Spin-correlation effects in tau-lepton pair induced by anomalous magnetic and electric dipole moments

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Outline of the talk

- Anomalous magnetic dipole moment (AMDM) and electric dipole moment (EDM) of τ lepton:
  - theory
  - experiment
- Weak magnetic and electric dipole moments of τ lepton
- Spin correlations in the τ-lepton pair:
  - $q\,\bar{q} \to \tau^-\tau^+$ (for LHC and FCC),
  - $e^-e^+ \to \tau^-\tau^+$ (for Belle II)
  - $\gamma + \gamma \to \tau^-\tau^+$ (for heavy-ion collisions and *pp* collisions)
- Weak dipole moments of τ lepton in *pp* collisions at the LHC

Magnetic and electric dipole moments of the τ

Electromagnetic vertex γff in a covariant form is

$$\Gamma^{\mu} = -ieQ_f \left\{ F_1(s)\gamma^{\mu} + \frac{\sigma^{\mu\nu} q_{\nu}}{2m_f} \left[i\, F_2(s) + \gamma_5\, F_3(s) \right] \right\}$$

where $s = q^2$ and q is the photon four-momentum.
At the real-photon point

$$F_2(0) = \frac{1}{2}(g_f - 2) \equiv a_f, \qquad F_3(0) = \frac{2m_f}{eQ_f} d_f, \qquad F_1(0) = 1$$

Motivations to study AMDM of the τ lepton

In the Standard Model (SM), AMDM a for τ lepton is calculated with high accuracy [S. Eidelman, M. Passera, 2007]:

$$a_{SM} = a_{QED} + a_{EW} + a_{hadron+light-by-light} = 0.00117721(5)$$

Effects of New Physics (NP) due to new heavy particles in the loops can be

$$a_{NP} = \mathcal{C}\, \frac{m_\tau^2}{\Lambda^2}, \qquad \mathcal{C} \sim \mathcal{O}(1),\ \mathcal{O}(\tfrac{\alpha}{\pi}),\ \text{or}\ \mathcal{O}(\tfrac{\alpha^2}{\pi^2})$$

[e.g. W. Marciano, 1994, 1995], where Λ is the scale of NP.

Compared to the muon, for τ lepton effects of NP can be enhanced by a factor of $m_\tau^2/m_\mu^2 \approx 280$!

Magnetic dipole moment of the τ: experiment

Measurements for the τ are extremely difficult because of its very short lifetime $\tau = 2.903(5) \times 10^{-13}\,\mathrm{s}$. This does not allow applying methods used in the electron and muon "$g-2$" experiments.

The limit on AMDM a was obtained by DELPHI collaboration at LEP2 in 2003 from the $e^+e^- \to e^+e^-\tau^+\tau^-$ cross section. At 95 % CL

$$a = (-0.052, +0.013)$$

Recently ATLAS Collaboration presented new constraints [PRL 131, 151802 (2023)] from the $Pb + Pb \to Pb\,(\gamma\gamma \to \tau^-\tau^+)\,Pb$ collisions at $\sqrt{s_{NN}} = 5.02$ TeV:

$$-0.057 < a < 0.024$$

and also CMS Collaboration [PRL 131, 151803 (2023)] presented the value:

$$a = 0.001^{+0.055}_{-0.089}$$

Also one should mention very recent CMS result with unprecedented precision [CMS PAS SMP-23-005]:

$$a = 0.0009^{+0.0032}_{-0.0031}$$

The experimental uncertainties are larger than the SM prediction $a_{SM} = 0.00117721(5)$.

Electric dipole moment of the τ: theory and experiment

EDM of a lepton can take nonzero values only if the parity P, time reversal T, and CP symmetries are violated. This induces great interest to EDM of the τ lepton.

Lepton EDM is extremely small in the SM because it originates from the 4-loop diagrams [I. Khriplovich, M. Pospelov, 1991], and additionally due to the smallness of CP violation in the CKM matrix.

Theoretical estimation of the τ lepton EDM:

$$d_{SM} \lesssim 3.5 \cdot 10^{-35}\, e \cdot cm$$

The experimental constraints are obtained at KEKB e^+e^- collider [Belle Collaboration, JHEP 04 (2022) 110]:

$$\mathrm{Re}(d) = (-0.62 \pm 0.63) \times 10^{-17} \mathrm{e} \cdot \mathrm{cm}, \quad \mathrm{Im}(d) = (-0.40 \pm 0.32) \times 10^{-17} \mathrm{e} \cdot \mathrm{cm}$$

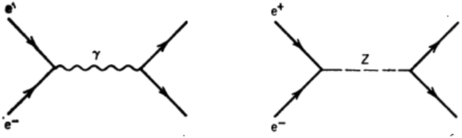
which are 18 orders of magnitude (!) off the SM estimate. Also we mention the CMS measurement [CMS PAS SMP-23-005]:

$$-1.7 < d < +1.7 \times 10^{-17}\ \mathrm{e} \cdot \mathrm{cm}$$

In any case, extremely small SM value of EDM is hardly reachable in experiments. Any observation in experiments of τ EDM will be indication of CP violation beyond the SM.

High energies $\sqrt{s} \sim M_Z$: weak dipole moments

At high energies Z-boson interaction with fermions becomes important



The $Z\tau^-\tau^+$ interaction vertex has the structure including additional contributions

$$\Gamma^{\mu}_{Z}(q) = -i\frac{g_Z}{2}\left\{\gamma^{\mu}(C_V - \gamma_5 C_A) + \frac{\sigma^{\mu\nu}q_{\nu}}{2m_{\tau}}\left[iX(s) + \gamma_5 Y(s)\right]\right\},$$

where $g_Z = e/(s_W c_W) = 2M_Z(\sqrt{2}G_F)^{1/2}$, $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, θ_W is the weak mixing angle, G_F is Fermi constant and vector and axial-vector constants for τ are $C_V = -1/2 + 2s_W^2$ and $C_A = -1/2$.

$X(s)$ is called weak anomalous magnetic (CP conserving) form-factor, while $Y(s)$ is weak electric (CP violating) form-factor.
On the Z-boson mass shell they are related to the weak dipole moments

$$X(M_Z^2) = a_w, \qquad Y(M_Z^2) = d_w$$

which are somewhat similar to electromagnetic dipole moments a and $\frac{2m_\tau}{e}d$.

High energies $\sqrt{s} \sim M_Z$: weak dipole moments

What is known at present about weak dipole moments of τ lepton?

Experiment ALEPH [Eur. Phys. J. C 30, 291-304, 2003] obtained the following constraints on the real and imaginary parts of the weak magnetic a_w and electric d_w dipole moments

$$|\mathrm{Re}(a_w)|_{exp} < 0.96 \times 10^{-3}, \qquad |\mathrm{Im}(a_w)|_{exp} < 2.23 \times 10^{-3},$$
$$|\mathrm{Re}(d_w)|_{exp} < 0.76 \times 10^{-3}, \qquad |\mathrm{Im}(d_w)|_{exp} < 1.69 \times 10^{-3}.$$

The SM prediction [Bernabeu et al. Nucl. Phys. B 436, 474–486, 1995] for the weak magnetic moment is

$$a_w^{(SM)} = -(1.77 + i\,0.51) \times 10^{-6}$$

while prediction for the weak electric moment is not available.

Spin effects in $f\bar{f}$ production of τ-lepton pair

Consider quark-antiquark (or electron-positron) annihilation to a pair of polarized τ leptons

$$f(k_1) + \bar{f}(k_2) \to \tau^-(p_-) + \tau^+(p_+), \qquad f = (q,\, e^-,\, \mu^-)$$

with the polarization four-vectors of the τ^- and τ^+ in their rest frames:

$$S^-_{rest} = (0,\, \vec{S}^-), \quad and \quad S^+_{rest} = (0,\, \vec{S}^+)$$

Then the cross section in the CM frame can be expressed through these polarizations:

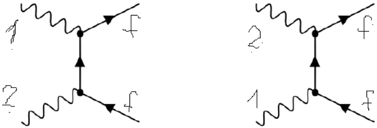
$$\frac{d\sigma}{d\Omega}(f\,\bar{f} \to \tau^-\tau^+) = \frac{d\sigma}{d\Omega}(f\,\bar{f} \to \tau^-\tau^+)\Big|_{\mathrm{unpol}}$$
$$\times \frac{1}{4}\Big(1 + \sum_{i=1}^{3} r_{i,4}\, S^-_i + \sum_{j=1}^{3} r_{4,j}\, S^+_j + \sum_{i,j=1}^{3} r_{i,j}\, S^-_i\, S^+_j\Big), \qquad (i, j = 1,\, 2,\, 3)$$

15 coefficients carry information on the dipole moments of the τ lepton:
(a) 6 elements – polarizations of $\tau^\mp$ leptons $\mathcal{P}_i(\tau^-) = r_{i,4}$ and $\mathcal{P}_j(\tau^+) = r_{4,j}$,
(b) 9 elements $r_{i,j}$ called spin-spin correlations (they are also related to entanglement).

Spin correlations in two-photon production of τ leptons

Similar approach is used for production of τ pairs in photon-photon collisions

$$\gamma(k_1) + \gamma(k_2) \to \tau^-(p_-) + \tau^+(p_+), \qquad (\textit{for almost real photons } k_1^2 \approx 0,\ k_2^2 \approx 0)$$

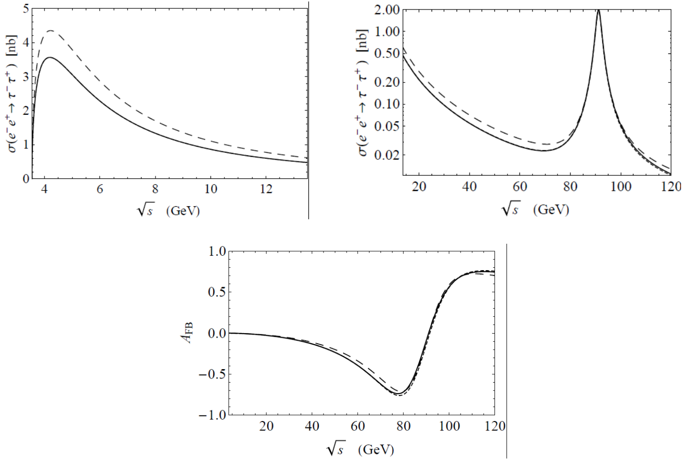


$$\frac{d\sigma}{d\Omega}(\gamma\gamma \to \tau^-\tau^+) = \frac{d\sigma}{d\Omega}(\gamma\gamma \to \tau^-\tau^+)\Big|_{\rm unpol} \frac{1}{4}\Big(1 + \sum_{i,j=1}^{3} r_{i,j}^{\gamma\gamma}\, s_i^- s_j^+\Big)$$

with spin correlation matrix $r_{i,j}^{\gamma\gamma}$.

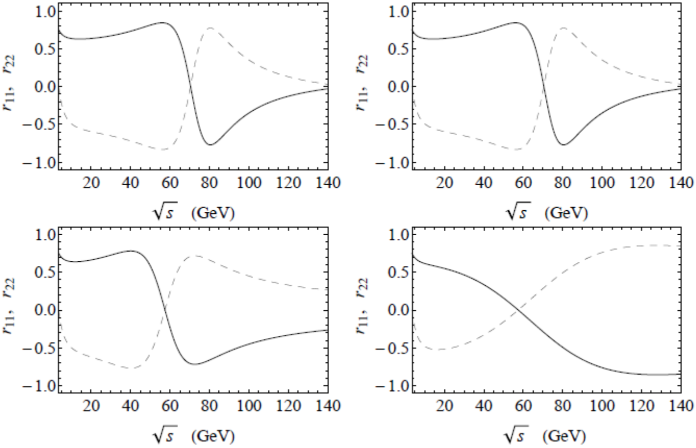
Spin correlations are expected to be more sensitive to the dipole moments than the cross section for unpolarized τ leptons, and can reduce limits on a, d or a_w, d_w.

Cross section and FB asymmetry in $e^- e^+ \to \tau^- \tau^+$



Cross section and forward-backward asymmetry in $e^- e^+ \to \tau^- \tau^+$ near threshold (for Belle) and at large energies.
Solid lines - SM, dashed - magnetic FF $\mathrm{Re}(F_2(s)) = 0.1$, dotted - weak magnetic FF $\mathrm{Re}(X(s)) = 0.1$.

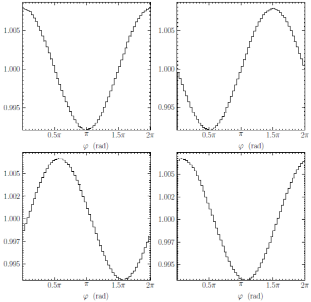
Transverse-transverse spin correlations in $q\bar{q} \to \tau^- \tau^+$



Transverse-transverse spin correlation elements: r_{11} (solid lines) and r_{22} (dashed lines) for the $u\,\bar{u}$ (top) and $d\,\bar{d}$ (bottom) in the SM.
The angle θ of quark vs τ^- is $\pi/3$ in the left plots and $2\pi/3$ in the right plots.

Weak dipole moments at Z-boson peak

It is not possible to measure directly $\tau^{\pm}$ polarizations, but only via measurement of the $\tau^{\pm}$ decay products.
We choose the two-step process: $e^- + e^+ \to \tau^- + \tau^+ \to \pi^- + \pi^+ + \nu_\tau + \bar{\nu}_\tau$, and construct observable which is acoplanarity angle φ between the two planes: one is $e^-\tau^-$ reaction plane and another is plane built on $\pi^-\tau^-$.



Ratio of number of events with/without weak dipole moments as a function of acoplanarity angle φ, the energy $\sqrt{s} = M_Z$.
Values of weak form-factors: top left – $\mathrm{Re}(a_w) = 4 \times 10^{-4}$, top right – $\mathrm{Re}(d_w) = 4 \times 10^{-4}$, bottom left – $\mathrm{Im}(a_w) = 4 \times 10^{-4}$, and bottom right – $\mathrm{Im}(d_w) = 4 \times 10^{-4}$

The LHC: spin correlations in the τ-lepton pair

In the condition of experiments at the LHC we need to weigh individual $q_f + \bar{q}_f \to \tau^- + \tau^+$ cross sections with the PDFs for various quark/antiquark flavors:

$$\frac{d\sigma(p\,p \to X^-\,X^+\,\nu_\tau\,\bar{\nu}_\tau + \ldots)}{dY_Z\,dy^*\,dM^2\,d\Omega_-\,d\Omega_+} \sim$$

$$\sim \sum_{q=u,d,s,\ldots} \Big[x_1\,F_q(x_1,\,M^2)\,x_2\,F_{\bar{q}}(x_2,\,M^2) \sum_{i,j=1}^{4} R_{i,j}^{(q\bar{q})}(M,\theta^*)\,h_i^-\,h_j^+$$

$$+\,x_1\,F_{\bar{q}}(x_1,\,M^2)\,x_2\,F_q(x_2,\,M^2) \sum_{i,j=1}^{4} \tilde{R}_{i,j}^{(q\bar{q})}(M,\pi-\theta^*)\,h_i^-\,h_j^+ \Big]$$

$$\sim 1 + \sum_{i=1}^{3} r_{i,4}^{(pp)} h_i^- + \sum_{j=1}^{3} r_{4,j}^{(pp)} h_j^+ + \sum_{i,j=1}^{3} r_{i,j}^{(pp)}\,h_i^-\,h_j^+$$

in terms of the PDFs $F_{q/\bar{q}}(x,\,M^2)$ and parton fractions

$$x_1 = \frac{M}{\sqrt{s}}\,\exp\,(Y_Z), \qquad x_1 = \frac{M}{\sqrt{s}}\,\exp\,(-Y_Z), \qquad M \text{ is invariant mass of tau pair}$$

In the proton-proton collision we introduce effective spin-correlation elements and τ polarization:

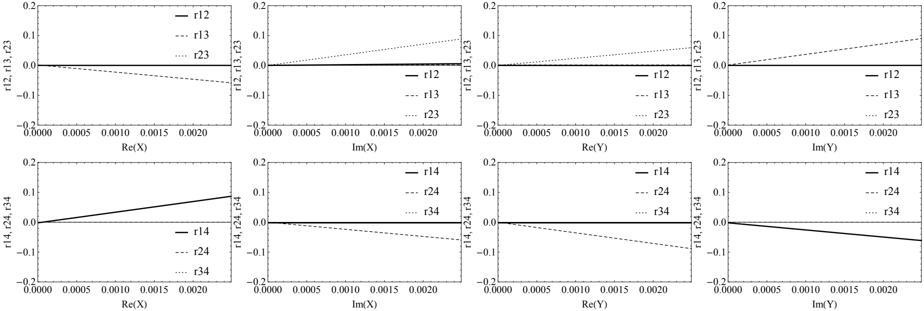
$$r_{i,j}^{(pp)}, \qquad r_{i,4}^{(pp)}, \qquad r_{4,j}^{(pp)}$$

Here h_i^- and h^+ are the so-called *polarimetric vectors* depending on the decay channel of τ^- and τ^+ leptons, respectively. For example, for $\tau^- \to \pi^- \nu_\tau$ one has $\vec{h}^- = \alpha_- \vec{n}_{\pi^-}$ and $\alpha_- = 1$.

The LHC: dipole moment contributions at Z-boson peak

Preliminary results for proton-proton energy $\sqrt{s} = 13.6$ TeV and τ-pair invariant mass $M = M_Z$.

The rapidity of τ pair is chosen $Y_Z = 2$, and scattering angle in the $q\bar{q} \to \tau^- \tau^+$ CM frame is 60°.
The most sizable correlations are **transverse-longitudinal**: one τ is polarized perpendicularly to direction of its movement OZ, another one – along OZ, or individual polarizations of τ perpendicular to OZ axis.



Dependence of $r_{ij}^{(pp)}$ on the real and imaginary parts of a_w and d_w.

Conclusions

- Formalism for calculation of effects of electromagnetic and weak dipole moments of the τ lepton in simulations of events in the processes: $q\bar{q} \to \tau^-\tau^+$, $e^-e^+ \to \tau^-\tau^+$, and $\gamma\gamma \to \tau^-\tau^+$, with the τ decays, is developed.
- It is prepared to work with Monte Carlo generators: `KKMC` for $e^-e^+ \to \tau^-\tau^+$ and `TauSpinner` for $pp \to \tau^-\tau^+$. They can be used at the Belle II energies ($\sqrt{s} = 10.58$ GeV) and at higher energies of the LHC and Future Circular Collider.
- Contributions from dipole moments are included on top of precise calculation in the Standard Model. In particular, electro-weak radiative corrections including WW- and ZZ-box diagrams are accounted for in framework of the Improved Born Approximation [D. Bardin et al. Comp. Phys. Comm. 133, 229 (2001); E. Richter-Was, Z. Was. Eur. Phys. J. C 74, 3177 (2014)]
- Calculations of observables in conditions of the LHC experiments are in progress.

Publications:

Sw. Banerjee, A.Yu. Korchin, Z. Was. Phys. Rev. D 106 (2022) 11, 113010
Sw. Banerjee, A.Yu. Korchin, E. Richter-Was, Z. Was. Phys. Rev. D 109 (2024) 1, 013002
A.Yu. Korchin, Acta Phys. Pol. B (2024) (to be published)
A.Yu. Korchin, E. Richter-Was, Yu. Volkotrub, Z. Was (in preparation)

Thank you for attention!