3.5 Axions

3.5.1 The strong CP Problem

$$\mathcal{L}_{QCD} = i\bar{\Psi}\not{D}\Psi - \bar{\Psi}_L M \Psi_R - \bar{\Psi}_R M \Psi_L$$

$$- \frac{1}{4} \operatorname{tr} F^a_{\mu\nu} F^{\mu\nu,a} + \frac{\Theta g^2}{16\pi^2} \operatorname{tr} F^a_{\mu\nu} \tilde{F}^{\mu\nu,a}$$

where

generators

$$F^a_{\mu\nu} \equiv \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu\right) t^a$$

structure constants

$$\tilde{F}^a_{\mu\nu} \equiv \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma,a}$$

$$D_\mu \equiv \partial_\mu - i g A^a_\mu t^a$$

Consequences of the extra term:

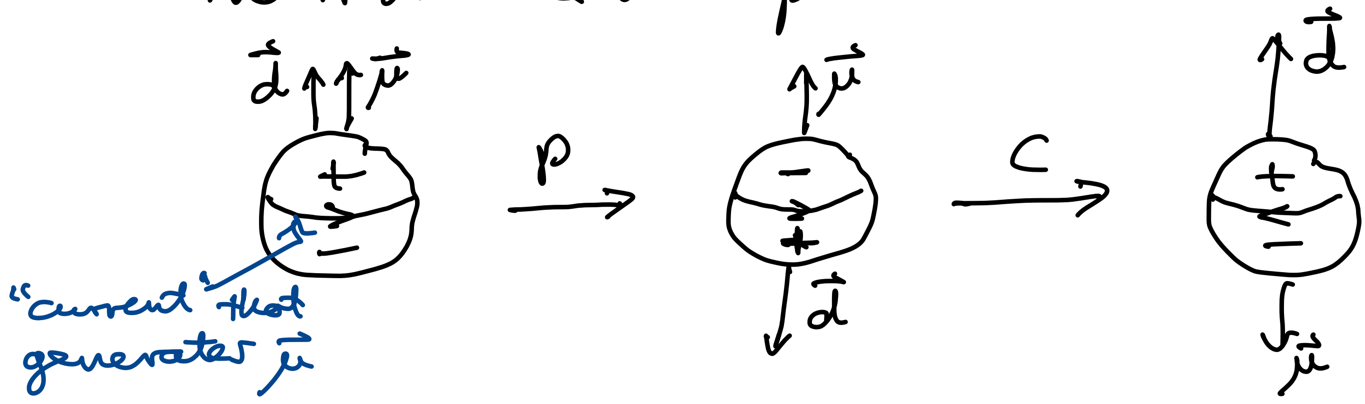
- P and CP odd: $F^{\mu\nu} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \neq 0$

only μ, ν, ρ, σ are all different.

$\rightarrow$ 3 spatial components, which flip sign under P. G does not affect A^a_μ, which is a real field.

- Observable sensitive to $\not{CP}$ in QCD:

Neutron electric dipole moment



$$\vec\mu\cdot\vec d \xrightarrow{CP} -\vec\mu\cdot\vec d$$

We know that $\vec\mu \neq 0$, therefore to preserve CP, $\vec d = 0$ would be needed.

Experiments constrain $|\vec d| < 0{,}29\times 10^{-25}$ e·cm

One can show that this corresponds to

$$\boxed{\theta \ll 10^{-10}}$$

<u>Strong CP Problem:</u> why is θ so small?

Relation to axial transformations

infinitesimal

$$\psi \to \psi' = (1 + i\alpha\gamma^5)\psi$$

$$\bar{\psi} \to \bar{\psi}' = \bar{\psi}(1 + i\alpha\gamma^5)$$

$$\mathcal{L}_{QCD} \to \mathcal{L}'_{QCD} = i\bar{\psi}(1+i\alpha\gamma^5)\gamma^\mu D_\mu \cdot (1+i\alpha\gamma^5)\psi$$

$$- \left[M\bar{\psi}(1+i\alpha\gamma^5)\frac{1+\gamma^5}{2}(1+i\alpha\gamma^5)\psi + h.c. \right]$$

$$- \text{gauge-kinetic term} - \theta\text{-term}$$

$$= \mathcal{L}_{QCD} - \left[2i\alpha M \bar{\psi}_L \psi_R + h.c. \right] \quad (*)$$

$\to$ massless QCD ~~is~~ would seem invariant under axial trafos

However: quantum effects break this classical symmetry

($\to$ <u>axial anomaly</u>)

Deeper reason: $S = \int d^4x \, \mathcal{L}_{QCD, M=0}$

is invariant, but the functional measure in the path integral is not

$$Z_\psi = \int \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \, \exp\left[i \int d^4x \, \bar{\psi} i \not{D} \psi \right]$$

consider only fermions as A_μ^a are not affected by $U(1)_A$ axial trafos

$$Z_\psi \xrightarrow{U(1)_A} Z_{\psi'} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \, \exp\left[i \int d^4x \left(\bar{\psi} i \not{D} \psi + i \alpha \frac{g^2}{8\pi^2} F_{\mu\nu}^a \tilde{F}^{\mu\nu, a} \right) \right]$$

In massless QCD, θ-term can be removed by a $U(1)_A$ transformation.

In massive QCD, θ can be traded for complex phases in the mass matrix. (see eq. (*) which shows how $\mathcal{L}_{QCD}$ transforms under $U(1)_A$).

