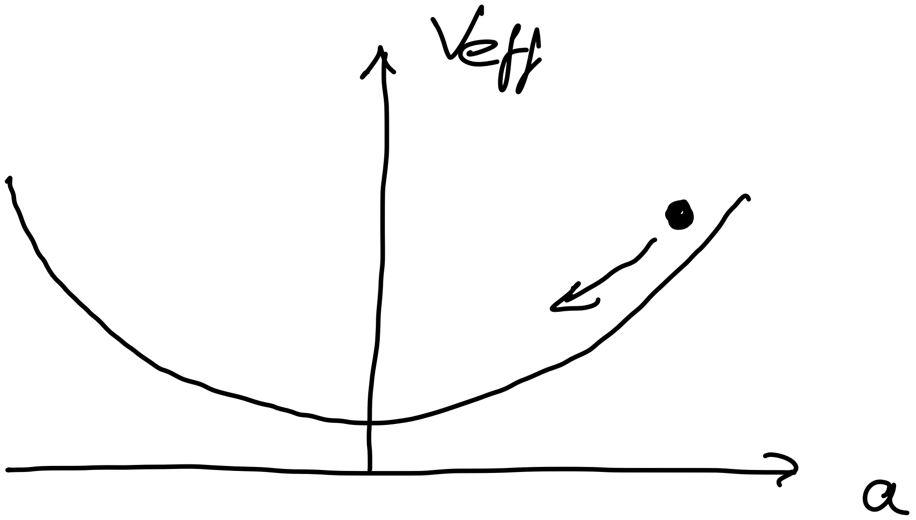
3.5.4 Production of Axion DM



$$\mathcal{L} = \sqrt{-g}\left[(D_\mu a)(D^\mu a) - V(a) \right]$$

determinant of $(g_{\mu\nu})$

$$D_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu a)} - \frac{\delta \mathcal{L}}{\delta a} = 0$$

covariant derivative

$$D_\mu a = \partial_\mu a$$

$$D_\mu X^\mu \equiv \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, X^\mu \right)$$

$$\Rightarrow D_\mu \left(\sqrt{-g}\, \partial^\mu a \right) + \sqrt{-g} \frac{\delta V}{\delta a} = 0$$

$$\Leftrightarrow \quad \frac{1}{\sqrt{-g}}\,\partial_\mu\left(-g\,\underset{\uparrow \sim R^3(t)}{\partial^\mu a}\right) + \sqrt{-g}\,\frac{\delta V}{\delta a} = 0$$

$\vec{\nabla} a \simeq 0$

$$\Leftrightarrow \quad \sqrt{-g}\,\ddot{a} + \frac{1}{\sqrt{-g}}\,\dot{a}\,3R^2\dot{R} + \sqrt{-g}\,\frac{\delta V}{\delta a} = 0$$

$$\Leftrightarrow \quad \boxed{\ddot{a} + 3H\dot{a} + \frac{\delta V}{\delta a} \simeq 0}$$

At early times: H is very large

$\rightarrow \dot{a} = 0$

Later: a "rolls down" the potential and oscillates

Potential is shallow

$\rightarrow$ slow oscillations

amplitude = # of particles

osc. frequency = particle energy

$\Rightarrow$ production of <u>many</u>, <u>non-relativistic</u> axions (in spite of small axion mass)

"Misalignment Mechanism"

4. Neutrinos

4.1 Neutrino Masses

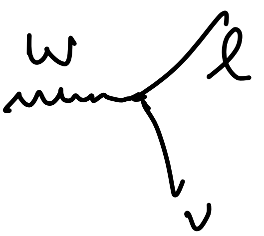
4.1.1 Neutrinos in the SM

Quarks	u	c	t
	d	s	b
Leptons	ν_e	ν_μ	ν_τ
	e	μ	τ

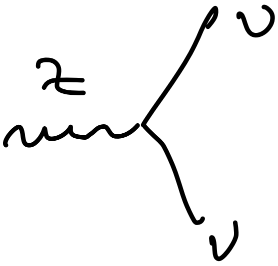
Why study neutrinos

- understand origin of flavor
- explain baryon asymmetry
- possible connections to DM

$$\mathcal{L} \supset \sum_{\alpha=e,\mu,\tau} \Big[\overline{\nu_{\alpha L}}\, i\not\partial\, \nu_{\alpha L}$$



$$+ \frac{g}{\sqrt{2}} \left(W^{\mu +} \overline{\nu_{\alpha L}} \gamma_\mu e_{\alpha L} + h.c. \right)$$



$$+ \frac{g}{2\cos\theta_W} Z^\mu \bar{\nu}_{\alpha L} \gamma_\mu \nu_{\alpha L} \Big]$$

$+$ mass terms

4.1.2 Dirac Masses

In the SM, only LH neutrinos exist

$$\nu_L = \frac{1-\gamma^5}{2}\nu \mathrel{\hat{=}} \begin{pmatrix} * \\ * \\ 0 \\ 0 \end{pmatrix}$$

