



UNIVERSITÄT  
HEIDELBERG  
ZUKUNFT  
SEIT 1386

COMETA WG2: Normalizing Flows in Particle Physics and Beyond  
28.03.2024

# Generative Unfolding

with conditional invertible neural networks

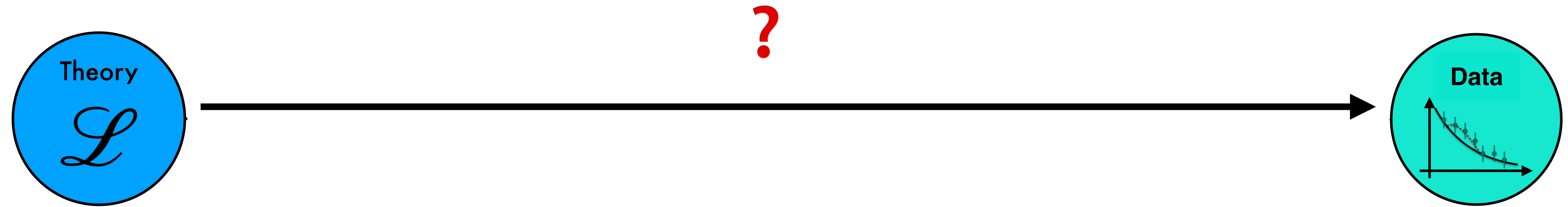
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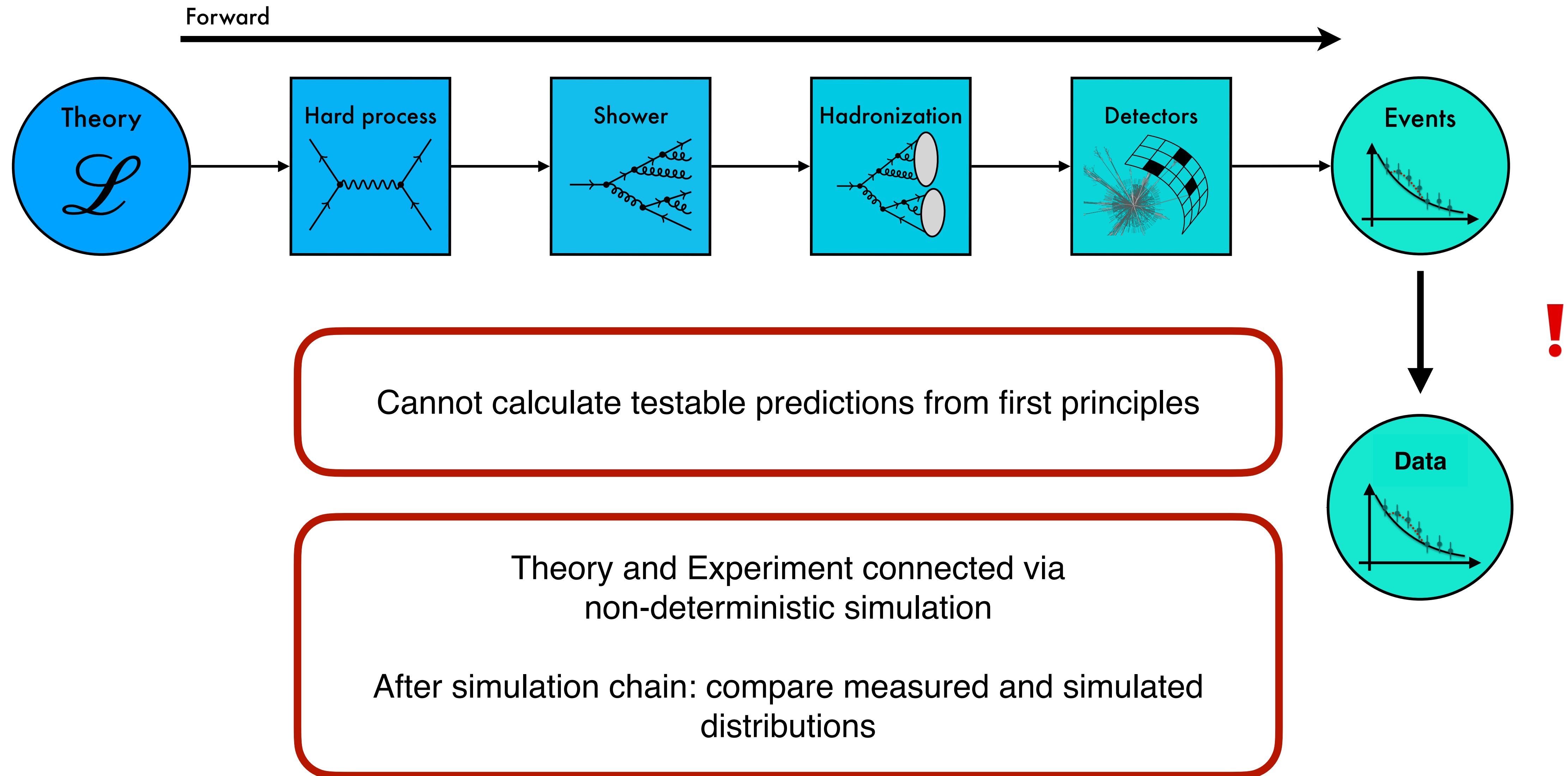
*Sofia Palacios Schweitzer with Tilman Plehn, Anja Butter  
Institute for theoretical Physics — Heidelberg*

# Simulation Chain

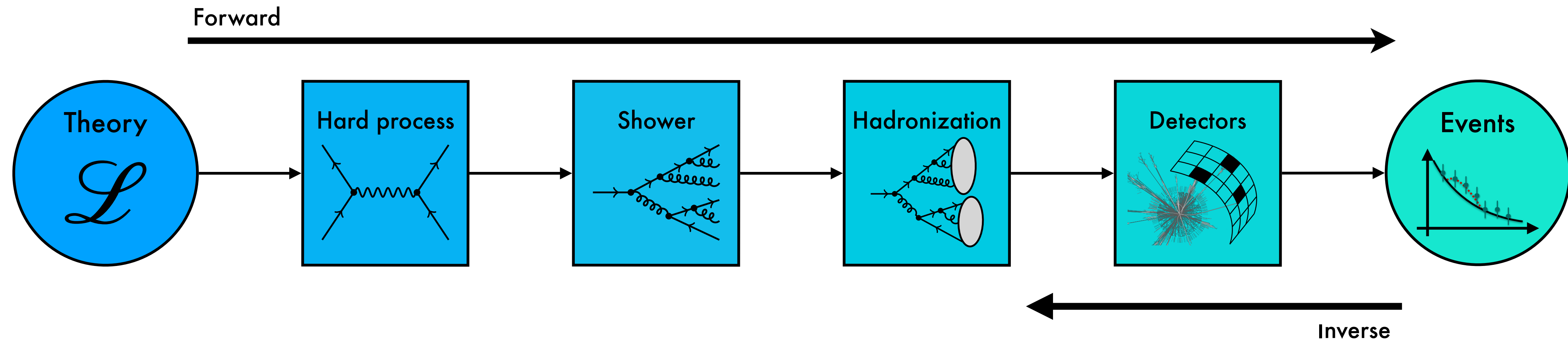


Cannot calculate testable predictions from first principles

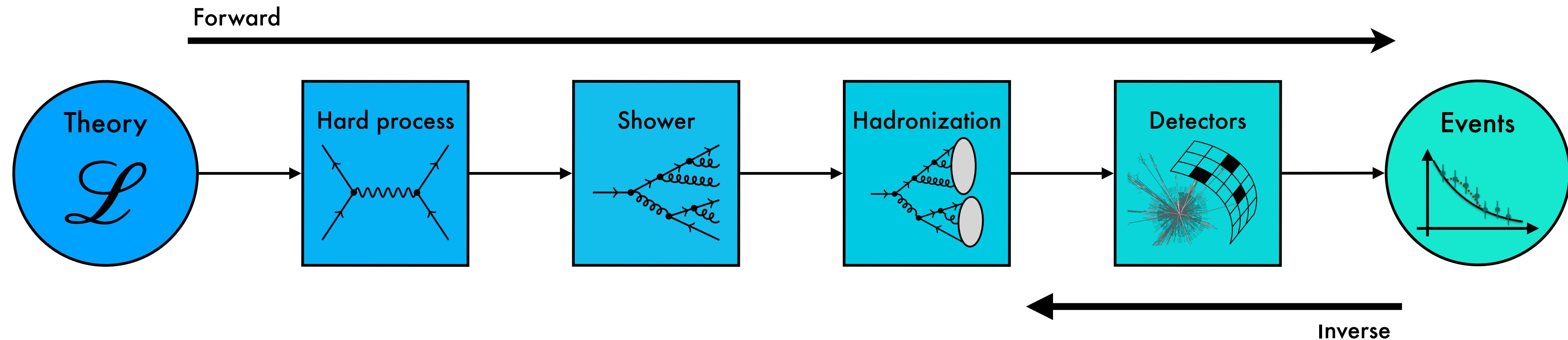
# Simulation Chain



# Simulation Chain — Inversion



# Why unfolding?



Theory analyses don't care about detectors

Comparing data from different experiments (Global Analysis)

For some analysis direct access to theory parameters

Resolution

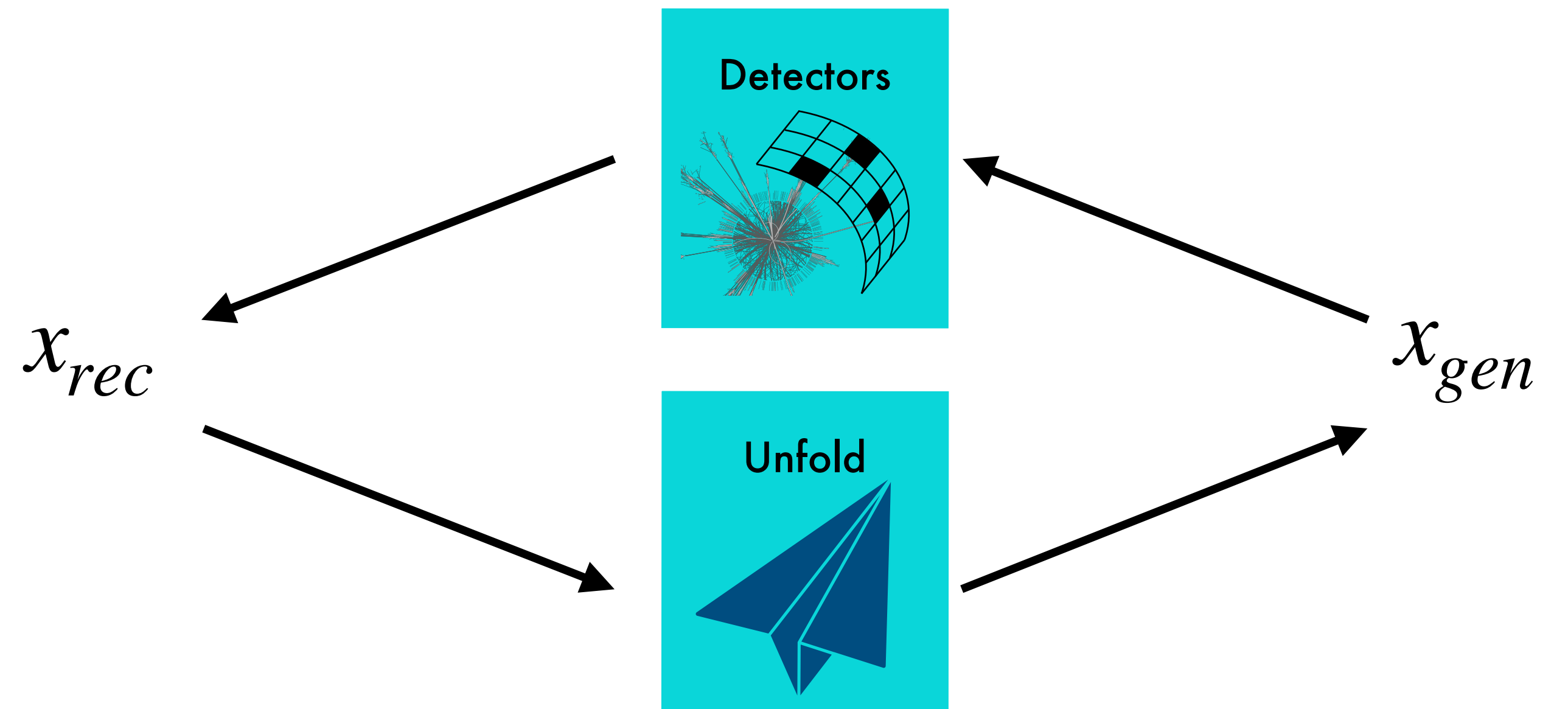
Data preservation

# Unfolding — a toy example

Consider binned distributions at  
gen and rec level

Describe detector effects by  
response matrix  $R$

Unfold by (pseudo-)inverting the  
matrix



$$h_{rec}^i = R^{ij} h_{gen}^j$$

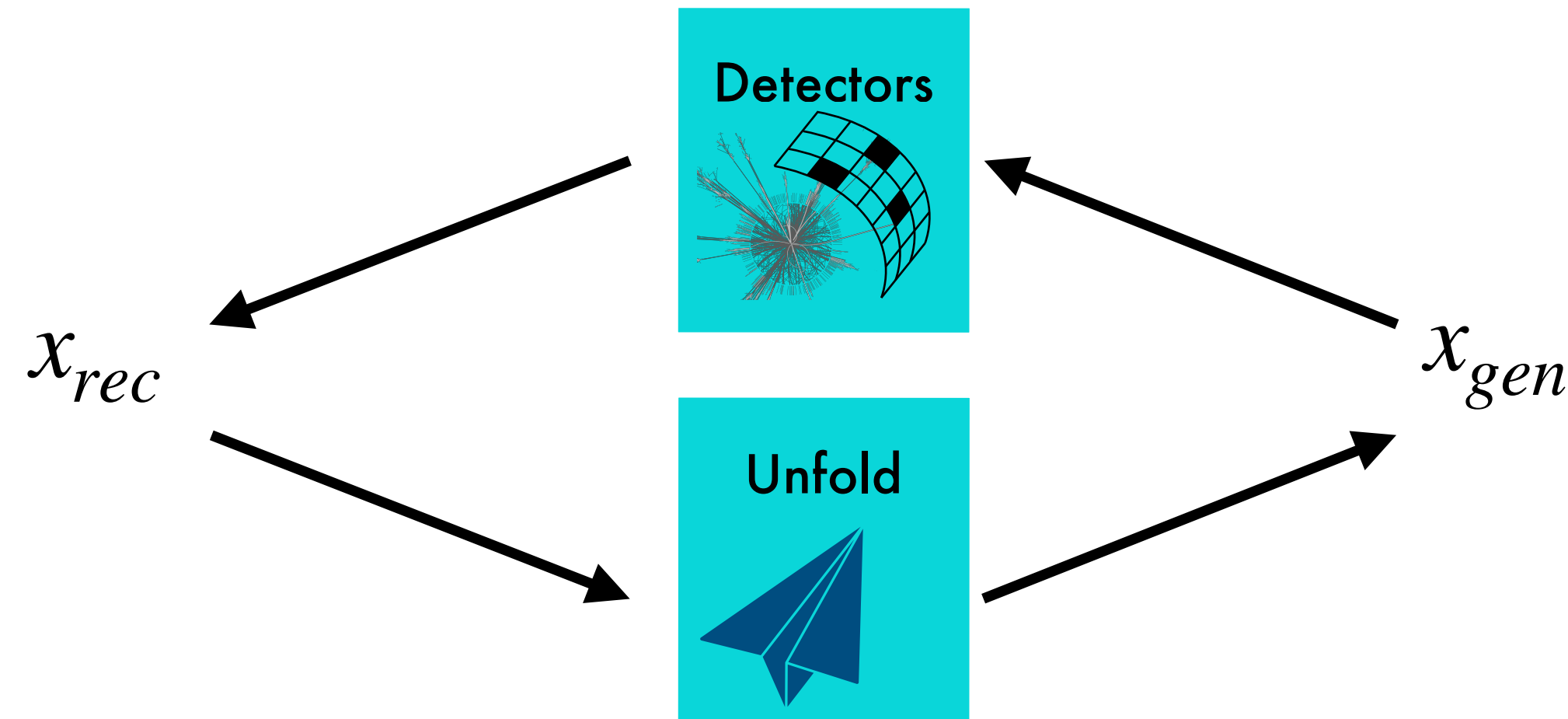
$$h_{gen}^j = (R^{-1})^{ji} h_{rec}^i$$

# Unfolding — a toy example

**Detector: local, linear smearing**

$$R = \begin{pmatrix} 1 - \epsilon & \epsilon & 0 \\ \epsilon & 1 - 2\epsilon & \epsilon \\ 0 & \epsilon & 1 - \epsilon \end{pmatrix}$$

$$h_{rec} = R h_{gen}$$



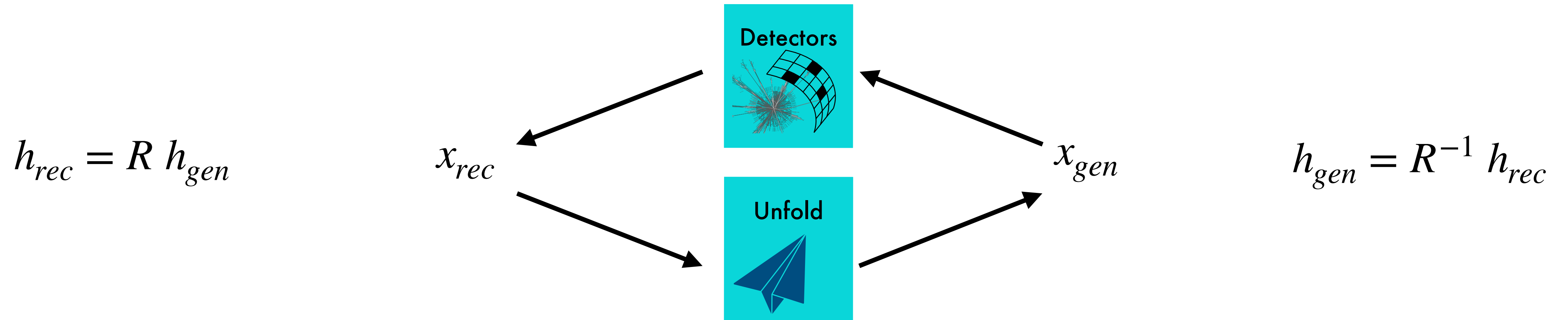
$$h_{gen} = R^{-1} h_{rec}$$

**Inversion: non-local, non-linear**

$$R^{-1} \approx \frac{1}{1 - 4\epsilon} \begin{pmatrix} 1 - 3\epsilon & -\epsilon & \epsilon^2 \\ -\epsilon & 1 - 2\epsilon & -\epsilon \\ \epsilon^2 & -\epsilon & 1 - 3\epsilon \end{pmatrix}$$

(squared terms dropped when linear term non zero)

# Unfolding — classical methods



Classical unfolding methods: binned, one-dimensional

Using inverse matrix gives very high variance

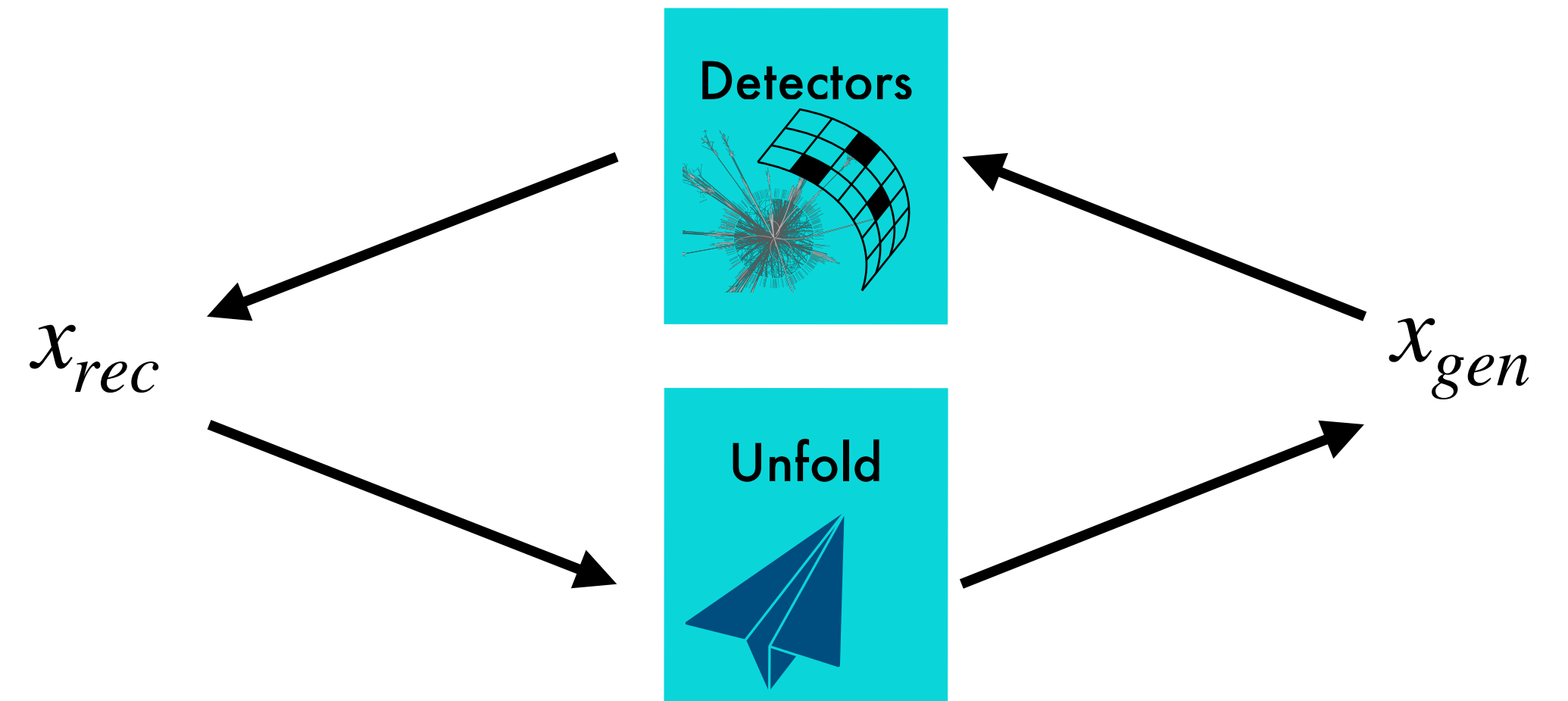
In praxis: regularised methods are used



# Unfolding — unchained

$$p(x_{rec}) = \int p(x_{gen}) \underbrace{R(x_{rec}, x_{gen})}_{\mathbf{P}(x_{rec} | x_{gen})} dx_{gen}$$

$$p(x_{gen}) = \int p(x_{rec}) \underbrace{p(x_{gen} | x_{rec})}_{\text{target probability}} dx_{rec}$$



