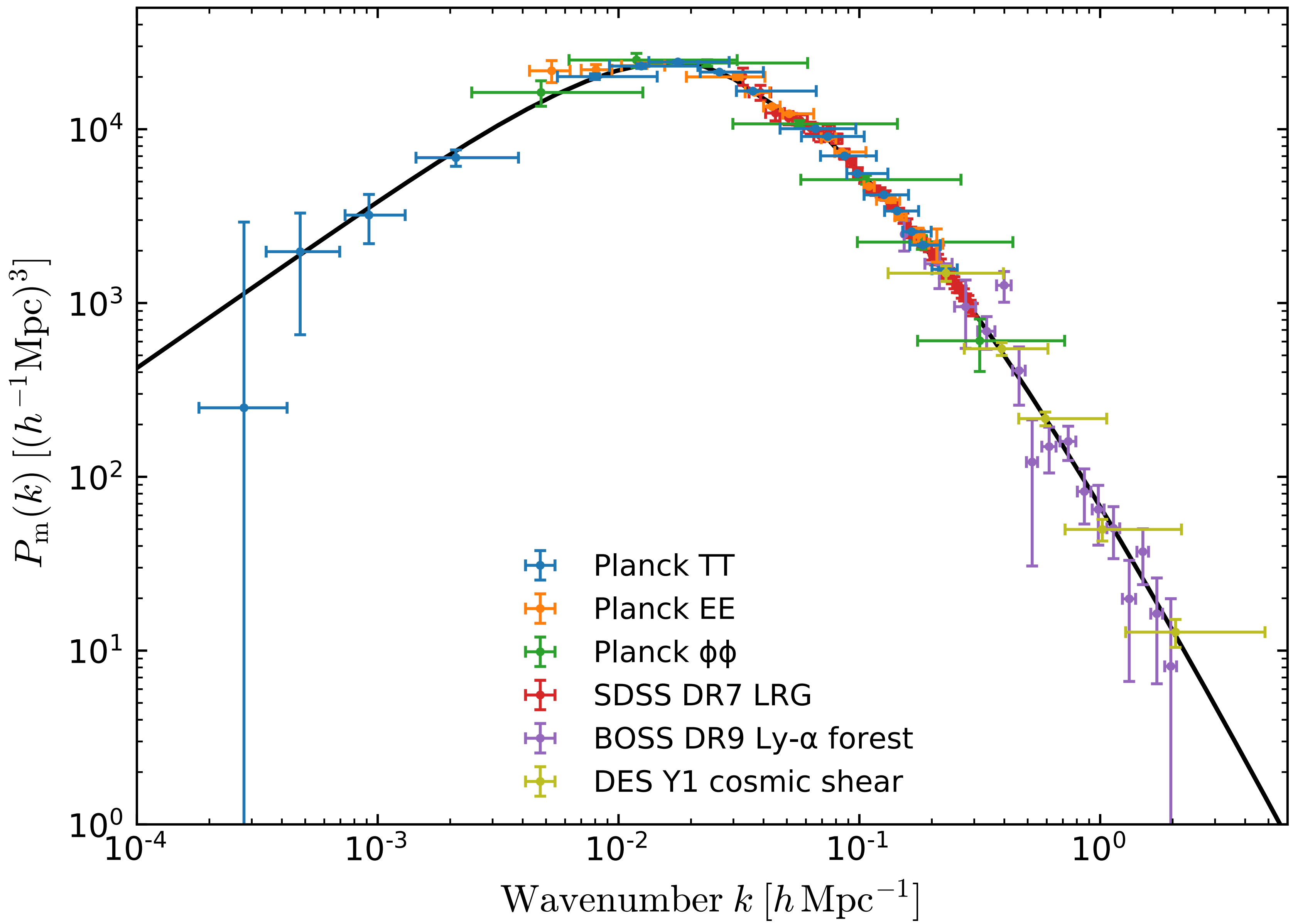


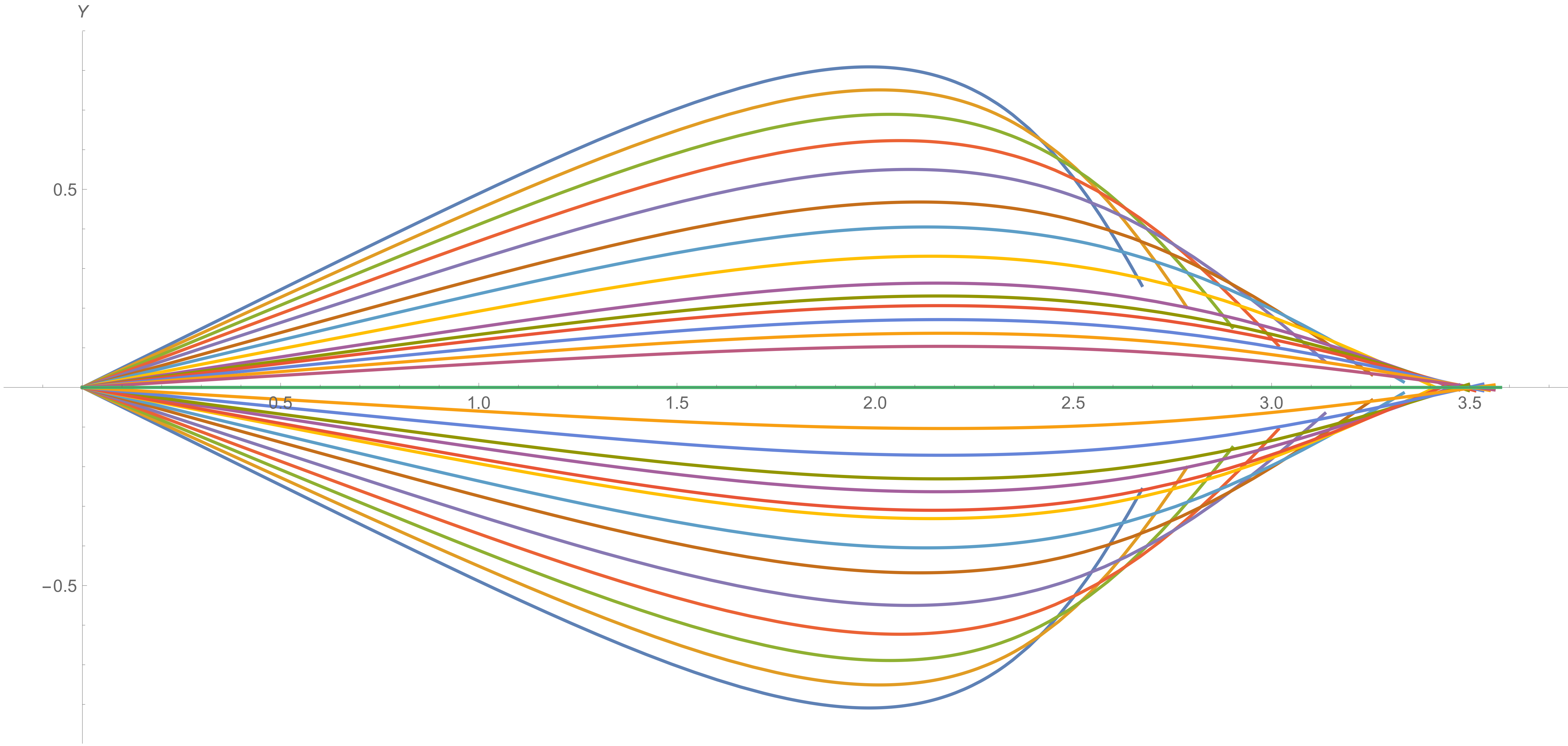
Figure 3: The matter power spectrum (at z = 0) inferred from different cosmological probes showing how CMB, LSS, clusters, weak lensing, and Ly-α forest all constrain matter power spectrum P(k). The spectrum measures the power of matter fluctuations on a given scale k. For the long wave length perturbations it has power-law behaviour $P(k) \propto k^{n_S}$ with the scalar spectral index $n_S = 0.967 \pm 0.004$, tilted away from the scale invariant $n_S = 1$ Harisson- Zeldovich spectrum. The sound waves diminish the strength of small scale fluctuations, and power spectrum tends to fall as $P(k) \propto k^{-3}$ for $k \geq 2 \times 10^{-2}$[h Mps^{-1}].

Galaxies are not distributed uniformly in space and time, as it can be seen in Fig. 1 and Fig. 2 representing the data of the Sloan Digital Sky Survey and of the Dark Energy Spectroscopic Instrument collaboration. *Extended galaxy redshift surveys revealed that at a large-scale the Universe consists of matter concentrations in the form of galaxies and clusters of galaxies of Mpc scale, as well as filaments of galaxies that are larger than 10 Mpc in length and vast regions devoid of galaxies*. The James Webb ST telescope and the Euclid mission will observe the first stars and galaxies that formed in the Universe from the epoch of recombination to the present day. The Large Scale Structure (LSS) of the Universe is this pattern of galaxies that provides information about the spectrum of matter density fluctuations shown in Fig. 3

The prevailing theoretical paradigm regarding the existence of LSS is that the initial density fluctuations of the early Universe seen as *temperature deviations* in the Cosmic Microwave Background (CMB) grow through *gravitational instability into the structure seen today in the galaxy density field*. The best constraints on the matter density fluctuations come from the study of the CMB *temperature fluctuations generated at the epoch of the last scattering of the radiation*. *The LSS of galaxies provides independent measurements of density fluctuations of similar physical scale, but at the late epoch*. The combination of CMB measurements with measurements of LSS provide independent probes of the matter power spectrum in complementary regions shown in Fig.3.

