



Feynman Diagrams

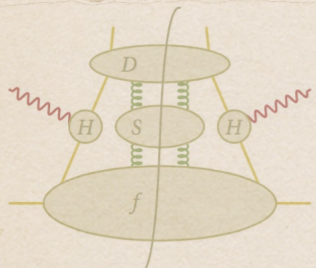


Figure 8.11: Factorisation in SIDIS: the bull diagram. All IR divergences are absorbed in the soft factor S, that hence only interacts with the TMD and FF. Note that the radiation comes from the hard subprocess.

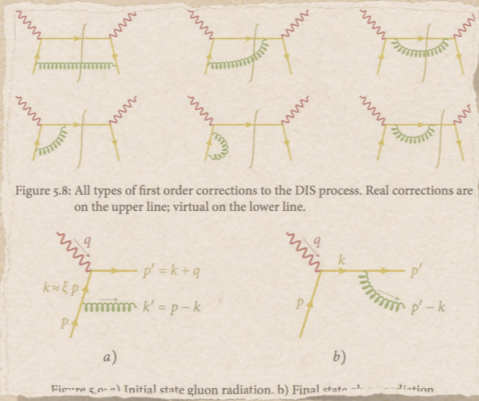


Figure 5.8: All types of first order corrections to the DIS process. Real corrections are on the upper line; virtual on the lower line. (a) Initial state gluon radiation. (b) Final state gluon radiation.

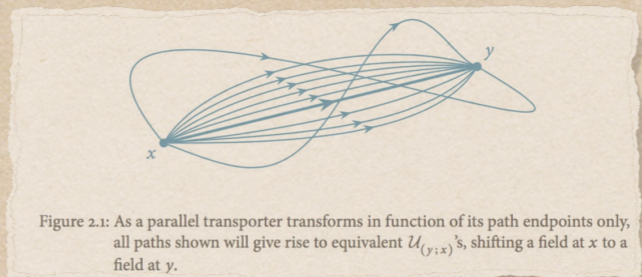
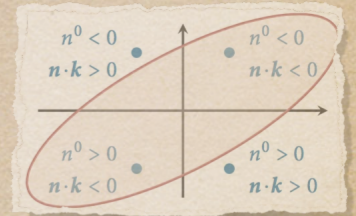


Figure 2.1: As a parallel transporter transforms in function of its path endpoints only, all paths shown will give rise to equivalent $U_{(y;x)}$'s, shifting a field at x to a field at y.



and stuff

The Mikowskian loop integrals are then the same as the Euclidian ones, up to a possible sign difference:

$$\int \frac{d^\omega k}{(2\pi)^\omega} \frac{1}{(k^2 - \Delta)^n} = i \frac{(-)^n}{(4\pi)^{\frac{\omega}{2}}} \frac{\Gamma(n - \frac{\omega}{2})}{\Gamma(n)} \Delta^{\frac{\omega}{2} - n},$$

$$\left(\begin{array}{l} d \geq 2n \\ d \text{ even} \end{array} \right) = i \frac{\Delta^{\frac{d}{2} - n}}{(4\pi)^{\frac{d}{2}}} \frac{(-)^{\frac{d}{2}}}{(n-1)! (\frac{d}{2} - n)!} \left(\frac{1}{\epsilon} - \gamma_E + \sum_j \frac{1}{j} + \ln 4\pi - \ln \Delta \right),$$

$$\int \frac{d^\omega k}{(2\pi)^\omega} \frac{k^2}{(k^2 - \Delta)^n} = i \frac{(-)^{n+1}}{(4\pi)^{\frac{\omega}{2}}} \frac{\omega \Gamma(n - \frac{\omega}{2} - 1)}{2 \Gamma(n)} \Delta^{\frac{\omega}{2} + 1 - n},$$

$$\left(\begin{array}{l} d \geq 2n - 2 \\ d \text{ even} \end{array} \right) = i \frac{\Delta^{\frac{d}{2} + 1 - n}}{(4\pi)^{\frac{d}{2}}} \frac{\omega}{2} \frac{(-)^{\frac{d}{2}}}{(n-1)! (\frac{d}{2} + 1 - n)!} \left(\frac{1}{\epsilon} - \gamma_E + \sum_j \frac{1}{j} + \ln 4\pi - \ln \Delta \right)$$

$$\int \frac{d^\omega k}{(2\pi)^\omega} \frac{k^4}{(k^2 - \Delta)^n} = i \frac{(-)^n}{(4\pi)^{\frac{\omega}{2}}} \frac{\omega(\omega+2) \Gamma(n - \frac{\omega}{2} - 2)}{4 \Gamma(n)} \Delta^{\frac{\omega}{2} + 2 - n},$$

$$\left(\begin{array}{l} d \geq 2n - 4 \\ d \text{ even} \end{array} \right) = i \frac{\Delta^{\frac{d}{2} + 2 - n}}{(4\pi)^{\frac{d}{2}}} \frac{\omega(\omega+2)}{4} \frac{(-)^{\frac{d}{2}}}{(n-1)! (\frac{d}{2} + 2 - n)!} \left(\frac{1}{\epsilon} - \gamma_E + \sum_j \frac{1}{j} + \ln 4\pi - \ln \Delta \right)$$

We list some other common Minkowskian integrals:

$$\int \frac{d^\omega k}{(2\pi)^\omega} \ln(k^2 - a) = -\frac{i}{(4\pi)^{\frac{\omega}{2}}} \Gamma\left(-\frac{\omega}{2}\right) a^{\frac{\omega}{2}},$$

$$\int \frac{d^\omega k}{(2\pi)^\omega} e^{ak^2 - ib \cdot k} = \frac{i}{(4\pi)^{\frac{\omega}{2}}} a^{-\frac{\omega}{2}} e^{\frac{b^2}{4a}},$$

$$\int \frac{d^\omega k}{(2\pi)^\omega} \frac{1}{(-k^2)^\alpha} e^{-ib \cdot k} = \frac{i}{4^\alpha \pi^{\frac{\omega}{2}}} \frac{\Gamma(\frac{\omega}{2} - \alpha)}{\Gamma(\alpha)} \frac{1}{(-b^2)^{\frac{\omega}{2} - \alpha}}.$$

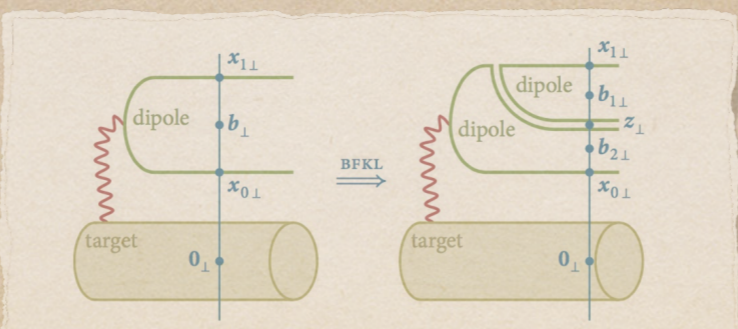
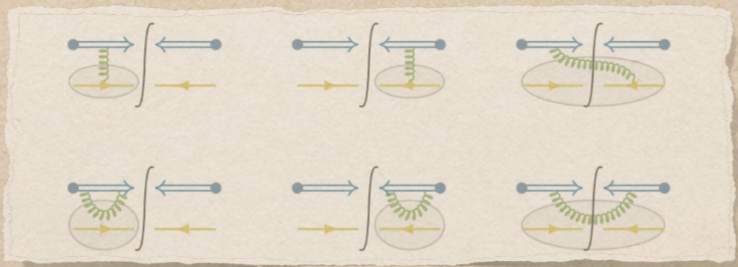


Figure 9.6: In the dipole picture, the BFKL evolution is an evolution in dipoles, i.e. new dipoles are created during the evolution. A gluon that is radiated from the dipole can be represented as two fundamental lines (see Equation 10.13). This essentially splits the dipole in two at the point z_1 , as is illustrated in the second diagram.

$$\oint_C dx \cdot A = \int_\Sigma d\sigma \cdot (\partial \wedge A)$$



$$\text{tr}(t^a t^x t^b t^x) = -\frac{1}{4N_c} \delta^{ab},$$

$$\text{tr}(t^b t^x t^y) f^{axy} = -i \frac{N_c}{4} \delta^{ab},$$

$$\text{tr}(t^y t^z) f^{axy} f^{bzx} = -\frac{N_c}{2} \delta^{ab},$$

$$f^{xay} f^{ycz} f^{zbw} f^{wcx} = \frac{N_c^2}{2} \delta^{ab},$$

$$f^{awv} f^{xby} f^{yvw} f^{zvx} = \frac{N_c^2}{2} \delta^{ab},$$

$$f^{awv} f^{zbw} f^{xzy} f^{yvx} = N_c^2 \delta^{ab},$$

$$f^{xay} f^{ycz} f^{zbw} f^{wcx} = \frac{N_c^2}{2} \delta^{ab},$$

$$f^{vaw} f^{wbz} f^{xzy} f^{yvx} = N_c^2 \delta^{ab},$$

and similarly for the seven remaining diagrams.



Quick recap

Fundamental Forces

THERE ARE FOUR FUNDAMENTAL FORCES BETWEEN PARTICLES:
(1) GRAVITY, WHICH OBEYS THIS INVERSE SQUARE LAW:

$$F_{\text{gravity}} = G \frac{m_1 m_2}{d^2}$$



OK...

(2) ELECTROMAGNETISM, WHICH OBEYS THIS INVERSE-SQUARE LAW:

$$F_{\text{static}} = k_e \frac{q_1 q_2}{d^2}$$

AND ALSO MAXWELL'S EQUATIONS



ALSO WHAT?

(3) THE STRONG NUCLEAR FORCE, WHICH OBEYS, UH...

...WELL, UMM...

...IT HOLDS PROTONS AND NEUTRONS TOGETHER.



I SEE.

IT'S STRONG.

AND (4) THE WEAK FORCE. IT [MUMBLE MUMBLE] RADIOACTIVE DECAY [MUMBLE MUMBLE]

THAT'S NOT A SENTENCE. YOU JUST SAID 'RADIO-
-AND THOSE ARE THE FOUR FUNDAMENTAL FORCES!



Elementary Particles

- Three types:
 - **Fermions:** matter particles
 - **Bosons:** force carriers ("exchange particles")
 - **Higgs:** special guy
- Difference lies in **spin**

Elementary Particles

- Two types of matter particles:
 - **Leptons:** electrons, muons, taus, and neutrinos
 - **Quarks:** don't exist alone, but combine to form hadrons (composite particles)
- Four fundamental forces:
 - **Electromagnetic:** exchanged by photon
 - **Weak:** exchanged by W^+ , W^- , Z^0
 - **Strong:** exchanged by gluons
 - **Gravity:** exchanged by graviton

Elementary Particles

- Two types of matter particles:
 - **Leptons:** electrons, muons, taus, and neutrinos
 - **Quarks:** don't exist alone, but combine to form hadrons (composite particles)

Three very cool and quantisable and not 'totally ignorable'

- ~~Four fundamental forces:~~
 - **Electromagnetic:** exchanged by photon
 - **Weak:** exchanged by W^+ , W^- , Z^0
 - **Strong:** exchanged by gluons
 - ~~**Gravity:** exchanged by graviton~~

Particle Properties

- Every force comes with an associated charge. If a certain particle does not have this charge, it will not interact with this force.
 - Electromagnetic charge
 - Weak hypercharge
 - Colour (strong force)
- Fermions come in 3 families, the difference between the families being the mass.

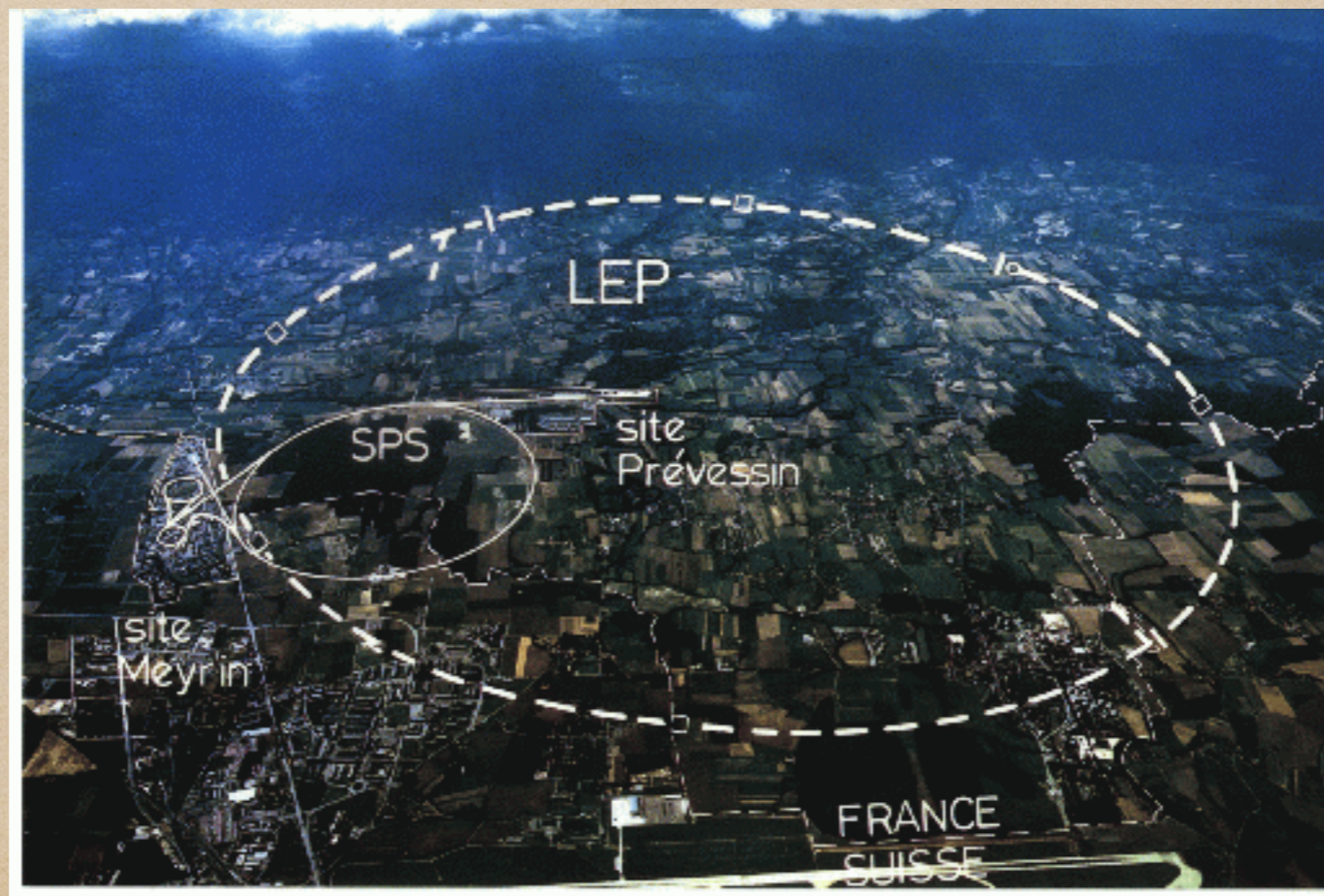


What Are Feynman Diagrams



Feynman Diagrams

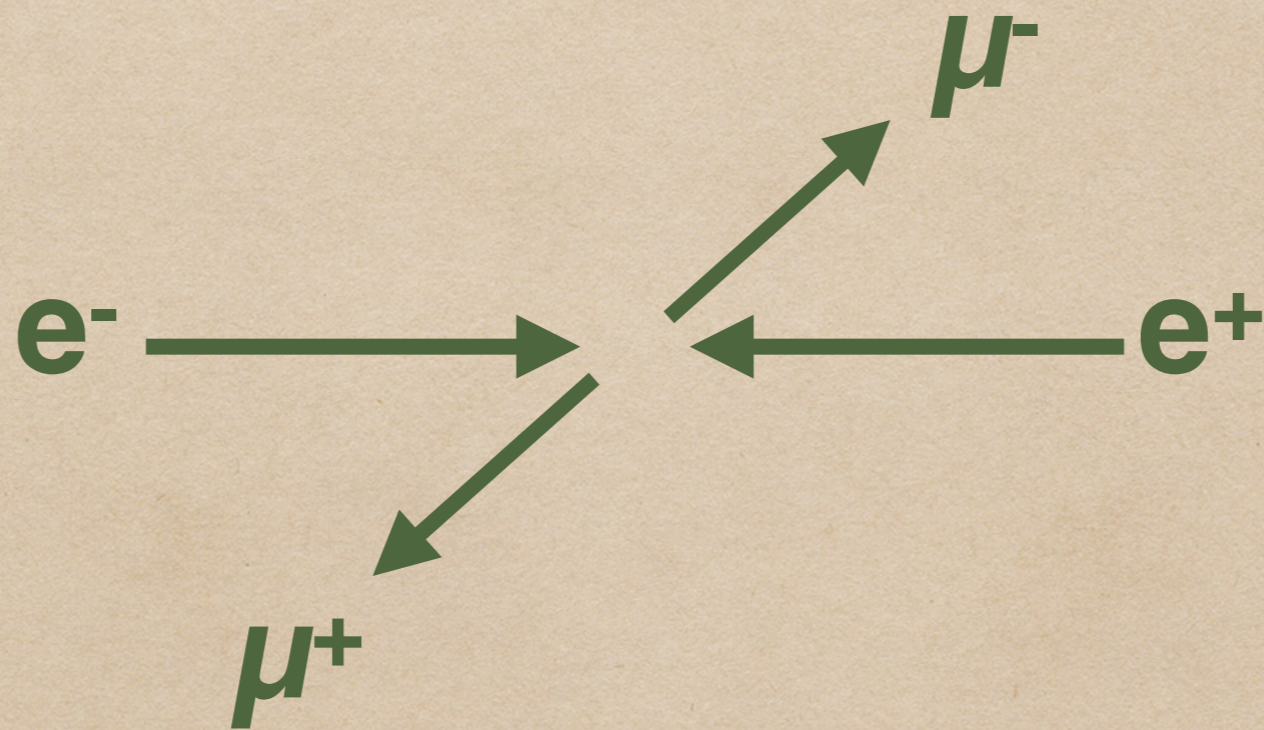
- Are an easy visual way to calculate elementary processes





Feynman Diagrams

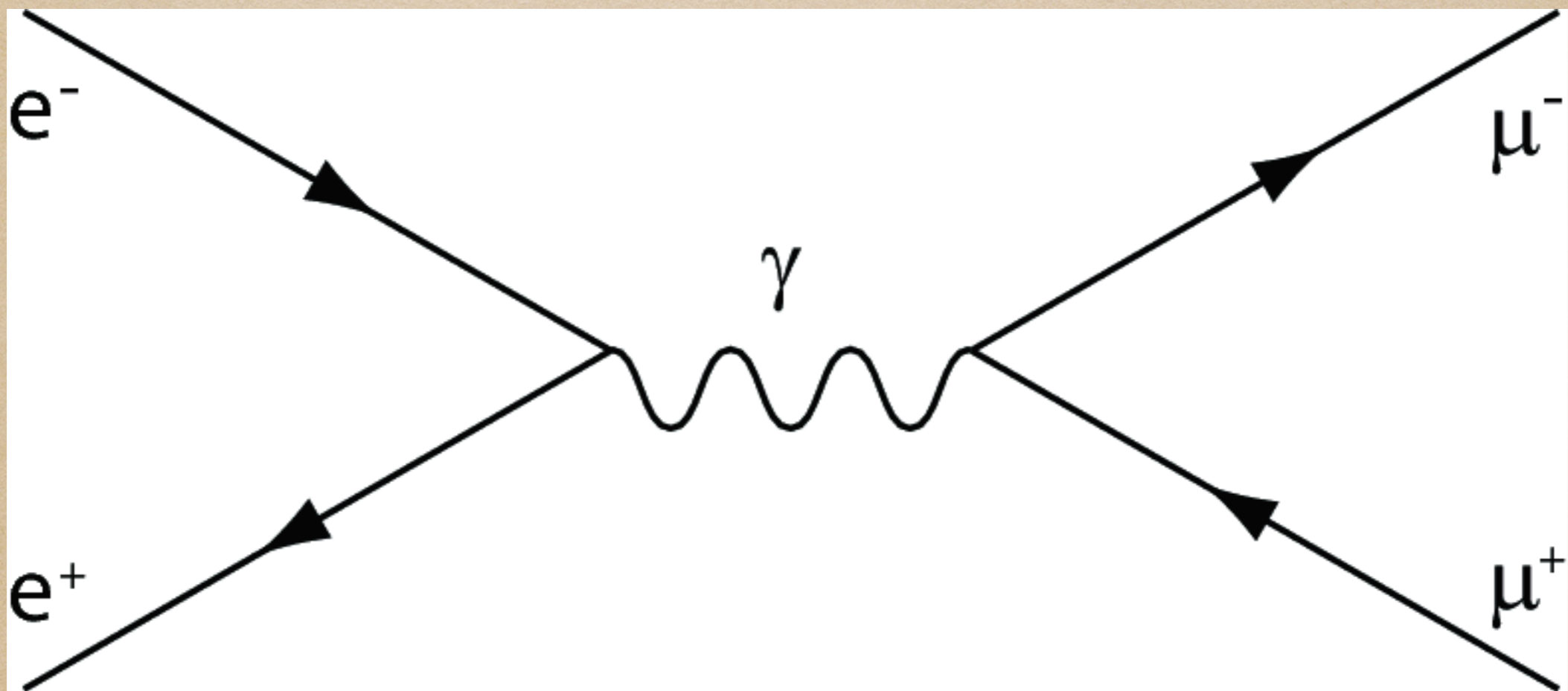
- Are an easy visual way to calculate elementary processes





Feynman Diagrams

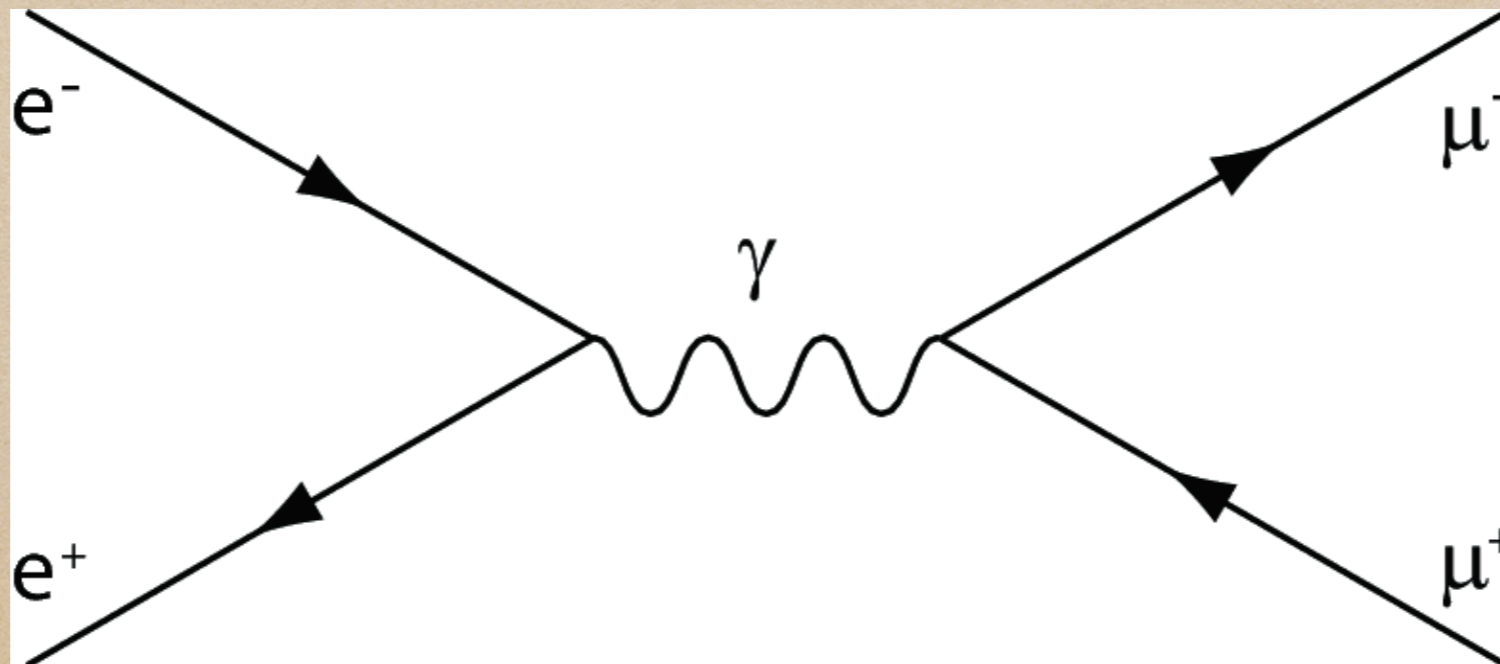
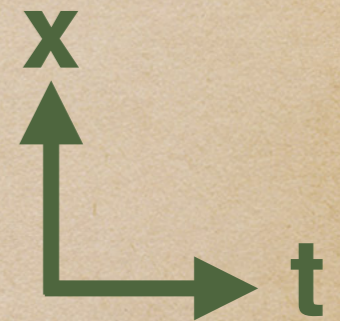
- Are an easy visual way to calculate elementary processes





Feynman Diagrams

- Are an easy visual way to calculate elementary processes
- Particles are drawn on space-time plane





How to Draw Feynman Diagrams



Feynman Diagrams

Different lines for different particle types:

- fermion (matter particle)



- antifermion (antimatter particle)





Feynman Diagrams

Different lines for different particle types:

● fermion (matter particle)



● antifermion (antimatter particle)



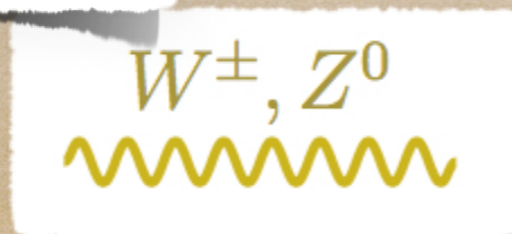
● photon



● gluon



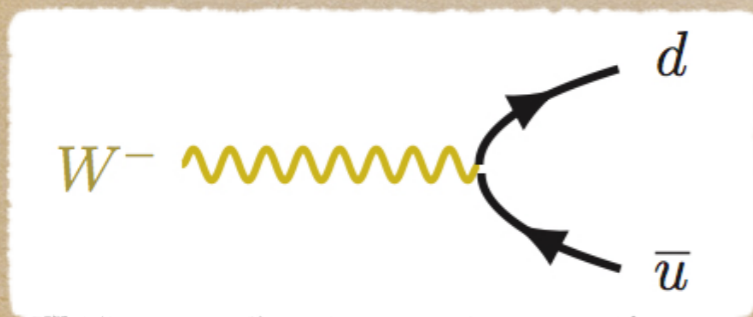
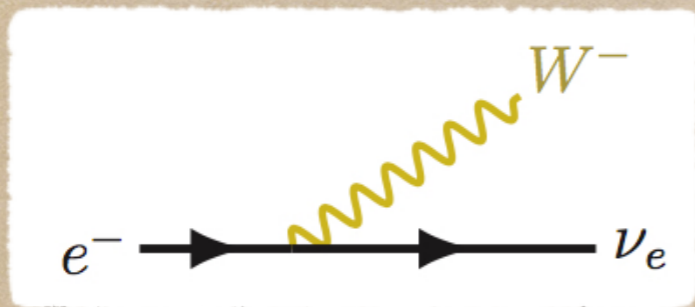
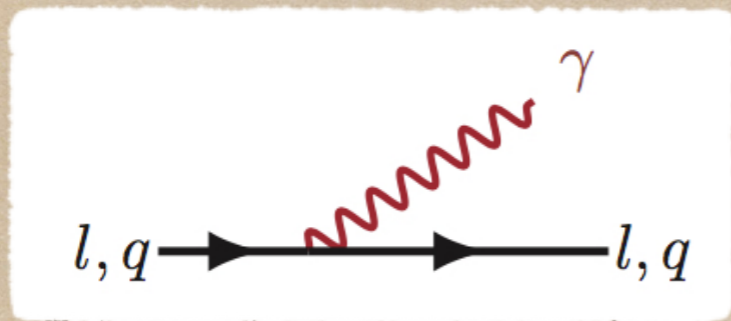
● weak boson





Feynman Diagrams

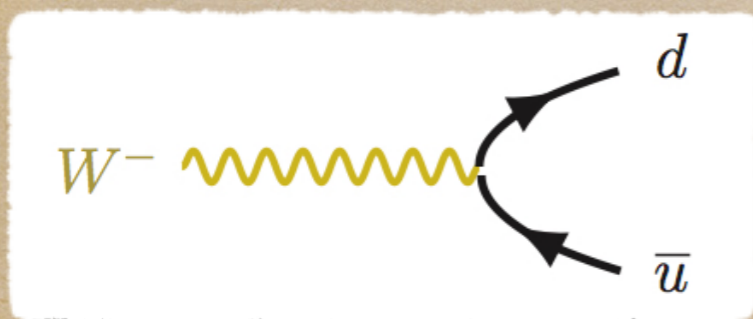
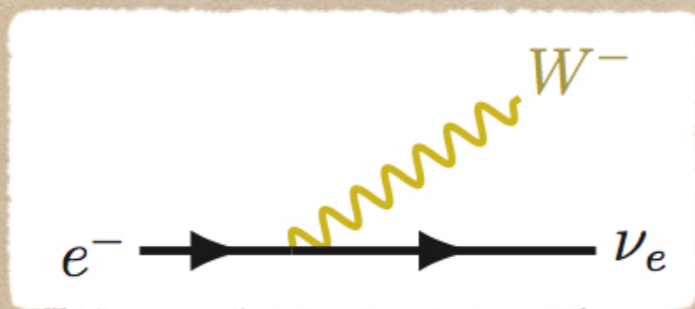
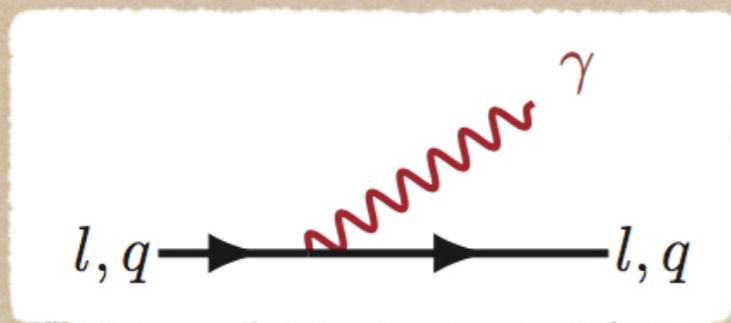
boson - fermion interactions





Feynman Diagrams

boson - fermion interactions



gluon self-interactions





Electromagnetic Interactions

- Photons only interact with **charged** particles
- Electromagnetic charge is **always** conserved

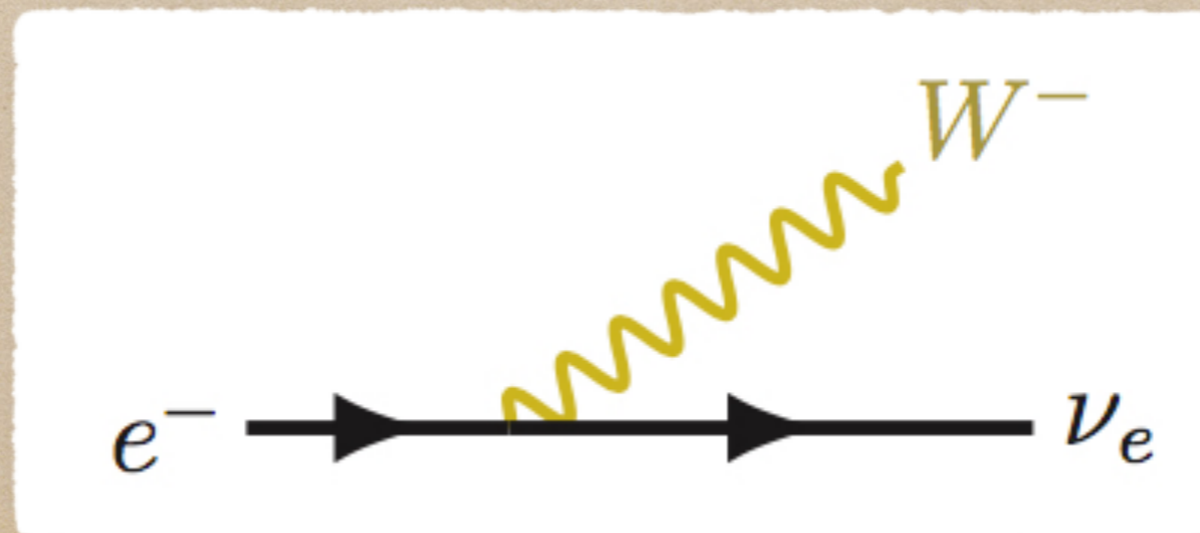
Exercises:

- $e^-e^- \longrightarrow e^-e^-$
- $e^-e^+ \longrightarrow \mu^-\mu^+$
- $e^-e^+ \longrightarrow e^-e^+$
- $\pi^0 \longrightarrow \gamma\gamma$
- $\gamma \longrightarrow e^-e^+ \longrightarrow \gamma$



Weak Interactions

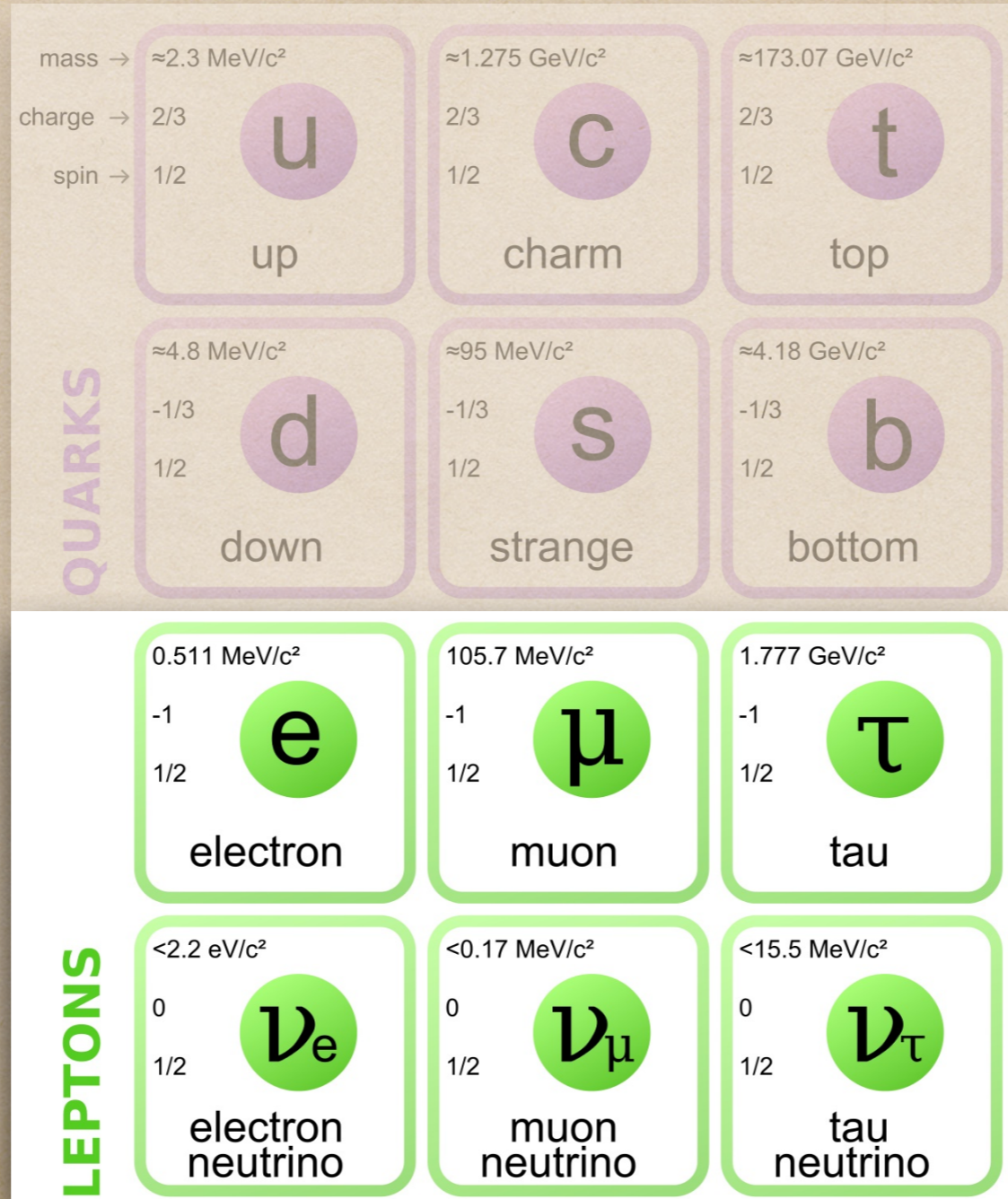
- The weak interaction is carried by Z^0 , W^+ , W^-
- The weak interaction is special because it's the only interaction that changes particle types



The Standard Model

	<p>mass → $\approx 2.3 \text{ MeV}/c^2$</p> <p>charge → $2/3$</p> <p>spin → $1/2$</p> <p>u</p> <p>up</p>	<p>mass → $\approx 1.275 \text{ GeV}/c^2$</p> <p>charge → $2/3$</p> <p>spin → $1/2$</p> <p>c</p> <p>charm</p>	<p>mass → $\approx 173.07 \text{ GeV}/c^2$</p> <p>charge → $2/3$</p> <p>spin → $1/2$</p> <p>t</p> <p>top</p>
QUARKS	<p>mass → $\approx 4.8 \text{ MeV}/c^2$</p> <p>charge → $-1/3$</p> <p>spin → $1/2$</p> <p>d</p> <p>down</p>	<p>mass → $\approx 95 \text{ MeV}/c^2$</p> <p>charge → $-1/3$</p> <p>spin → $1/2$</p> <p>s</p> <p>strange</p>	<p>mass → $\approx 4.18 \text{ GeV}/c^2$</p> <p>charge → $-1/3$</p> <p>spin → $1/2$</p> <p>b</p> <p>bottom</p>
	<p>mass → $0.511 \text{ MeV}/c^2$</p> <p>charge → -1</p> <p>spin → $1/2$</p> <p>e</p> <p>electron</p>	<p>mass → $105.7 \text{ MeV}/c^2$</p> <p>charge → -1</p> <p>spin → $1/2$</p> <p>μ</p> <p>muon</p>	<p>mass → $1.777 \text{ GeV}/c^2$</p> <p>charge → -1</p> <p>spin → $1/2$</p> <p>τ</p> <p>tau</p>
	LEPTONS	<p>mass → $< 2.2 \text{ eV}/c^2$</p> <p>charge → 0</p> <p>spin → $1/2$</p> <p>ν_e</p> <p>electron neutrino</p>	<p>mass → $< 0.17 \text{ MeV}/c^2$</p> <p>charge → 0</p> <p>spin → $1/2$</p> <p>ν_μ</p> <p>muon neutrino</p>

The Standard Model

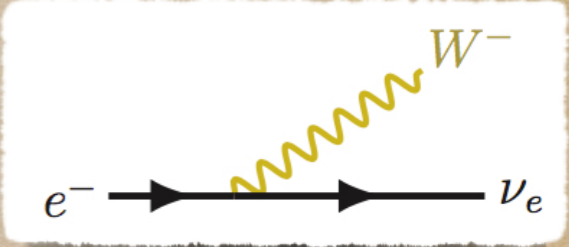




The Standard Model

	mass →	charge →	spin →
QUARKS	$\approx 2.3 \text{ MeV}/c^2$	$2/3$	$1/2$
	$\approx 1.275 \text{ GeV}/c^2$	$2/3$	$1/2$
	$\approx 173.07 \text{ GeV}/c^2$	$2/3$	$1/2$
	$\approx 4.8 \text{ MeV}/c^2$	$-1/3$	$1/2$
	$\approx 95 \text{ MeV}/c^2$	$-1/3$	$1/2$
	$\approx 4.18 \text{ GeV}/c^2$	$-1/3$	$1/2$
	up	charm	top
	down	strange	bottom
LEPTONS	$0.511 \text{ MeV}/c^2$	-1	$1/2$
	$105.7 \text{ MeV}/c^2$	-1	$1/2$
	$1.777 \text{ GeV}/c^2$	-1	$1/2$
	$< 2.2 \text{ eV}/c^2$	0	$1/2$
	$< 0.17 \text{ MeV}/c^2$	0	$1/2$
	$< 15.5 \text{ MeV}/c^2$	0	$1/2$
	electron	muon	tau
	electron neutrino	muon neutrino	tau neutrino

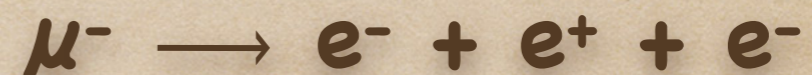
W^+
 W^-



Leptons

- The lepton number of a system is the total number of leptons minus the total number of anti-leptons. Lepton number is always conserved, **even per lepton family!**

- Exercise:



$$0 \neq 1 - 1 + 1 \quad \text{NOK}$$

$$1 \neq 0 + 0 + 0 \quad \text{NOK}$$

$$1 = 1 - 1 + 1 \quad \text{OK}$$

Leptons

- The lepton number of a system is the total number of leptons minus the total number of anti-leptons. Lepton number is always conserved, **even per lepton family!**

- Exercise:



$$L_e: 0 = 1 - 1 + 0 \quad \text{OK}$$

$$L_\mu: 1 = 0 + 0 + 1 \quad \text{OK}$$

$$L: 1 = 1 - 1 + 1 \quad \text{OK}$$



$$0 \neq 1 - 1 + 1 \quad \text{NOK}$$

$$1 \neq 0 + 0 + 0 \quad \text{NOK}$$

$$1 = 1 - 1 + 1 \quad \text{OK}$$



Quarks

- Number of baryons (3 quarks) is conserved



$$B: 1 = 1 + 0 + 0 \quad \text{OK}$$



$$B: 1 = 0 + 0 \quad \text{NOK}$$

- Number of mesons ($q + \bar{q}$) is **not** conserved

Exercise:

- $pp \rightarrow \Delta^{++} \bar{\Delta}^0 \dots$ (add missing + draw diagram)



Quarks

- In the weak interaction, quarks behave different than leptons, as they can **mix** between each other



The Standard Model

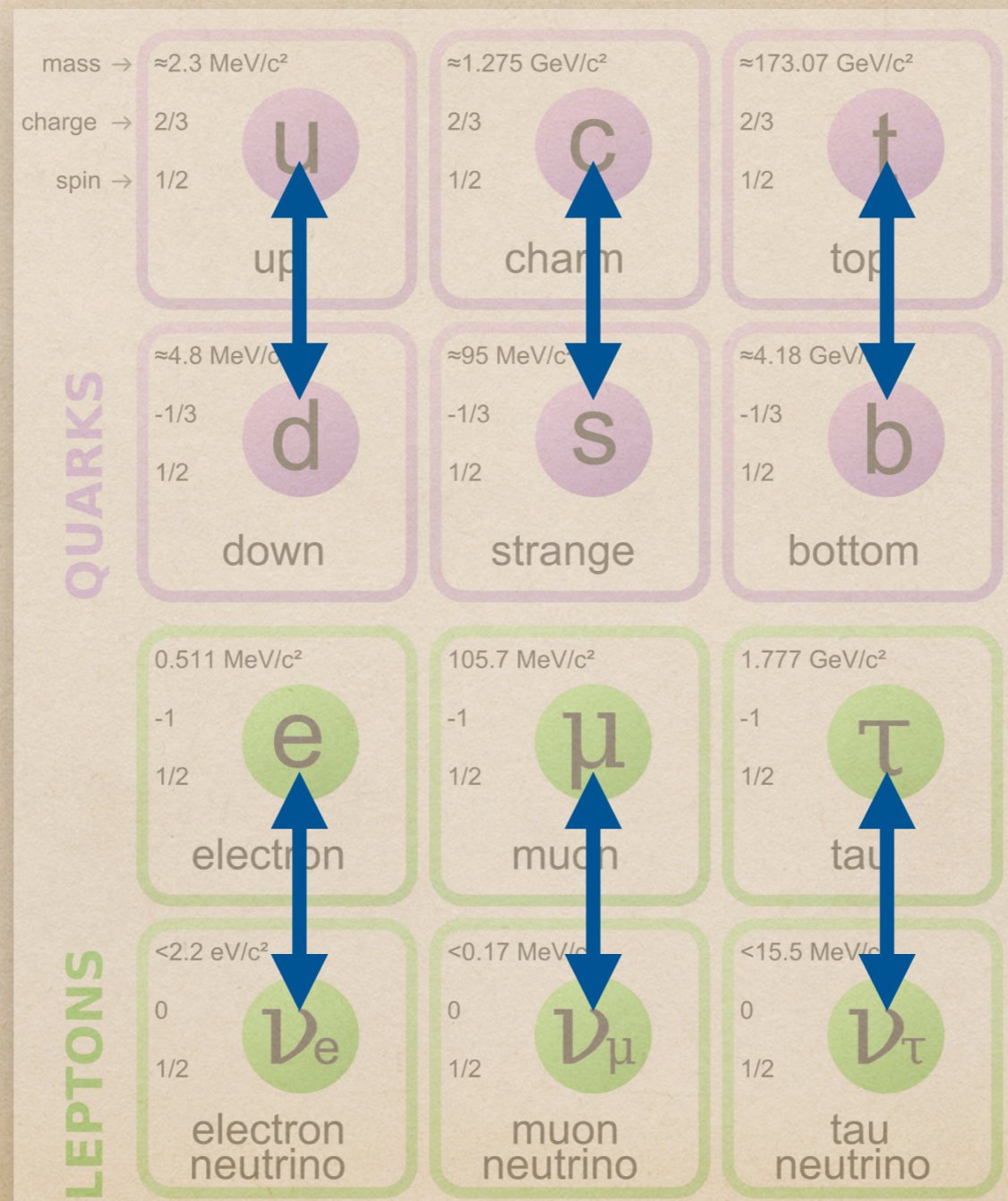
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charge →	$2/3$	$2/3$	$2/3$
spin →	$1/2$	$1/2$	$1/2$
	u	c	t
	up	charm	top
	$\approx 4.8 \text{ MeV}/c^2$	$\approx 95 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$
	$-1/3$	$-1/3$	$-1/3$
	$1/2$	$1/2$	$1/2$
	d	s	b
	down	strange	bottom
	$0.511 \text{ MeV}/c^2$	$105.7 \text{ MeV}/c^2$	$1.777 \text{ GeV}/c^2$
	-1	-1	-1
	$1/2$	$1/2$	$1/2$
	e	μ	τ
	electron	muon	tau
	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$
	0	0	0
	$1/2$	$1/2$	$1/2$
	ν_e	ν_μ	ν_τ
	electron neutrino	muon neutrino	tau neutrino

QUARKS

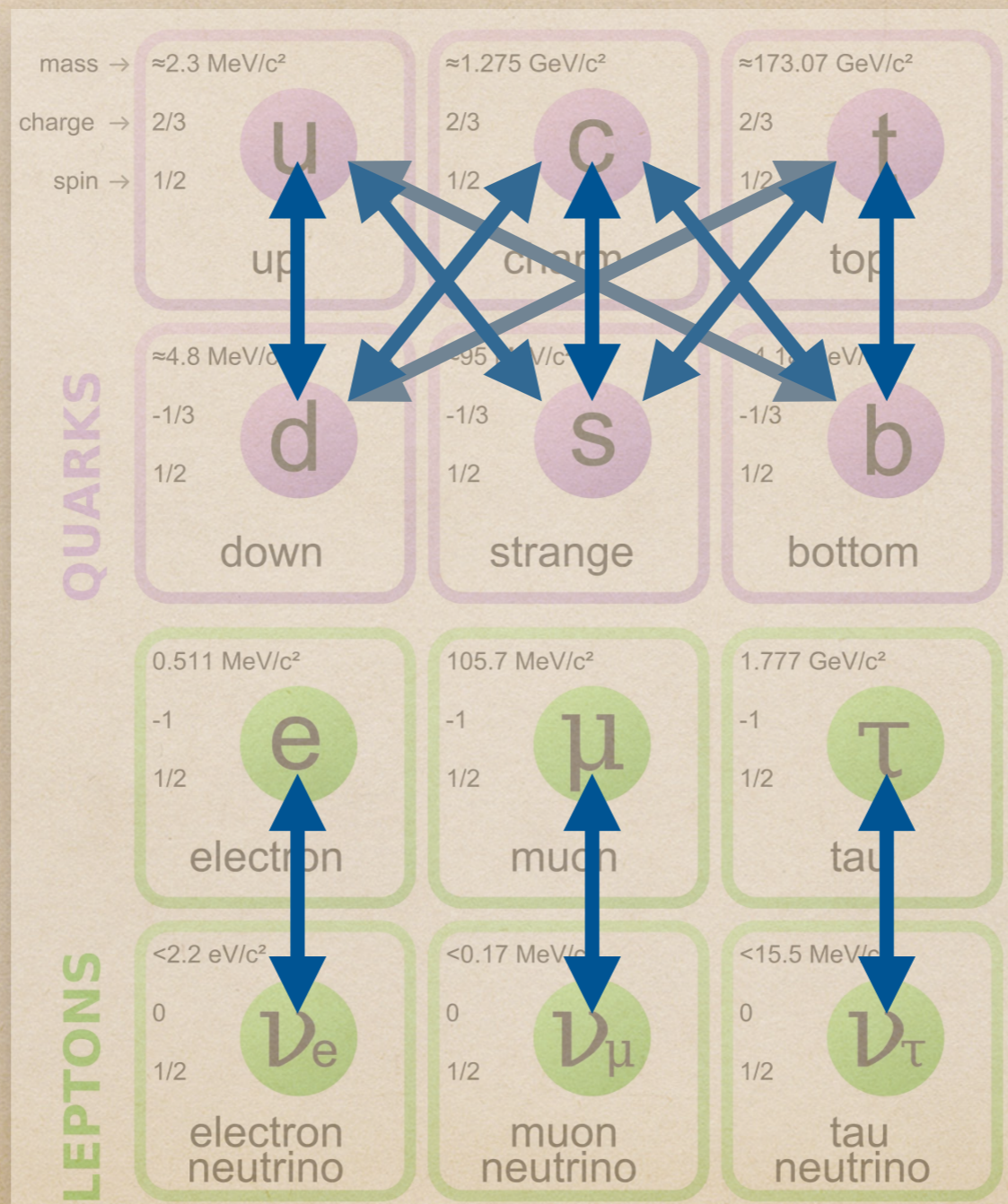
LEPTONS



The Standard Model



The Standard Model





Let's Try

Exercise:

- $n \longrightarrow p e^- \dots$ (add missing + draw diagram)
- $\pi^+ \longrightarrow \mu^+ \dots$ (add missing + draw diagram)
- $K^- \longrightarrow \pi^- \dots$ (add missing + draw diagram)
- $Z^0 \longrightarrow \gamma\gamma$ (draw diagram, be careful!)
- $K^0 \longrightarrow \dots \longrightarrow \gamma\gamma$ (add missing + draw diagram)



Weak Bosons

- Weak bosons can interact with themselves (like gluons), as long as charge is conserved
- Diagram with $W^+W^-Z^0$ (try)
- Diagram with $W^+W^-Z^0\gamma$ (try)
- LEP100 vs LEP200

Exercises

- Draw Feynman diagrams for the following processes using the weak interaction:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\Lambda \rightarrow p + e^- + \bar{\nu}_e$$

$$K^0 \rightarrow \pi^+ + \pi^-$$

$$\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$$

- Draw Feynman diagrams for the following processes using the strong interaction:

$$\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0$$

$$\rho^0 \rightarrow \pi^+ + \pi^-$$

$$\Delta^{++} \rightarrow p + \pi^+$$



Order in the Particle Zoo

Spin



Wikipedia:

“In quantum mechanics and particle physics, spin is an intrinsic form of **angular momentum** carried by elementary particles, composite particles (hadrons), and atomic nuclei.

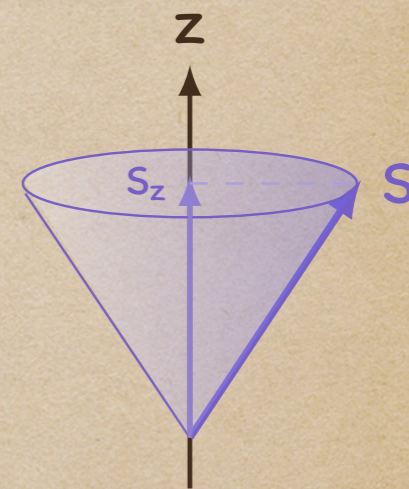
In some ways, spin is like a **vector quantity**; it has a definite magnitude, and it has a ‘direction’ (but quantisation makes this ‘direction’ different from the direction of an ordinary vector).

All elementary particles of a given kind have the same magnitude of spin angular momentum, which is indicated by assigning the particle a **spin quantum number**.”

Spin



- Spin is a **vector**, however, due to uncertainty in quantum mechanics, we cannot know all three components S_x , S_y , and S_z at the same time
- But we can know the **length** S and the **z-component** S_z simultaneously
- But spin is a **quantum** vector, which puts some restrictions on its possible values, as they are quantised (which means values go in steps)..



Spin

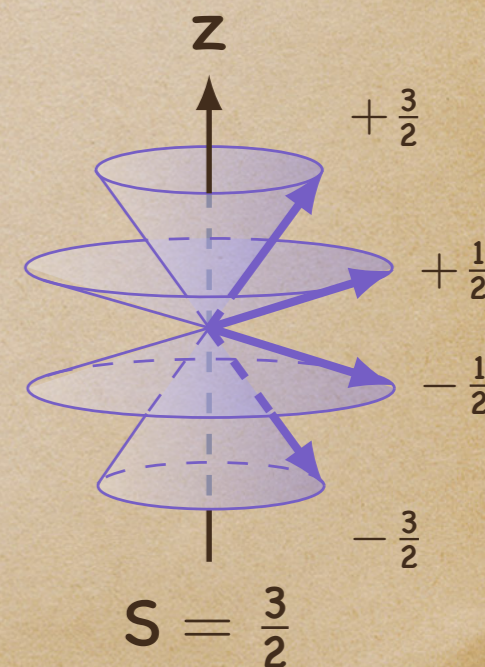
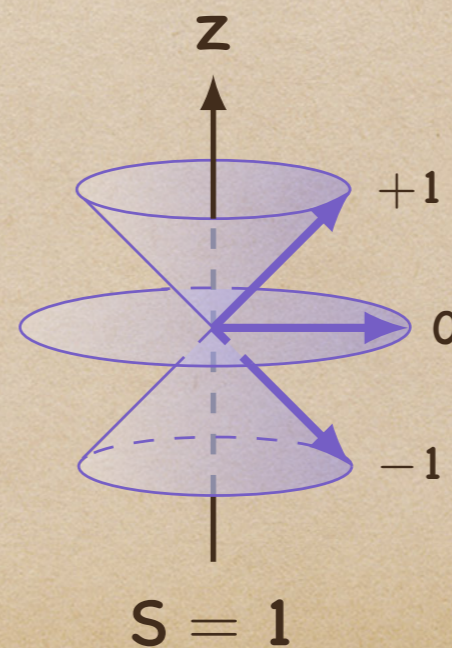
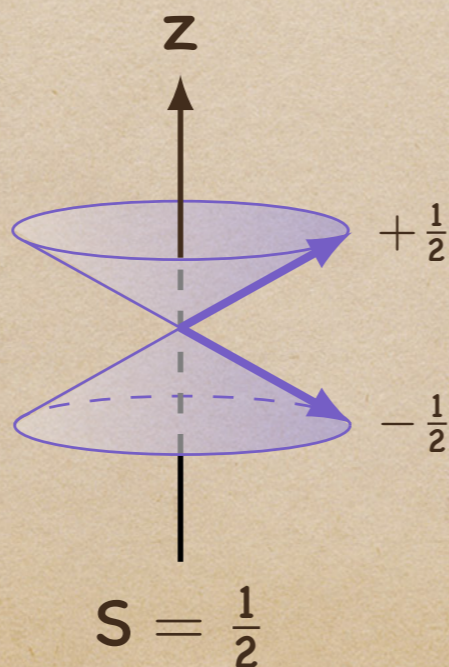


- The **length** needs to be a positive multiple of $1/2$, so $S = 0, 1/2, 1, 3/2, \dots$
- S_z can be anything between $-S, -S+1, \dots, S-1, S$
This means that spin states come in **multiplets**.

$$S = 0 \quad \rightarrow \quad S_z = 0$$

$$S = 1/2 \quad \rightarrow \quad S_z = -1/2, +1/2$$

$$S = 1 \quad \rightarrow \quad S_z = -1, 0, +1$$

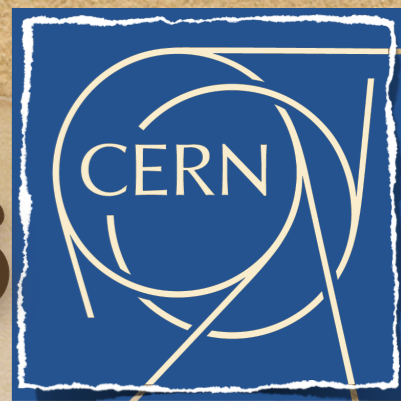


Spin



- Typical example is the electron: it has spin $1/2$, which means it has two possible states:
 $-1/2$ or $+1/2$
 also known as 'up' or 'down'
- Now back to elementary particles..

Elementary Particles



- Three types:
 - **Fermions:** matter particles \Rightarrow spin $1/2$
 - **Bosons:** force carriers \Rightarrow spin 1
 - **Higgs:** special guy \Rightarrow spin 0

Baryons

- So, using only u and d quarks, we can make a proton (uud) or a neutron (udd), but also two other particles which have the same quarks but different spin:
 - Δ^+ : uud but spin 3/2
 - Δ^0 : udd but spin 3/2
- We can even make two more combinations:
 - Δ^{++} : uuu (spin 3/2)
 - Δ^- : ddd (spin 3/2)

Baryons

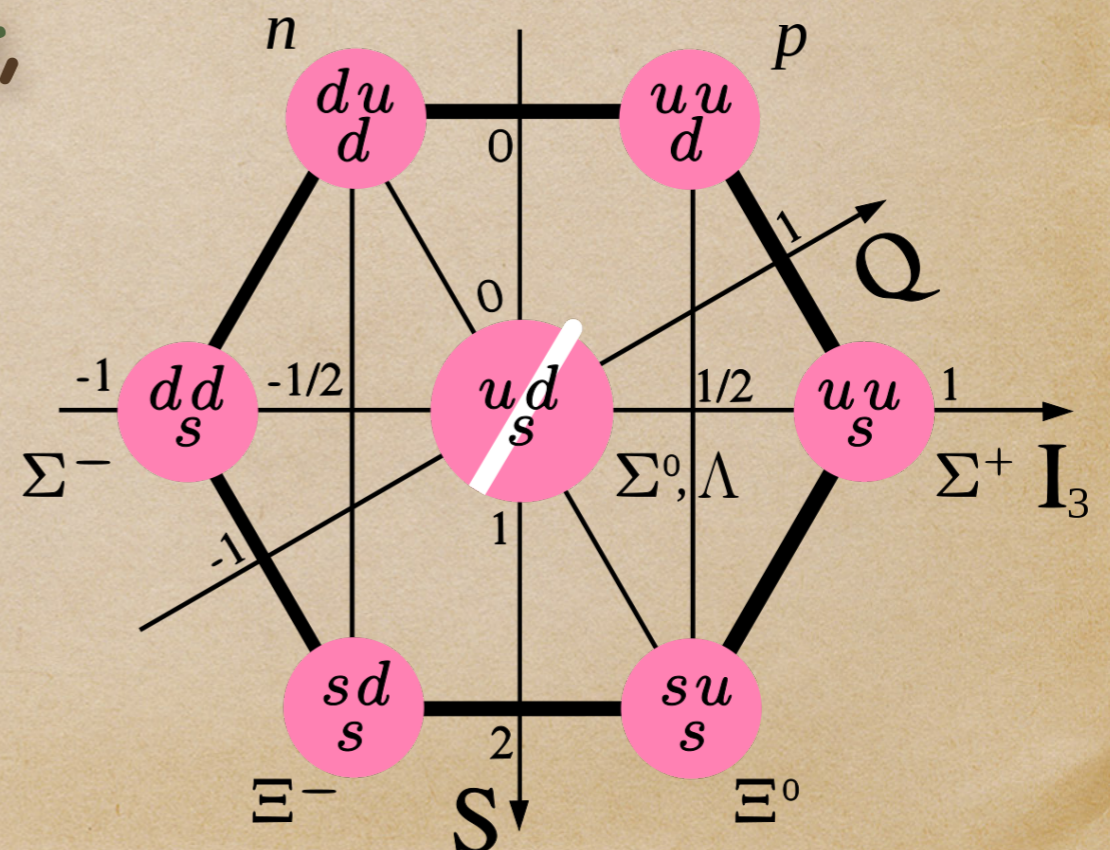
- The other quarks are way heavier than u and d:
 $m_s = 20 m_d = 40 m_u$, $m_c = 250 m_d = 500 m_u$
which means that the resulting baryons will be much heavier as well
- Before quarks were discovered, only hadrons built from u, d, and s quarks were found. Some were acting 'normal', like a proton, but some were acting 'strange' (because - we know now - they contain an s quark). They were given a **Strangeness**

Baryons

- We know today that a hadron with Strangeness -1 contains exactly one s quark (similarly, Strangeness -2 implies two s quarks etc).

- The simplest spin $1/2$ baryons can be organised in an octet, called "The eightfold way"

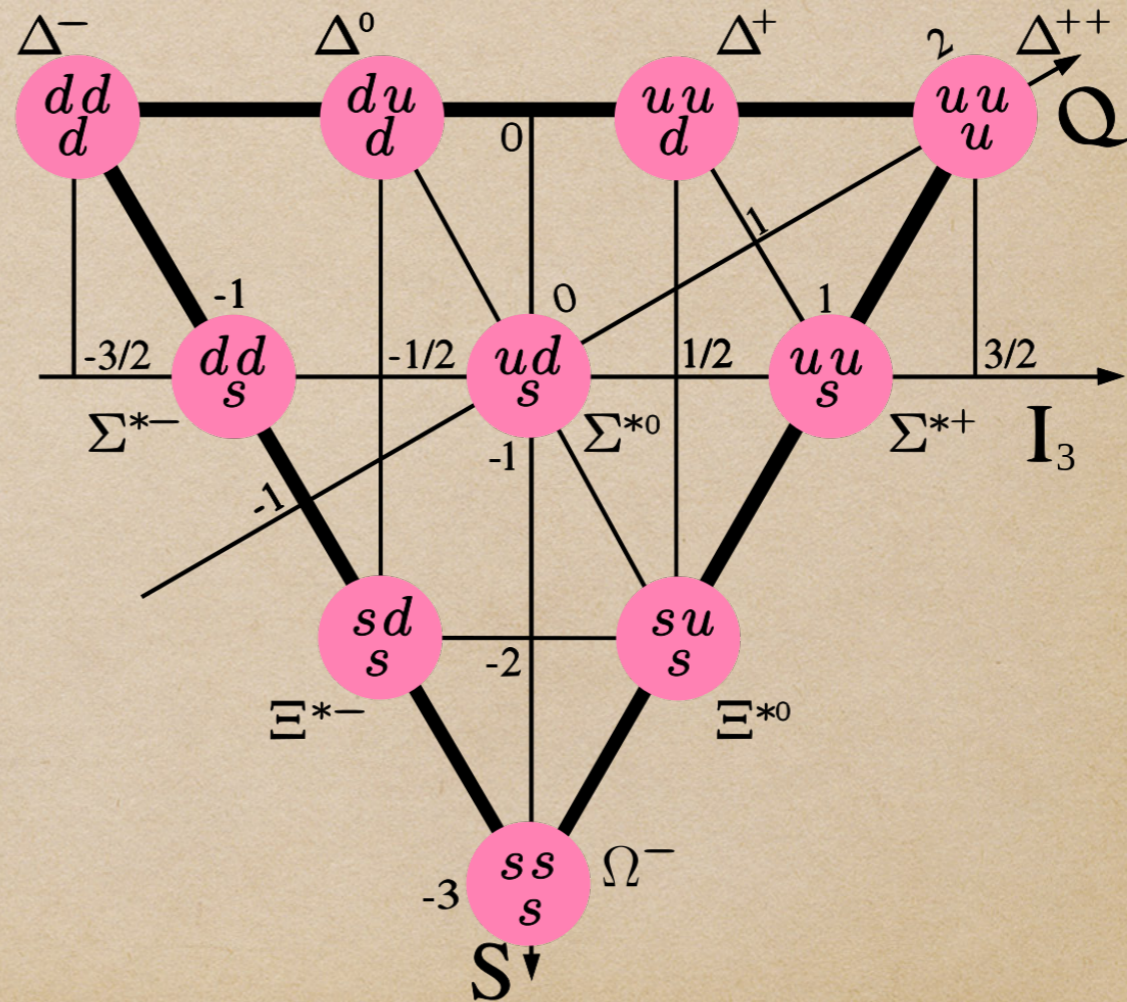
- Strangeness 0: n, p
Strangeness -1 : Σ, Λ
Strangeness -2 : Ξ



Baryons

- Similarly, the simplest spin 3/2 baryons can be organised in a **decuplet**

- Strangeness 0: Δ
- Strangeness -1: Σ^*
- Strangeness -2: Ξ^*
- Strangeness -3: Ω



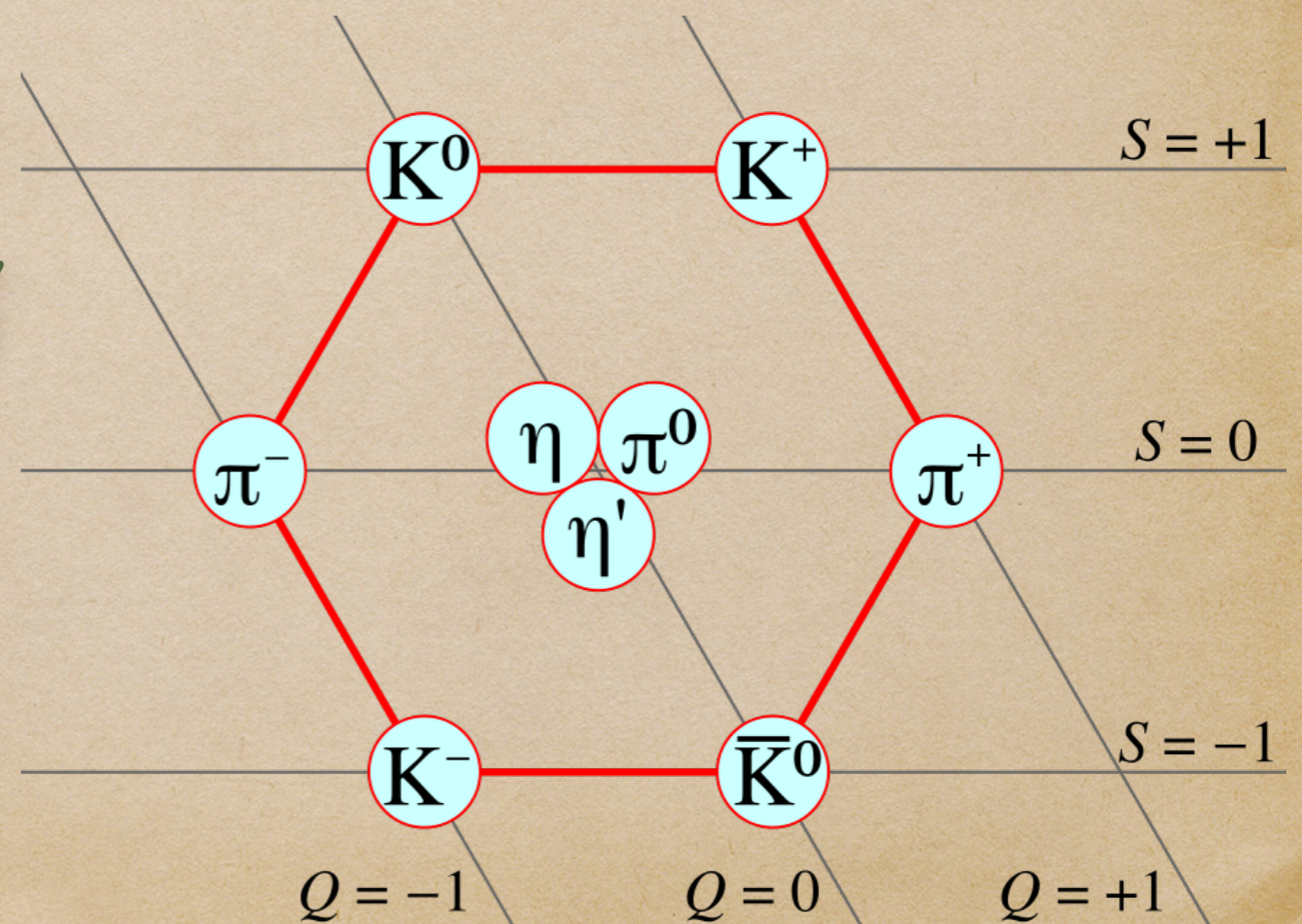
Mesons

- Mesons are built from a quark and an antiquark, and hence lighter than baryons.
- As they are built from two quarks, their spin is $1/2 + 1/2 = 0$ or 1 .
- They are classified similarly to baryons, in function of their Strangeness.

Mesons

- The simplest spin 0 mesons can be organised in a **nonet**, originally called "The eightfold way" as well (because η' wasn't found yet)

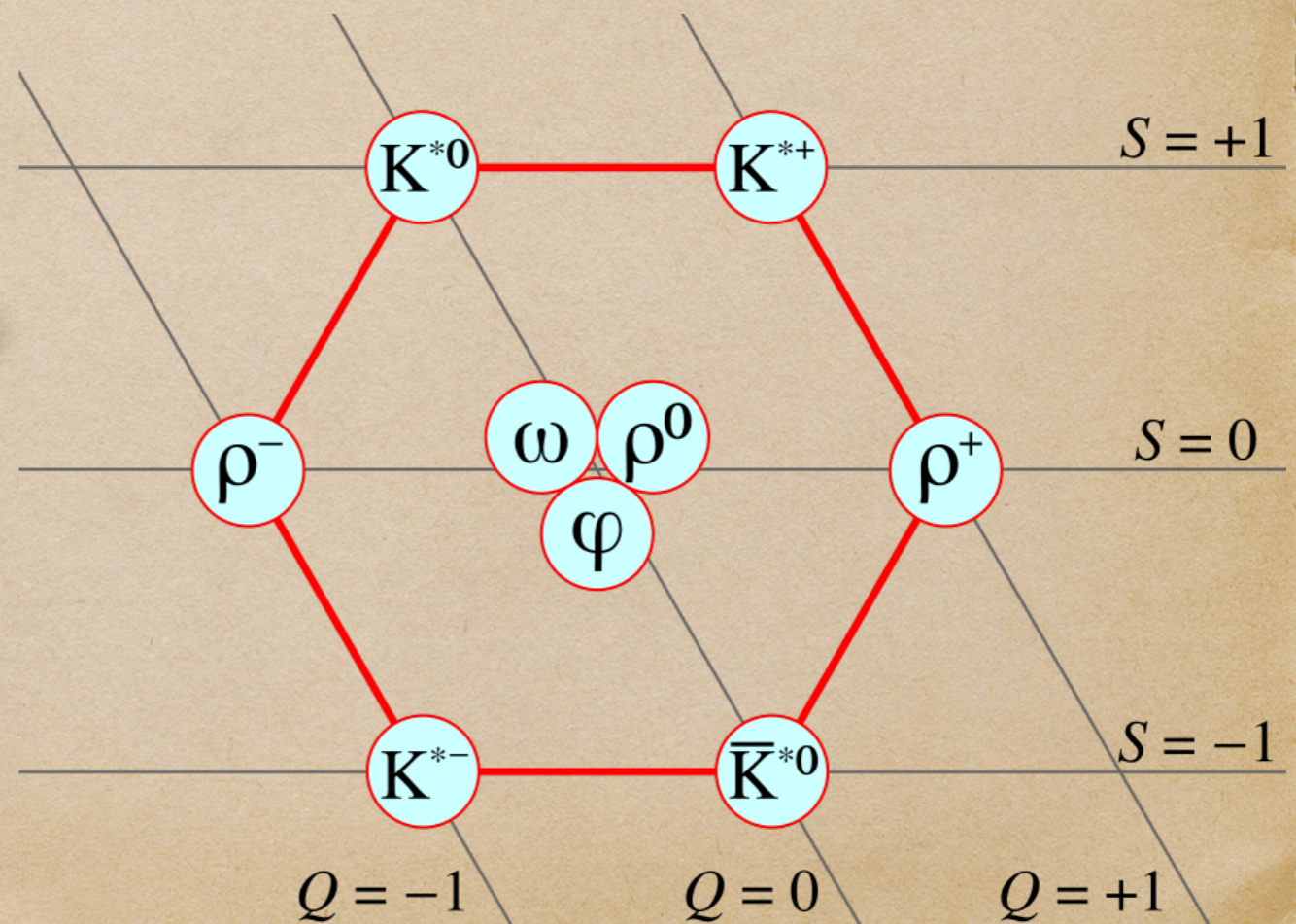
- Strangeness +1: **K**
- Strangeness 0: **π , η , η'**
- Strangeness -1: **K**



Mesons

- And the simplest spin 1 mesons can be organised in another **nonet**

- Strangeness +1: K^*
- Strangeness 0: ρ , ω , φ
- Strangeness -1: K^*





Recap: Conservation Laws

Conservation Laws

- Conservation laws are an important concept in quantum physics. They tell us that a certain quantity is **conserved**
- This means that its values before and after an interaction have to be the same
- For example EM charge is conserved. Not for each particle separately, but the sum of all particles' charges before an interaction has to equal the sum of all particles' charges after that interaction

Conservation Laws

- Conservation laws help us determine if a certain process can take place or not
- Example: $p + p \rightarrow n + \pi^+ + \pi^- + p$ is forbidden (initial charge is 2, final charge is 1)
- Certain laws are **exact**: it is believed that they always hold. Others are **approximately exact** (they hold in say 99% of cases), or are valid under certain **conditions** only (for example only when the process is without the weak force)

Conservation Laws

- Conservation of 4-momentum
- Conservation of charges (EM, weak, colour)
- Conservation of baryon number
- Conservation of lepton number (total & individual)
- Conservation of flavour
-

Mass

- Important for decays (where one particle transforms into several). In rest frame of initial particle: mass of initial particle equals sum of energies of final particles. This gives the following condition:

$$m_{\text{initial}} \geq \sum m_{\text{final}}$$

- Probability of decay becomes larger, the larger the mass difference!

Charge Conservation

- All charges are always 100% conserved!
- Conservation of **electromagnetic charge** is extremely important and an **easy check** of process validity
- Conservation of **colour charge** is automatically satisfied as long as quarks are combined correctly
- Conservation of weak charge is more complex to check, use rules of weak interaction

Weak interaction?

- Weak interaction changes **flavour**
- This means between one family:
 $e^- \leftrightarrow \nu_e$ $u \leftrightarrow d$ etc
- Charge is not the same \Rightarrow need other particles to correct this
- Quarks can show **mixing** between families:
 $u \leftrightarrow s$ or $u \leftrightarrow b$
but probability is rather small
No mixing between leptons!

Weak interaction?

- No other interaction can change flavour, so if we see a flavour change in a certain process, we know that this process is governed by the weak force
- We say that all interactions except the weak conserve flavour

Summary

- In decays, the sum of masses of final products cannot be larger than the initial mass
- EM charge is conserved
- Baryon number is conserved
- Lepton number is conserved (total & individual)
- Flavour is conserved unless the process is weak
- Antiparticles have opposite numbers and charges

Exercises

- Determine if the following processes are possible, and if yes, with which interaction:

$$p + \bar{p} \rightarrow \pi^+ + \pi^- + \pi^0 + \pi^+ + \pi^-$$

$$p + K^- \rightarrow \Sigma^+ + \pi^- + \pi^+ + \pi^- + \pi^0$$

$$p \rightarrow \Lambda^0 + \bar{\Sigma}^0 + \pi^+$$

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + n$$

$$\bar{\nu}_e + p \rightarrow e^+ + \Lambda^0 + K^0$$

$$\Sigma^0 \rightarrow \Lambda^0 + \gamma$$



The Higgs Boson

Production



Higgs boson production at LEP

- Direct production
- Higgs strahlung

Production

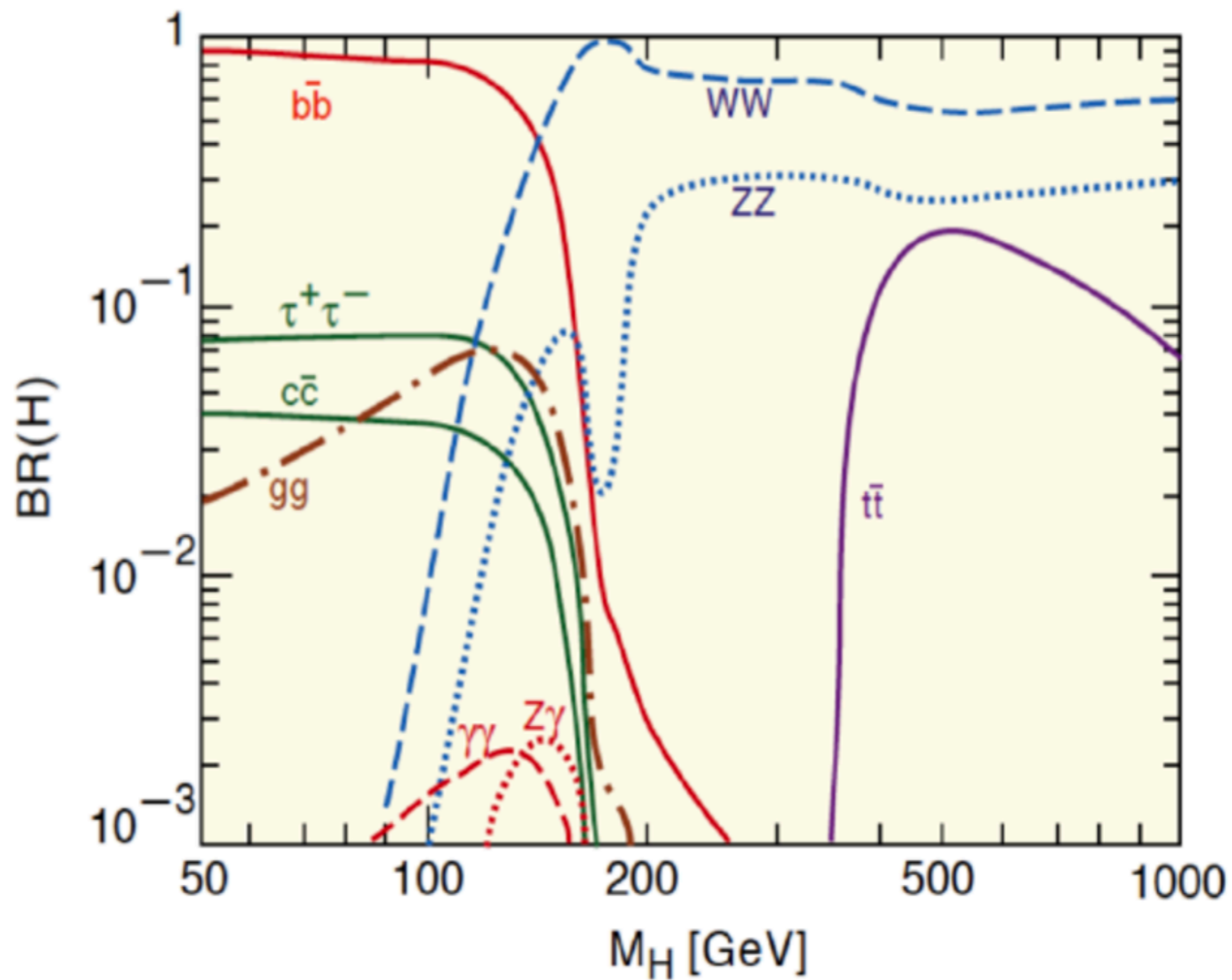
Higgs boson production at LEP

- Direct production
- Higgs strahlung

Higgs boson production at LHC

- Gluon fusion
- Vector boson fusion
- Higgs strahlung
- Associated $t\bar{t}$ production

Decay





Feynman Rules



Symmetries:

Building the Lagrangian



Basics

- basic components are **fields**
 - just mathematical tools
 - will give rise to particles
- principal quantity is the **action**, which is the integral of the Lagrangian:

$$S = \int d^4x \mathcal{L}(x, \phi, \partial\phi)$$



- all paths possible (simultaneous), but path with **least action** is favoured
- minimising action leads to **equations of motion**



Lagrangian?

- is kinetic energy minus potential energy

$$\mathcal{L} = T - V$$

- classical example: spring



$$\mathcal{L} = \frac{1}{2}m \left(\frac{dx}{dt} \right)^2 - \frac{1}{2}kx^2 \quad \Rightarrow \quad x = x_0 \cos \sqrt{\frac{k}{m}}t$$

- field example: free electron field

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi \quad \Rightarrow \quad (i\not{\partial} - m)\psi = 0 \quad (\text{QM})$$



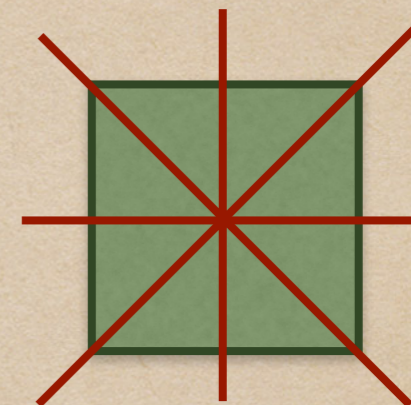
Lagrangian!

- kinetic terms are **quadratic** and have **derivatives** $\bar{\psi}\not{\partial}\psi$, $\partial_{\mu}\phi\partial^{\mu}\phi$, ...
- potential terms are what is left
 - special type: **mass terms**: $m\bar{\psi}\psi$, $m^2|\phi|^2$, ...
quadratic without derivatives
 - others are **interaction terms** $\bar{\psi}A\psi$, ...



Symmetries

- leave theory unchanged
- **symmetry** \Rightarrow **conservation**



Emmy Noether

- homogeneity of space
 - \Rightarrow translational invariance
 - \Rightarrow momentum conservation
- isotropy of space
 - \Rightarrow rotational invariance
 - \Rightarrow angular momentum conservation





Symmetries

- in quantum mechanics, ψ is an amplitude
 - not physical
 - $|\psi|^2$ is probability, physical

- **phase** is undetermined, because we can scale $\psi \rightarrow e^{ia}\psi$, then $\bar{\psi} \rightarrow e^{-ia}\bar{\psi}$, such that $|\psi|^2 \rightarrow |\psi|^2$

=> invariant!



Symmetries

- similar in quantum field theories:

$$i\bar{\psi}\not{\partial}\psi \rightarrow i\bar{\psi}\not{\partial}\psi \quad m\bar{\psi}\psi \rightarrow m\bar{\psi}\psi$$

in other words $\mathcal{L} = \mathcal{L}'$

- free electron field

=> conservation of electric charge

- BUT... phase α can depend on spacetime

coordinates: $\alpha = \alpha(x)$

$$i\bar{\psi}\not{\partial}\psi \rightarrow i\bar{\psi}\not{\partial}\psi - \bar{\psi}(\not{\partial}\alpha)\psi \quad \mathcal{L}' = \mathcal{L} - \bar{\psi}(\not{\partial}\alpha)\psi$$

=> no longer invariant!



Lagrangian

- add term $g\bar{\psi}\not{A}\psi$ to the Lagrangian
 - with property $A_\mu \rightarrow A_\mu + 1/g \partial_\mu a$
 - because then $g\bar{\psi}\not{A}\psi \rightarrow g\bar{\psi}\not{A}\psi + \bar{\psi}(\not{\partial}a)\psi$
 - invariant !
- but new field also needs kinetic terms
 - symmetry \Rightarrow conserved tensor:
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$
 - its square will be kinetic term:
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



QED:

**From the Lagrangian
to a full theory**



QED

Full Lagrangian for Quantum Electro Dynamics:

$$\mathcal{L}_{\text{QED}} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi + g\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

electron kinetic term

electron mass term

electron-photon interaction term

photon kinetic term



QED

Full Lagrangian for Quantum Electro Dynamics:

$$\mathcal{L}_{\text{QED}} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi + g\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

'free' theory

'interaction' theory

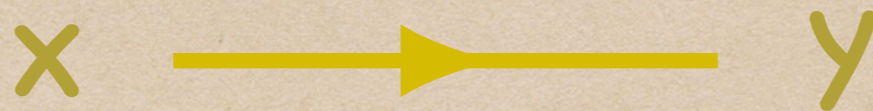
$$\mathcal{L}_{\text{QED}} = \mathcal{L}_0 + \mathcal{L}_I$$

Keep \mathcal{L}_0 exact, but expand $\mathcal{L}_I \Rightarrow$ perturbation theory



Free Theory

Propagation of an electron from x to y :



$$\langle 0 | \psi(y) \bar{\psi}(x) | 0 \rangle = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \psi(y) \bar{\psi}(x) e^{iS_0}$$

Propagation of a photon from x to y :

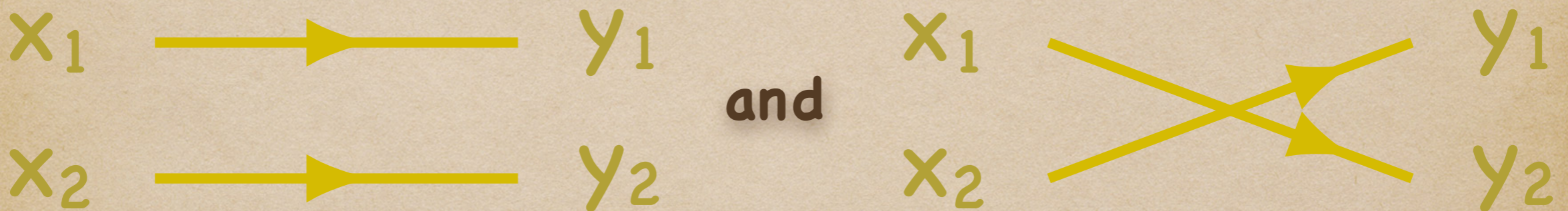


$$\langle 0 | A_\mu(x) A_\mu(y) | 0 \rangle = \int \mathcal{D}A_\mu A_\mu(x) A_\mu(y) e^{iS_0}$$



Free Theory

Easily generalised to more points:



$$\langle 0 | \psi_{y_1} \psi_{y_2} \bar{\psi}_{x_1} \bar{\psi}_{x_2} | 0 \rangle = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \psi_{y_1} \psi_{y_2} \bar{\psi}_{x_1} \bar{\psi}_{x_2} e^{S_0}$$



Interaction

- Add interaction part from action (Lagrangian) to the exponential $e^{S_0 + S_I}$
- Equations are not solvable anymore

=> expand interaction part:

$$e^{S_I} \approx 1 + S_I + \frac{1}{2} S_I^2 + \dots$$

- Propagation of electron from x to y is now:

$$\langle \psi(y) \bar{\psi}(x) \rangle = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_\mu \psi(y) \bar{\psi}(x) e^{S_0} \left(1 + S_I + \frac{1}{2} S_I^2 + \dots \right)$$

Interaction

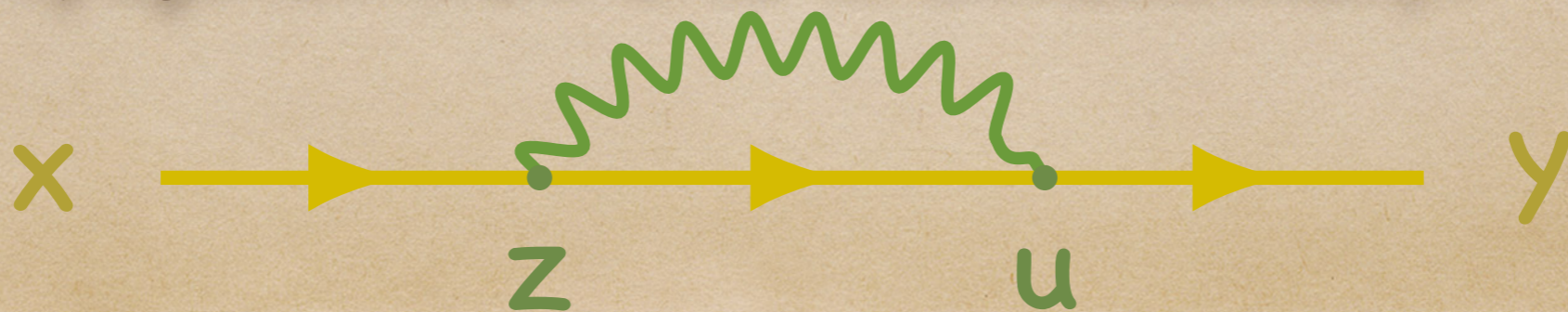
- Propagation of electron from x to y is now:

$$\langle \Psi(y) \bar{\Psi}(x) \rangle = \int \mathcal{D}\bar{\Psi} \mathcal{D}\Psi \mathcal{D}A_\mu \Psi(y) \bar{\Psi}(x) e^{S_0} \left(1 + S_I + \frac{1}{2} S_I^2 + \dots \right)$$

- Take the second order as example:

$$S_I^2 = g^2 \int dz du (\bar{\Psi} \not{A} \Psi)_z (\bar{\Psi} \not{A} \Psi)_u$$

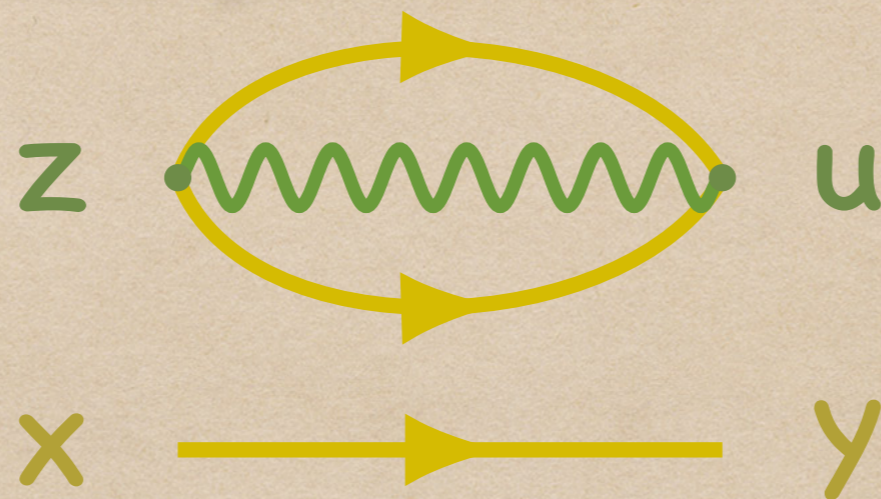
- So we have electron propagation from x to z , from z to u , and from u to y . We also have photon propagation from z to u . Schematically:





Interaction

- Other possibility:



- Vacuum diagrams

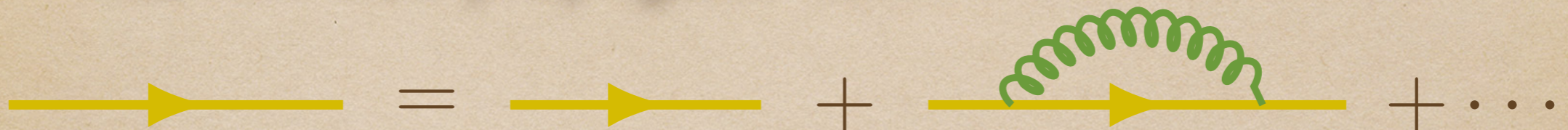
=> These are unwanted and have to be cancelled:

$$\langle \psi(y) \bar{\psi}(x) \rangle = \frac{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \psi(y) \bar{\psi}(x) e^{\mathcal{S}}}{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{\mathcal{S}}}$$

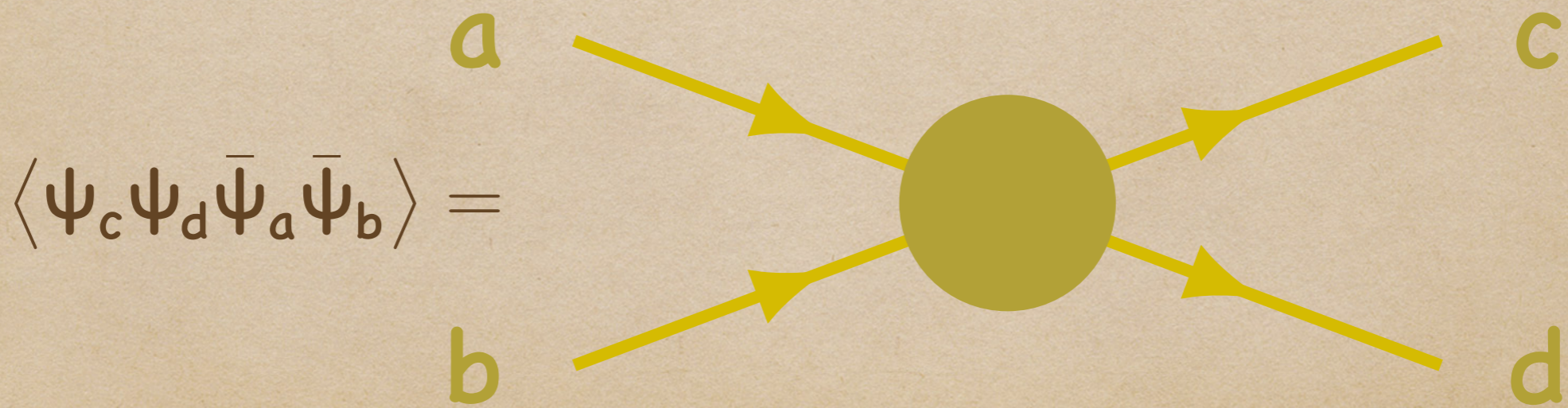


Interaction

- So the full propagator is:



- This can be extended to any number of particles, i.e. electron-electron collision:

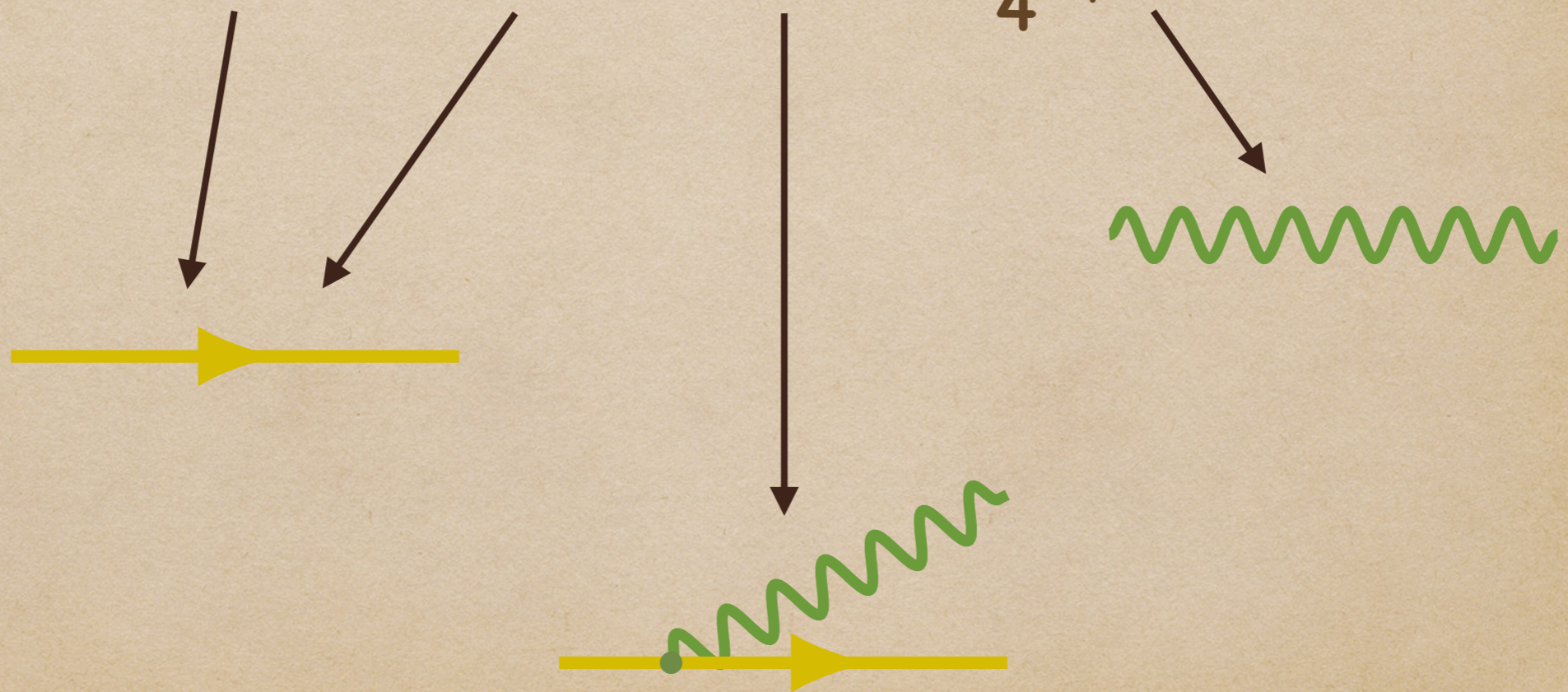




Feynman Rules

Full Lagrangian for Quantum Electro Dynamics:

$$\mathcal{L}_{\text{QED}} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi + g\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$





SM:

**The Standard Model
of particle physics**

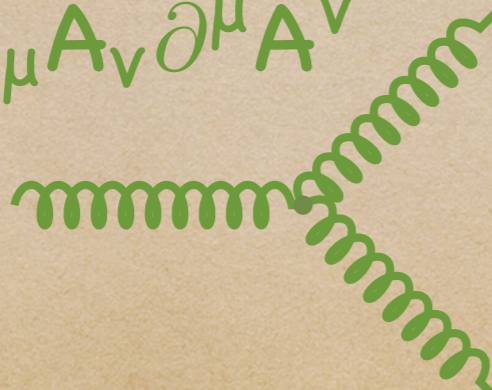


Lagrangian

- We can easily extend our theory by adding new parts to our Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QFD}} + \mathcal{L}_{\text{QCD}} + \dots$$

- QFD (weak force) and QCD (strong force) are very similar to QED. They only add two different types of interaction terms:

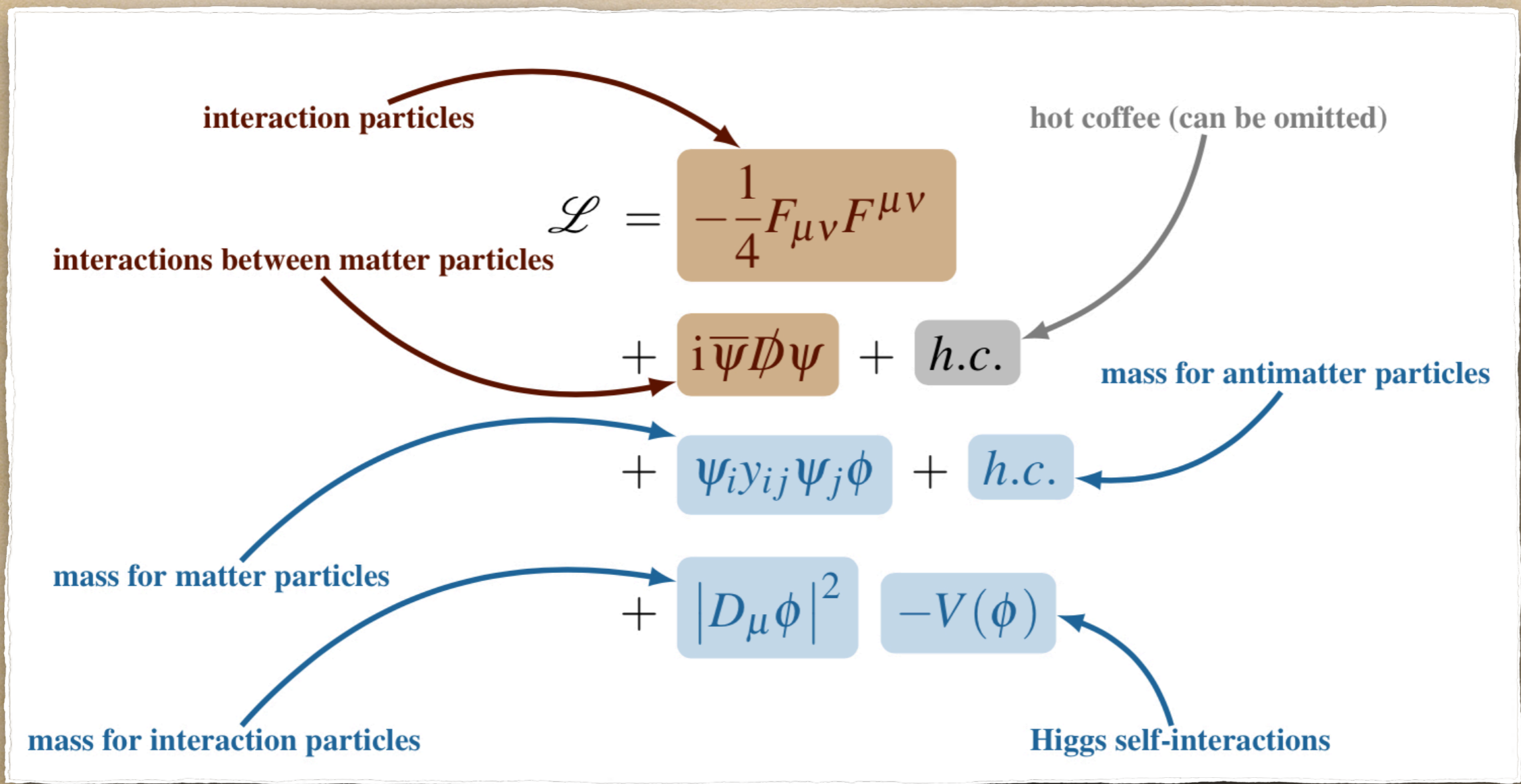
$$A_\mu A_\nu \partial^\mu A^\nu$$
A Feynman diagram representing the interaction term $A_\mu A_\nu \partial^\mu A^\nu$. It consists of a single green wavy line that starts from the bottom left and curves upwards and to the right, ending at the top right.

$$A_\mu A_\nu A^\mu A^\nu$$





Lagrangian



Let's have a coffee with the Standard Model of particle physics!

J Woithe, GJ Wiener, FF VdV, Phys Educ 52 (2017) no 3, 034001