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Italiadomani
PIANO NAZIONALE DI RIPRESA E RESILIENZA

INFN
Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati

Theory of baryon number violation

Vasja Susič

Laboratori Nazionali di Frascati (LNF), INFN

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Introduction

- Motivation: baryon (and lepton) number
- A (biased) overview of **proton decay**:

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  (1) Experiment: bounds and prospects
  (2) Calculating the decay rate
  (3) Embedding into a GUT framework
  (4) Uncertainties in the decay rate prediction

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Note: references are in grey, clickable links are in purple.

Motivation — Theoretical

- **Standard Model** (SM): $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
  - Fermions: 3 families of

$$Q \sim (3, 2, +\tfrac{1}{6}), \qquad u^c \sim (\bar{3}, 1, -\tfrac{2}{3}), \qquad d^c \sim (\bar{3}, 1, +\tfrac{1}{3}), \tag{1}$$

$$L \sim (1, 2, -\tfrac{1}{2}), \qquad e^c \sim (1, 1, +1). \tag{2}$$

  - Scalars: $H \sim (1, 2, +\tfrac{1}{2})$.
  - 19 real parameters: $(g)_{3\times}$, $(m_f)_{9\times}$, $(\mathrm{CKM})_{4\times}$, m_H, λ, θ_{QCD}

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- Consider SM Yukawa sector:

$$\mathcal{L}_Y = Y_u\, Q\, u^c\, H \;+\; Y_d\, Q\, d^c\, H^* \;+\; Y_e\, L\, e^c\, H^* + h.c. \tag{3}$$

Phase redefinitions: 5 fermion irreps, 3 terms (and irrep H, $\mathrm{U}(1)_Y$) . . .

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SM preserves 2 (flavor universal) global $\mathrm{U}(1)$ charges

(1) Baryon number B: $[Q]_B = +\frac{1}{3}$, $[u^c]_B = -\frac{1}{3}$, $[d^c]_B = -\frac{1}{3}$

(2) Lepton number L: $[L]_L = +1$, $[e^c]_L = -1$

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- At **low energies**: quarks q confined in hadrons, but B still preserved

$$[qqq]_B = +1, \qquad [\bar{q}\bar{q}\bar{q}]_B = -1, \qquad [q\bar{q}]_B = 0. \tag{4}$$

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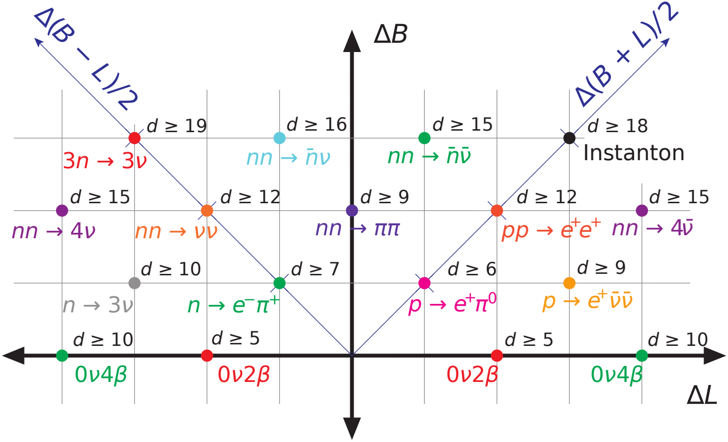
  - Global symmetries (B,L) expected to be eventually broken (by gravity at M_{Pl} if nothing else)
  - Non-renormalizable operators in SM effective theory (SMEFT):

$d = 5$:	$\frac{\kappa^{IJ}}{\Lambda} L_I L_J H^2$	$\not{L}$	**neutrino masses** (*beyond* SM!) $\Lambda \simeq 10^{14}\,\mathrm{GeV}$ for $\kappa = \mathcal{O}(1)$
$d = 6$:	$\frac{C}{\Lambda^2}\, qqql$ (multiple)	$\not{B}$, $\not{L}$	induce **proton decay** not observed (**yet?**) preserve $B - L$

Motivation — Theoretical

- $\not{B}$ and $\not{L}$ operators in SMEFT more generally: credit: [1]



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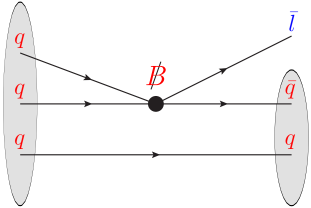
- Experimental search for $\not{B}$:

  decay of p or n (in a nucleus)

baryon $\to$ meson $+$ anti-lepton



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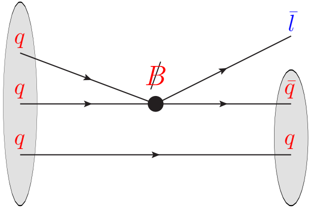
$$\boxed{\tau_p \gtrsim 10^{34}\,\mathrm{y} \;\to\; \Lambda \gtrsim 4 \cdot 10^{15}\,\mathrm{GeV}}$$



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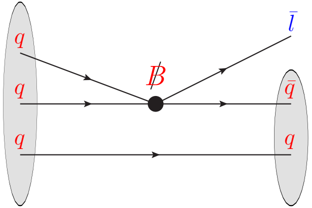
  $\to$ Observe 1 proton for 10^N years
  $\leftrightarrow$ observe 10^N protons for 1 year



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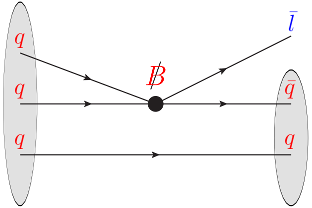

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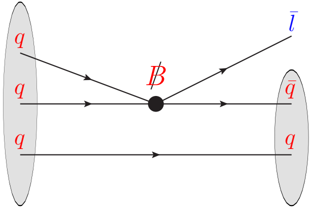
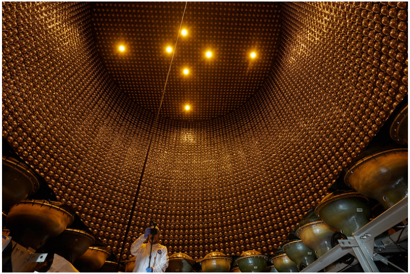
  - $\to$ $1\,\mathrm{kg} \sim 6 \cdot 10^{26}\,p$ (Avogadro)
  - $\to$ $1\,\mathrm{kt} \sim 6 \cdot 10^{32}\,p \sim (10\,\mathrm{m})^3$ water





The $22.5\,\mathrm{kt}$ tank of Super-K [flickr]

Experimental bounds and prospects

- Main current experiment: **Super-K**
  - $\rightarrow$ running for almost 30 years
  - $\rightarrow$ exposure $\mathcal{E}$: now $> 450\,\mathrm{kt}\cdot\mathrm{y}$

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- Bounds from Super-K ($90\,\%$ CL):

decay	$> \tau/\mathcal{B}\,[\mathrm{y}]$	$\mathcal{E}[\mathrm{kt}\cdot\mathrm{y}]$	year
$p \to \pi^0 e^+$	$2.4\cdot 10^{34}$	450	2020 [2]
$p \to \pi^0 \mu^+$	$1.6\cdot 10^{34}$	450	2020 [2]
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  - → fiducial mass (f.m.): $8\times$ Super-K

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- Experiment summary:

experiment	f.m. [kt]	year	technology
Super-Kamiokande	22.5 → 27.2	1996 – present	water Cherenkov
Hyper-Kamiokande	187	? 2027 –	water Cherenkov
DUNE	40	? 2030s –	liquid Ar TPC
JUNO	20	? 2025 –	liquid scintillator

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Baryon number violation at dim6 in SMEFT

- There are 4 independent dim6 $\not{B}$-operators $qqql$ in SMEFT [10]

$\mathcal{O}_{RL}$	$(du)(QL)$	$(d^{\alpha a} u^{\beta b})(Q^{\gamma i c} L^{j d})\, \epsilon_{\alpha\beta\gamma}\, \epsilon_{ij}$
$\mathcal{O}_{LR}$	$(QQ)(ue)$	$(Q^{\alpha i a} Q^{\beta j b})(u^{\gamma c} e^{d})\, \epsilon_{\alpha\beta\gamma}\, \epsilon_{ij}$
$\mathcal{O}_{LL}$	$(QQ)(QL)$	$(Q^{\alpha i a} Q^{\beta j b})(Q^{\gamma k c} L^{l d})\, \epsilon_{\alpha\beta\gamma}\, \epsilon_{il}\, \epsilon_{jk}$
$\mathcal{O}_{RR}$	$(du)(ue)$	$(d^{\alpha a} u^{\beta b})(u^{\gamma c} e^{d})\, \epsilon_{\alpha\beta\gamma}$

Conventions for table: using Weyl fermions. $\epsilon_{12} = 1$.
$x^{cc} \equiv x$, so $u \sim (3, 1, +2/3)$, $d \sim (3, 1, -1/3)$, $e \sim (1, 1, -1)$ are right-chiral.

Index types: $ijkl \to \mathrm{SU}(2)_L$, $\alpha\beta\gamma \to \mathrm{SU}(3)_C$, $abcd \to$ generations.

- All we need to know: coefficients C^{I}_{abcd}, where $I \in \{LR, RL, LL, RR\}$

$$\mathcal{L}_{\not{B}} = \sum_I C^{I}_{abcd}\, \mathcal{O}^{abcd}_{I}, \qquad C^I \propto \frac{1}{\Lambda^2} \text{ are dimensionful.} \tag{5}$$

Computing the decay rates for different channels — procedure

- Use coefficients $C^{(i)}$ in the following procedure: [11, 12, 13]

(1) At EW scale: match $\not{B}$-operators $\mathcal{O}_I$ in SMEFT to $\tilde{\mathcal{O}}_J$ in LEFT.
LEFT ("low-energy EFT"): $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{EM}$ with h, $W^{\pm}_{\mu}$, Z^0_{μ} and t integrated out.

(2) Rotate flavor indices in $\tilde{\mathcal{O}}^{abcd}_J$ to mass eigenbasis

(3) Run operators $\tilde{\mathcal{O}}_J$ via RGE down to proton-mass-scale m_p

(4) Insert coefficients of $\tilde{\mathcal{O}}_J$ into expression derived from chiPT
In this step we need hadronic matrix elements from lattice QCD.

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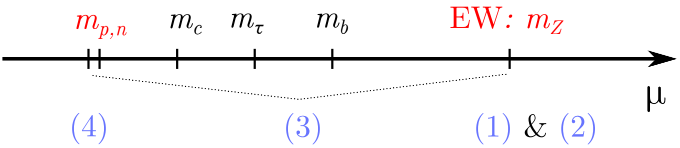

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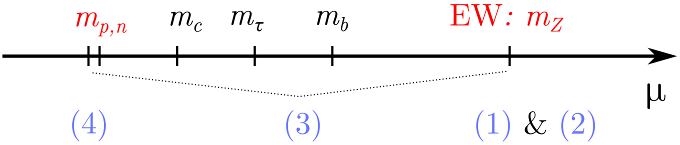

- Computes steps (3)–(4): *Mathematica* package `ProtonDecay` [14]

Computing the decay rates for different channels — matching

- Matching for step (1):

$$\mathcal{L}_{\not B} = \sum_I C^I_{abcd} O^{abcd}_I = \frac{1}{16\pi^2} \sum_J \tilde{C}^J_{abcd} \tilde{O}^{abcd}_J \tag{6}$$

$\tilde{\mathcal{O}}_{RL}(udue) = ({u_R}^{\alpha a} {d_R}^{\beta b})({u_L}^{\gamma c} {e_L}^{d})\epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{RL(udue)}_{abcd} = -16\pi^2\, C^{RL}_{bacd}$
$\tilde{\mathcal{O}}_{LR}(udue) = ({u_L}^{\alpha a} {d_L}^{\beta b})({u_R}^{\gamma c} {e_R}^{d})\epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{LR(udue)}_{abcd} = 16\pi^2 (C^{LR}_{abcd} + C^{LR}_{bacd})$
$\tilde{\mathcal{O}}_{LL}(udue) = ({u_L}^{\alpha a} {d_L}^{\beta b})({u_L}^{\gamma c} {e_L}^{d})\epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{LL(udue)}_{abcd} = 16\pi^2 (- C^{LL}_{abcd} + C^{LL}_{acbd} - C^{LL}_{cabd})$
$\tilde{\mathcal{O}}_{RR}(udue) = ({u_R}^{\alpha a} {d_R}^{\beta b})({u_R}^{\gamma c} {e_R}^{d})\epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{RR(udue)}_{abcd} = -16\pi^2\, C^{RR}_{bacd}$
$\tilde{\mathcal{O}}_{RL}(udd\nu) = ({u_R}^{\alpha a} {d_R}^{\beta b})({d_L}^{\gamma d} {\nu_L}^{d})\, \epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{RL(udd\nu)}_{abcd} = 16\pi^2\, C^{RL}_{bacd}$
$\tilde{\mathcal{O}}_{LL}(udd\nu) = ({u_L}^{\alpha a} {d_L}^{\beta b})({d_L}^{\gamma d} {\nu_L}^{d})\, \epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{LL(udd\nu)}_{abcd} = 16\pi^2 (C^{LL}_{bacd} + C^{LL}_{cbad} - C^{LL}_{bcad})$
$\tilde{\mathcal{O}}_{RL}(ddu\nu) = \frac{1}{2}({d_R}^{\alpha a} {d_R}^{\beta b})({u_L}^{\gamma c} {\nu_L}^{d})\, \epsilon_{\alpha\beta\gamma}$	$\tilde{C}^{RL(ddu\nu)}_{abcd} = 0$

Mediation of proton decay

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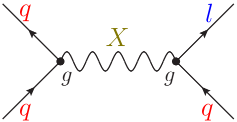
(a) Mediation by **gauge boson** X

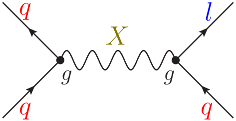


$$\Gamma_{p^+} \sim \frac{g^4 m_p^5}{m_X^4}$$

Mediation of proton decay

- Simplest way for $\mathcal{O}_I$ to arise from BSM physics: tree-level mediation

(a) Mediation by **gauge boson** X (b) Mediation by **scalar** T



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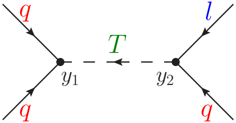

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Mediation of proton decay

- Simplest way for $\mathcal{O}_I$ to arise from BSM physics: tree-level mediation

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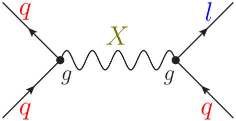

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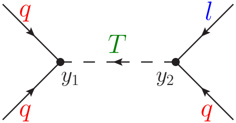

X	induced $\mathcal{O}$
$(3, 2, -\frac{5}{6})$	$\mathcal{O}_{RL,LR}$
$(3, 2, +\frac{1}{6})$	$\mathcal{O}_{RL}$

$\rightarrow$ gauge current: L to R

T	induced $\mathcal{O}$
$(3, 1, -\frac{1}{3})$	$\mathcal{O}_{RL,LR,LL,RR}$
$(3, 1, -\frac{4}{3})$	$\mathcal{O}_{RR}$
$(3, 3, -\frac{1}{3})$	$\mathcal{O}_{LL}$

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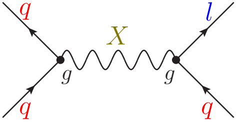

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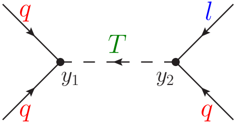

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- If $m_X \approx m_T$, $g \approx 0.5$, $y \lesssim 10^{-3}$ (like 1st/2nd gen Yukawa to H):

gauge contribution dominates!

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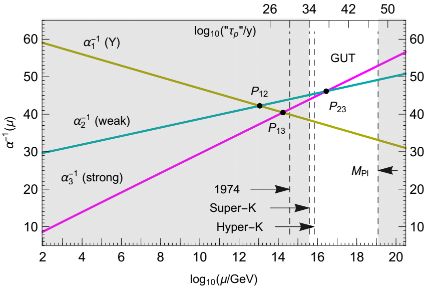

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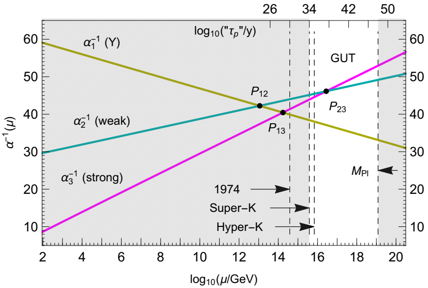

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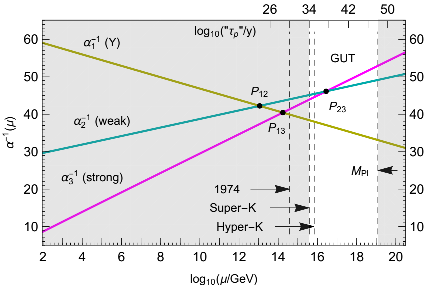

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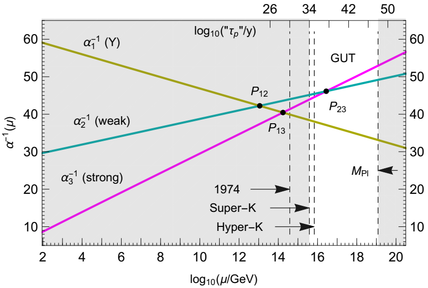

- Alternatives: $X \sim (3, 2, \frac{Y}{2})$, so at least $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ unify

→ flipped $\mathrm{U}(1)_Y$ embedding: $\boxed{\mathrm{SU}(5)' \times \mathrm{U}(1)', \quad \mathrm{SO}(10)' \times \mathrm{U}(1)'}$

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Embeddings of irreducible representations in GUT

- **Fermions**:

$\rightarrow$ Neat fit of SM to G-irreps

$\rightarrow$ This feature non-trivial! [15]

SM	Q	u^c	e^c	d^c	L	
SU(5)	$\mathbf{10}$			$\overline{\mathbf{5}}$		
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group	$(3, 2, -\frac{5}{6})$	$(3, 2, +\frac{1}{6})$
SU(5)	✓	
SO(10)	✓	✓
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- **Scalars**: $T \sim (3,1,-\frac{1}{3})$ always present in true GUTs

→ $\mathrm{SU}(5)$: Higgs doublet $H \sim (1,2,+\frac{1}{2})$ in irreps 5, 45 or 70

→ T also present in 5, 45, 70

→ Since $\mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6$: H comes with T in all cases

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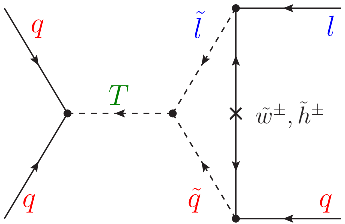

$\rightarrow$ Left: one example diagram

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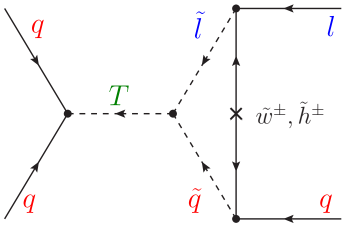

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- If $m_{\mathrm{SUSY}} \ll m_T$: couplings qqT and qlT must be small ($\ll 1$)

  → In GUT: yes, they are related to SM Yukawas

A quick guide to computing dim5 proton decay in SUSY GUT

- Procedure [12, 13]:
  1. Integrate T out of the superpotential at M_{GUT}.
  2. Determine the effective dim5 coefficients C^{5L} and C^{5R}.
  3. Use RGE to run down $C^{5L,5R}$ to m_{SUSY}.
  4. Dress dim5 operators with SUSY particles.
     Add this contribution to SM operators $\tilde{\mathcal{O}}_i$ in LEFT.
  5. Continue as before: run $\tilde{\mathcal{O}}_i$ down to m_p, use chiPT.
- Step (2): the 2 $\not{B}$-operators superpotential operators are [16]

$$W \supset -\tfrac{1}{2} C^{5L}_{abcd}\, Q^{\alpha i a} L^{jb} Q^{\beta k c} Q^{\gamma l d}\, \epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{kl} - C^{5R}_{abcd}\, (u^c)_\alpha{}^a (d^c)_\beta{}^b (e^c)^c (u^c)_\gamma{}^d\, \epsilon^{\alpha\beta\gamma} \tag{8}$$

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- *Mathematica* tools for this computation [14]:
  → For steps (3)–(4): `SusyTCProton`
  → For step (5): `ProtonDecay`

Predictions for proton decay in GUTs

- Typical dominant decay modes (caveats always possible!):

Non-SUSY GUT	gauge mediated	$p \to \pi^0 e^+$	(π-modes)
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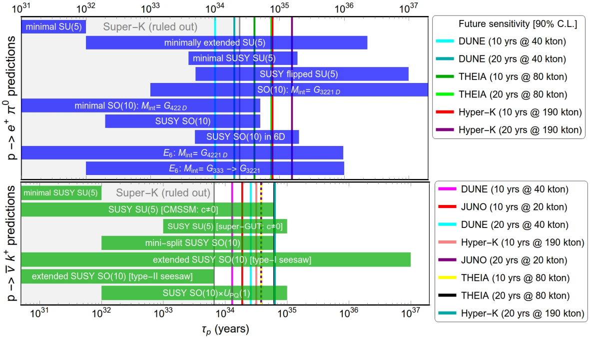
- For a given model: predictions have wide span [1]



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(d) **Hadronic matrix elements**: parameters α, β extracted from lattice [20]

$\rightarrow$ $\Gamma \cdot \mathcal{B}_i \propto |\alpha \mathcal{A}_1 + \beta \mathcal{A}_2|^2$, error in α, β estimated at 10–$20\,\%$

Robustness of the calculation (in GUT)

- Controlling flavor uncertainties:
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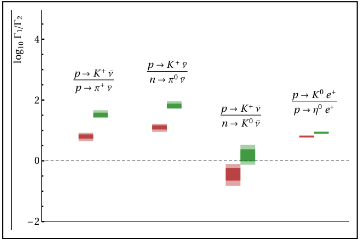

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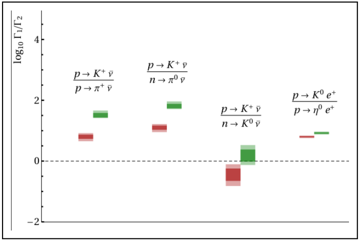

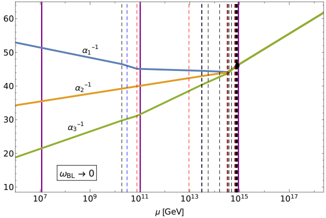

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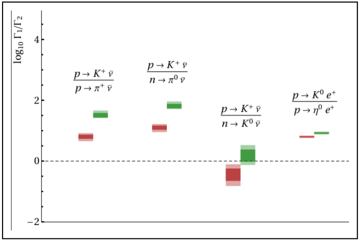
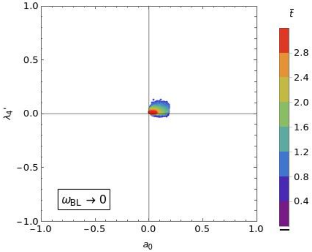
  - → Can perturbativity (stability of RGE) provide constraints? [21]





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Conclusions

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(2) Hyper-K will increase τ_p bound to $\sim 10^{35}$ years (after 20 years)

$\rightarrow$ Bulk and time matters

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Thank you for your attention!

References I

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