Theoretical Astroparticle Physics 3

3. Dark Matter (contd.)

3.3 WIMPs

Assume DM particle χ with weak but non-negligible interactions with the SM. At early times: chemical equilibrium:

$$\underset{\text{eq. number density}}{n_{eq}} = \underset{\substack{\text{number of d.o.f.} \\ \text{(e.g. 4 for Dirac fermion)}}}{g} \int \frac{d^3 p}{(2\pi)^3}\, e^{-(E-\mu)/T}$$

μ: chemical potential $= 0$

$$= \frac{g}{2\pi^2} \int dE\, E \sqrt{E^2 - m^2}\, e^{-E/T}$$

Later: T drops below m_χ

$\Rightarrow$ n_{eq} exponentially suppressed

eventually, n_{eq} becomes so small that $\chi\chi \to SM\, SM$ becomes inefficient and "freezes out"

Boltzmann Eq:

number of χ in given volume element

$$\frac{dN_\chi}{dt} = \Gamma(SM\, SM \to \chi\bar{\chi}) - \Gamma(\chi\bar{\chi} \to SM\, SM)$$

Detailed balance:

$$\Gamma^{eq}(SM\, SM \to \chi\bar{\chi}) = \Gamma^{eq}(\chi\bar{\chi} \to SM\, SM)$$

As $\Gamma(SM\, SM \to \chi\bar{\chi})$ is determined by the thermal distributions of SM particles (which remain in thermal equilibrium), we can replace

$$\Gamma(SM\, SM \to \chi\bar{\chi}) = \Gamma^{eq}(SM\, SM \to \chi\bar{\chi})$$

Moreover, express rate in terms of x-section:

$$\text{cross section} = \frac{\text{rate}}{(\#\text{ of targets}) \cdot (\text{beam flux})}$$

$$\hookrightarrow \Gamma(\chi\bar{\chi} \to SM\,SM) = \Big\langle \sigma(\chi\bar{\chi} \to SM\,SM) \cdot (n_\chi V)(n_\chi v_{rel}) \Big\rangle$$

thermal average

$$V_c R^3(t)$$

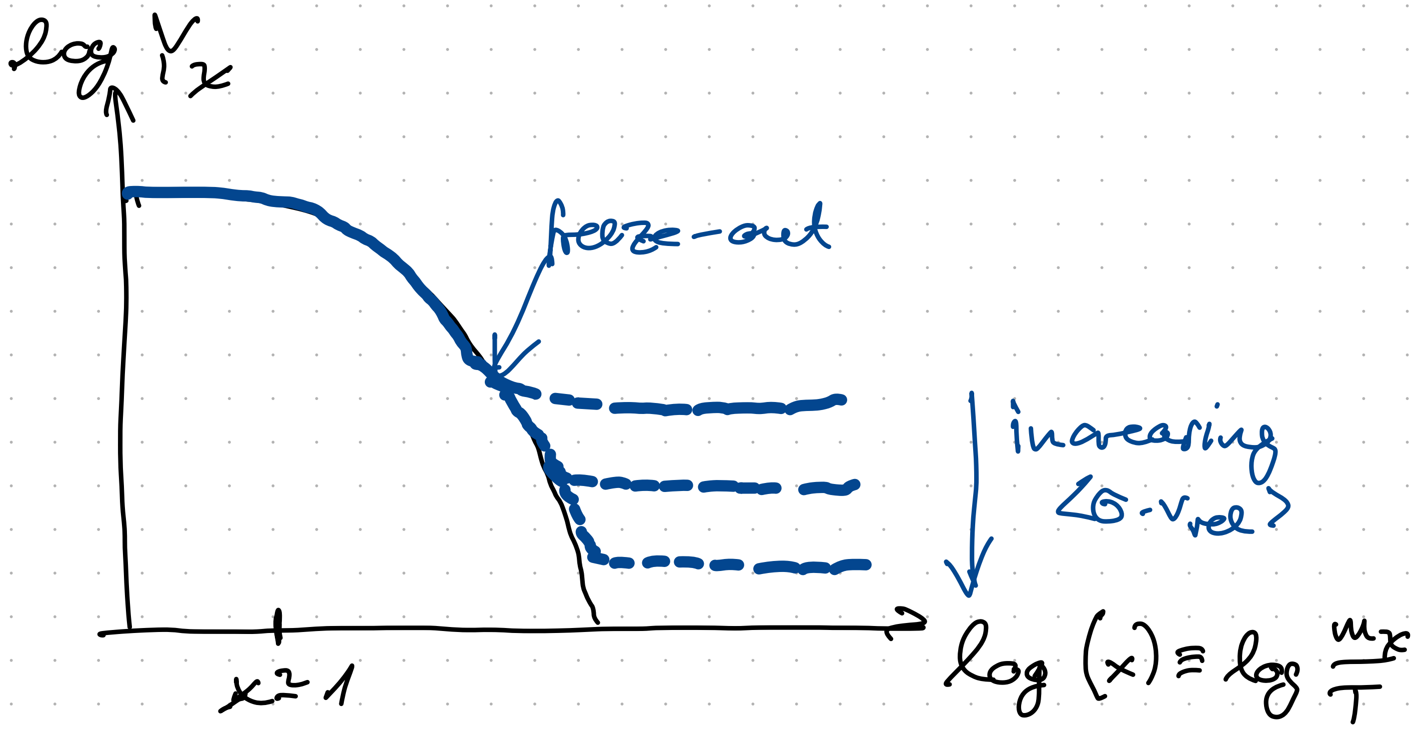
$$\Rightarrow \frac{dN_\chi}{dt} = \frac{d(n_\chi V)}{dt} = \langle \sigma(\chi\bar{\chi} \to SM\,SM) \cdot v_{rel} \rangle \cdot (n_{\chi,eq}^2 - n_\chi^2) \cdot V$$

$$\boxed{\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma(\chi\bar{\chi} \to SM\,SM) \cdot v_{rel} \rangle \cdot (n_\chi^2 - n_{\chi,eq}^2)}$$

Often used definition:

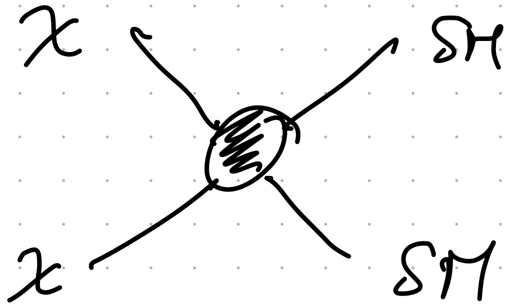
$$Y_\chi \equiv \frac{n_\chi}{s}$$

entropy density ($s \cdot R^3(t) \equiv \text{const}$)

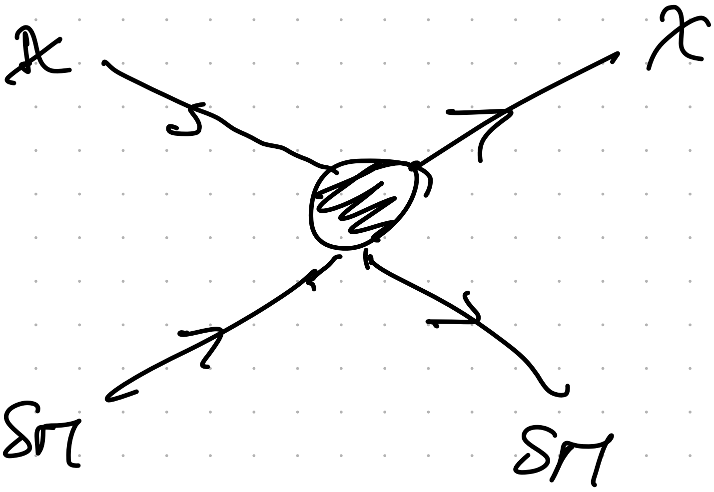


WIMP Detection today

Freeze-out requires interactions of the form

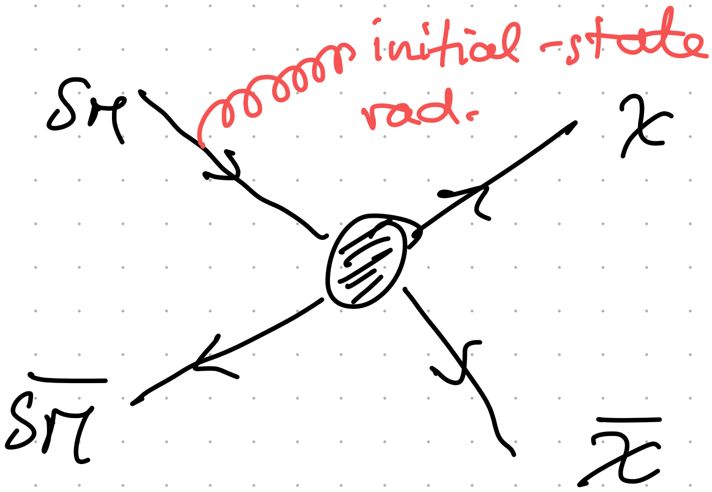


Turn diagram around

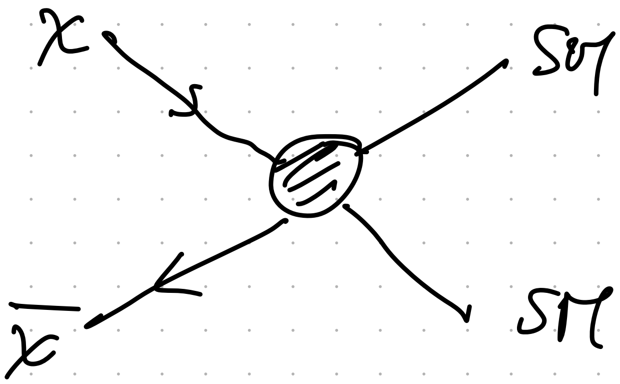


DM – nucleus scattering

"Direct DM detection"



Collider production



DM annihilation

↳ Indirect detection

3.4 Primordial Black Holes

Production

- Must have happened very early to satisfy CMB observations (BHs from stellar collapse not an option)
- upward fluctuation of plasma density collapses into BH
- Criterion: "collapse should be faster than rebound"

  - collapse timescale: $\frac{1}{(G\,\delta\rho)^{1/2}}$ overdensity

    (based on $R \sim \frac{GMt^2}{R^2}$)

  - rebound timescale $\frac{R}{c_{sound}} \sim \frac{R}{\sqrt{w}}$

    e.o.s. param

- $$\frac{1}{G\rho} \lesssim \frac{R^2}{w}$$

$$R \gtrsim \sqrt{\frac{w}{G\,\delta\rho}}$$

- set $R \sim \frac{1}{H}$ (Hubble radius or Hubble horizon)

  $H \sim \frac{T^2}{m_{Pl}}$

- use $G \sim \frac{1}{m_{Pl}}$

- Leads to $$\boxed{\frac{\delta\rho}{T^4} > w}$$

  T^4 ~ radiation density

Conclusion: an $O(1)$ overdensity is required to form PBHs.

Mechanisms:

- Inflation
- Phase transitions