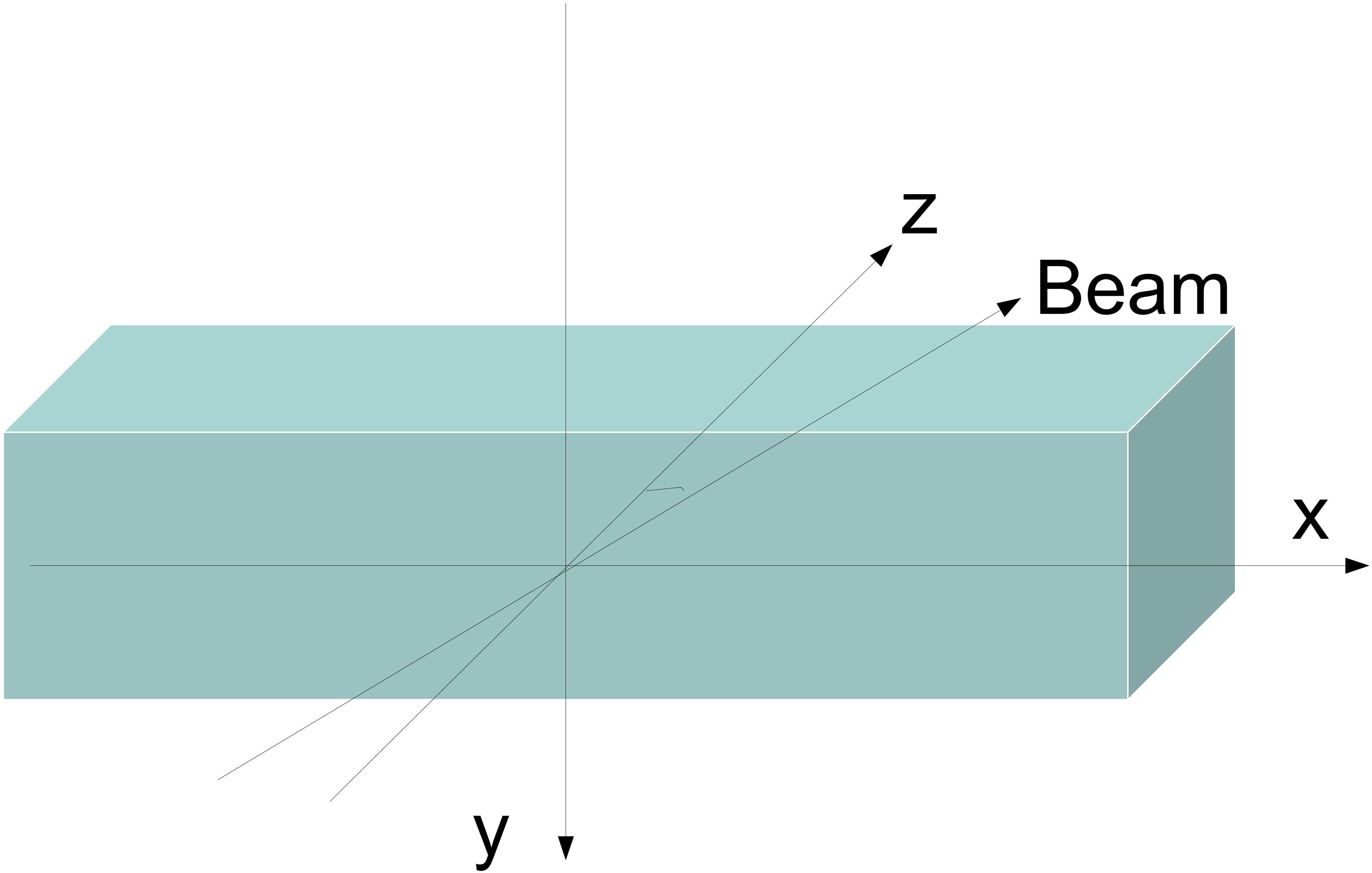
Algebra Exercises

Reference Frame



Beam Direction

- At 0 degree:
  - $p(0) = (0, 0, 1)$
- After a rotation in xz plane (of an angle ω):
  - $p(\omega) = (\sin \omega, 0, \cos \omega)$