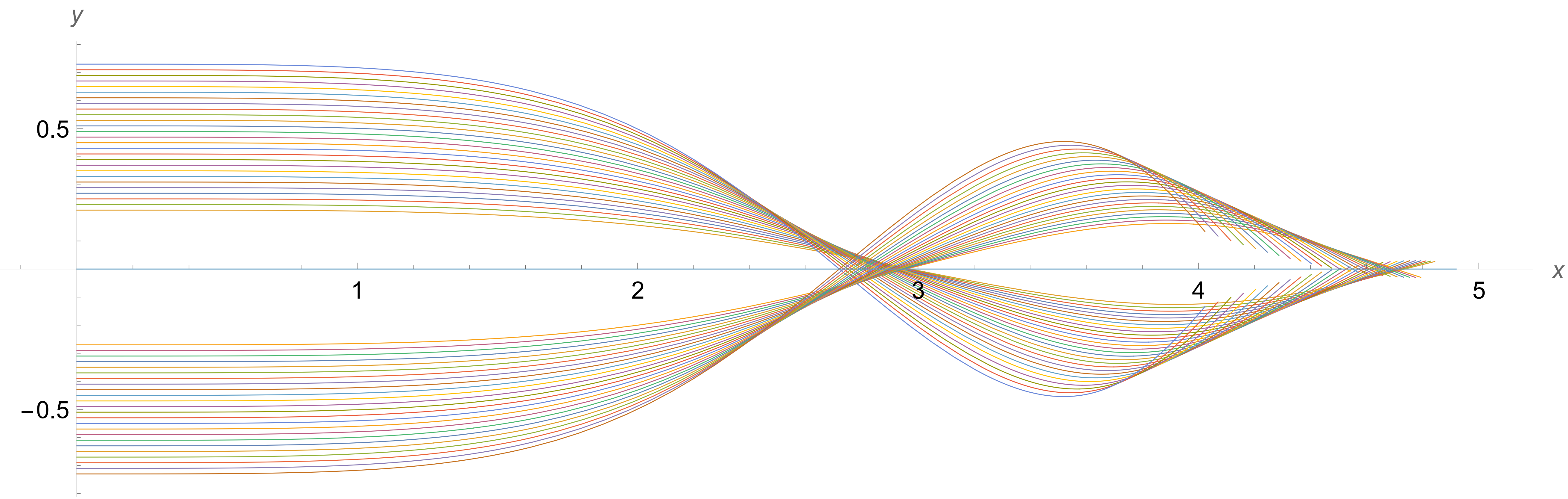
Light caustics on a seabed

Caustics in Yang Mills Classical Mechanics



Yang-Mills mechanical system $\mathcal{L}_{YM} = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}x^2y^2$

Caustics in Yang Mills Classical Mechanics



Yang-Mills mechanical system $\mathcal{L}_{YM} = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}x^2y^2$

Geometrisation of Self-Gravitating N-body System

Let us consider a system of N massive particles with masses M_α and the coordinates

$$q^\alpha(s) = (M_1^{1/2}\vec{r}_1, ..., M_N^{1/2}\vec{r}_N), \quad \alpha = 1, ..., 3N, \tag{2.6}$$

that are defined on a Riemannian coordinate manifold $q^\alpha(s) \in Q^{3N}$ and have the velocity vector

$$u^\alpha(s) = \frac{dq^\alpha}{ds}, \tag{2.7}$$

$$ds^2 = g_{\alpha\beta} dq^\alpha dq^\beta, \qquad g_{\alpha\beta} = \delta_{\alpha\beta}(E - U(q)) = \delta_{\alpha\beta} W(q),$$

geodesic trajectories on the Riemannian manifold Q^{3N} are defined by the following equation:

$$\frac{d^2 q^\alpha}{ds^2} + \Gamma^\alpha_{\beta\gamma} \frac{dq^\beta}{ds}\frac{dq^\gamma}{ds} = 0, \tag{2.10}$$

In terms of the coordinate system ($\boxed{2.6}$) introduced above ($\vec{r}_a$, $a = 1, ..., N$) this equation reduces to the Euler-Lagrangian equation for massive particles interacting though the potential function $U(\vec{r}_1, ..., \vec{r}_N)$:

$$M_a \frac{d^2 \vec{r}_a}{dt^2} = -\frac{\partial U}{\partial \vec{r}_a}, \qquad a = 1, ..., N. \tag{2.14}$$

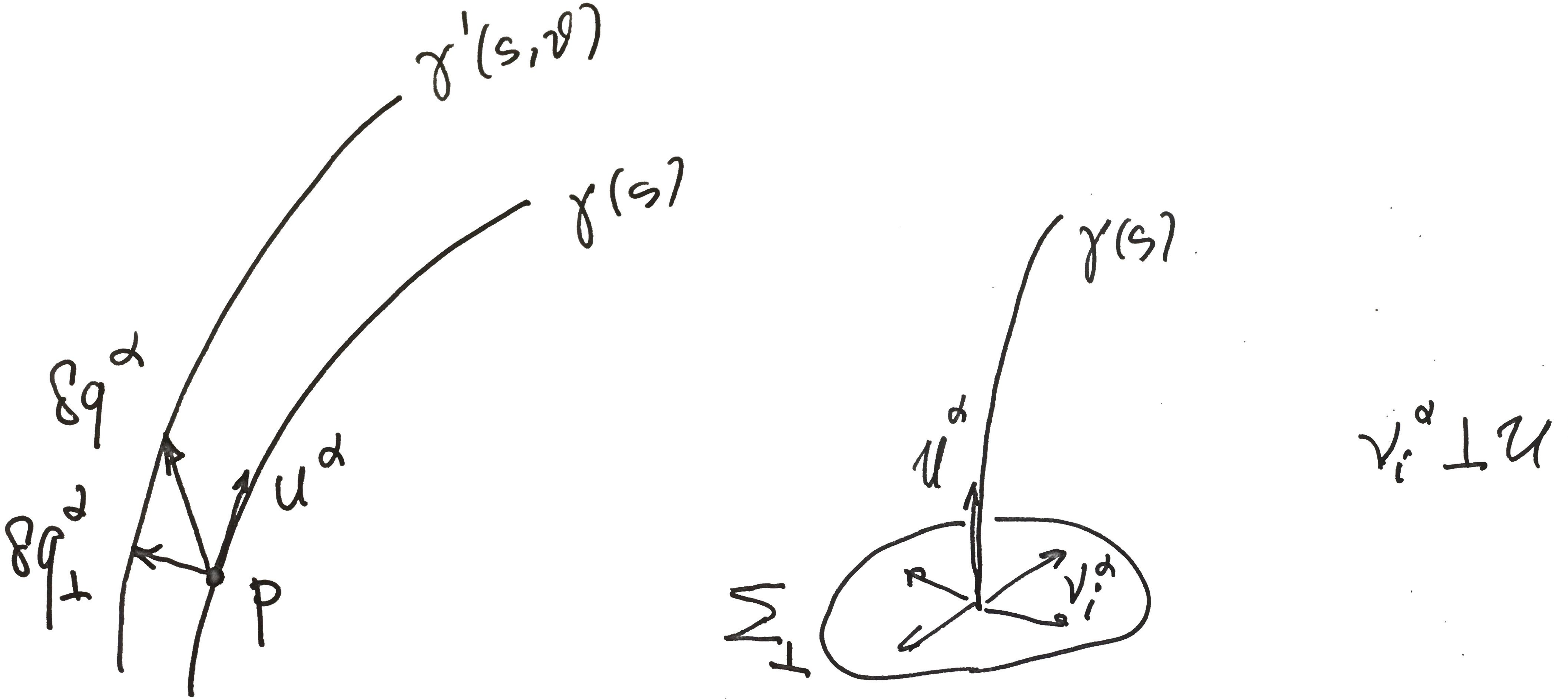
Geometrisation of Self-Gravitating N-body System

$$ds^2 = g_{\alpha\beta} dq^\alpha dq^\beta, \qquad g_{\alpha\beta} = \delta_{\alpha\beta}(E - U(q)) = \delta_{\alpha\beta} W(q),$$

$$U = -G \sum_{a<b} \frac{M_a M_b}{r_{ab}}, \qquad r^i_{ab} = r^i_a - r^i_b \qquad i = 1, 2, 3 \qquad a, b = 1, ..., N$$

$$\frac{d^2 q^\alpha}{dt^2} = -\frac{\partial U}{\partial q^\alpha}$$

the benefit of using q coordinates !



The vector field δq^α defined along the geodesic $\gamma(s)$ and satisfying the above equations is called a *Jacobi field*. The equation can be written also in an alternative first-order form:

$$\frac{D\delta q^\alpha}{ds} = \delta u^\alpha, \qquad \frac{D\delta u^\alpha}{ds} = -R^\alpha_{\beta\gamma\sigma} u^\beta \delta q^\gamma u^\sigma. \tag{3.27}$$

$$\frac{d}{ds}|\delta q_\perp|^2 = 2\delta q_\perp^\alpha u_{\alpha;\beta}\delta q_\perp^\beta$$

$$\frac{d^2}{ds^2}|\delta q_\perp|^2 = -2K(q,u,\delta q_\perp)|\delta q_\perp|^2 + 2|\delta u_\perp|^2.$$

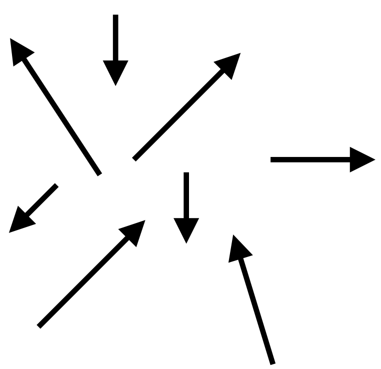
$$K(q,u,\delta q_\perp) = \frac{R_{\alpha\beta\gamma\sigma}\delta q_\perp^\alpha u^\beta \delta q_\perp^\gamma u^\sigma}{|\delta q_\perp|^2}$$

The sign of the sectional curvature defines the stability of geodesic trajectories in different parts of the phase space (q,u)

In the regions where the sectional curvature is negative the trajectories of particles are unstable, are exponentially diverging, and the self-gravitating system is in a phase of deterministic chaos. In the regions where the sectional curvature is positive the trajectories are stable, exhibit geodesic focusing, generating caustics.

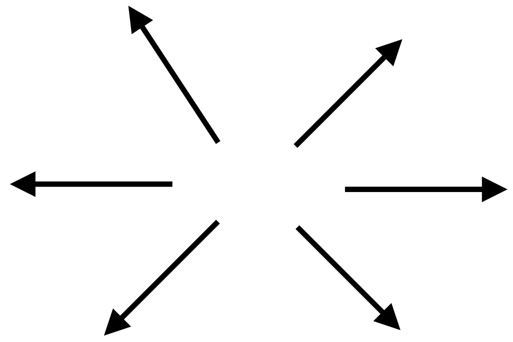
A self-gravitating N-body system can be assigned to these distinguished regions of the phase space depending on the initial distribution of particles velocities and quadrupole momentum of the system.

Particle distribution in the phase space and the corresponding sign of the sectional curvature



$$K(q,u,\delta q_\perp) < 0$$

chaotic behaviour



$$K(q,u,\delta q_\perp) > 0$$

focusing behaviour that generating caustics

$$\tau_{collective} = \sqrt{\frac{W}{(\nabla W)^2}} = \gamma \frac{\langle v^2 \rangle^{1/2}}{\pi G M n^{2/3}},$$

$$\tau_{galaxies} \simeq 6.14 \times 10^9 \left(\frac{\langle v^2 \rangle^{1/2}}{100 \frac{km}{s}} \right) \left(\frac{1pc^{-3}}{n} \right)^{2/3} \left(\frac{M_\odot}{M} \right) \, years.$$

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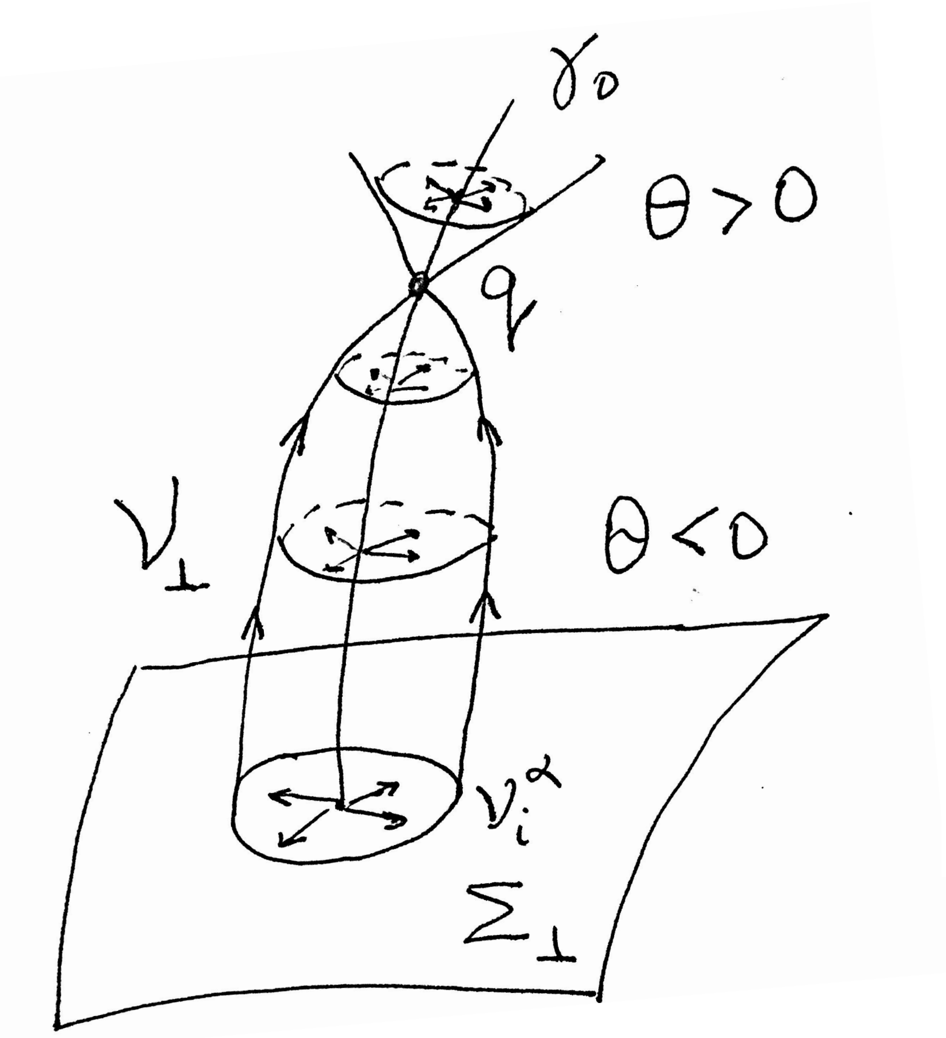
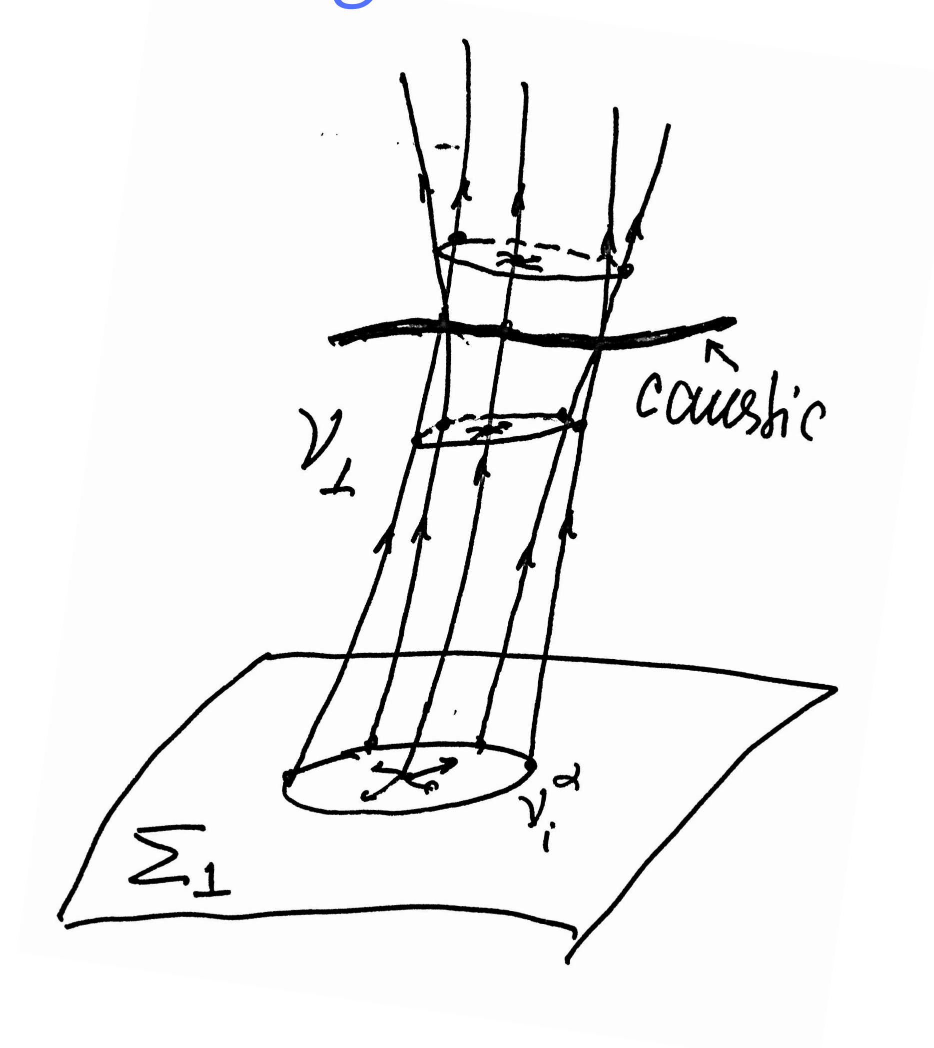
160 (1986) 203--210

Raychaudhuri Equation and focusing



$$\theta = \frac{d \ln \mathcal{V}}{ds}$$



$$\frac{d\theta}{ds} = \underline{-R_{\alpha\beta}u^\alpha u^\beta} - \frac{1}{3N-1}\theta^2 - \theta^{\alpha\beta}\theta_{\alpha\beta} + \omega^{\alpha\beta}\omega_{\alpha\beta}.$$

The scalar θ measures the expansion of a small cloud of neighbouring geodesic trajectories forming a congruence and as such measures the expansion if $\theta > 0$ or the contraction $\theta < 0$ of the system of particles.

$$\frac{1}{\mathcal{V}}\frac{d\mathcal{V}}{ds} = \frac{d\ln\mathcal{V}}{ds} = \theta. \tag{9.130}$$

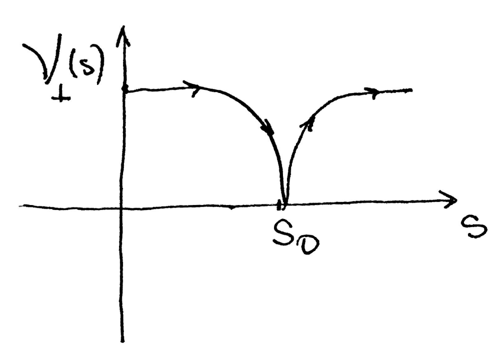
Thus the expansion scalar θ measures the fractional rate at which the volume of a small ball of particles forming a congruence is changing with respect to the time measured along the trajectory $\gamma(s)$. One can calculate the second derivative of the transversal volume:

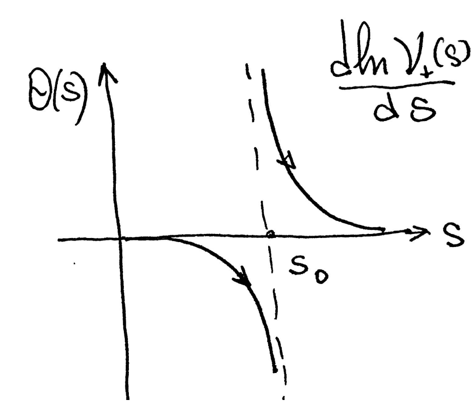
$$\ddot{\mathcal{V}} = (\dot{\theta} + \theta^2)\ \mathcal{V}. \tag{9.131}$$

The vanishing of the volume element at q characterises q as a conjugate point. It follows that the expansion scalar θ given by a logarithmic derivative of the volume element (9.130)

$$\theta = \frac{d\ln\mathcal{V}}{ds} \tag{10.146}$$

is a continuous function at all points of $\gamma_{(}s)$ at which $\mathcal{V} \neq 0$, while θ becomes unbounded near point q at which $\mathcal{V} = 0$ with large and positive just to the future of q and large and negative just to the past of q on $\gamma_{(}s)$





Raychaudhuri Equation for N-body system

$$ds^2 = g_{\alpha\beta} dq^\alpha dq^\beta, \qquad g_{\alpha\beta} = \delta_{\alpha\beta}(E - U(q)) = \delta_{\alpha\beta} W(q),$$

The self-gravitating system of N particles interacts through the gravitational potential function of the form

$$U = -G \sum_{a<b} \frac{M_a M_b}{r_{ab}}, \qquad r^i_{ab} = r^i_a - r^i_b \qquad i = 1,2,3 \qquad a,b = 1,...,N \tag{7.74}$$

The Raychaudhuri equation (10.157) will take the following form:

$$\frac{d\theta}{ds} = -\frac{3(3N-2)}{4W^2}\Big((uW')^2 - \frac{3N-4}{3(3N-2)}|W'|^2\Big) - \frac{1}{3N-1}\theta^2 - \theta_{\alpha\beta}\theta^{\alpha\beta} + \\ + \frac{3N-2}{2W}\Big((uW''u) + \frac{1}{3N-2}|W''|\Big). \tag{11.166}$$

In the case of spherically symmetric evolution $\theta^{\alpha\beta} = 0$ [106] and the equation will take the following form:

$$\frac{d\theta}{ds} = -(3N-1)\frac{(\nabla W)^2}{2W^3} - \frac{1}{3N-1}\theta^2, \tag{11.171}$$

Raychaudhuri Equation for N-body system

When the number of particles is large $N \gg 1$ we will have

$$\frac{d\theta}{ds} = -3N \frac{(\nabla W)^2}{2W^3} - \frac{1}{3N}\theta^2.$$

It is convenient to introduce the function B^2

$$B^2(s) = (3N)^2 \frac{(\nabla W)^2}{2W^3}$$

so that the equation (11.172) will take the following form:

$$\frac{d\theta}{ds} = -\frac{1}{3N}(\theta^2 + B^2(s)).$$

Solution of Raychaudhuri Equation for N-body system

$$\theta(s) = B \tan\left(\arctan\frac{\theta(0)}{B} - \frac{B}{3N}s\right),$$

The expansion scalar $\theta(s)$ becomes singular at the proper times s_n:

$$s_{caustics} = \frac{3N}{B}\left(\arctan\frac{\theta(0)}{B} + \frac{\pi}{2} + \pi n\right), \qquad n = 0, \pm 1, \pm 2, \ldots \tag{11.176}$$

As far as the expansion scalar $\theta(s)$ tends to infinity at a certain epoch $s_{caustics}$, it follows that the volume element that is occupied by the galaxies decreases and tends to zero creating the regions in space of large galactic densities.

$$\mathcal{V}(s) = \mathcal{V}(0)\left[\frac{\cos\left(\arctan\frac{\theta(0)}{B} - \frac{B}{3N}s\right)}{\cos\left(\arctan\frac{\theta(0)}{B}\right)}\right]^{3N}.$$

Solution of Raychaudhuri Equation for N-body system

The ratio of densities during the evolution from the initial volume $\mathcal{V}(0)$ to the volume $\mathcal{V}(s)$ at the epoch s will give us the density contrast:

$$\delta_{caustics}(s) + 1 = \frac{\rho(s)}{\rho(0)} = \frac{\mathcal{V}(0)}{\mathcal{V}(s)} = \left[\frac{\cos\left(\arctan\frac{\theta(0)}{B}\right)}{\cos\left(\arctan\frac{\theta(0)}{B} - \frac{B}{3N}s\right)}\right]^{3N}. \qquad (11.179)$$

As one can see, at the epoch (11.176) where the expansion scalar $\theta(s)$ becomes singular, the trigonometric function in the denominator tends to zero and the density contrast is increasing and tends to infinity, the phenomenon similar to the spherical top-hat model.

In terms of physical time (2.12) the characteristic time scale of generation of gravitational caustics is

$$\tau_{caustics} = \frac{3\pi N}{2B\sqrt{2}W} = \frac{\pi}{2}\sqrt{\frac{W}{(\nabla W)^2}}. \qquad (11.186)$$

Newtonian Cosmological Mechanics

Let us consider the evolution of a spherical shell of radius R_0 that expands with the Universe, so that $R = R_0\ a(t)$ and $a(t)$ is the scale factor in the Newtonian cosmological model of the expanding Universe. One can derive the evolution of $a(t)$ by using mostly the Newtonian mechanics and accepting two results from the general relativity: The Birkhoff's theorem stated that for a spherically symmetric system the force due to gravity at radius R is determined only by the mass interior to that radius and that the energy contributes to the gravitating mass density through the matter density ρ_m at zero pressure, $p = 0$, and the energy density of radiation/relativistic particles, $\rho_r = 3p/c^2$, where $p = \epsilon/3$ is pressure and $\epsilon = \rho_r c^2$ is energy density. The expansion of the sphere will slow down due to the gravitational force of the matter inside:

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2} = -\frac{G}{R^2}\frac{4\pi}{3}R^3\rho = -\frac{4\pi G}{3}R\rho, \tag{8.99}$$

where $\rho = \rho_m + 3P/c^2$. Since $R = R_0\ a(t)$ and R_0 is a constant, one can get the evolution equation for the scale factor $a(t)$ that reproduces the Friedmann equation:

$$\ddot{a} = -\frac{4\pi G}{3}(\rho_m + \frac{3P}{c^2})a. \tag{8.100}$$

$$W(t) = T = \sum_{g=1}^{N} \frac{M_g v_g^2}{2} = \frac{N M_g R_0^2 \dot{a}^2(t)}{2}.$$

The square of the force acting on a unit mass of the galaxies is

$$(\nabla W)^2(t) = \sum_{g=1}^{N} \frac{1}{M_g} F_g^2 = \frac{N}{M_g} \left(\frac{G M M_g}{R_0^2 a^2(t)} \right)^2 = \frac{N}{M_g} \left(\frac{4\pi G M_g}{3} R_0 a(t) \rho(t) \right)^2,$$

We can evaluate the quantities entering into this equation by considering a self-gravitating system of N galaxies of the mass M_g each. The kinetic energy W of the galaxies was found in (8.101) and the square of the force acting on a unit mass of the galaxies $(\nabla W)^2$ in (8.102). Thus we will obtain

$$\tau_{caustics} = \frac{\alpha'}{4\pi G \rho(t)} H(t), \tag{11.187}$$

where the numerical coefficient $\alpha = \sqrt{1/10}$. This general result for the characteristic time scale of the appearance of galactic caustics, the regions of the space where the density of galaxies is large, means that the appearance of caustics depends on the given epoch of the Universe expansion. The formula has a universal character and depends only on the density of matter and the Hubble parameter[18]. These are time-dependent parameters that are varying during the evolution of the Universe from the recombination epoch to the present day. Let us calculate this time scale during the *matter-dominated epoch* when

$$\rho_m(t) = \rho_0 \frac{a_0^3}{a^3(t)}. \tag{8.104}$$

In that case the equation (8.100) has the following form:

$$\dot{a}^2 = \frac{A^2}{a} - kc^2, \qquad A^2 = \left(\frac{8\pi G}{3}\right)\rho_0 a_0^3, \qquad k = 1, 0, -1, \tag{8.105}$$

and for the flat Universe, $k = 0$, we will get:

$$a_m(t) = \left(\frac{3A}{2}\right)^{2/3} t^{2/3}, \qquad H_m(t) = \frac{2}{3t}, \qquad \rho_m(t) = \frac{1}{6\pi G t^2}. \tag{8.106}$$

By substituting these values into the general formula (8.103) we will find that $\tau_{caustics}$ is proportional to the given epoch t:

$$\tau_{caustics} = \alpha \, \frac{2}{3H(t)} = \alpha \, t. \tag{8.107}$$

This result means that the time required to generate galactic caustics is very short at the early stages of the Universe expansion, at the recombination epoch, and linearly increases with the expansion time. At the present epoch, $a = a_0$, this time scale is large and is proportional to the Hubble time:

$$\tau_{0\ caustics} = \alpha \frac{2}{3H_0}, \tag{8.108}$$

where for a flat, matter-dominated Universe we substituted the expression for the matter density equal to the critical density:

$$\rho_c = \frac{3H_0^2}{8\pi G}. \tag{8.109}$$

Considering the *radiation-dominated epoch* one can obtain the identical functional time dependence, with $\alpha = \sqrt{2/5}$.

Let us compare the above time scales with the Jeans gravitational instability of a uniformly distributed matter. Jeans developed a Newtonian theory of instability of a uniformly distributed matter in a non-expanding infinite space, and Lifshitz considered small perturbations of a homogeneously expanding Universe in the theory of the general relativity. Bonnor demonstrated that in the Newtonian cosmological model of an expanding Universe

the Jeans *exponential growth of density perturbation* $\delta(t) \sim Ae^{t/\tau_{Jeans}} + Be^{-t/\tau_{Jeans}}$ transforms into a slower *power-growth rate* $\delta(t) \sim At^{2/3} + Bt^{-1} = Aa(t) + Ba(t)^{-3/2}$ and that his result coincides with the Lifshitz' exact solution for the long wave length perturbations.

$$\tau_{Jeans} \sim \frac{1}{\sqrt{4\pi G\rho_c}}, \qquad \tau_{collapse} \sim 2t_{turn} \propto \sqrt{\frac{2}{G\rho_{lump}}}.$$

where for a flat, matter-dominated Universe we substituted the expression for the matter density equal to the critical density:

$$\rho_c = \frac{3H_0^2}{8\pi G}. \tag{8.110}$$

The gravitational collapse time scale in the spherical top-hat model

Polarisation of the Vacuum

Dark Energy and Cosmological Inflation

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QCD Vacuum Structure and Confinement
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Gauge field theory vacuum and cosmological inflation without scalar field
Annals Phys. **436** (2022) 168681

Stability of the Yang Mills Vacuum State
Nucl.Phys. ***B*** **990** (2023) 116187

What is the Influence of the

Vacuum Energy Density

on the Cosmological Evolution?

Vacuum

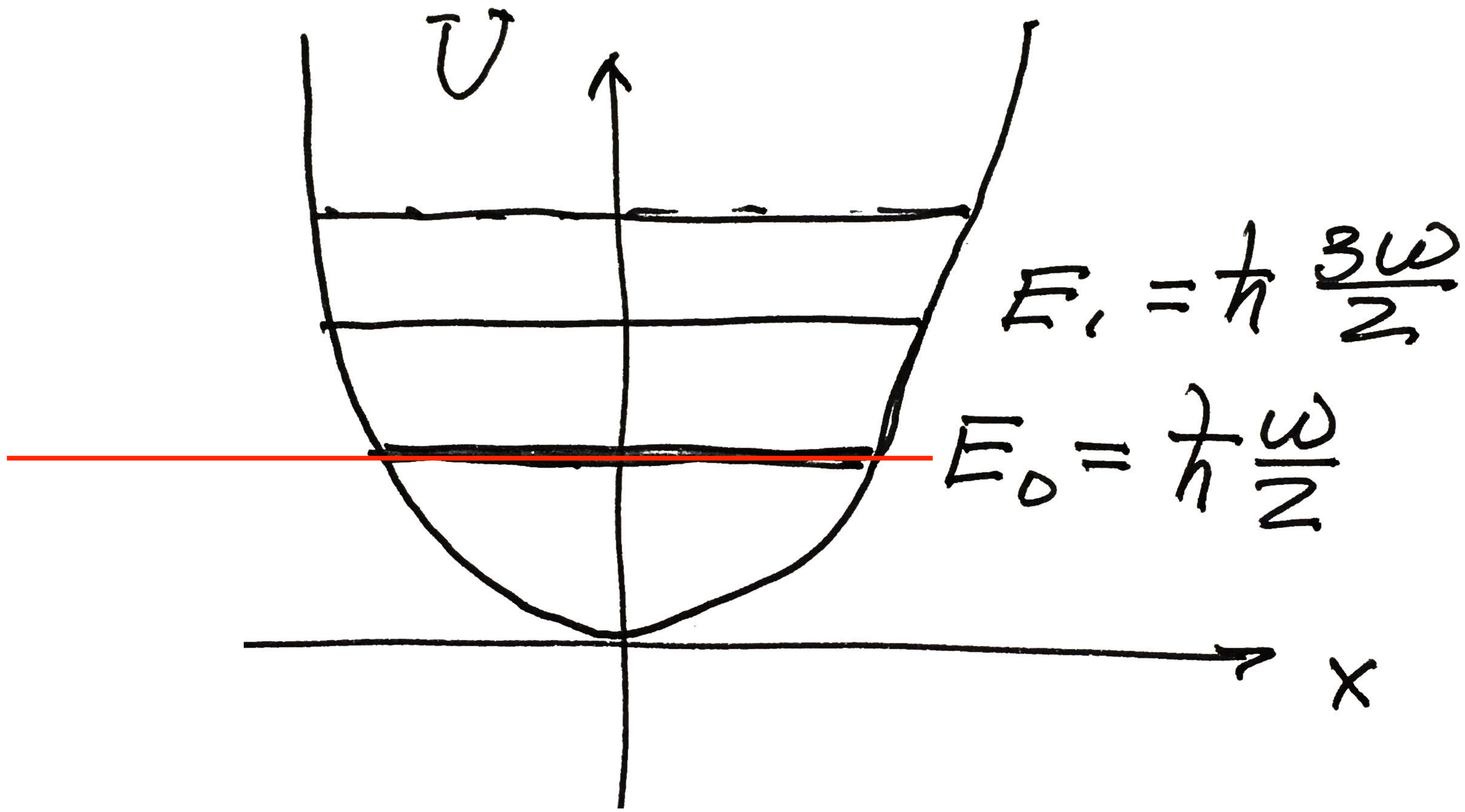
Y. B. Zel'dovich, *The Cosmological constant and the theory of elementary particles,* Sov. Phys. Usp. **11** (1968) 381

S. Weinberg, *The Cosmological constant problem,* Rev. Mod. Phys. **61** (1989) 1-23

V. Mukhanov, Physical Foundations of Cosmology, Cambridge University Press, New York, 2005.

Is there Energy Density in the Vacuum ?

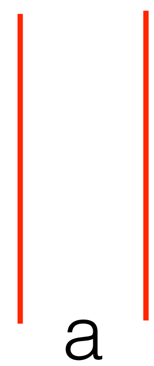
Zero Point Energy of a Quantised Field



$$U_{vacuum} = \hbar \sum_k \frac{\omega_k}{2}$$

There is Energy Density in the Vacuum, it is Zero Point Energy

Lamb shift - 1947 Casimir effect 1948



$$U_\gamma^\infty = \sum \frac{1}{2}\hbar\omega_k e^{-\gamma\omega_k}$$

$$\lim_{\gamma\to 0}[\ U_\gamma^\infty(J) - U_\gamma^\infty(0)\ ] = U_{phys} \qquad U_{phys} = \hbar\ c\pi^2\frac{Area}{720a^3}$$

The Cosmological vacuum energy density from a quantum field

$$E_0 = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\omega_p \sim \frac{1}{16\pi^2}\Lambda^4 \qquad \approx 1.44\times 10^{110}\frac{g}{s^2cm}$$

Critical Energy Density in Universe

$$\epsilon_{crit} = 3\frac{c^4}{8\pi G}\left(\frac{H_0}{c}\right)^2 \approx 7.67\times 10^{-9}\frac{g}{s^2cm}$$

Vacuum energy contribution to the energy density of the universe

$$\epsilon_{crit} = 3\frac{c^4}{8\pi G}\left(\frac{H_0}{c}\right)^2 \approx 7.67 \times 10^{-9}\frac{g}{s^2 cm}$$ 100%

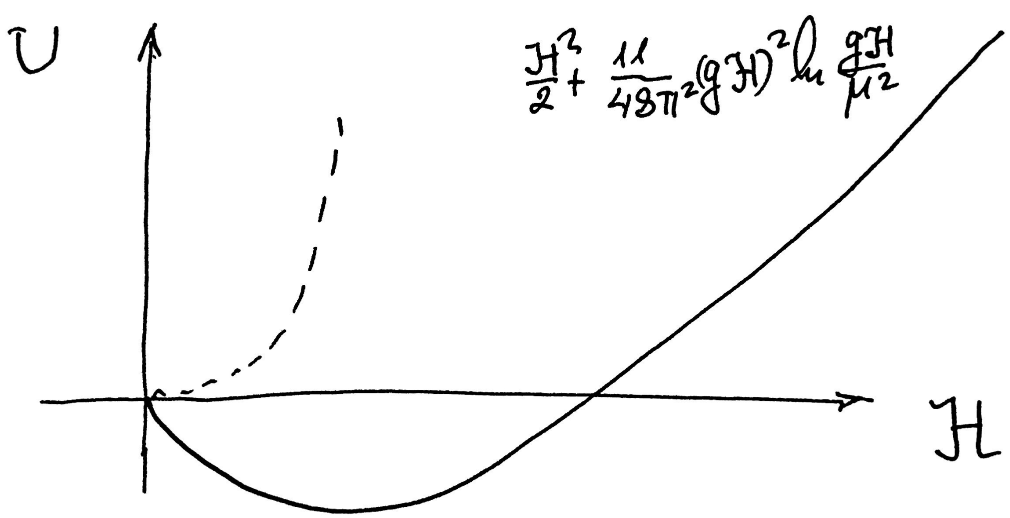
$$\epsilon_\Lambda = 3\frac{c^4}{8\pi G}\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda \approx 5.28 \times 10^{-9}\frac{g}{s^2 cm}$$ 68%

The Yang-Mills Theory Vacuum Energy Density

G.S. 1977, 2020

$$\mathcal{L}_g = -\mathcal{F} - \frac{11N}{96\pi^2} g^2 \mathcal{F}\Big(\ln \frac{2g^2\mathcal{F}}{\mu^4} - 1\Big) , \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}_a^2 - \vec{\mathcal{E}}_a^2}{2} > 0, \quad \mathcal{G} = \vec{\mathcal{E}}_a \vec{\mathcal{H}}_a = 0 \ .$$

$$\mathcal{L}_q = -\mathcal{F} + \frac{N_f}{48\pi^2} g^2 \mathcal{F}\Big[\ln(\frac{2g^2\mathcal{F}}{\mu^4}) - 1\Big]$$



$$2g^2\mathcal{F}_{vac} = \mu^4 \exp\big(-\frac{96\pi^2}{b\ g^2(\mu)}\big) = \Lambda_{YM}^4,$$

where $b = 11N - 2N_f$.

$$T_{\mu\nu} = T_{\mu\nu}^{YM}\Big[1 + \frac{b\ g^2}{96\pi^2} \ln \frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0.$$

