Cosmological Implications of Electroweak Monopole
–Topological Avatar of New Physics–

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Motivation

- **What is the electroweak monopole?**

1. It is the only realistic monopole that we can find in nature, predicted by the standard model.

2. It is the electroweak generalization of the Dirac monopole. So it is this monopole, not the Dirac monopole, which exists in nature.

3. It is different from the Dirac monopole. The magnetic charge is twice bigger.

- **Why is this so important?**

1. The detection of the electroweak monopole, not the Higgs particle, should be the final (and topological) test of the standard model.

2. If detected, it will be the first absolutely stable magnetically charged topological elementary particle (the true God's particle) in human history, which has a huge mass.

3. It could play important roles in physics in general, in particular in cosmology. Coupled to gravity, it becomes the primordial magnetic blackhole which could become the seed of stars and galaxies and the source of intergalactic magnetic field.

Within the standard model!

- **Contents**

Electroweak Monopole

A. History of Monopole

- Ever since Dirac predicted the Dirac monopole in 1931, the monopole has become an obsession, theoretically and experimentally.
- After Dirac we have had Wu-Yang (1969), 'tHooft-Polyakov (1974), and grand unification (Dokos-Tomaras; 1980) monopoles. But they are unrealistic or unphysical.
- Strangely, it has been asserted that the standard model has no monopole.

- No-Go theorem: The standard model has no monopole topology

$$\pi_2(G/H) = \pi_2\Big(\frac{SU(2)\times U(1)_Y}{U(1)_{(em)}}\Big) = 0.$$

This topology, however, is not the only possible monopole topology.

1. With $U(1)_Y$ the Higgs doublet becomes CP^1 which has S^2 topology.
2. $U(1)_{(em)}$ has non-trivial Abelian monopole topology.

- In 1997 the existence of the electroweak monopole was established.

B. Electroweak Monopole: A Review

- Start from the Weinberg-Salam Lagrangian

$$\mathcal{L} = -|\mathcal{D}_\mu \phi|^2 - \frac{\lambda}{2}\big(|\phi|^2 - \frac{\mu^2}{\lambda}\big)^2 - \frac{1}{4}\vec{F}_{\mu\nu}^2 - \frac{1}{4}G_{\mu\nu}^2,$$

$$\mathcal{D}_\mu \phi = \big(\partial_\mu - i\frac{g}{2}\vec{\tau}\cdot\vec{A}_\mu - i\frac{g'}{2}B_\mu\big)\phi.$$

- Choose the hybrid ansatz with non-trivial $U(1)_Y$

$$\phi = \frac{1}{\sqrt{2}}\rho(r)\ \xi, \quad \xi = i\begin{pmatrix} \sin(\theta/2)\ e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix},$$

$$\vec{A}_\mu = \frac{1}{g}A(r)\partial_\mu t\ \hat{r} + \frac{1}{g}(f(r)-1)\ \hat{r}\times\partial_\mu \hat{r}, \quad (\hat{r} = -\xi^\dagger \vec{\tau}\ \xi)$$

$$B_\mu = \frac{1}{g'}B(r)\partial_\mu t - \frac{1}{g'}(1-\cos\theta)\partial_\mu \varphi.$$

- In the physical fields the ansatz becomes

$$
\begin{aligned}
A_\mu^{(\mathrm{em})} &= \frac{e}{gg'}\Big(\frac{g'}{g}A(r) + \frac{g}{g'}B(r)\Big)\partial_\mu t - \frac{1}{e}(1-\cos\theta)\partial_\mu\varphi, \\
W_\mu &= \frac{i}{g}\frac{f(r)}{\sqrt{2}}e^{i\varphi}(\partial_\mu\theta + i\sin\theta\partial_\mu\varphi), \\
Z_\mu &= \frac{e}{gg'}\big(A(r) - B(r)\big)\partial_\mu t.
\end{aligned}
$$

With $A = B = 0$, this describes the electroweak monopole.

Dyon ansatz with W and Z dressing

- With this we have the equations of motion

$$\ddot{\rho} + \frac{2}{r}\dot{\rho} - \frac{f^2}{2r^2}\rho + \frac{1}{4}(A-B)^2\rho = \frac{\lambda}{2}\big(\rho^2 - \rho_0^2\big)\rho,$$
$$\ddot{f} - \frac{f^2-1}{r^2}f = \big(\frac{g^2}{4}\rho^2 - A^2\big)f,$$
$$\ddot{A} + \frac{2}{r}\dot{A} - \frac{2f^2}{r^2}A = \frac{g^2}{4}\rho^2(A-B),$$
$$\ddot{B} + \frac{2}{r}\dot{B} = -\frac{g'^2}{4}\rho^2(A-B).$$

This has the point monopole solution with $q_m = 4\pi/e$,

$$\rho = \rho_0, \quad f = 0, \quad A = B = 0,$$
$$A_\mu^{(\mathrm{em})} = -\frac{1}{e}(1-\cos\theta)\partial_\mu\varphi.$$

- With $\rho(0)=0,\ f(0)=1,\ \rho(\infty)=\rho_0,\ f(\infty)=0,\ A=B=0$, we have the topologically stable Cho-Maison monopole which can be viewed as hybrid between Dirac and 'tHooft-Polyakov

$$A_\mu^{(\mathrm{em})} = -\frac{1}{e}(1-\cos\theta)\partial_\mu\varphi,$$

$$\rho \simeq \rho_0 + \frac{\rho_1}{r}\exp(-M_H\ r), \quad f \simeq f_1 \exp(-M_W\ r),$$

$$M_H = \sqrt{\lambda}\rho_0, \quad M_W = g\rho_0/2, \quad M_Z = \sqrt{g^2+g'^2}\rho_0/2.$$

The monopole size is set by the W boson mass.

- The solution can be generallized to a dyon (and anti-dyon) solution.

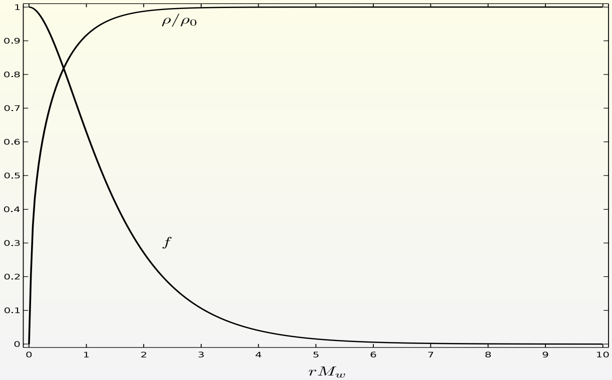
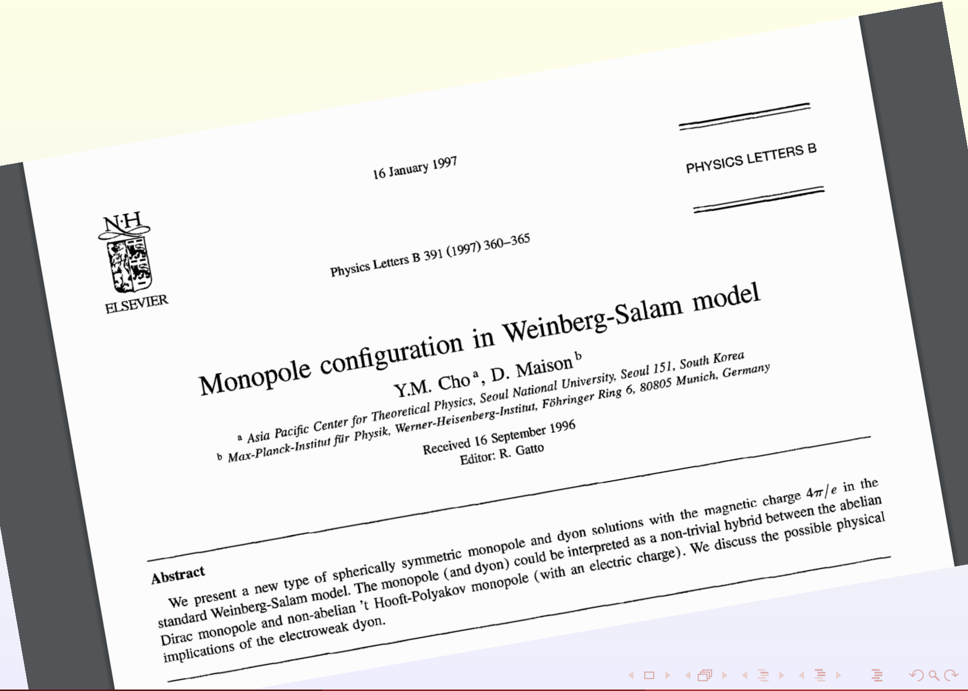


Figure: The electroweak monopole solution. Here we have chosen $\sin^2\theta_{\rm w} = 0.2312$ and $M_H/M_W = 1.56$.



16 January 1997

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Monopole configuration in Weinberg-Salam model

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Abstract

We present a new type of spherically symmetric monopole and dyon solutions with the magnetic charge $4\pi/e$ in the standard Weinberg-Salam model. The monopole (and dyon) could be interpreted as a non-trivial hybrid between the abelian Dirac monopole and non-abelian 't Hooft-Polyakov monopole (with an electric charge). We discuss the possible physical implications of the electroweak dyon.

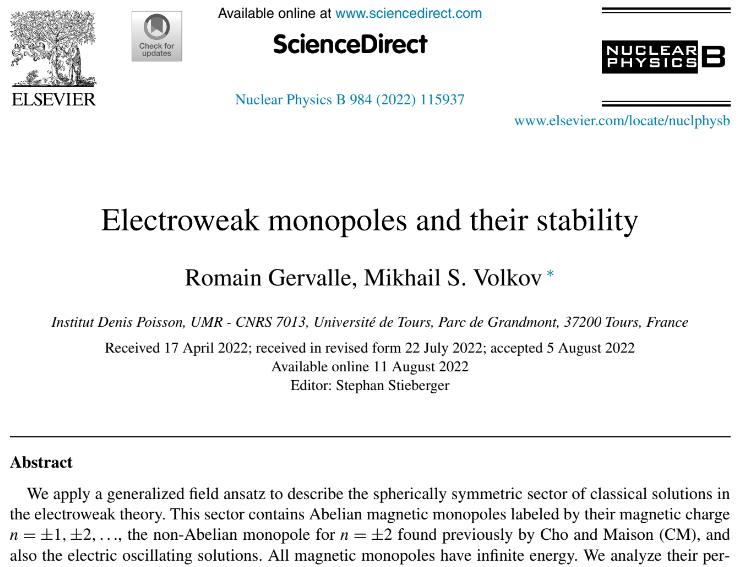
C. Charactristic Features of Electroweak Monopole

- It is absolutely stable.

- The magnetic charge is $4\pi/e$, which changes the Dirac quantization rule to $eg = 4\pi n$.

- Classically the energy of the monopole solution is infinite, but we can estimate the mass. Intuitively, we have $M \simeq M_W/\alpha \simeq 11 \text{ TeV}$.

- There are different ways to estimate the mass, the dimensional argument, the scaling argument, and the quantum correction. All of them predict the mass to be around 4 to 11 TeV.



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Electroweak monopoles and their stability

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Abstract

We apply a generalized field ansatz to describe the spherically symmetric sector of classical solutions in the electroweak theory. This sector contains Abelian magnetic monopoles labeled by their magnetic charge $n = \pm 1, \pm 2, \ldots$, the non-Abelian monopole for $n = \pm 2$ found previously by Cho and Maison (CM), and also the electric oscillating solutions. All magnetic monopoles have infinite energy. We analyze their per-

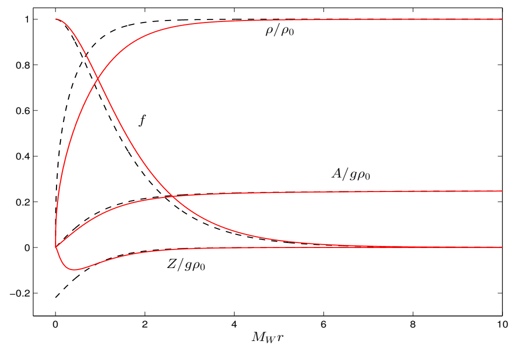
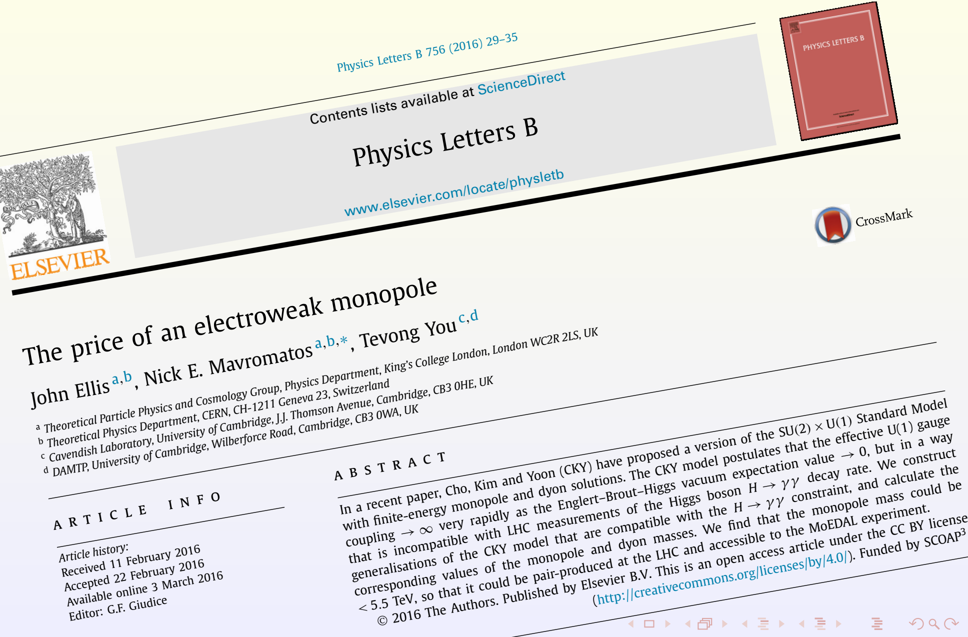


Figure: The finite energy electroweak dyon. The solid line (red) represents the regularized dyon and the dotted (blue) line represents the singular dyon.



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The price of an electroweak monopole

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ABSTRACT

In a recent paper, Cho, Kim and Yoon (CKY) have proposed a version of the $SU(2) \times U(1)$ Standard Model with finite-energy monopole and dyon solutions. The CKY model postulates that the effective U(1) gauge coupling $\to \infty$ very rapidly as the Englert–Brout–Higgs vacuum expectation value $\to 0$, but in a way that is incompatible with LHC measurements of the Higgs boson $H \to \gamma\gamma$ decay rate. We construct generalisations of the CKY model that are compatible with the $H \to \gamma\gamma$ constraint, and calculate the corresponding values of the monopole and dyon masses. We find that the monopole mass could be < 5.5 TeV, so that it could be pair-produced at the LHC and accessible to the MoEDAL experiment.

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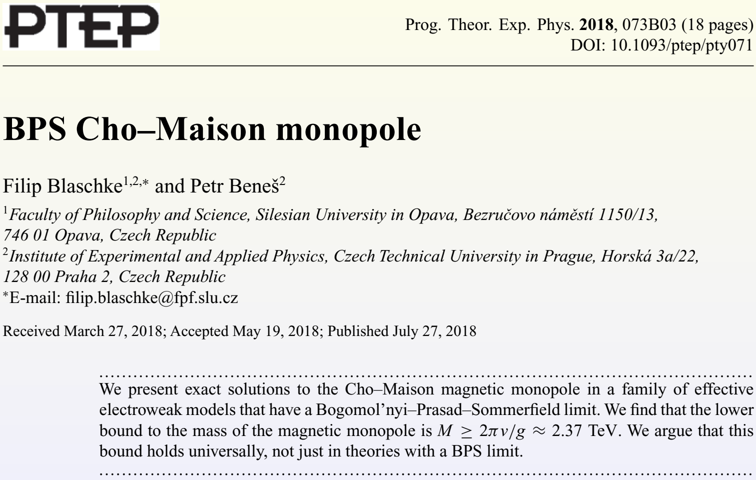

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BPS Cho–Maison monopole

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We present exact solutions to the Cho–Maison magnetic monopole in a family of effective electroweak models that have a Bogomol'nyi–Prasad–Sommerfield limit. We find that the lower bound to the mass of the magnetic monopole is $M \geq 2\pi v/g \approx 2.37$ TeV. We argue that this bound holds universally, not just in theories with a BPS limit.

- This strongly implies that the standard model has the electroweak monopole whose mass could be 4 to 10 TeV. But ultimately this has to be confirmed by experiment.

- If the monopole mass is less than 7 TeV, the present LHC should be able to produce the monopole pair, and the MoEDAL could actually detect it.

- If the mass is larger, however, the 14 TeV LHC can not produce the monopole, and we may have to look for the remnant monopole in the present universe.

SSC???!

Cosmological Production of Electroweak Monopole

A. Electroweak Phase Transition

- The electroweak phase transition is controlled by the effective potential of the standard model

$$V_T(\rho) \simeq \frac{\lambda}{8}(\rho^2-\rho_0^2)^2 - \frac{C_1}{12\pi}\rho^3\ T + \frac{C_2}{2}\rho^2\ T^2 - \frac{\pi^2}{90}g_* T^4,$$

$$C_1 = \frac{6M_W^3+3M_Z^3}{\rho_0^3} \simeq 0.36,$$

$$C_2 = \frac{4M_W^2+2M_Z^2+M_H^2+4m_t^2}{8\rho_0^2} \simeq 0.36,$$

which is charactrized by three temperatures, T_1, T_c, and T_2.

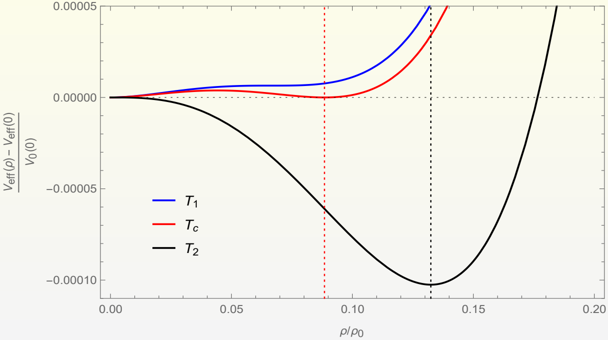


Figure: The effective potential of the standard model at $T_1,\ T_c,\ T_2$. Here the unit of V_{eff} is chosen to be $V_0=(\lambda/8)\rho_0^4=1$.

- The phase transition is first order:

  1. Above $T_1 \simeq 146.7$ GeV the standard model is in the symmetric state. But it develops a second unstable vacuum ρ_+ at T_1, while $\rho_0 = 0$ remains the true vacuum.
  2. At $T_c \simeq 146.6$ GeV we have $\rho_+ \simeq 21.8$ GeV, and the two vacua become degenerate, and ρ_+ becomes the true vacuum below T_c.
  3. Below $T_2 \simeq 146.4$ GeV ρ_+ becomes the only vacuum, which approaches to the Higgs vacuum at zero temperature.

- However, since $T_1,\ T_c,\ T_2$ are very close the phase transition is very mildly first order ("cross-over"), and effectively becomes second order.

- It has been asserted that he cosmic monopole production depends on the type of phase transition.

  1. In the first order phase transition, the monopoles are supposed to be produced during the phase transition through the vacuum bubble collisions.

  2. In the second order phase transition, the monopoles produced by the Kibble-Zurek mechanism around the critical temperature.

- This popular view, however, may have critical defects because the monopoles are produced by thermal fluctuation of the Higgs vacuum to $\langle \rho \rangle = 0$, which induces the change of topology.

- This thermal fluctuation continues till the Ginzburg temperature T_G, as far as $\xi^3 \Delta F \leq T \ (\Delta F = V(\rho_s) - V(\rho_+))$, where $\xi(T)$ is the correlation length of the Higgs field.

- So the electroweak monopole production starts at $T_c \simeq 146.6$ GeV and stops at $T_G \simeq 57.6$ GeV.

- What is important in the monopole production is not the type of the phase transition but the Ginzburg temperature. In particular, the monopole production in the first order phase transition has no exponential suppression as far as T_G becomes less than T_2.

Ginzburg Temperature!

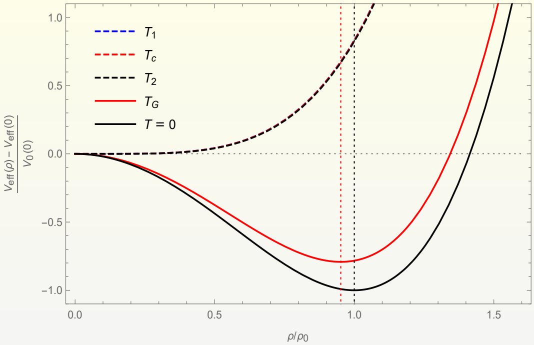


Figure: The macroscopic view of the effective potential (19). The effective potentials at $T_1,\ T_c,\ T_2$ are shown in dotted lines, and the effective potentials at the Ginzburg temperature T_G and $T=0$ are shown in red and black curves. Notice that V_{eff} is almost indistinguishable at $T_1,\ T_c$, and T_2.

B. Initial Monopole Density

- The effective potential gives the temperature-dependent Higgs mass which determines the correlation length $\xi \simeq 1/\alpha^{1/3}\bar{M}_H$,

  $$\bar{M}_H^2 = \begin{cases} \big[(T/T_1)^2 - 1\big] M_H^2/2, & T > T_c, \\ \big[(\rho_+/\rho_0)^2 + 1 - (T/T_1)^2\big] M_H^2/2, & T \leq T_c. \end{cases}$$

  and the W boson mass which determines the monopole mass $M \simeq \bar{M}_W/\alpha$,

  $$\bar{M}_W^2 = \begin{cases} 0, & T > T_c, \\ \dfrac{g^2}{4}\rho_+^2, & T \leq T_c. \end{cases}$$

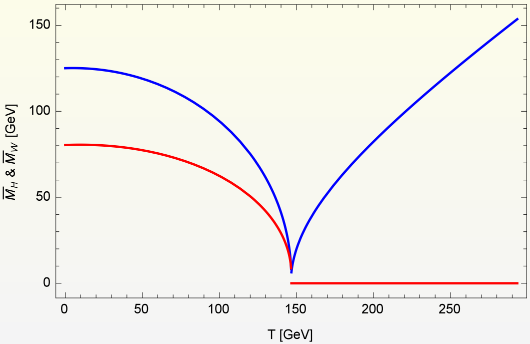


Figure: The temperature-dependent Higgs and W boson masses. The blue and red curves represent the Higgs and W-boson masses.

- So in cosmology the monopole mass depends on the production temperature. The universe starts to produce baby monopole with mass 1.4 TeV at T_c and stops the monopole production with mass 10 TeV at T_G.

- The monopole mass approaches to the adolescent mass (the zero temperature mass) 11.0 TeV as the universe cools down.

- In time scale the monopole production starts at $1.8 \times 10^{-11} sec$ and stops at $1.2 \times 10^{-10} sec$ after the big bang, for the period of $1.03 \times 10^{-10}\ sec$.

Baby Monopole and Adolescent Monopole!

- We can estimate how mamy times the Higgs vacuum fluctuate during this period. Since the average ρ_+ between T_c and T_G is 187.8 GeV, the time for one fluctuation is given by

$$\Delta t \simeq \frac{1}{\Delta E} \simeq 3.34 \times 10^{-27}\ sec.$$

- From this we have

$$N_f \simeq \frac{\bar{t}}{\Delta t} \simeq 3.1 \times 10^{16}.$$

This assures that we have (more than) enough fluctuations of the Higgs vacuum to produce the monopoles.

- Assuming the monopoles are produced between T_c and T_G, around $T_i \simeq (T_c + T_G)/2 \simeq 102$ GeV, with the correlation length $\xi_i \simeq (\xi_c + \xi_G)/2 \simeq 9.3 \times 10^{-16}$ cm, we have the initial monopole density n_i

  $$(n_m)_i \simeq \frac{g_P}{\xi_i^3} \simeq 7.6 \times 10^{44} \times g_p \; cm^{-3},$$

  where $g_P \simeq 0.1$ is the probability that one monopole is produced in one correlation volume.

C. Evolution of Monopole

- The cosmic evolution of the initial monopoles are described by the Boltzmann equation

$$\frac{dn_m}{dt} + 3Hn_m = -\sigma n_m^2,$$

  where H and σ are the Hubble expansion parameter and the monopole annihilation cross section.

- The annihilation cross section is affected by two factors, the mean free length l_{free} of the Brownian motion of the monopole and the capture radius r_{capt} of the monopole-antimonopole attraction.

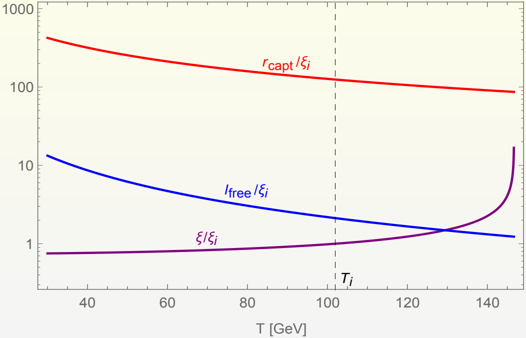


Figure: The relevant scales, ξ in purple, l_{free} in blue, and r_{capt} in red, against T. They are normalized by the correlation length ξ_i at T_i. Here we set $M_m = 5\ \text{TeV}$.

- Solving the Boltzmann equation we have the following result:

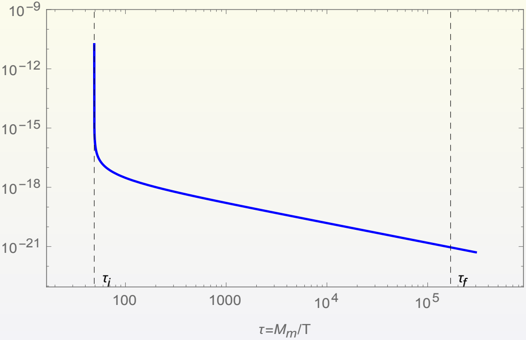


Figure: The evolution of the monopole density n_m against $\tau = M_m/T$. The final value of the monopole density is independent of the initial value.

- Most of the initial monopoles are quickly annihilated since r_{capt} becomes much bigger (by the factor 10^2) than l_{free}.
- The annihilation lasts very long, and stops around $T_f \simeq 29.5$ MeV when l_{free} becomes bigger than r_{capt}. The terminal density at T_f becomes

$$(n_m)_f \simeq 4.95 \times 10^{14}\ cm^{-3}.$$

- The number of monopole within the comoving volume is conserved thereafter. But they interact with the electron pairs before decouple around $T_d \simeq 0.5$ MeV, when the interaction rate becomes less than the expansion rate.

D. Relic Monopole Density

- Assuming that the expansion is adiabatic we have the current density parameter of monopole

  $$n_0 \simeq 1.86 \times 10^{-20}\ cm^{-3},$$
  $$\Omega_m\ h^2 = \frac{\rho_{m,0}}{\rho_{\mathsf{c},0}}\ h^2 \simeq 4.3 \times 10^{-8},$$

  where $\rho_{\mathsf{c},0}$ is the critical density of present universe and $h \simeq 0.678$ is the scaled Hubble parameter.

- This is about about 0.7×10^{-13} of the baryon number density (roughly 2×10^7 monopoles per every volume of the earth in the universe).

- So the electroweak monopole does not alter the standard cosmology. In particular, it can not be the dark matter, but there are enough monopoles left over in the universe that we could detect.

- The free streaming monopole density, however, colud be much lower:

  1. Many of them might have been buried in stellar objects and galactic center.
  2. Most of the leftover monopoles could have been trapped and filtered out by stellar objects when they collide with them.

- Moreover, they generate a huge density perturbation

$$\frac{\delta\rho}{\rho} \simeq 10^{43},$$

so that many of them evolve to primordial magnetic black holes (PMBHs) and become the seed of stellar objects and galaxies. Moreover, they could generate the intergalactic magnetic field.

- In fact the monopoles, coupled to gravity, automatically turn to the Reissner-Nordstrom type PMBHs. This enhances the possibility that they become the seed of the stellar objects and galaxies, and could even account for the dark matter of the universe.

Experimental Detection of Electroweak Monopole

A. Monopole Detection at LHC

- Since the electroweak monopole has unique characters, MoEDAL at LHC could detect it without much problem.
- There are three monopole production mechanisms, the Drell-Yan (and two photon production) process, the Schwinger mechanism, and the topological production mechanism.
- If LHC produces the monopole by Drell-Yan or Schwinger mechanism, the present 14 TeV LHC may not be able to produce when the mass becomes over 7 TeV.

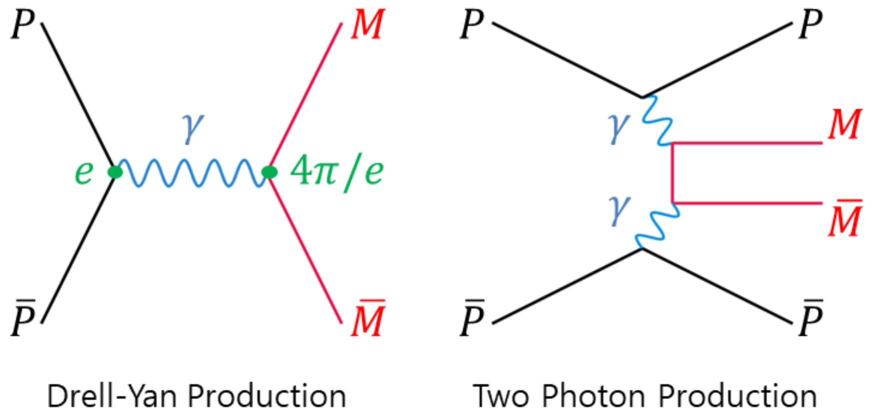


Figure: The Drell-Yan and two photon monopole production mechanism.

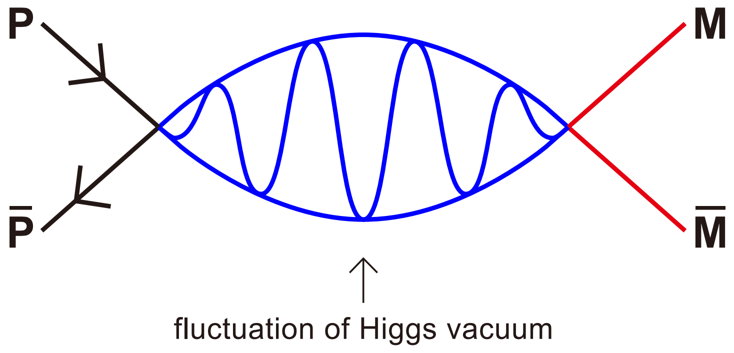


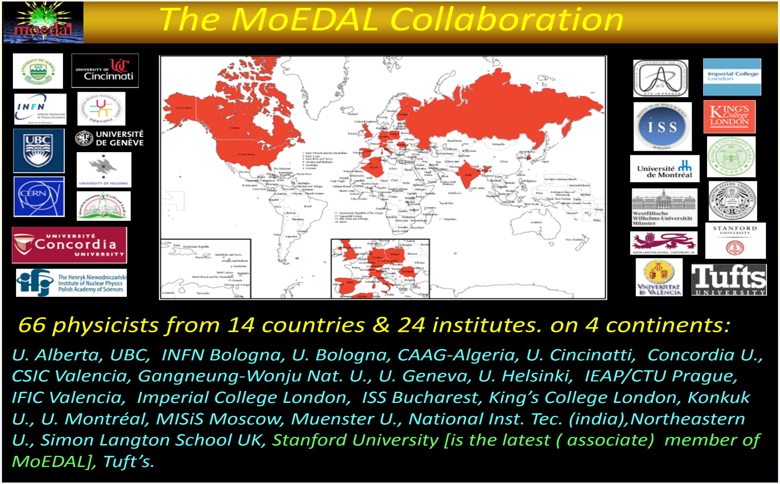
Figure: The topological monopole production mechanism by thermal fluctuation of Higgs vacuum which can not be expressed by Feynman diagram.

- It has been asserted that LHC could reproduce the early universe. If so, it could produce the monopole topologically. In this case the monopole mass depends on the production temperature, with $M(T) \simeq M_W(T)/\alpha$.

- So LHC could produce the infant electroweak monopoles with mass around 5.3 TeV, lighter than the adolescent monopole mass expected to be around 11.0 TeV.

- LHC could also produce the monopolium bound state with mass around 2.5 TeV, even when the mass of the monopole-antimonopole pair becomes around 10.6 TeV. This enhance the possibility for LHC to produce the monopole greatly.

Infant Monopole and Adolescent Monopole



The MoEDAL Collaboration

66 physicists from 14 countries & 24 institutes. on 4 continents:

U. Alberta, UBC, INFN Bologna, U. Bologna, CAAG-Algeria, U. Cincinatti, Concordia U., CSIC Valencia, Gangneung-Wonju Nat. U., U. Geneva, U. Helsinki, IEAP/CTU Prague, IFIC Valencia, Imperial College London, ISS Bucharest, King's College London, Konkuk U., U. Montréal, MISiS Moscow, Muenster U., National Inst. Tec. (india),Northeastern U., Simon Langton School UK, Stanford University [is the latest (associate) member of MoEDAL], Tuft's.

Figure: The MoEDAL (The Magnificent Seventh) Collaboration at CERN, now 70 physicists from 30 institutes.

B. Cosmic Monopole Detection

- Ultra High Energy Cosmic Ray: Recently Telescope Array detected a 244 EeV (244×10^{18} eV) cosmic ray, following the 320 EeV cosmic ray discovered in 1991.

  1. There appears no known particle which could explain such high energy cosmic ray.
  2. It comes from nowhere, i.e., from the void.

Oh-My-God / Amaterasu Particle

"Incomplete Knowledge of Particle Physics???"

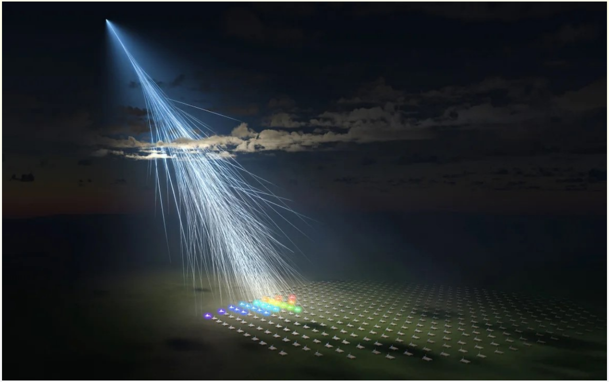


Figure: The Amaterasu Particle: The ultra high energy cosmic ray detected by the Surface Detector of Telescope Array which covers 700 km^2, Utah West Desert, 1370 m above the sea level.

- Relativistic protons moving through the 3K cosmic microwave background have the resonant scattering

  $$p + \gamma \to \Delta^* \to N + \pi,$$

  with 6 Mpc mean free path in average. This prevent them to acquire the energy above 5×10^{19} eV. This is the Greisin-Zatsepin-Kuzmin (GZK) energy limit.

- With mass around 11 TeV, the electroweak monopoles have no such resonant scattering ($M + \gamma \to M + \gamma$), and thus could generate ultra high energy cosmic ray beyond the GZK limit.

Monopole Has No GZK Limit!

- With the intergalactic magnetic field $B \simeq 3 \times 10^{-6}\ G$ and the coherent length $L \simeq 300\ pc$, the monopole energy gain becomes

$$\Delta E \simeq \frac{4\pi}{e}\ BL \simeq 1.2 \times 10^{20}\ \text{eV},$$

  which is precisely the energy range of the Oh-My-God particle.

- The remnant electroweak monopoles become highly relativistic and move through the intergalactic space and come from the void.

- The grand unification monopole cannot explain UHECR, because of the huge mass of 10^{15} GeV.
- So the electroweak monopole becomes a most natural candidate for the Oh-My-God particle. There is no other known stable elementary particle which can decribe such cosmic ray.
- To confirm this, we have to measure the magnetic charge of the cosmic ray.

Has TA discovered the electroweak monopole???

Cho-Maison Monopole and Primordial Magnetic Blackhole

A. Importance of Primordial Blackhole

- Zeldovich and Novikov predicted that the density perturbation in early universe near Planck time could produce the primordial blackhole (PBH), which could grow to superheavy blackholes.

- But Hawking and others argued that they could have been evaporated completely by Hawking radiation.

- The PBH has become important because with mass range 10^{17} to 10^{22} g produced between 10^{-21} to 10^{-16} sec after the big bang could account for the dark matter of the universe.

- Moreover, they could carry color charge, because they are produced before $T \simeq \Lambda_{QCD}$.

- It has been asserted that "these featherweight PBHs can't be produced by any known physical process, and if we find them it will shake up theoretica physics". And NASA plans to hunt for them with the new Roman Space Telescope.

- When coupled to gravity, however, our electroweak monopole turns to a primordial magnetic blackhole (PMBH). This PMBH could account for the dark matter and has deep implications in cosmology.

B. Primordial Magnetic Blackhole from Electroweak Monopole

- With the static spherically symmetric metric

$$ds^2 = -N^2(r)A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d^2\theta + \sin^2\theta d\varphi^2),$$

the Einstein-Weinberg-Salam Lagrangian reduces to

$$S = -\int \Big[\frac{r\dot{A} + A - 1}{8\pi G} + AK + U\Big] N dr,$$

$$K = \frac{\dot{f}^2}{g^2} + \frac{r^2}{2}\dot{\rho}^2,$$

$$U = \frac{(1-f^2)^2}{2g^2r^2} + \frac{\lambda}{8}r^2(\rho^2 - \rho_0^2)^2 + \frac{1}{2g'^2r^2} + \frac{1}{4}f^2\rho^2.$$

- From this we have

$$\frac{\dot N}{N} = 8\pi G \frac{K}{r},$$
$$\dot A + \frac{A-1}{r} = -\frac{8\pi G}{r}(AK+U),$$
$$A\ddot f + \Big(\dot A + A\frac{\dot N}{N}\Big)\dot f + \frac{1-f^2}{r^2} f - \frac{1}{4} g^2 \rho^2 f = 0,$$
$$A\ddot\rho + \Big(\frac{2A}{r} + \dot A + A\frac{\dot N}{N}\Big)\dot\rho - \frac{f}{2r^2}\rho = \frac{\lambda}{2}(\rho^2 - \rho_0^2)\ \rho.$$

- This has two types of solutions, the gravitating Cho-Maison monopole and magnetic blackhole.

Gravitating Cho-Maison Monopole

- With the boundary condition

$$f(0)=1,\quad \rho(0)=0,\quad m(0)=0,$$
$$f(\infty)=0,\quad \rho(\infty)=\rho_0,\quad N(\infty)=1,$$

we have the gravitating monopole solution very similar to Cho-Maison monopole which has the following expansion near $r=0$,

$$f(r)=1-f_1x^2+...,\quad \rho(r)=h\rho_0x^{\delta}+...,\quad \delta=\frac{\sqrt{3}-1}{2},$$
$$A(r)=1-\frac{16\pi}{e^2}\sin^2\theta_W\Big(\frac{M_W}{M_P}\Big)^2\frac{\delta^2}{\sqrt{3}}h^2x^{2\delta}+..., \tag{1}$$

where f_1 and h are constants, $x=M_Wr$, and M_P is the Planck mass.

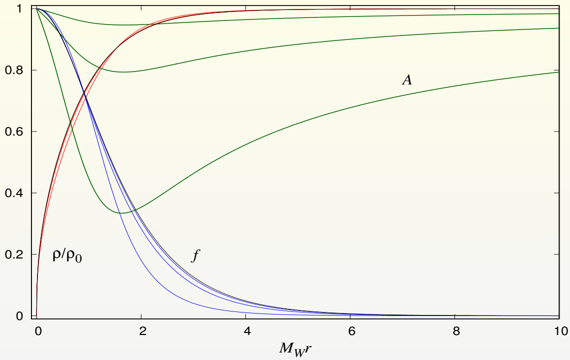


Figure: The W-boson f (blue), Higgs field ρ (red), and the metric function A (green) profiles of the gravitating monopole, obtained with $M_W/M_P = 0.3,\ 0.6$, and 1.2. Notice that in real world $M_W/M_P \simeq 6.6 \times 10^{-18}$, and the metric is almost flat.

Primordial Magnetic Blackhole

- With $f = 0$ and $\rho = \rho_0$, we have the RN blackhole,

$$N = 1, \quad A(r) = 1 - \frac{2GM}{r} + \frac{4\pi G}{e^2}\frac{1}{r^2},$$

where M is the ADM mass of the blackhole.

- This has the outer horizon

$$r_H = r_+ = MG + \sqrt{M^2G^2 - 4\pi G/e^2},$$
$$r_H^{eRN} = \frac{L_P}{\sqrt{\alpha}} \simeq 1.87 \times 10^{-32} cm,$$

where $L_P \simeq 1.6 \times 10^{-33} cm$ is the Planck length.

- The mass of the extremal RN blackhole is fixed by

  $$M \geq M_{eRN} = \frac{M_P}{\sqrt{\alpha}} \simeq 1.43 \times 10^{20}\mathrm{GeV}.$$

  So the gravity sets the minimum mass of the naked Cho-Maison monopole of the order of ten times the Planck mass.

- This confirms that the naked Cho-Maison monopole without the weak boson dressing, when coupled to gravity, becomes the RN blackhole with no limit on the maximum mass.

- We can also have the modified RN blackhole which has the weak boson dressing whose size and mass are bounded by

$$\cos\theta_W \frac{L_P}{\sqrt{\alpha}} < r_H \leq \frac{1}{M_W},$$

$$1.3 \times 10^{17}\ \text{TeV} < M_{MRN} \leq 9.3 \times 10^{32}\ \text{TeV}.$$

- Our PMBH has different production mechanism and different production time, and can not evaporate. Nevertheless, our PMBH could also account for the dark matter, and could carry the color charge.

- Do we need two PBHs in cosmology? Which one is more realistic?

Important Question!

Prospect

A. Experimental

- In spite of the huge efforts the search for the monopole has not been successful. Most were the blind searches in the dark room. Focusing on the electroweak monopole, we should be able to do better.

- A major concern at LHC has been whether it can produce the monopole. If it can, MoEDAL has the best chance.

- The existing remnant monopole detectors (IceCube, ANTARES, Auger, etc.) have a serious trouble. We need “Cosmic” MoEDAL installed in high mountains.

B. Theoretical

- Can the monopoles indeed be the seed of the primordial black holes and galaxies? Can it explain the dark matter? Do they generate the inter-galactic magnetic field? Could they trigger the electroweak baryogenesis?

- How can we justify the perturbative expansion in the presence of monopole? Can we construct the quantum field theory of monopole which generalizes QED?

- If detected, it will open the new era in physics.

Topological Avatar of New Physics!

The Importance of High Risk Research

"So many centuries after the Creation, it is unlikely that anyone could find hitherto unknown lands of any value." - Spanish Royal Commission, rejecting Christopher Columbus' proposal to sail west.

Figure: The Columbus proposal was denied, but he prevailed!

- References

1. Y.M. Cho and D. Maison, Phys. Lett. B391, 360 (1997).
2. Yisong Yang, Proc. Roy. Soc. A454, 155 (1998); *Solitons in Field Theory and Nonlinear Analysis,* Springer-Verlag, 2001.
3. Y.M. Cho, Kyoungtae Kimm, and J.H. Yoon, Euro. Phys. J. C75, 67 (2015).
4. J. Ellis, N. Mavromatos, and T. You, Phys. Lett. B756, 29 (2016).
5. Y.M. Cho, Kyoungtae Kimm, and J.H. Yoon, Euro. Phys. J. C77, 88 (2017).
6. F. Blaschke and P. Benes, Prog. Theo. Exp. Phys. 073B03 (2018).
7. Y.M. Cho, Phil. Trans. Roy. Soc. **A377**, 0038 (2019).