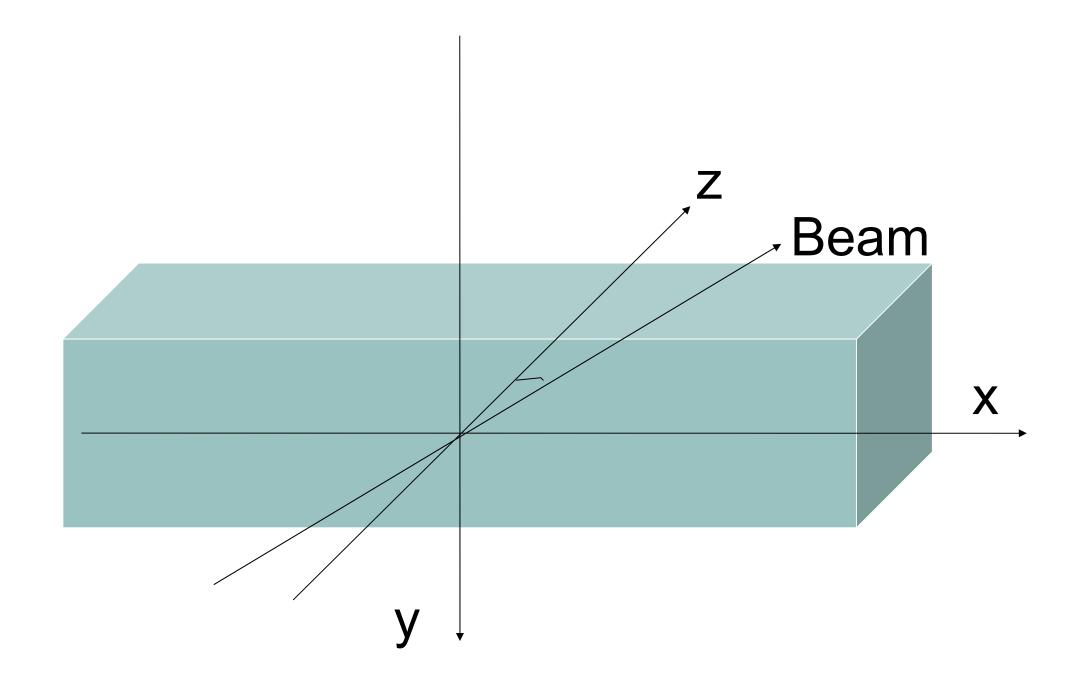
Algebra Exercises

psfile ngeoacceptance

15/03/2007 Roberto Ferrari

Reference Frame



Beam Direction

- At 0 degree:
 - p(0) = (0, 0, 1)
- After a rotation in xz plane (of an angle ω):
 - $p(\omega) = (\sin \omega, 0, \cos \omega)$

Cherenkov Light

- At 0 degree:
- C(0) Chesienthows angle in $c \cdot \sin \varphi$, $\cos c$)

 = critical angle with air, = $\cos \alpha \cdot \sin \varphi$, $\sin \alpha$)
- After beam rotation:

$$\alpha$$

$$\pi/2 - c$$

$$\mathbf{C}(\omega) = (\cos\alpha \cdot \cos\varphi \cdot \cos\omega + \sin\alpha \cdot \sin\omega, \\ \cos\alpha \cdot \sin\varphi, \\ -\cos\alpha \cdot \cos\varphi \cdot \sin\omega + \sin\alpha \cdot \cos\omega)$$

Geometrical Acceptance

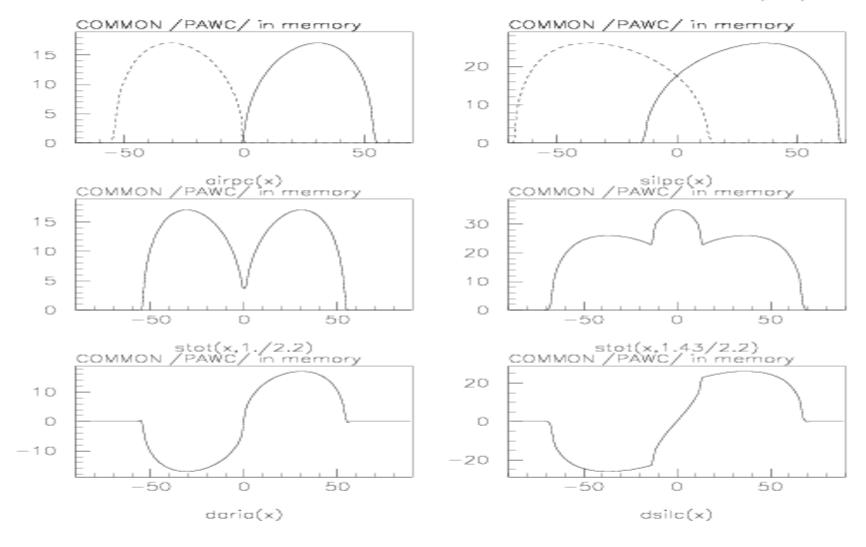
$$\mathbf{C} = (\cos \theta_{\mathbf{x}}, \cos \theta_{\mathbf{y}}, \cos \theta_{\mathbf{z}})$$

$$\cos \theta_x > \cos \eta \qquad (\eta : critical \ angle)$$

$$\cos \alpha \cdot \cos \varphi \cdot \cos \omega + \sin \alpha \cdot \sin \omega > \cos \eta$$

$$\cos \varphi > \frac{\cos \eta - \sin \alpha \cdot \sin \omega}{\cos \alpha \cdot \cos \omega} = \cos \lambda$$

$$\lambda = f(\omega, \alpha, \eta) \dots \alpha, \eta : "known" \dots$$



Wrapping

• From the unitarity constraint:

$$\cos \theta_{\mathbf{y}}, \cos \theta_{\mathbf{z}} \leq \sin \theta_{\mathbf{x}} < \sin \eta < \cos \eta_{\mathbf{air}}$$

$$\sin \eta_{air} = 1.0/2.2 = 0.45$$

 $\sin \eta_{sil} = 1.43/2.2 = 0.65$

 $\cos\eta_{ extbf{air}} = extbf{0.89}$

so:

$$\theta_{\mathbf{x}}, \theta_{\mathbf{z}} > \eta_{\mathbf{air}}$$

anyway totally reflected on the non – instrumented faces
Reflective wrapping doesn't improve light collection.

Polarization

 Cherenkov light 100% linearly polarized in plane containing photon and incident particle momenta:

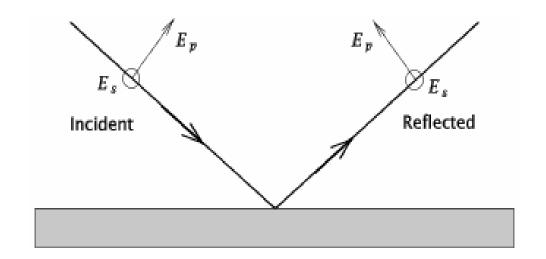
$$\overrightarrow{E(0)} = (\cos c \cdot \cos \varphi, \cos c \cdot \sin \varphi, -\sin c)$$
$$= (\sin \alpha \cdot \cos \varphi, \sin \alpha \cdot \sin \varphi, -\cos \alpha)$$

After rotation:

$$\overrightarrow{E(\omega)} = \left(\sin \alpha \cdot \cos \varphi \cdot \cos \omega - \cos \alpha \cdot \sin \omega, \\
\sin \alpha \cdot \sin \varphi, \\
- \sin \alpha \cdot \cos \varphi \cdot \sin \omega - \cos \alpha \cdot \cos \omega \right)$$