Cherenkov Light

- At 0 degree:

$$\mathbf{C}(\mathbf{0}) = (\sin\mathbf{c}\cdot\cos\varphi,\ \ \sin\mathbf{c}\cdot\sin\varphi,\ \ \cos\mathbf{c})$$

$$= (\cos\alpha\cdot\cos\varphi,\ \ \cos\alpha\cdot\sin\varphi,\ \ \sin\alpha)$$

- c = Cherenkov angle
  α = critical angle with air = $\pi/2 - c$

- After beam rotation:

$$\mathbf{C}(\omega) = (\cos\alpha\cdot\cos\varphi\cdot\cos\omega + \sin\alpha\cdot\sin\omega,$$
$$\cos\alpha\cdot\sin\varphi,$$
$$-\cos\alpha\cdot\cos\varphi\cdot\sin\omega + \sin\alpha\cdot\cos\omega)$$

Geometrical Acceptance

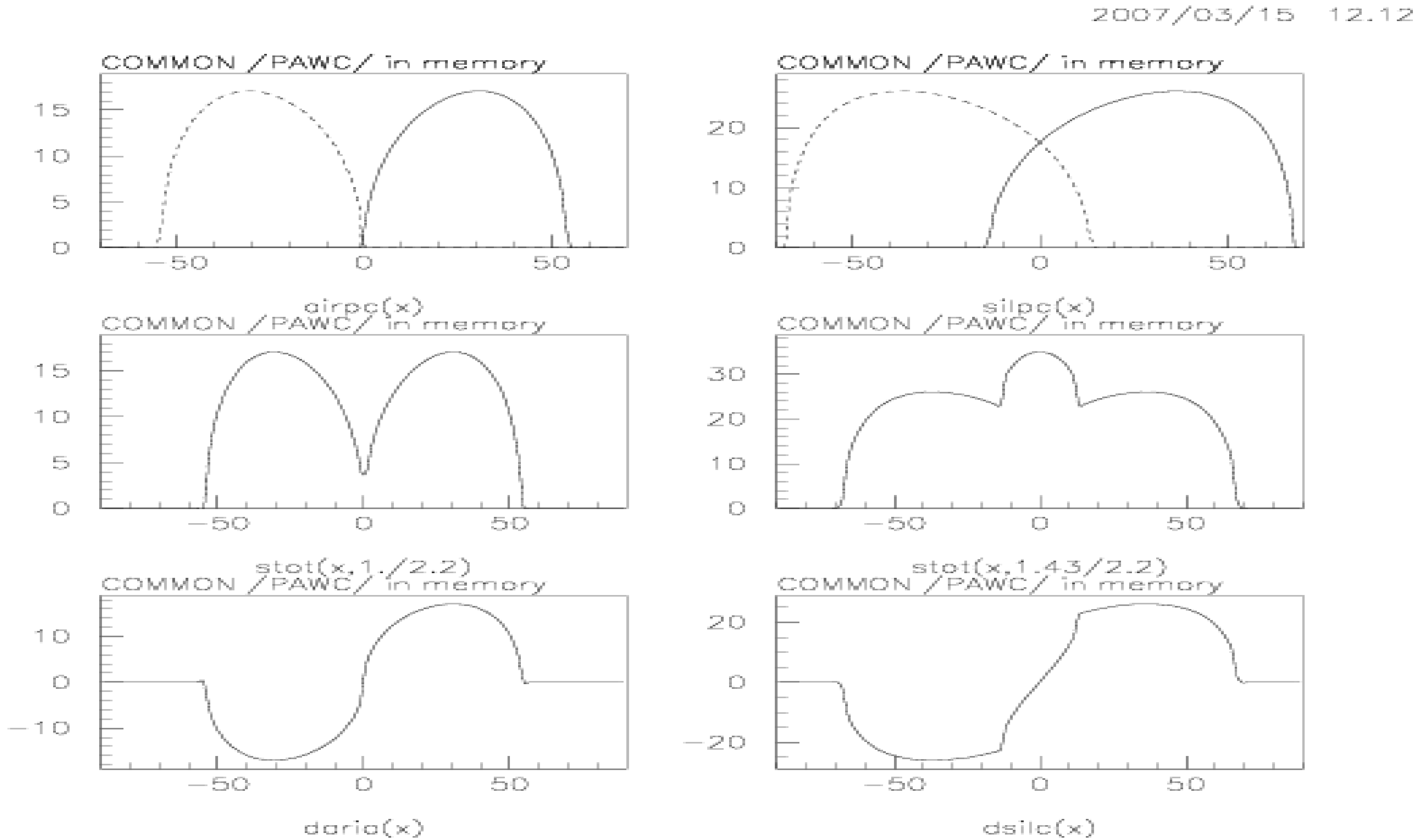
$$\mathbf{C} \;=\; (\cos\theta_{\mathbf{x}}, \cos\theta_{\mathbf{y}}, \cos\theta_{\mathbf{z}})$$

$$\cos\theta_x \;>\; \cos\eta \qquad (\eta \;:\; \mathit{critical\ angle})$$

$$\cos\alpha\cdot\cos\varphi\cdot\cos\omega \;+\; \sin\alpha\cdot\sin\omega \;>\; \cos\eta$$

$$\cos\varphi \;>\; \frac{\cos\eta \;-\; \sin\alpha\cdot\sin\omega}{\cos\alpha\cdot\cos\omega} \;=\; \cos\lambda$$

$$\lambda \;=\; f(\omega,\alpha,\eta)\ \ldots\ \alpha,\ \eta : \text{"}\mathit{known}\text{"}\ \ldots$$



2007/03/15 12.12
COMMON /PAWC/ in memory
airpc(x)
COMMON /PAWC/ in memory
silpc(x)
COMMON /PAWC/ in memory
stot(x,1./2.2)
COMMON /PAWC/ in memory
stot(x,1.43/2.2)
COMMON /PAWC/ in memory
dario(x)
COMMON /PAWC/ in memory
dsilc(x)

Wrapping

- From the unitarity constraint:

$$\cos\theta_{\mathbf{y}}, \cos\theta_{\mathbf{z}} \leq \sin\theta_{\mathbf{x}} < \sin\eta < \cos\eta_{\mathbf{air}}$$

being :

$$\sin\eta_{\mathbf{air}} = \mathbf{1.0/2.2 = 0.45}$$
$$\sin\eta_{\mathbf{sil}} = \mathbf{1.43/2.2 = 0.65}$$
$$\cos\eta_{\mathbf{air}} = \mathbf{0.89}$$

so :

$$\theta_{\mathbf{x}}, \theta_{\mathbf{z}} > \eta_{\mathbf{air}}$$

anyway totally reflected on the non – instrumented faces

Reflective wrapping doesn't improve light collection.

Polarization

- Cherenkov light 100% linearly polarized in plane containing photon and incident particle momenta:

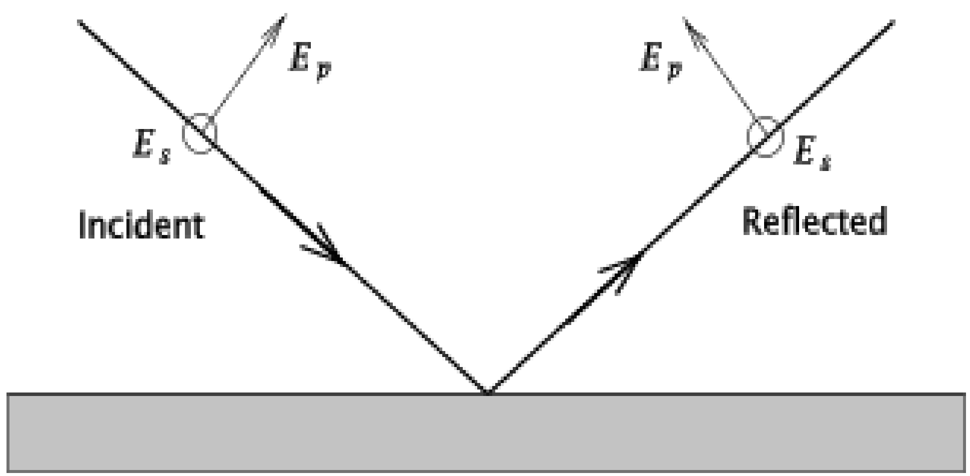
$$\overrightarrow{E(0)} = (\cos c \cdot \cos\varphi, \ \cos c \cdot \sin\varphi, \ -\sin c) \\ = (\sin\alpha \cdot \cos\varphi, \ \sin\alpha \cdot \sin\varphi, \ -\cos\alpha)$$

- After rotation:

$$\overrightarrow{E(\omega)} = (\sin\alpha \cdot \cos\varphi \cdot \cos\omega - \cos\alpha \cdot \sin\omega, \\ \sin\alpha \cdot \sin\varphi, \\ - \sin\alpha \cdot \cos\varphi \cdot \sin\omega - \cos\alpha \cdot \cos\omega)$$

Polarization

- Two components: Es and Ep



$$\overrightarrow{E}_p = (\ E_x,\ a \cdot E_y,\ b \cdot E_z\ )$$

$$\overrightarrow{E}_s = (\ 0,\ (1-a) \cdot E_y,\ (1-b) \cdot E_z\ )$$

Transmission

$$|\mathbf{E_p}| \;=\; \frac{\mathbf{E_x}}{\sqrt{1-\mathbf{C_x^2}}}$$

$$|\mathbf{E_s}| \;=\; \sqrt{1-\mathbf{E_p^2}} \;=\; \frac{\sin\ \varphi\cdot\cos\ \omega}{\sqrt{1-\mathbf{C_x^2}}}$$

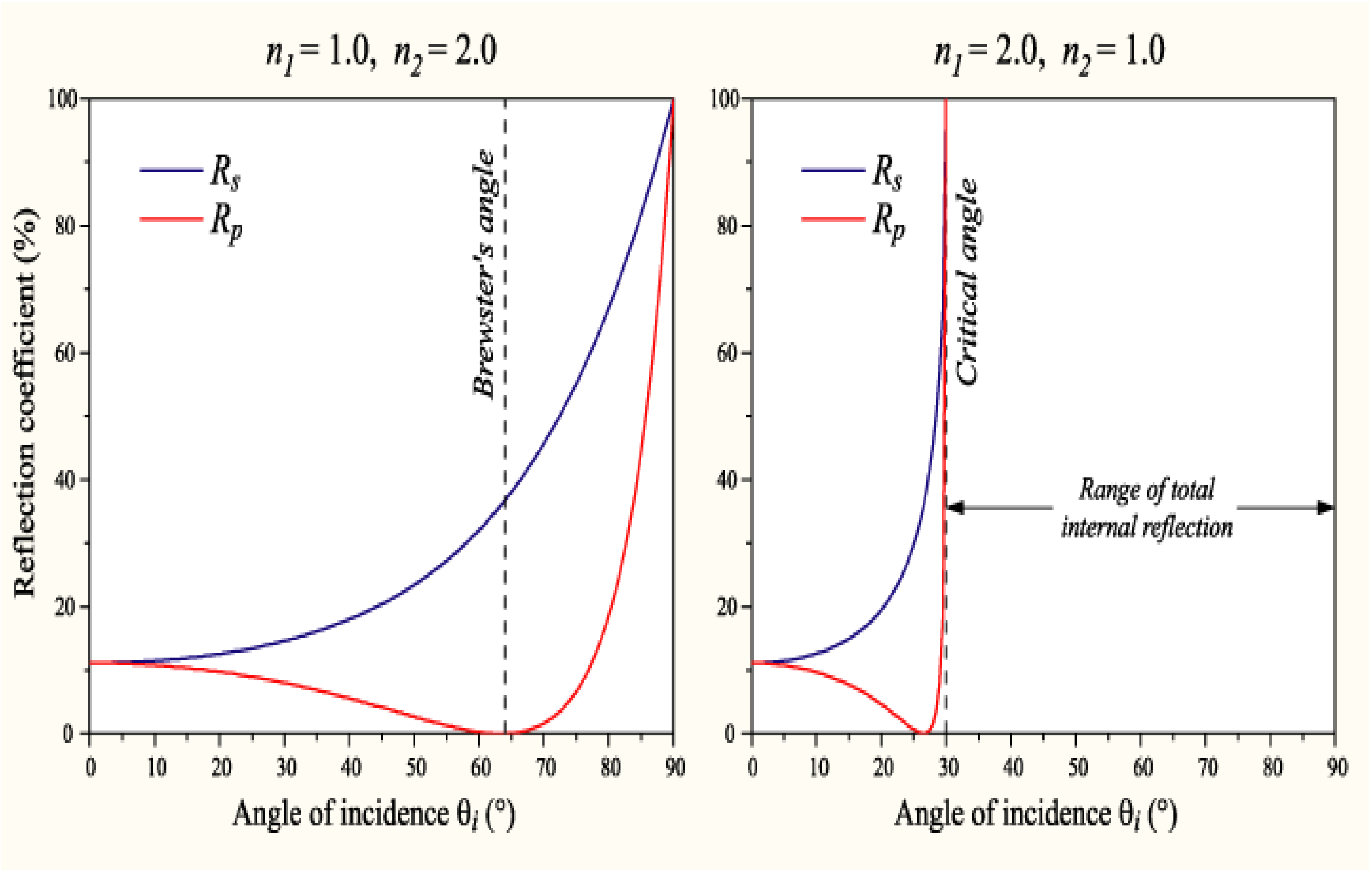
$$\mathbf{T_p} \;=\; \mathbf{1}-\left[\frac{\tan(\theta_\mathbf{t}-\theta_\mathbf{x})}{\tan(\theta_\mathbf{t}+\theta_\mathbf{x})}\right]^\mathbf{2}$$

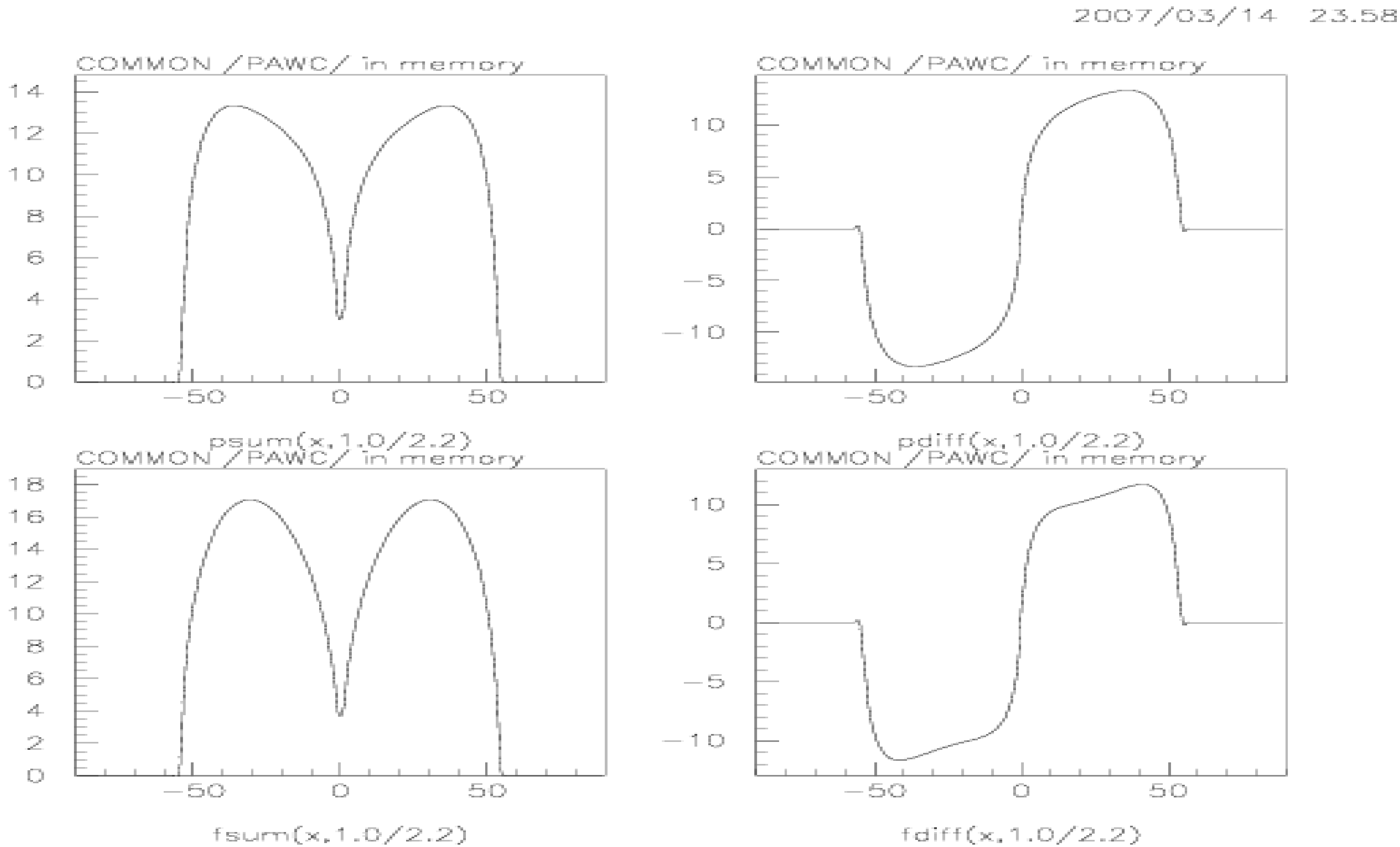
$$\sin\theta_\mathbf{t} \;=\; \frac{\sin\theta_\mathbf{x}}{\sin\eta}$$

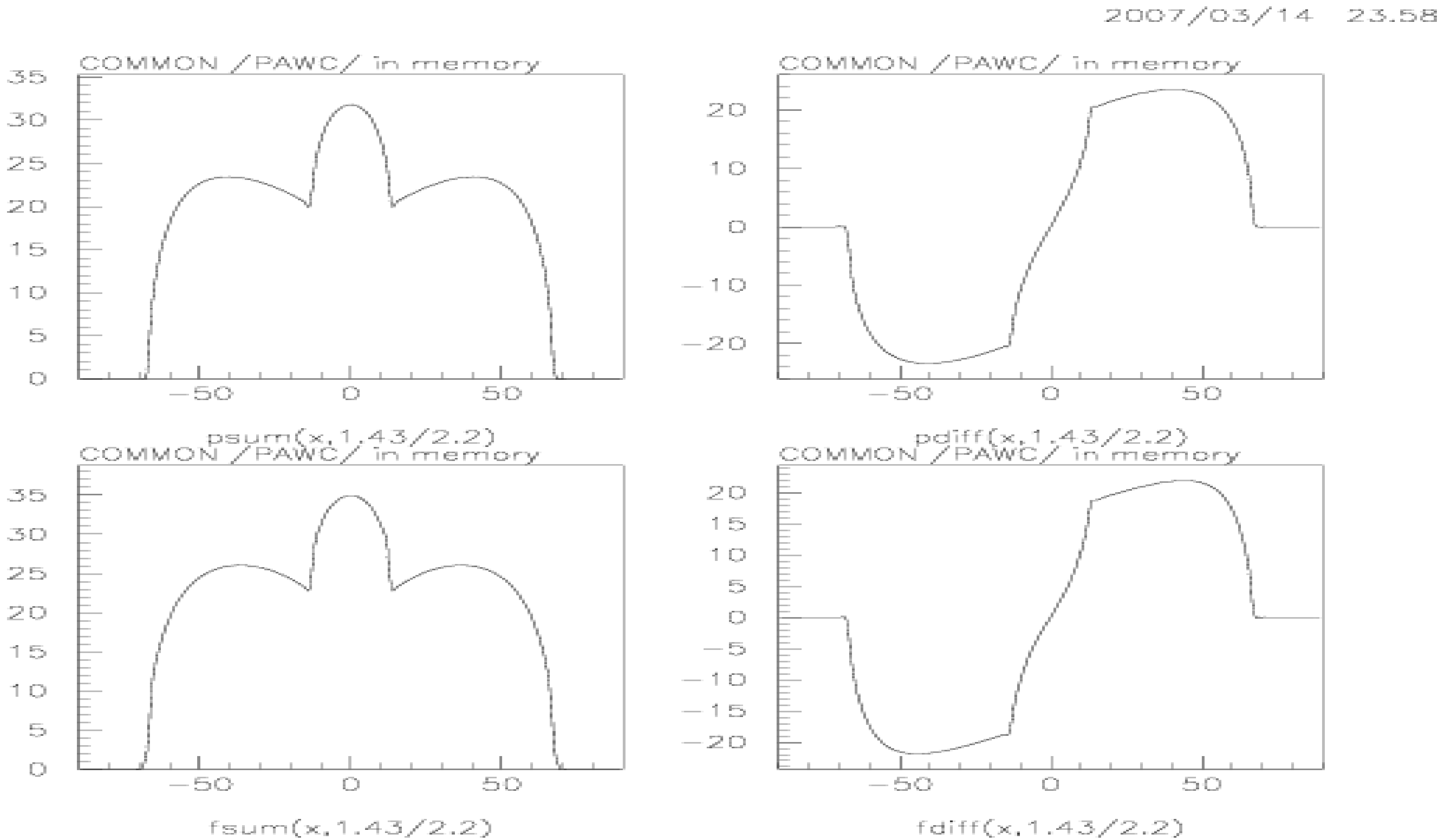
$$\mathbf{T_s} \;=\; \mathbf{1}-\left[\frac{\sin(\theta_\mathbf{t}-\theta_\mathbf{x})}{\sin(\theta_\mathbf{t}+\theta_\mathbf{x})}\right]^\mathbf{2}$$

$$\mathbf{Prob(T)} \;=\; \mathbf{T_p}\cdot|\mathbf{E_p}|^\mathbf{2}+\mathbf{T_s}\cdot|\mathbf{E_s}|^\mathbf{2}$$

Reflection Coefficients




2007/03/14 23.58
COMMON /PAWC/ in memory
psum(x,1.0/2.2)
COMMON /PAWC/ in memory
pdiff(x,1.0/2.2)
COMMON /PAWC/ in memory
fsum(x,1.0/2.2)
COMMON /PAWC/ in memory
fdiff(x,1.0/2.2)
−50
0
50


2007/03/14 23.58
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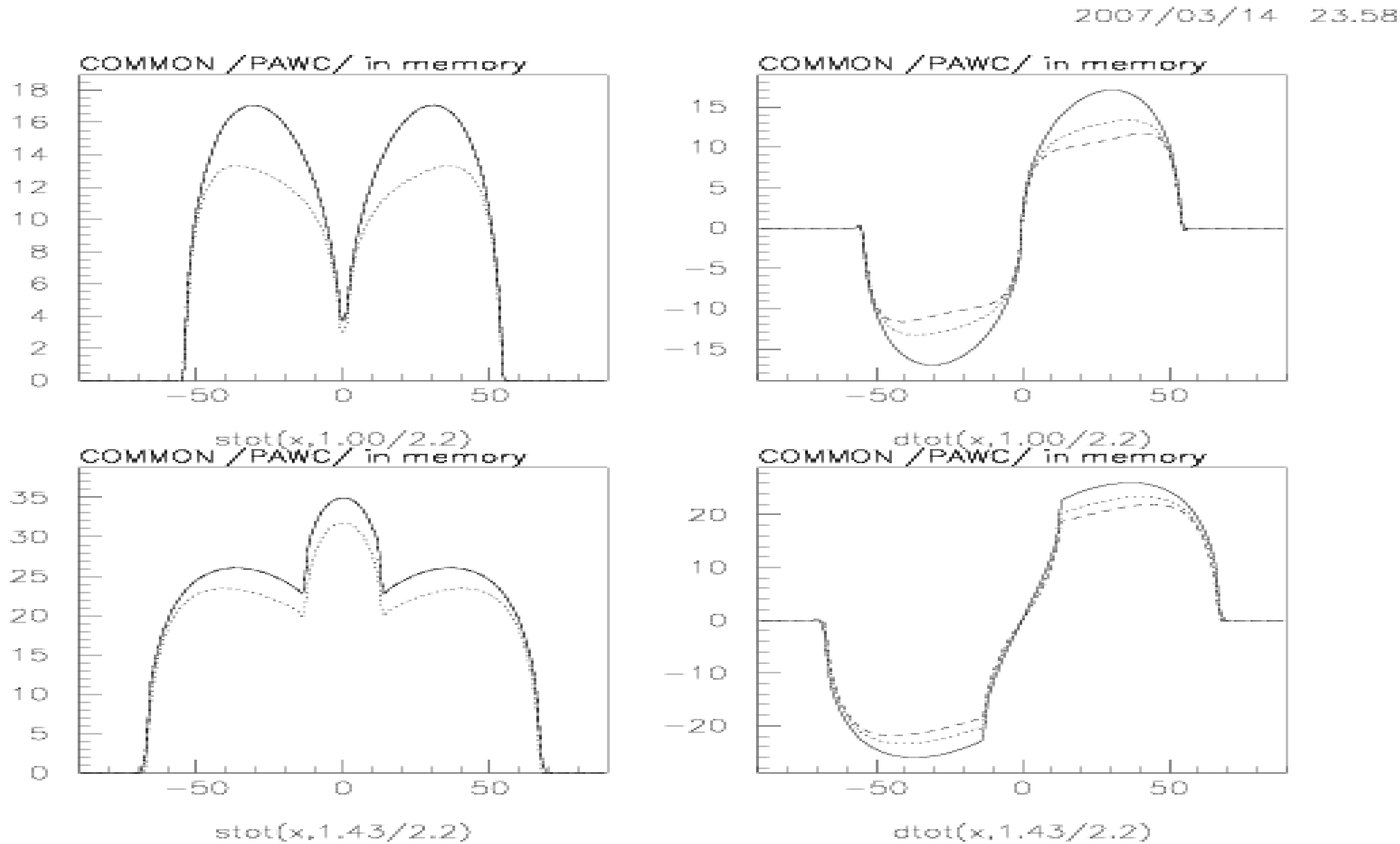
COMMON /PAWC/ in memory
fdiff(x,1.43/2.2)


2007/03/14 23.58
COMMON /PAWC/ in memory
stot(x,1.00/2.2)
COMMON /PAWC/ in memory
dtot(x,1.00/2.2)
COMMON /PAWC/ in memory
stot(x,1.43/2.2)
COMMON /PAWC/ in memory
dtot(x,1.43/2.2)

Scintillating Light

- Calculations for scintillating light, for (TOTAL) geometrical acceptance, transmittance and prompt collection give:
- AIR : 11, 81, 9 %
- SILICON : 24, 90, 22 %

Conclusions

- ... I am convinced that likely geometry could (should ?) be improved :