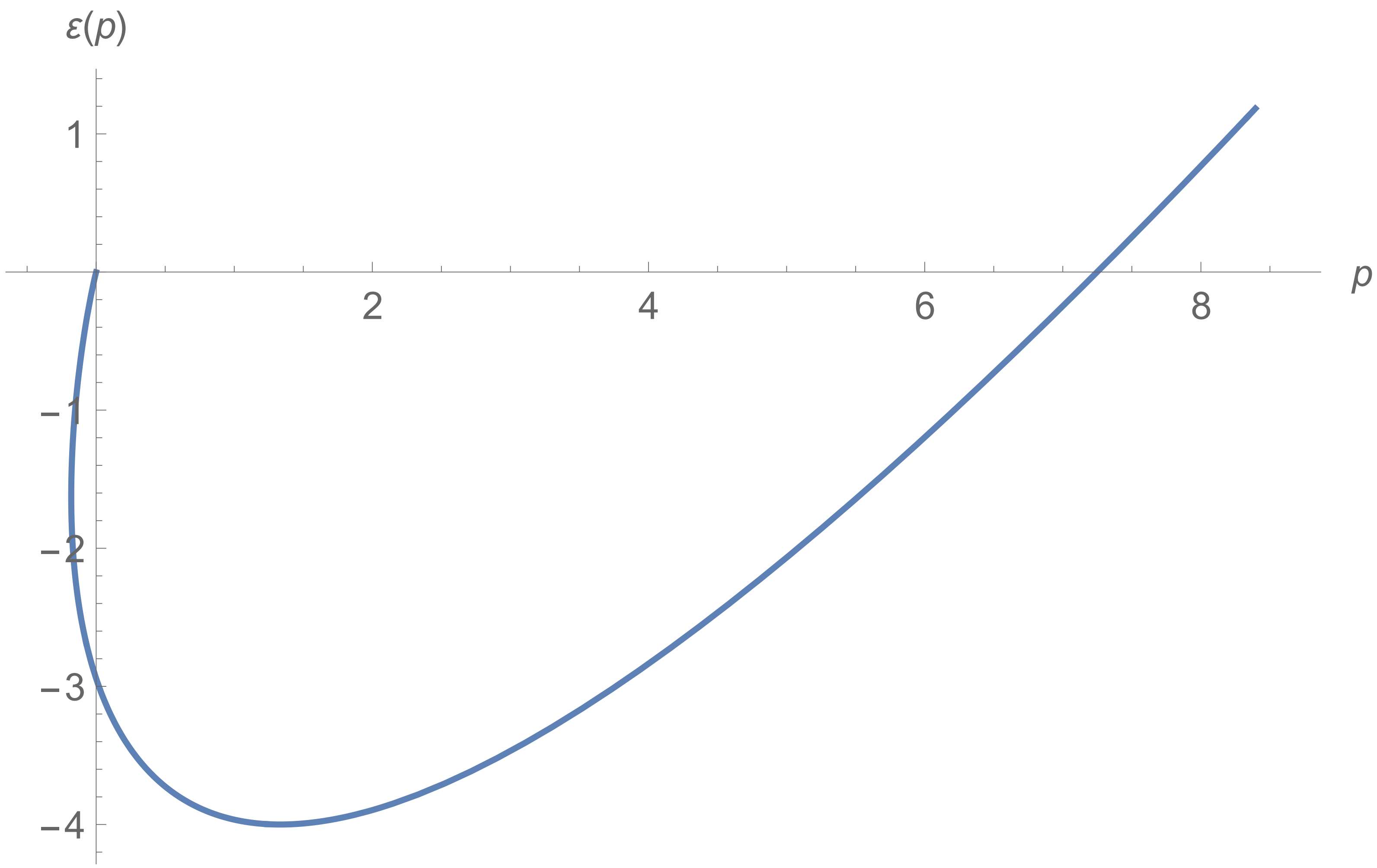
YM Quantum Energy Momentum Tensor

$$T_{\mu\nu} = T_{\mu\nu}^{YM}\left[1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\right] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0,$$

$$\epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

$$\mathcal{F} = \tfrac{1}{4}g^{\alpha\beta}g^{\gamma\delta}G^a_{\alpha\gamma}G_{\beta\delta} \geq 0 \qquad \mathcal{G} = G^*_{\mu\nu}G^{\mu\nu} = 0$$

Yang-Mills Quantum Equation of State



$$\epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

Yang-Mills Quantum Equation of State

$$p = \frac{1}{3}\epsilon + \frac{4}{3}\frac{b\ g^2\mathcal{F}}{96\pi^2}\Lambda_{YM}^4 \qquad \text{and} \qquad w = \frac{p}{\epsilon} = \frac{\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 3}{3\left(\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} - 1\right)}$$

general parametrisation of the equation of state $p = w\epsilon$

Friedman Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p).$$

General Relativity and Yang-Mills Vacuum Energy Density

$$S = -\frac{c^3}{16\pi G}\int R\sqrt{-g}d^4x + \int \left(\mathcal{L}_q + \mathcal{L}_g\right)\sqrt{-g}d^4x.$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\Big[T_{\mu\nu}^{YM}\Big(1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big) - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big].$$

The contribution of the YM vacuum field to the energy balance of the universe

Friedmann Evolution Equations in YM, QCD Vacuum

$$a(\tau) = a_0 \, \tilde{a}(\tau), \qquad ct = L \, \tau,$$

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

$$\frac{1}{L^2} = \frac{8\pi G}{3c^4} \, \mathcal{A} \, \Lambda_{YM}^4 \equiv \Lambda_{eff} \ ,$$

$$\mathcal{A} = \frac{b}{192\pi^2} = \frac{11N - 2N_f}{192\pi^2}.$$

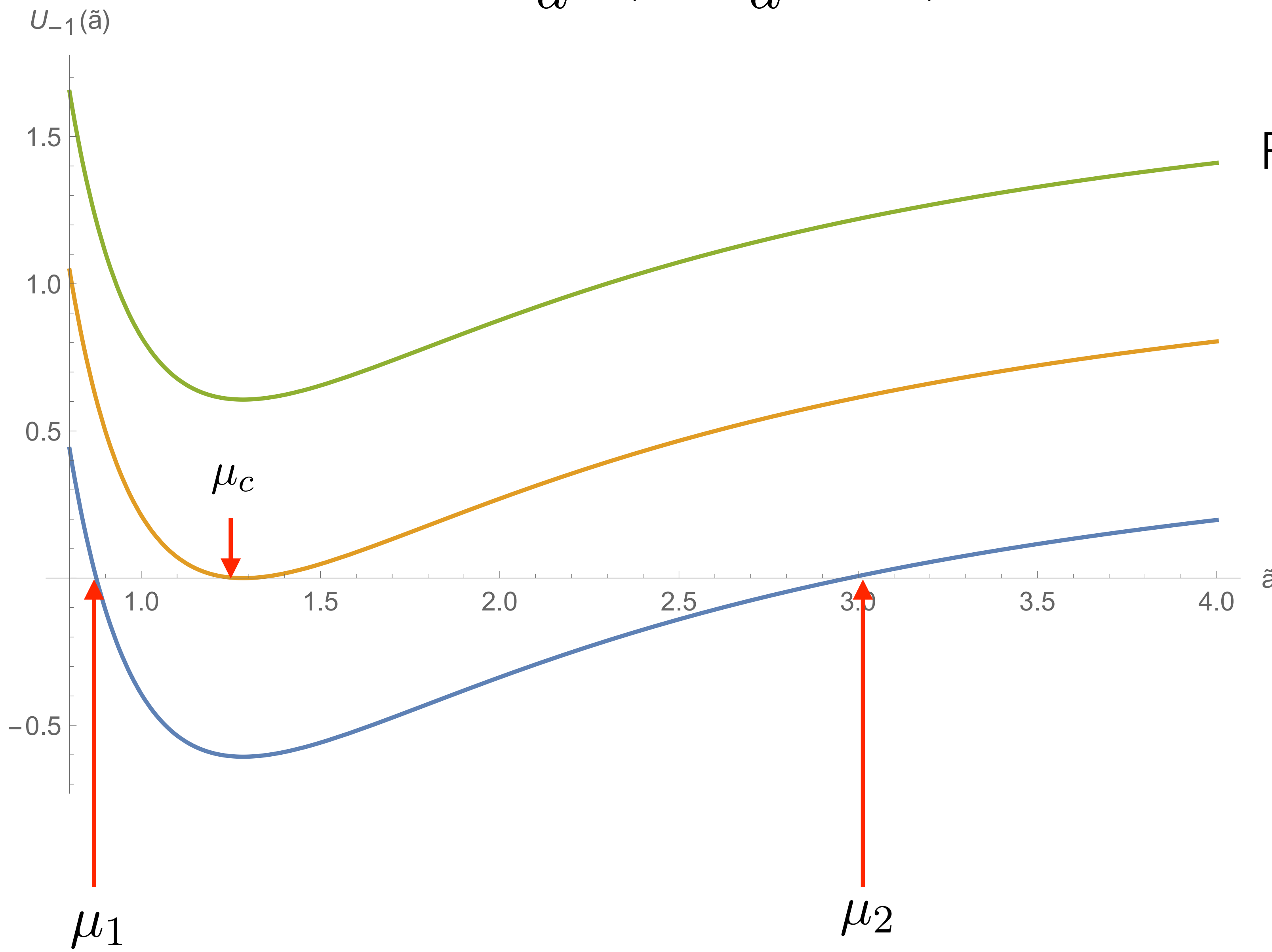
$$U_{-1}(\tilde{a}) \equiv \frac{1}{\tilde{a}^2}\left(\log\frac{1}{\tilde{a}^4} - 1\right) + \gamma^2.$$

Region III

$(\tilde{a} \leq \mu_c)$

Region I

$(\tilde{a} \leq \mu_1)$



Region IV

$(0 \leq \tilde{a})$

Region II

$(\mu_2 \leq \tilde{a})$

$$0 \leq \gamma^2 < \gamma_c^2$$

$$\gamma^2 = \gamma_c^2 = \frac{2}{\sqrt{e}}$$

$$\gamma_c^2 < \gamma^2$$

Polarisation of the YM vacuum and the Effective Lagrangians

$$\epsilon_{YM} = 3\frac{c^4}{8\pi G}\frac{1}{L^2}, \qquad \frac{1}{L^2} = \frac{8\pi G}{3c^4}\,\frac{11N - 2N_f}{196\pi^2}\,\Lambda^4_{YM}$$

Λ^4_{YM} is the dimensional transmutation scale of YM theory

$$\epsilon_{YM} = 3\frac{c^4}{8\pi G}\frac{1}{L^2} = \begin{cases} 9.31 \times 10^{-3} & eV \\ 9.31 \times 10^{29} & QCD \\ 9.31 \times 10^{97} & GUT \\ 9.31 \times 10^{110} & Planck \end{cases} \qquad \frac{g}{s^2 cm}$$

the YM vacuum energy density is well defined and is finite

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Type II Solution — Initial Acceleration of Finite Duration