What remains unchanged under $U(1)_A$ is the combination

$$\theta_{eff} \equiv N_f\, \theta + \arg\det M$$

when generalizing to more than one quark species

3.5.2 The Peccei – Quinn Mechanism

Idea: Add a dynamical field that contributes to arg det M and show that its vev dynamically evolves to a value where $\theta_{eff} = 0$.

Toy model:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi$$

complex scalar

$$+ y\, \phi\, \bar{\psi}_L \psi_R + y^\dagger \phi^\dagger \bar{\psi}_R \psi_L$$

$$- (\partial_\mu \phi)(\partial^\mu \phi) - V(\phi, \phi^\dagger)$$

$$+ \frac{\theta g^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Under axial $U(1)_{PQ}$ transformations

$$\psi_L \to e^{i\alpha} \psi_L$$

$$\psi_R \to e^{-i\alpha} \psi_R$$

$$\phi \to e^{2i\alpha} \phi$$

- as above: absorb θ into complex phase of y
- assume ϕ develops vev
- show that $\boxed{\arg \langle\phi\rangle + \arg y = 0}$

Outline of proof:

Goal: compute effective potential (potential including quantum corrections)

Partition function

$$Z[J] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\phi \mathcal{D}A \; e^{i\int d^4x \, (\mathcal{L} + J\phi)}$$

$$\Rightarrow \langle\phi\rangle = -i \frac{\delta \log Z[J]}{\delta J}$$

Effective action:

$$\Gamma[\langle\phi\rangle] \equiv -i \log Z[J] - \int d^4y\, J(y) \langle\phi\rangle(y)$$

$$\Rightarrow \frac{\delta \Gamma[\langle\phi\rangle]}{\delta\langle\phi\rangle} = -i \int d^4y \frac{\delta \log Z[J]}{\delta J(y)} \frac{\delta J(y)}{\delta\langle\phi\rangle}$$

$$- J(x) + \int d^4y \frac{\delta J(y)}{\delta\langle\phi\rangle(x)} \langle\phi\rangle(y)$$

$$= -J(x)$$

$\Rightarrow$ for zero $J(x)$, $\langle\phi\rangle$ minimizes the effective action, $\Gamma[\langle\phi\rangle]$

Effective potential

$$V_{eff}[\langle\phi\rangle] \equiv -\frac{1}{V \cdot T} \Gamma[\langle\phi\rangle]$$

volume time interval

Here: - terms depending on avg $\langle\phi\rangle$ have to come from the Yukawa couplings

- assume $y \ll 1$ and expand

$$V_{eff}[\langle\phi\rangle] \sim V_{class}(\langle\phi\rangle) + y\langle\phi\rangle \cdot K + y^*\langle\phi\rangle^* K$$

unknown constant, has to be real because the rest of $\mathcal{L}$ is symmetric under $\psi_L \leftrightarrow \psi_R$

$$\sim -|y| \cdot |\langle\phi\rangle| \cdot K \cdot \cos(\arg\langle\phi\rangle + \arg y)$$

non-trivial

$$\begin{cases} y\langle\phi\rangle = |y|\,|\langle\phi\rangle|\, e^{i \arg y}\, e^{i \arg\langle\phi\rangle} \\ y^*\langle\phi\rangle^* = |y|\,|\langle\phi\rangle|\, e^{-i \arg y}\, e^{-i \arg\langle\phi\rangle} \end{cases}$$

$\Rightarrow$ $V_{eff}[\langle\phi\rangle]$ is extremal at

$$\boxed{\arg\langle\phi\rangle + \arg y = 0}$$

3.5.3 The axion

- Write $\phi = |\phi| \cdot e^{i\left(\underbrace{\alpha}_{\arg\langle\phi\rangle} + \frac{a}{f_a}\right)}$ dynamical real scalar field

If $|\phi|$ has very large mass, we cannot observe it

↳ we can write

$$\phi = f_a \, e^{i(\alpha + \frac{a}{f_a})}$$

Excitations of $a \iff$ chiral rotations

Using the axial anomaly again, we can made the transformation

$$y f_a e^{i(\alpha + \frac{a}{f_a})} \bar{\psi}_L \psi_R$$

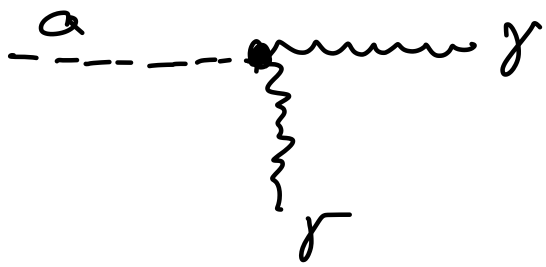
$$\downarrow$$

$$y f_a e^{i\alpha} \bar{\psi}_L \psi_R + \frac{a g^2}{16\pi^2 f_a} \operatorname{tr} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

low-E phenomenology of a (the axion) depends on $a - A_\mu$ couplings because $|\phi|$, ψ are very heavy

If ψ is charged under several gauge group factors, we obtain an $aF_{\mu\nu}\tilde{F}^{\mu\nu}$ coupling for each of them, in particular

$$\boxed{\frac{a\, e^2 \overbrace{q^2}^{\text{em charge of } \psi}}{16\pi^2 f_a}\, \underbrace{F_{\mu\nu}F^{\mu\nu}}_{\text{em field strength tensor } (F_{\mu\nu} = \tilde{F}_{\mu\nu})}}$$



for instance: axions traveling through a strong B-field can convert into photons