4.2 Neutrino Oscillations

4.2.1 Three Flavors of Neutrinos

$$\mathcal{L} \supset \sum_{\alpha=e,\mu,\tau} \Big[\bar{\nu}_{\alpha L}\, i\not{\partial}\, \nu_{\alpha L}$$

$$+ \frac{g}{\sqrt{2}} \left(W^{\mu +}\, \bar{\nu}_{\alpha L} \gamma_\mu e_{\alpha L} + h.c. \right)$$

$$+ \frac{g}{2\cos\theta_W} Z^\mu\, \bar{\nu}_{\alpha L} \gamma_\mu \nu_{\alpha L} \Big]$$

$$- \frac{1}{2} \sum_{\alpha,\beta} \Big(\underbrace{m_{\alpha\beta}}_{\text{in general off-diagonal}} \overline{\nu^c_{\alpha L}}\, \nu_{\beta L} + h.c. \Big)$$

Diagonalize $m_{\alpha\beta}$

$$\underbrace{\nu_{\alpha L}}_{\text{flavor eigenstates}} = \underbrace{U_{\alpha j}}_{\text{unitary } 3\times 3 \text{ matrix}} \underbrace{\nu_{j2}}_{\text{mass eigenstates}}$$

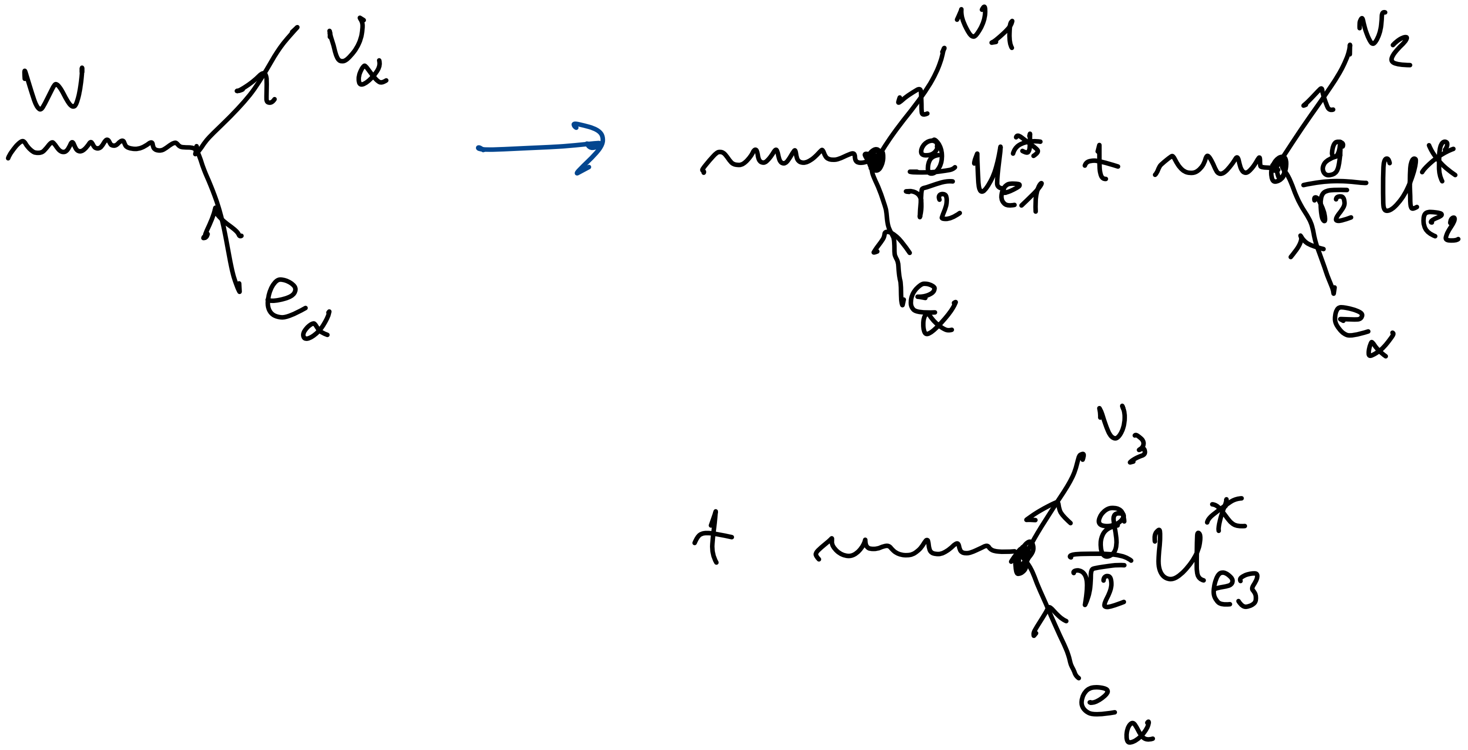
In the mass basis

$$\mathcal{L} = \sum_j \Big[\overbrace{\overline{\nu_{jL}}\, i\not{\partial}\, \nu_{jL}}^{\text{because } U^\dagger U = \mathbb{1}}$$

$$+ \frac{g}{\sqrt{2}} \sum_j W^{\mu +}\, \overline{\nu_{jL}}\, U^*_{\alpha j}\, \gamma_\mu\, e_{\alpha L} + h.c.)$$

$$+ \frac{g}{2\cos\theta_W} Z^\mu\, \overline{\nu_{jL}}\, \gamma_\mu\, \nu_{jL} \Big]$$

$$- \left(\frac{1}{2} \sum_j m_j\, \overline{\nu^c_{jL}}\, \nu_{jL} + h.c. \right)$$



4.2.2 Neutrino Oscillations

A CC interaction produces a superposition

$$|\nu_\alpha\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle$$

$$\left[\begin{aligned} \nu_\alpha &= U_{\alpha j} \nu_j \\ |\nu_\alpha\rangle &= \nu^\dagger_\alpha |0\rangle = U^*_{\alpha j} \nu^\dagger_j |0\rangle \\ &= U^*_{\alpha j} |\nu_j\rangle \end{aligned}\right.$$

Detection via another CC interaction maps the time-evolved neutrino state onto $\langle \nu_\beta| = U_{\beta k} \langle \nu_k|$

Amplitude:

$$\mathcal{A} = \langle \nu_\beta | e^{-i\hat{H}t} | \nu_\alpha\rangle$$

$$= \sum_{j,k} \langle \nu_k | U_{\beta k} U^*_{\alpha j} | \nu_j \rangle \, e^{-iE_j t}$$

$$= \sum_j U_{\beta j} U^*_{\alpha j} \, e^{-iE_j t}$$

$$\boxed{\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= |\mathcal{A}|^2 \\ &= \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \\ &\qquad \cdot e^{-i(E_j - E_k)t} \end{aligned}}$$

More accurately: include evolution in space (typically, neutrino propagation time is not accurately measured, but distance is,

$$e^{-i\hat{H}t} \longrightarrow e^{-i\hat{H}t + i\hat{p}L}$$

source-detector distance "baseline"

$$P(\nu_\alpha \to \nu_\beta) = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \cdot e^{-i(E_j - E_k)t + i(p_j - p_k)t}$$

<u>Note:</u> neutrinos with different E and p can interfere because the corresponding differences in E and p of their interaction partners in the source and in the detector are smaller than the Heisenberg uncertainties on E and p

Typically, we do not know t precisely
$\rightarrow$ integrate over it

$$\bar{P}(\nu_\alpha \to \nu_\beta) \equiv \frac{1}{N} \int_{-\infty}^{\infty} dt \; P(\nu_\alpha \to \nu_\beta)$$

$\uparrow$ normalization factor

$$= \cancel{\frac{1}{N}} \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \; \cancel{2\pi \, \delta(E_j - E_k)}$$

$$\cdot \exp\left[i\left(\sqrt{E^2-m_j^2}-\sqrt{E^2-m_k^2}\right)L\right]$$

smallness of

$$\underset{m_j,\, m_k}{=} \sum_{j,k} U^*_{\alpha j}\, U_{\beta j}\, U_{\alpha k}\, U^*_{\beta k}$$

$$\cdot \exp\left[-i\,\frac{\overbrace{\Delta m^2_{jk}}^{\equiv\, m_j^2 - m_k^2} L}{2E}\right]$$

Consider as toy example a model with only two ν flavors

$$\hookrightarrow\ U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\hookrightarrow\ P(\nu_e \to \nu_\mu)\Big|_{2\,\text{flavor}} = |U_{e1}|^2\,|U_{\mu 1}|^2$$

$$+\ |U_{e2}|^2\,|U_{\mu 2}|^2$$

$$+\ U_{e1}U_{\mu 1}U_{e2}U_{\mu 2}\left(e^{-i\frac{\overbrace{\Delta m^2}^{\equiv\, m_2^2 - m_1^2} L}{2E}} + e^{i\frac{\Delta m^2 L}{2E}}\right)$$

$$= 2\sin^2\theta\cos^2\theta \ - 2\cos^2\theta\sin^2\theta\cos\frac{\Delta m^2 L}{2E}$$

$$= \frac{1}{2} \sin^2 2\theta \left(1 - \cos \frac{\Delta m^2 L}{2E}\right)$$

$$= \underbrace{\sin^2 2\theta}_{\text{mixing angle determines osc. amplitude}} \cdot \underbrace{\sin^2 \frac{\Delta m^2 L}{4E}}_{\Delta m^2 \text{ determines the oscillation length}}$$

mixing angle determines osc. amplitude

Δm^2 determines the oscillation length $L_{osc} = \frac{4\pi E}{\Delta m^2}$

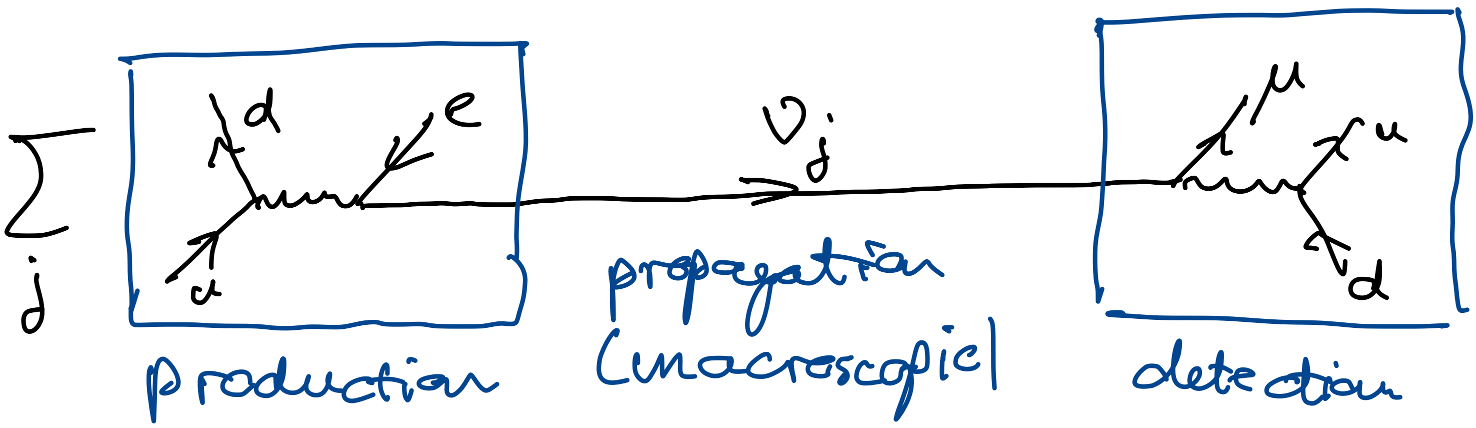
$\Delta m^2_{31} \sim 2 \cdot 10^{-3}\, eV^2 \rightarrow L_{osc} \sim 1\, km$

(at $E \sim MeV$)

$\Delta m^2_{21} \sim 8 \cdot 10^{-5}\, eV^2 \rightarrow L_{osc} \sim 60\, km$

(at $E \sim MeV$)

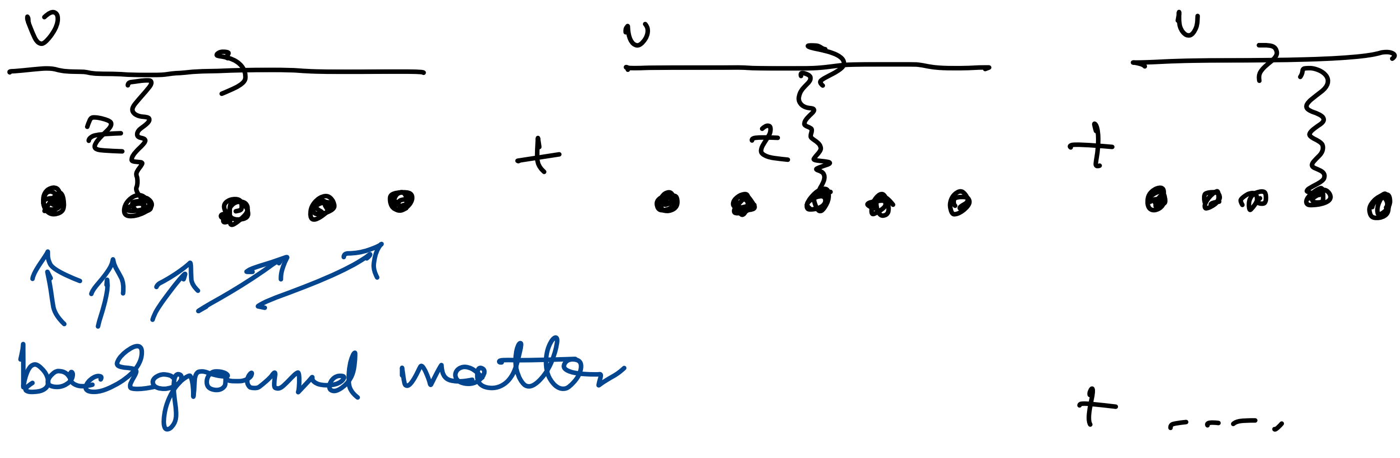
Note: in QFT, ν oscillation can be described as a Feynman diagram



- treat external states as wave packets (with plane waves, no interference would be possible)
- approximate ν propagator at large L
- calculate

4.2.3 Neutrino Oscillations in Matter

Coherent forward scattering (exchange of W or Z boson without exchange of momentum)



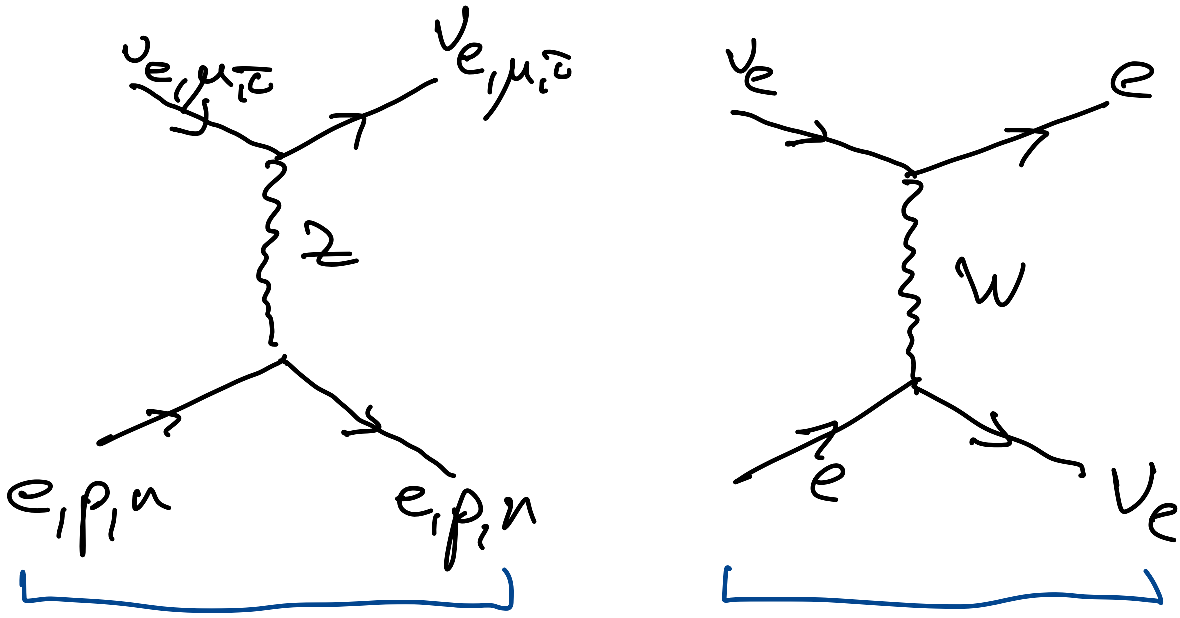
As initial and final quantum states in coherent forward scattering are identical, need to sum coherently over all of the above diagrams.

[Analogy: photon traveling through matter]

$$\mathcal{M} \sim n \, G_F$$

matter density

Fermi constant



for any ν_α | only for ν_e

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} \left[\bar{e} \gamma^\mu (1-\gamma^5) \nu_e \right] \left[\bar{\nu}_e \gamma_\mu (1-\gamma^5) e \right]$$

Fierz trafo

$$= \frac{G_F}{\sqrt{2}} \left[\bar{e} \gamma^\mu (1-\gamma^5) e \right] \left[\bar{\nu}_e \gamma_\mu (1-\gamma^5) \nu_e \right]$$

Take expectation value of the electron current:

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} \underbrace{\langle \bar{e} \gamma^\mu (1-\gamma^5) e \rangle}_{= \begin{cases} n_e & \text{for } \mu = 0 \\ 0 & \text{for } \mu = 1,2,3 \end{cases}} [\bar{\nu}_e \gamma_\mu (1-\gamma^5) \nu_e]$$

$$= \underbrace{\sqrt{2}\, G_F n_e}_{V_{CC} \text{ (MSW potential)}} \bar{\nu}_{eL} \gamma^0 \nu_{eL}$$

In the derivation of $P(\nu_\alpha \to \nu_\beta)$, we had e^{ipL}

$$\phi = \hat{p} \cdot L = \sqrt{(\hat{H} - \hat{V})^2 - \hat{M}^2}$$

$\hat{M}$: 2x2 matrix

$\hat{H}$: 2x2 matrix (in 2-flavor approximation)

$\hat{V}$: $\begin{pmatrix} V_{CC} & 0 \\ 0 & 0 \end{pmatrix}$

Diagonalize $\hat{H} - \frac{\hat{M}^2}{2H} - \hat{V}$

in flavor basis

$$= E \cdot \mathbb{1}_{2\times 2} - U \begin{pmatrix} \frac{m_1^2}{2E} & \\ & \frac{m_2^2}{2E} \end{pmatrix} U^+$$

$$- \begin{pmatrix} V_{cc} & 0 \\ 0 & 0 \end{pmatrix}$$

Eigenvalues :

$$\lambda_{1,2} = \pm \frac{1}{2} \sqrt{(V_{cc} - \underbrace{\Delta}_{\Delta m^2/2E} \cos 2\theta)^2 + \Delta^2 \sin^2 \theta}$$

Effective mixing angle (the angle that parameterizes the unitary 2x2 matrix that diagonalizes the above matrix :

$$\sin 2\theta_{eff} = \frac{\Delta \sin 2\theta}{\sqrt{(V_{cc} - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}}$$

- If $V_{cc} \ll \Delta \Rightarrow$ recover vacuum case $\theta_{eff} = \theta$

- If $V_{CC} \gg \Delta \Longrightarrow \theta_{eff} \to 0$, oscillations suppressed
- If $V_{CC} = \Delta \cos 2\theta \Longrightarrow \sin 2\theta_{eff} = 1$

  "MSW resonance"