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

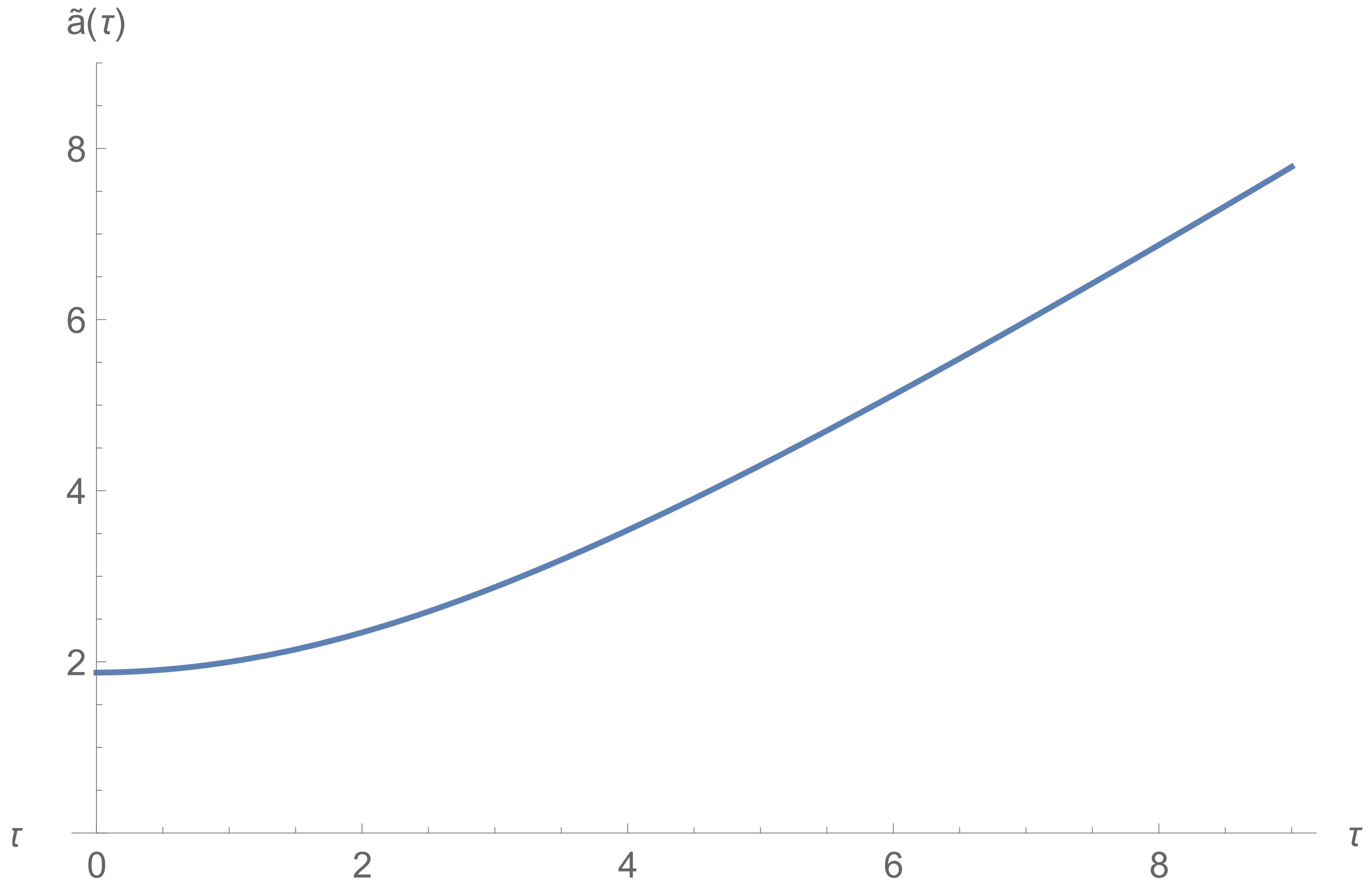
$$\tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty],$$

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\ e^{-\frac{b^2}{2}}\Big(\frac{\gamma^2\mu_2^2}{b^2}(e^{\frac{b^2}{2}} - 1) - 1\Big)^{1/2}.$$

$$\mu_2^2 = -\frac{2}{\gamma^2}\ W_-\Big(-\frac{\gamma^2}{2\sqrt{e}}\Big), \qquad 0 \le \gamma^2 < \frac{2}{\sqrt{e}} \text{ and } \tilde{a} \ge \mu_2.$$

Type II Solution *Initial Acceleration of Finite Duration*

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\ e^{-\frac{b^2}{2}} \Big(\frac{\gamma^2 \mu_2^2}{b^2} (e^{\frac{b^2}{2}} - 1) - 1 \Big)^{1/2}. \qquad \tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty],$$

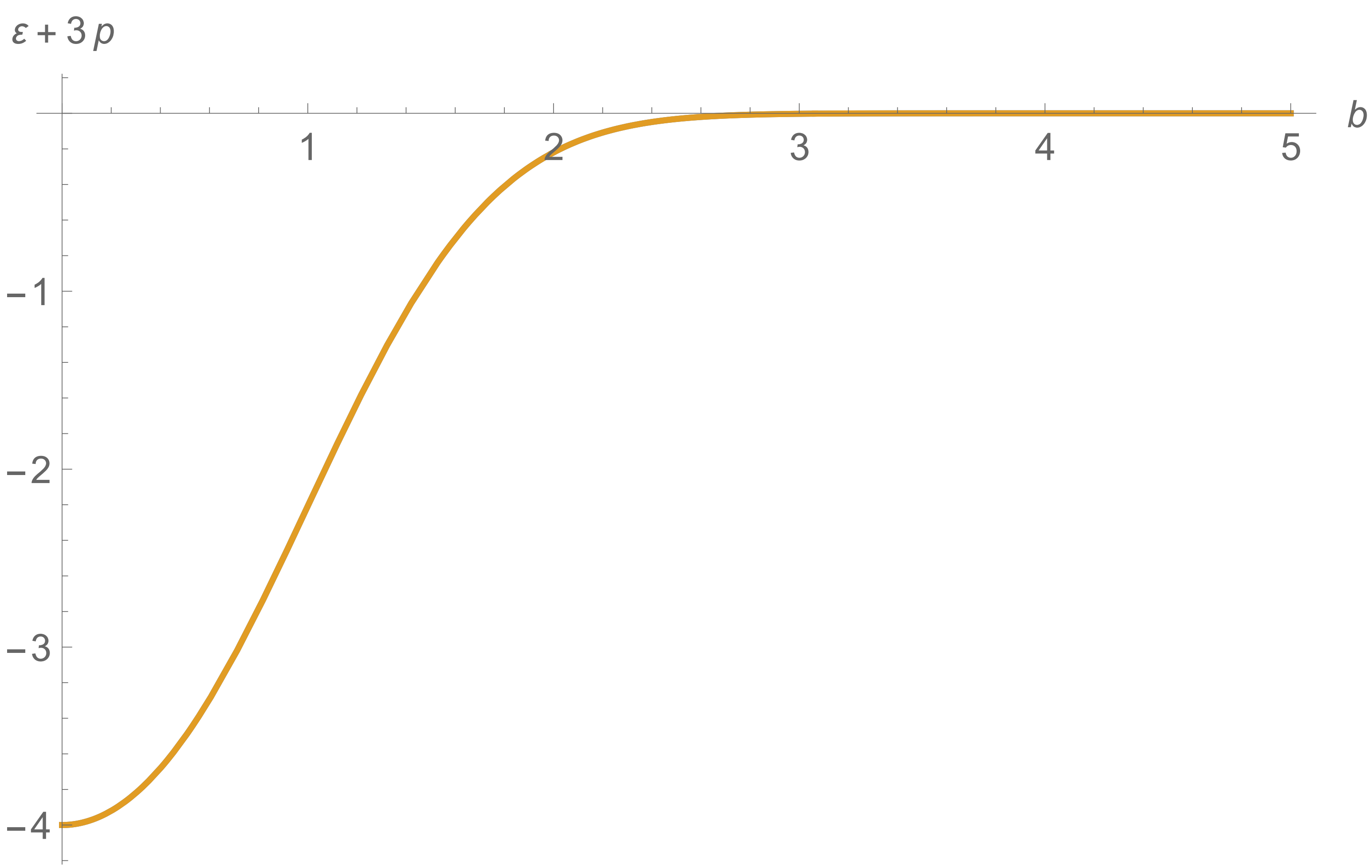


The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq \ ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \tag{5.87}$$

Type II Solution — Initial Acceleration of Finite Duration

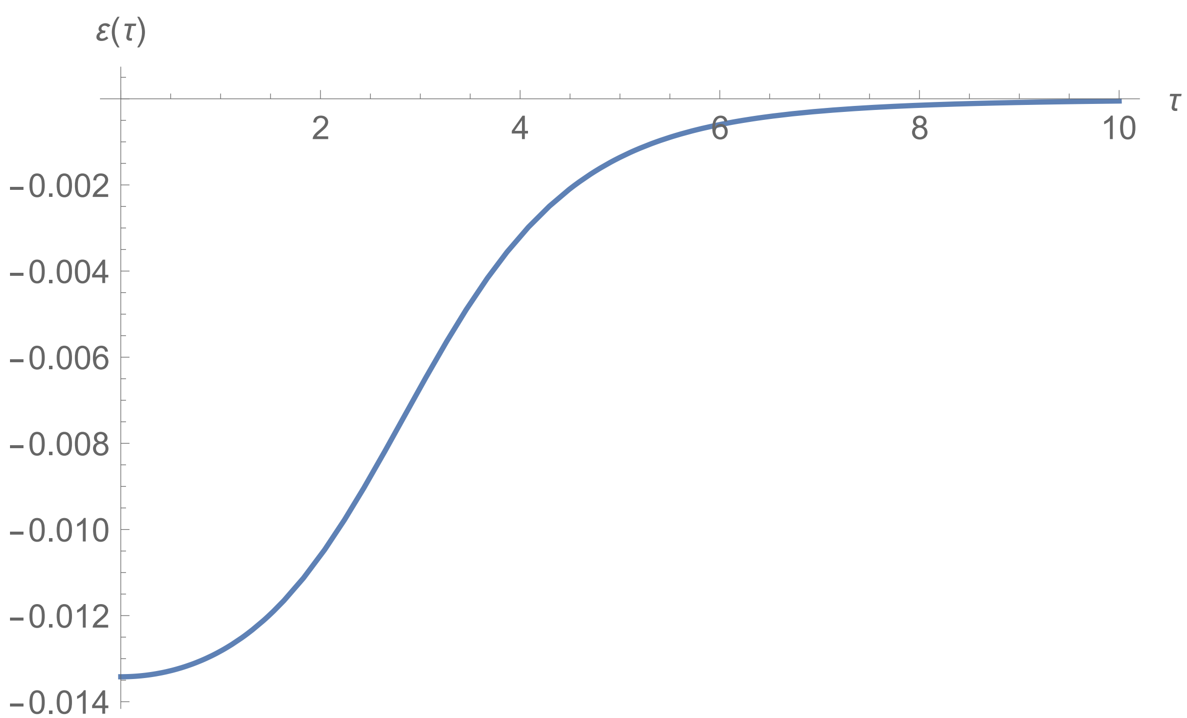
$$\epsilon + 3p = -\frac{2\mathcal{A}}{\mu_2^4}\; e^{-b^2(\tau)}(b^2(\tau) + \gamma^2\mu_2^2 - 2)\Lambda_{YM}^4, \qquad b \in [0, +\infty],$$

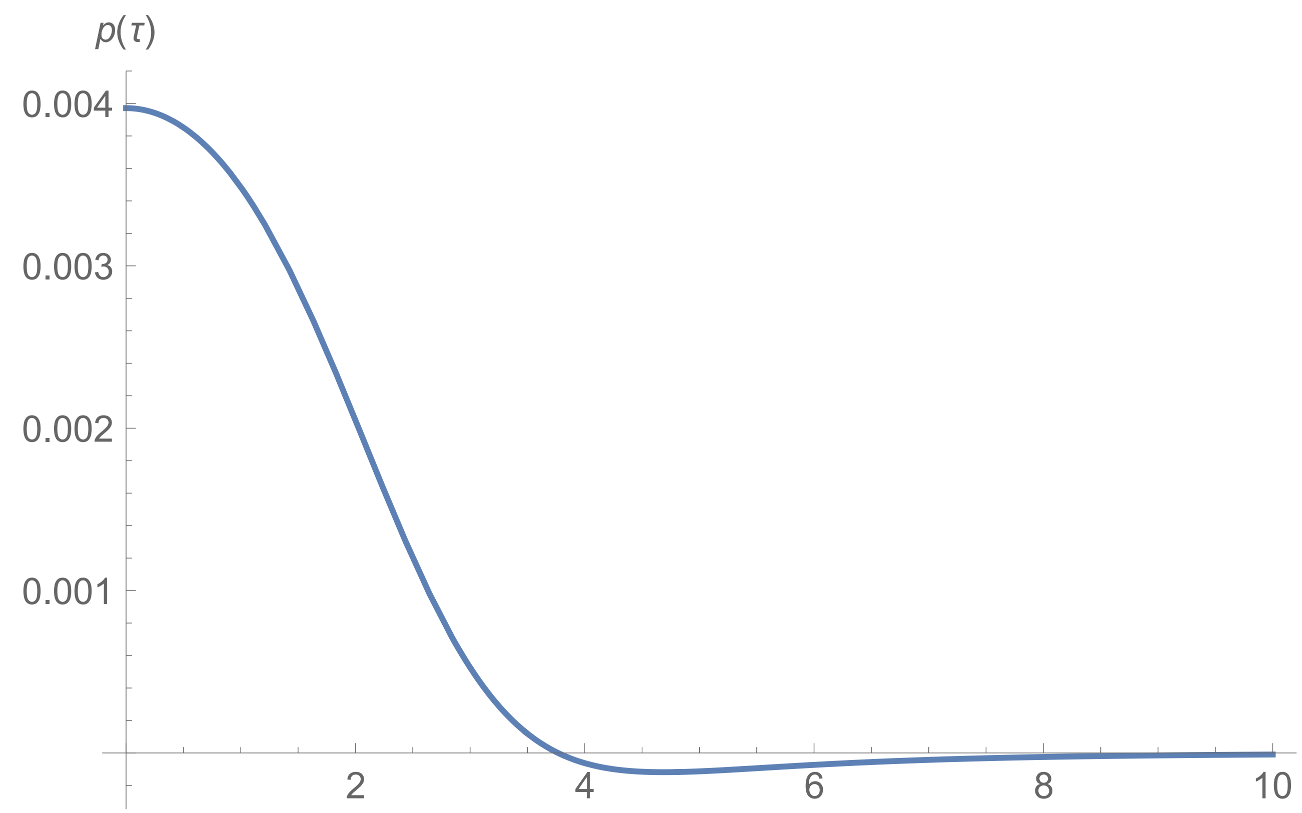


The r.h.s $\epsilon + 3p$ of the Friedmann acceleration equation (1.4) always negative

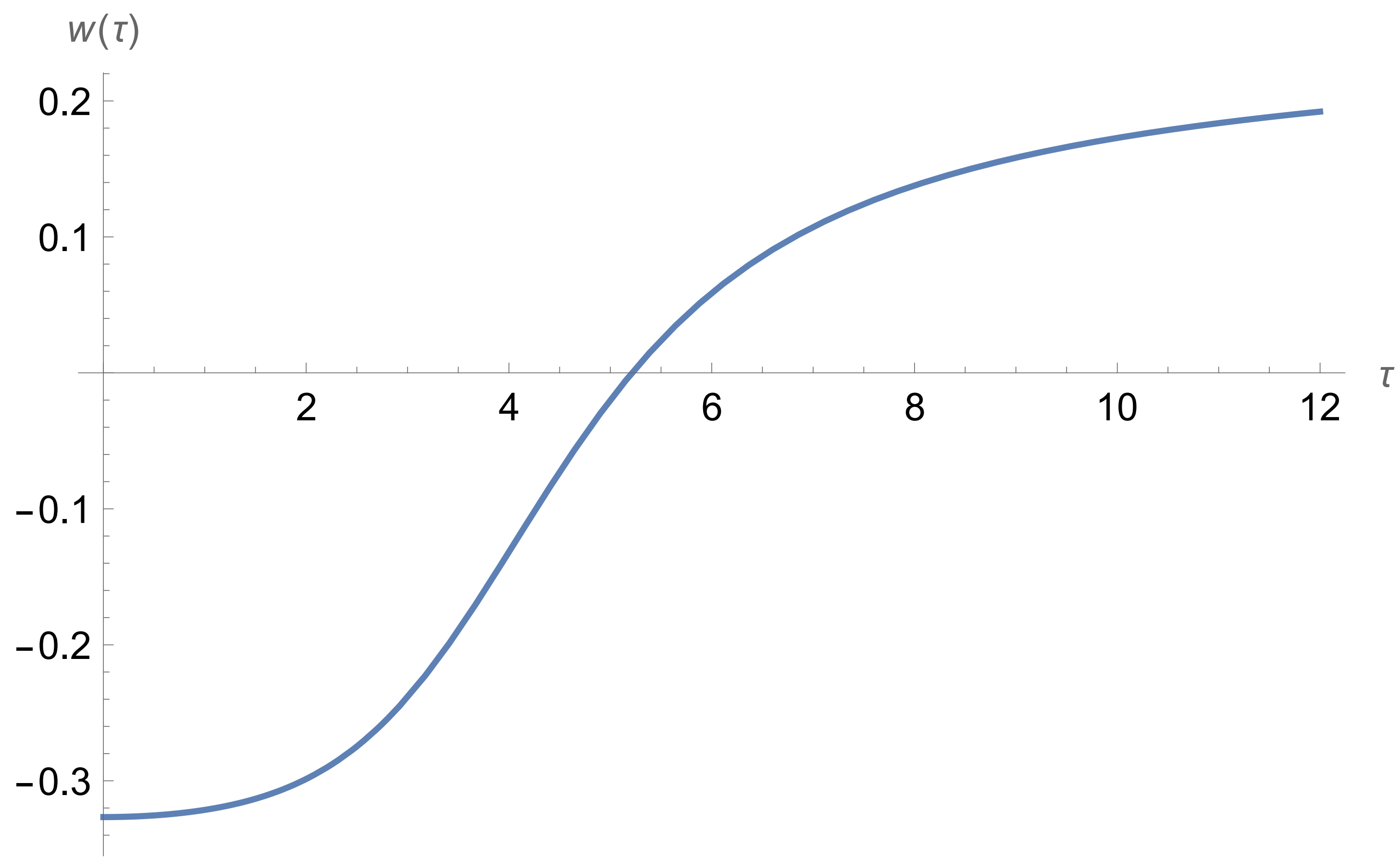
Evolution of Energy Density and Pressure

$$\epsilon = \frac{\mathcal{A}}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)\Lambda_{YM}^4, \qquad p = \frac{\mathcal{A}}{3\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} + 3\Big)\Lambda_{YM}^4.$$





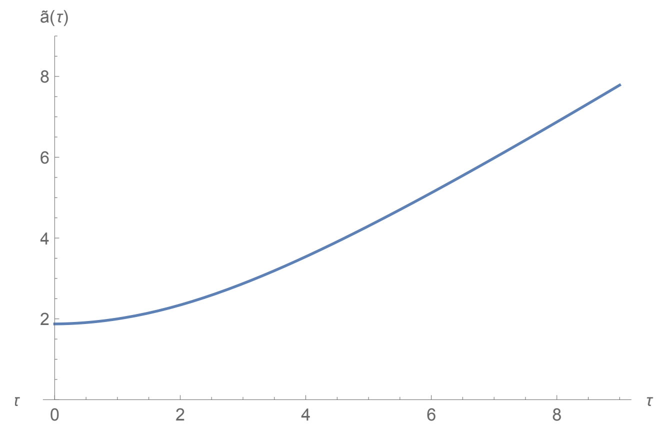
Type II Solution — Effective Parameter w



For the equation of state $p = w\epsilon$ one can find the behaviour of the effective parameter w

$$w_{II} = \frac{b^2(\tau) + \gamma^2\mu_2^2 - 4}{3\Big(b^2(\tau) + \gamma^2\mu_2^2\Big)}, \qquad -\frac{1}{3} < w_{II},$$

Type II Solution *Initial Acceleration of Finite Duration*



The number of e-foldings

typical parameters around $\gamma^2 = 1.211$, $\mu_2^2 \simeq 1.75$ we get $\tau_s = 10^{23}$ and $\mathcal{N} \simeq 53$. $\quad \mathcal{N} = \ln \frac{a(\tau_s)}{a(0)}$.

$$t_s^{GUM} = \frac{L_{GUM}}{c} \tau_s \simeq 4.2 \times 10^{-13} \ sec, \qquad \text{where } L_{GUM} \simeq 1.25 \times 10^{-25} cm$$

$$a(0) = L_{GUM} \frac{\mu_2}{\gamma} \simeq 1.5 \times 10^{-25} cm, \qquad a(t_s) = L_{GUM} \frac{\mu_2}{\gamma} e^{\mathcal{N}} \simeq 1.25 \times 10^{-2} cm,$$

The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq \ ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \tag{5.87}$$

Thank You !