AXION DETECTION USING SUPERCONDUCTING QUBIT SYSTEM

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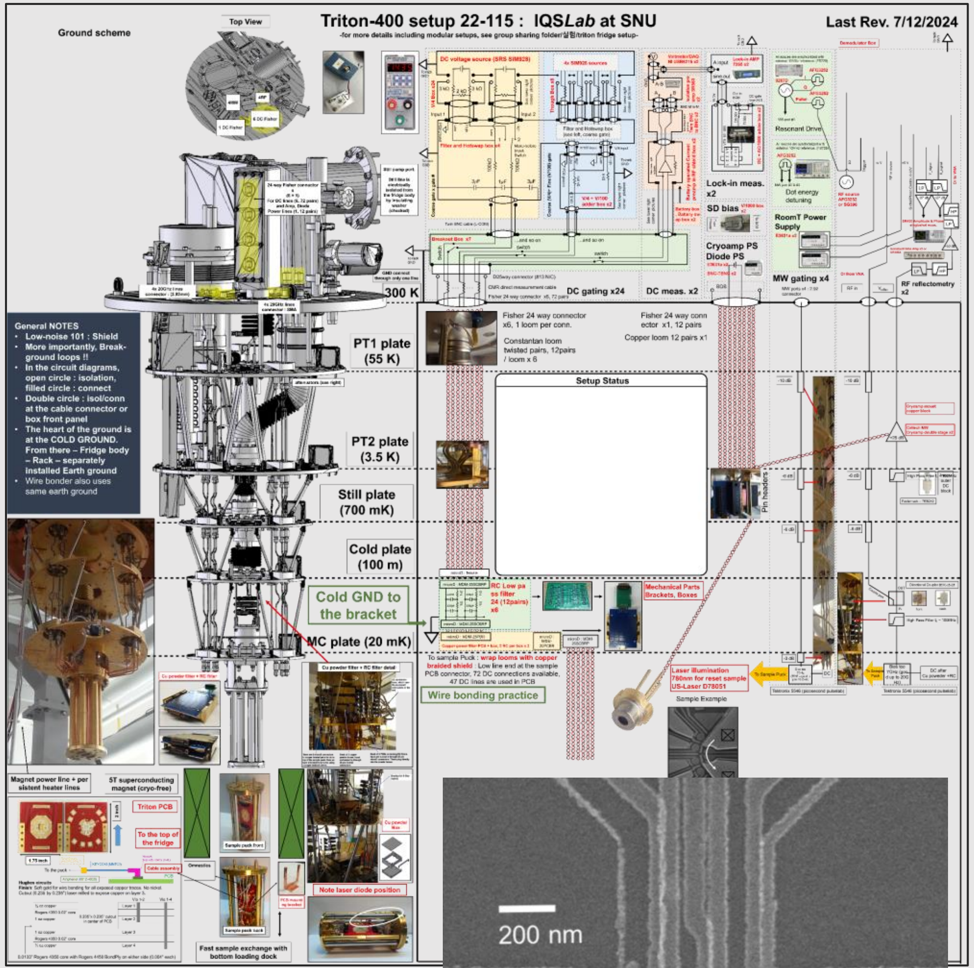
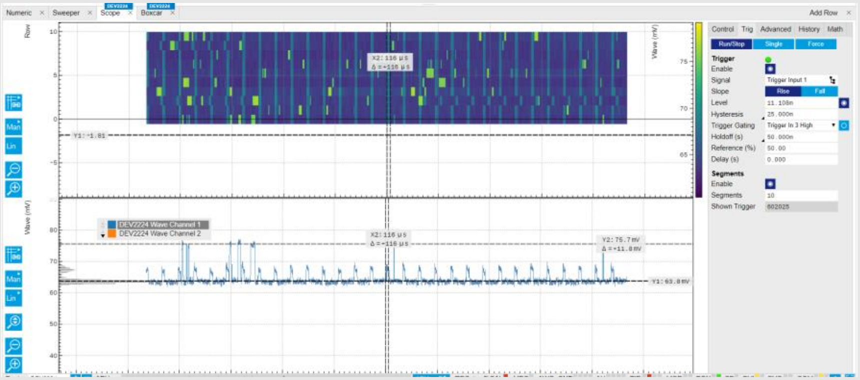
Integrated Quantum System Lab

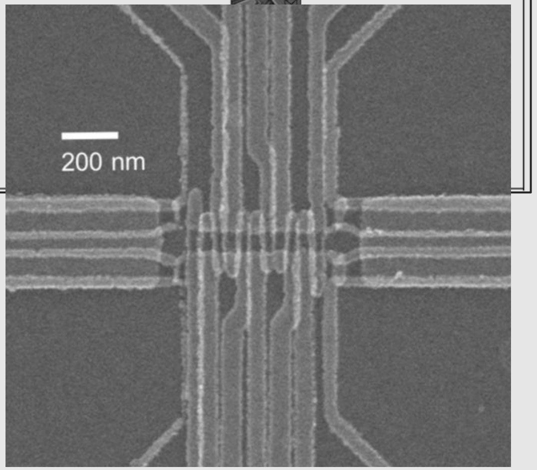
관심분야: Quantum Computing (QD, SC, Hybrid)

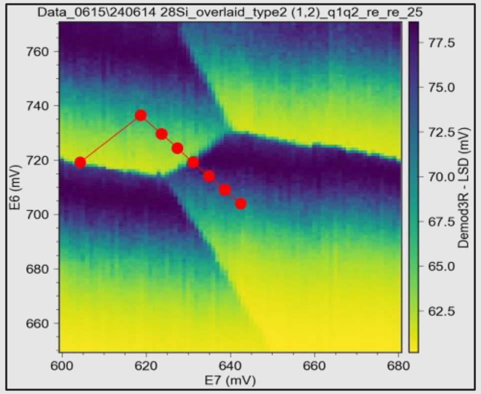
MBTI: INTJ (MBTI 안 믿음)

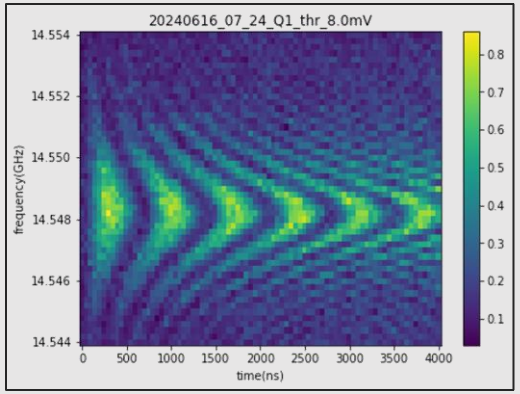


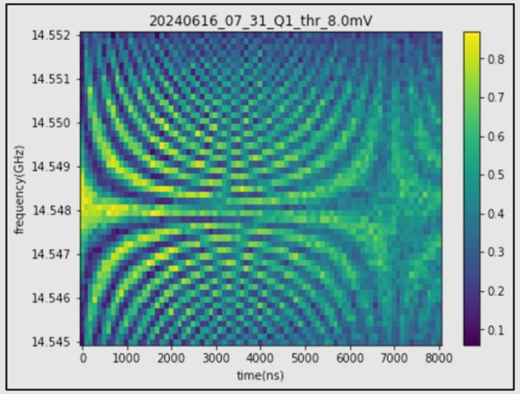


```python
import numpy as np
from scipy.signal.windows import gaussian
from qm.qua import *


def gauss(amplitude, mu, sigma, length):
    t = np.linspace(-length / 2, length / 2, length)
    gauss_wave = amplitude * np.exp(-((t - mu) ** 2) / (2 * sigma**2))
    return [float(x) for x in gauss_wave]

def IQ_imbalance(g, phi):
    c = np.cos(phi)
    s = np.sin(phi)
    N = 1 / ((1-g**2)*(2*c**2-1))
    return [float(N * x) for x in [(1-g)*c, (1+g)*s, (1-g)*s, (1+g)*c]]


################
# CONFIGURATION:
################
opx_ip = "192.168.1.14"
opx_port = 10254
octave_ip = "192.168.1.14"
octave_port = 11253
con = "con1"
octave = "octave1"

####### Lock-in readout ##########################################################
lockin_freq = 0
lockin_amp = 0.01
readout_pulse_length = 2000*1000 # ns  1800
readout_pulse_length22 = 2000*1000 # ns
readout_pulse_length2 = 10 * 1000  #readout duration for initialization check
slice_size = 2500 #3500 #1250 #50 # 4ns (max readout array length ~ 100)
slice_size2 = 2500 #1250 #50 # 4ns (max readout array length ~ 100)

####### faster scan ##############################################################
step_num = 200  # number of dc values per gate
scan_vpp = 0.5

###### MIXER PARAMETER ####################
octave_gain = 4 # can set the gain from -10dB to 20dB
IF = 100e6      # The IF frequency
LO = 15.70e9    # The LO frequency data set {13.0e9 ~ 17.9e9, step : 0.1e9}
if_freq = IF
lo_freq = LO
```









CONTENTS

INTRODUCTION

We are living in Noisy-Intermediate Scale Quantum (NISQ) Era!

> **SUMMARY AND CONCLUSIONS**
>
> We have demonstrated computational supremacy in the quantum simulation of nonequilibrium magnetic spin dynamics. We simulated square, cubic, diamond, and biclique topologies that are relevant to applications in materials science and AI, and are amenable to scaling

A. D. King, et al., Computational supremacy in quantum simulation, arXiv:2403.00910 (Mar 2024)

Energy Scale of Qubit System

Conversion factor: $1\text{GHz} \simeq 50\text{mK} \simeq 4.13\mu\text{eV}$

$$T_{Qubit} \sim 10\text{mK} \sim 0.8\mu\text{eV}$$

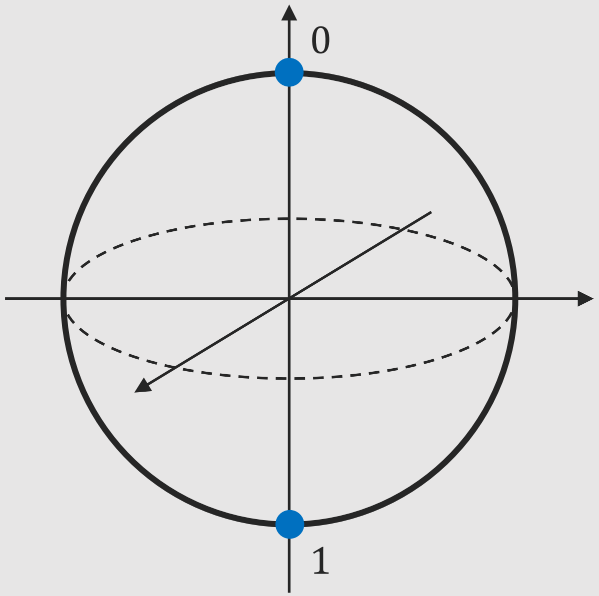
$$\Delta E = E_e - E_g \sim 10\text{GHz} \sim 40\mu\text{eV}$$

BASICS OF THE QUBIT SYSTEM

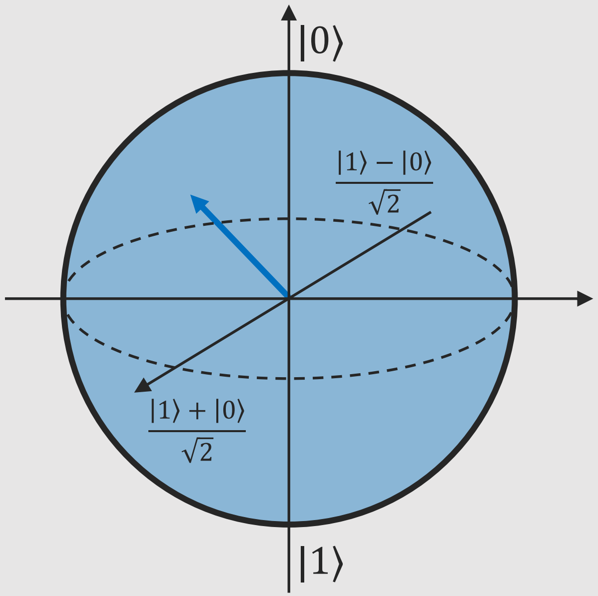
What is Qubit?

Qubit is an abbreviation for Quantum Bit.

It is **a quantum two level system whose states can be well manipulated**.



Classical Bit

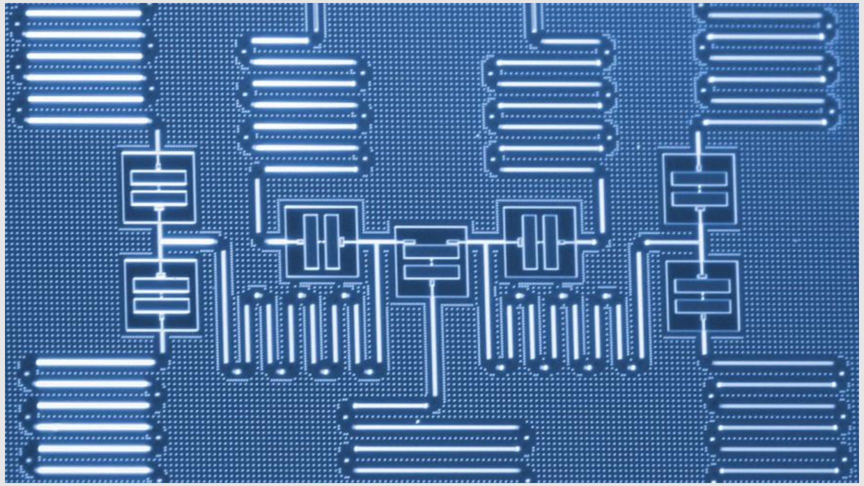


Qubit

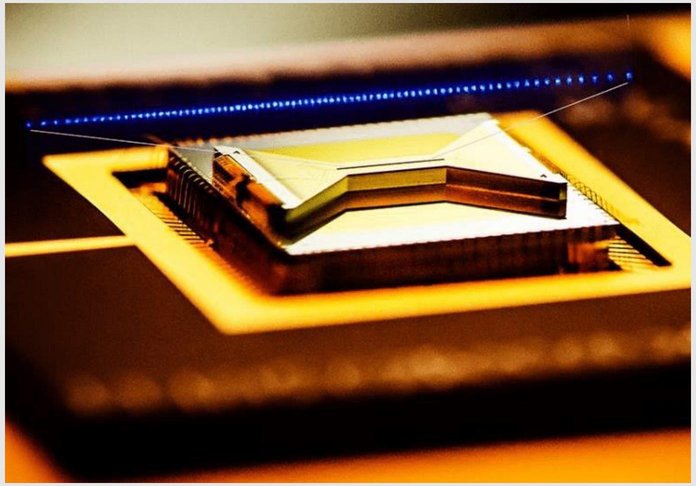
BASICS OF THE QUBIT SYSTEM

Various Platforms of Qubit Systems

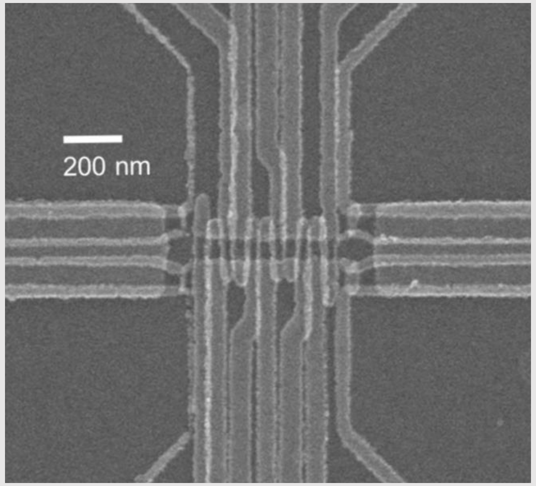
Superconducting circuit, Ion trap, Quantum dot, Neutral atom, NV center, Photonic devices, etc.



Superconducting Circuit



Ion Trap



Semiconductor Quantum Dot

BASICS OF THE QUBIT SYSTEM

Density Operator

Necessity of density operator arises from the question:

How to distinguish $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and mixture of $|0\rangle, |1\rangle$ with same proportion?

If we measure these states in the basis $\{|0\rangle, |1\rangle\}$, we can't distinguish them since the probability to measure $|0\rangle, |1\rangle$ is $50:50$.

However, if we set new basis $\left\{|0'\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}, |1'\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}\right\}$, probability to measure $|0'\rangle$ is 1 for the first case but 0.5 for the last case since $|0\rangle = \frac{|0'\rangle+|1'\rangle}{\sqrt{2}}, |1\rangle = \frac{|0'\rangle-|1'\rangle}{\sqrt{2}}$.

BASICS OF THE QUBIT SYSTEM

Density Operator

A good tools that can represent this difference is the density operator ρ.

$$\rho \equiv \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

Only when the state is pure, $\mathrm{Tr}\{\rho^2\} = 1$. (For the further reading, refer to Sakurai CH.3.4)

One important property of the density operator is that the trance of it is always 1.

$$\mathrm{Tr}\{\rho\} = \sum_j \langle\psi_j| \sum_i p_i |\psi_i\rangle\langle\psi_i| \, |\psi_j\rangle = \sum_{i,j} p_i \delta_{ij} = 1$$

From this fact, we can let $\rho = \mathbf{1} + \sum_i a_i \sigma_i$.

BASICS OF THE QUBIT SYSTEM

Bloch Sphere and Bloch Vector

In quantum two level system, it is possible to display information about all possible states in the 3-dim space only except the global phase.

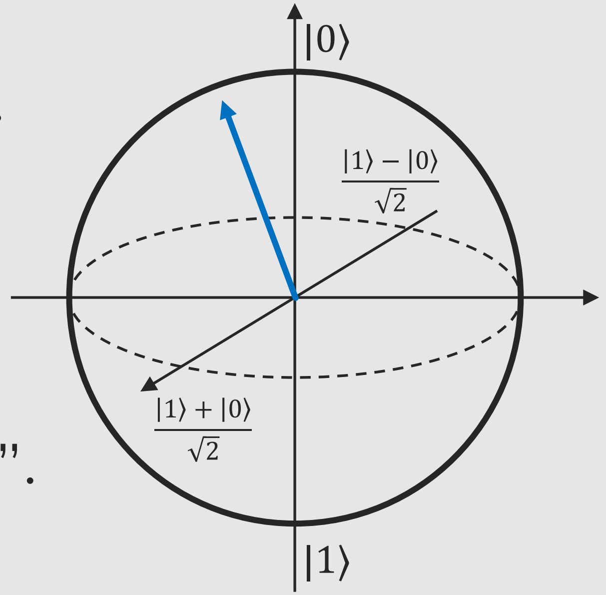
From $\rho = \mathbf{1} + \sum_i a_i \sigma_i$, we get 3-dim vector a_i which is called Bloch vector.

If the state is pure, Bloch vector is on the surface of the sphere.

If the state is not pure, norm of Bloch vector is smaller than 1.

Since the density matrix is observable, Bloch vector can be

Measured experimentally and this is called “state tomography”.



BASICS OF THE QUBIT SYSTEM

Open Quantum Mechanics

The theoretical base of the open quantum mechanics is quantum statistical mechanics.

Unlike conventional quantum mechanics where only consider isolated systems, open quantum mechanics is theory to explain open quantum system where it can interact with environments.

In open quantum mechanics, dynamics of the system is described by the density operator $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ and Master equation instead of $|\psi\rangle$ and Schrodinger equation.

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \sum_i \gamma_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\} \right)$$

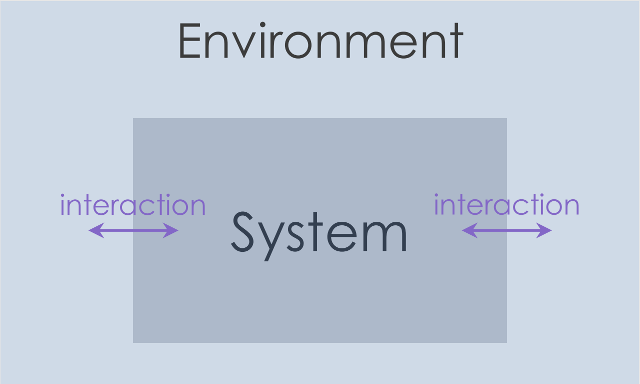
BASICS OF THE QUBIT SYSTEM

Open Quantum Mechanics

The energy of the whole universe is conserved. Thus, it can be described by the conventional quantum mechanics.

However, the energy of system isn't conserved, so open quantum mechanics must be applied.

Cf) Micro-canonical ensemble & Canonical ensemble

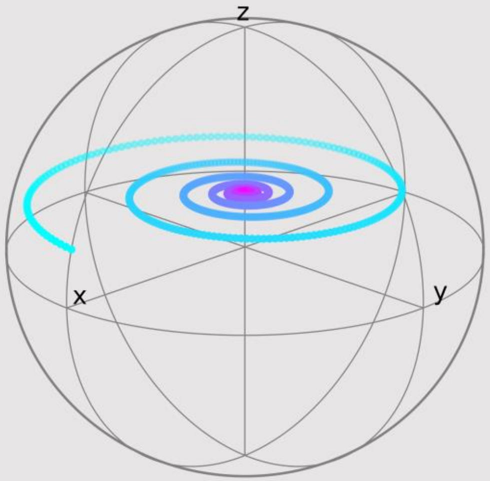


BASICS OF THE QUBIT SYSTEM

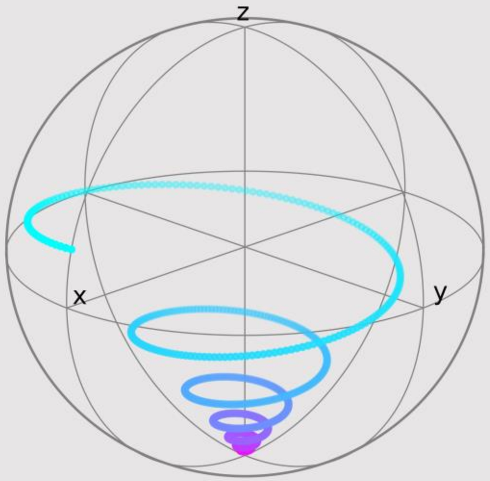
Open Quantum Mechanics

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \sum_i \gamma_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\} \right)$$

In open quantum mechanics, pure state can't last forever because of the interaction with environments. This mechanism is called decoherence.



Pure dephasing
$L = \sigma_+ \sigma_-$



Relaxation
$L = \sigma_-$

BASICS OF THE QUBIT SYSTEM

Open Quantum Mechanics (Summary)

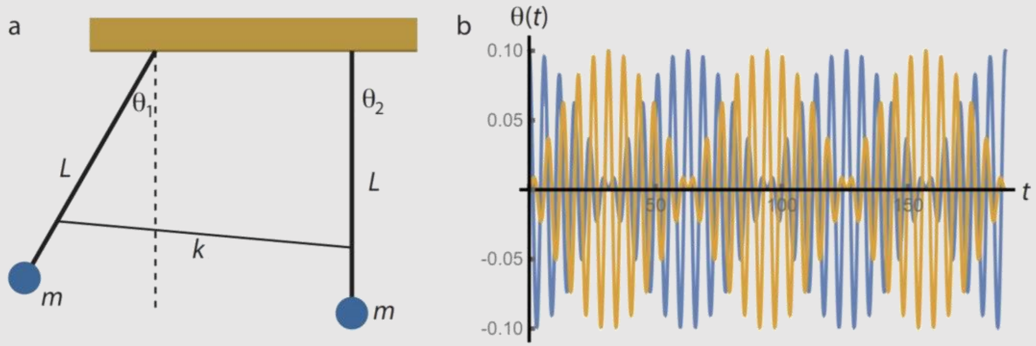
	Conventional Quantum Mechanics	Open Quantum Mechanics
State	$\vert\psi\rangle$	$\rho = \sum_i p_i \vert\psi_i\rangle\langle\psi_i\vert$
Sys./Env.	Isolated / Closed	Open
Coherence	Infinite	Finite
Operator	Hermitian	Non-Hermitian
Dynamics	$i\hbar\partial_t\vert\psi\rangle = H\vert\psi\rangle$	$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \sum_i \gamma_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\} \right)$

BASICS OF THE QUBIT SYSTEM

Quantum Coupling

In quantum mechanics, without coupling, dynamics can't be made.

The quantum coupling is basically equal to the classically coupled oscillator.



For some arbitrary two-level system (TLS) with basis $\{|0\rangle, |1\rangle\}$, Hamiltonian can be represented as 2×2 matrix. If there is no quantum coupling between two states, only diagonal terms (energy of each state) exist. However, if coupling exist, non-trivial off-diagonal terms arises, and this generates the dynamics of the quantum system.

$$H \doteq \begin{pmatrix} \langle 0|H|0\rangle & \langle 0|H|1\rangle \\ \langle 1|H|0\rangle & \langle 1|H|1\rangle \end{pmatrix} = \begin{pmatrix} \epsilon_0 & t \\ t & \epsilon_1 \end{pmatrix}$$

BASICS OF THE QUBIT SYSTEM

Quantum Coupling

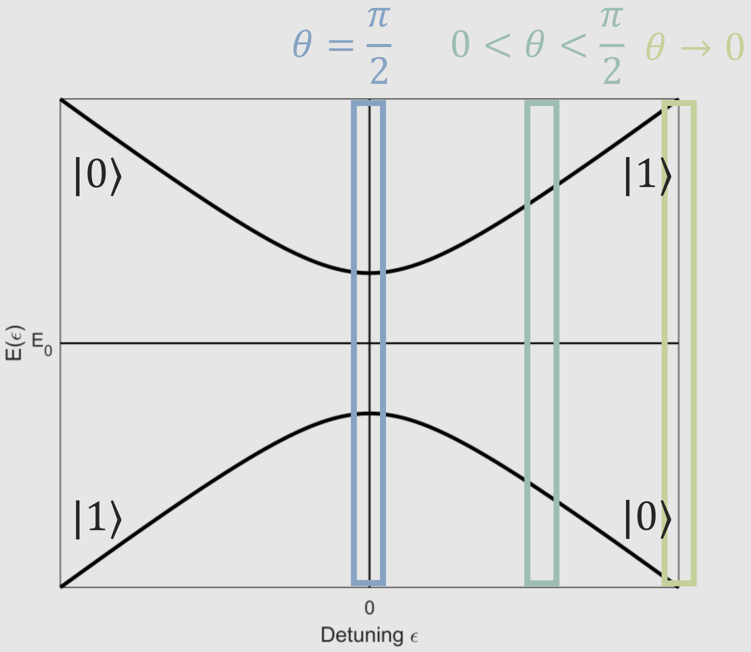
$$H \doteq \begin{pmatrix} \epsilon_0 & t \\ t & \epsilon_1 \end{pmatrix}, \ \epsilon \equiv \frac{\epsilon_1 - \epsilon_0}{2}$$

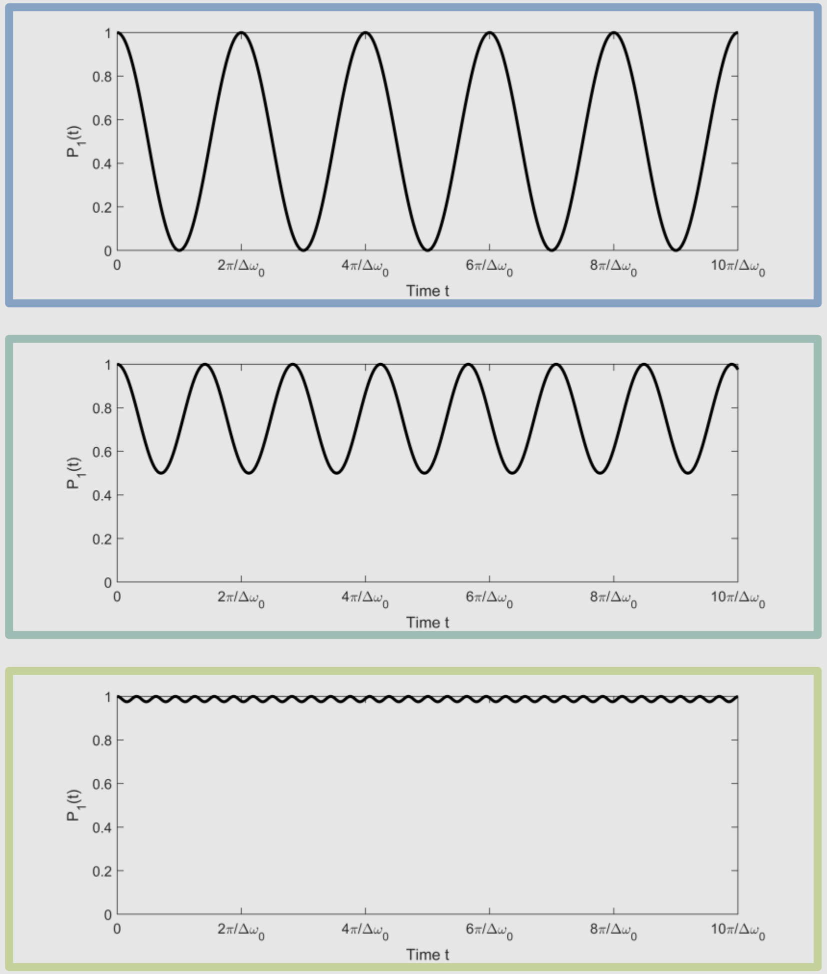
$$E_\pm = \pm\sqrt{\epsilon^2 + t^2}$$

$$|+\rangle = C_{\theta/2}|1\rangle + S_{\theta/2}|0\rangle$$

$$|-\rangle = S_{\theta/2}|1\rangle - C_{\theta/2}|0\rangle$$

where $t_\theta = t/\epsilon$





BASICS OF THE QUBIT SYSTEM

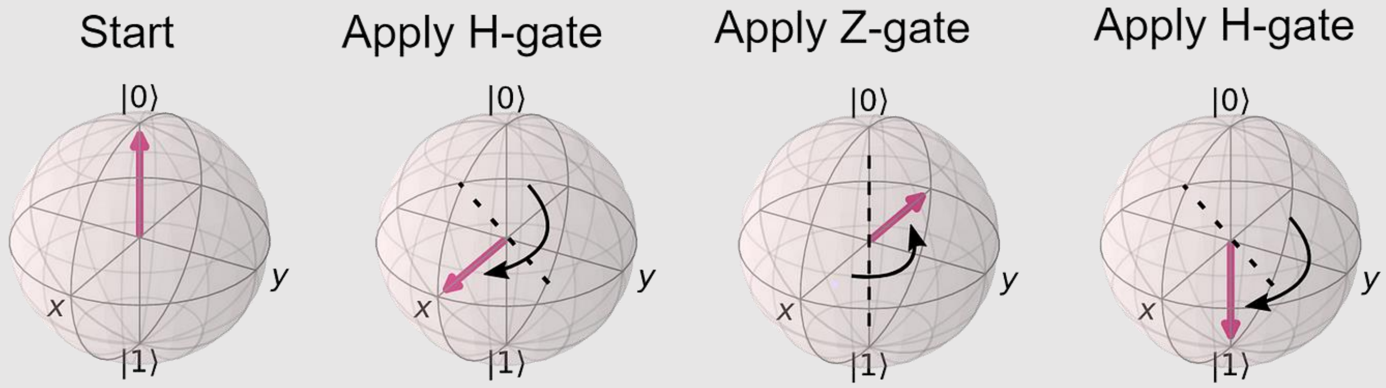
Quantum Gate

By making proper coupling between $|0\rangle$ and $|1\rangle$, we can make quantum gate.

Quantum gate is basically time evolution generated by Hamiltonian H.

$$\frac{H}{\hbar} = h_0\mathbf{1} + h_x\sigma_x + h_y\sigma_y + h_z\sigma_z = h_0\mathbf{1} + \boldsymbol{h}\cdot\boldsymbol{\sigma}$$

$$U(t) = e^{-iHt/\hbar} = e^{-ih_0t}e^{-i\boldsymbol{h}\cdot\boldsymbol{\sigma}} = e^{-ih_0t}e^{-\frac{2i|\boldsymbol{h}|t}{2}\hat{\boldsymbol{h}}\cdot\boldsymbol{\sigma}} \qquad \text{cf) } R_{\hat{\boldsymbol{n}}}(\phi) = e^{-i\frac{\phi}{2}\hat{\boldsymbol{n}}\cdot\boldsymbol{\sigma}}$$



BASICS OF THE QUBIT SYSTEM

Jaynes-Cummings Model

This is a Hamiltonian model for the 'two-level atom'-'photon' interaction.

$$H = \hbar\omega_\gamma\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger + a)(b^\dagger + b)$$

Here, if the coupling is not strong enough, we can neglect $a^\dagger b^\dagger$ and ab term. (This is called 'Rotating Wave Approximation' or RWA)

Therefore, it becomes

$$H_{JC} = \hbar\omega_\gamma\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger b + ab^\dagger)$$

In this Hamiltonian, basis is given as a coupled form $\{|e, n\rangle, |g, n+1\rangle\}$.

This Hamiltonian can be solved quite easily.

BASICS OF THE QUBIT SYSTEM

Jaynes-Cummings Model

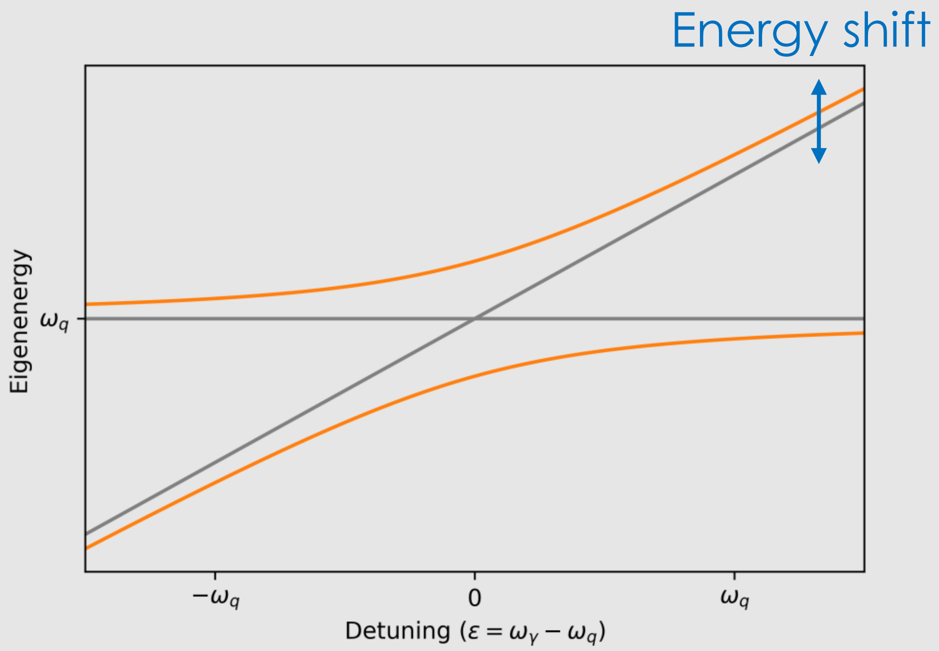
This can be further simplified if $\epsilon \gg 1$ (dispersive regime).

Using block-wise perturbation theory (Schrieffer-Wolff Transformation), we get

$$H_{disp} = \hbar(\boldsymbol{\omega_\gamma} + \boldsymbol{\chi\sigma_z})a^\dagger a + \frac{\hbar}{2}(\omega_q + \chi)\sigma_z + const.$$

where $\chi \equiv \frac{g^2}{\epsilon}$.

When compared to the original Hamiltonian, coupling term has disappeared. Instead, frequency of cavity and qubit has shifted.
Here note that **the frequency shift of the cavity is qubit state dependent**.



*AXION

Standard Model and Strong CP Problem

The Lagrangian for the QCD can be given as

$$\mathcal{L}_{QCD} = \cdots \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi \cdots$$

But this can be more generalized to

$$\mathcal{L}_{QCD} = \cdots \bar{\psi}(i\gamma^{\mu}D_{\mu} - m\boldsymbol{e^{i\theta'\gamma_5}})\psi \cdots$$

where θ' is arbitrary phase. This θ-term generates CP-violation.

Although non-trivial θ' is allowed by SM, but experimental results states that $\theta' \simeq 0$.

This fine-tuning problem is called **strong CP problem.**

*AXION

Peccei-Quinn (PQ) Theory

To solve strong CP problem, Peccei and Quinn introduced hypothetical boson called **axion**.

Researchers expect that axion could be highly stable and invisible (or, barely couples to other matters) because we can't find the evidence of its existence until now.

Due to this property, **axion is considered as good candidate for the cold dark matter**.

However, this implies **detection of axion requires ultra sensitive devices**.

*AXION

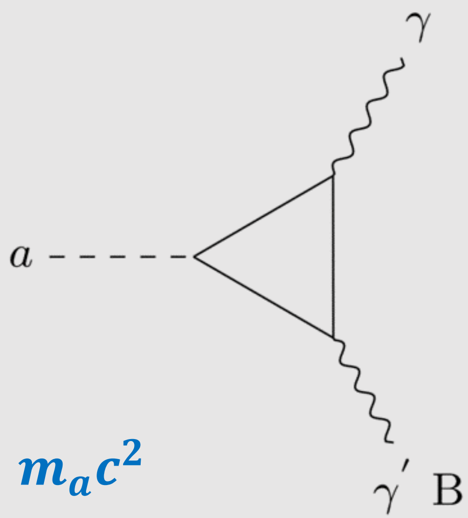
Axion-Photon Interaction

Through the process called Primakoff effect, **axion can produce photon** by interacting with strong magnetic field

$$\mathcal{L} = -\frac{g_{a\gamma\gamma}}{4} aF\tilde{F} = g_{a\gamma\gamma} a\boldsymbol{E} \cdot \boldsymbol{B}$$

There are two primary models for axion-photon interaction:

KSVZ model ($g_{a\gamma\gamma} = -0.97$), DFSZ model ($g_{a\gamma\gamma} = 0.36$)



Frequency of produced photon is equal to the mass of axion $\boldsymbol{\hbar\omega_\gamma \simeq m_a c^2}$

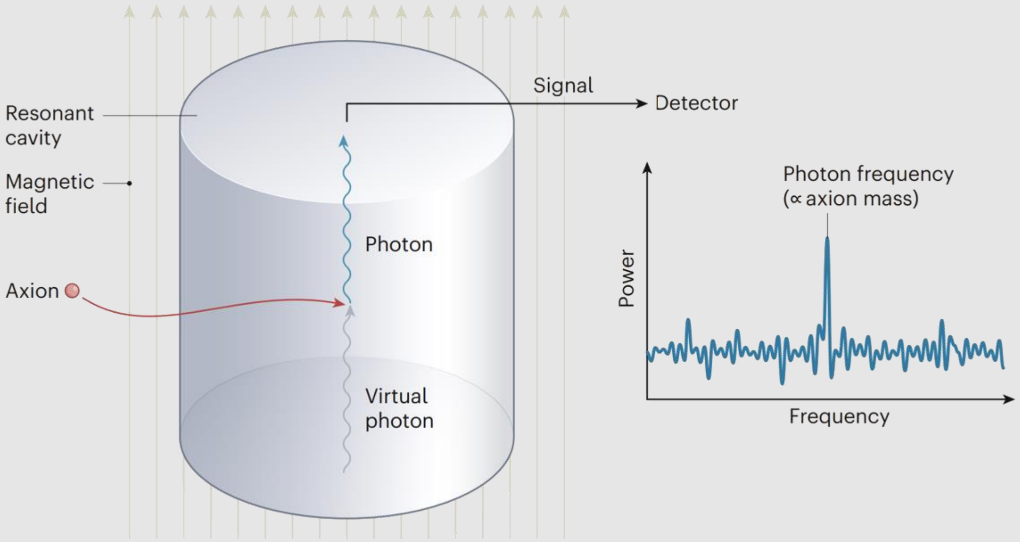
Mass of axion is expected to range from $1\mu\text{eV}$ to 1meV

Ref: I. G. Irastorza, *Nature* **590**, 226-227 (2021).

ADMX USING SC QUBIT SYSTEM

Axion Dark Matter Experiment (ADMX)

Using axion-photon interaction in strong static magnetic field, **we can detect the signal of axion**.



In ADMX, researchers use **microwave cavity** to accumulate EM power produced by axion-photon interaction. The power in resonance $\hbar\omega_c \simeq m_a c^2$ is given as below.

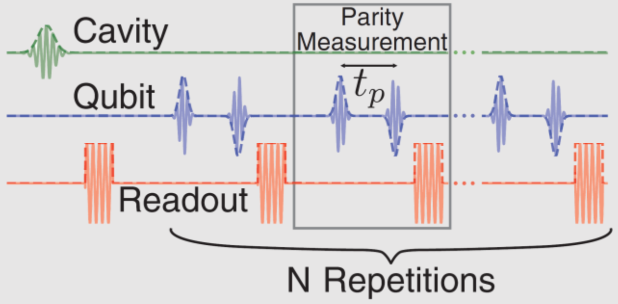
$$P_{sig} = \hbar\omega_c \dot{N} = (3 \times 10^{-26} W)\left(\frac{V}{1\text{L}}\right)\left(\frac{B}{1\text{T}}\right)^2\left(\frac{C_{nlm}}{0.4}\right)\left(\frac{g_{a\gamma\gamma}}{0.97}\right)^2\left(\frac{\rho_a}{0.45\ \text{GeV/cm}^3}\right)\left(\frac{f_a}{650\text{MHz}}\right)\left(\frac{Q}{50000}\right)$$

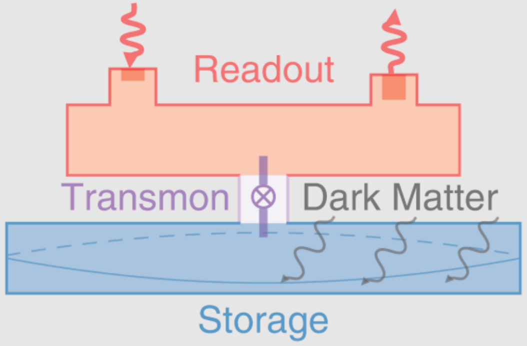
$$\dot{N} \simeq 0.44 s^{-1}$$

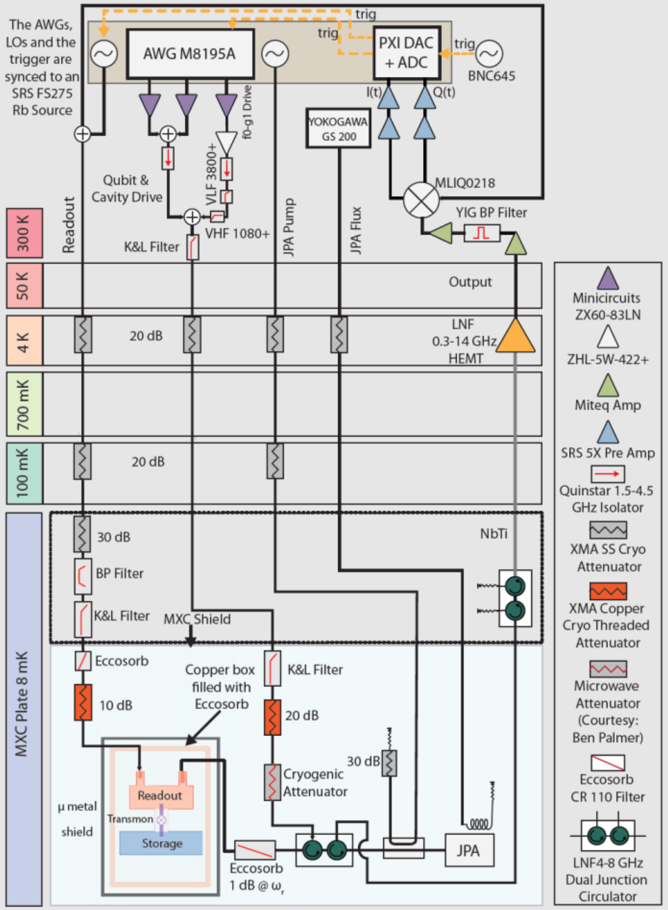
Ref: R. Khatiwada et. al., *Rev. Sci. Instrum* **92** 124502 (2021).

ADMX USING SC QUBIT SYSTEM

Experimental Setup



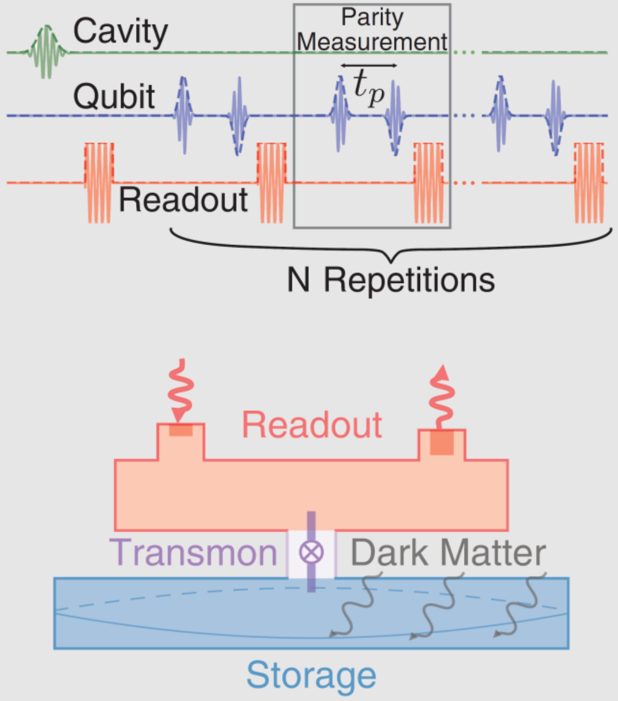


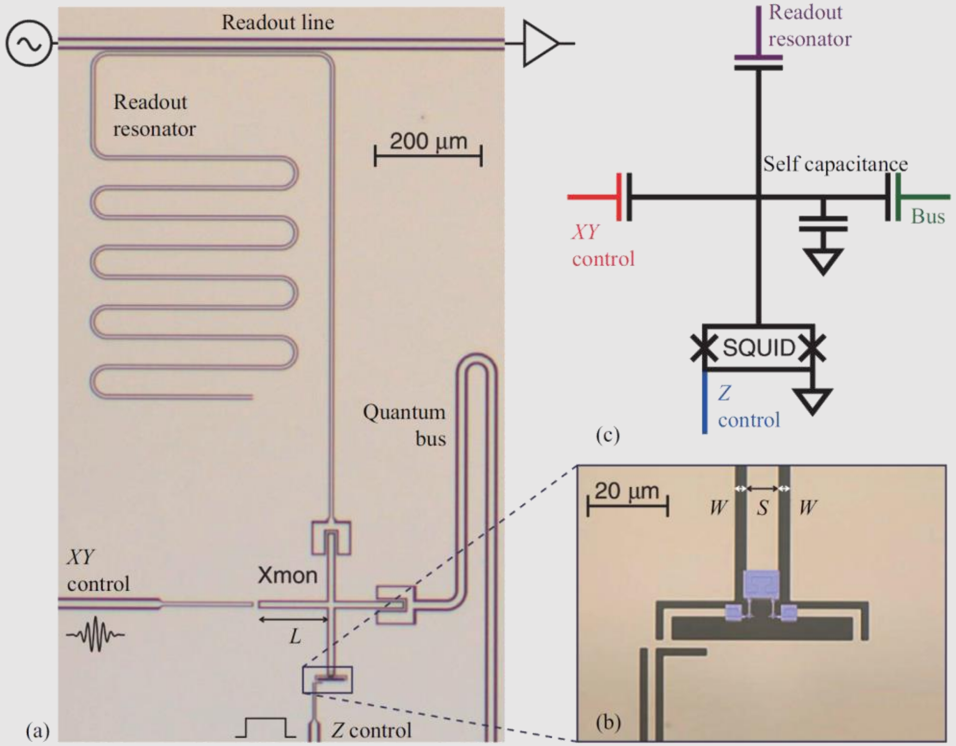


Ref: Akash V. Dixit et al., *PRL* **126**, 141302 (2021).

ADMX USING SC QUBIT SYSTEM

Experimental Setup





Ref: Akash V. Dixit et al., *PRL* **126**, 141302 (2021).

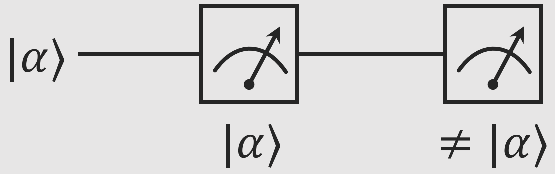
ADMX USING SC QUBIT SYSTEM

Quantum Non-Demolition (QND) Measurement

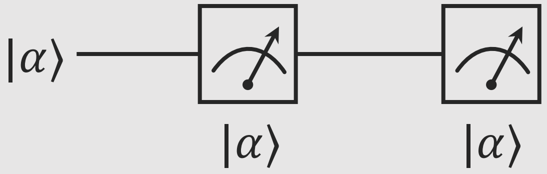
Conventional measurement using amplification of cavity photon signal destroys cavity state.

On the other hand, **QND measurement conserves Fock state**. Therefore, **repetitive measurement of the same state is possible**.

Iterative photon number measurement using QND enables exponential suppress of readout error with linear time cost.



Conventional Measurement



QND Measurement

Ref: Akash V. Dixit et al., *PRL* **126**, 141302 (2021).

ADMX USING SC QUBIT SYSTEM

Single-Photon Parity Readout

Using dispersive coupling of SC qubit and cavity, we can measure weather the cavity has even/odd number of photons.

$$|\psi\rangle = |g\rangle|0\rangle$$

$\downarrow$ $Y_{\pi/2}$

$$|\psi\rangle = \frac{|g\rangle+|e\rangle}{\sqrt{2}}|0\rangle$$

$\downarrow$ $U(\Delta t = \pi/\chi)$ by $H_{int} = -\chi a^{\dagger} a|e\rangle\langle e|$

$$|\psi\rangle = \frac{|g\rangle+|e\rangle}{\sqrt{2}}|0\rangle$$

$\downarrow$ $Y_{-\pi/2}$

$$|\psi\rangle = |g\rangle|0\rangle$$

$$|\psi\rangle = |g\rangle|1\rangle$$

$\downarrow$ $Y_{\pi/2}$

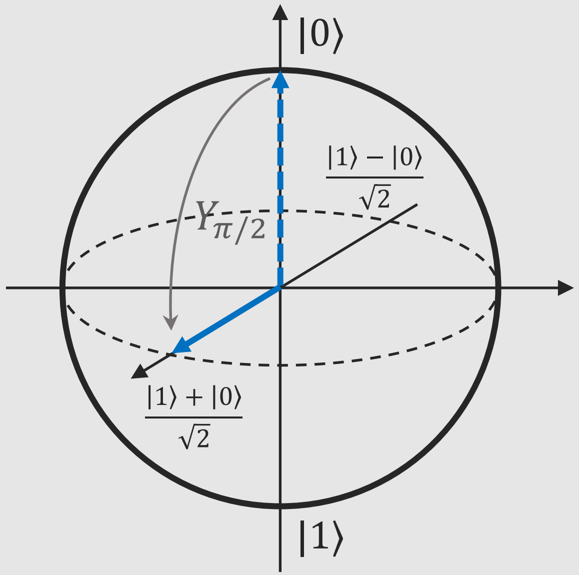
$$|\psi\rangle = \frac{|g\rangle+|e\rangle}{\sqrt{2}}|1\rangle$$

$\downarrow$ $U(\Delta t = \pi/\chi)$ by $H_{int} = -\chi a^{\dagger} a|e\rangle\langle e|$

$$|\psi\rangle = \frac{|g\rangle-|e\rangle}{\sqrt{2}}|1\rangle$$

$\downarrow$ $Y_{-\pi/2}$

$$|\psi\rangle = |e\rangle|1\rangle$$



Ref: L. Sun et al. *Nature* **511**, 7510, 444–448 (2014).

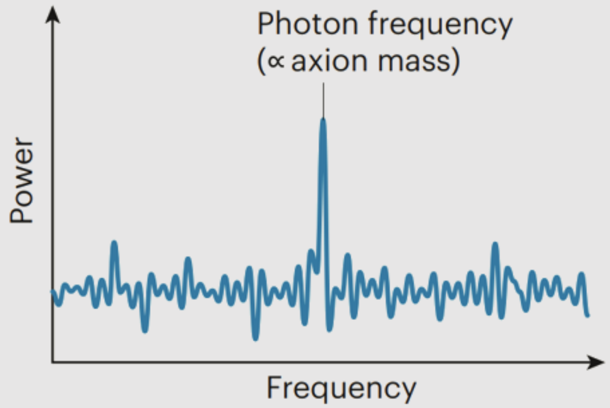
ADMX USING SC QUBIT SYSTEM

Single-Photon Parity Readout

The measurement must be done fast since the longer measurement results error induced by photon decay of cavity and decoherence of the qubit. In general, measurement time is $\sim$ns scale.

Noting $\dot{N} \simeq 0.44s^{-1}$, most of measured photon number would be $|0\rangle$ or $|1\rangle$. Thus, **if parity readout reads $|g\rangle$ / $|e\rangle$, cavity state is $|0\rangle$ / $|1\rangle$.**

By repeating measurement many times, accumulate data to remove background photon noise. Plot the average number of photon in cavity for varying cavity frequency. Then we can find the mass of axion if it exists.



ADMX USING SC QUBIT SYSTEM

Results and Limitations

The measurement procedure itself is not difficult but the biggest problem is that we don't know the exact mass of axion.

Thus, we must test wide frequency region($200\mathrm{MHz} - 2\mathrm{GHz}$) with high resolution.

Also, the signal of axion is so weak that many times of repetition is required to find the signal of axion out from the noise.

Consequently, **finding an axion takes an enormous amount of time**.

REFERENCE

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