Polarization

Two components: Es and Ep



$$\overrightarrow{E}_p = (E_x, a \cdot E_y, b \cdot E_z)$$

$$\overrightarrow{E}_s = (0, (1-a) \cdot E_y, (1-b) \cdot E_z)$$

Transmission

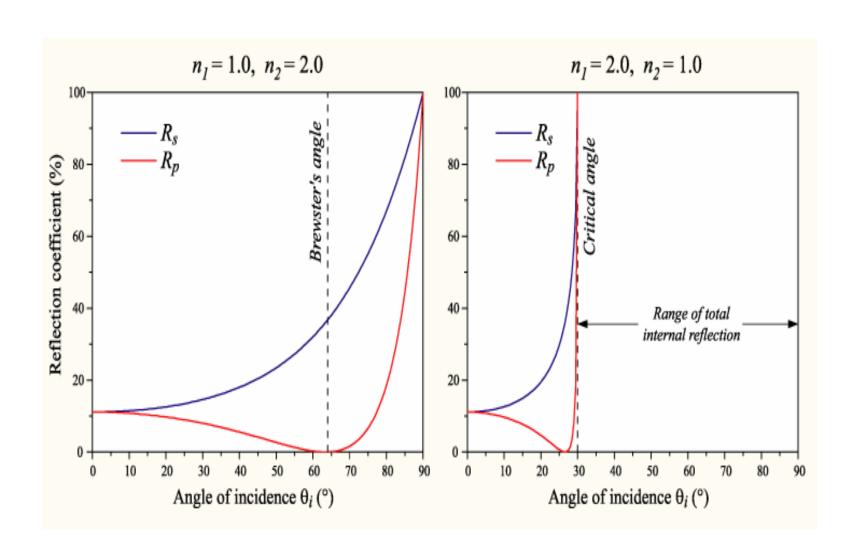
$$egin{array}{lll} |\mathbf{E_p}| &= & \dfrac{\mathbf{E_x}}{\sqrt{\mathbf{1} - \mathbf{C_x^2}}} \ |\mathbf{E_s}| &= & \sqrt{\mathbf{1} - \mathbf{E_p^2}} &= & \dfrac{\sin \, arphi \cdot \cos \, \omega}{\sqrt{\mathbf{1} - \mathbf{C_x^2}}} \end{array}$$

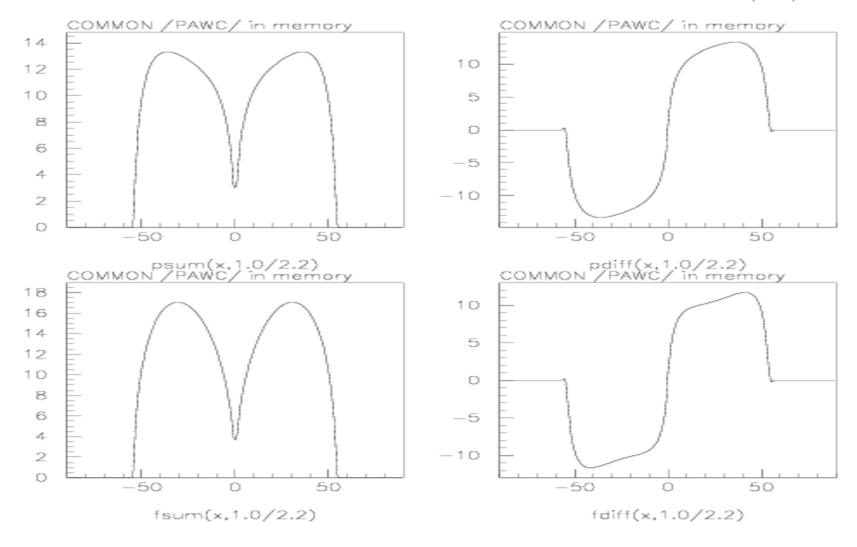
$$\mathbf{T_{p}} = \mathbf{1} - \left[\frac{\tan(\theta_{t} - \theta_{x})}{\tan(\theta_{t} + \theta_{x})}\right]^{2} \qquad \sin \theta_{t} = \frac{\sin \theta_{x}}{\sin \eta}$$

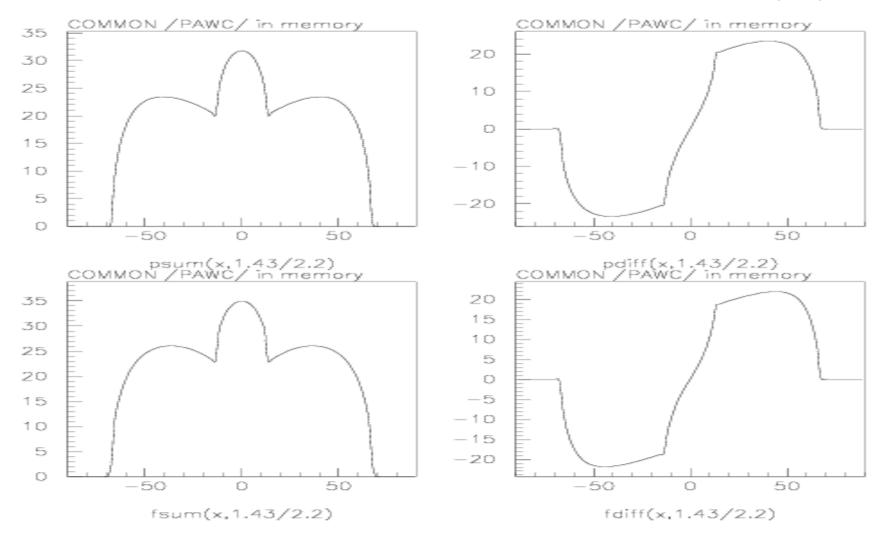
$$\mathbf{T_s} = 1 - \left[\frac{\sin(\theta_{\mathbf{t}} - \theta_{\mathbf{x}})}{\sin(\theta_{\mathbf{t}} + \theta_{\mathbf{x}})} \right]^2$$

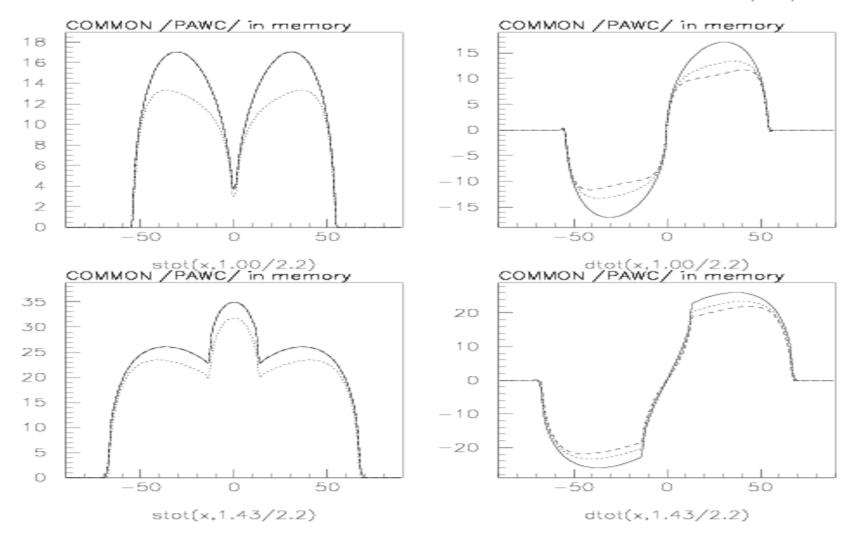
$$\mathbf{Prob}(\mathbf{T}) = \mathbf{T_p} \cdot |\mathbf{E_p}|^2 + \mathbf{T_s} \cdot |\mathbf{E_s}|^2$$

Reflection Coefficients









Scintillating Light

 Calculations for scintillating light, for (TOTAL) geometrical acceptance, transmittance and prompt collection give:

• AIR: 11, 81, 9 %

• SILICON: 24, 90, 22 %

Conclusions

•	I am convinced that geometry should likely be
	improved:

