



# International Conference on QCD Vacuum Structure and Confinement (QCD-VSC-2024)

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Naxos Island of Cyclades, Greece



## **Manifestations of High-Energy QCD Asymptotics**

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NRC Kurchatov Institute, Gatchina**



## **Outline:**

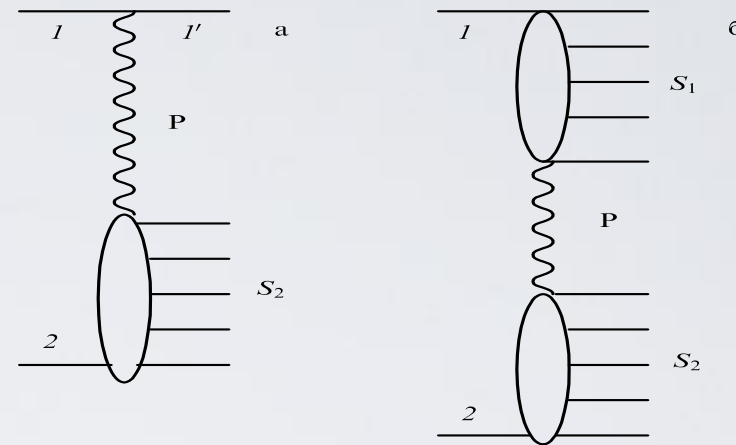
- **Introduction & Motivation**
- **High-energy asymptotics of QCD: BFKL evolution**
- **Manifestation of BFKL evolution at LHC**
- **Summary**

# High-Energy Asymptotics: Pomeron



Pomeron at high energies is responsible for:

- elastic scattering
- diffractive scattering
- inelastic scattering
- total x-section



# Pomeron before QCD: original foundations



asymptotic theorem:

particle and antiparticle x-section equality

I. Pomeranchuk 34 (1958) 725

non-relativistic scattering: Regge poles

T. Regge (1959, 1960)

relativistic scattering: Regge poles

V. Gribov Nucl. Phys. 22 (1961) 249

Gribov-Froissart representation

M. Froissart Phys. Rev. 123 (1961) 1053

Pomeron: vacuum pole and trajectory  $\alpha_{IP}(t) = \alpha_{IP}(0) + \alpha'_{IP} \cdot t$

V. Gribov ZhETP 41 (1961) 667 [JETP 14 (1962) 472]

G. Chew, S. Frautschi PRL 7 (1961) 394

$$-1, C = -1, G = -1, I$$

$$\frac{d\sigma_{el}}{dt} = F(t) s^{2\alpha(0)-2} e^{-2\alpha'|t| \ln s}$$

➡ elastic & diffractive cone shrinkage

➡ x-section: constant with energy IHEP (Protvino) U70 data since 1967

ories discussed in Sect. 5.7 have inte



## Unitarity condition:

Froissart-Martin x-section asymptotic bound  $\leq \log^2(s)$

M. Foissart Phys. Rev. 123 (1961) 1053

## Reggeon field theory

V.N. Gribov (1967)

## multi-Pomeron exchanges

V.N. Gribov, A.A. Migdal (1968-1970)

K.A. Ter-Martirosyan, A.A. Migdal, A.M. Polyakov 1972-1975

A.B. Kaidalov K.A. Ter-Martirosyan 1973-1979

## supercritical Pomeron $\alpha_{\text{IP}}(0) > 1$

V.N. Gribov, A.A. Migdal, A.M. Polyakov 1970-1975

## strongly-interacting supercritical Pomeron

V.N. Gribov, A.A. Migdal, A.M. Polyakov 1969



## Born approximation: two-gluon Pomeron

F.E. Low, Phys. Rev. D12 (1975) 163

S. Nussinov, Phys. Rev. Lett. 34 (1975) 1286

## Leading logarithmic approximation: LL BFKL Pomeron

V.S. Fadin, E.A. Kuraev, L.N. Lipatov, Phys. Lett. B 60 (1975) 50

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, ZhETF 71 (1976) 840 [JETP 45 (1977) 79]

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, ZhETF 72 (1977) 377 [JETP 45 (1977) 79]

I.I. Balitsky, L.N. Lipatov, Yad. Fiz. 28 (1978) 1597

## Next-to-leading logarithmic approximation: NLL BFKL Pomeron

V.S. Fadin, L.N. Lipatov, Phys. Lett. B 429 (1998) 127

E.A. Camici, L.N. Ciafaloni, Phys. Lett. (1998)

S.J. Brodsky V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov, Pisma ZhETF 70 (1999) 161 (BFKLP)



## - Large-angle scattering (hard processes):

### QCD in Bjorken limit

- **GLAPD: V. Gribov & L. Lipatov (71-72); L. Lipatov (74); G. Altarelli & G. Parisi (77); Yu. Dokshitzer (77)**

## - Small-angle scattering (“semi-hard” processes):

### QED in Gribov-Regge limit

- **V. Gribov, V. Gorshkov, L. Lipatov & G. Frolov (67-70)  
H. Cheng & T. Wu (66-70)**

### QCD in Gribov-Regge limit

- **BFKL: V. Fadin, E. Kuraev & L. Lipatov (75-78)  
I. Balitsky & L. Lipatov (78)**

# High-energy QCD asymptotics: GLAPD and BFKL



$$s=(p_1+p_2)^2$$
$$t=(p_1-p_3)^2 \quad Q^2=-t$$

Scattering in the Standard Model (QCD) at high energies:

Large logarithms: as  $\log(s)$ , as  $\log(Q^2)$

**Bjorken limit (large-angle scattering):**

$$s \sim Q^2 \gg m^2$$

$$Q^2/s = x \sim 1$$

Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (GLAPD):

$(a_s \log(Q^2))^n$  resummation

Inclusive cross section  $\sim 1/Q^4$

**Gribov-Regge limit (small-angle scattering):**

$$s \gg Q^2 \gg m^2$$

$$Q^2/s = x \Rightarrow 0$$

Balitsky-Fadin-Kuraev-Lipatov (BFKL):

$(a_s \log(s))^n$  resummation

Total cross section  $\sim s^{(a_P-1)}$

$a_P$  – Pomeron intercept

soft scattering data:  $a_P = 1.1$



# pQCD x-section asymptotics



**Bjorken limit (GLAPD):**

$$s \sim Q^2 \gg m^2$$

$$Q^2/s = x \sim 1$$

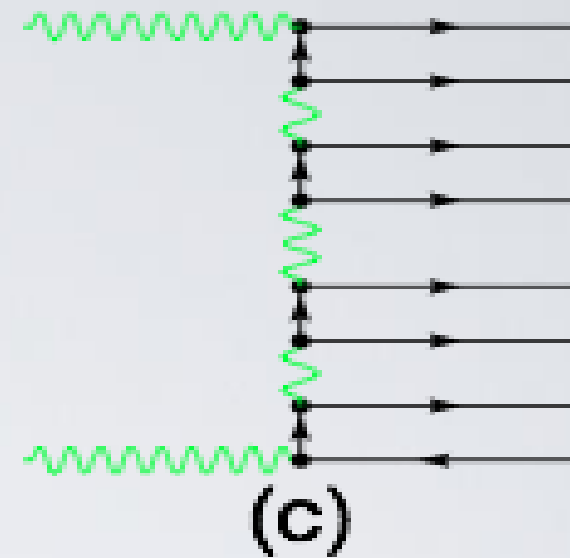
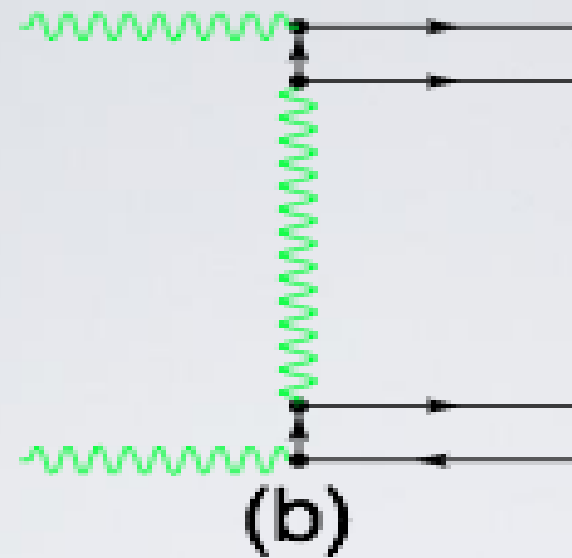
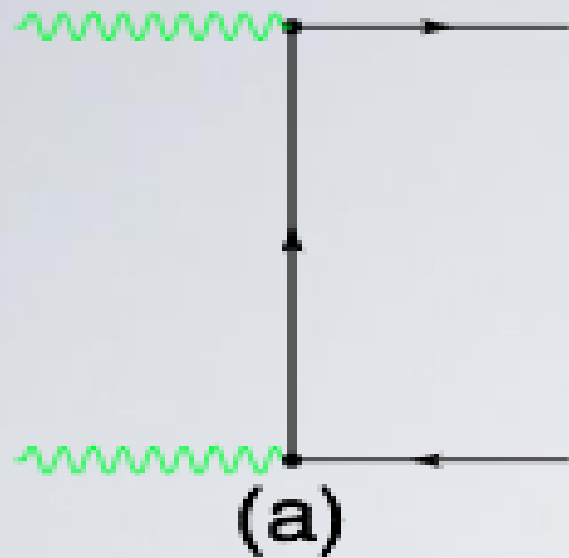
**Large-angle (large-x) scattering**

**Gribov-Regge limit (BFKL):  $s \gg Q^2 \gg m^2$**

$$Q^2/s = x \rightarrow 0$$

**Small-angle (small-x) scattering**

# Asymptotics of QED cross sections



$$\sigma \sim (\alpha_{\text{QED}})^2 \log(s)/s$$

$$\sigma \sim (\alpha_{\text{QED}})^4 \text{const}(s)$$

All orders: V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)  
 H. Cheng & T.T. Wu (69-70)

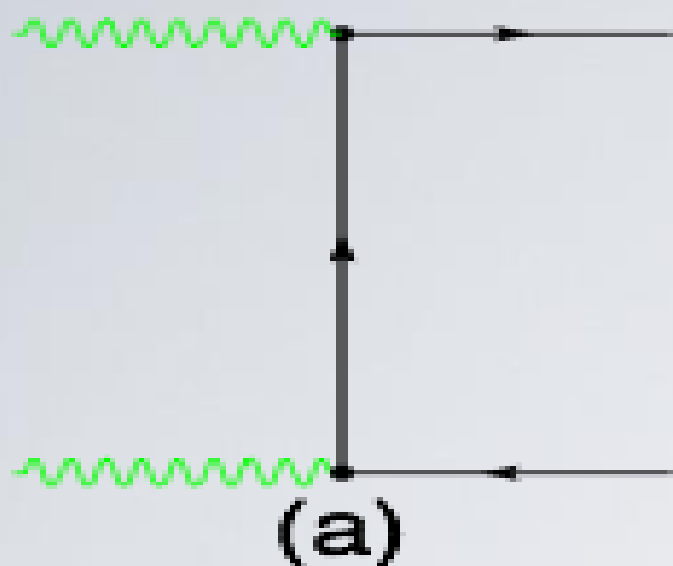
Cross section at  $s \rightarrow \infty$ :  $\sim (\alpha_{\text{QED}})^4 (S/S_0)^{a_P-1}$

$$a_P = 1 + C (\alpha_{\text{QED}})^2 \approx 1.002$$

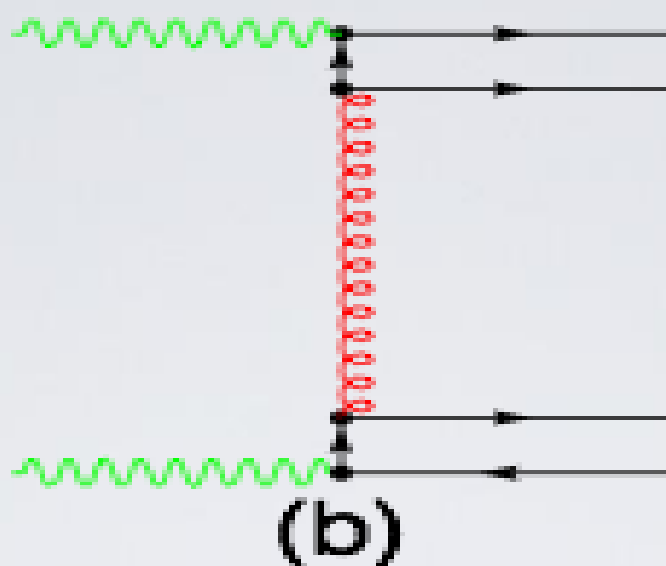
## photon: no reggeization!



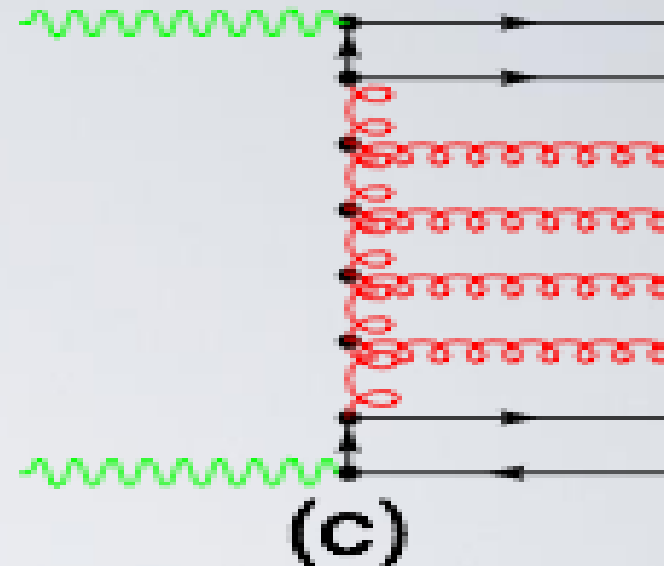
# High-energy limit pQCD as LL BFKL: $\gamma\gamma$



$$\sigma \sim (\alpha_{\text{QED}})^2 \log(s)/s$$



$$\sigma \sim (\alpha_{\text{QED}})^2 (\alpha_s)^2 \text{const}(s)$$



Resummation of all leading logarithms: LL BFKL

## gluon: reggeization!

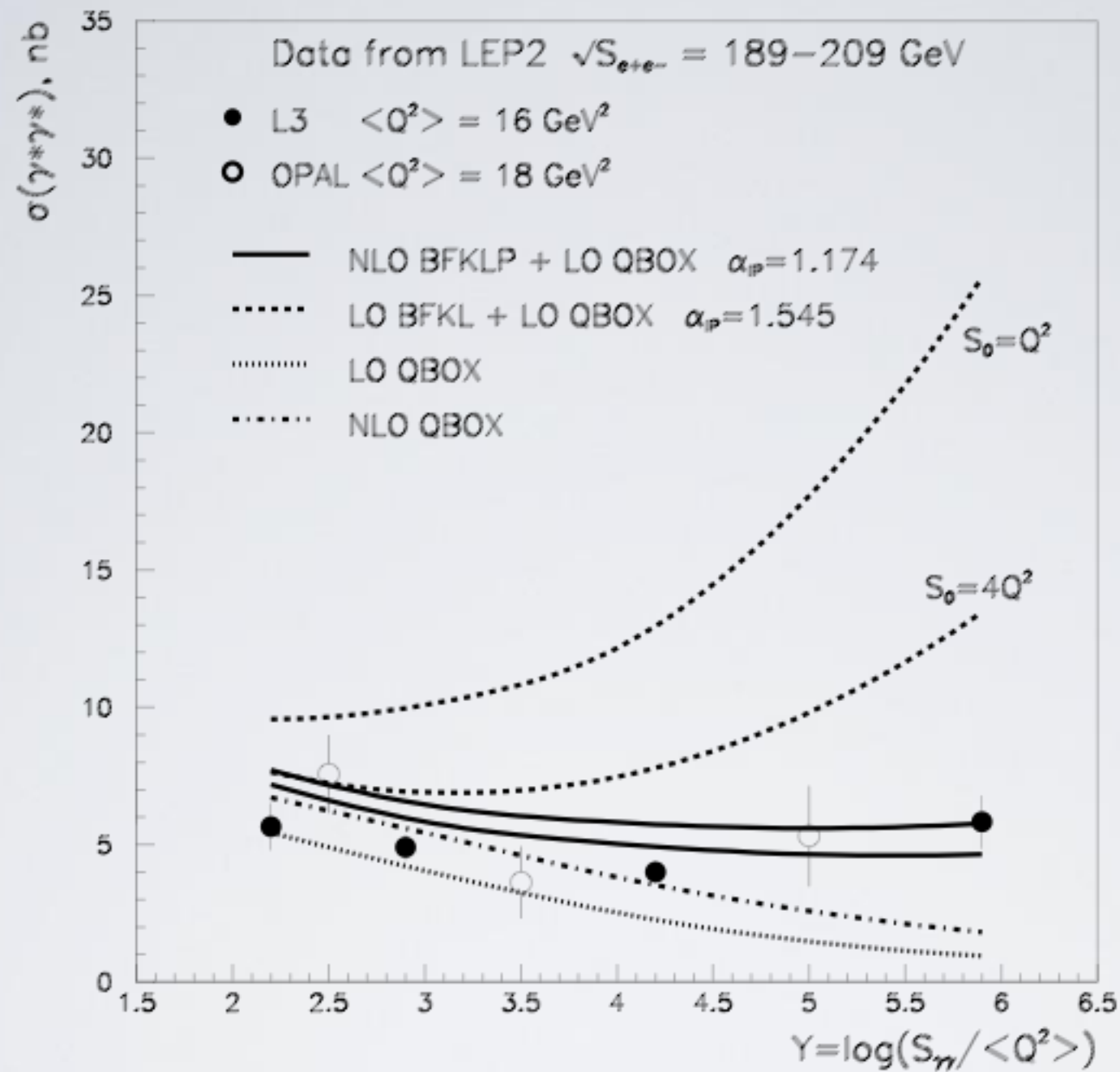
Cross section at  $s \rightarrow \infty$ :  $\sim (\alpha_{\text{QED}})^2 (\alpha_s)^2 (S/S_0)^{a_P-1}$

$a_P = 1 + C \alpha_s \approx 1.5$  LL BFKL S. Brodsky & F. Hautmann (96)

$a_P = 1 + C \alpha_s \approx 1.2$  NLL BFKL

S. Brodsky, V Fadin, VK, L. Lipatov, G. Pivovarov (2001-02)

# Highly virtual photon scattering at LEP-2



S.J. Brodsky, V.K., L.N. Lipatov, V.S. Fadin & G.B. Pivovarov (2002)

BFKLP: NLL BFKL + generalized BLM

LO Impact factor

Full NLL BFKL calculations: require extra studies



# LL BFKL: problems

LL BFKL: designed for infinite collision energies  
multi-Regge-kinematics

LL BFKL problems (at finite energies):

- fixed (non-running) coupling  $\alpha_s$
- energy-momentum conservation
- transverse momentum conservation

Cross section in LL BFKL:

$$\sigma = \sigma_0 (S/S_0)^{a_P-1} \quad a_P = 1 + C \alpha_s \approx 1.5-1.6$$

Data:  $a_P \approx 1.2-1.3$



# **BFKL: next-to-leading logs (NLL) improved by running $a_s$**

next-to-leading log approximation (NLL) BFKL  
MSbar-renormalization scheme: large corrections

V.S. Fadin & L.N. Lipatov (89-98)

C.Camici & M. Ciafaloni (96-98)

BFKLP: NLL BFKL + resummation of running coupling  $a_s$   
generalized for the case with non-Abelian LO

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP

→ BLM approach Brodsky, Lepage & Mackenzie – 1983

→ works only (!) for the case with Abelian LO



# BFKLP: generalized BLM for non-Abelian case

**S. Brodsky, P. Lepage, P. Mackenzie (1983) BLM**

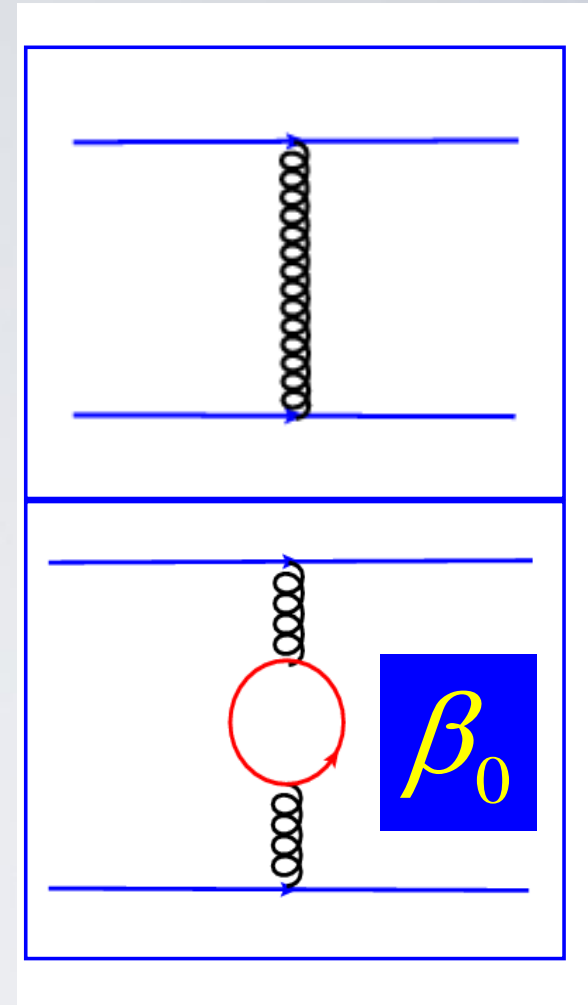
propagator insertions. It seems difficult if not impossible to separate the divergent part of the vertex, which realizes  $\alpha_s$  from the finite process dependent part  $i$

$$\beta_0 = \frac{11}{3} N_C - \frac{2}{3} n_F$$

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \beta(\alpha_s) = - \sum_{i \geq 0} \beta_i(n_f) \alpha_s^{i+2}$$

(d) Equation (11a) is a particularly convenient perturbative results since all flavor dependence is explicit in the definition of  $\alpha_{\overline{MS}}$ .

The  $\beta$  function is defined in terms of any (c) The leading-order  $\beta_0$  which comes from quadratic divergences. This is usually all that



LO Abelian -> LO non-Abelian  
MSbar-scheme -> MOM scheme 3g-vertex

**S. Brodsky, V. Fadin, V.K., L. Lipatov, G. Pivovarov(99) BFKLP**



# **BFKLP (generalized BLM) works for non-Abelian cases**

## **NLL BFKL and $Y \rightarrow ggg$ decay**

**Naïve BLM application does not work (!):**

- **NLL BFKL in  $\overline{MS}$  scheme**
- **Upsilon  $\rightarrow ggg$  decay in NLO in  $\overline{MS}$  scheme**

**$\overline{MS}$ -scheme: nonphysical RG scheme (!)**

**numerically close to V-scheme (heavy quark potential) – Abelian in LO**

**physical RG scheme: MOM scheme (gauge dependent)**

- **NLL BFKL**                       **$\leftarrow$  non-Abelian in LO**
- **Upsilon  $\rightarrow ggg$  decay**  **$\leftarrow$  non-Abelian in LO**

**one can use MOM-scheme based on ggg-vertex non-Abelian in LO**

**BLM generalized for non-Abelian case:**

**S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP**

**BFKLP: NLL BFKL + resummation of running coupling as**

**BLM resummation depends on non-Abelian structure in LO**



# BFKLP: NLL BFKL within generalized BLM



S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP

$$\omega_{\overline{MS}}(Q_1^2, \nu) = \int d^2 Q_2 K_{\overline{MS}}(\mathbf{Q}_1, \mathbf{Q}_2) \left( \frac{Q_2^2}{Q_1^2} \right)^{-\frac{1}{2} + i\nu}$$

$$\sigma \sim s^{\alpha_{IP} - 1} = s^{\omega_{\max}}$$

$$= N_C \chi_L(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \left[ 1 + r_{\overline{MS}}(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \right],$$

$$\chi_L(\nu) = 2\psi(1) - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)$$

$$r_{\overline{MS}}(\nu) = r_{\overline{MS}}^{\beta}(\nu) + r_{\overline{MS}}^{\text{conf}}(\nu)$$

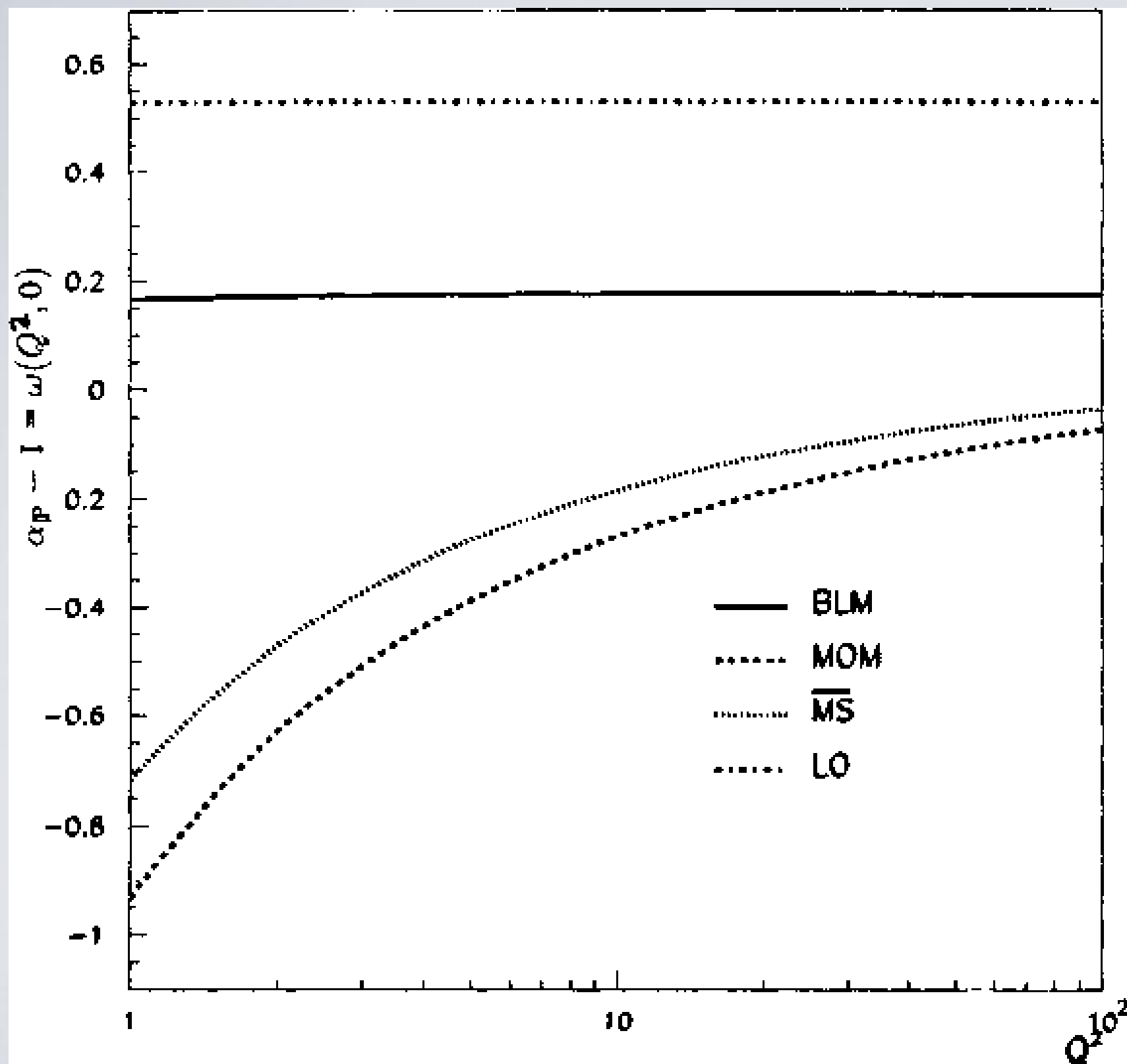
$$r_{\overline{MS}}^{\beta}(\nu) = -\frac{\beta_0}{4} \left[ \frac{1}{2} \chi_L(\nu) - \frac{5}{3} \right]$$

$$r_{\overline{MS}}^{\text{conf}}(\nu) = -\frac{N_C}{4\chi_L(\nu)} \left[ \frac{\pi^2 \sinh(\pi\nu)}{2\nu \cosh^2(\pi\nu)} \left( 3 + \left( 1 + \frac{N_F}{N_C^3} \right) \frac{11 + 12\nu^2}{16(1 + \nu^2)} \right) - \chi_L''(\nu) + \frac{\pi^2 - 4}{3} \chi_L(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} - 6\zeta(3) + 4\varphi(\nu) \right]$$

CD N=4 A.V. Kotikov, L.N. Lipatov (2000)



# BFKLP: NLL BFKL within generalized BLM



$$\sigma \sim S^{\alpha_{IP} - 1} = S^{\omega^{\max}}$$

# BFKLP: NLL BFKL within generalized BLM



V.S. Fadin & L.N. Lipatov (89-98)

C. Camici & M. Ciafaloni (96-98)

next-to-leading log approximation (NLL) BFKL

$\overline{\text{MS}}$ -renormalization scheme: large corrections

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov (98-99) BFKLP

BFKLP: NLL BFKL + resummation of running coupling  $\alpha_s$

in physical renormalization scheme

BFKLP: Conformal BFKL kernel in NLL  $\rightarrow$  SUSY N=4

Pomeron intercept:  $a_P = 1.2 - 1.3$

Cross section:  $\sigma_0 (S/S_0)^{a_P-1}$   $a_P = 1 + C \alpha_s$

L.N. Lipatov, A.V. Kotikov et al. (2000-06)

SUSY N=4 BFKL Pomeron

Anomalous dimensions: test of AdS/CFT



# **BFKL observables**

**Heavy quark production**

I.I. Balitsky, L.N. Lipatov (1978)

**Inclusive jet**

M.G. Ryskin (1980)

**Lepton pair production**

M.G. Ryskin, E.M. Levin (1981)

**Deep inelastic processes -> small-x physics  
unitarization -> small-x shadowing**

L.V. Gribov, M.G. Ryskin, E.M. Levin (1981-83)

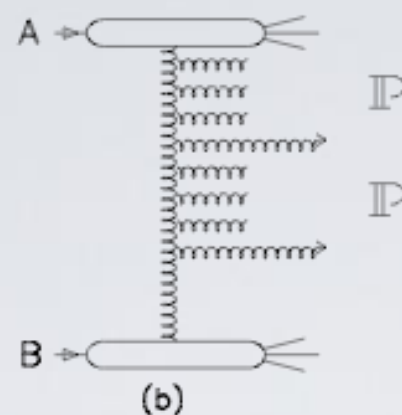
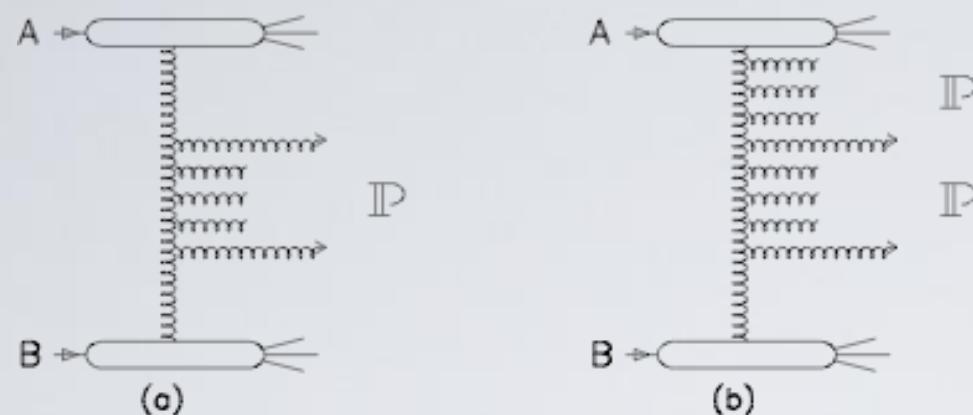
**Most forward/backward (Mueller-Navelet) dijets:**

**x-section  $\sim \exp(|\Delta|y)$**

A. Mueller & H. Navelet, Nucl. Phys. B (1987)



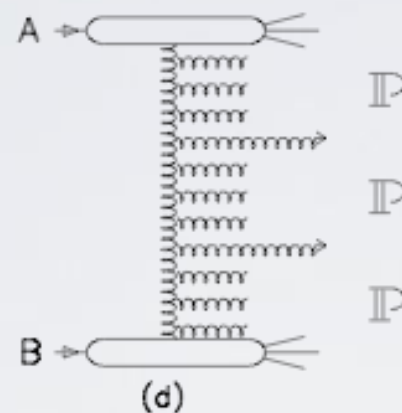
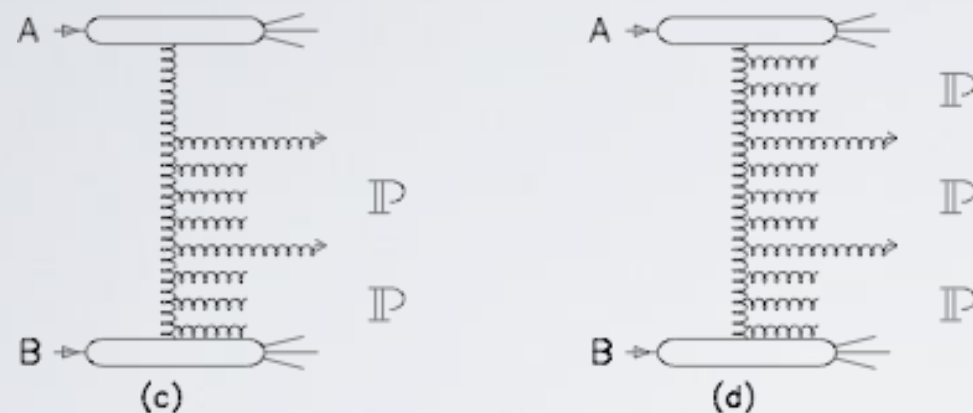
# BFKL direct observable: dijet with large rapidity separation between jets



Jet production

GLAPD: ordering on  $\kappa T$   
 $y$  – no ordering

BFKL: ordering on  $y$   
 $\kappa T$  – no ordering



Most forward/backward (Mueller-Navelet) dijets: x-section  $\sim \exp(|\Delta|y)$

A. Mueller & H. Navelet, Nucl. Phys. B (1987)

Most forward/backward (Mueller-Navelet) dijets: azimuthal decorrelations

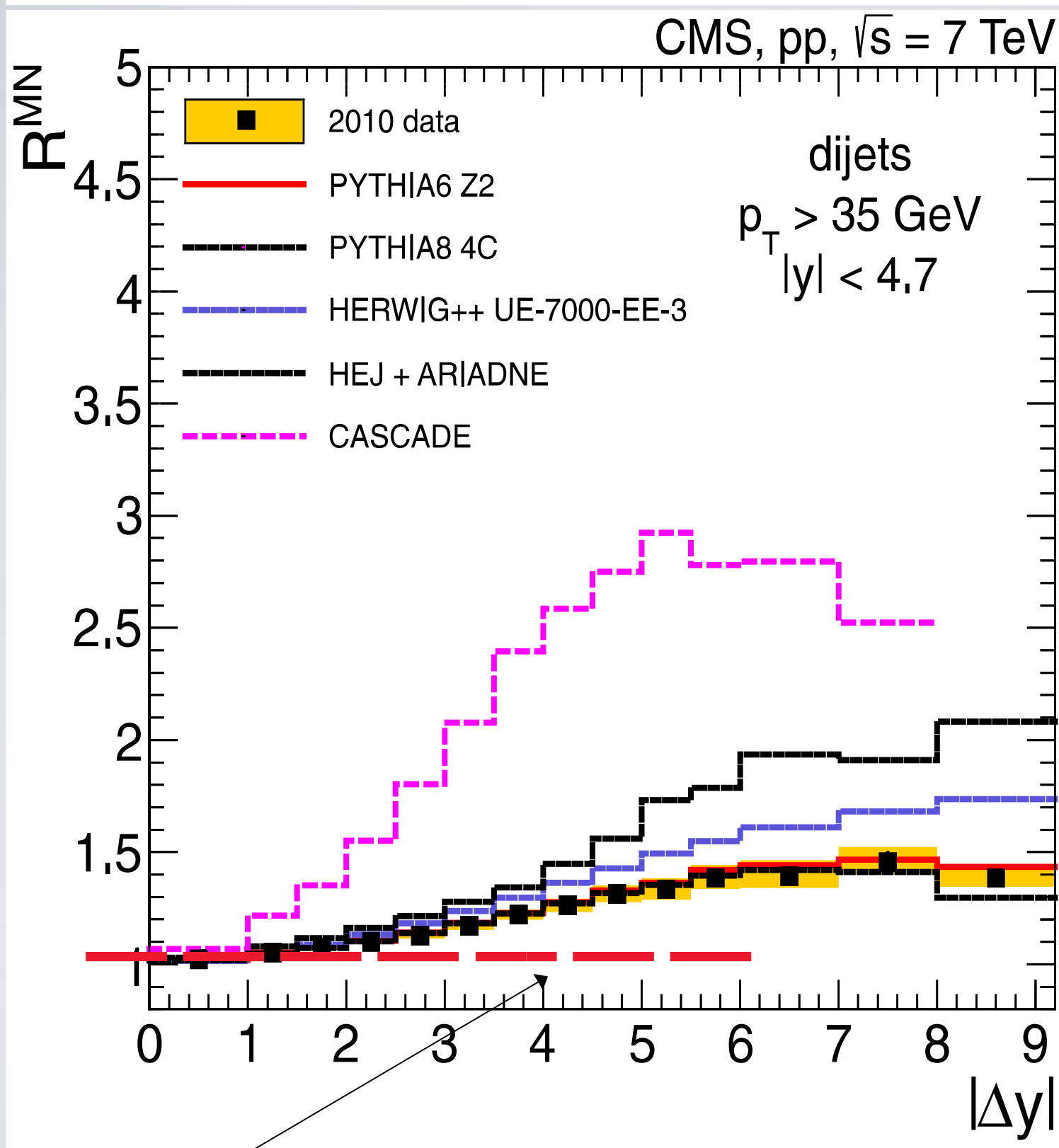
V. Del Duca & C. Schmidt, Phys. Rev. D (1994)

W.J. Stirling, Nucl. Phys. B (1994)

Inclusive dijets

VK & G.B. Pivovarov, Phys. Rev. D (1996)

# CMS: dijet “K-factor”

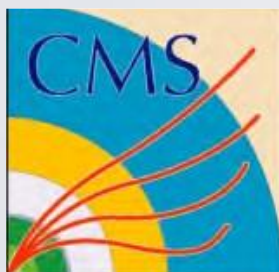
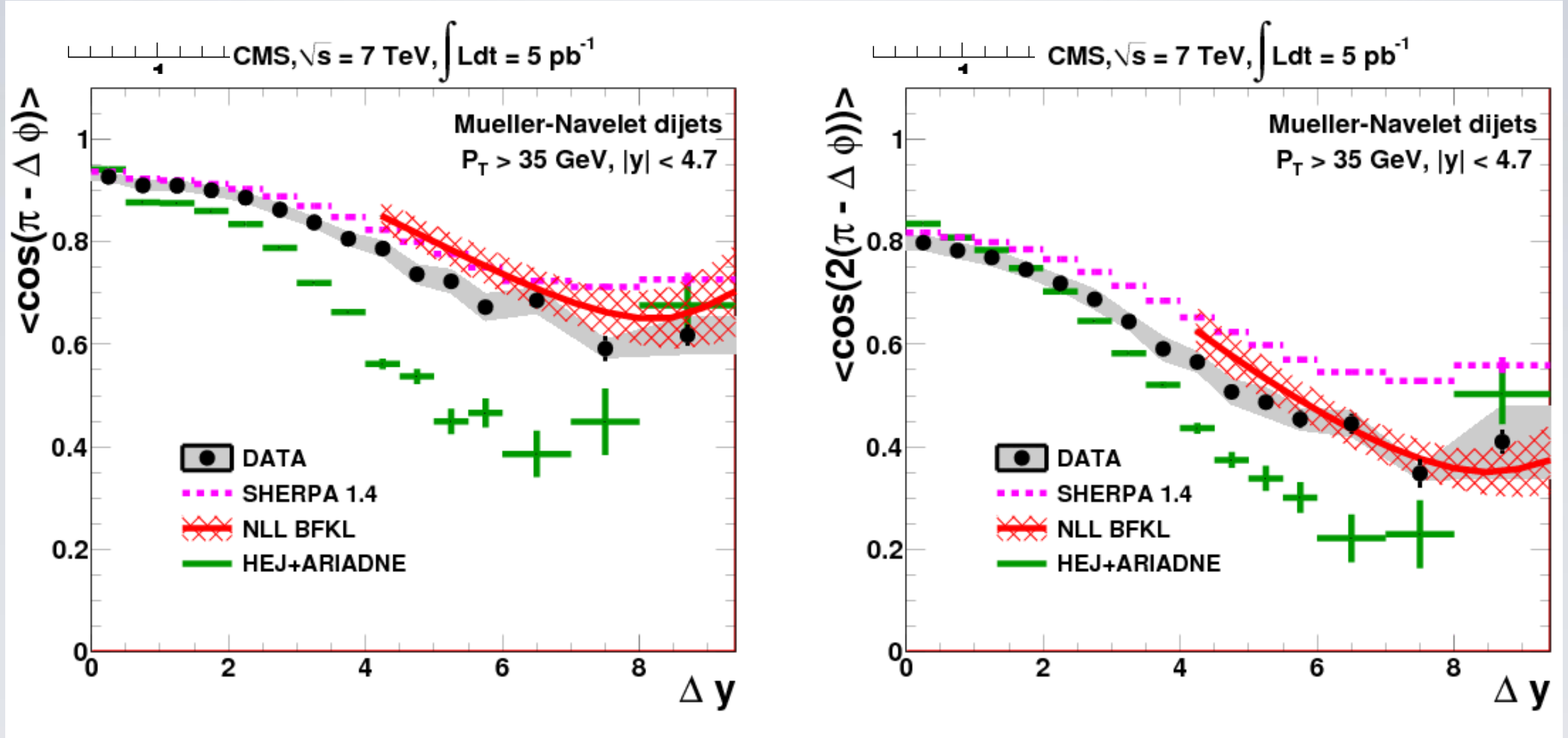


**EPJ C 72 (2012) 2216**  
**7 TeV,  $p_{T\_min} = 35$  GeV**  
 **$\Delta y = | | < 9.4$**

**MC generators:  
contain terms  
beyond GLAPD**

**GLAPD**

# Dijets: $\langle \cos \rangle$ vs NLL BFKL+BFKLP



**CMS (2016)**  
**7 TeV,  $p_{T\_min} = 35 \text{ GeV}$**   
 **$\Delta y = | | < 9.4$**

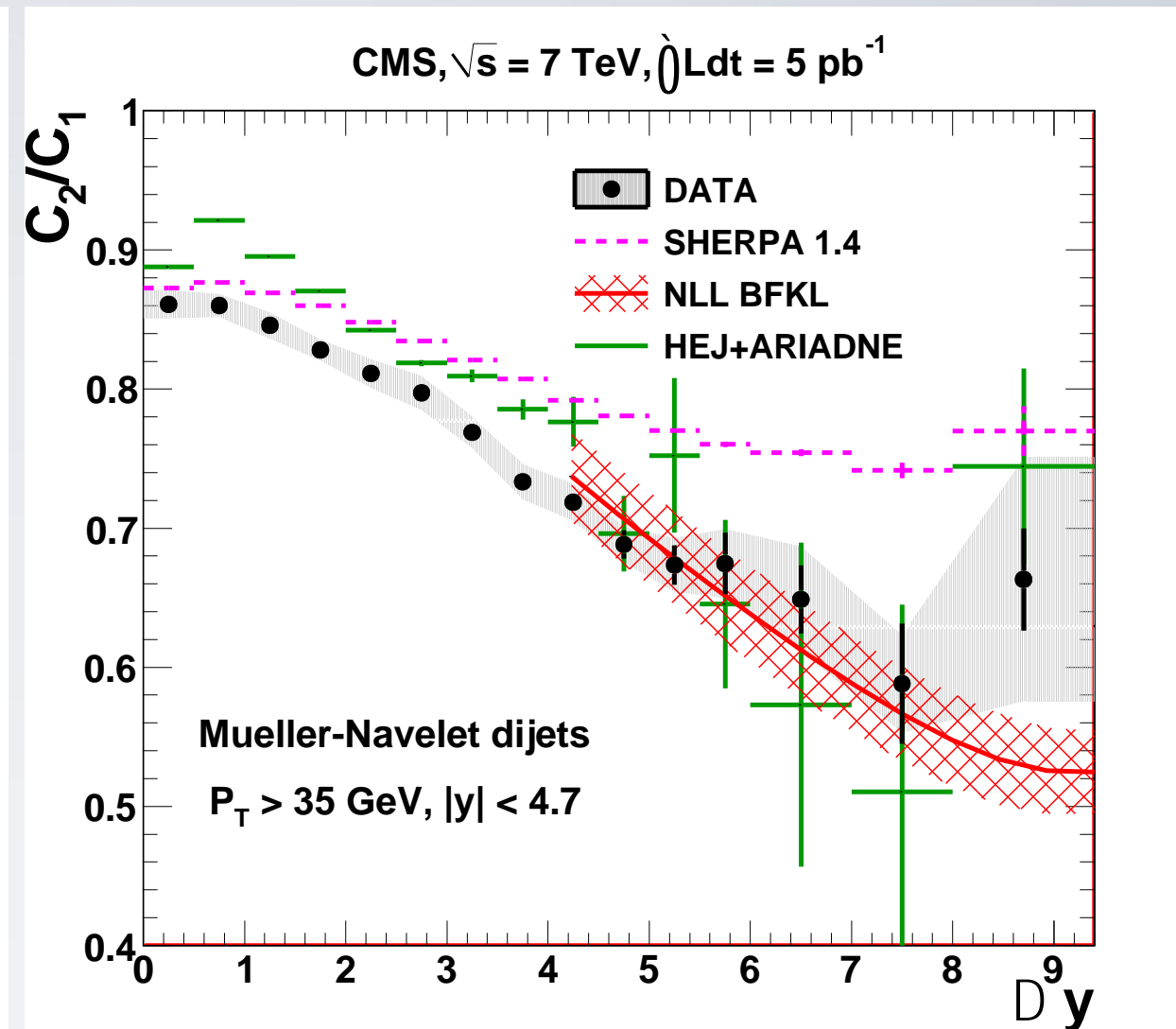
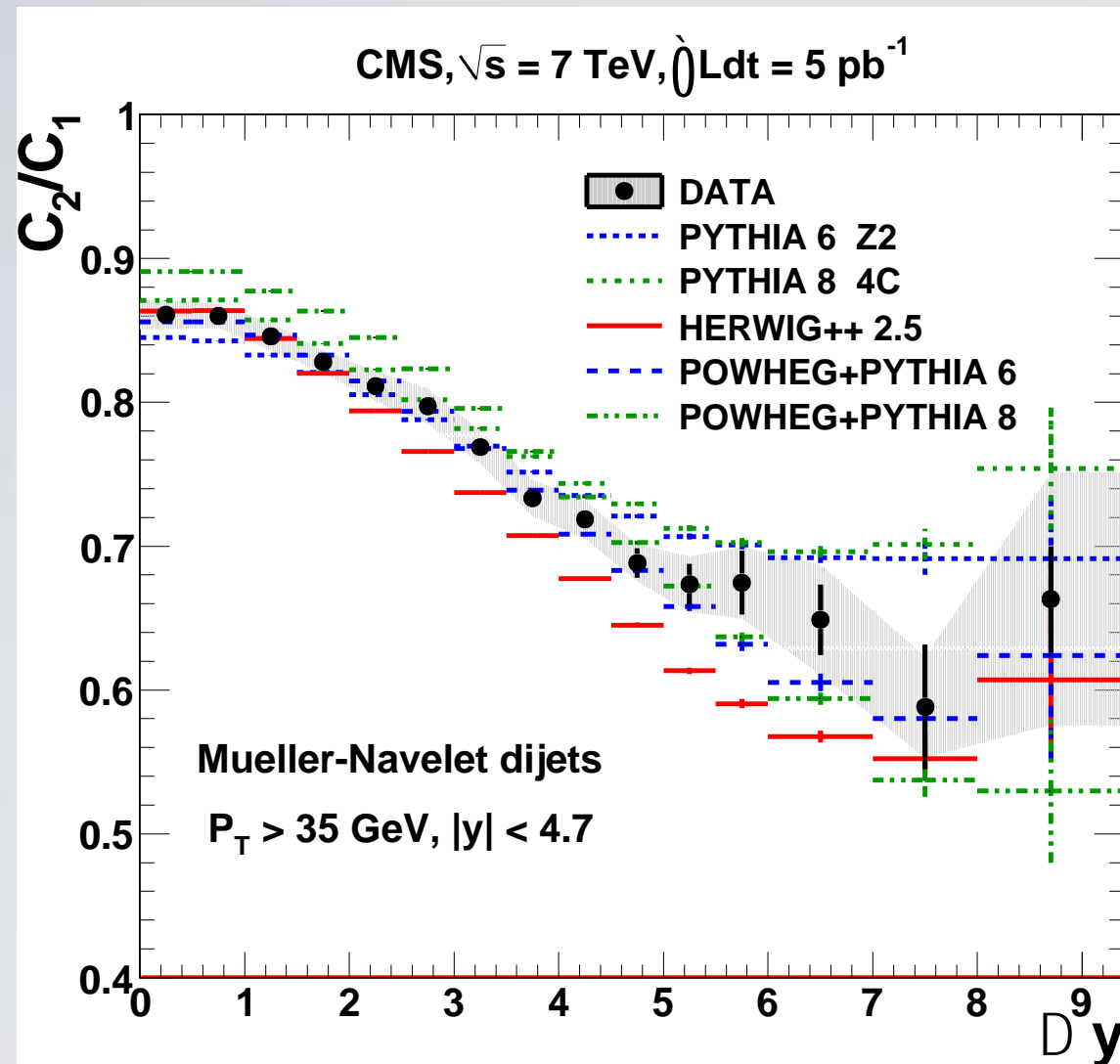
**NLL BFKL + BFKLP (2014)**  
**B. Ducloue, L. Szymanowski & S. Wallon**



# Dijets: $\langle \cos 2\phi \rangle / \langle \cos \phi \rangle$ vs NLL BFKL + BFKLP

## BFKL conformal feature: cosine ratio

A. Sabio Vera et al (2007)



**CMS (2016)**  
**7 TeV,  $p_{T\_min} = 35$  GeV**  
 **$\Delta y < 9.4$**

**NLL BFKL + BFKLP (2014)**  
**B. Ducloue, L. Szymanowski & S. Wallon**

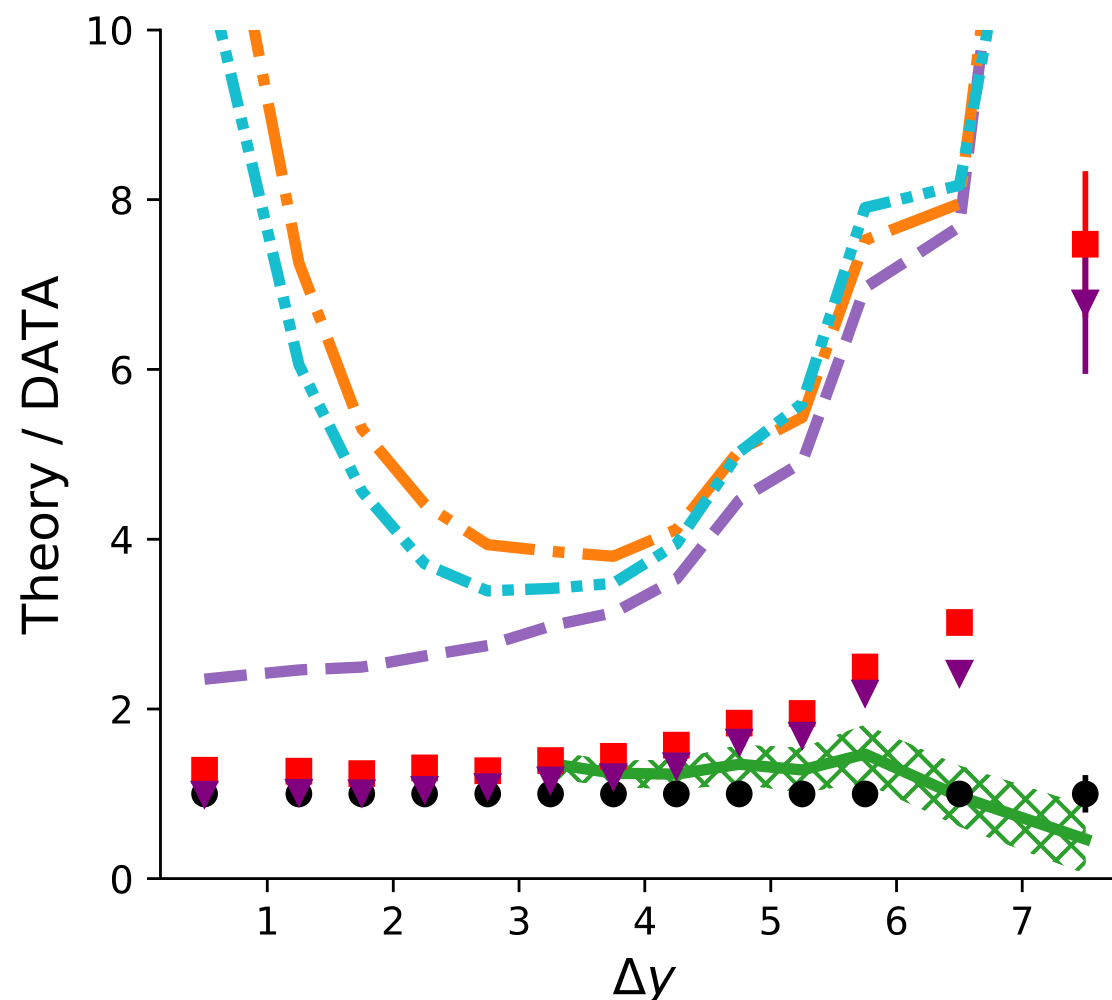
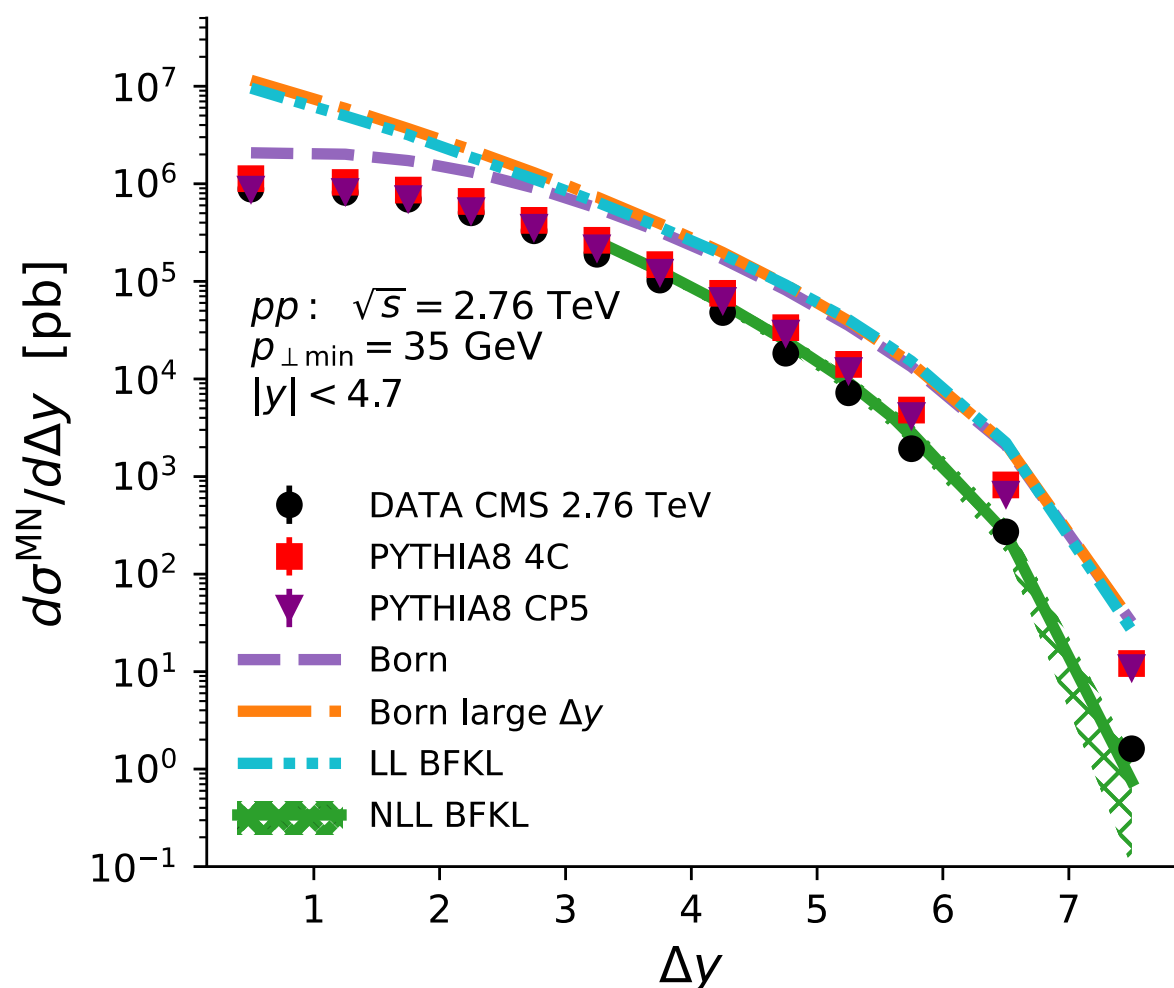




# MN dijets within NLL BFKL improved by BFKLP

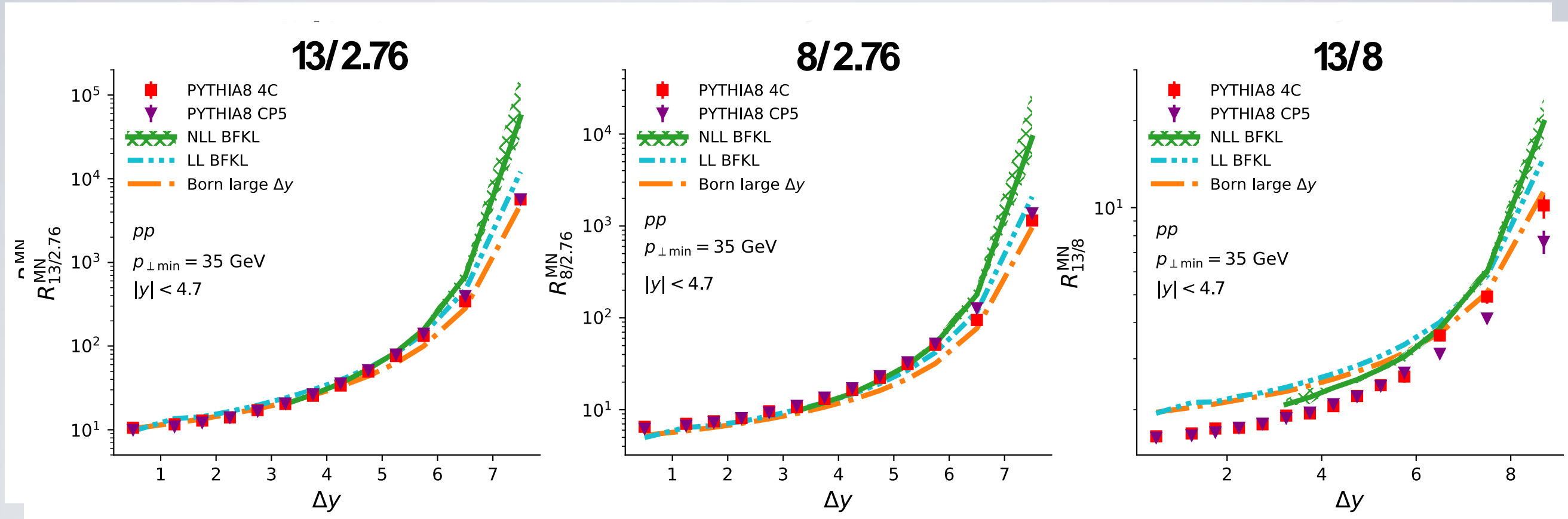
**BFKL with BFKLP** F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, *Phys. Rev. D* (2015)

**BFKL with BFKLP: 2.76 TeV dijet x-section** A. Egorov & VK *Phys. Rev. D* (2023)



**CMS (2022)**  
**2.76 TeV,  $p_{T\_min} = 35 \text{ GeV}$**

# MN dijet x-section ratio within NLL BFKL with BFKLP: collision energy dependence at LHC



**A. Egorov & VK, Phys. Rev. D (2023)**

**NLL BFKL with BFKLP prediction: strong energy dependence**

# Direct NLL BFKL manifestation in dijets



## CMS dijet production (2022) with large rapidity separation between jets A. Egorov & VK, Phys. Rev. D (2023)

- > Some indication on BFKL in exclusive dijets production
  - at LHC 13 TeV at CMS:
  - **Mueller-Tang (MT) dijets**
- > Some indication with NLL BFKL (BFKLP improved) in Mueller-Navelet (MN) and inclusive dijet in x-section ratios and azimuthal decorrelations at LHC 7 TeV
  - **MN and inclusive dijet**
- > The new observation of NLL BFKL (BFKLP improved) in dijets
  - in at LHC 2.76 TeV
  - **MN dijet x-sections A. Egorov & VK, Phys. Rev. D (2023)**
- **Prediction for dijet observables:**
  - - MN dijet x-section energy ratios  $8/2.76$ ,  $13/2.76$   $13/8$
  - - K-factor with extra jet veto, number of extra jets, ... ?
  - **LHC Run 3 at 13.6 TeV ?!**

# **Pomeron in pQCD: established NLL BFKL in dijets**



## **New Physics:**

- **new particles and interactions beyond SM**
- **new dynamics within SM**

## **New dynamics within SM:**

- **phase transitions at dense baryon matter**
- 

**NB. New Physics beyond SM should manifest above  
new high energy SM dynamics!**

# LL BFKL remarkable properties



**2D-conformal properties**

**BFKL(Schredinger eq) as "quantization" of RG-DGLAP (Euler-Lagrange eq)**

**L.N. Lipatov (1986)**

**Effective action for reggeized gluons**

**L.N. Lipatov (1995)**

**LL BFKL 2D-conformal block symmetry:  
Feynman-like rules for inclusive x-sections**

**VK, G.B. Pivovarov (1997)**

**LL BFKL 2D-conformal block symmetry**

**H. Navelet, R. Peschanski (1998-1999)**

**Effective Regge QCD: gluon intercept as RG constant**

**VK, G.B. Pivovarov (1997)**

**Feynman rules for Reggeized gluons**

**E.N. Antonov, E.A. Kuraev, L.N. Lipatov, I. Cherednikov (2005)**



## LL BFKL Pomeron

2D conformal symmetry and  $1/N$  expansion

↳ factorization into integrable theory

**high-energy QCD -> integrable system!**

L.N. Lipatov (1994)

L.D. Faddeev, G.P. Korchemsky (1994)

## LL BFKL Pomeron with $1/N$ expansion

### Dipole Pomeron

A.H. Mueller (1994)

N.N. Nikolaev, B.G. Zakharov (1994)

## Reggeon field theory with BFKL Pomeron

E.M. Levin, A. Kovner, M. Lublinsky (2024)



## kT-factorization

S. Catani, M. Ciafaloni, F. Hautmann (1991)

J.C. Collins, R.K. Ellis (1991)

E.M. Levin, M.G. Ryskin, Yu. Shabelski, M.G. Shuvaev (1991)

G. Salam, H. Jung, N. Raicevic

S.P. Baranov, A.V. Lipatov, M.A. Malyshev, N.P. Zotov, G.I. Lykasov,

V.A. Saleev, A. Shipilova, A. Nefedov, ...

## CCFM evolution: interpolates with color coherence between LL BFKL and DGLAP

M. Ciafaloni (1988), S. Catani, F. Fiorani, G. Marchesini (1990)

## KMR evolution: interpolates between LL BFKL and DGLAP

M.A. Kimber, A.D. Martin, M.G. Ryskin (1999)

# BFKL generalizations in QCD



Non-planar Pomeron in QCD with  $1/N$  expansion:

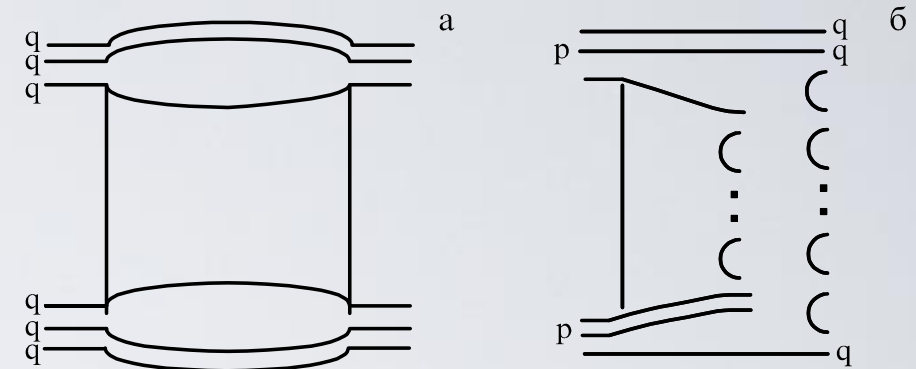
G.Veneziano (1977)

- dual parton model

A.Capella, J. Tran Thanh Van (1981)

- quark-gluon string model (QGSM)

A.B. Kaidalov, K.A. Ter-Martirosyan (1982)



Unitarity with  $1/N$  expansion for saturation limit

Balitsky-Kovchegov equation with  $\alpha_s \rightarrow 0$ : reproduces BFKL

I.I. Balitsky (1996) Yu. Kovchegov (1999, 2000)

Color Glass Condensate evolution for saturation limit

with  $\alpha_s \rightarrow 0$ : reproduces BFKL

L. McLerran, R. Venugopalan (1994) H. Weigert, A. Kovner, A. Leonidov (2001)

F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan (2010)



# NLL BFKL motivated approaches



SUSY N=2 NLL BFKL Pomeron

**A.V. Kotikov, L.N. Lipatov (2000)**

AdS/CFT-correspondence test with anomalous dimensions

**A.V. Kotikov, L.N. Lipatov, A. Onischenko, V. Velizhanin (2002-2006)**

Graviton-Pomeron duality

**C.-I. Tan, C. Brower (2006)**

**L. Alvarez-Gaume et al. (2007)**

# Summary



- FKL reproduces main classical Pomeron properties bringing new remarkable features: conformality, integrability, AdS/CFT duality, holographic properties ...
- New Physics beyond SM should manifest within BFKL: the new high energy SM dynamics!
- The first direct observation:  
NLL BFKL manifests in dijet production with large rapidity separation in CMS data at LHC 2.76 TeV