# Unfolding — unchained

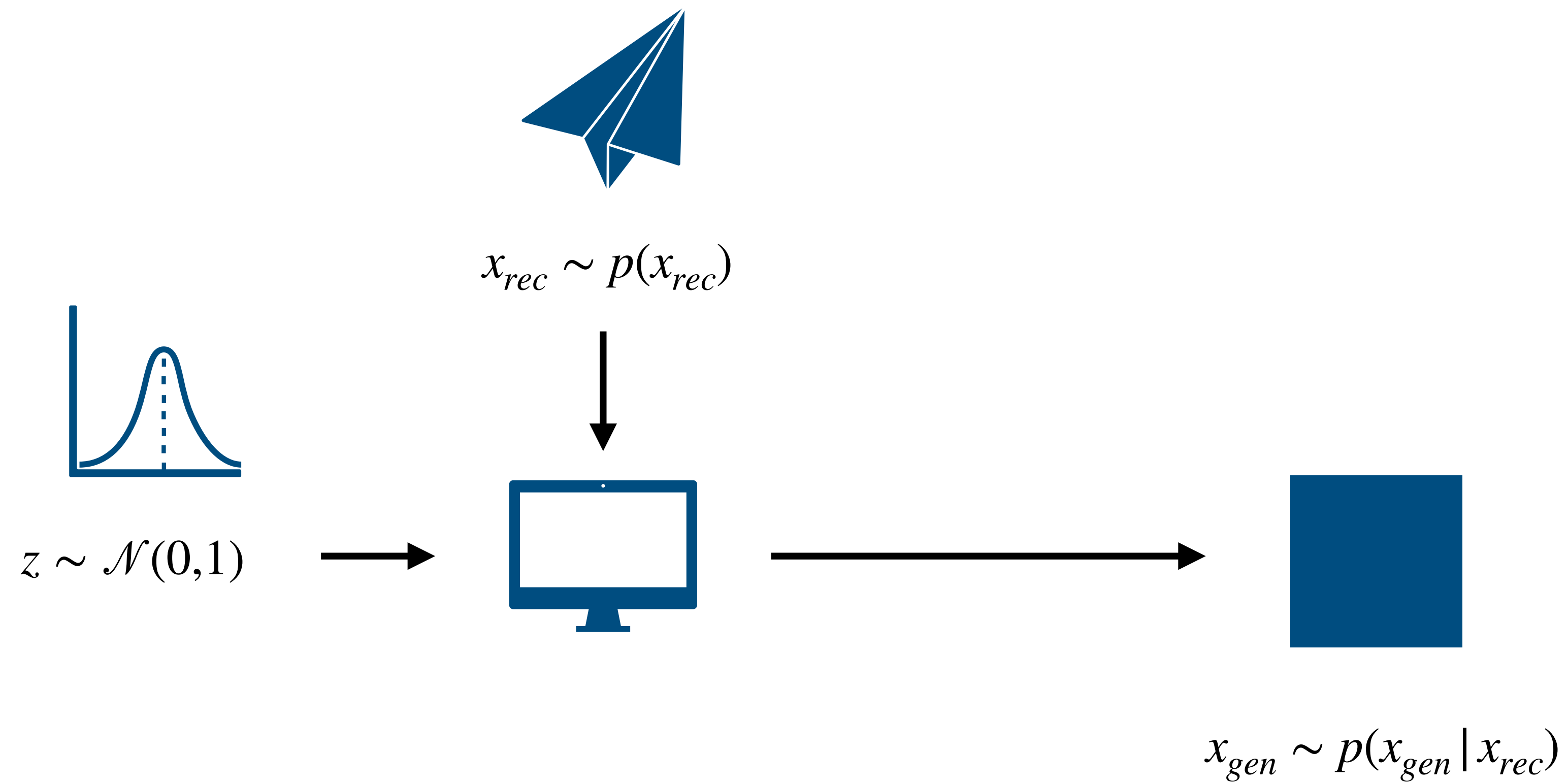
$$p(x_{rec}) = \int p(x_{gen}) \underbrace{R(x_{rec}, x_{gen})}_{\mathbf{p}(\mathbf{x}_{rec} | \mathbf{x}_{gen})} dx_{gen}$$

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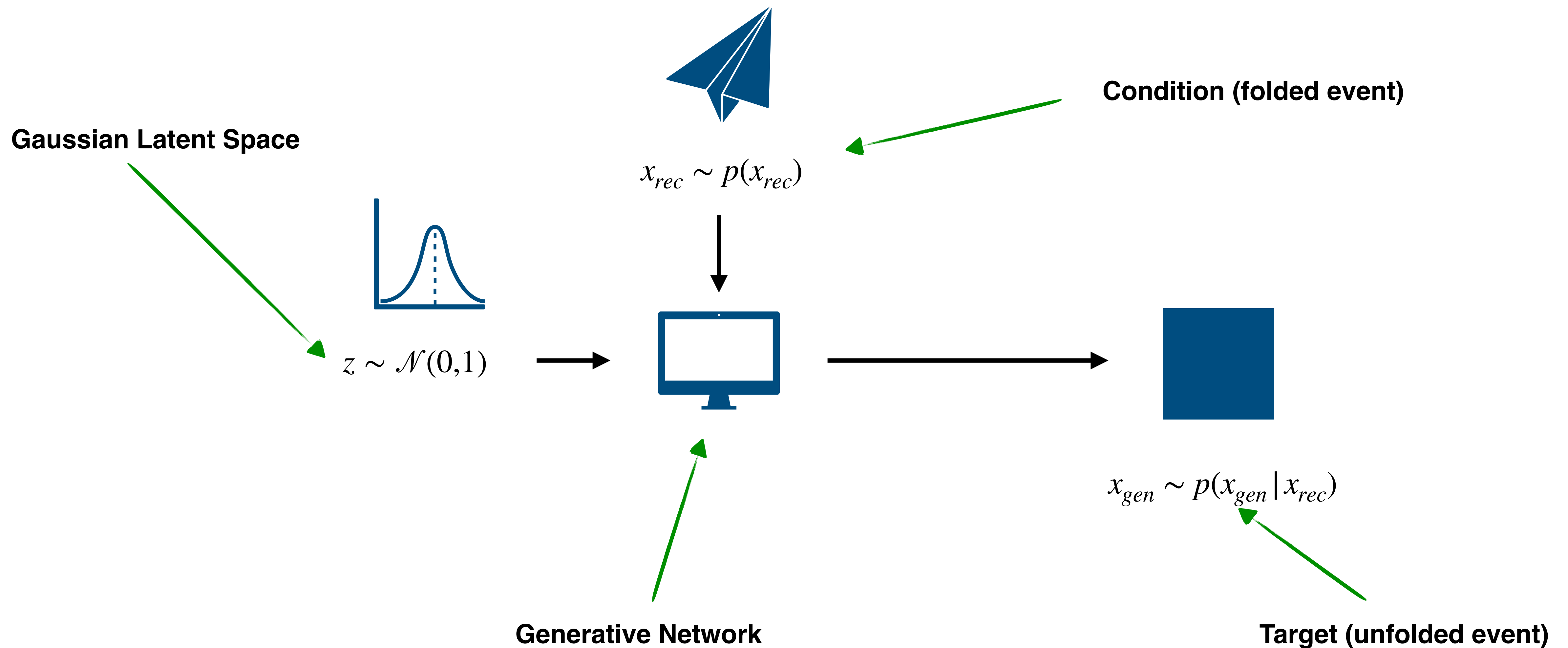
Classical methods are restricted to binned, one-dimensional distributions

We would like to learn high-dimensional, unbinned unfolding probability

# Unfolding — generative methods



# Unfolding — generative methods

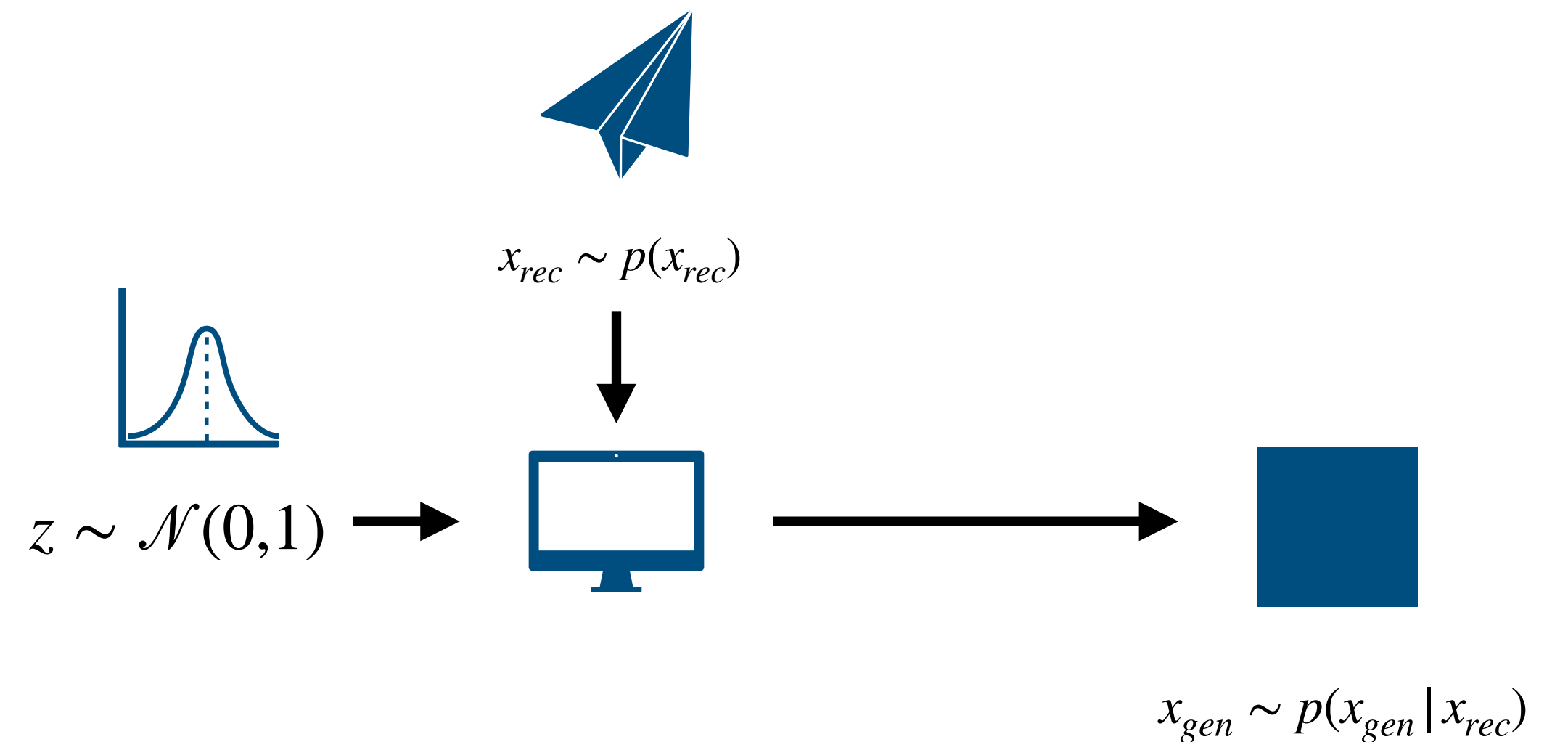


# Unfolding — generative methods

**Goal:** learn transformation latent  $\rightarrow$  gen phase space conditioned on rec event

During training, use paired events of forward simulation

After training, repeated sampling from latent space with constant condition allows probabilistic single event unfolding



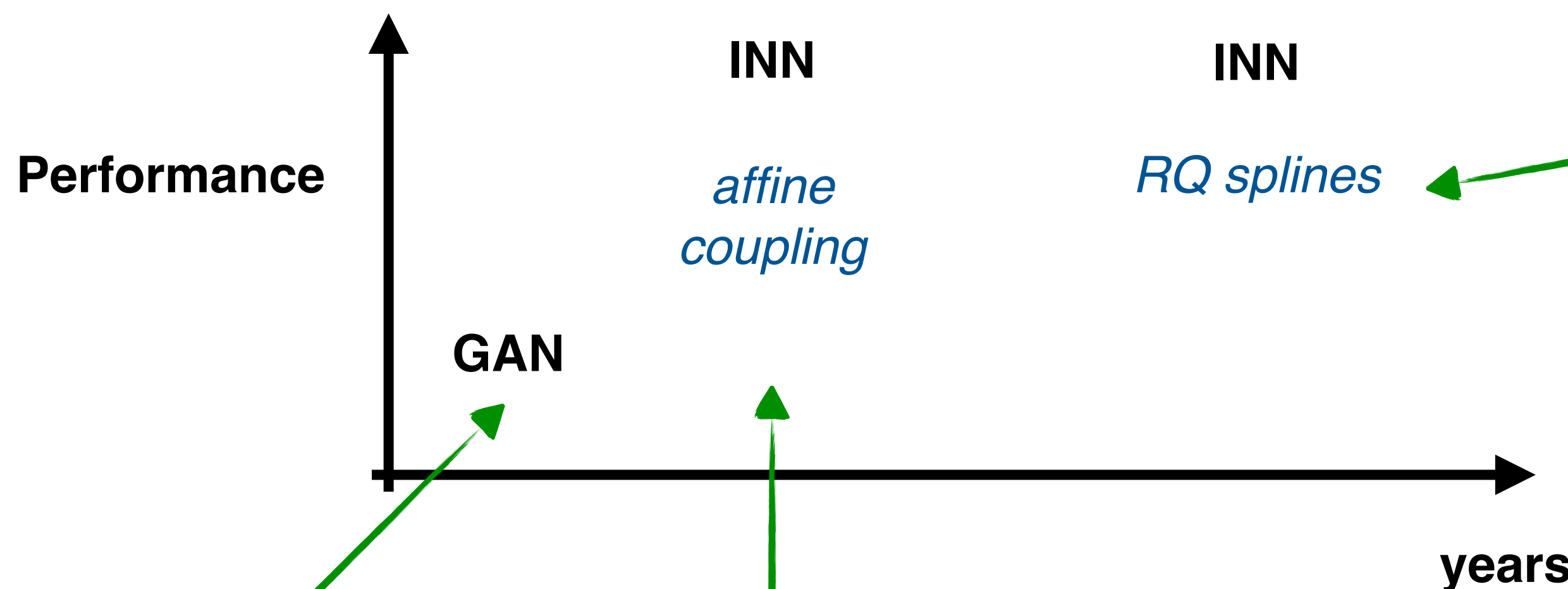
# Intermezzo — Generative Networks



Which generative Network?

*How to GAN away Detector Effects*

Bellagente et al.  
arXiv:1912.00477



*Invertible Networks or from Partons to Detector and back again*

Bellagente et al.  
arXiv:2006.06685

*An unfolding method based on Conditional Invertible Neural Networks (cINN) using iterative training*

Backes et al.  
arXiv:2212.08674

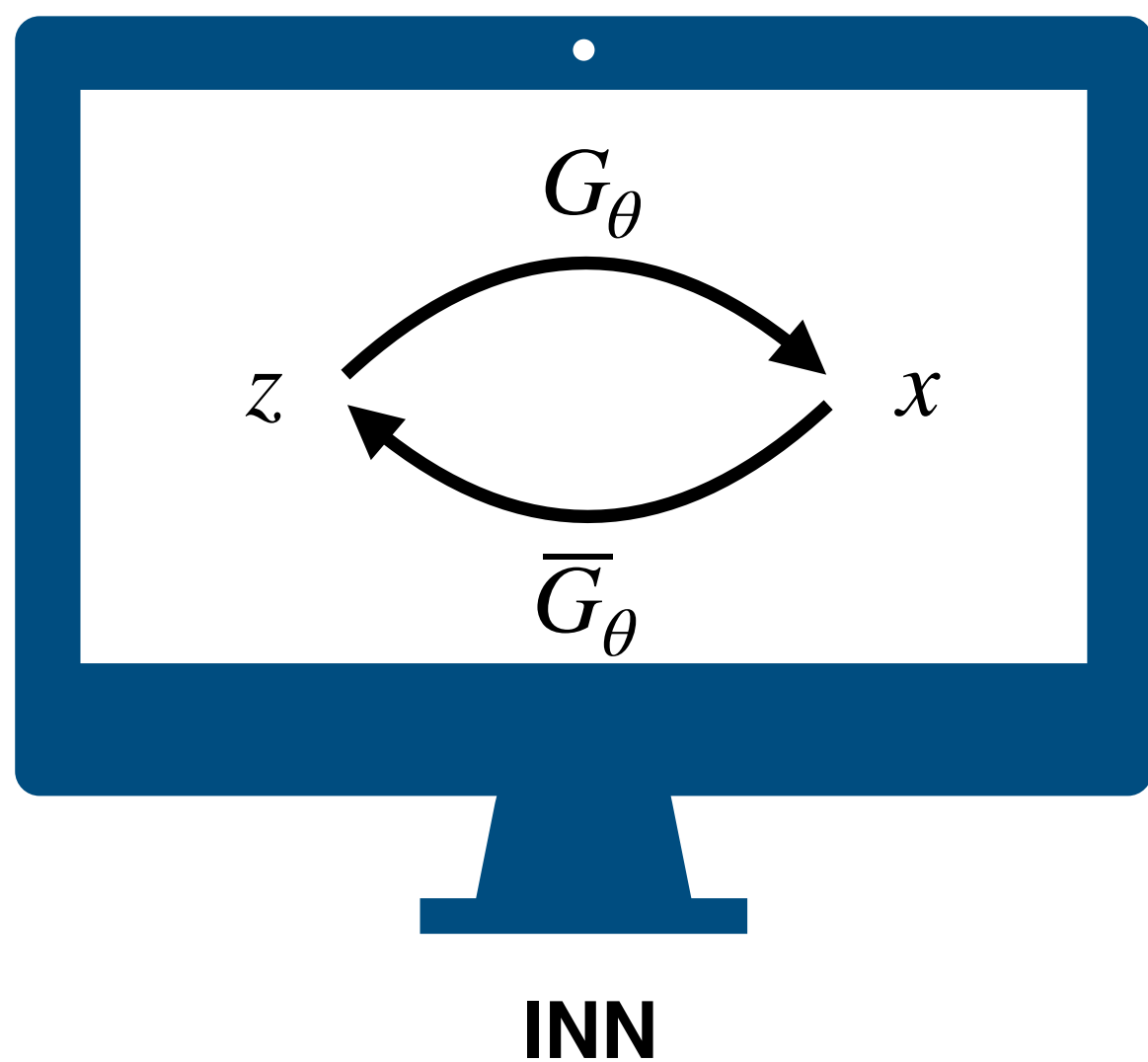
*Returning CP - Observables to the Frames they Belong*

Ackerschott et al.  
arXiv:2308.00027

*Event-by-event Comparison between Machine-Learning- and Transfer-Matrix-based Unfolding Methods*