Introduce

$$\nu_R = \frac{1+\gamma^5}{2}\nu \mathrel{\hat{=}} \begin{pmatrix} 0 \\ 0 \\ * \\ * \end{pmatrix}$$

$$\Rightarrow \mathcal{L}_m \equiv \sum_{\underbrace{\alpha,\beta}_{e,\mu,\tau}} \underbrace{m_{\alpha\beta}}_{y_{\alpha\beta}\langle\phi\rangle} \bar{\nu}_{\alpha L} \nu_{\beta R}$$

$y_{\alpha\beta}\langle\phi\rangle$ arises from $\mathcal{L} \sim -y \bar{L}_\alpha \tilde{H} \nu_{R\beta}$

We have introduced two additional degrees of freedom

Problem: $m_{\alpha\beta}$ is $< eV$

$\Rightarrow y_{\alpha\beta}$ need to be tiny

It would be nice to have a deeper explanation for this

4.1.3 Majorana Masses

Mass terms couple LH to RH fields

The antiparticle of ν_L <u>is</u> a RH field

Could it play the role of ν_R?

More formally: charge conjugation

$$\hat{C}: \Psi \to \Psi^c \equiv -i\gamma^2\gamma^0\overline{\Psi}^T$$

Effect on chirality:

$$\gamma^5\psi^c = \gamma^5\left(-i\gamma^2\gamma^0\overline{\psi}^T\right)$$

$$= \gamma^5 \left(-i\gamma^2 \gamma^0 \gamma^0 \psi^*\right)$$

$$= + i \gamma^2 \gamma^5 \psi^*$$

$$= - (\gamma^5 \psi)^c$$

shows that $\hat{C}$ flips chirality

Identify $\boxed{\nu_R \equiv \nu_L^c}$

In 4-component notation

$$\nu_L = \begin{pmatrix} \chi_1 \\ \chi_2 \\ 0 \\ 0 \end{pmatrix} \hat{=} \begin{pmatrix} \chi \\ 0 \end{pmatrix}$$

$$\nu_L^c = -i \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix} \begin{pmatrix} \chi^* \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i\sigma^2 \chi^* \end{pmatrix}$$

A new type of mass term:

$$\mathcal{L}_m = -\frac{1}{2} \sum_{\alpha,\beta} \underbrace{m_{\alpha\beta}}_{\text{complex symmetric}} \overline{\nu_{L\alpha}^c} \, \nu_{L\beta} + h.c.$$

- Problem: why are the $m_{\alpha\beta}$ so small?
- how can this be made $SU(2)$-invariant (under $U(1)$, $\nu_L \to e^{i\alpha}\nu_L$, $\overline{\nu_L^c} \to e^{i\alpha}\overline{\nu_L^c}$. This is not a problem for the neutrinos, who do not carry any $U(1)$ charges. But it forbids Majorana mass terms for all other SM fermions, in particular the charged leptons who are in the same $SU(2)$ multiplets as the neutrinos)

4.1.4 The seesaw mechanism

Augment the SM with 3 RH neutrinos N_R, singlet under all SM gauge groups.

For simplicity consider only one flavor.

$$\mathcal{L} \supset -\underbrace{m_D}_{\substack{\text{from Higgs mechanism} \\ \text{via Yukawa coupling} \\ -y\,\bar{L}\tilde{H}N_R \\ \text{natural scale is} \\ m_D \sim m_H}} \bar{\nu}_L N_R - \frac{1}{2}\underbrace{m_M}_{\substack{\text{no natural} \\ \text{scale.} \\ \text{Assume} \\ m_M \gg m_H}} \overline{N_R^c} N_R + \text{h.c.}$$

Write $n = \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix}$

$$\rightarrow \mathcal{L} \supset -\frac{1}{2} n^c \underbrace{M}_{\begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix}} n + \text{h.c.}$$

$$= -\frac{1}{2}\left(\overline{\nu_L^c}\, m_D N_R^c + m_M \overline{N_R} N_R^c\right) + \text{h.c.}$$

Diagonalize M to obtain physical states. To do so, introduce trafo

$$\begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix}$$

$\hookrightarrow$ mass matrix goes to

$$\begin{pmatrix} \cos\theta & +\sin\theta \\ \mp\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

require that the off-diagonal elements vanish

$$\hookrightarrow \quad \tan 2\theta = \frac{2m_D}{m_M}$$

Eigenvalues:

$$m_{1,2} = \frac{m_M}{2} \mp \sqrt{\frac{m_M^2}{4} + m_D^2}$$

$$m_2 \simeq m_M$$

$$m_1 \simeq \frac{m_D^2}{m_M}$$

$\Rightarrow$ we obtain one neutrino state ($\simeq \nu_L$) with small mass $\frac{m_D^2}{m_M}$ and one super-heavy state (N_R) with mass $\simeq m_M$.

With $m_D \sim 100$ GeV, $m_M \sim 10^{14}$ GeV, we obtain $m_1 \sim 0.1$ eV

4.1.5 Measuring ν masses

Kinematics

β-decay $\;{}^3H \to {}^3He + e^- + \bar{\nu}_e$

$$E_{e,max} = \underbrace{Q}_{m_{^3H} - m_{^3He}} - m_\nu$$

Measure e^- spectrum very precisely look at endpoint: