Semileptonic decays in lattice QCD

Alejandro Vaquero

Universidad de Zaragoza

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~~Semileptonic decays~~ $B \to D^* \ell \nu$ in lattice QCD

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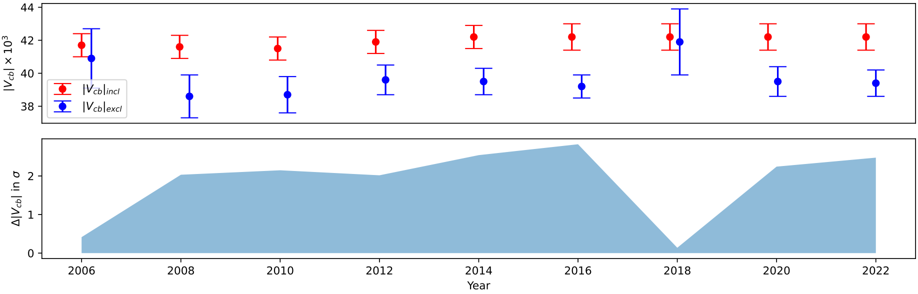
The motivation

The CKM matrix



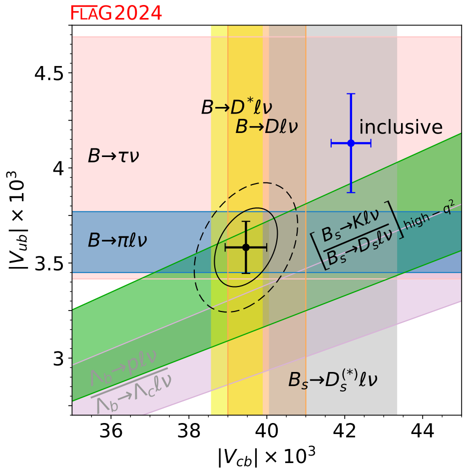
- Current values (PDG 2024):

$$|V_{cb}|_{\text{excl}} \times 10^{-3} = 39.8(6)$$

$$|V_{cb}|_{\text{incl}} \times 10^{-3} = 42.2(5)$$

- The 3σ difference between these two values shows that we have not improved much

Motivation: CKM matrix elements

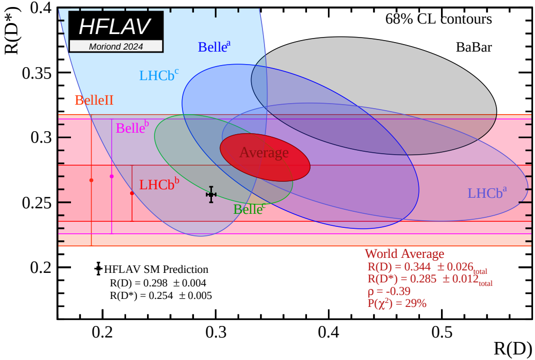


Strong arguments disfavoring new physics

Phys. Rev. Lett. 114, 011802 (2015)

Motivation: Tensions in LFU ratios

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current $\approx 3.3\sigma$ combined tension with the SM (HFLAV)
  - Tension in $R(D) \approx 1.6\sigma$ Tension in $R(D^*) \approx 2.5\sigma$

The theory

Semileptonic B decays on the lattice: Exclusive $|V_{cb}|$

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \underbrace{K_1(w, m_\ell\approx 0)}_{\text{Known factors}}\underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}}|V_{cb}|^2, \quad w = v_{D^*}\cdot v_B$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative
- Can use effective theories (HQET) to say something about $\mathcal{F}$
  - Separate light (non-perturbative) and heavy degrees of freedom as $m_Q\to\infty$
  - $\lim_{m_Q\to\infty}\mathcal{F}(w)=\xi(w)$, which is the Isgur-Wise function
  - **We don't know what $\xi(w)$ looks like, but we know $\xi(1)=1$**
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- Reduction in the phase space $(w^2-1)^{\frac{1}{2}}$ limits experimental results at $w\approx 1$
  - Need to extrapolate $|V_{cb}|^2\,|\eta_{ew}\mathcal{F}(w)|^2$ to $w=1$
  - This extrapolation is done using well established parametrizations

Semileptonic B decays on the lattice: Universality ratios

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \left[\underbrace{K_1(w,m_\ell)}_{\text{Known factors}}\underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}} + \underbrace{K_2(w,m_\ell)}_{\text{Known factors}}\underbrace{|\mathcal{F}_2(w)|^2}_{\text{Theory}}\right] \times |V_{cb}|^2$$

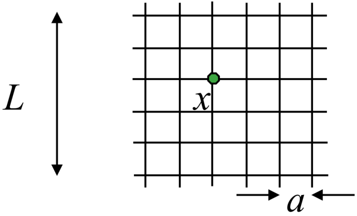
- The amplitudes $\mathcal{F}$,$\mathcal{F}_2$ must be calculated in the theory
- Since $K_2(w,0)=0$, $\mathcal{F}_2$ only contributes significantly with the τ
- Knowing these amplitudes, one can extract $|V_{cb}|$ from experiment
    - It is possible to extract $R(D^*)$ without experimental data!

$$R(D^*) = \frac{\int_1^{w_{\mathrm{Max},\tau}} dw\,\left[K_1(w,m_\tau)\,|\mathcal{F}(w)|^2 + K_2(w,m_\tau)\,|\mathcal{F}_2(w)|^2\right] \times \cancel{|V_{cb}|^2}}{\int_1^{w_{\mathrm{Max}}} dw\,\left[K_1(w,0)\,|\mathcal{F}(w)|^2\right] \times \cancel{|V_{cb}|^2}}$$

- $|V_{cb}|$ cancels out

Semileptonic B decays on the lattice: Introduction to Lattice QCD

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f \left(\gamma^\mu D_\mu + m_f\right) \psi_f + \frac{1}{4}\mathrm{tr} F_{\mu\nu} F^{\mu\nu}$$



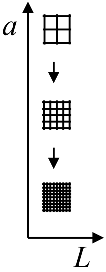
- Discretize space-time in a computer
  - Finite lattice spacing a
  - Finite spatial volume L
  - Finite time extent T

- Perform simulations in an unphysical setup and approach the physical limit
  - Enlarge the volume and reduce a
  - Quark masses $\Longrightarrow$ Pion masses (hadrons are matched)
  - Number of sea quarks $n_f = 2+1, \quad 2+1+1, \quad 1+1+1+1\ldots$

Semileptonic B decays on the lattice: Introduction to Lattice QCD

The systematic error analysis is based on **EFT** descriptions of QCD
The EFT description:

- provides functional form for different extrapolations (or interpolations)
- can be used to construct improved actions
- can estimate the size of the systematic errors



In order to keep the systematic errors under control we must repeat the calculation for several lattice spacings, volumes, light quark masses... and use the EFT to extrapolate to the physical theory

- Heavy quark treatment in Lattice QCD
    - For heavy quarks $(m_Q > \Lambda_{QCD})$, discretization errors typically grow as $\sim a^2 m_Q^2$
    - Two treatments
        - EFTs: Physical heavy masses, but requires matching and renormalization
        - Same action as the light quarks: Unphysical heavy masses, requires extrapolation
    - Not all actions perform equally well
        - Typical action $\sim a^2 m_Q^2$ VS HISQ action $\sim \alpha_s a^2 m_Q^2$

Semileptonic B decays on the lattice: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,\left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\ \ \rho\sigma}v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,\left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

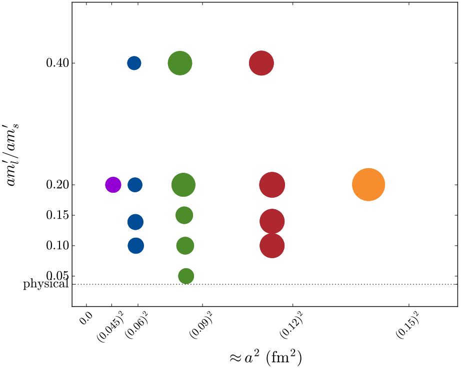
$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}\left(1+w\right)\boldsymbol{h_{A_1}}(w) - v_B^\nu\left(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w)\right)\right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$
- We can calculate the h_X directly from the lattice

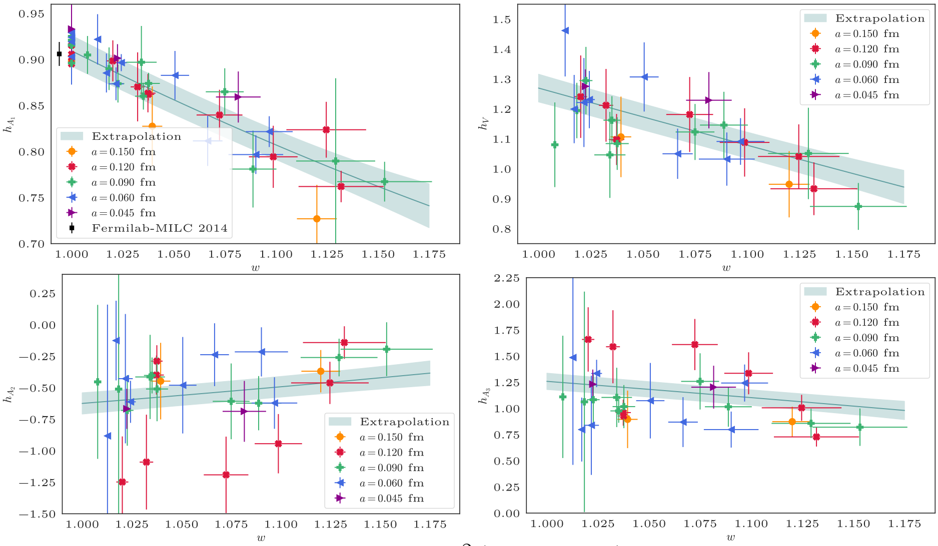
The calculation

$B \to D^*\ell\nu$: Setup

- Using 15 $N_f = 2+1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action
- Lightest $m_\pi \approx 180$ MeV

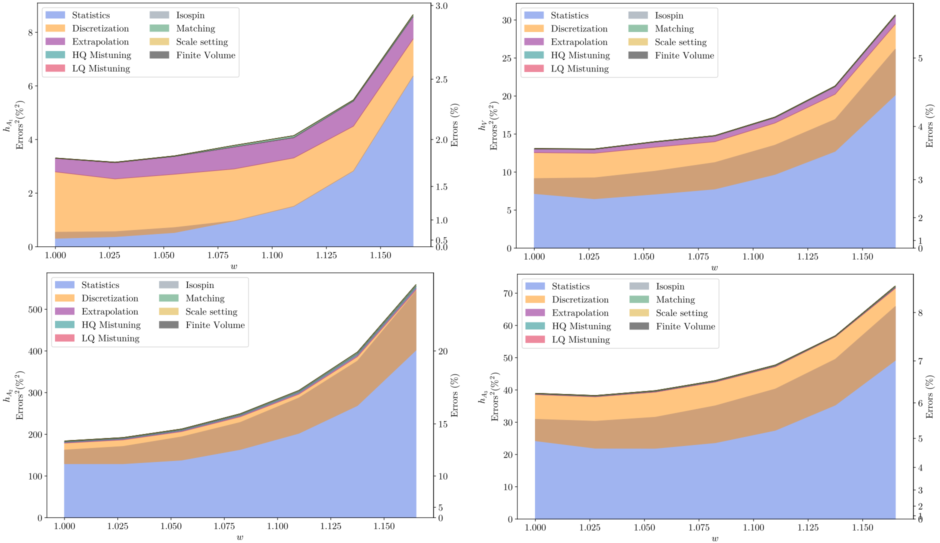


$B \to D^* \ell \nu$: Chiral-continuum extrapolation



Combined fit $\chi^2/\text{dof} = 85.2/95$

$B \to D^*\ell\nu$: Chiral-continuum extrapolation



Largest systematic errors come from discretization

$B \to D^*\ell\nu$: z expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w)h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j$$

$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

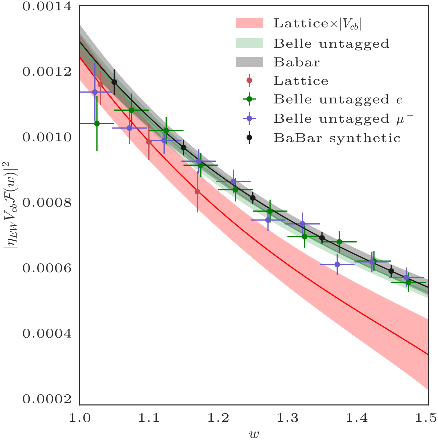
$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- Constraint $(1+w)m_B^2(1-r)\mathcal{F}_1(z=z_{\text{Max}}) = (1+r)\mathcal{F}_2(z=z_{\text{Max}})$
- BGL (weak) unitarity constraints

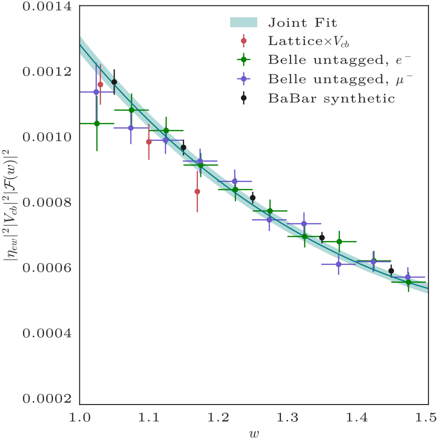
$$\sum_j a_j^2 \le 1, \qquad \sum_j b_j^2 + c_j^2 \le 1, \qquad \sum_j d_j^2 \le 1$$

$B \to D^* \ell \nu$: BGL fits

Separate fits



Joint fit

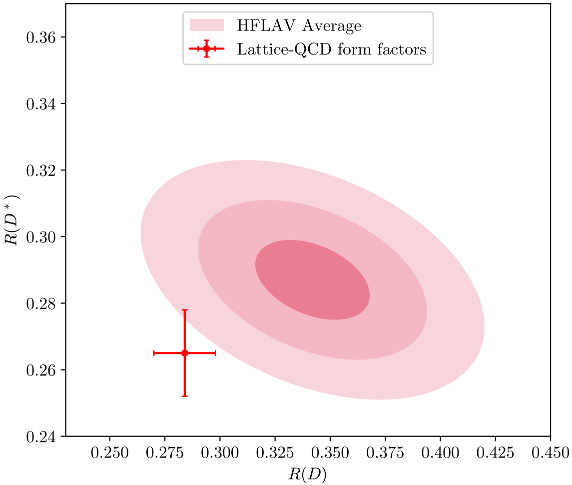


Fit	Lattice	Exp	Lat + Belle	Lat + BaBar	Lat + Exp
χ^2/dof	0.63/1	104/76	111/79	8.50/4	126/84

Unblinded, final result $|V_{cb}| = 38.40(78) \times 10^{-3}$

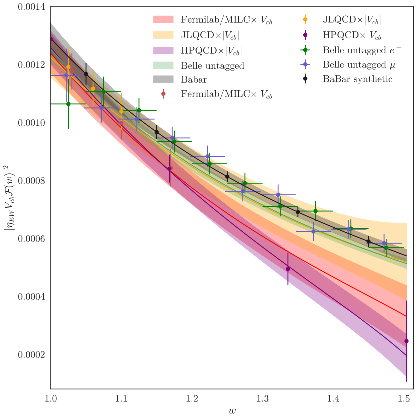
$B \to D^*\ell\nu$: $R(D^*)$

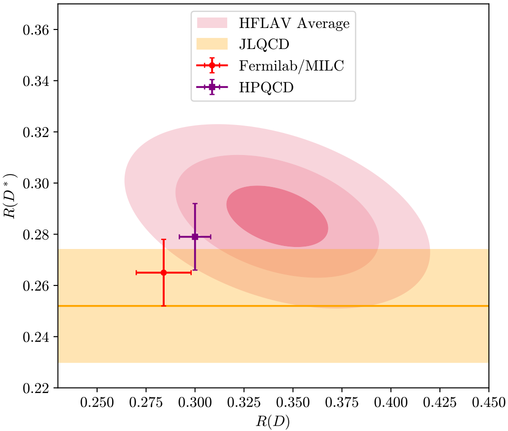
$$R(D^*)_{\mathbf{Lat}} = 0.265(13)$$



The mess

The mess: Lattice results





$$|V_{cb}|^{\rm FM} = 38.40(78) \times 10^{-3}$$

$$|V_{cb}|^{\rm JLQCD} = 39.19(90) \times 10^{-3}$$

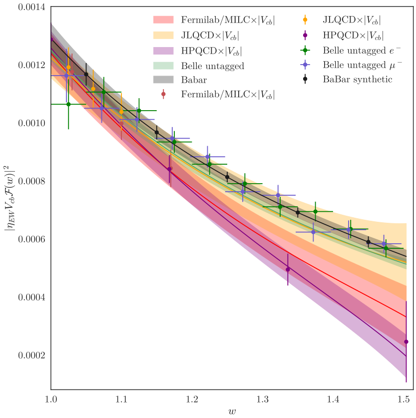
$$|V_{cb}|^{\rm HPQCD} = 39.31(74) \times 10^{-3}$$

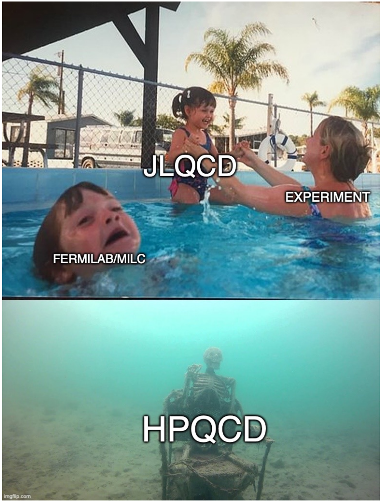
$$R(D^*)^{\rm FM} = 0.265(13)$$

$$R(D^*)^{\rm JLQCD} = 0.252(22)$$

$$R(D^*)^{\rm HPQCD} = 0.279(13)$$

The mess: Lattice results





$$|V_{cb}|^{\mathrm{FM}} = 38.40(78) \times 10^{-3}$$

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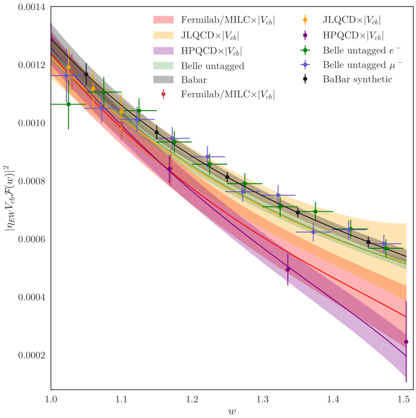
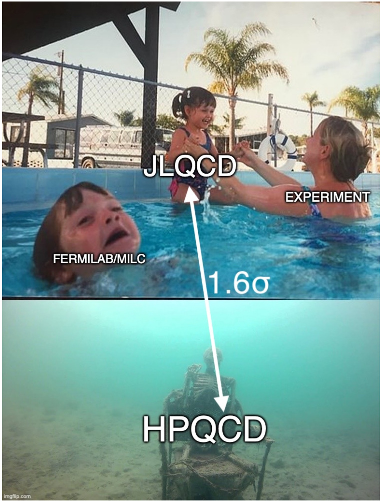
The mess: Lattice results





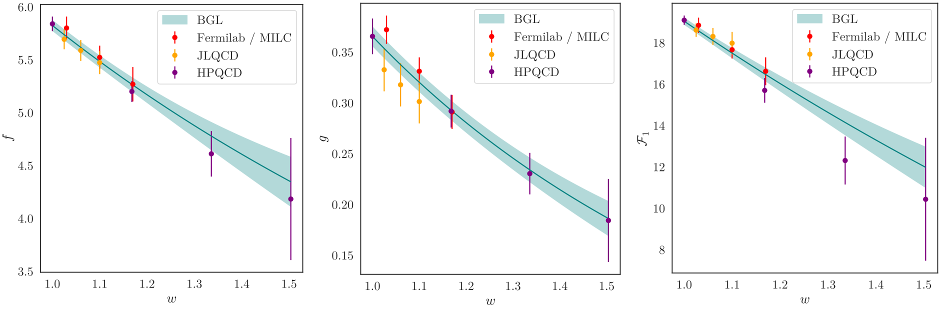
$$|V_{cb}|^{\text{FM}} = 38.40(78) \times 10^{-3}$$

$$|V_{cb}|^{\text{JLQCD}} = 39.19(90) \times 10^{-3}$$

$$|V_{cb}|^{\text{HPQCD}} = 39.31(74) \times 10^{-3}$$

- Combined BGL 2222 fits with priors 0(1)
- p-value of Belle untagged + BaBar BGL fit 223 is ≈ 0.04
- Combined $R(D^*) = 0.2667(57)$

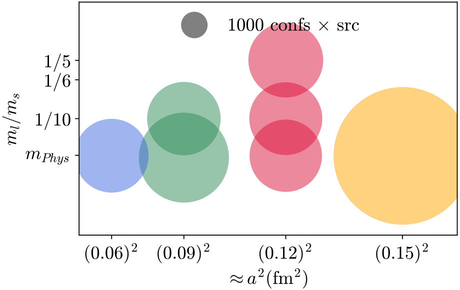
	p
MILC+JLQCD	0.40
MILC+HPQCD	0.44
JLQCD+HPQCD	0.73
All	0.56



The future

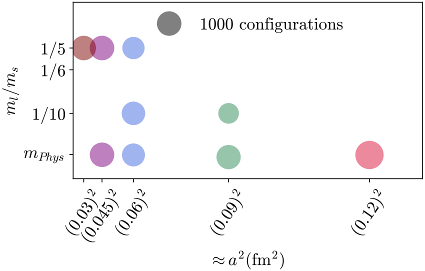
Future projects: HISQ + Fermilab

- Fermilab/MILC calculation
- Using 7 $N_f = 2 + 1 + 1$ ensembles of sea HISQ quarks
- The heavy quarks use the Fermilab effective action
  - Correlated with a $B \to L\ell\nu$ analysis using the same data
  - Four channels $B_{(s)} \to D^{(*)}_{(s)}\ell\nu$ in a single correlated analysis



Future projects: HISQ^2

- Fermilab/MILC calculation
- Planning to use 9 $N_f = 2+1+1$ ensembles of sea HISQ quarks
- The heavy quarks use the HISQ action
  - Physical bottom mass reachable with the finest ensembles
- m_π physical in several ensembles



Conclusions

- Great progress in LQCD calculations of $B \to D^* \ell \nu$ form factors
- Good agreement between different LQCD results
  - Not so good between LQCD and experiment
- New calculations are needed to clarify the situation

- Fermilab/MILC working on the next two calculations of $B \to D^*$

Thank you for your attention

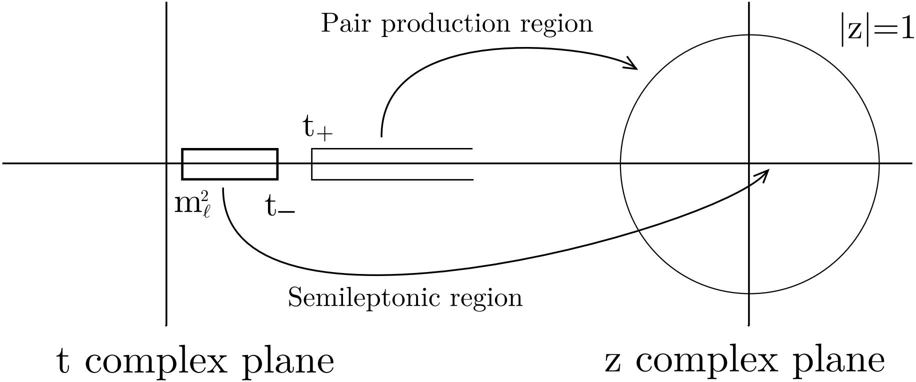
BACKUP SLIDES

Semileptonic B decays on the lattice: Parametrizations

Most parametrizations perform an expansion in the z parameter

$$\frac{1+z}{1-z} = \sqrt{\frac{t_+ - t}{t_+ - t_-}}, \qquad z = \frac{\sqrt{w+1} - \sqrt{2N}}{\sqrt{w+1} + \sqrt{2N}}$$

with $t_\pm = (m_B \pm m_{D^*})^2\,, \quad t = (p_B - p_{D^*})^2\,, \quad w = v_B \cdot v_{D^*}$



Semileptonic B decays on the lattice: Parametrizations

- Boyd-Grinstein-Lebed (BGL) — Phys. Rev. Lett. 74 (1995) 4603-4606; Phys.Rev. D56 (1997) 6895-6911; Nucl.Phys. B461 (1996) 493-511

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called outer function and must be computed for each form factor
  - Weak unitarity constraints $\sum_n |a_n|^2 \leq 1$

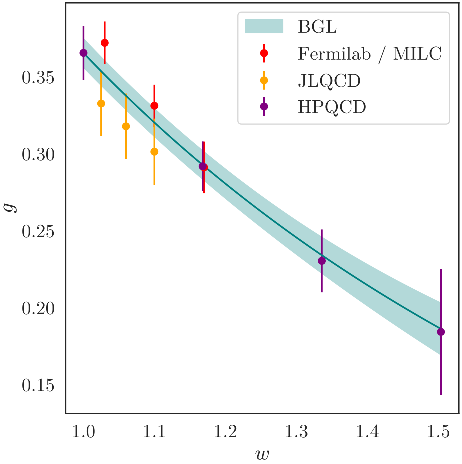
- Caprini-Lellouch-Neubert (CLN) — Nucl. Phys. B530 (1998) 153-181

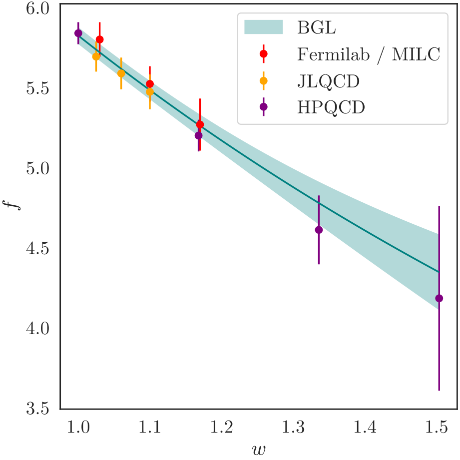
$$F(w) \propto 1 - \rho^2 z + c z^2 - d z^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $F(w)$: four independent parameters, one relevant at $w = 1$
- Current consensus: abandon CLN
  - Spiritual sucessors of CLN — Bernlochner et al. Phys.Rev.D 95 (2017) 115008, Phys.Rev.D 97 (2018) 059902

Bordone, Gubernari, Jung, Straub, Van Dyk... Eur.Phys.J.C 80 (2020) 74, Eur.Phys.J.C 80 (2020) 347, JHEP 01 (2019) 009

The mess: Combined lattice fits





The mess: Combined lattice fits