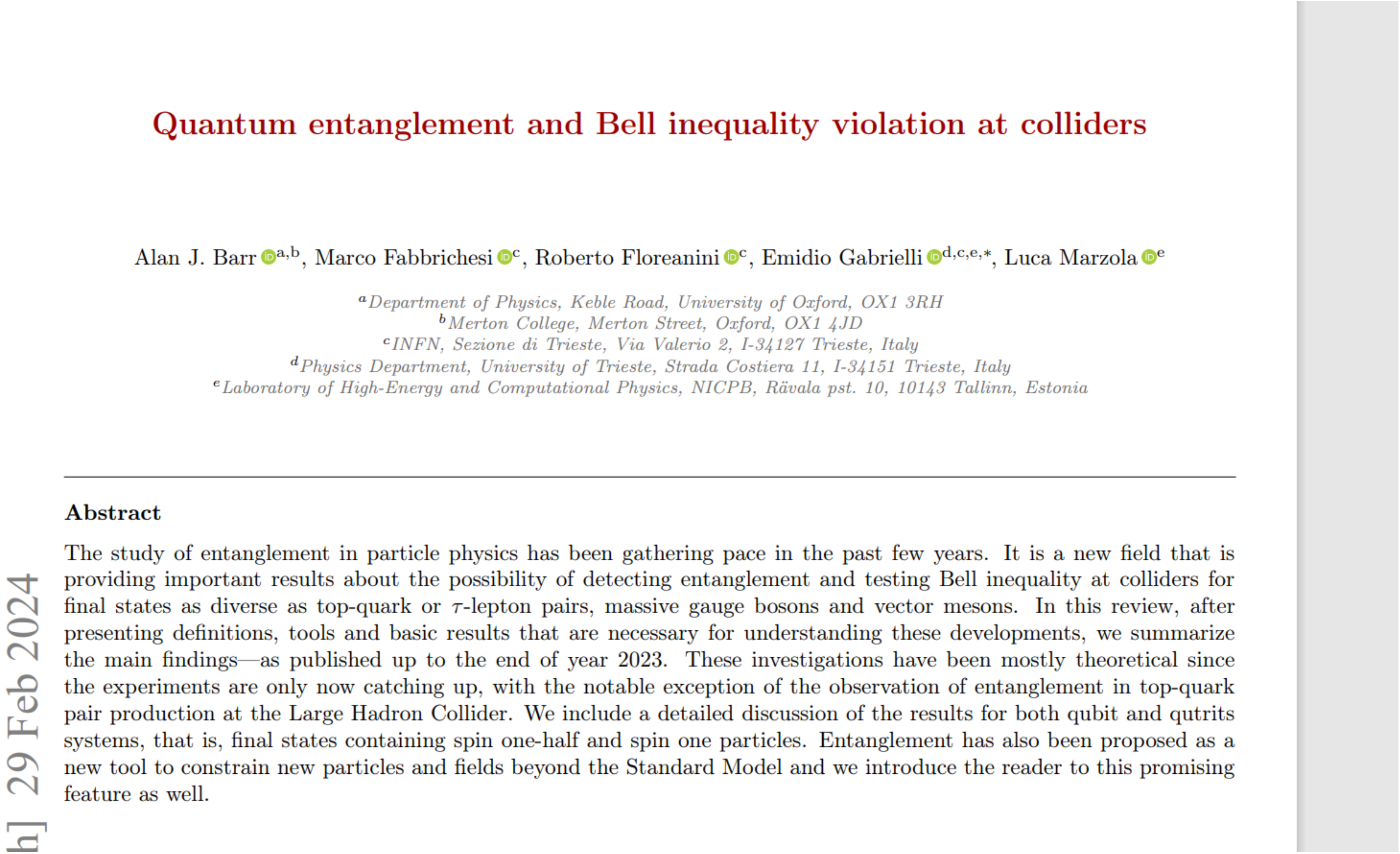

Quantum entanglement and Bell inequality violation at colliders

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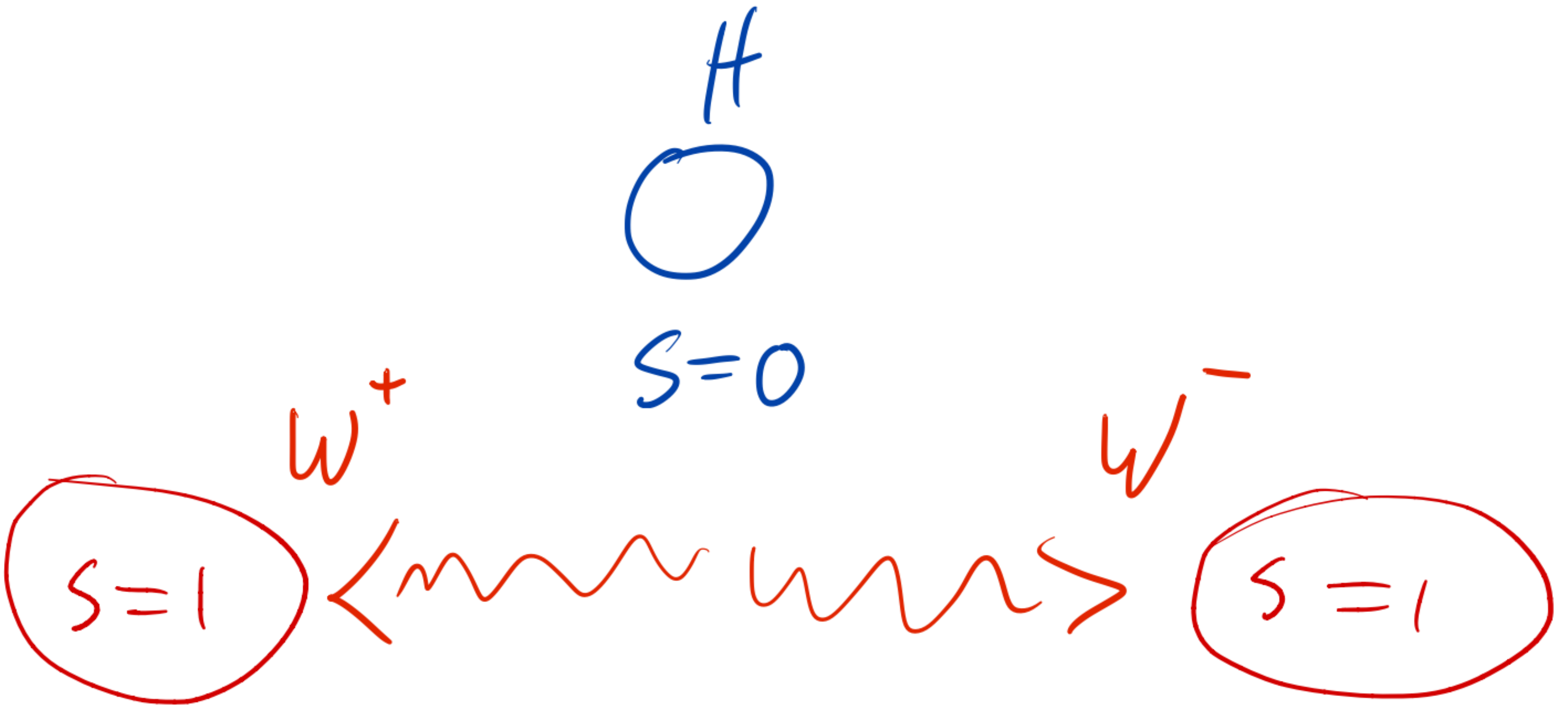
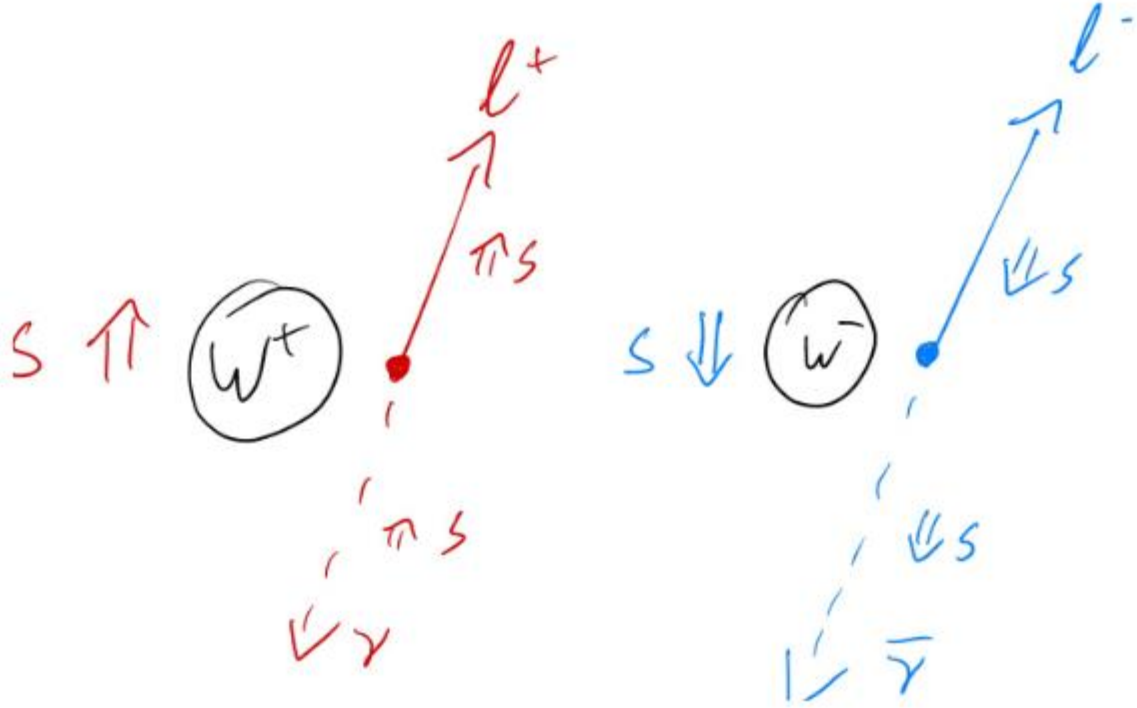
Abstract

The study of entanglement in particle physics has been gathering pace in the past few years. It is a new field that is providing important results about the possibility of detecting entanglement and testing Bell inequality at colliders for final states as diverse as top-quark or τ-lepton pairs, massive gauge bosons and vector mesons. In this review, after presenting definitions, tools and basic results that are necessary for understanding these developments, we summarize the main findings—as published up to the end of year 2023. These investigations have been mostly theoretical since the experiments are only now catching up, with the notable exception of the observation of entanglement in top-quark pair production at the Large Hadron Collider. We include a detailed discussion of the results for both qubit and qutrits systems, that is, final states containing spin one-half and spin one particles. Entanglement has also been proposed as a new tool to constrain new particles and fields beyond the Standard Model and we introduce the reader to this promising feature as well.

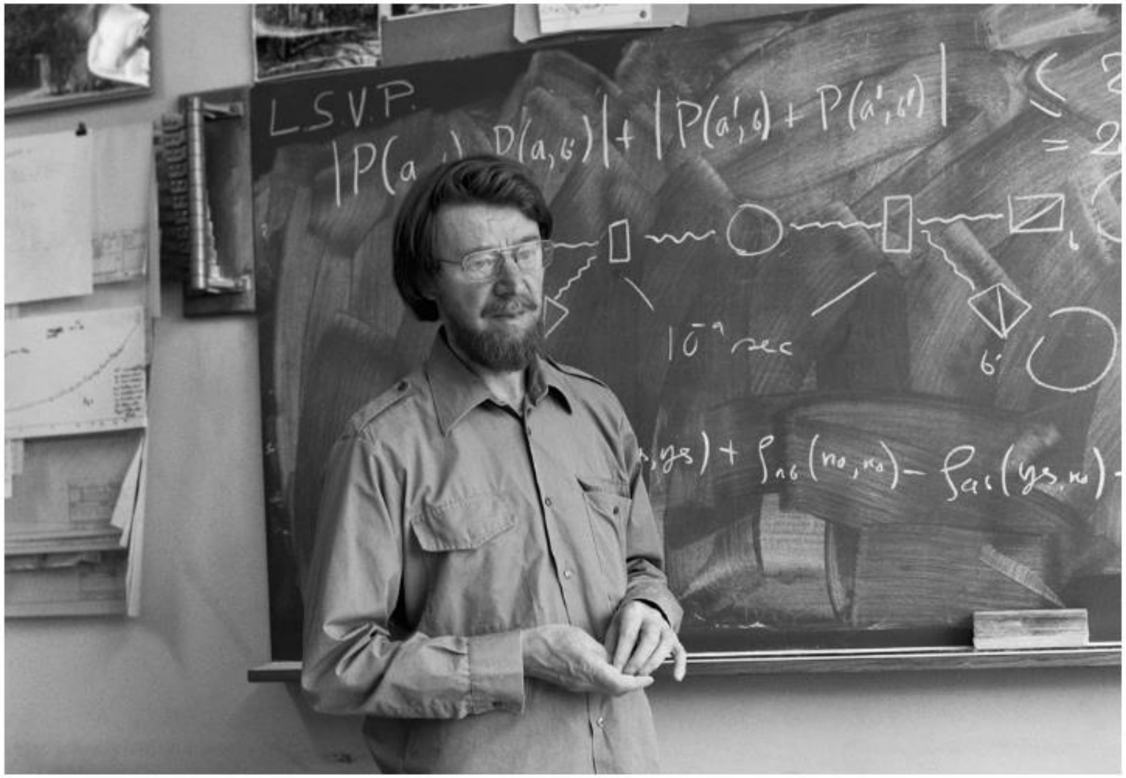
https://arxiv.org/pdf/2402.07972.pdf



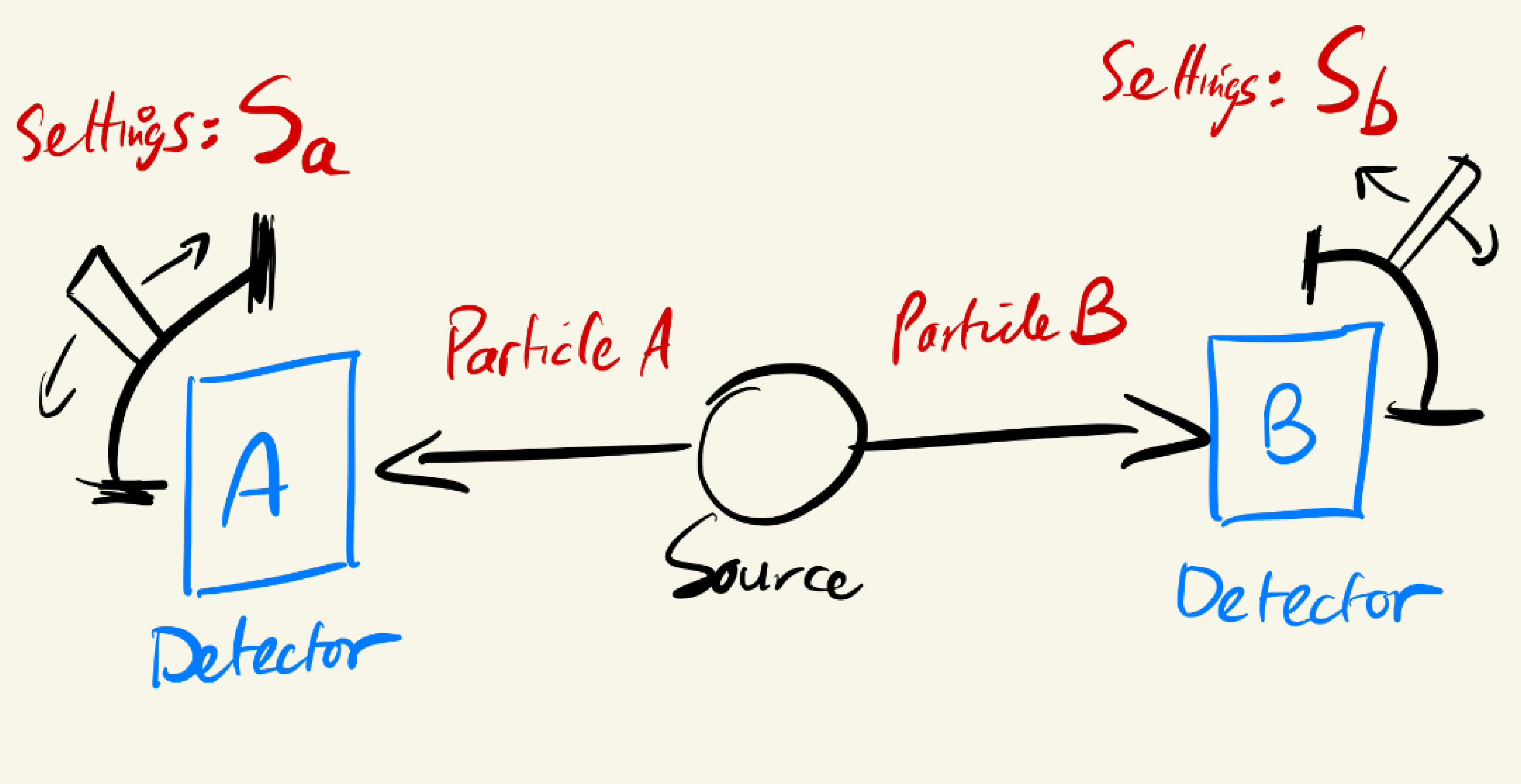
Bell state



Weak decays measure spin (projective measurement for W)



J.S. Bell *'On the Einstein Podolsky Rosen paradox'* (1964)

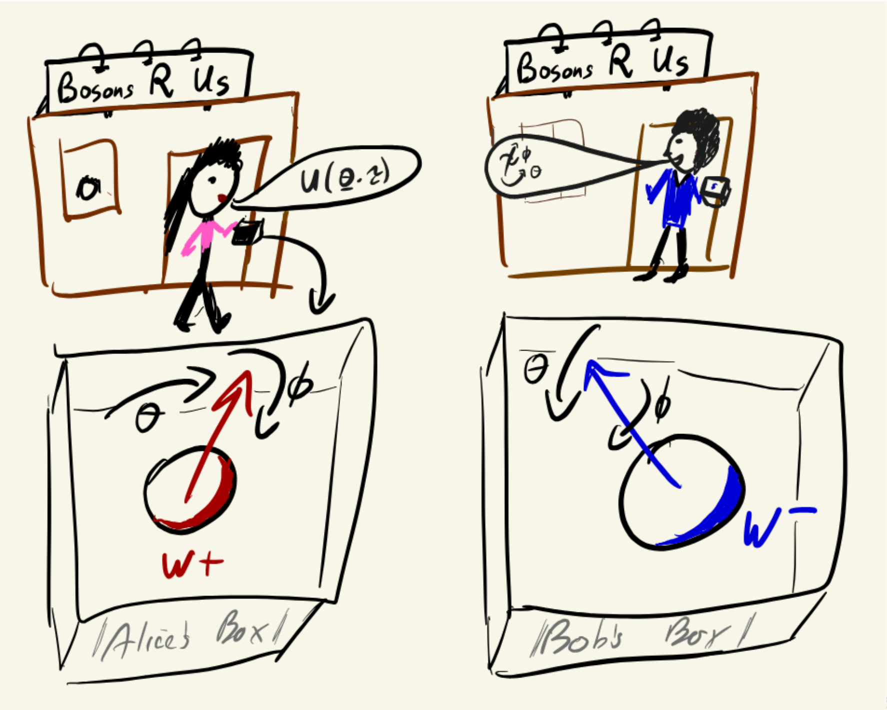

Settings: S_a
Settings: S_b
Particle A
Particle B
A
B
Source
Detector
Detector

Separable/Entangled states

A state (density matrix) ρ of S is called **separable** *if and only if it can be written as a linear convex combination of tensor products of density matrices:*

$$\rho = \sum_{ij} p_{ij}\, \rho_i^{(A)} \otimes \rho_j^{(B)} \ , \quad \text{with} \quad p_{ij} > 0 \quad \text{and} \quad \sum_{ij} p_{ij} = 1 \ , \tag{2.11}$$

where $\rho_i^{(A)}$ and $\rho_j^{(B)}$ are density matrices for the subsystems S_A and S_B. States ρ that cannot be written in the form of (2.11) are called **entangled** *or* **non-separable***, and exhibit quantum correlations.*



Energy scale × trillion

Qutrits rather than qubits

Spin self-measuring systems

Quantum – classical transition

And with virtual particles

Density matrix parameterisations

qubits

$$\rho = \frac{1}{4}\left[\mathbf{1}_2 \otimes \mathbf{1}_2 + \sum_{i=1}^{3} B_i^{+}(\sigma_i \otimes \mathbf{1}_2) + \sum_{i=1}^{3} B_j^{-}(\mathbf{1}_2 \otimes \sigma_j) + \sum_{i,j=1}^{3} C_{ij}(\sigma_i \otimes \sigma_j)\right] ,$$

qudits (e.g. qutrits)

$$\rho = \frac{1}{d^2}\left[\mathbf{1}_d \otimes \mathbf{1}_d + \sum_{i=1}^{d^2-1} \mathcal{A}_i^{(d)}(\tau_i \otimes \mathbf{1}) + \sum_{j=1}^{d^2-1} \mathcal{B}_j^{(d)}(\mathbf{1} \otimes \tau_j) + \sum_{i,j=1}^{d^2-1} \mathcal{C}_{ij}^{(d)}(\tau_i \otimes \tau_j)\right] ,$$

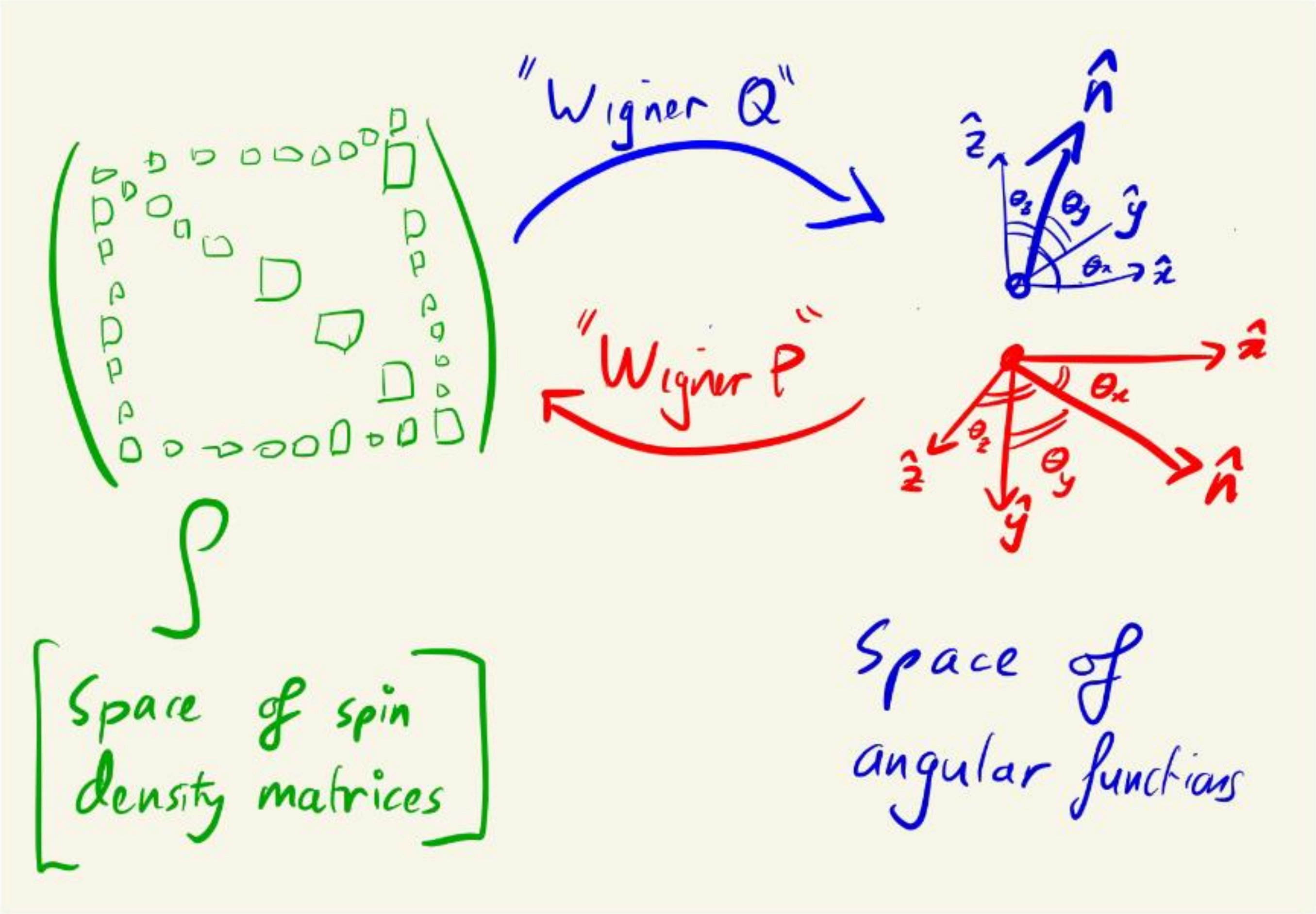
Reconstructing the parameters: (qubit)

$$\rho = \frac{1}{4}\left[\mathbf{1}_2 \otimes \mathbf{1}_2 + \sum_{i=1}^{3} B_i^{+}(\sigma_i \otimes \mathbf{1}_2) + \sum_{i=1}^{3} B_j^{-}(\mathbf{1}_2 \otimes \sigma_j) + \sum_{i,j=1}^{3} C_{ij}(\sigma_i \otimes \sigma_j)\right] ,$$

$$B_i^{\pm} = \frac{3}{\kappa_{\pm}} \frac{1}{\sigma} \int \mathrm{d}\Omega^{\pm} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^{\pm}} (\vec{n}^{\pm} \cdot \hat{e}_i),$$

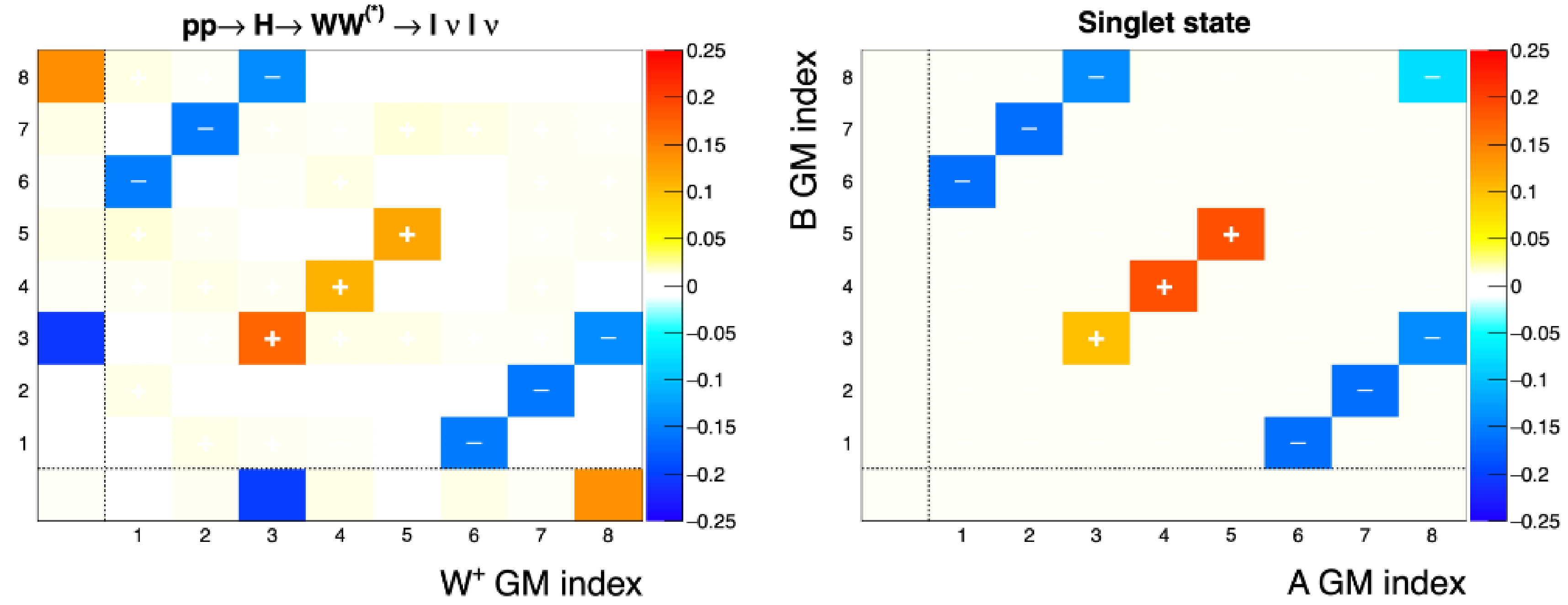
$$C_{ij} = \frac{9}{\kappa_{+}\kappa_{-}} \frac{1}{\sigma} \int \mathrm{d}\Omega^{+}\mathrm{d}\Omega^{-} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^{+}\mathrm{d}\Omega^{-}} (\vec{n}^{+} \cdot \hat{e}_i)(\vec{n}^{-} \cdot \hat{e}_j)$$

Reconstructing the parameters: (qutrit)



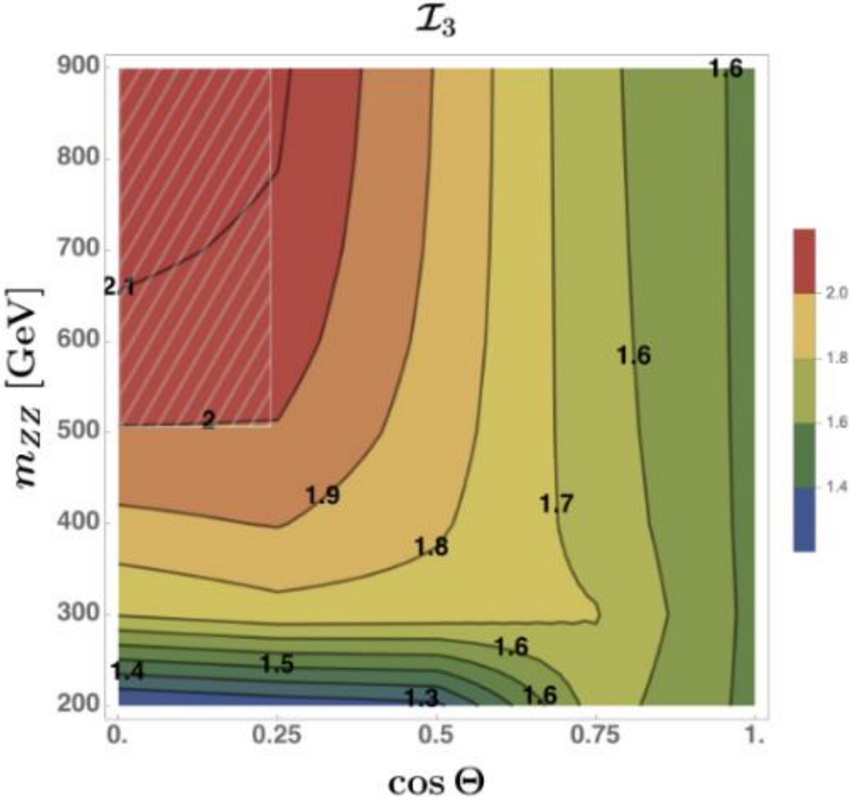
Quantum state tomography example

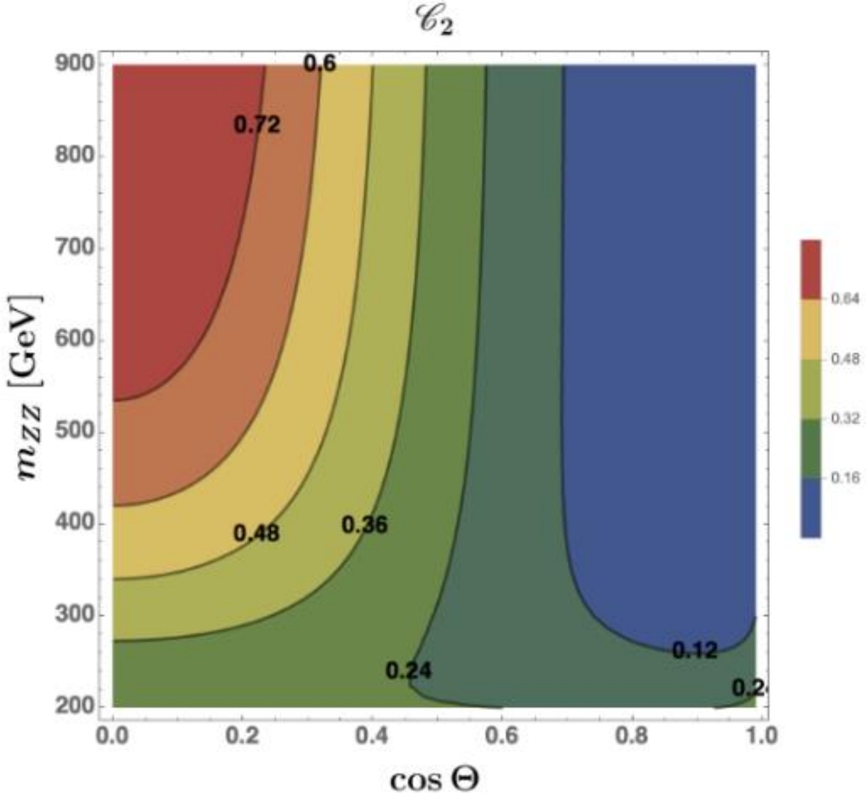
Higgs boson decays



Density matrix parameters from simulated Higgs boson decays to vector bosons (Madgraph, no background)

pp$\longrightarrow$ ZZ





Optimised Bell Operator

$> 2?$

Bound on the concurrence

$> 0?$

Fabbrichesi et al. 2302.00683

Most studies have been done for the LHC

- Advantages
  - Large yields of events (inc. Higgs events)
  - High energy

- Disadvantages
  - Large backgrounds
  - Pileup
  - Poorer kinematic reconstruction
    - e.g. H->WW)
  - Limited charm tagging

- Some ee studies e.g.

Entanglement variables can be highly sensitive to Wilson coefficients

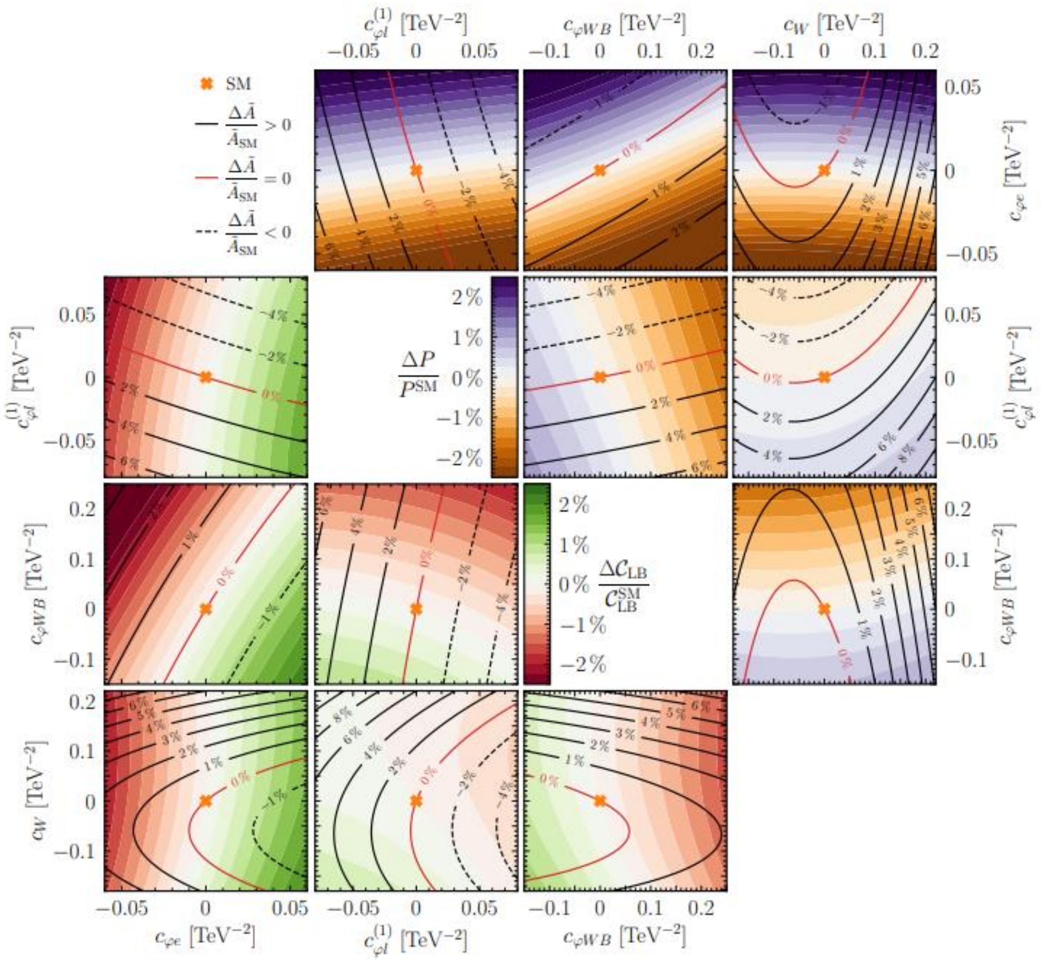


Figure 7: The relative changes in the marker $\mathcal{C}_{\rm LB}$ (lower triangle) and purity P (upper triangle) compared to the SM values $\mathcal{C}_{\rm LB}^{\rm SM} = 1.0$ and $P^{\rm SM} = 0.94$ as a function of the Wilson coefficients $c_{\varphi e}$, $c_{\varphi l}^{(1)}$, $c_{\varphi WB}$ and c_W for W^+W^- production at a lepton collider, at $m_{WW} = 500\,\text{GeV}$ and $\cos\theta = 0$. The lines indicate the relative change of the cross section.

Aoude, Madge, Maltoni, Mantani

2307.09675

Quantum information and CP measurement in $H \to \tau^+\tau^-$ at future lepton colliders

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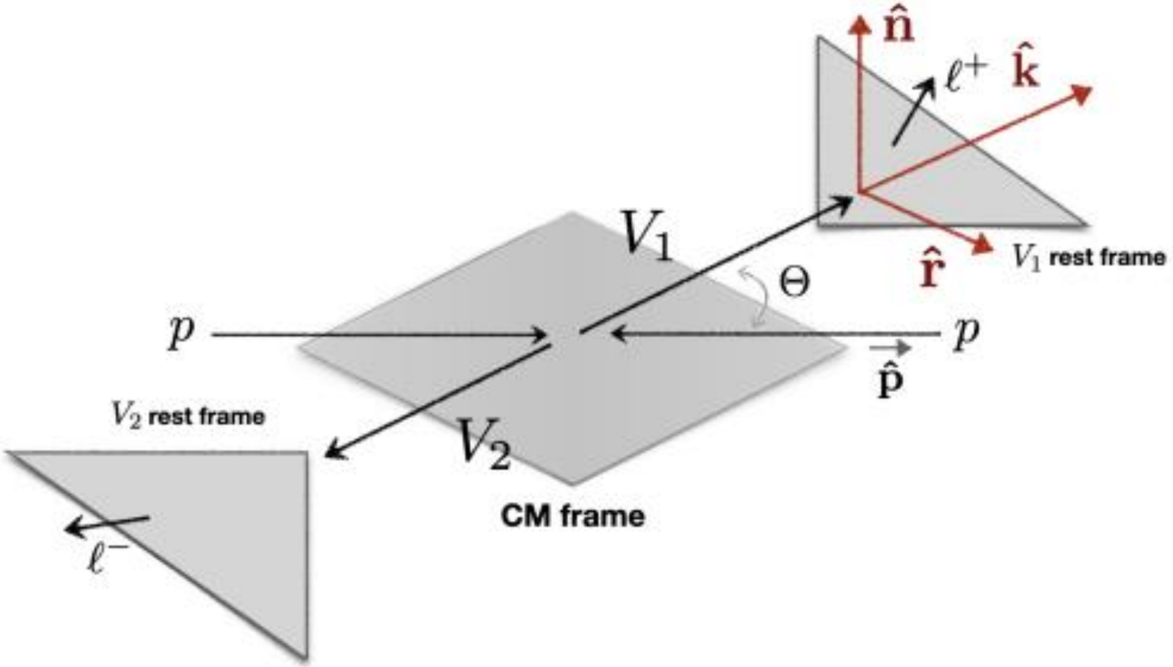
We introduce a methodology and investigate the feasibility of measuring quantum properties of tau lepton pairs in the $H \to \tau^+\tau^-$ decay at future lepton colliders. In particular, observation of entanglement, steerability and violation of Bell inequalities are examined for the ILC and FCC-ee. We find that detecting quantum correlation crucially relies on precise reconstruction of the tau lepton rest frame and a simple kinematics reconstruction does not suffice due to the finite energy resolution of the colliding beams and detectors. To correct for energy mismeasurements, a log-likelihood method is developed that incorporates the information of impact parameters of tau lepton decays. We demonstrate that an accurate measurement of quantum properties is possible with this method. As a by-product, we show that a novel model-independent test of CP violation can be performed and the CP-phase of $H\tau\tau$ interaction can be constrained with an accuracy comparable to dedicated analyses, i.e., up to 7.9° and 5.4° at ILC and FCC-ee, respectively.

What are you interested in exploring?



$\hat{\mathbf{n}}$
ℓ^+
$\hat{\mathbf{k}}$
V_1
$\hat{\mathbf{r}}$
V_1 rest frame
Θ
p
p
$\vec{\hat{\mathbf{p}}}$
V_2 rest frame
V_2
CM frame
ℓ^-