Backes et al.  
arXiv:2310.17037

# Intermezzo — Generative Networks

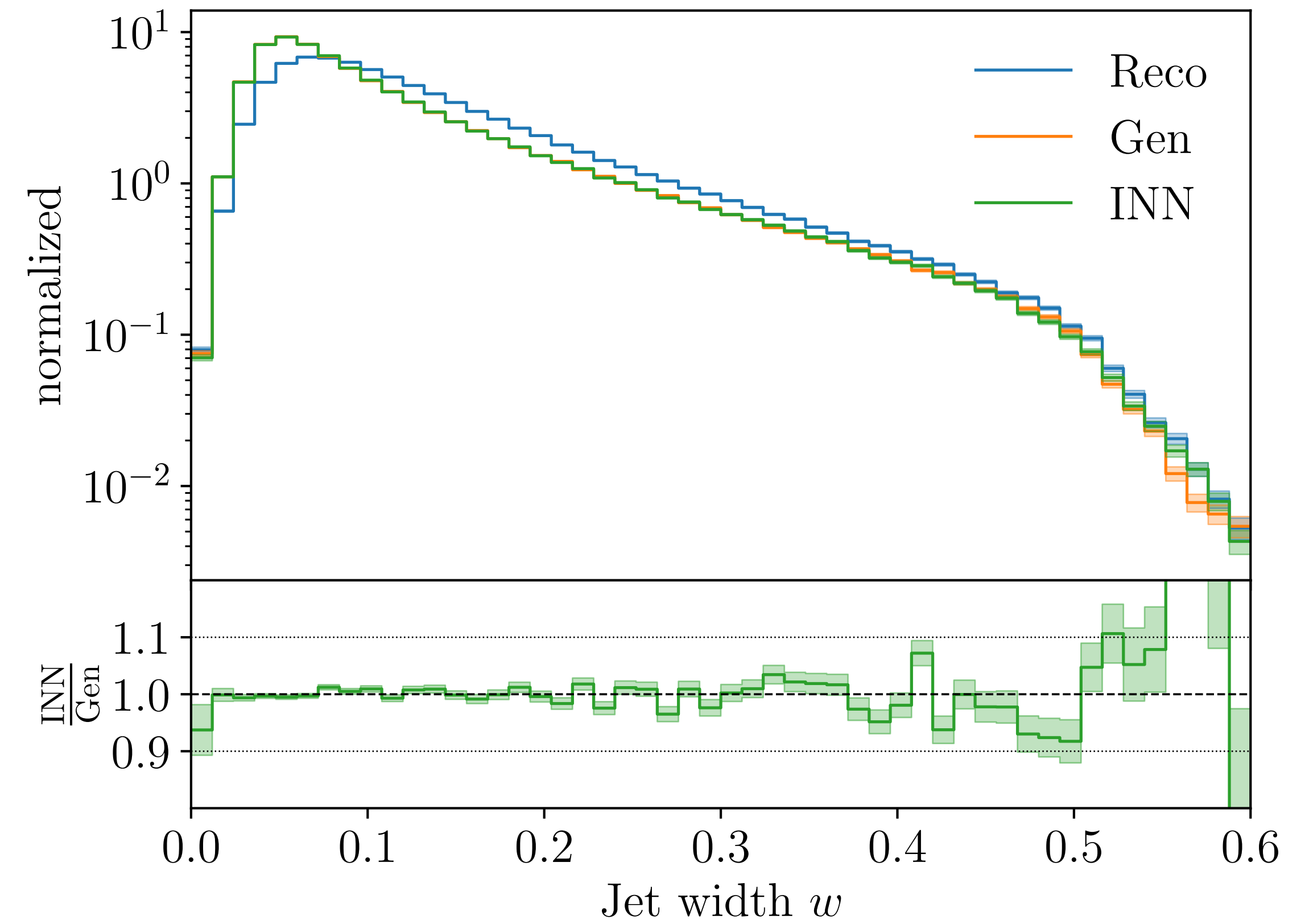
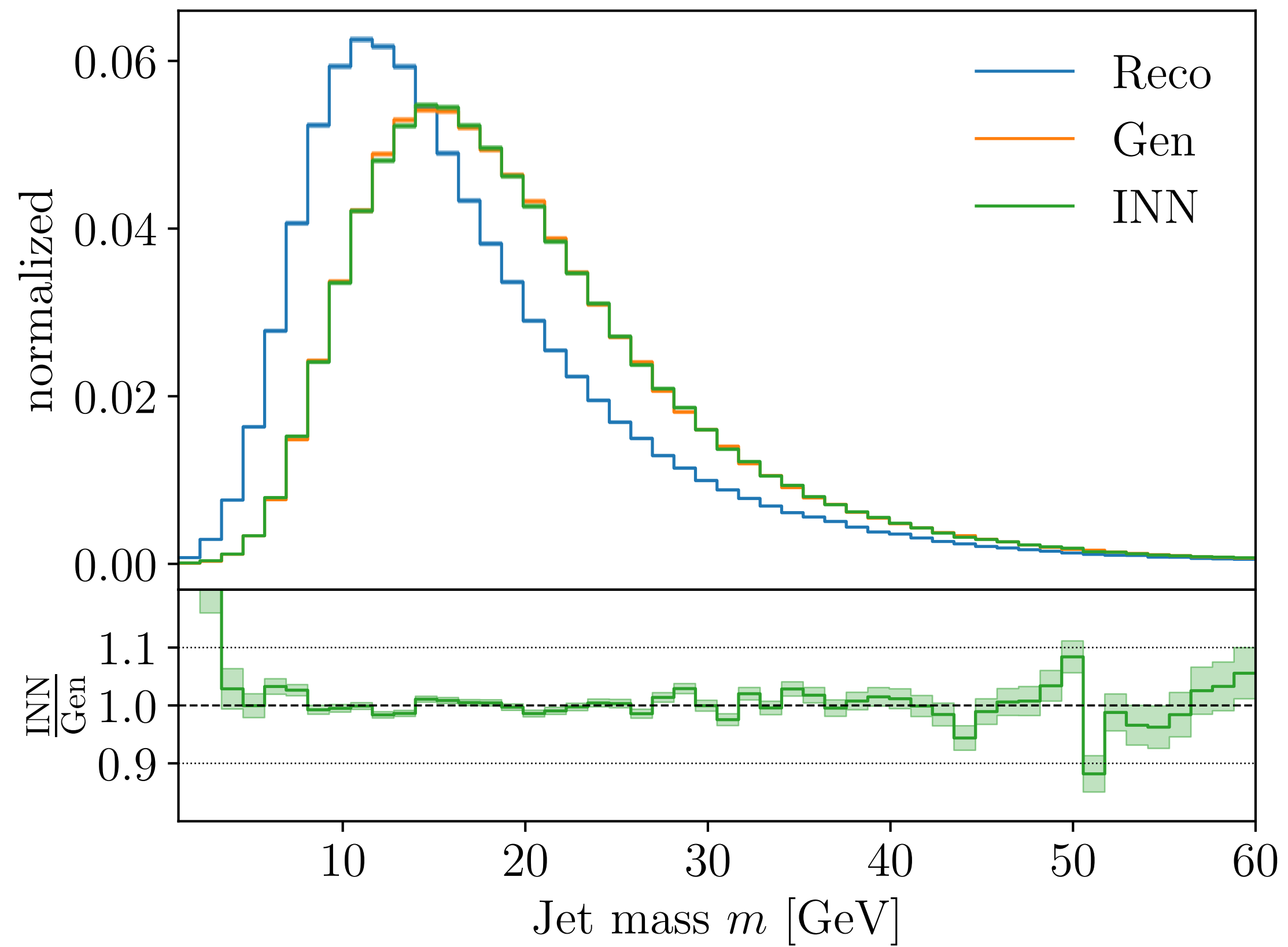


Target probability  $p(x_{gen} | x_{rec})$

Train on conditional likelihood loss

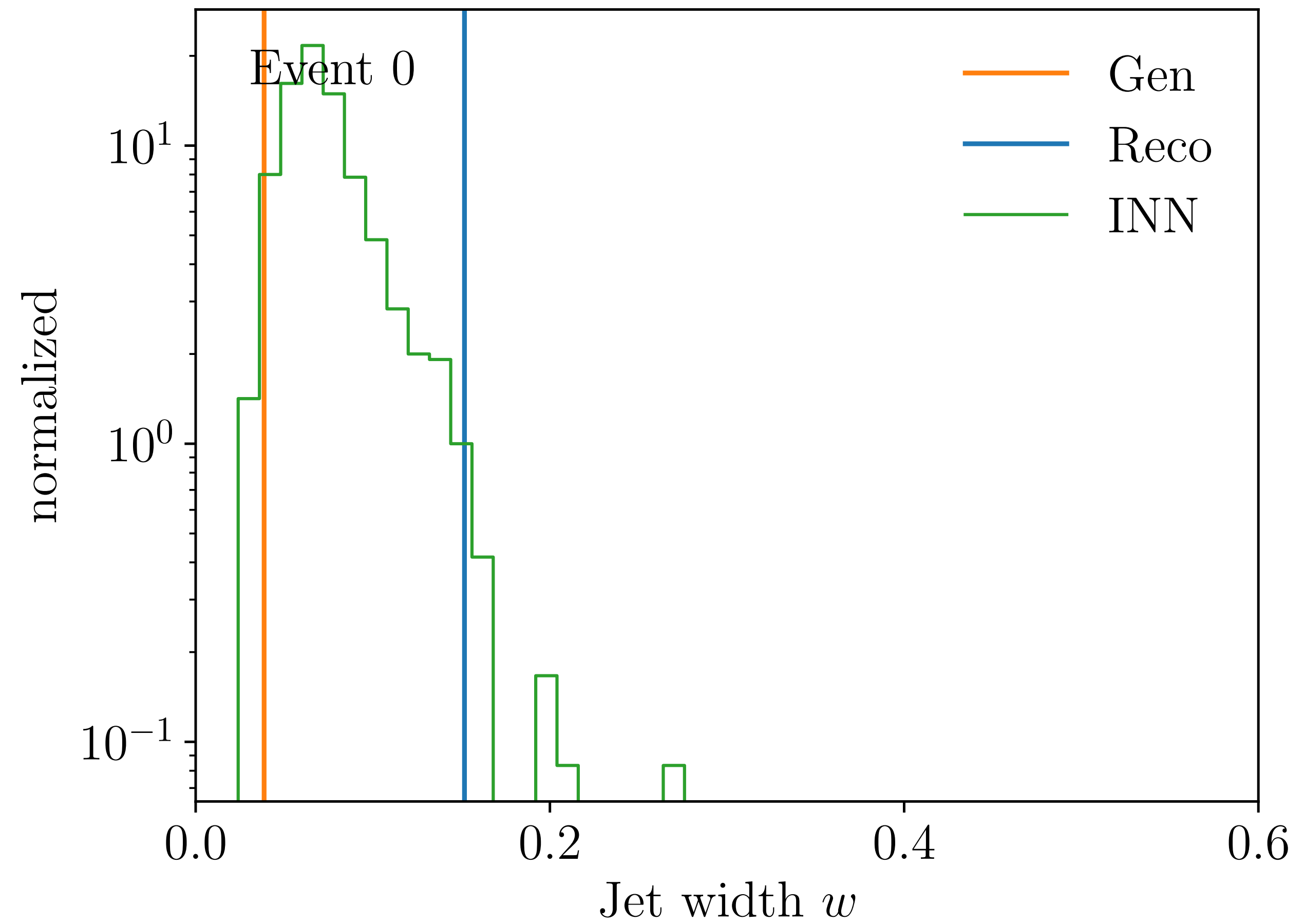
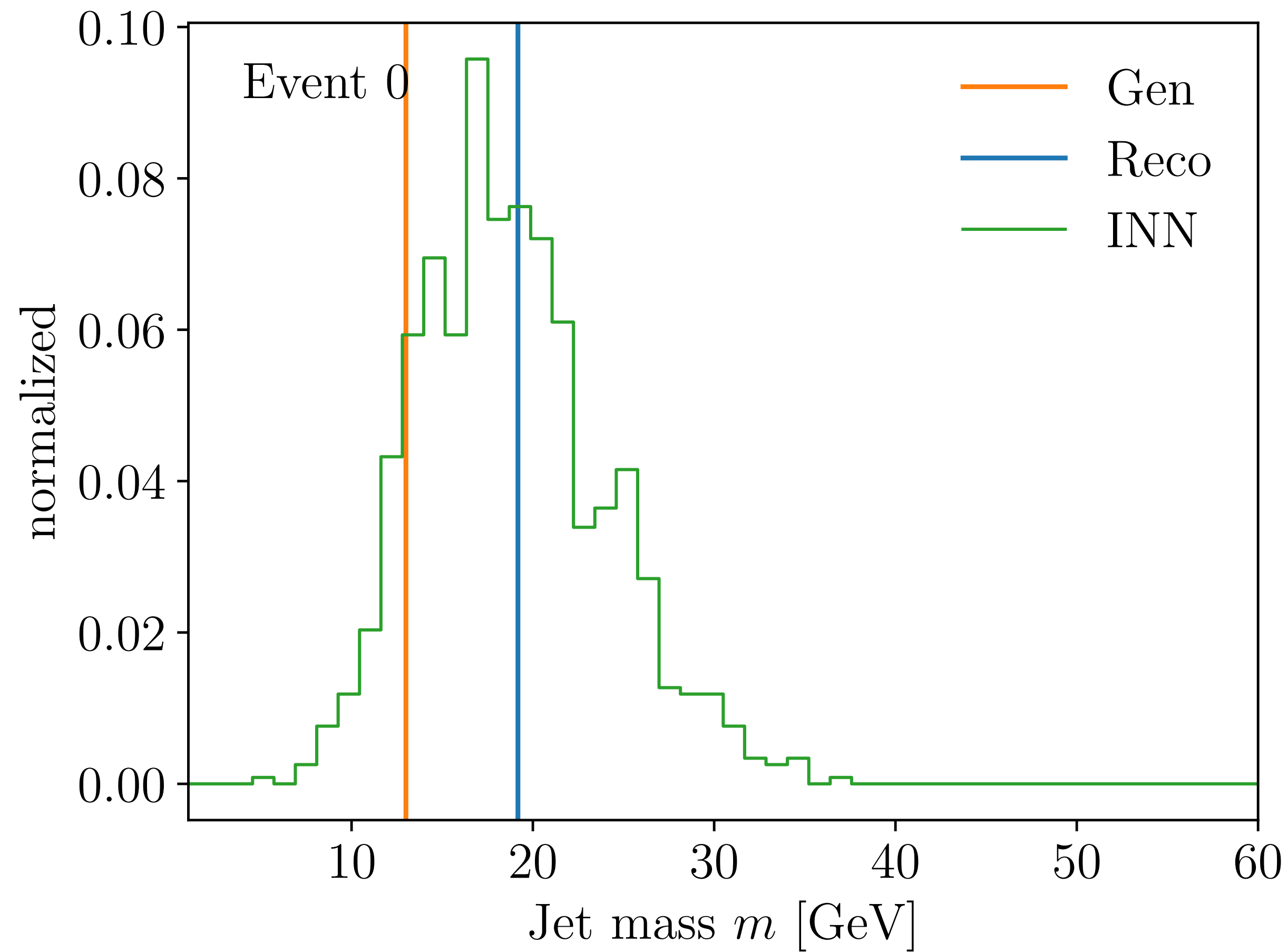
$$\begin{aligned} \mathcal{L} &= - \left\langle \log p_\theta(x_{gen} | x_{rec}) \right\rangle_{x_{gen}, x_{rec} \sim p(x_{gen}, x_{rec})} \\ &= - \left\langle \log \left( p_{latent} \left( \bar{G}_\theta(x_{gen} | x_{rec}) \right) \right) + \log \left| \frac{\partial \bar{G}_\theta(x_{gen} | x_{rec})}{\partial x_{gen}} \right| \right\rangle_{x_{gen}, x_{rec} \sim p(x_{gen}, x_{rec})} \end{aligned}$$

# Z + jet example

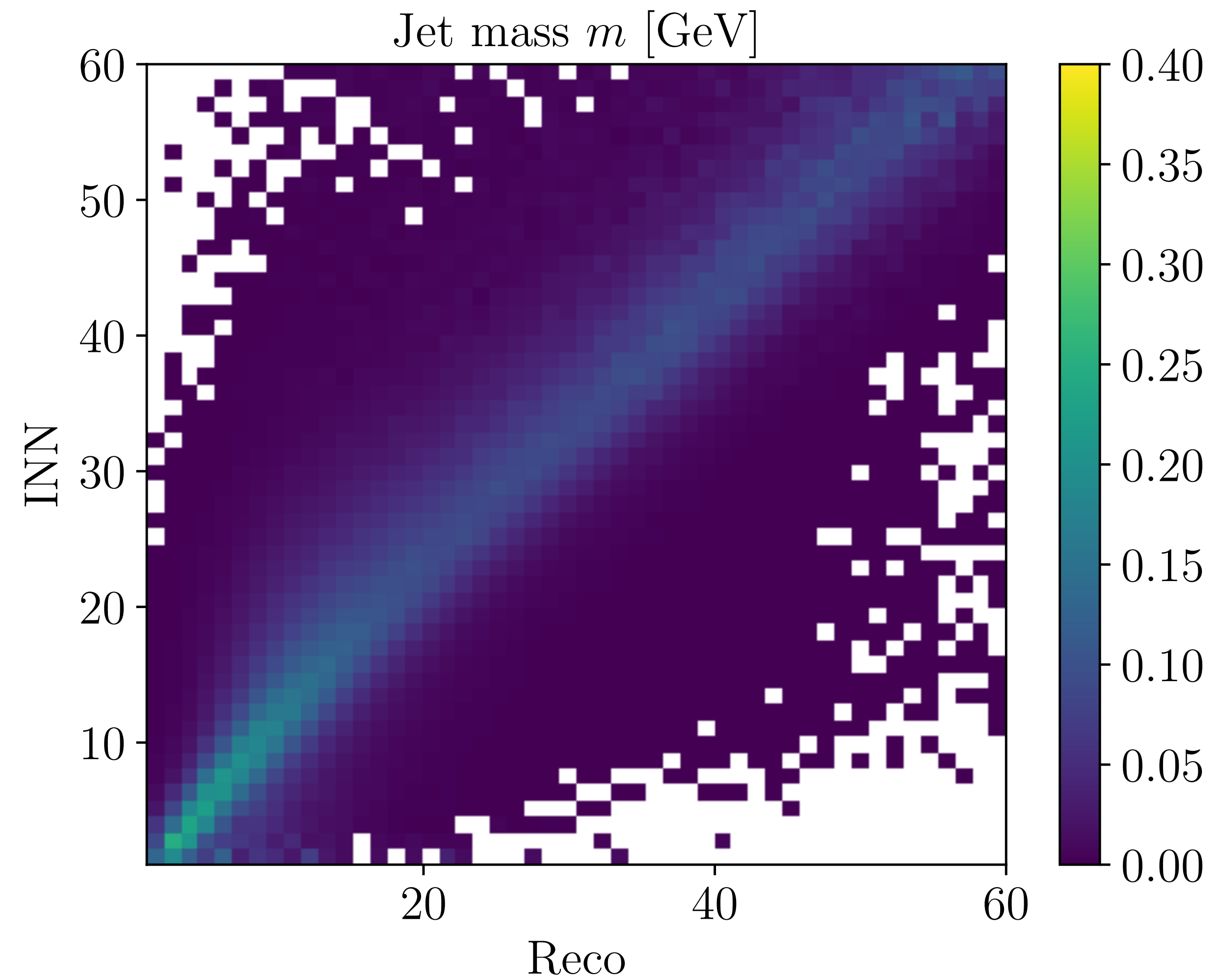
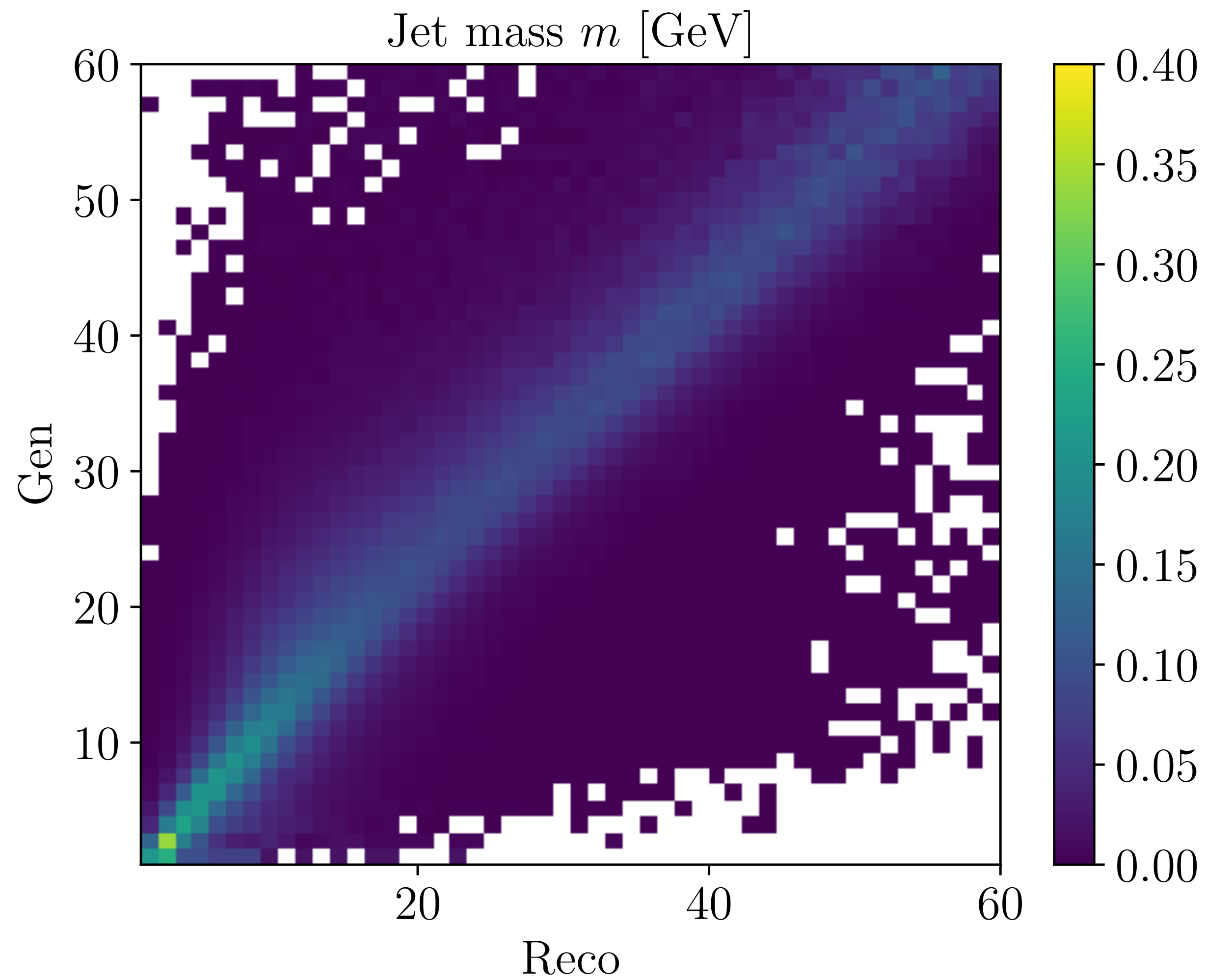




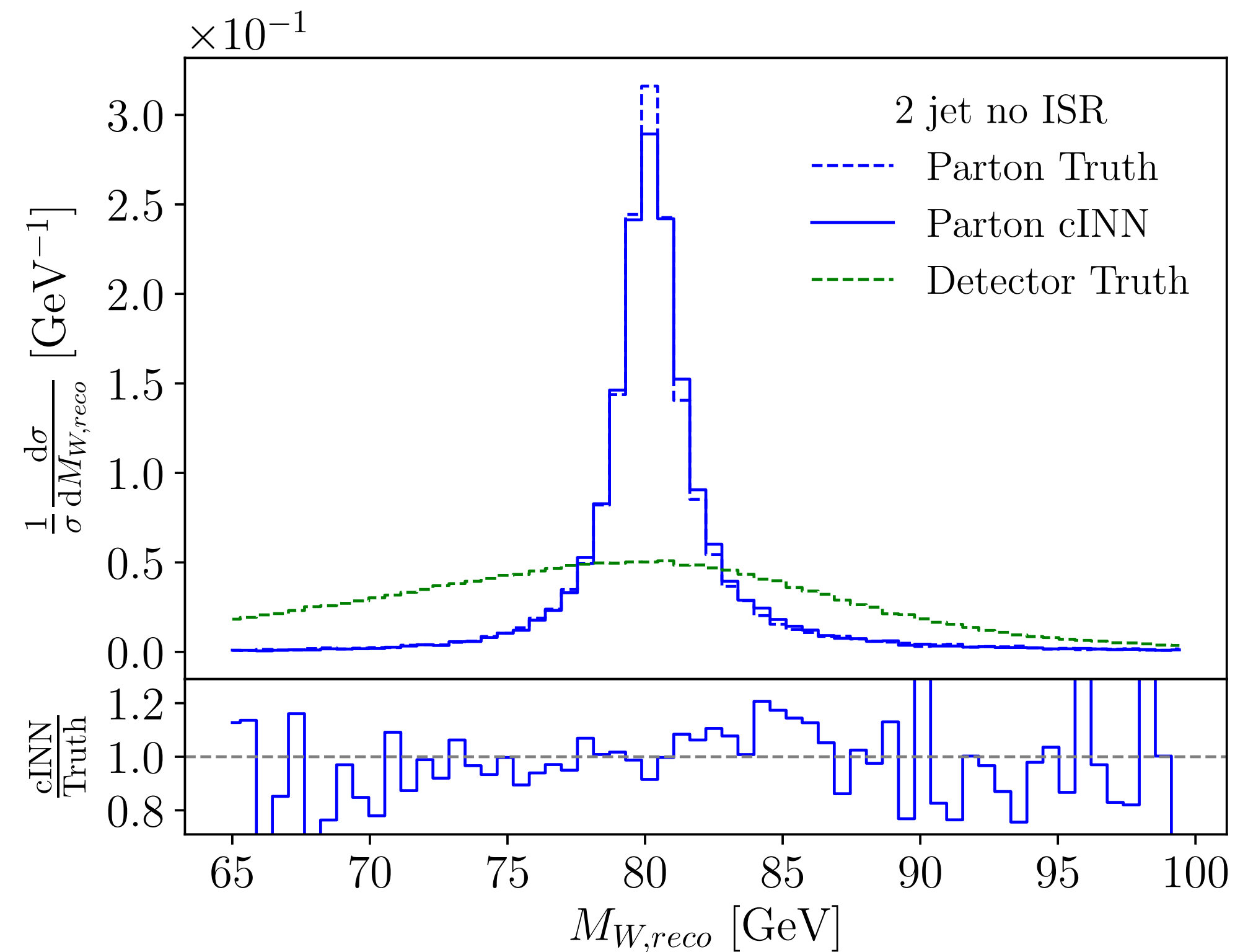
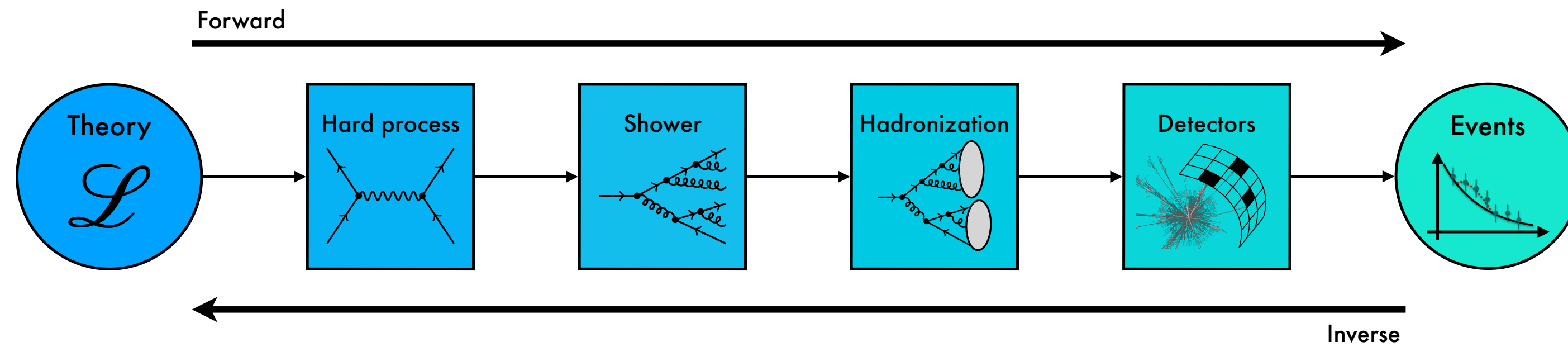
# Z + jet example — single event unfolding



# Z + jet example — migration



# Parton Level Unfolding — Z + 2 jets



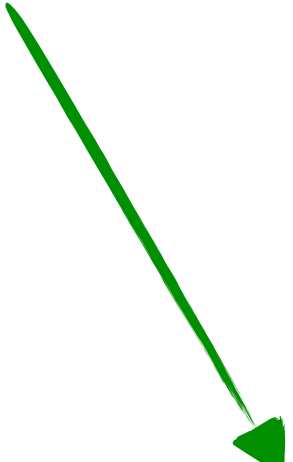
# What about model dependence?

Unfolding probability is Bayesian posterior =  
we have a prior

Prior is gen level distribution in training data  
(MC simulation)

Problematic for large MC-data differences

Prior


$$p(x_{gen} | x_{rec}) = \frac{p(x_{rec} | x_{gen})p(x_{gen})}{p(x_{rec})}$$

# Iterative Bayesian Unfolding (IBU)

Unfolding probability is Bayesian posterior =  
we have a prior

Prior is gen level distribution in training data  
(MC simulation)

Problematic for large MC-data differences

$$p(x_{gen}) = \int p(x_{rec})p(x_{gen} | x_{rec}) dx_{rec}$$

# Iterative Bayesian Unfolding (IBU)

Unfolding probability is Bayesian posterior =  
we have a prior

Prior is gen level distribution in training data  
(MC simulation)

Problematic for large MC-data differences

$$p(x_{gen}) = \int p(x_{rec}) \overset{\text{Unknown}}{p(x_{gen} | x_{rec})} dx_{rec}$$



Using Bayes' Theorem

**Prior**

$$p(x_{gen}) = \int p(x_{rec}) \frac{p(x_{rec} | x_{gen}) p(x_{gen})}{\int p(x_{rec} | \tilde{x}_{gen}) p(\tilde{x}_{gen}) d\tilde{x}_{gen}} dx_{rec}$$

# Iterative Bayesian Unfolding (IBU)

Unfolding probability is Bayesian posterior = we have a prior

Prior is gen level distribution in training data (MC simulation)

Problematic for large MC-data differences

IBU idea: Update your prior after each iteration

$$p(x_{gen}) = \int p(x_{rec}) \overset{\text{Unknown}}{p(x_{gen} | x_{rec})} dx_{rec}$$

Using Bayes' Theorem

$$p(x_{gen}) = \int p(x_{rec}) \frac{p(x_{rec} | x_{gen}) p(x_{gen})}{\int p(x_{rec} | \tilde{x}_{gen}) p(\tilde{x}_{gen}) d\tilde{x}_{gen}} dx_{rec}$$

Start from prior and use iterative approach

$$p^n(x_{gen}) = \int p(x_{rec}) \frac{p(x_{rec} | x_{gen}) p^{n-1}(x_{gen})}{\int p(x_{rec} | \tilde{x}_{gen}) p^{n-1}(\tilde{x}_{gen}) d\tilde{x}_{gen}} dx_{rec}$$

# Iterative Bayesian Unfolding (IBU)

Unfolding probability is Bayesian posterior = we have a prior

Prior is gen level distribution in training data (MC simulation)

Problematic for large MC-data differences

IBU idea: Update your prior after each iteration

$$p^n(x_{gen}) = \int p(x_{rec}) \frac{p(x_{rec} | x_{gen}) p^{n-1}(x_{gen})}{\int p(x_{rec} | \tilde{x}_{gen}) p^{n-1}(\tilde{x}_{gen}) d\tilde{x}_{gen}} dx_{rec}$$

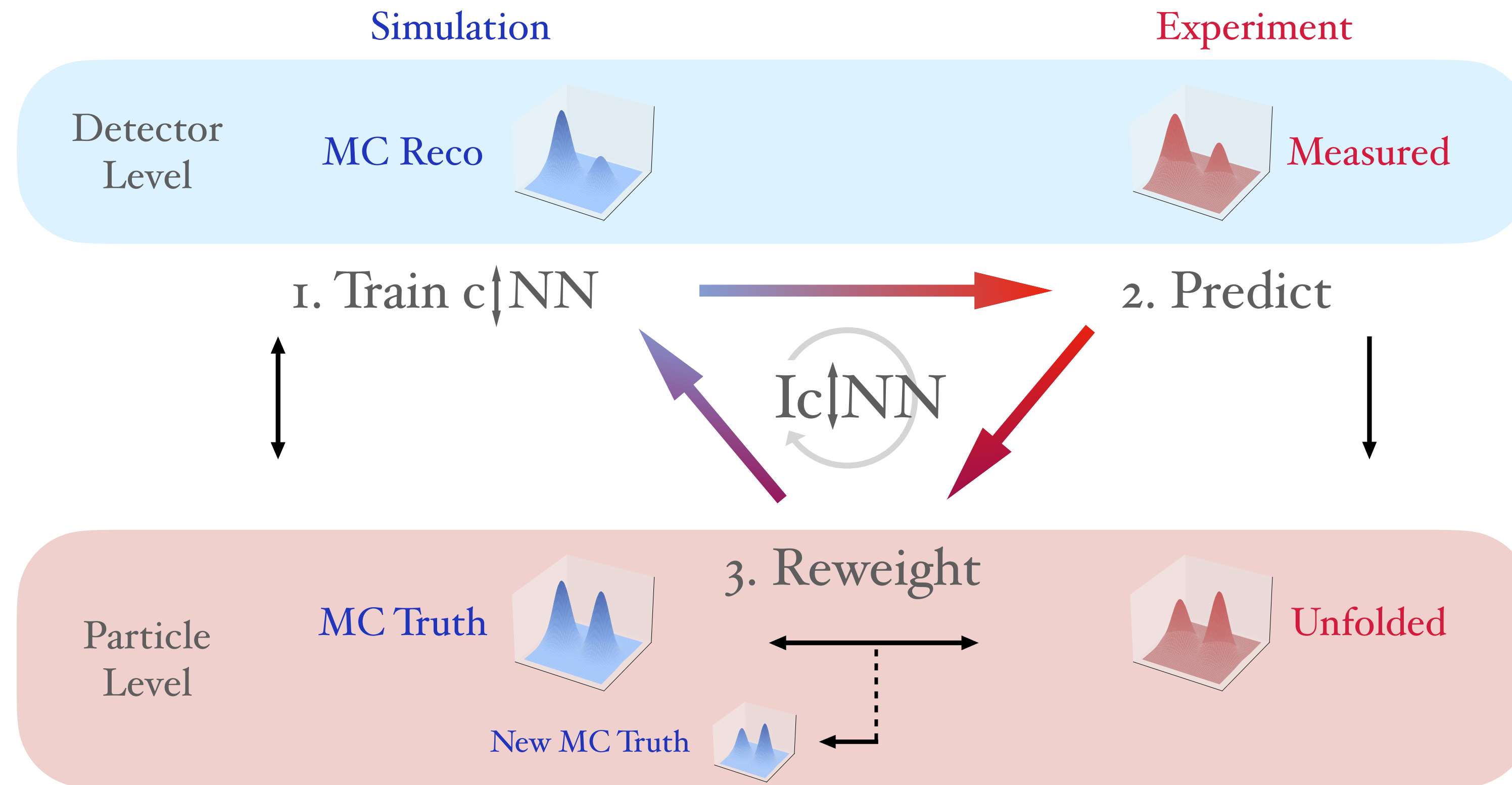
In the past for LHC-like phase spaces  
no access to those quantities

$$h_{gen,n}^j = \sum_i h_{rec}^i \frac{R_{ij} h_{gen,n-1}^j}{\sum_k R_{ik} h_{gen,n-1}^k}$$

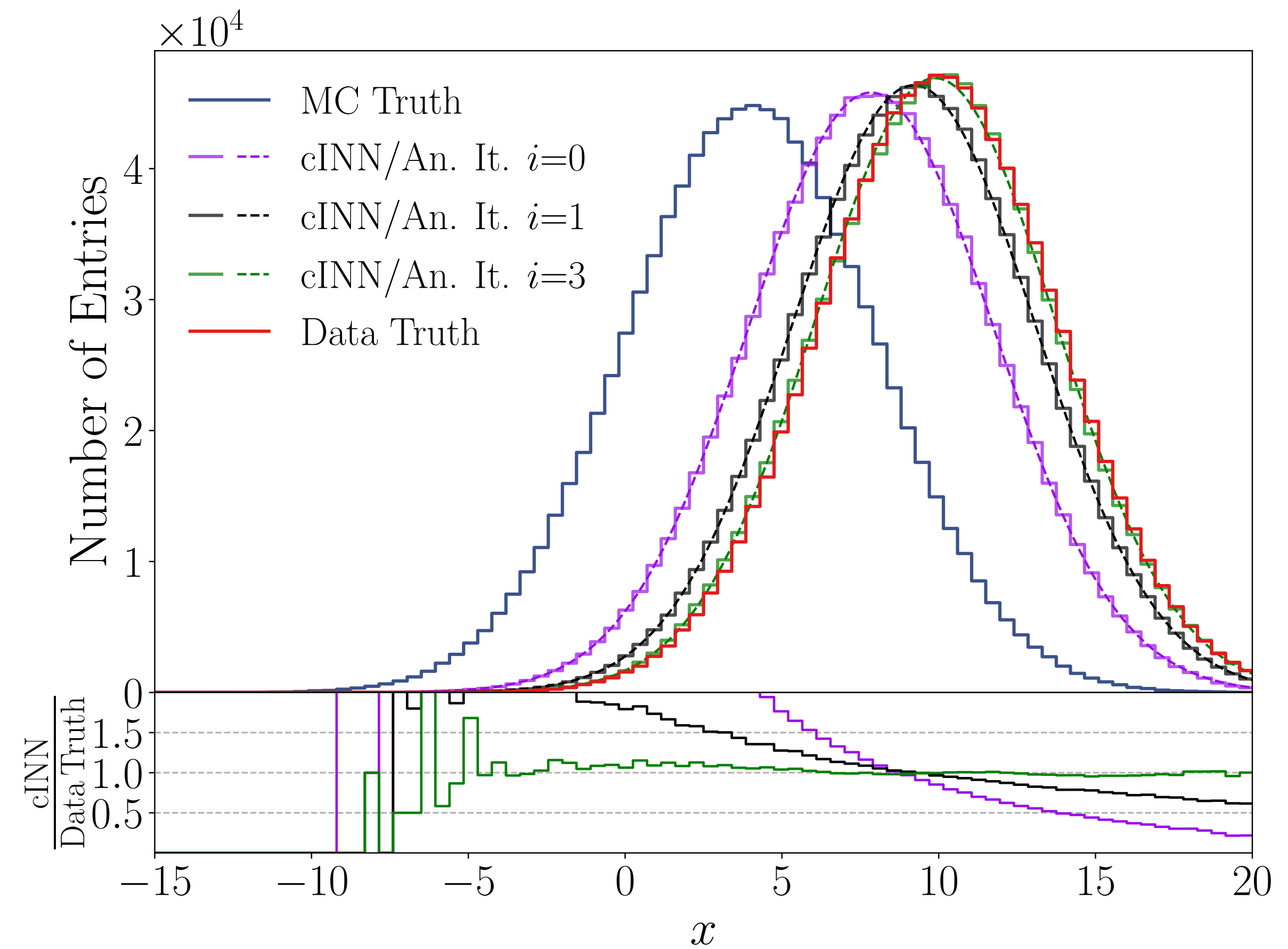
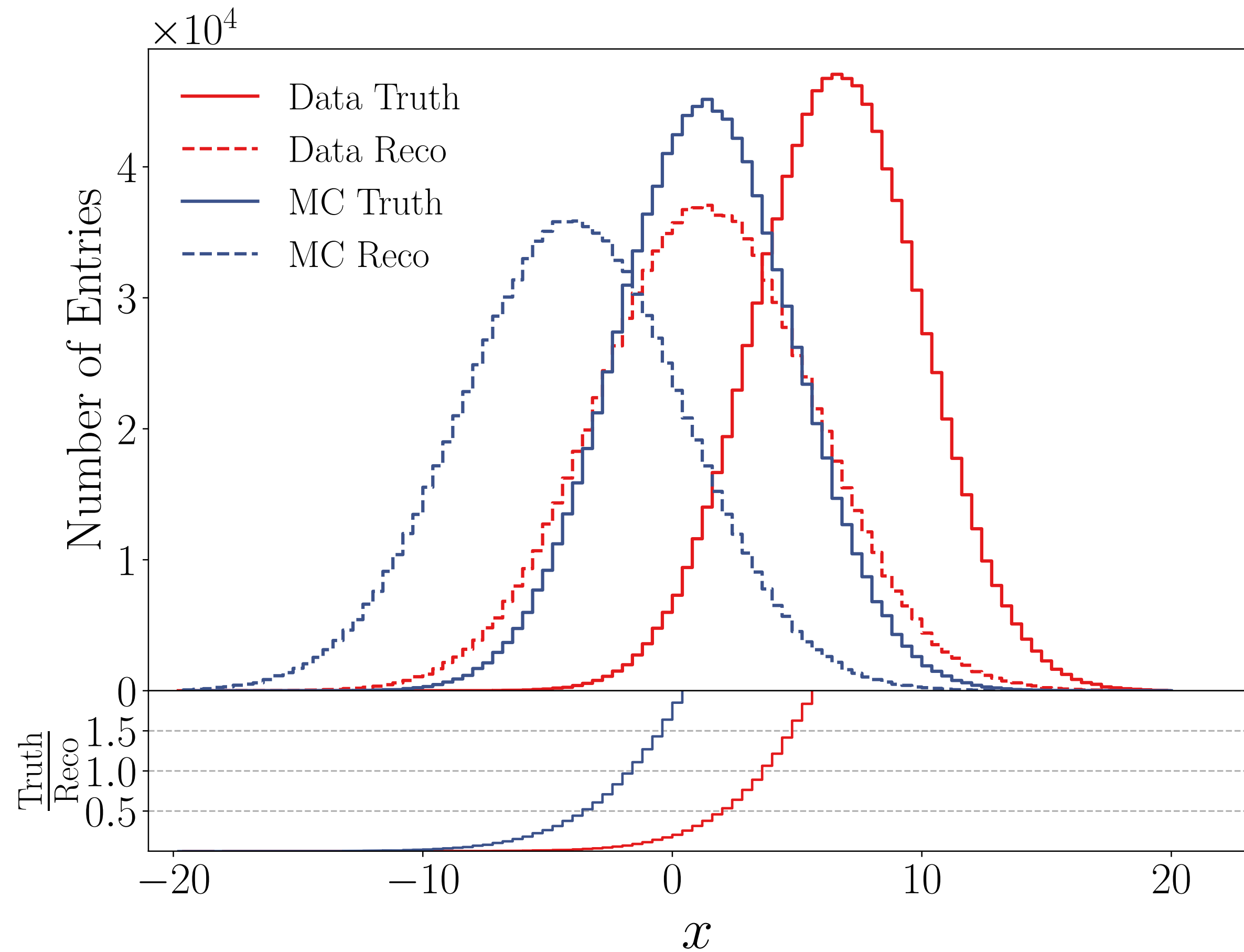
One dimensional & binned



# Iterative generative unfolding



# Iterative generative unfolding



# Generative Unfolding in experimental praxis

Failure modes in model dependency

All necessary information to unfold could be lost

Background subtraction ?

Finite detector efficiency

Sideband studies

Combinatorics

...



Currently, working together with experimentalist bridging gap between theory and praxis

# And now what?

Generative machine learning allows for unbinned, high dimensional unfolding

Methods to unbias unfolding networks exists and are currently tested to match experimental requirements

In parallel, there is an on going project comparing different ML based unfolding methods

**Stay tuned!**

