# Algebra Exercises 

psfile
ngeoacceptance

## Reference Frame



## Beam Direction

- At 0 degree:

$$
p(0)=(0,0,1)
$$

- After a rotation in xz plane (of an angle $\omega$ ):

$$
p(\omega)=(\sin \omega, 0, \cos \omega)
$$

## Cherenkov Light

- At 0 degree:

$=$ critical angle with air $=\overline{=} \cos \alpha \cdot \sin \varphi, \sin \alpha)$
- After ${ }_{c}$ beam rotation:
$\alpha$

$$
\pi / 2-c
$$

$\mathrm{C}(\omega)=(\cos \alpha \cdot \cos \varphi \cdot \cos \omega+\sin \alpha \cdot \sin \omega$, $\cos \alpha \cdot \sin \varphi$,
$-\cos \alpha \cdot \cos \varphi \cdot \sin \omega+\sin \alpha \cdot \cos \omega)$

## Geometrical Acceptance

## $\mathbf{C}=\left(\cos \theta_{\mathbf{x}}, \cos \theta_{\mathbf{y}}, \cos \theta_{\mathbf{z}}\right)$

$\cos \theta_{x}>\cos \eta \quad(\eta:$ critical angle $)$
$\cos \alpha \cdot \cos \varphi \cdot \cos \omega+\sin \alpha \cdot \sin \omega>\cos \eta$
$\cos \varphi>\frac{\cos \eta-\sin \alpha \cdot \sin \omega}{\cos \alpha \cdot \cos \omega}=\cos \lambda$

$$
\lambda=f(\omega, \alpha, \eta) \ldots \alpha, \eta: " k n o w n " \ldots
$$








## Wrapping

- From the unitarity constraint:

$$
\cos \theta_{\mathbf{y}}, \cos \theta_{\mathbf{z}} \leq \sin \theta_{\mathbf{x}}<\sin \eta<\cos \eta_{\mathbf{a i r}}
$$

being :

$$
\begin{aligned}
& \sin \eta_{\text {air }}=1.0 / 2.2=0.45 \\
& \sin \eta_{\text {sil }}=1.43 / 2.2=0.65 \\
& \cos \eta_{\text {air }}=0.89
\end{aligned}
$$

SO :

$$
\theta_{\mathbf{x}}, \theta_{\mathbf{z}}>\eta_{\text {air }}
$$

anyway totally reflected on the non - instrumented faces
Reflective wrapping doesn't improve light collection.

## Polarization

- Cherenkov light 100\% linearly polarized in plane containing photon and incident particle momenta:

$$
\begin{aligned}
\overrightarrow{E(0)} & =(\cos c \cdot \cos \varphi, \cos c \cdot \sin \varphi,-\sin c) \\
& =(\sin \alpha \cdot \cos \varphi, \sin \alpha \cdot \sin \varphi,-\cos \alpha)
\end{aligned}
$$

- After rotation:

$$
\begin{aligned}
\overrightarrow{E(\omega)}= & (\sin \alpha \cdot \cos \varphi \cdot \cos \omega-\cos \alpha \cdot \sin \omega \\
& \sin \alpha \cdot \sin \varphi \\
& -\sin \alpha \cdot \cos \varphi \cdot \sin \omega-\cos \alpha \cdot \cos \omega)
\end{aligned}
$$

## Polarization

- Two components: Es and Ep

$\vec{E}_{p}=\left(E_{x}, a \cdot E_{y}, b \cdot E_{z}\right)$

$$
\vec{E}_{s}=\left(0,(1-a) \cdot E_{y},(1-b) \cdot E_{z}\right)
$$

## Transmission

$$
\begin{gathered}
\left|\mathbf{E}_{\mathbf{p}}\right|=\frac{\mathbf{E}_{\mathbf{x}}}{\sqrt{1-\mathbf{C}_{\mathbf{x}}^{2}}} \\
\left|\mathbf{E}_{\mathbf{s}}\right|=\sqrt{1-\mathbf{E}_{\mathbf{p}}^{2}}=\frac{\sin \varphi \cdot \cos \omega}{\sqrt{1-\mathbf{C}_{\mathbf{x}}^{2}}} \\
\mathbf{T}_{\mathbf{p}}=1-\left[\frac{\tan \left(\theta_{\mathbf{t}}-\theta_{\mathbf{x}}\right)}{\tan \left(\theta_{\mathbf{t}}+\theta_{\mathbf{x}}\right)}\right]^{2} \quad \sin \theta_{\mathbf{t}}=\frac{\sin \theta_{\mathbf{x}}}{\sin \eta} \\
\mathbf{T}_{\mathbf{s}}=1-\left[\frac{\sin \left(\theta_{\mathbf{t}}-\theta_{\mathbf{x}}\right)}{\sin \left(\theta_{\mathbf{t}}+\theta_{\mathbf{x}}\right)}\right]^{2}
\end{gathered}
$$

$$
\operatorname{Prob}(\mathbf{T})=\mathbf{T}_{\mathbf{p}} \cdot\left|\mathbf{E}_{\mathbf{p}}\right|^{\mathbf{2}}+\mathbf{T}_{\mathbf{s}} \cdot\left|\mathbf{E}_{\mathbf{s}}\right|^{\mathbf{2}}
$$

## Reflection Coefficients







## Scintillating Light

- Calculations for scintillating light, for (TOTAL) geometrical acceptance, transmittance and prompt collection give:
- AIR :
- SILICON :

24, 90, 22 \%

## Conclusions

- I am convinced that geometry should likely be improved:


