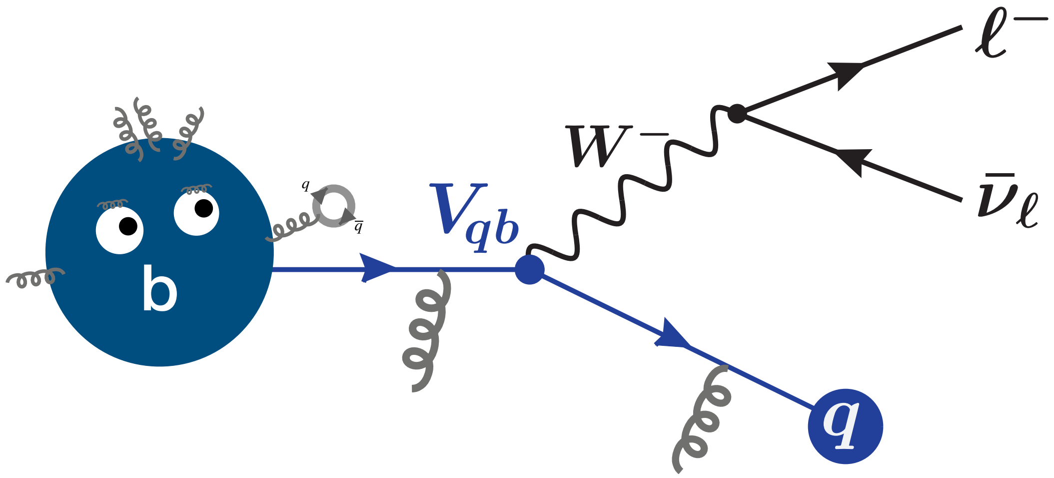
Semileptonic Measurements from Belle and Belle II

DISCRETE 2024

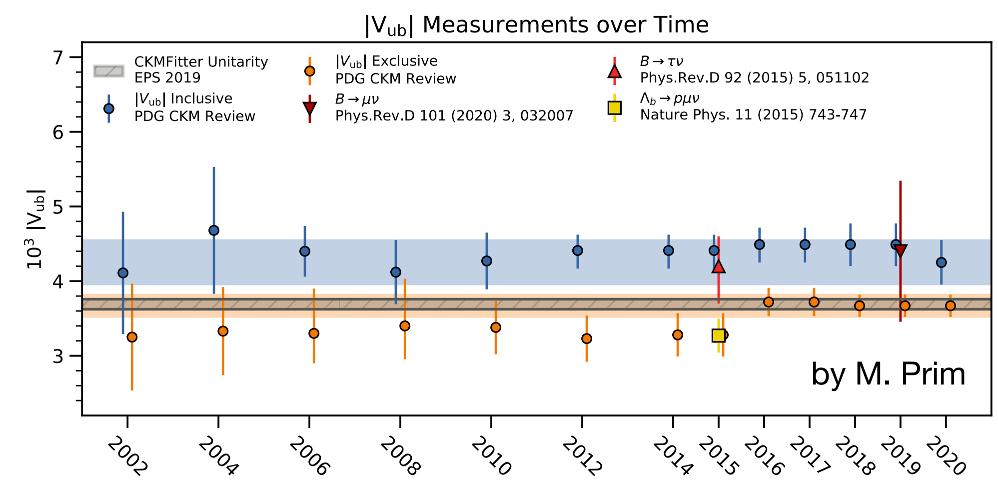
Florian Bernlochner

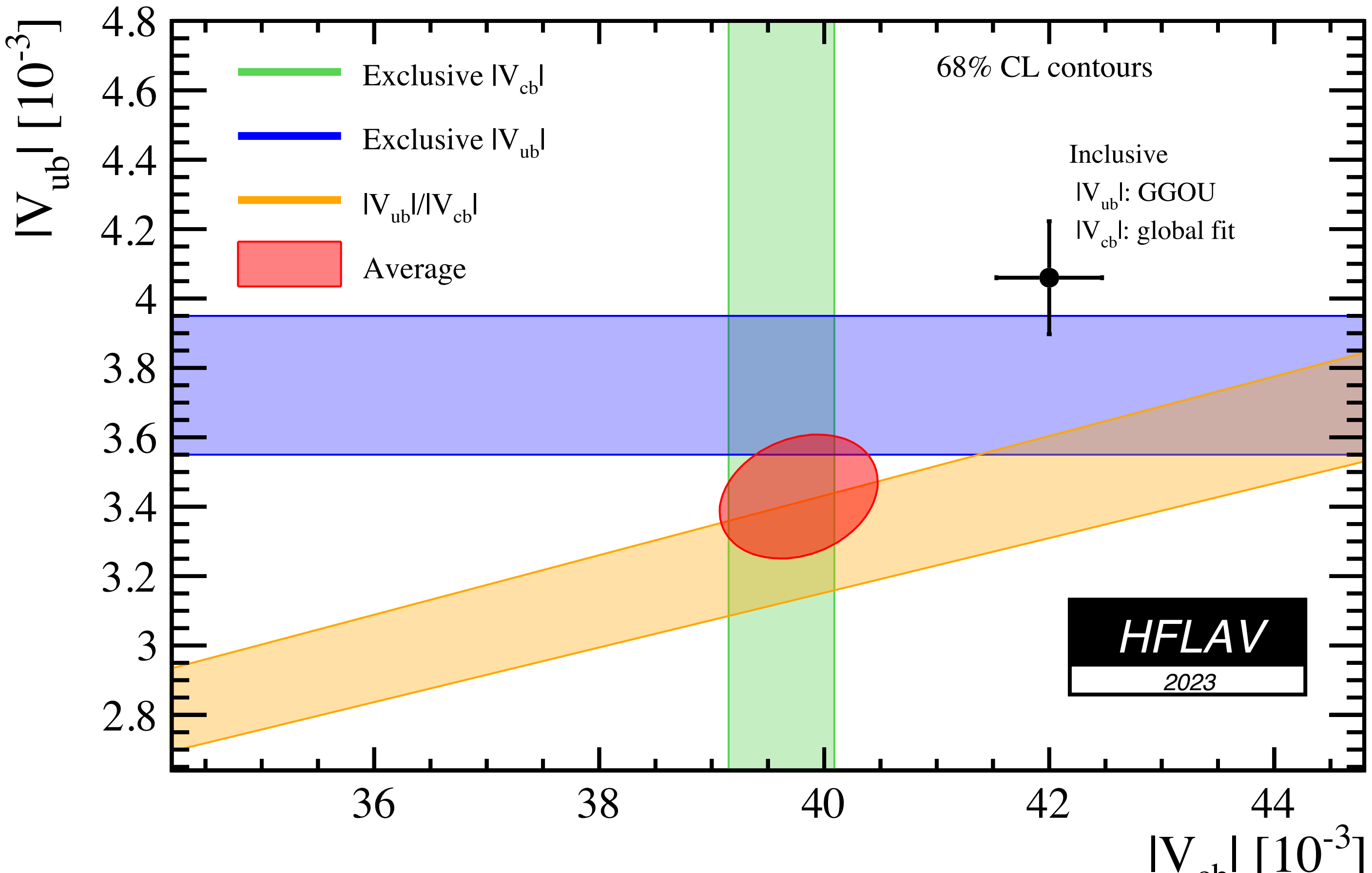
Puzzles…

It may look cute, but that might be deceiving…

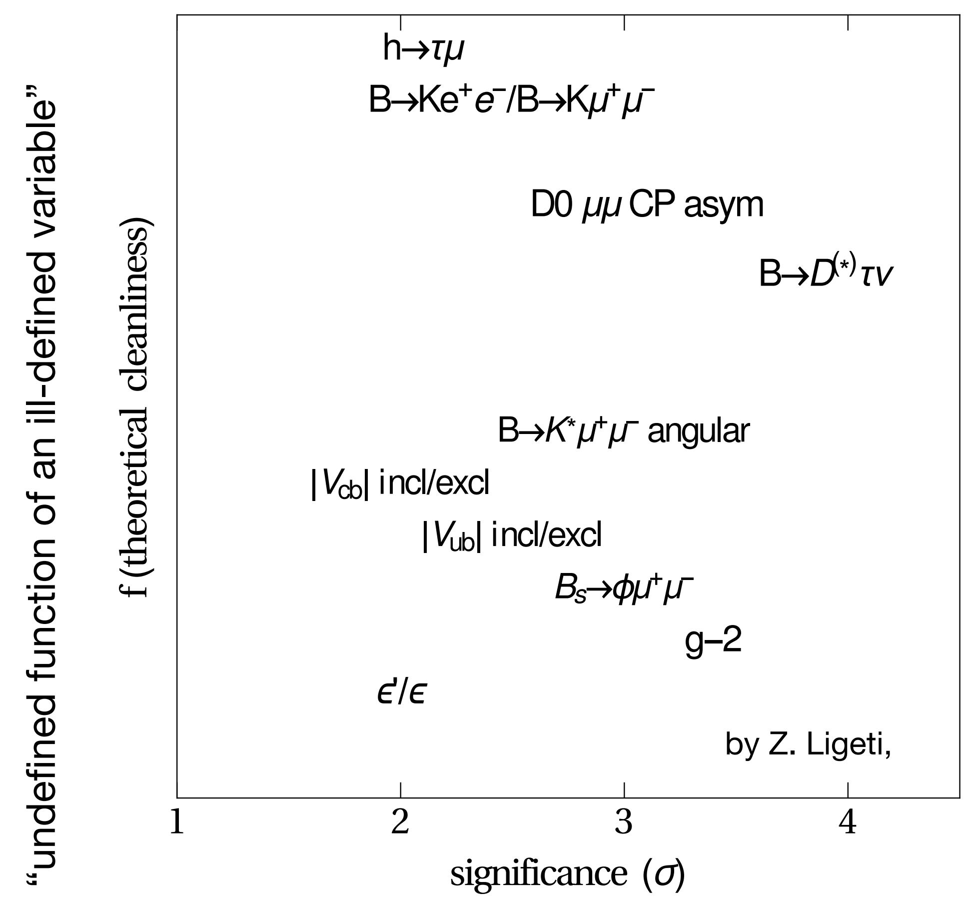


… Long-standing discrepancy since more than two decades





HFLAV report 2024, https://arxiv.org/abs/2411.18639



https://arxiv.org/pdf/1704.02938

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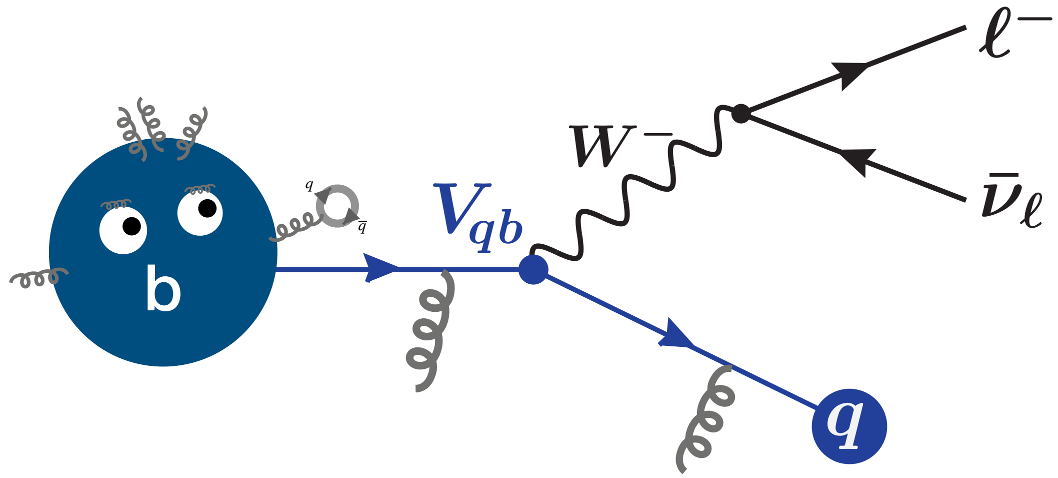
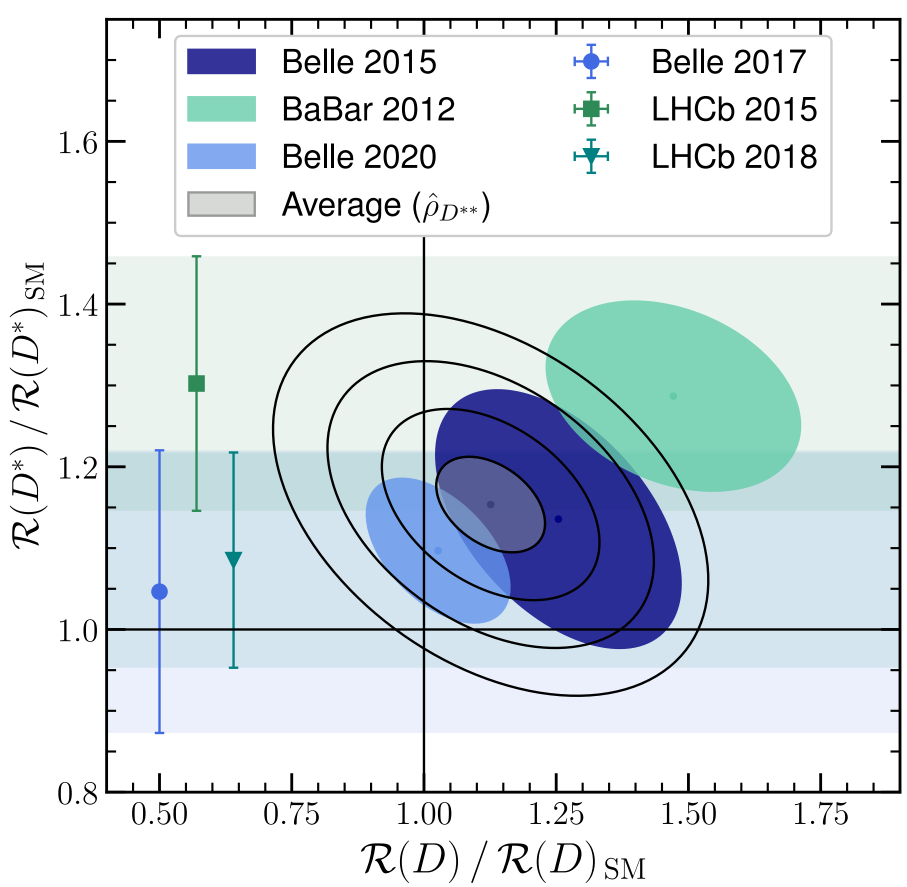


$$R = \frac{b \to q\,\tau\bar{\nu}_\tau}{b \to q\,\ell\bar{\nu}_\ell}$$

$\ell = e, \mu$

↓

Obs.	Current World Av./Data	Current SM Prediction	Significance	
$\mathcal{R}(D)$	0.340 ± 0.030	0.299 ± 0.003	1.2σ	3.1σ
$\mathcal{R}(D^*)$	0.295 ± 0.014	0.258 ± 0.005	2.5σ	
$P_\tau(D^*)$	$-0.38 \pm 0.51^{+0.21}_{-0.16}$	-0.501 ± 0.011	0.2σ	
$F_{L,\tau}(D^*)$	$0.60 \pm 0.08 \pm 0.04$	0.455 ± 0.006	1.6σ	
$\mathcal{R}(J/\psi)$	$0.71 \pm 0.17 \pm 0.18$	0.2582 ± 0.0038	1.8σ	
$\mathcal{R}(\pi)$	1.05 ± 0.51	0.641 ± 0.016	0.8σ	
$\mathcal{R}(D)$	$\mathbf{0.337 \pm 0.030}$	0.299 ± 0.003	1.3σ	$\mathbf{3.6\sigma}$
$\mathcal{R}(D^*)$	$\mathbf{0.298 \pm 0.014}$	0.258 ± 0.005	2.5σ	



F. Bernlochner, Manuel Franco Sevilla, Dean J. Robinson, Guy Wormser, Review of Modern Physics, arXiv:2101.08326 [hep-ex]

SL Analysis Methods

The question of **tagging**:

At e^+e^--B-Factories we can leverage the known initial collision kinematics

E.g. if just one final state particle is missing, then with $Y = X\ell$

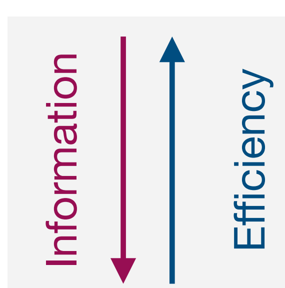
$$\cos\theta_{BY} = \frac{2E_B E_Y - m_B^2 - m_Y^2}{2\,|\mathbf{p}_B|\,|\mathbf{p}_Y|} \in [-1,1]$$

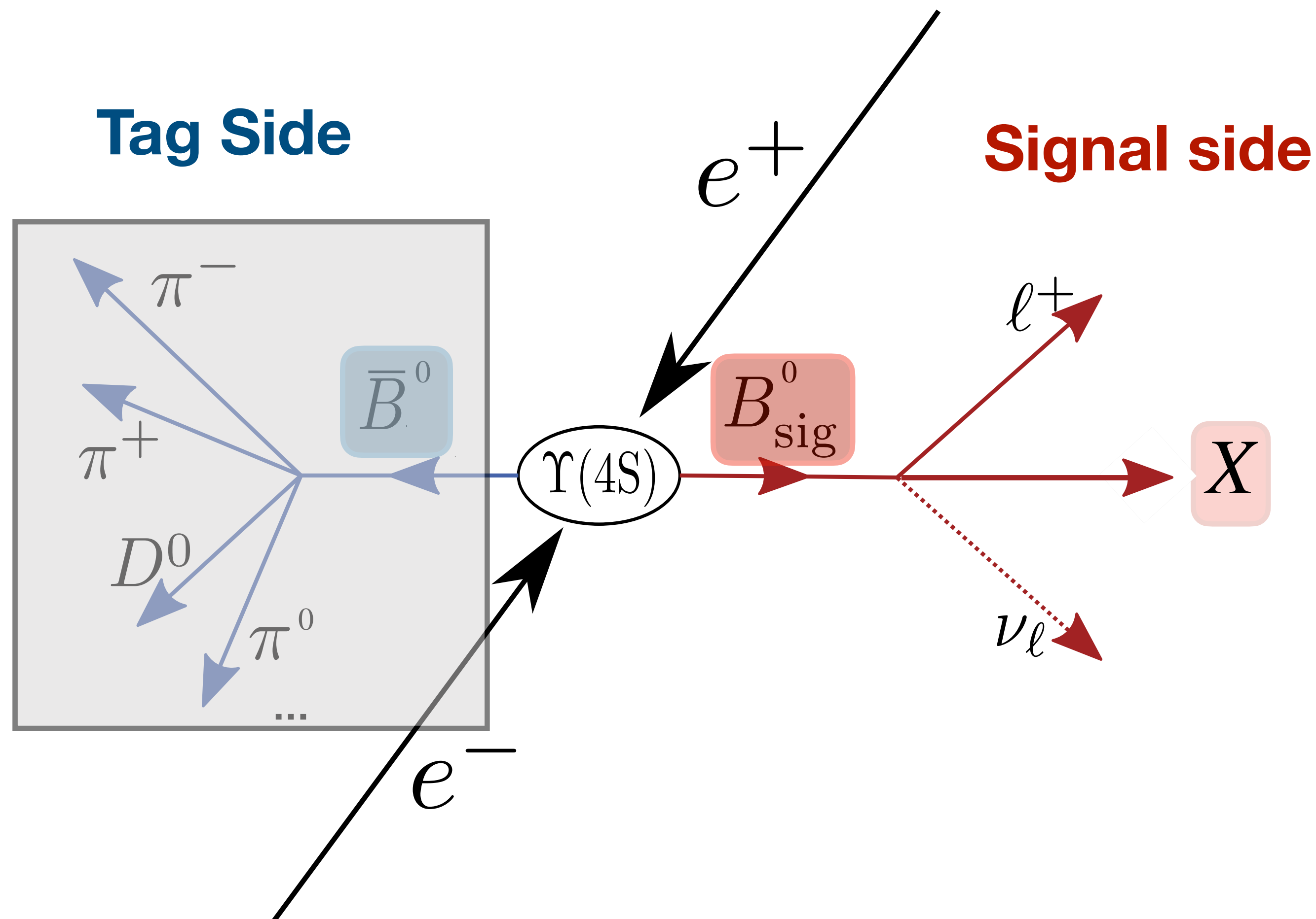
Can gain even more information, if we reconstruct

second B decay $\widehat{=}$ tagging

Idea comes in many flavors:

- Inclusive tagging
- SL tagging
- Hadronic tagging



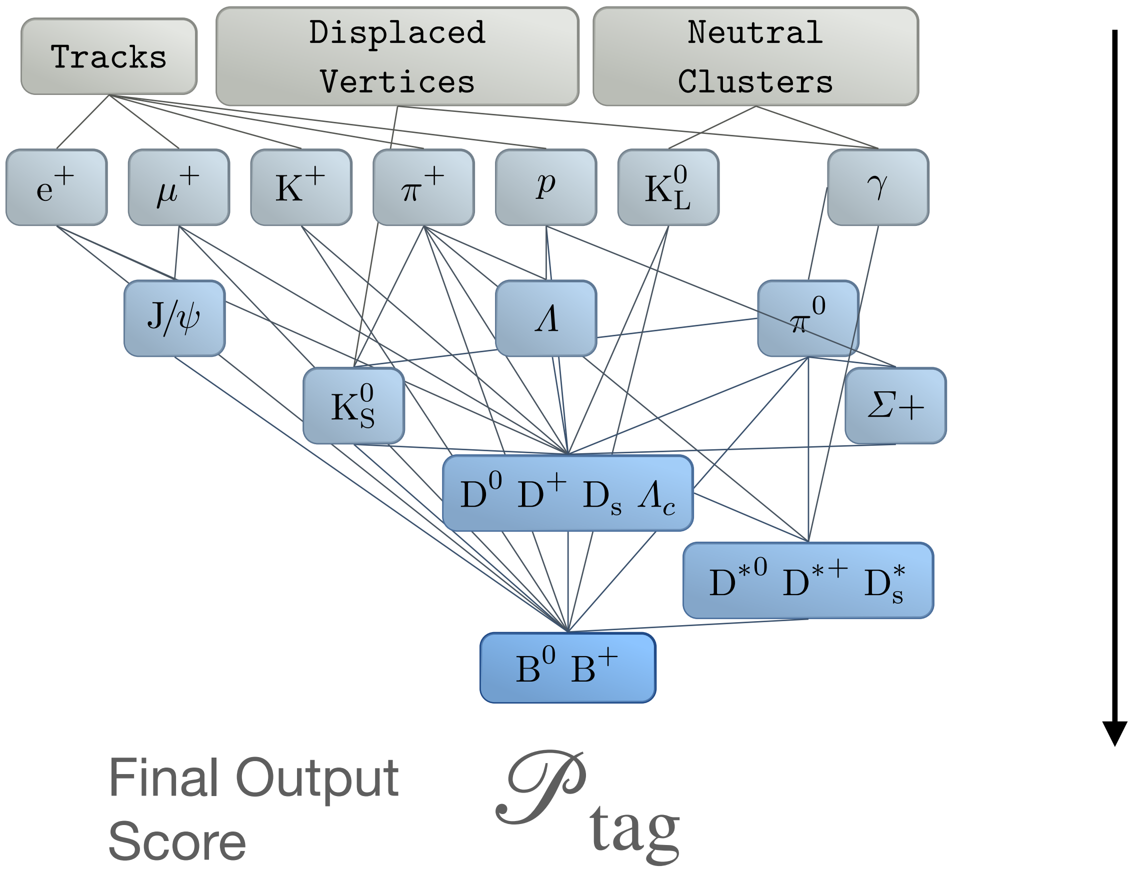


e.g. with hadronic tagging the full event kinematics but not the neutrino is reconstructed

$$M_\nu^2 \simeq M_{\text{miss}}^2 = \left(p_{e^+e^-} - p_{B_{\text{tag}}} - p_X - p_\ell\right)^2$$

Exclusive Tagging in a nutshell

From
arXiv:2008.06096 [hep-ex]



Reconstruct B-Mesons in **several stages**:

start with **detector stable particles**; then progress to **simple composite states**; combine the **composite** states to **build** more **complexity**

Each **stage** trains a **Boosted Decision Tree (BDT)** to identify good combinations;

each stage's BDT output is used as input for the next stage
+ all kinematic information
+ (particle identification scores)
+ vertex fit probabilities

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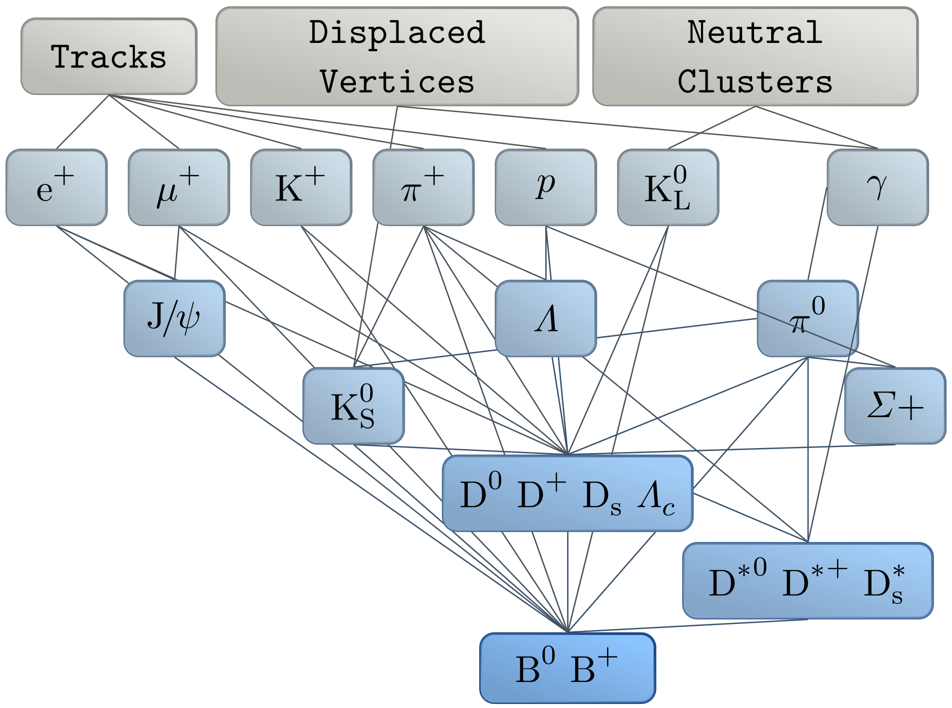

Final Output Score $\mathscr{P}_{\text{tag}}$

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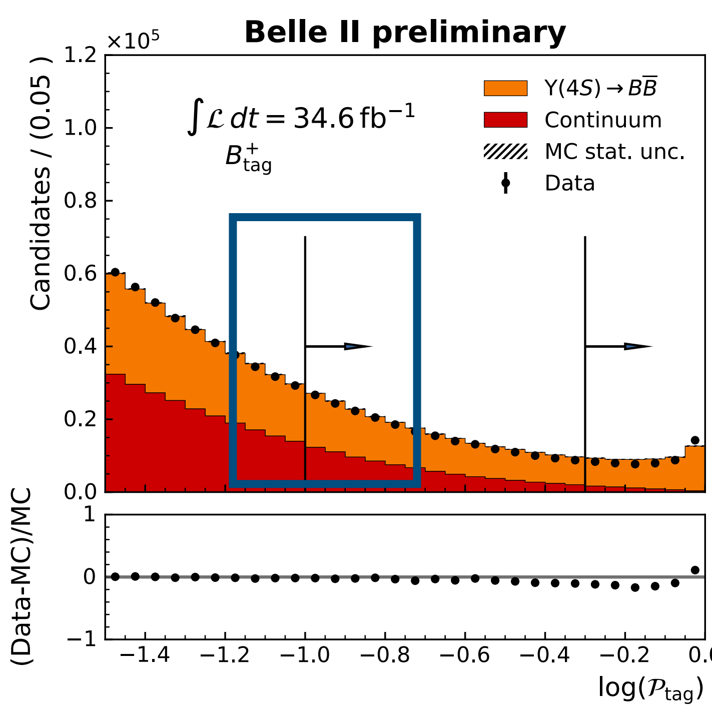

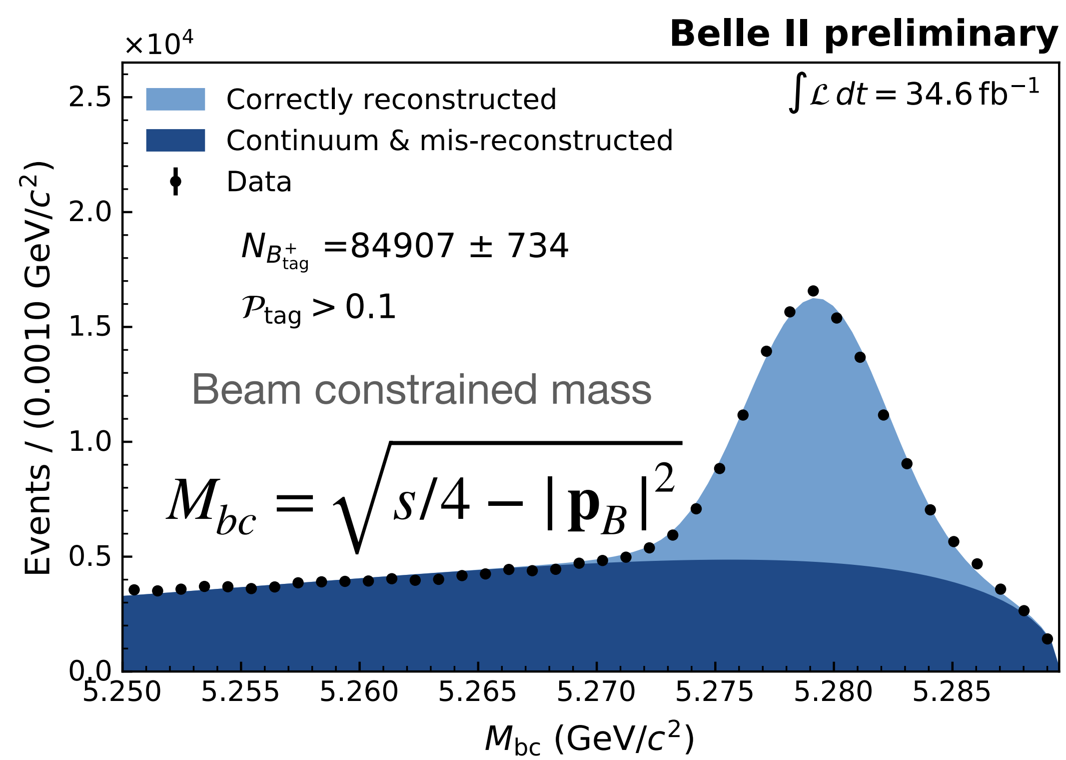

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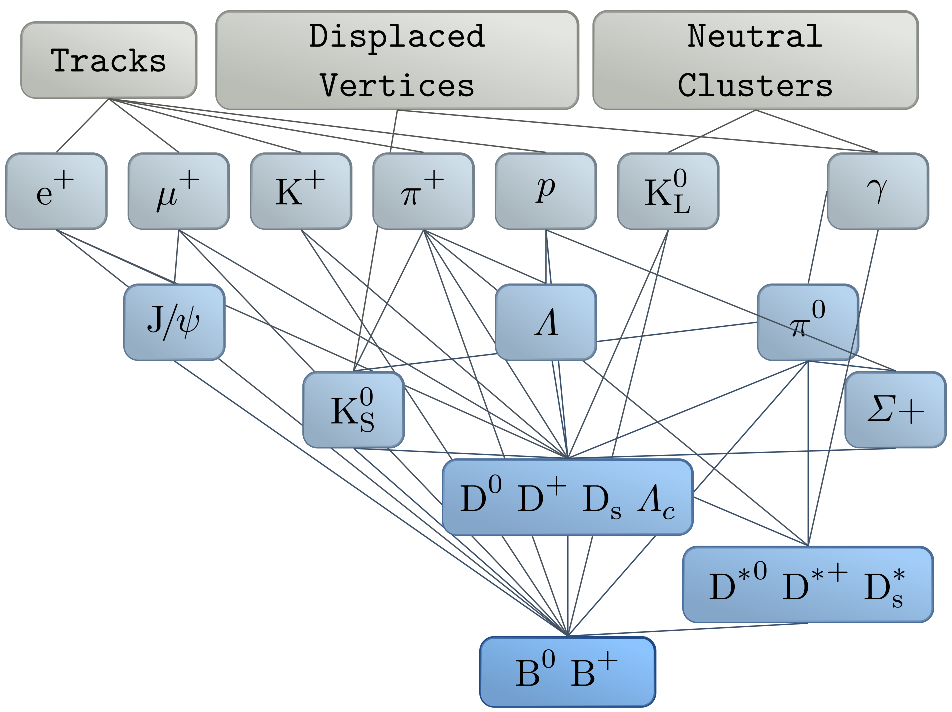

Final Output Score $\mathscr{P}_{tag}$

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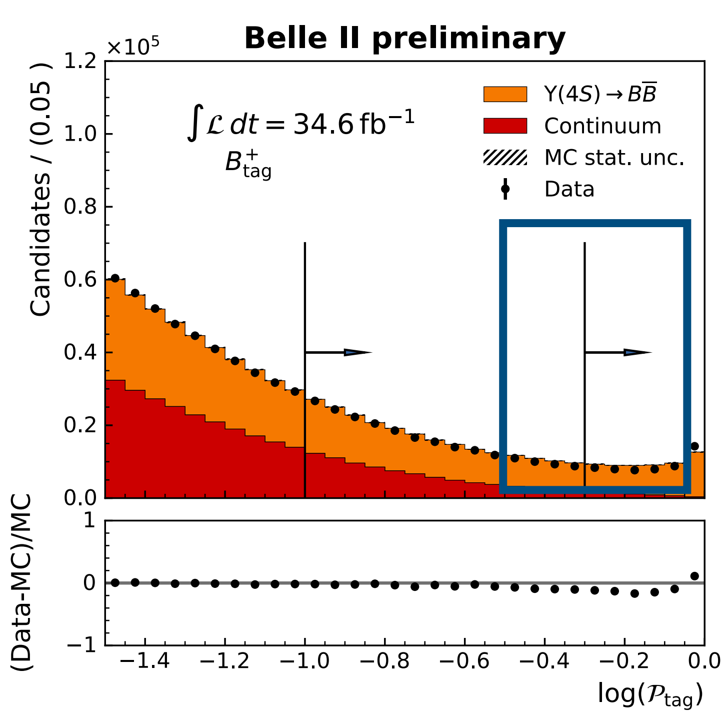
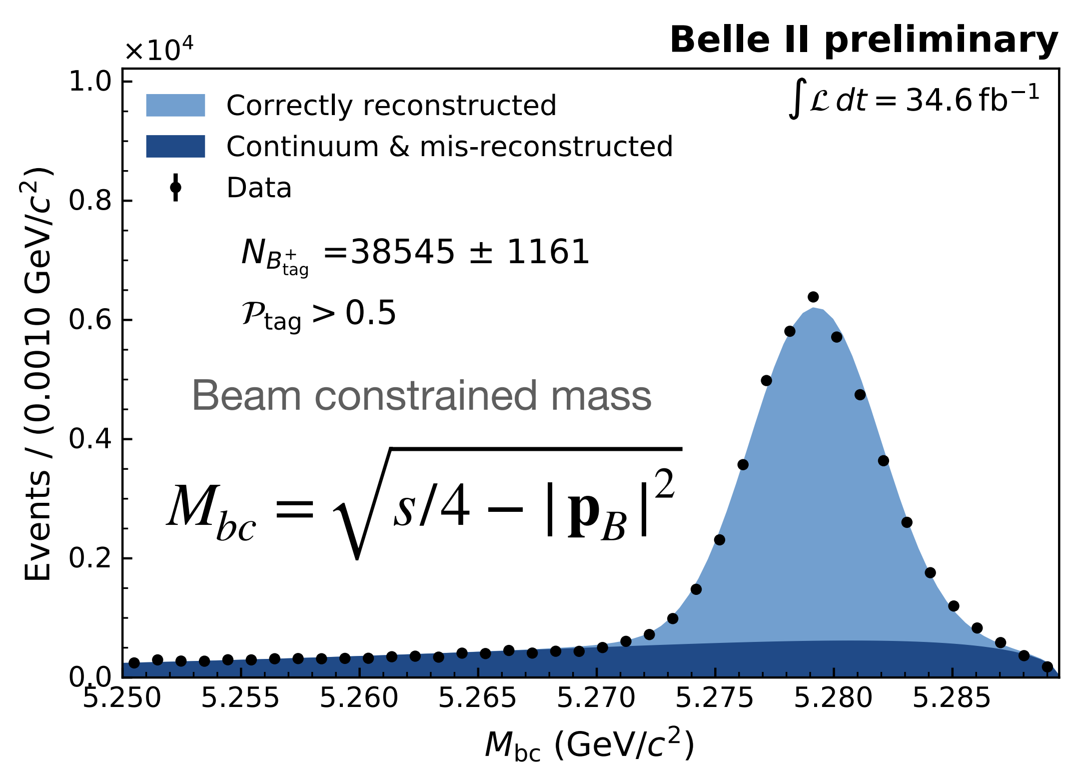




Recent Results Overview

Inclusive

1. Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment [Phys. Rev. D 107, 072002, arXiv:2205.06372]

2. **First Measurement** of $\mathscr{R}(X_{\tau/\ell})$ as an Inclusive Test of the $b \to c\tau\bar{\nu}_\tau$ Anomaly [Phys.Rev.Lett. 132 (2024) 21, 211804, arXiv:2311.07248]

3. First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$ [Phys.Rev.Lett. 131 (2023) 21, 211801, arXiv:2303.17309]

Exclusive

4. Determination of $|V_{cb}|$ using $\overline{B}^0 \to D^{*+}\ell^- \bar{\nu}_\ell$ with **Belle II**, [Phys Rev D. 108, 092013, arXiv:2310.01170]

5. Test of **light-lepton universality** in **angular asymmetries** of hadronically tagged $B^0 \to D^{*-}\{e^+, \mu^+\}\, \nu$ decays at Belle II, [Phys.Rev.Lett. 131 (2023) 18, 181801, arXiv:2308.02023]

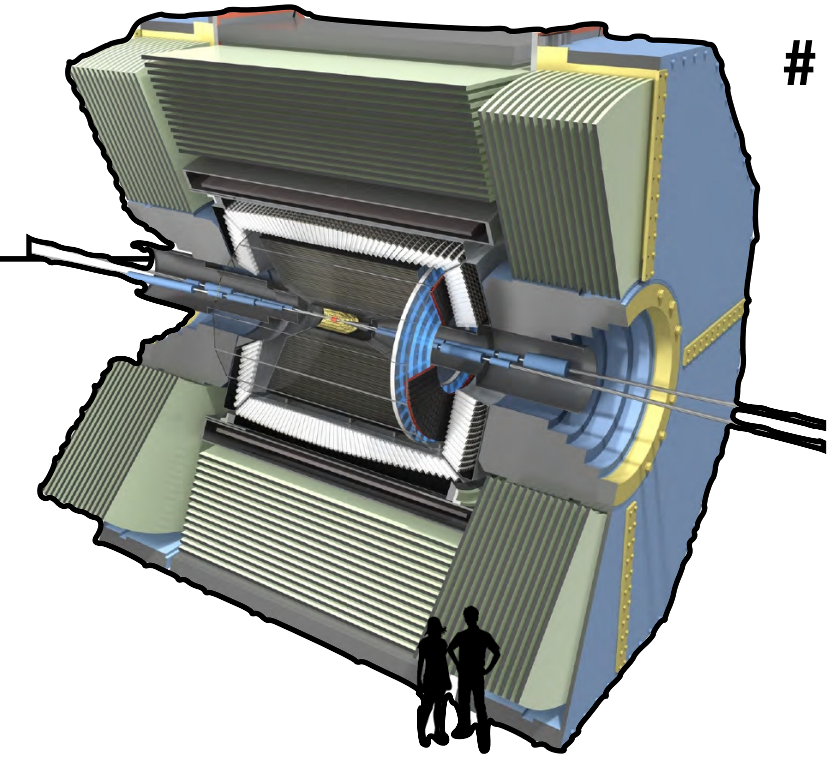
6. Measurement of **Angular Coefficients** of $B \to D^* \ell \bar{\nu}_\ell$, Implications on $|V_{cb}|$ and Tests of Lepton Flavor Universality, [Phys.Rev.Lett. 133 (2024) 13, 131801, arXiv: 2310.20286]

7. A test of **lepton flavor universality** with a measurement of $\mathscr{R}(D^*)$ using hadronic B tagging at the Belle II experiment, [Phys.Rev.D 110 (2024) 7, 072020, 2401.02840]

8. Determination of $\left|V_{ub}\right|$ from simultaneous measurements of untagged $B^0 \to \pi^- \ell \bar{\nu}_\ell$ and $B^+ \to \rho^0 \ell \bar{\nu}_\ell$ decays [Submitted to PRD, arXiv:2407.17403]

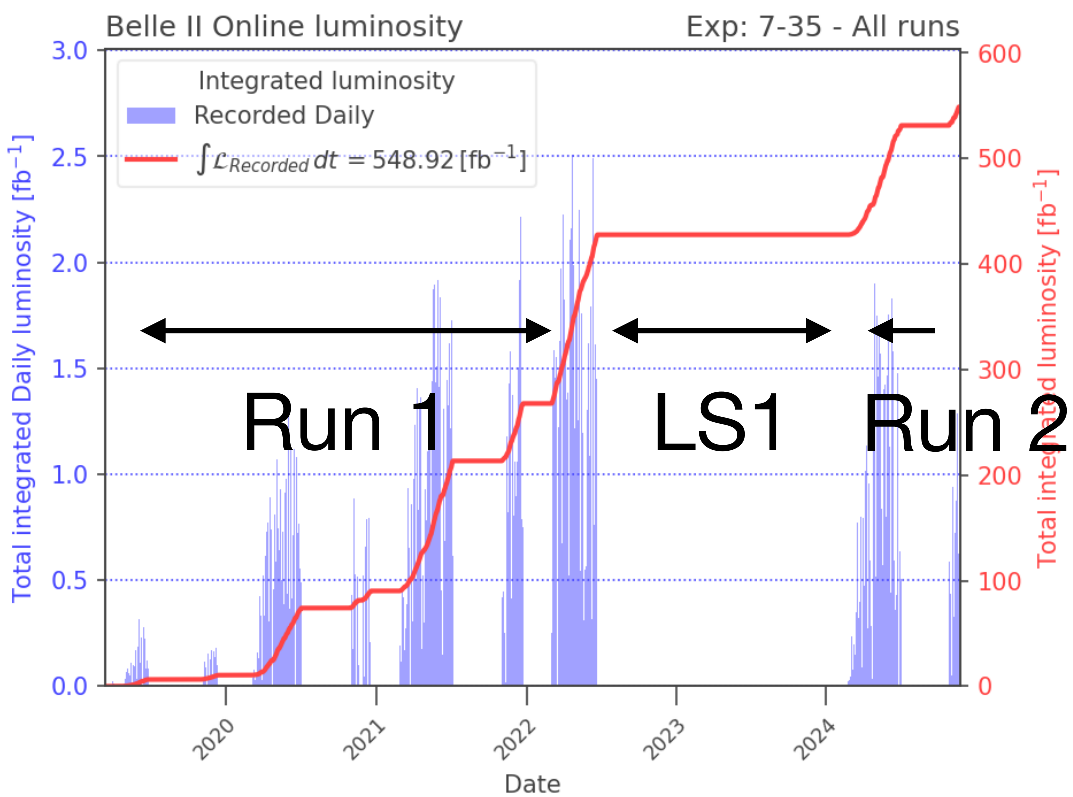
+ more, e.g. arXiv:2210.04224v2, arXiv:2301.07529, arXiv:2311.07248, arXiv:2211.09833

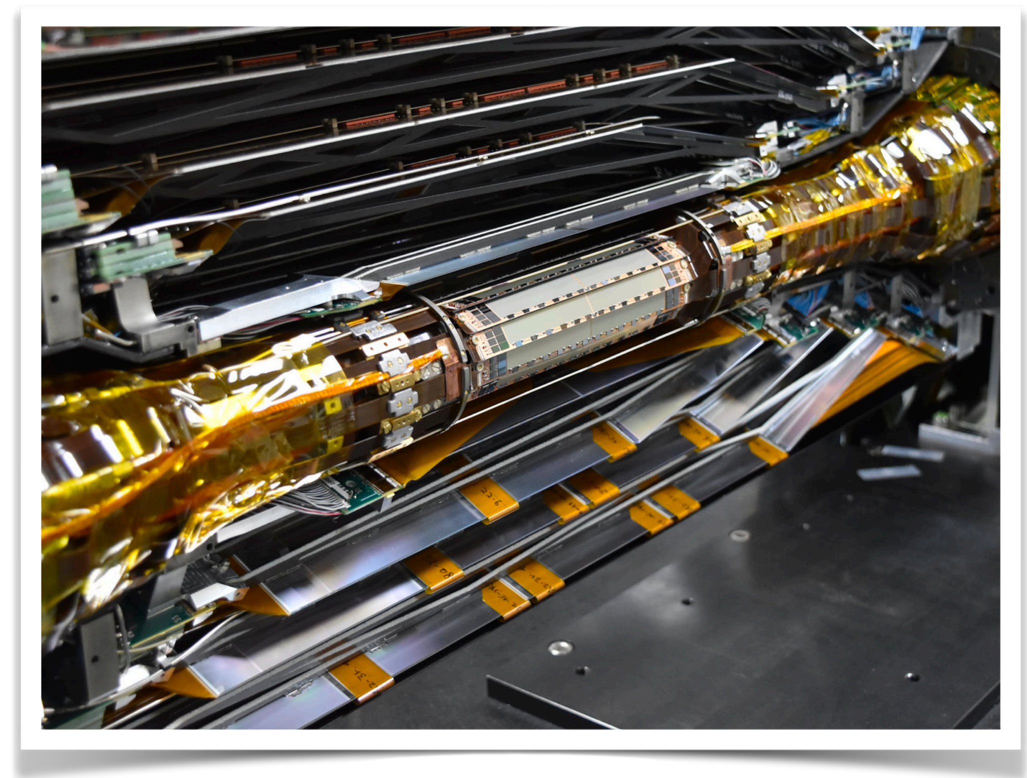
Belle II Status



Run 2 of experiment started Jan 29th 2024

Collected ca. 0.55/ab = BaBar





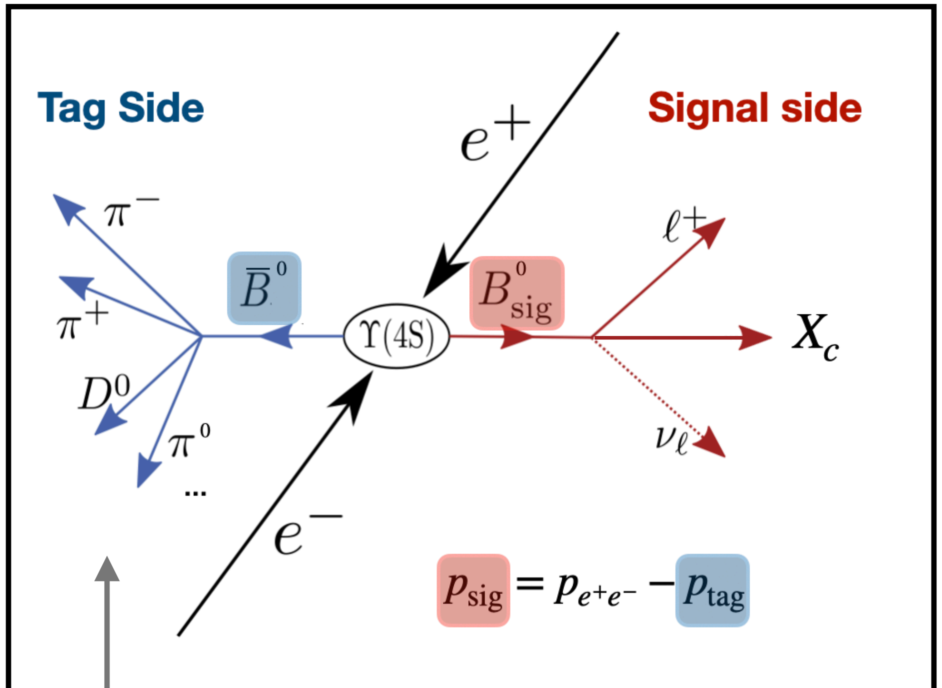
Current **status**:

Sudden beam losses of unknown origin hinder the collider to reach stable operations

→ Devoting significant fraction of running time for machine studies to understand instabilities

1. Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment [Phys. Rev. D 107, 072002, arXiv:2205.06372]

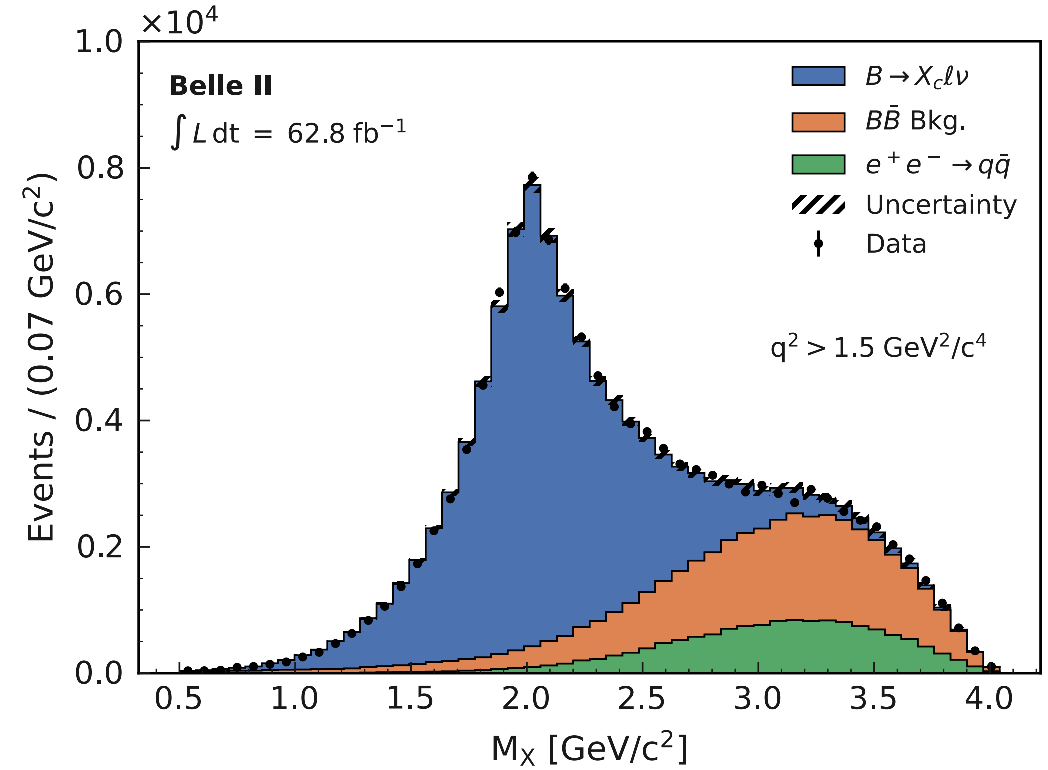
Key-technique: hadronic tagging

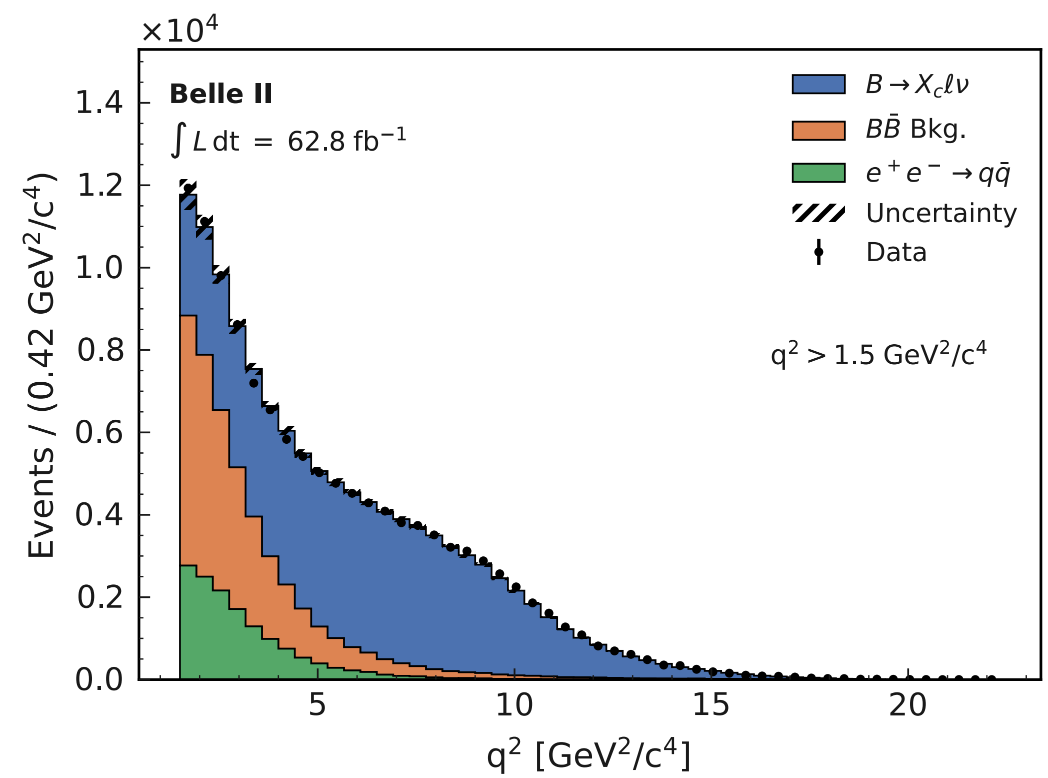


Can identify X_c constituents

$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$

$$q^2 = \left(p_{sig} - p_{X_c}\right)^2$$



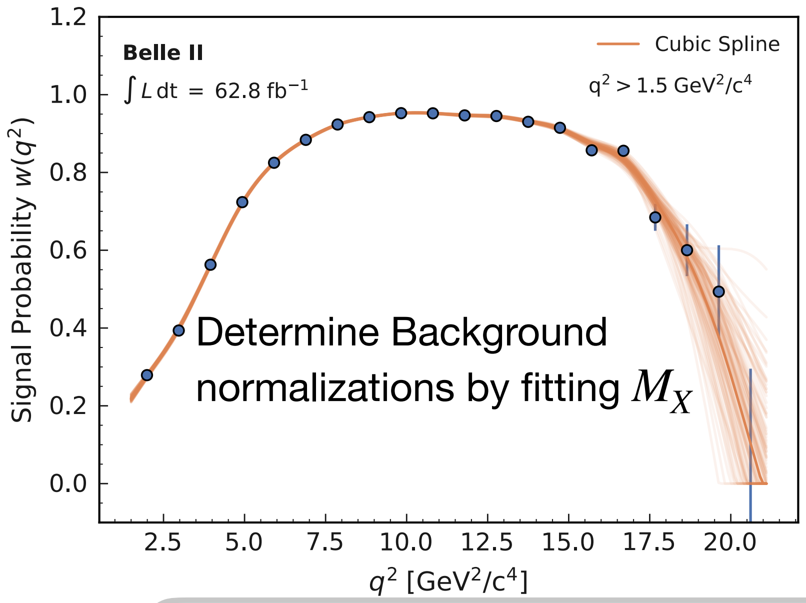


Improved Hadronic Tagging
using **Belle II** algorithm
(ca. 2 times more efficient)

[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]

1.

Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment [Phys. Rev. D 107, 072002, arXiv:2205.06372]



Step #1: Subtract Background

Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_\mathrm{data}} w(q_i^2) \times q_{\mathrm{calib},i}^{2n}}{\sum_j^{N_\mathrm{data}} w(q_j^2)} \times \mathcal{C}_\mathrm{calib} \times \mathcal{C}_\mathrm{gen}$$

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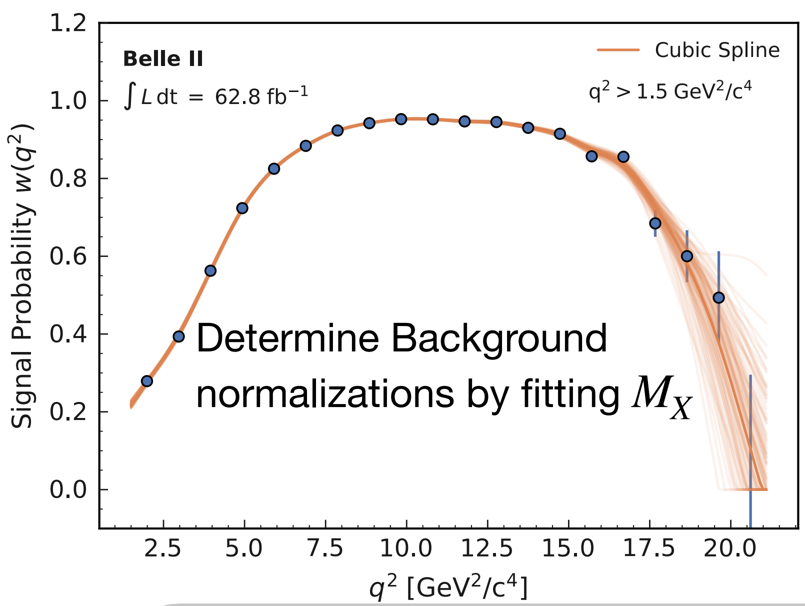

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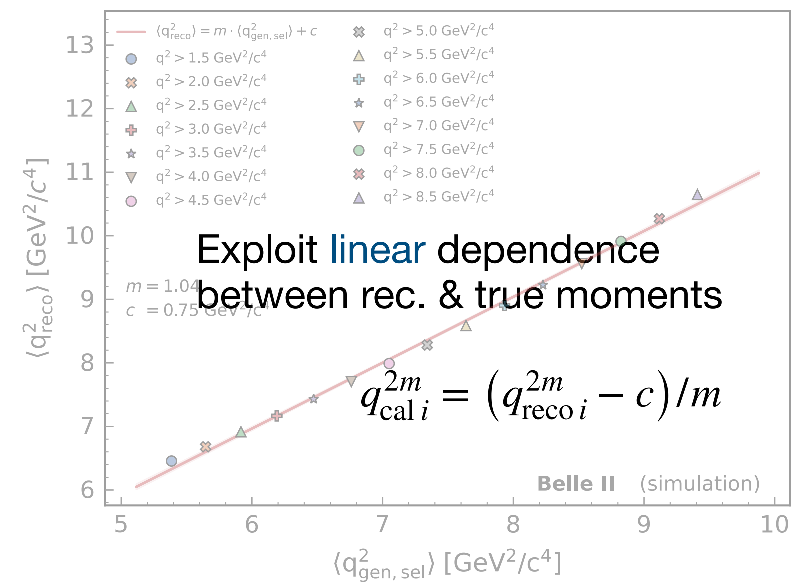

Step #2: Calibrate moment

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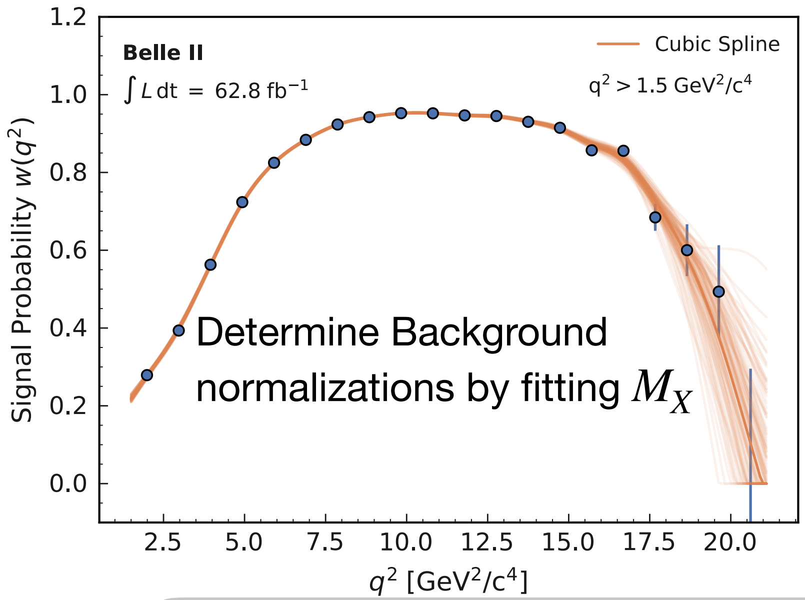

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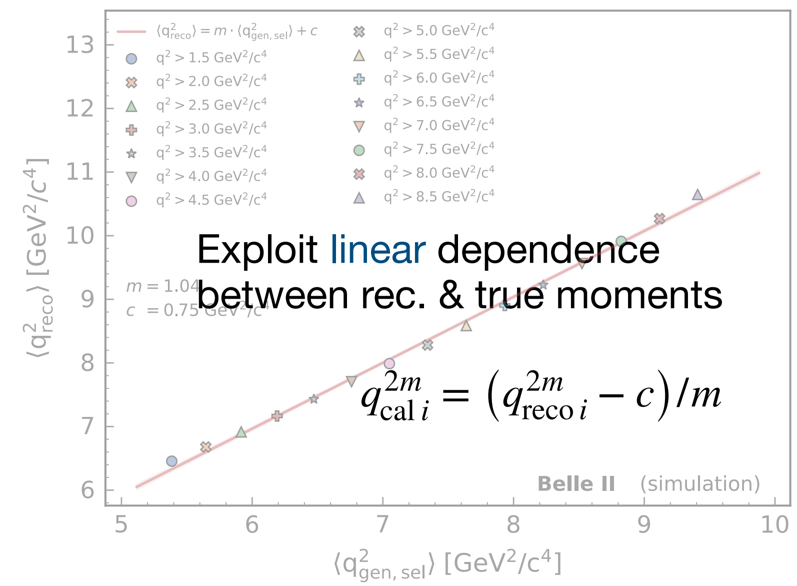

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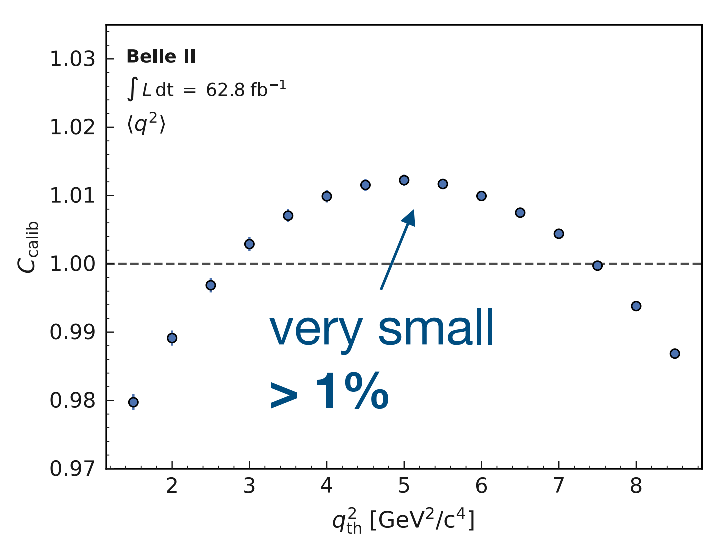
Step #3: If you fail, try again



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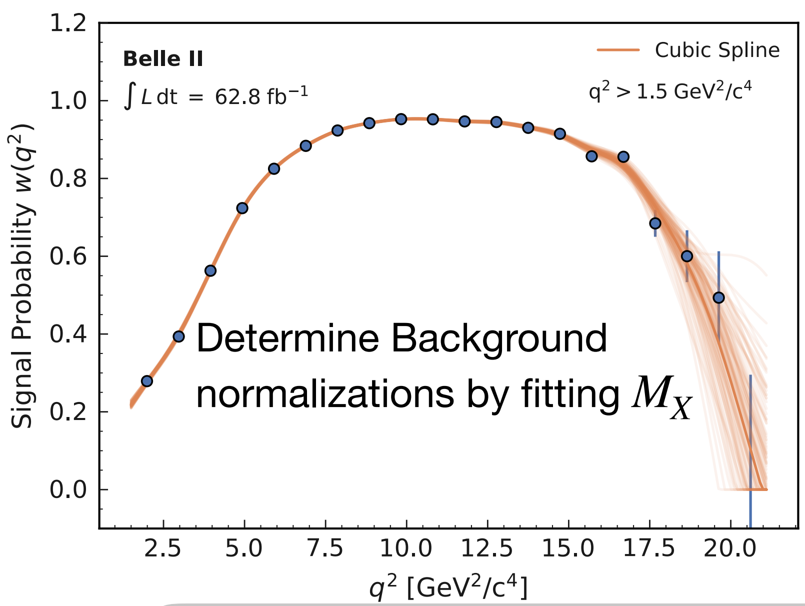

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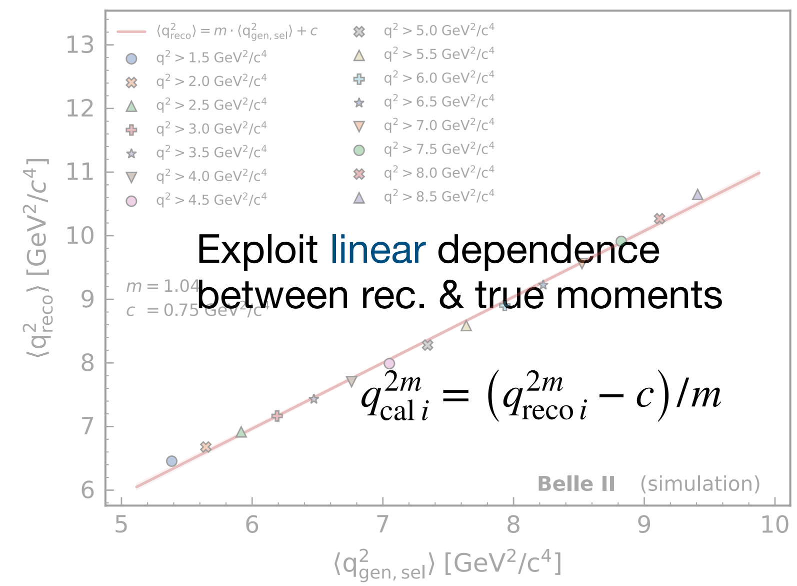

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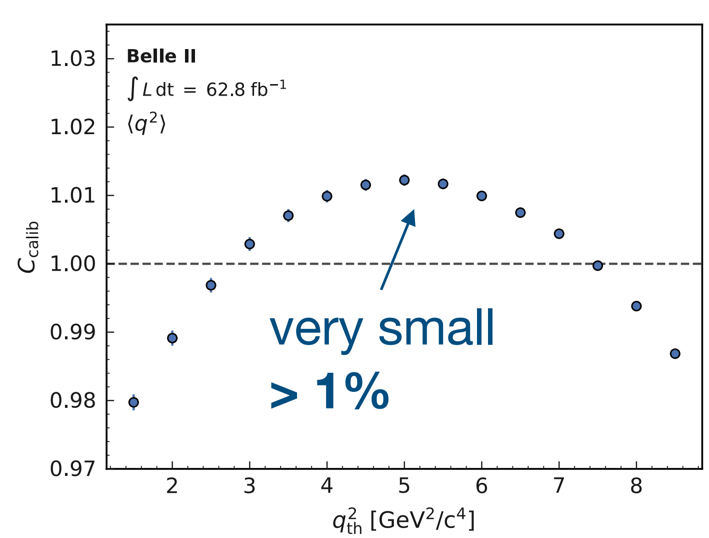
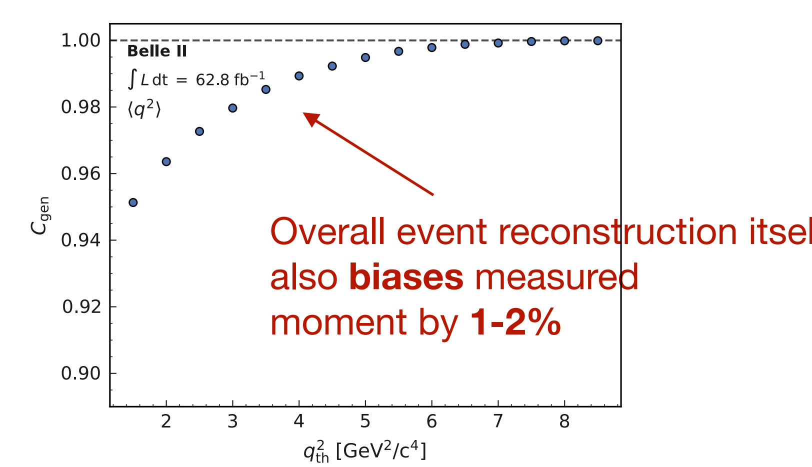
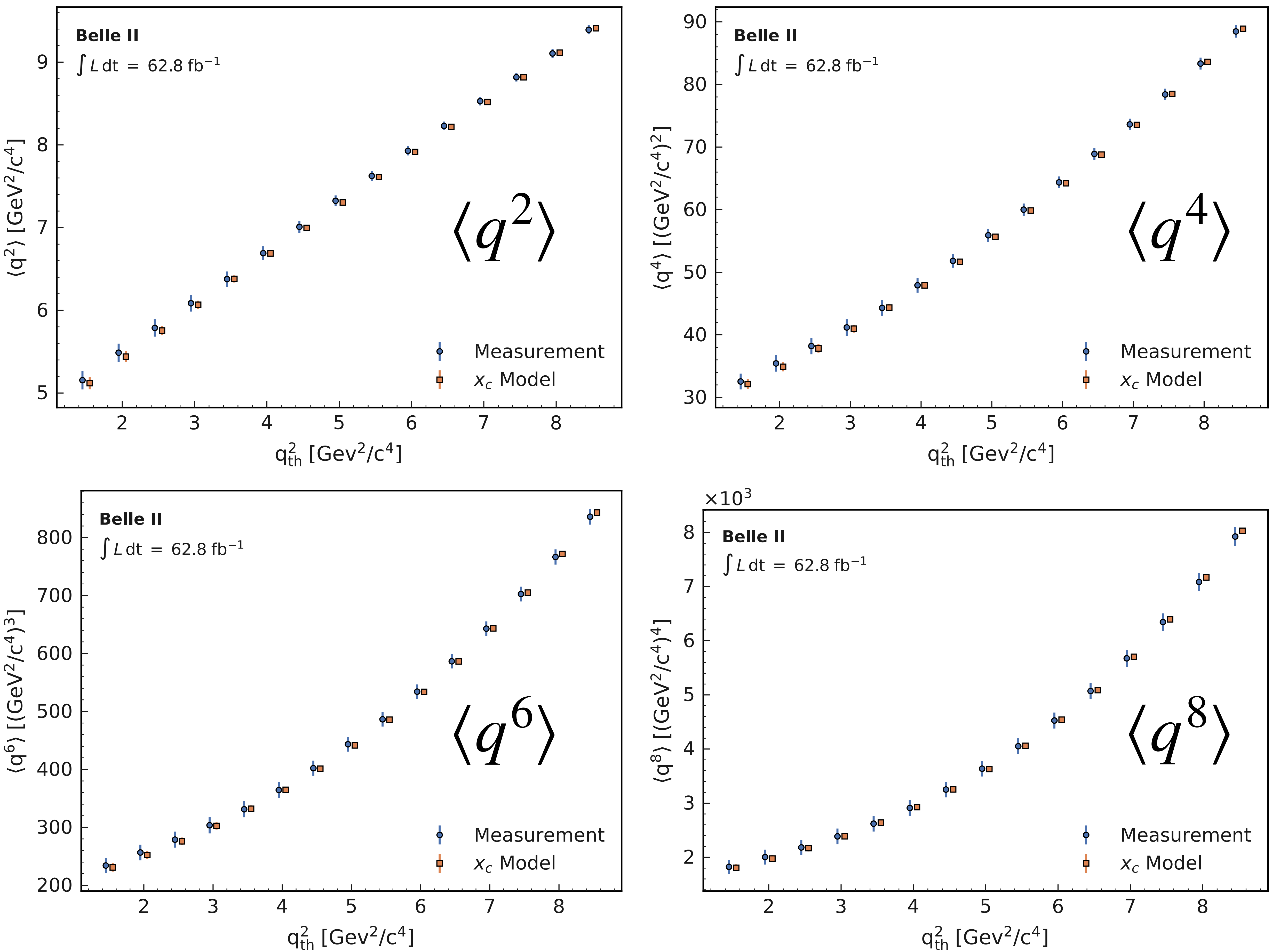
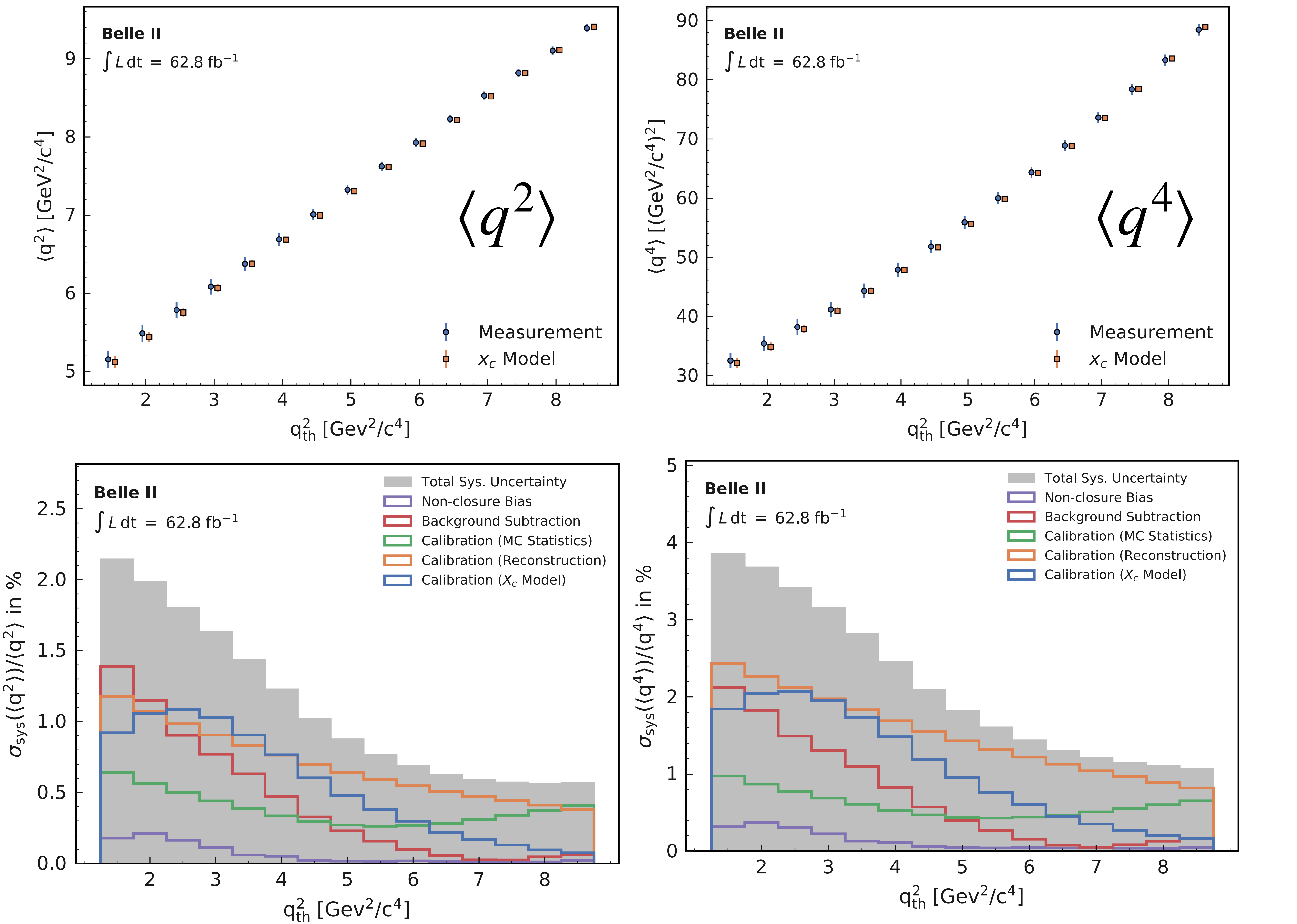


Step #4: Correct for selection effects




Belle II
$\int L\,dt = 62.8\ \text{fb}^{-1}$
$\langle q^2 \rangle$
$\langle q^2 \rangle$ [GeV^2/c^4]
q^2_{th} [Gev^2/c^4]
Measurement
x_c Model
Belle II
$\int L\,dt = 62.8\ \text{fb}^{-1}$
$\langle q^4 \rangle$
$\langle q^4 \rangle$ [(GeV^2/c^4)^2]
q^2_{th} [Gev^2/c^4]
Measurement
x_c Model
Belle II
$\int L\,dt = 62.8\ \text{fb}^{-1}$
$\langle q^6 \rangle$
$\langle q^6 \rangle$ [(GeV^2/c^4)^3]
q^2_{th} [Gev^2/c^4]
Measurement
x_c Model
$\times 10^3$
Belle II
$\int L\,dt = 62.8\ \text{fb}^{-1}$
$\langle q^8 \rangle$
$\langle q^8 \rangle$ [(GeV^2/c^4)^4]
q^2_{th} [Gev^2/c^4]
Measurement
x_c Model



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Total Sys. Uncertainty
Non-closure Bias
Background Subtraction
Calibration (MC Statistics)
Calibration (Reconstruction)
Calibration (X_c Model)
$\sigma_{sys}(\langle q^2 \rangle)/\langle q^2 \rangle$ in %
q^2_{th} [Gev^2/c^4]
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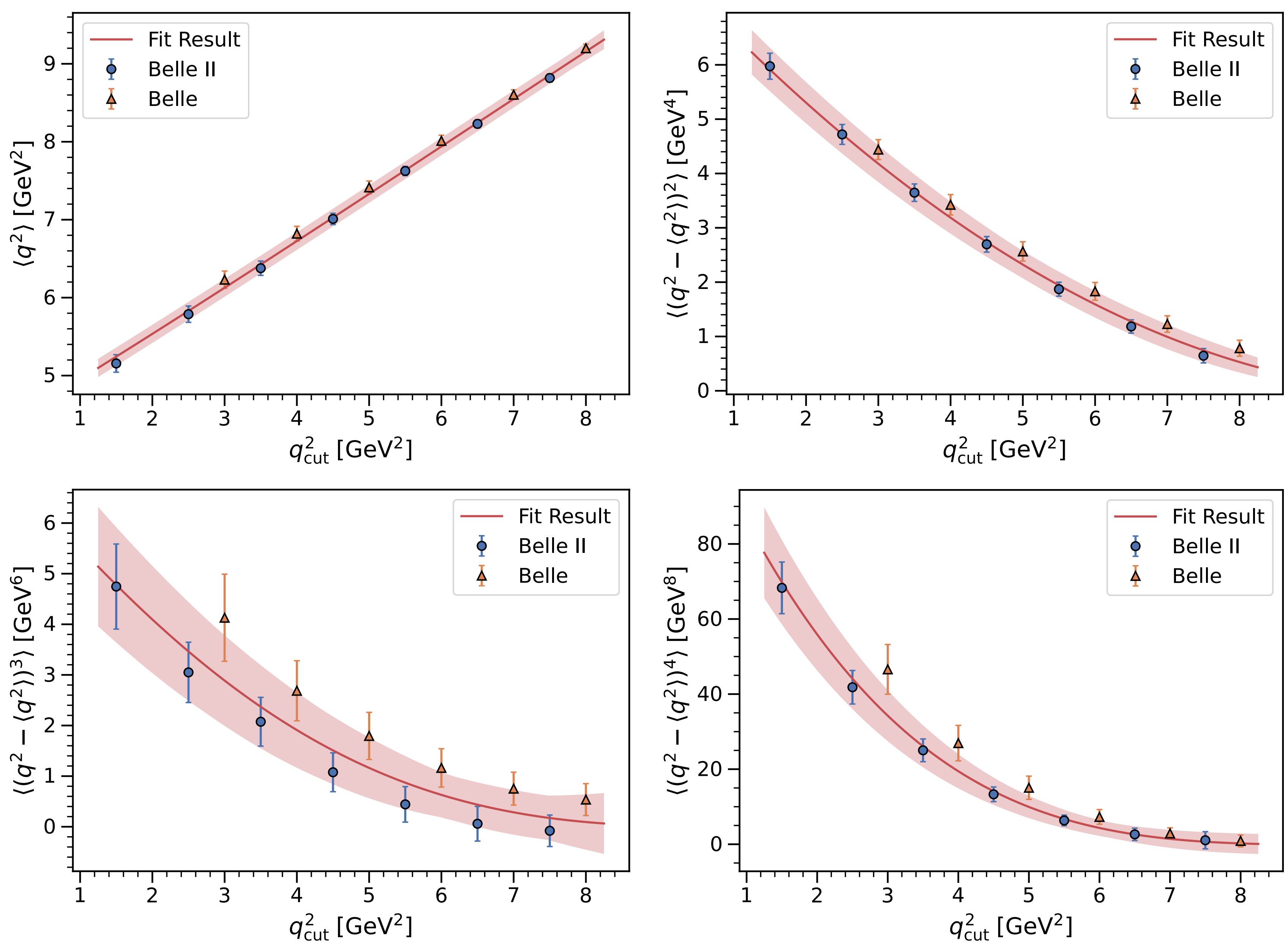
q^2_{th} [Gev^2/c^4]

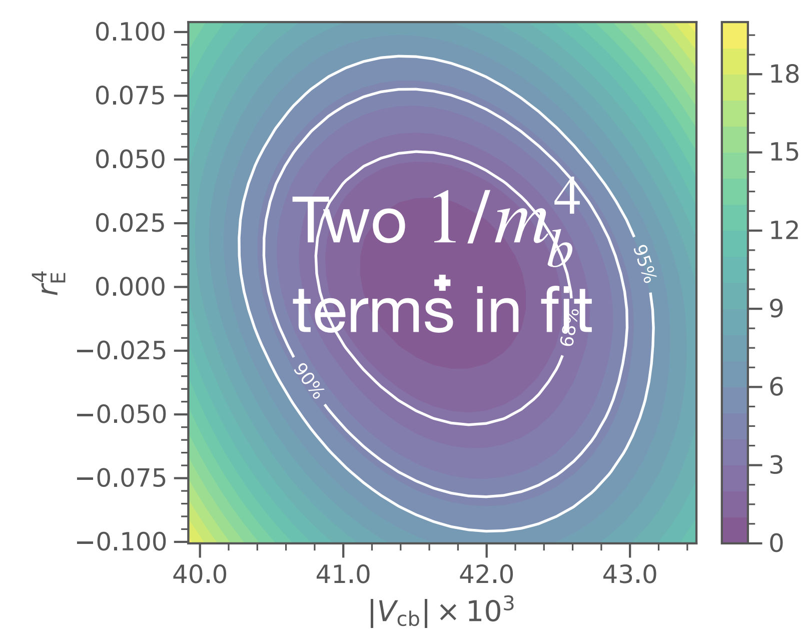
$|V_{cb}|$ from q^2 mom.

F. Bernlochner, M. Fael, K. Olschwesky, E. Persson,
R. Van Tonder, K. Vos, M. Welsch [JHEP 10 (2022) 068, arXiv:2205.10274]

First extraction of $|V_{cb}|$ from q^2 moments:



$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓		
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			

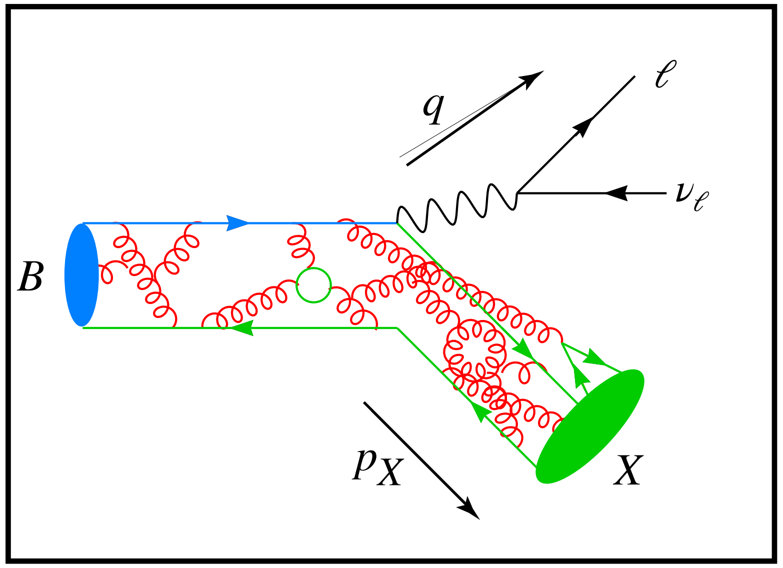


$$\longrightarrow \quad |V_{cb}| = (41.69 \pm 0.59|_{\mathrm{fit}} \pm 0.23|_{\mathrm{h.o.}}) \cdot 10^{-3} = (41.69 \pm 0.63) \cdot 10^{-3}$$

2. **First Measurement** of $\mathscr{R}(X_{\tau/\ell})$ as an Inclusive Test of the $b \to c\tau\bar{\nu}_\tau$ Anomaly

[Phys.Rev.Lett. 132 (2024) 21, 211804, arXiv:2311.07248]

Inclusive $B \to X\tau\bar{\nu}_\tau$ offer a great path to **cross check** anomalous behavior of $R(D^{(*)})$



$D, D^*, D^{**}, D^{(*)}\pi, \ldots$

Observable: $$R(X_{\tau/\ell}) = \frac{\mathscr{B}(B \to X\tau\bar{\nu}_e)}{\mathscr{B}(B \to X\ell\bar{\nu}_\mu)}$$

$\ell = e, \mu$

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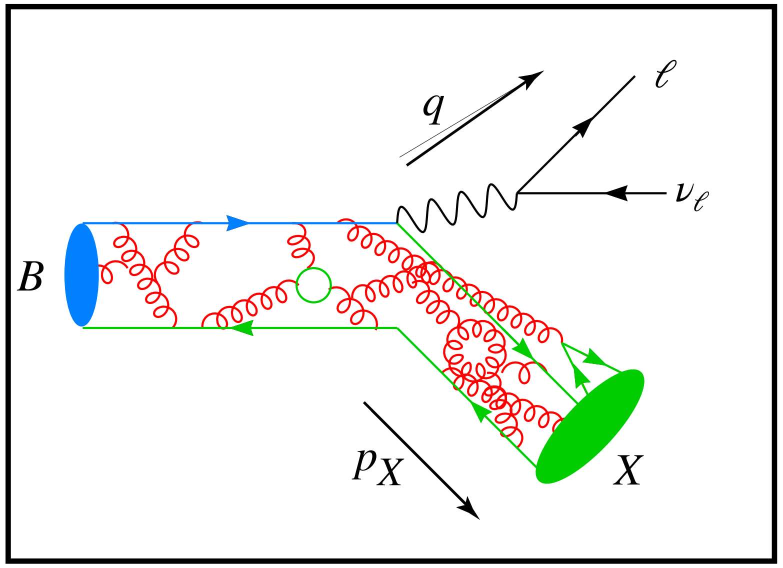

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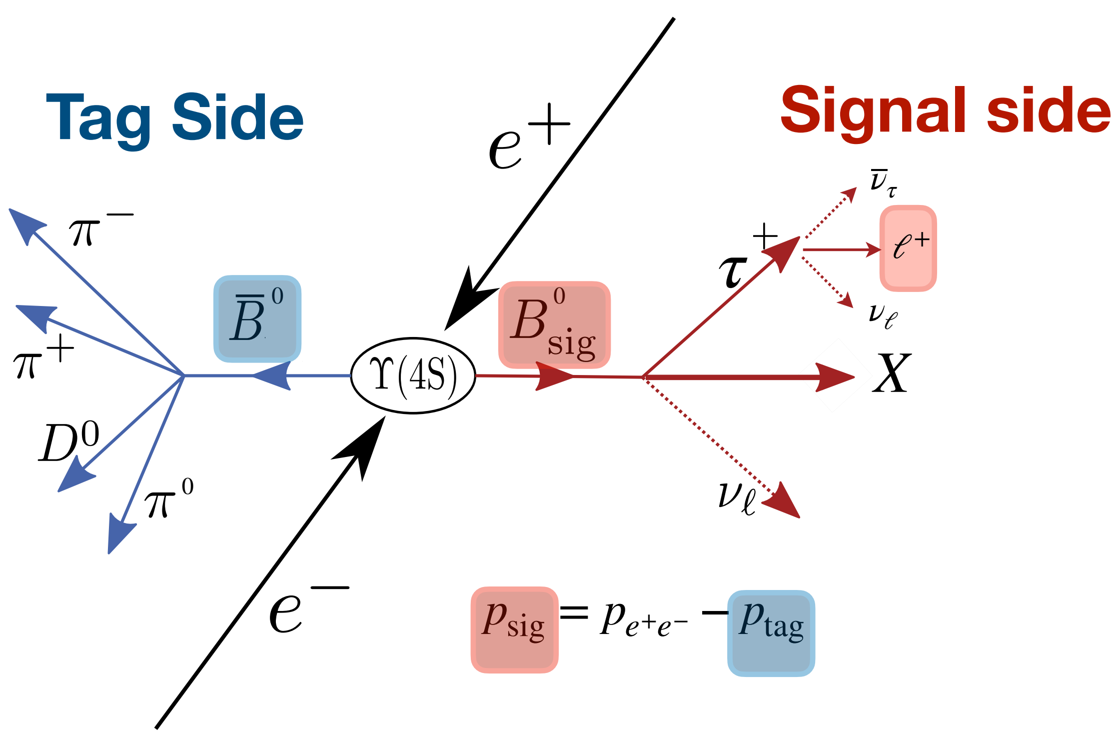
Observable: $$R(X_{\tau/\ell}) = \frac{\mathscr{B}(B \to X\tau\bar{\nu}_e)}{\mathscr{B}(B \to X\ell\bar{\nu}_\mu)}$$ $\ell = e, \mu$

Strategy: Use hadronic tagging to select sample of $B \to X\tau\bar{\nu}_\ell$ with $\tau \to \ell\bar{\nu}_\ell\nu_\tau$



Key variables:

p_ℓ^B : $M^2_{\text{miss}} = \left(p_{B_{\text{sig}}} - p_X - p_\ell\right)^2$

2. **First Measurement** of $\mathscr{R}(X_{\tau/\ell})$ as an Inclusive Test of the $b \to c\tau\bar{\nu}_\tau$ Anomaly

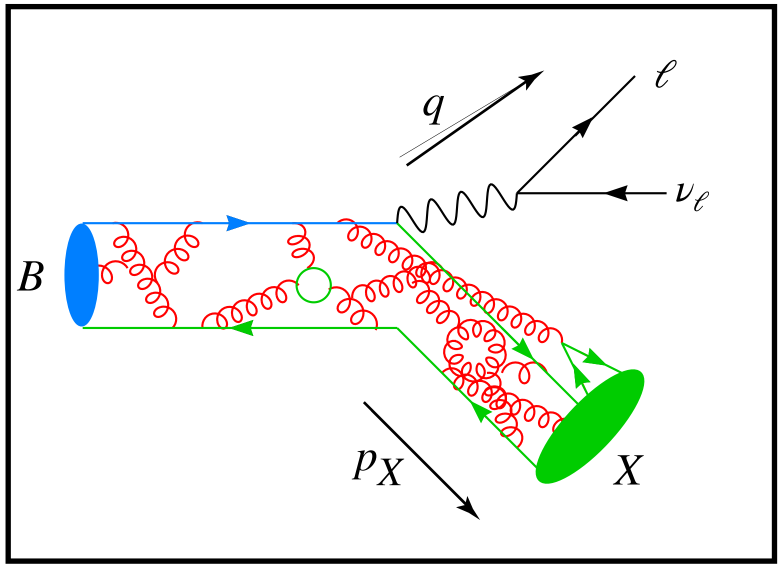
[Phys.Rev.Lett. 132 (2024) 21, 211804, arXiv:2311.07248]

Inclusive $B \to X\tau\bar{\nu}_\tau$ offer a great path to **cross check** anomalous behavior of $R(D^{(*)})$

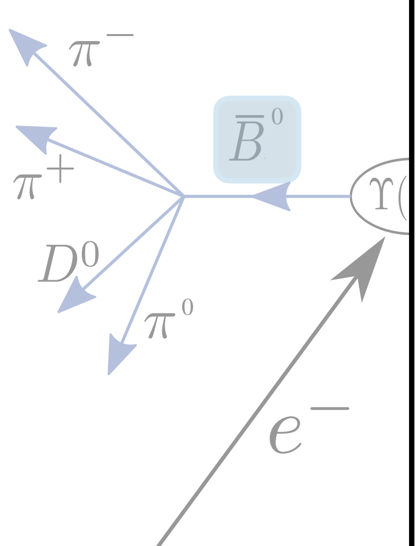
$D, D^*, D^{**}, D^{(*)}\pi, \ldots$

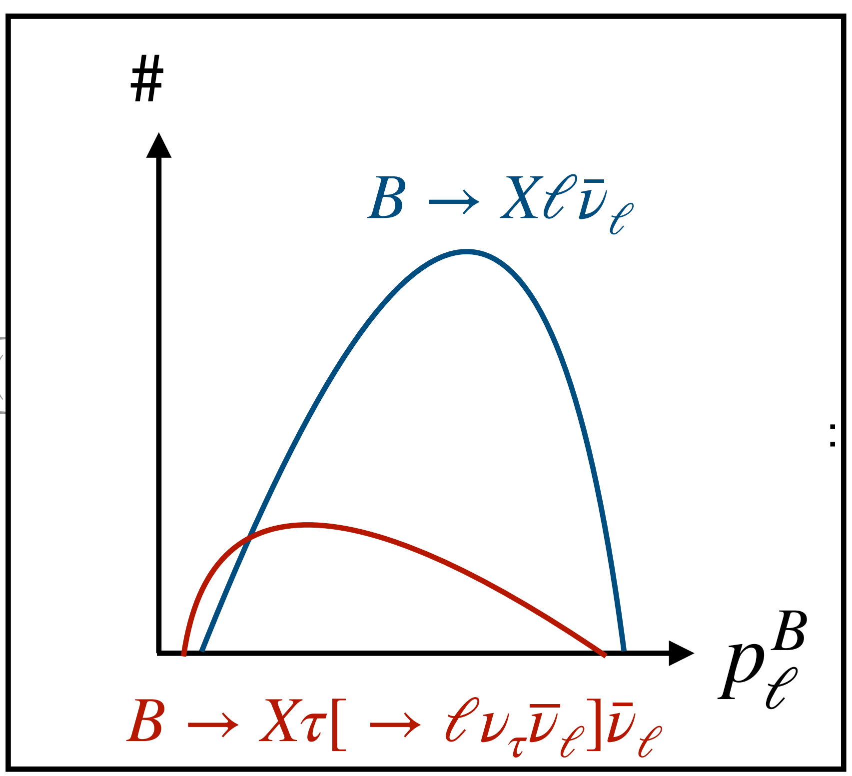
Observable: $$R(X_{\tau/\ell}) = \frac{\mathscr{B}(B \to X\tau\bar{\nu}_e)}{\mathscr{B}(B \to X\ell\bar{\nu}_\mu)}$$

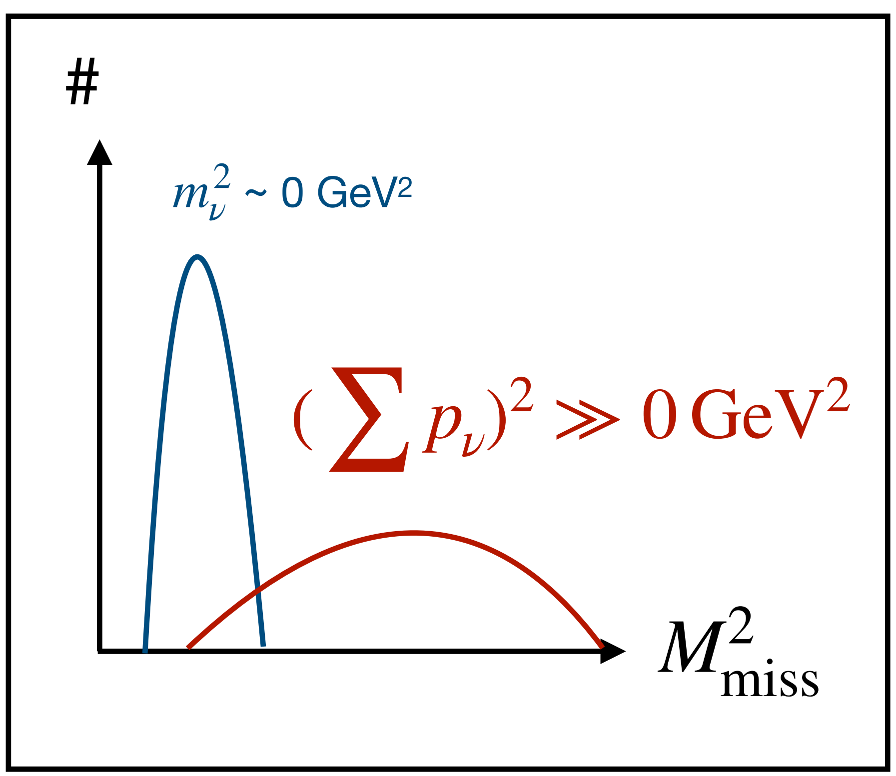
$\ell = e, \mu$

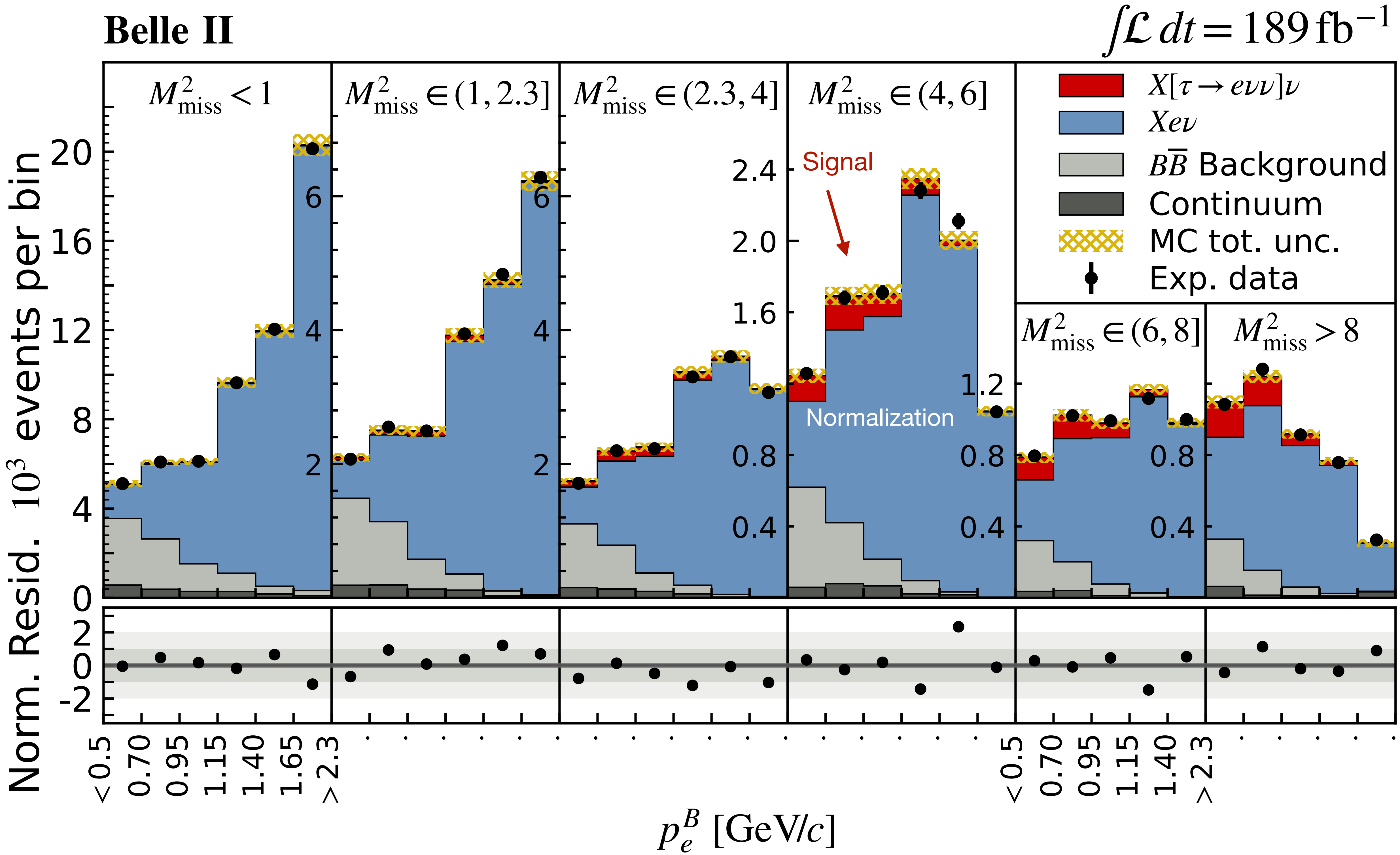


Strategy: Use hadronic tagging to select sample of $B \to X\tau\bar{\nu}_\ell$ with $\tau \to \ell\bar{\nu}_\ell\nu_\tau$



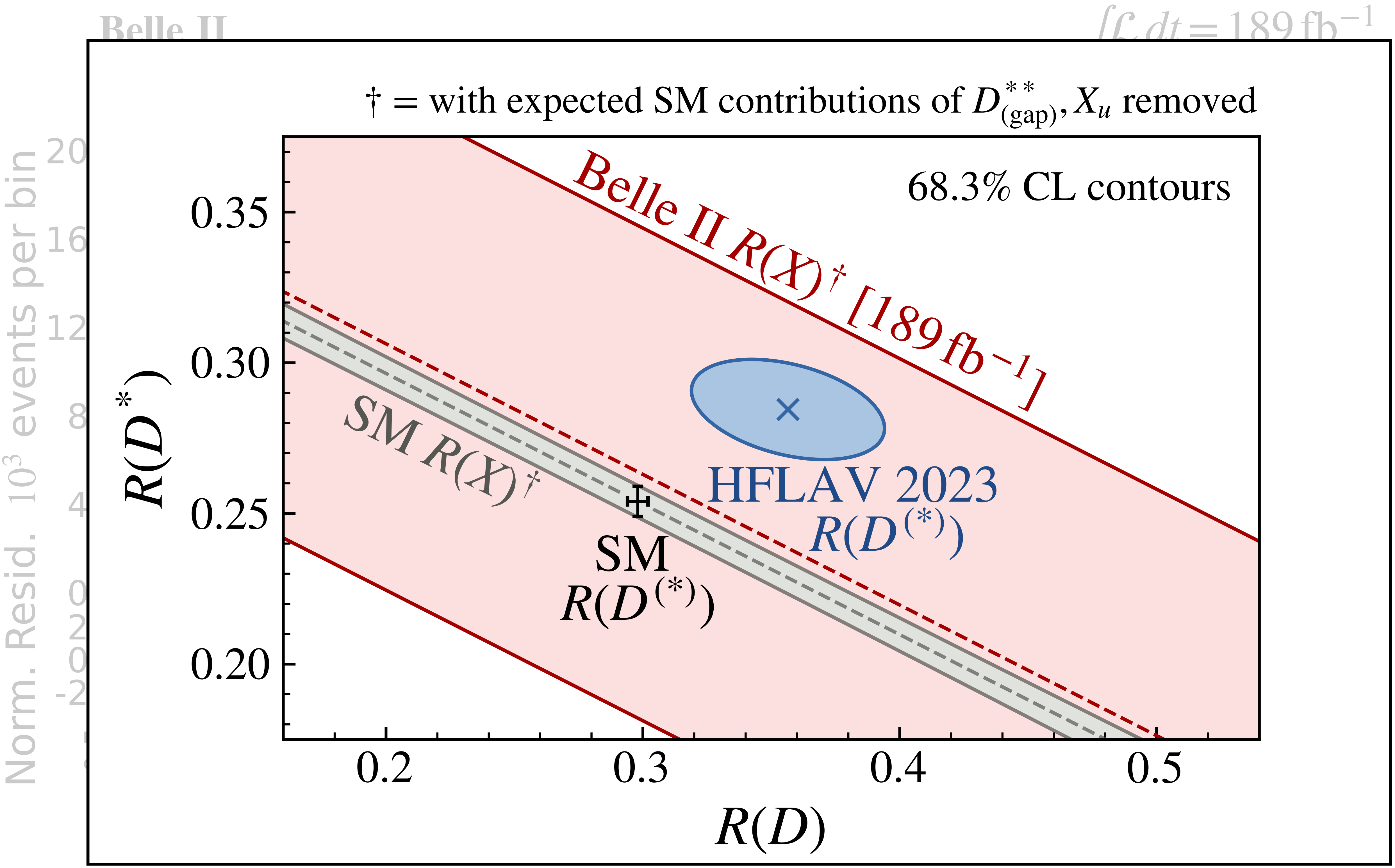






$$R(X_{\tau/\ell}) = \frac{\mathscr{B}(B \to X\tau\bar{\nu}_e)}{\mathscr{B}(B \to X\ell\bar{\nu}_\mu)} = 0.228 \pm 0.016(\text{stat}) \pm 0.036(\text{syst})$$

0.223 ± 0.005

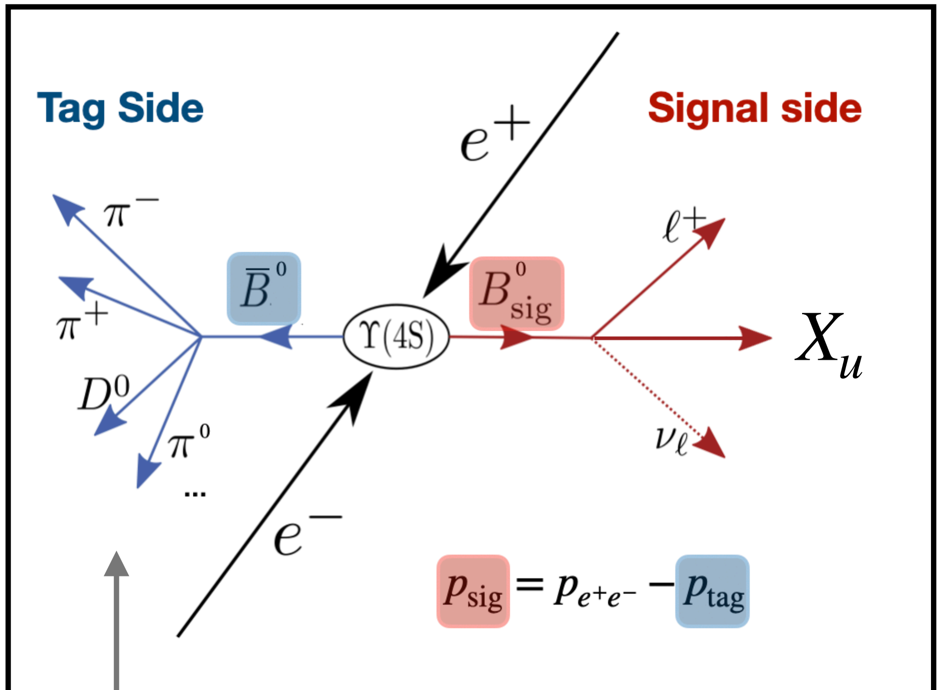


$$R(X_{\tau/\ell}) = \frac{\mathcal{B}(B \to X\tau\bar{\nu}_e)}{\mathcal{B}(B \to X\ell\bar{\nu}_\mu)} = 0.228 \pm 0.016(\text{stat}) \pm 0.036(\text{syst})$$

0.223 ± 0.005

3. First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$

[Phys.Rev.Lett. 131 (2023) 21, 211801, arXiv:2303.17309]



Belle | **Hadronic Tagging (FR)**

ca. factor of 2 less efficient,
but focus on cleaner tags

Hadronic **tagging** just is **fun**:
Capability to identify **kinematic** and **constituents** of X_u **system**

Charged Tracks | Neutral Clusters

$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j (E_j, \mathbf{k}_j)$$

$$q^2 = (p_{\rm sig} - p_X)^2$$

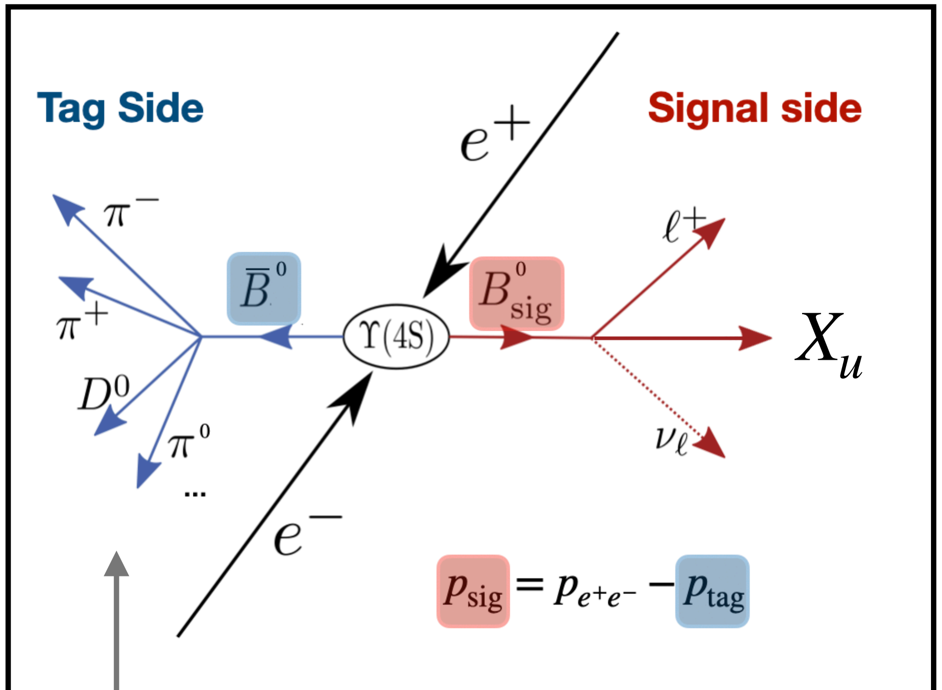
$$M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$$

$$m_{\rm miss}^2 = \left(p_{\rm sig} - p_X - p_\ell\right)^2 \approx m_\nu^2 = 0\,\mathrm{GeV}^2$$

But … this is still a pretty difficult measurement

3. First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$

[Phys.Rev.Lett. 131 (2023) 21, 211801, arXiv:2303.17309]

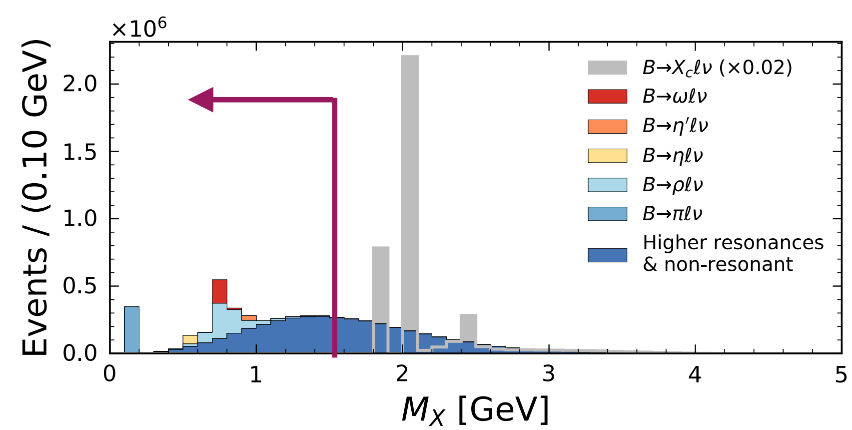


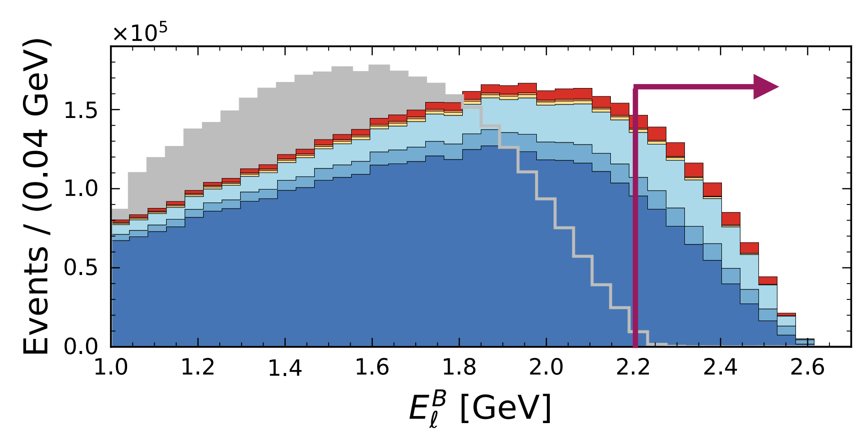
Belle I **Hadronic Tagging (FR)**

ca. factor of 2 less efficient,
but focus on cleaner tags

Inclusive $B \to X_u \ell \bar{\nu}_\ell$ measurements are extremely challenging due to dominant $B \to X_c \ell \bar{\nu}_\ell$ background

→ Clean **separation** only **possible** in certain **kinematic regions**, e.g. **lepton endpoint** or **low M_X**





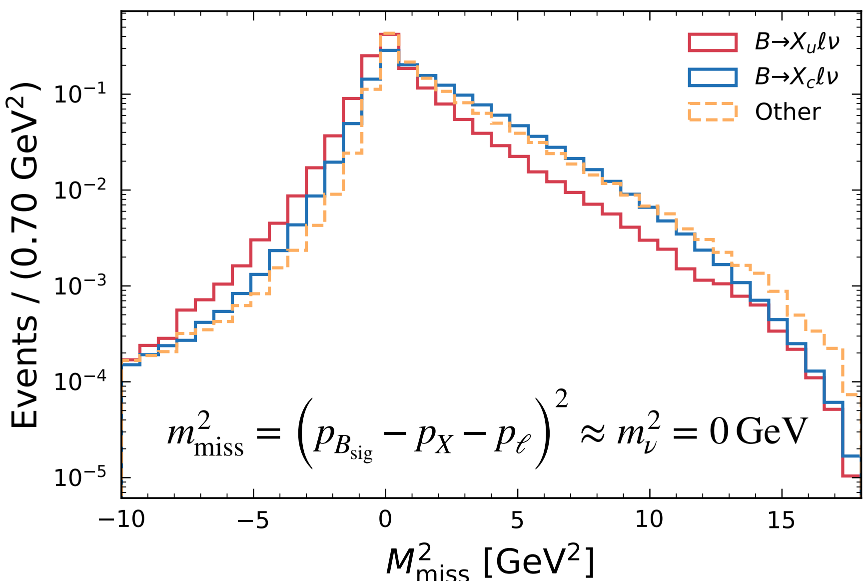
Multivariate Sledgehammer

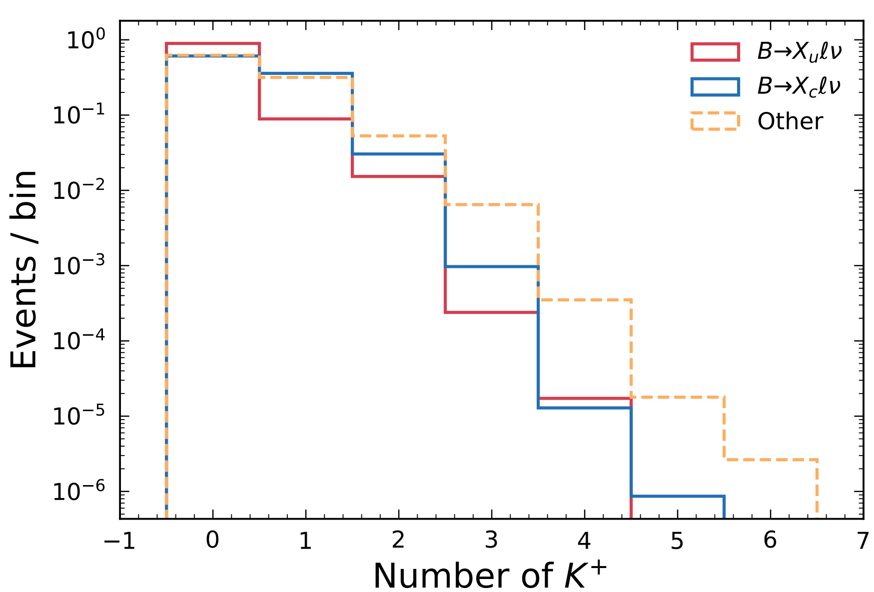
Can exploit that there are differences:

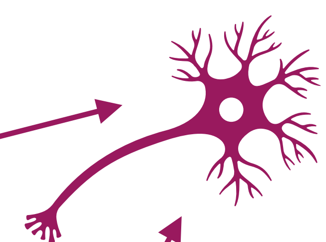
X_u ⟷ X_c

Direct cuts on m_X, E_ℓ **problematic** (i.e. direct theory / shape-function dependence)

Higher multiplicity
Often come with charged and neutral **Kaons**
D* decays (**slow pions**)
(**Slightly lower** E_ℓ)

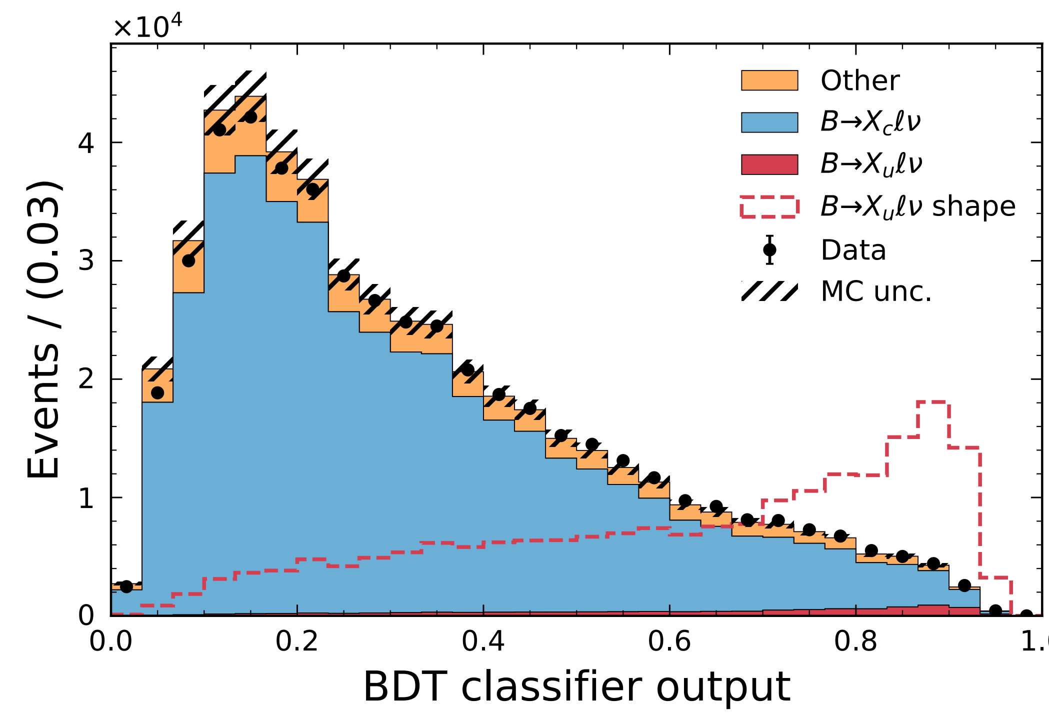






+ 9 other variables

=



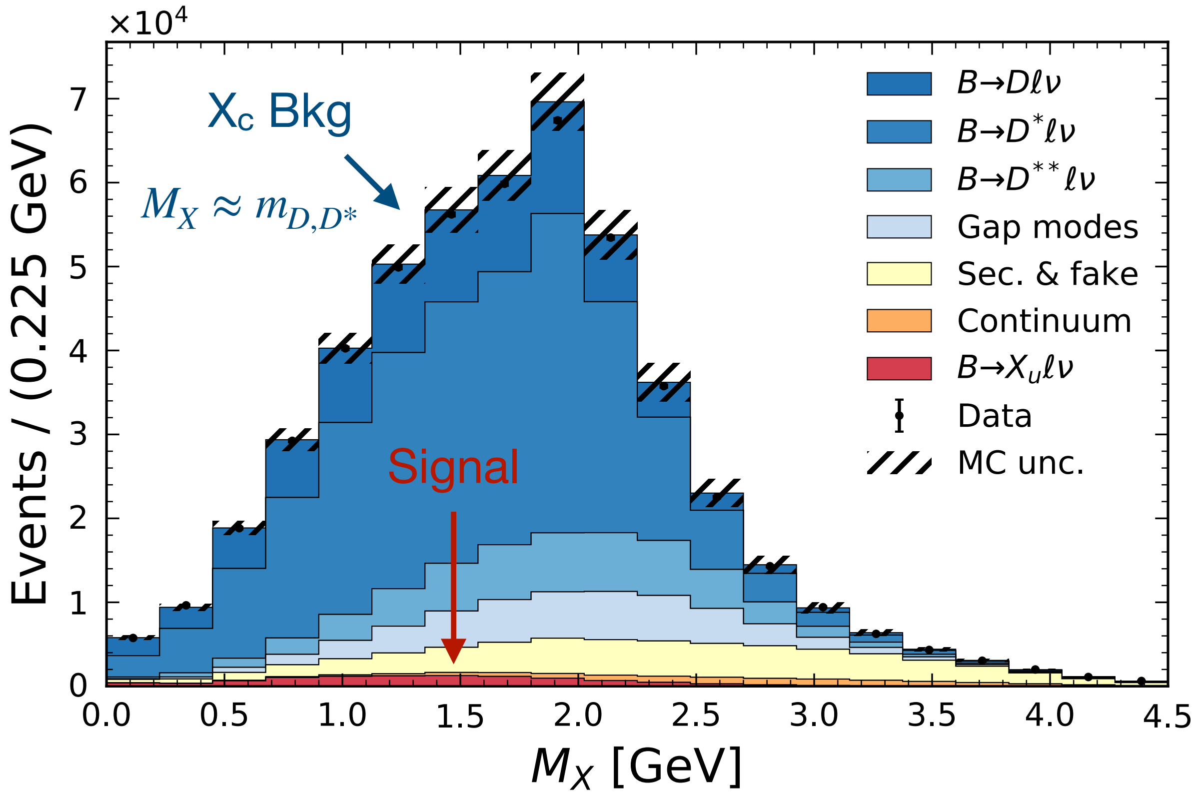
Can reject **98.7%** of X_c

Selection	$B \to X_u\,\ell^+\,\nu_\ell$	$B \to X_c\,\ell^+\,\nu_\ell$	Data
$M_{\text{bc}} > 5.27\,\text{GeV}$	84.8%	83.8%	80.2%
$\mathcal{O}_{\text{BDT}} > 0.85$	18.5%	1.3%	1.6%
$\mathcal{O}_{\text{BDT}} > 0.83$	21.9%	1.7%	2.1%
$\mathcal{O}_{\text{BDT}} > 0.87$	14.5%	0.9%	1.1%

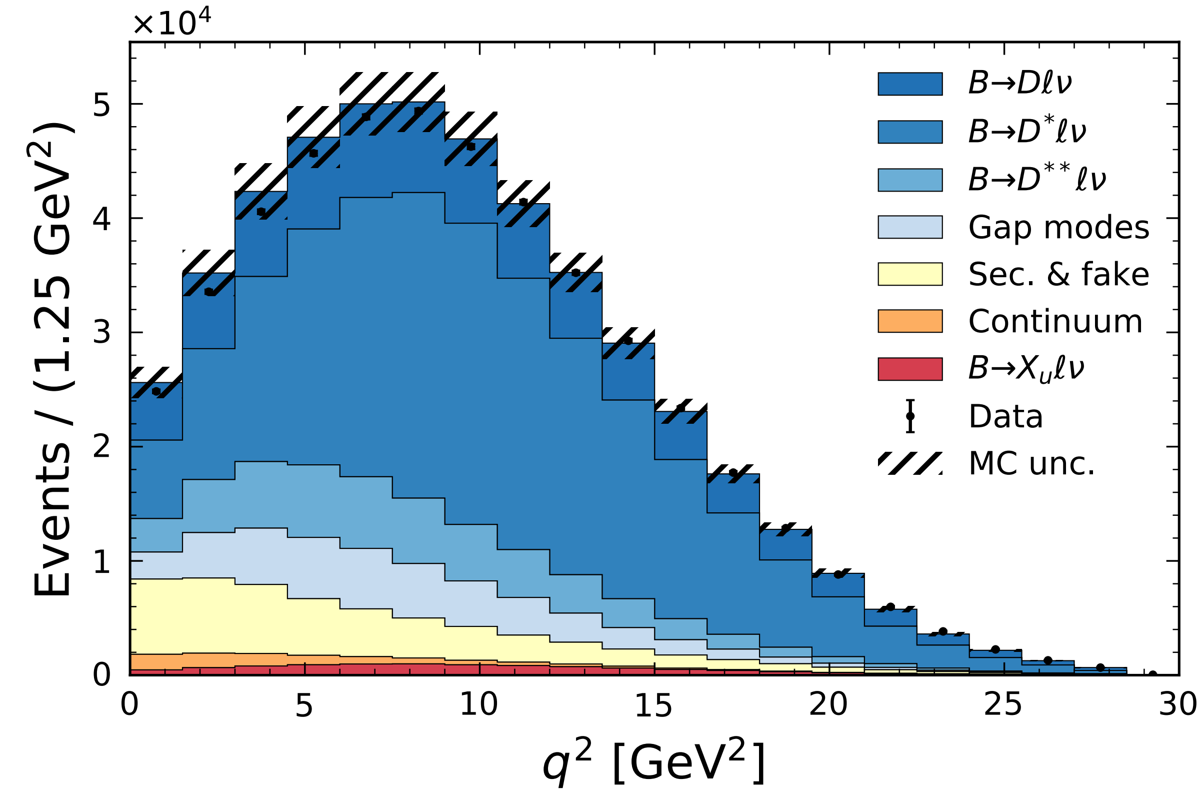
... and retain **18.5%** of X_u

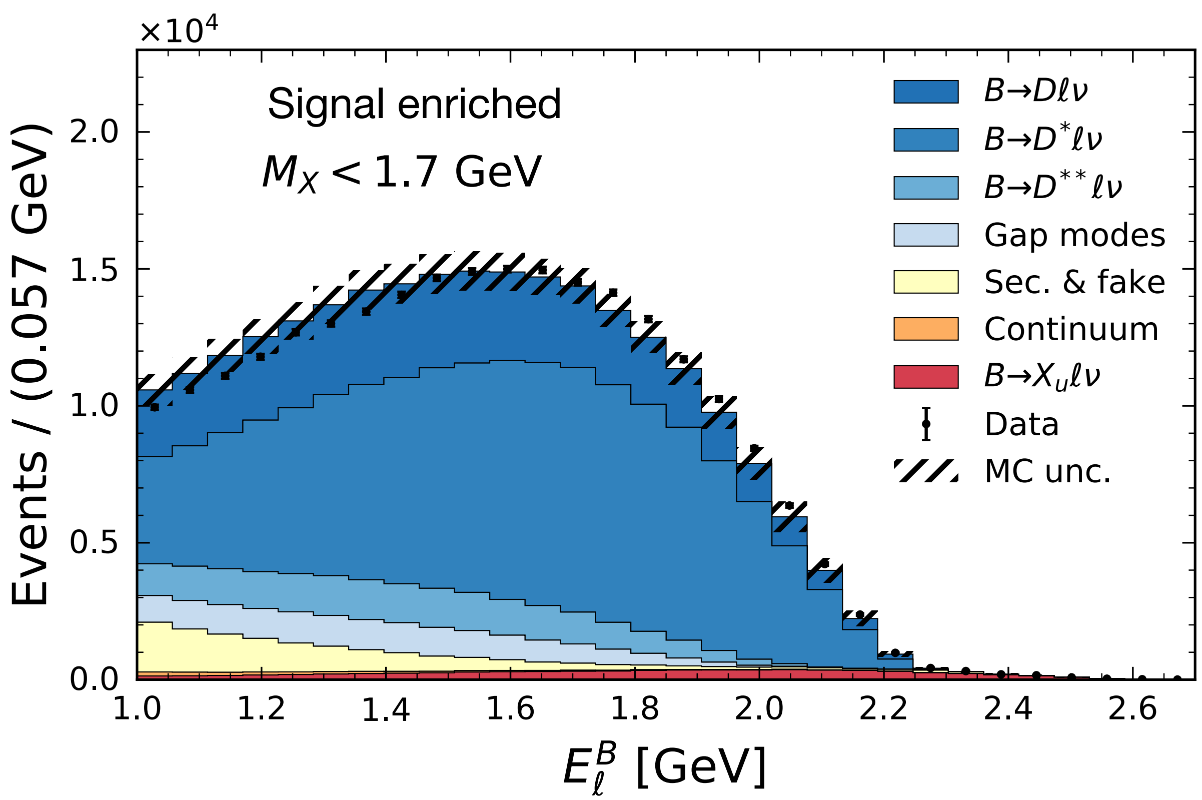
Before BDT selection

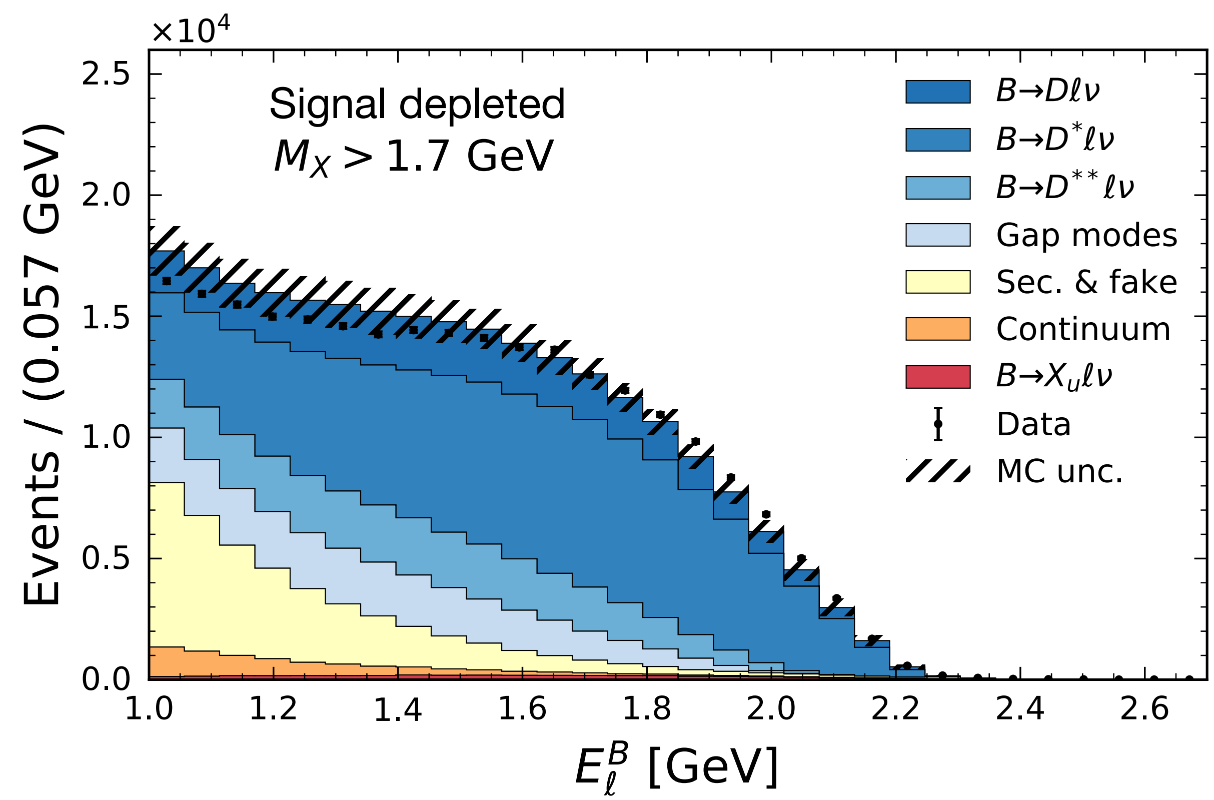
Hadronic Mass $M_X = \sqrt{p_X^2}$



Four-momentum transfer squared $q^2 = (p_B - p_X)^2$





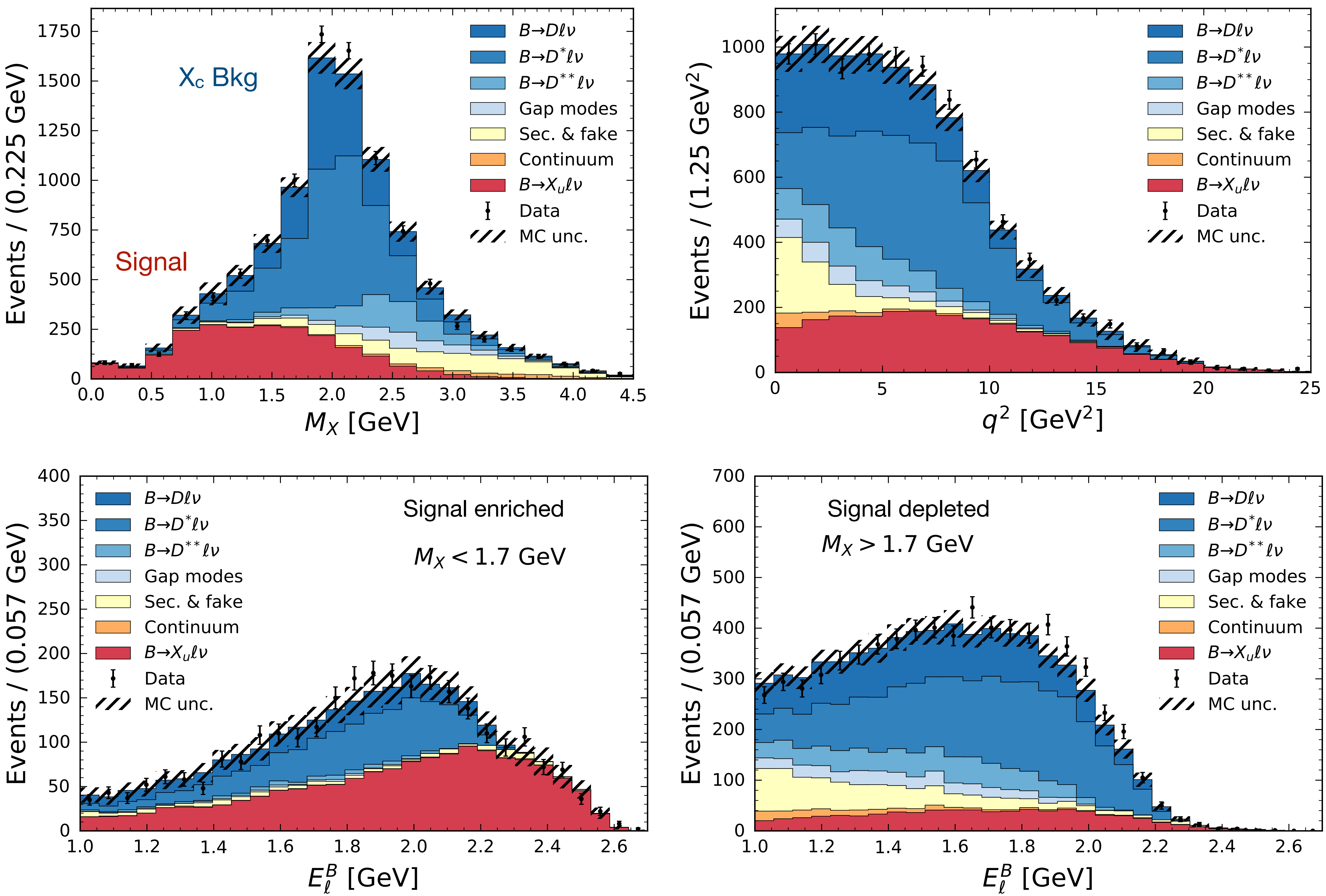


Lepton Energy in signal B rest frame E_ℓ^B

After BDT selection

Hadronic Mass $M_X = \sqrt{p_X^2}$

Four-momentum transfer squared $q^2 = (p_B - p_X)^2$

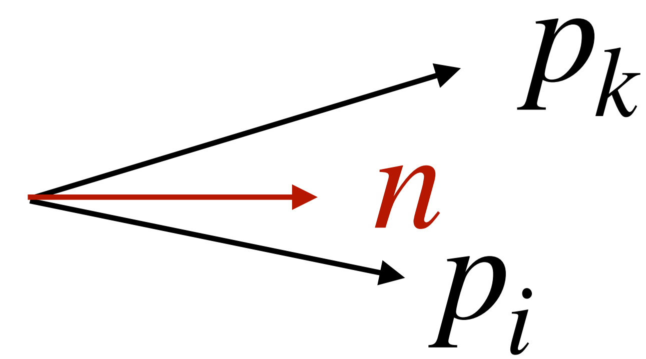


Lepton Energy in signal B rest frame E_ℓ^B

3. First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$

[Phys.Rev.Lett. 131 (2023) 21, 211801, arXiv:2303.17309]

New Idea: Exploit that exclusive X_u final states can be separated using the # of charged pions



X_u:

- $n_{\pi^+} = 0$: $B \to \pi^0 \ell \bar{\nu}_\ell$
- $n_{\pi^+} = 1$: $B \to \pi^+ \ell \bar{\nu}_\ell$
- $n_{\pi^+} = 2$: other $B \to X_u \ell \bar{\nu}_\ell$
- $n_{\pi^+} \geq 3$: other $B \to X_u \ell \bar{\nu}_\ell$

Use **'thrust'**, expect more collimated system for $B \to \pi^0 \ell \bar{\nu}_\ell$ and $B \to \pi^+ \ell \bar{\nu}_\ell$ than for other processes

$$\max_{|\mathbf{n}|=1} \left(\sum_i |\mathbf{p_i} \cdot \mathbf{n}| / \sum_i |\mathbf{p_i}| \right)$$

q^2 — Extraction of **BFs** and $B \to \pi$ **form factors**, in 2D fit of $q^2 : n_{\pi^+}$

M_X — Use high M_X to constrain $B \to X_c \ell \bar{\nu}_\ell$

3. First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$

[Phys.Rev.Lett. 131 (2023) 21, 211801, arXiv:2303.17309]

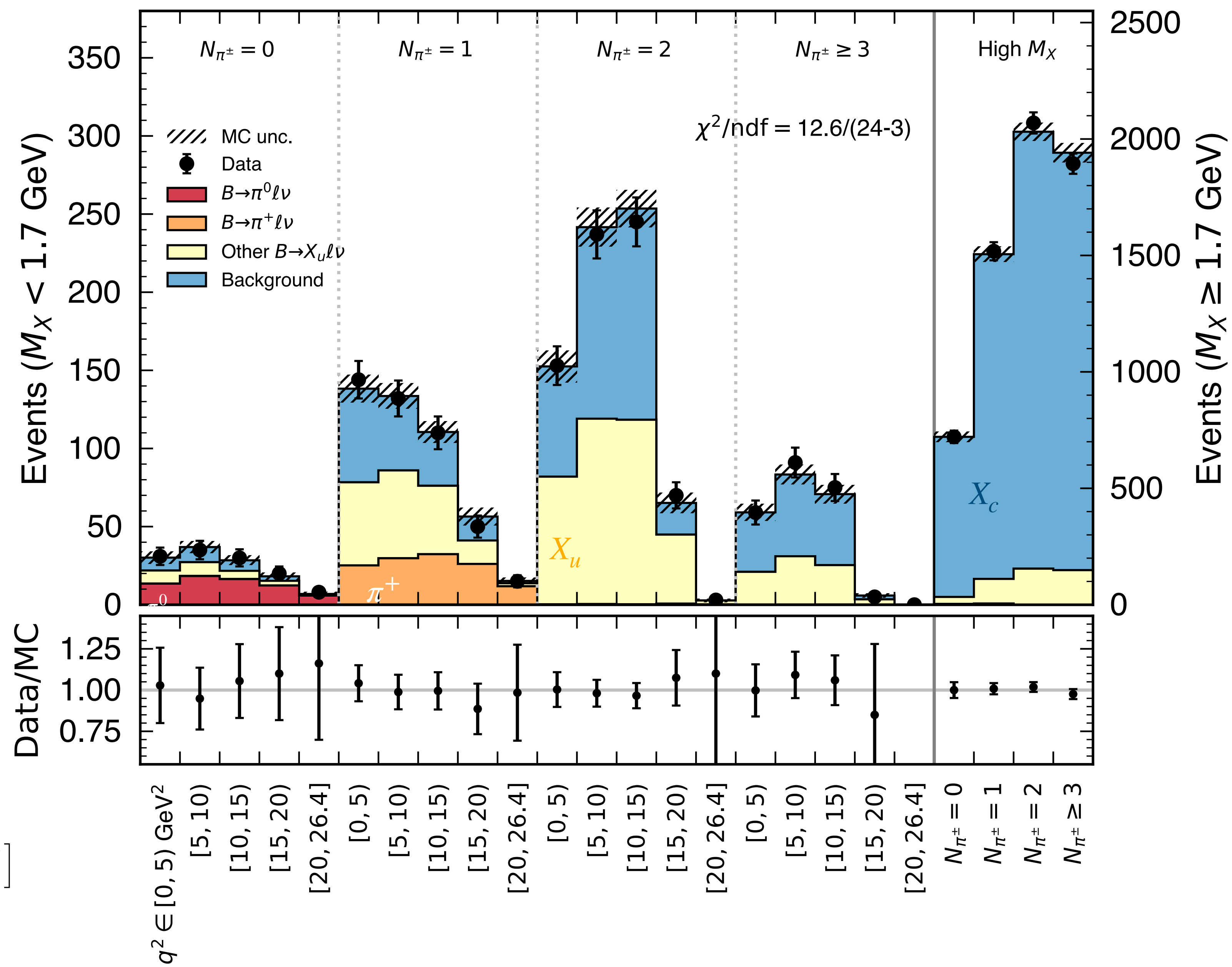
2D Categories :

For fit link

$B \to \pi^0 \ell \bar{\nu}_\ell$

$B \to \pi^+ \ell \bar{\nu}_\ell$

assuming **isospin**



Float BCL $B \to \pi$ FF constrained to **FLAG 2022** WA [Eur.Phys.J.C 82 (2022) 10, 869]

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{n=0}^{N^+-1} a_n^+ \left[z^n - (-1)^{n-N^+} \frac{n}{N^+} z^{N^+} \right]$$

$$f_0(q^2) = \sum_{n=0}^{N^0-1} a_n^0 z^n \,, \qquad (3)$$

→

$$\mathcal{B}(\overline{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell) = (1.43 \pm 0.19 \pm 0.13) \times 10^{-4} \,,$$
$$\Delta\mathcal{B}(B \to X_u \ell \bar{\nu}_\ell) = (1.40 \pm 0.14 \pm 0.23) \times 10^{-3} \,,$$

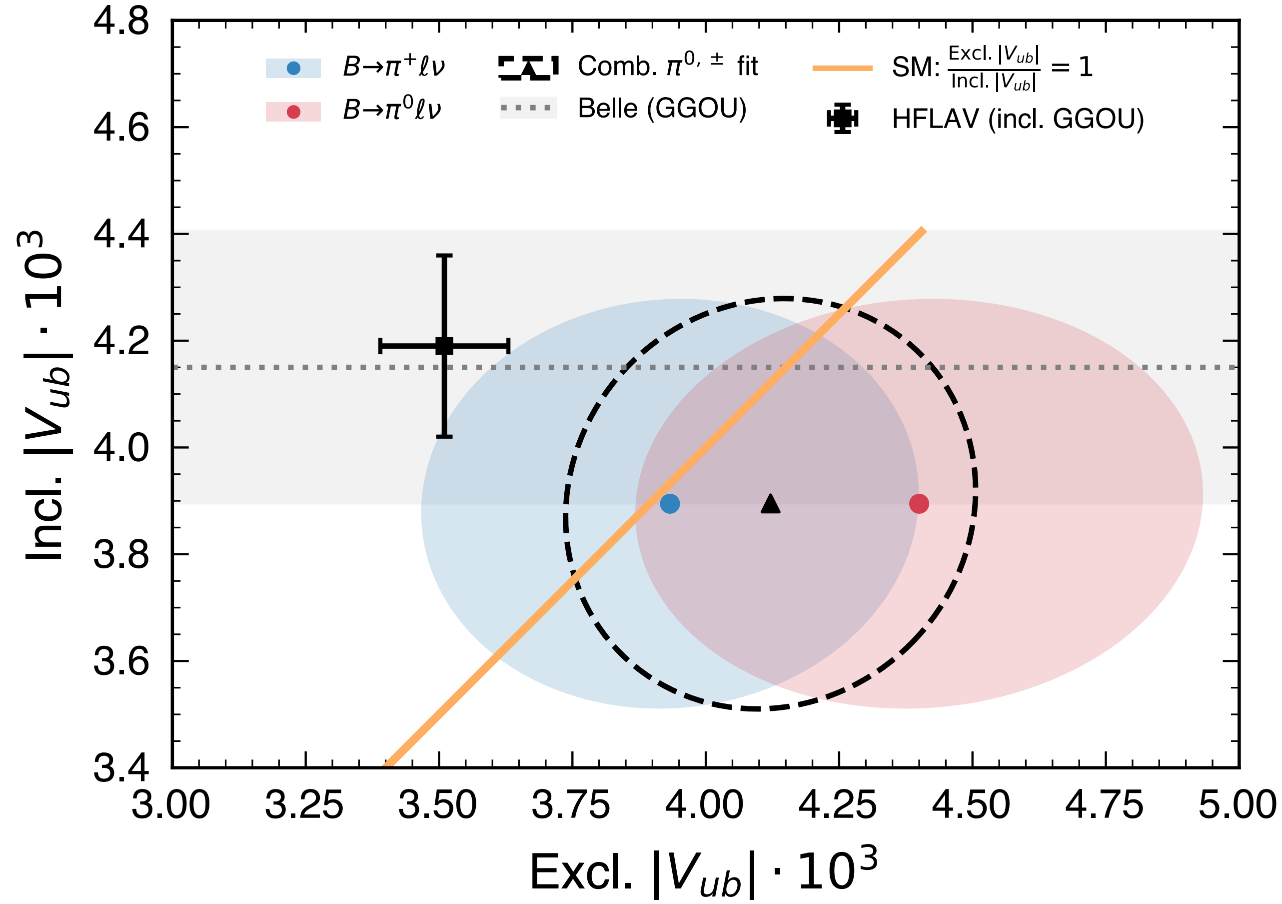
$\rho = 0.10$

Two sets of results:

1) FLAG 2022

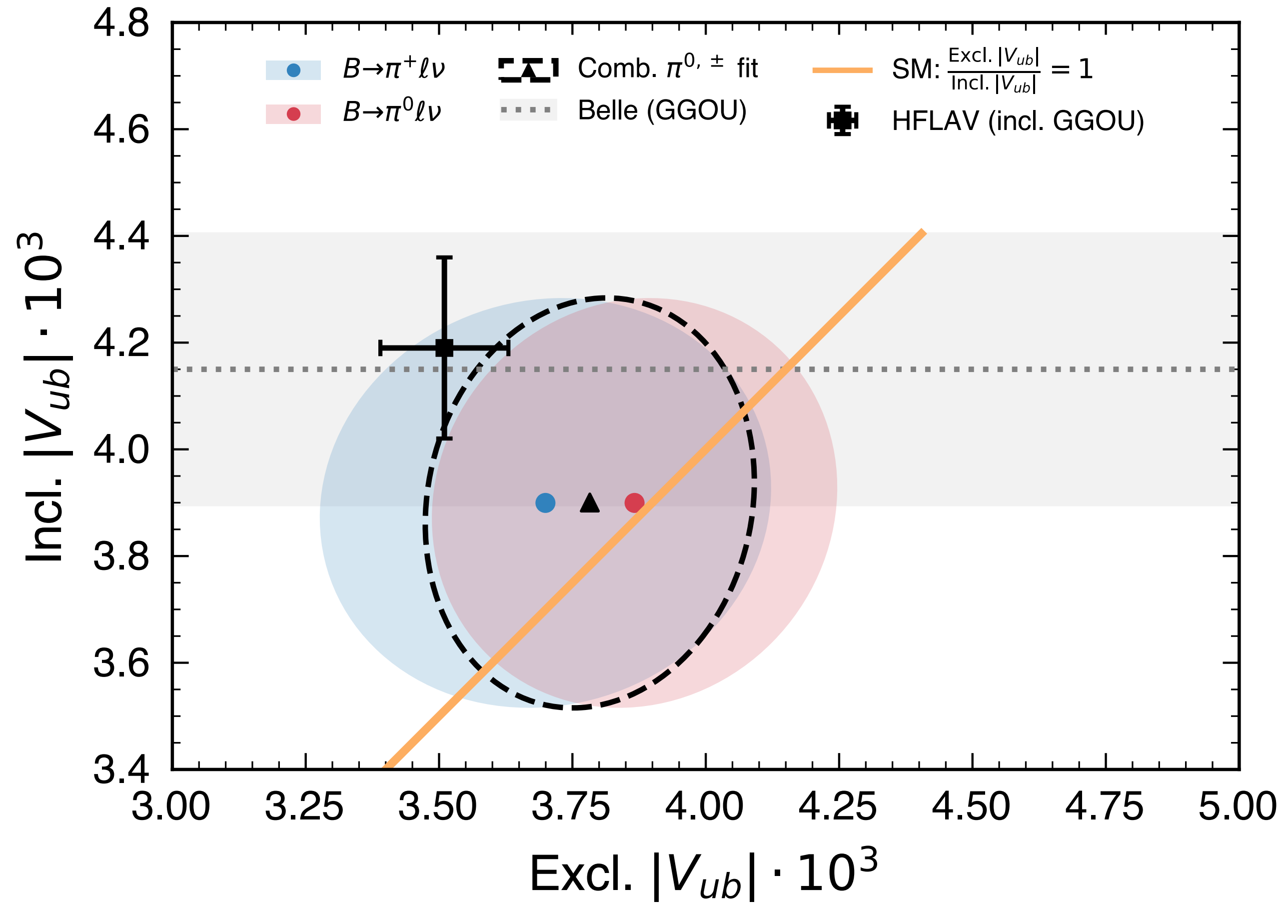
Flavor Lattice Averaging Group

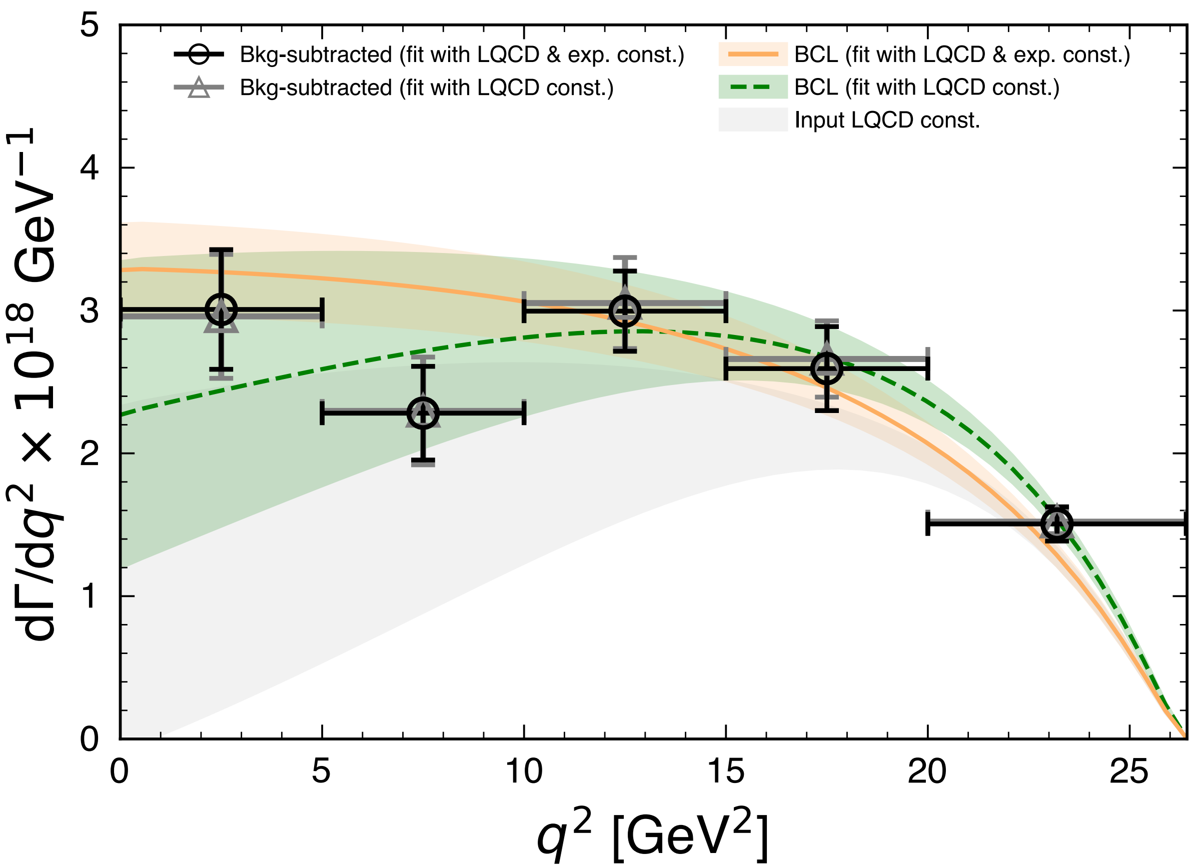
$$\left|V_{ub}^{\text{excl.}}\right| / \left|V_{ub}^{\text{incl.}}\right| = 1.06 \pm 0.14\,,$$



2) FLAG 2022 + all experimental information on $B \to \pi$ FF

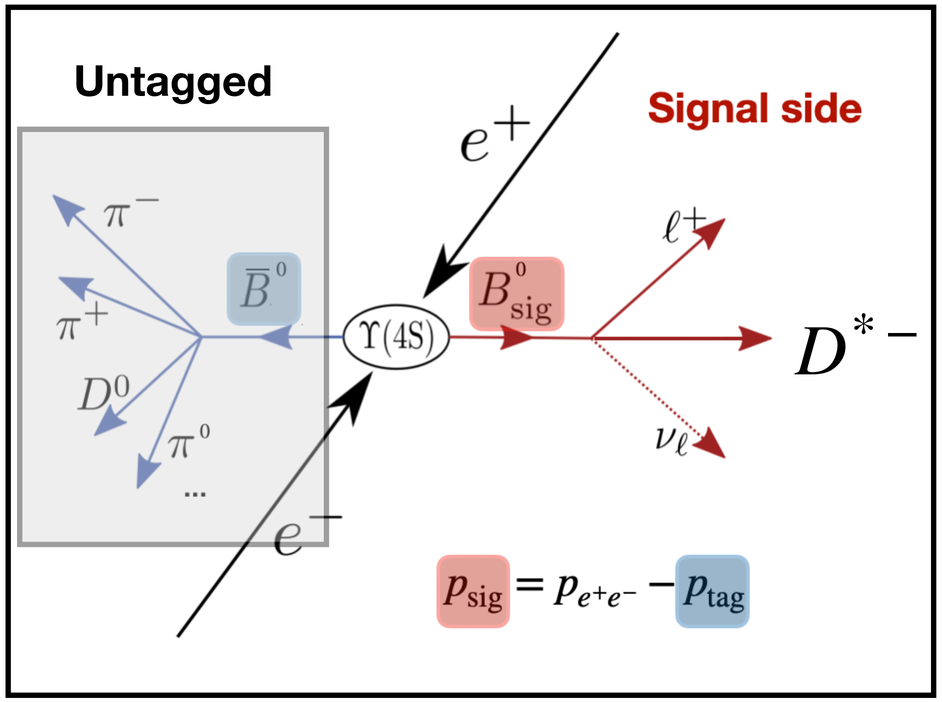
$$\left|V_{ub}^{\text{excl.}}\right| / \left|V_{ub}^{\text{incl.}}\right| = 0.97 \pm 0.12\,,$$





4. Determination of $|V_{cb}|$ using $\overline{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$ with **Belle II**,

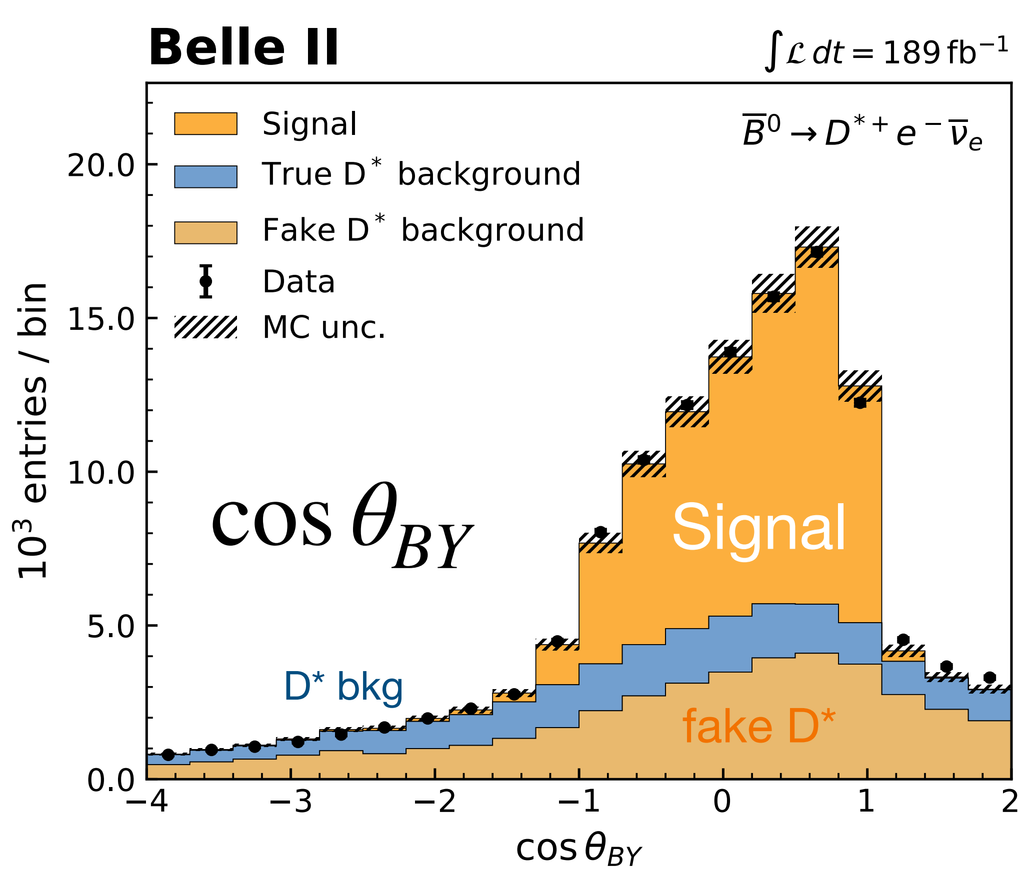
[Phys Rev D. 108, 092013, arXiv:2310.01170]

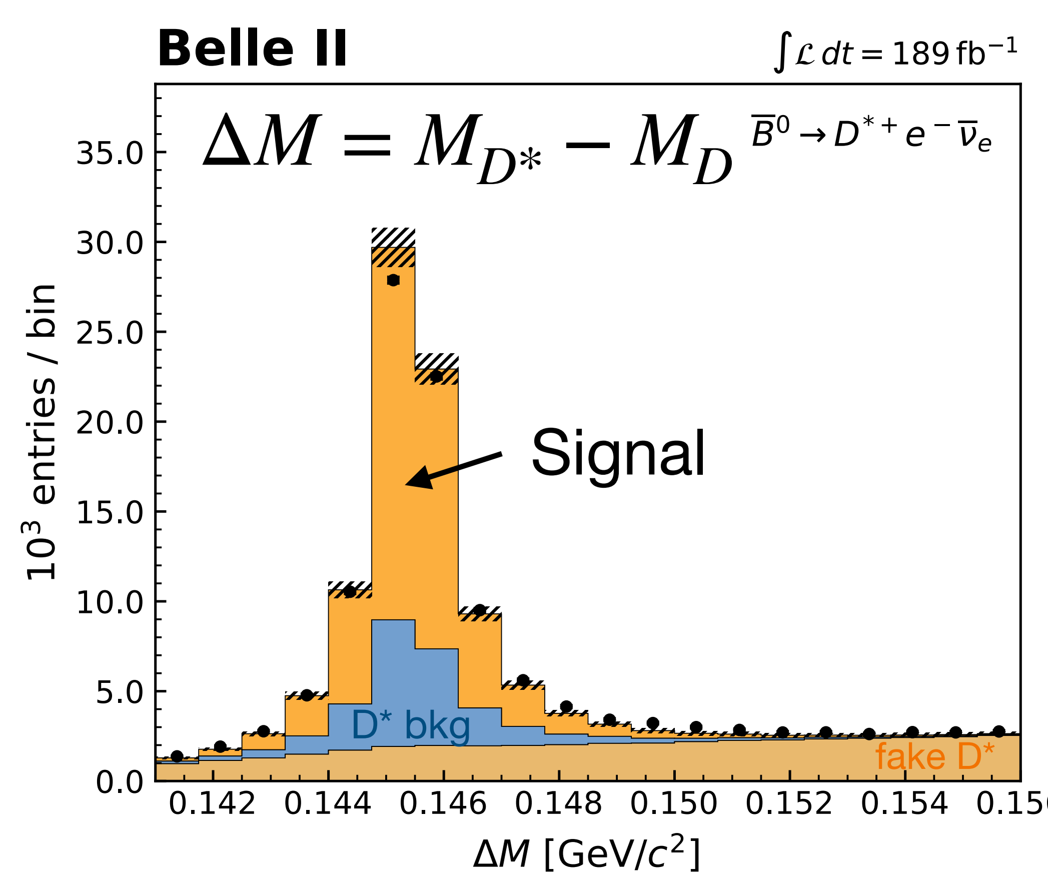


Untagged analysis focussing on experimentally **cleanest mode**:

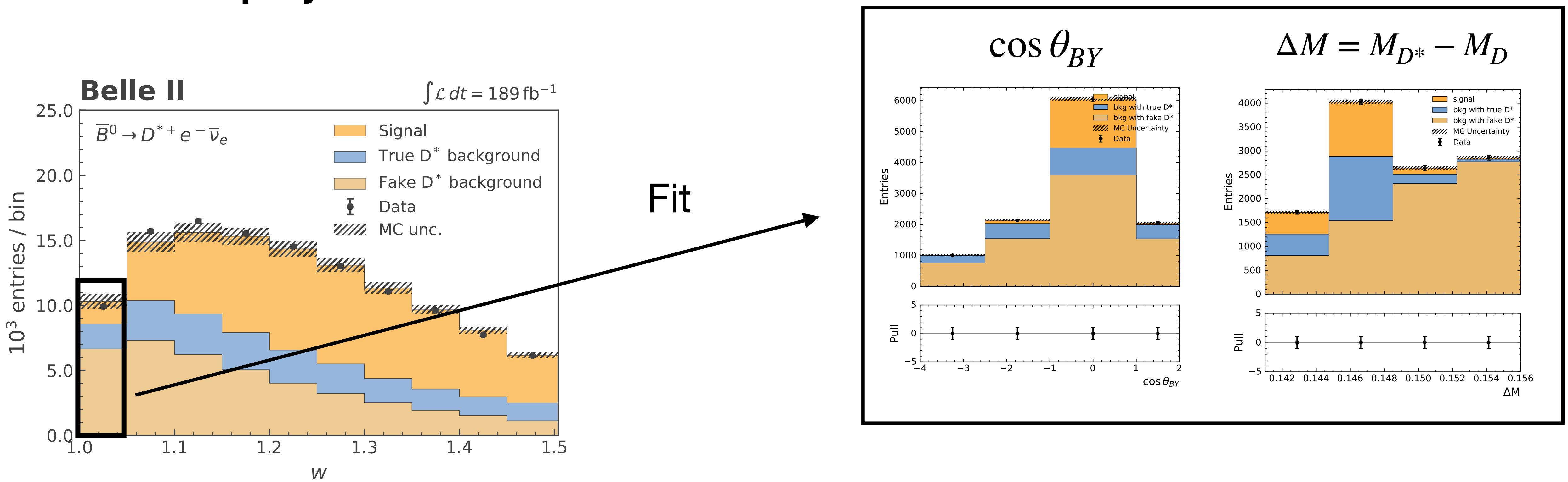
$$\overline{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$$
$$\hookrightarrow D^{*+} \to D^0\pi^+$$
$$\hookrightarrow D^0 \to K^-\pi^+$$

Extraction in **2D fit**:



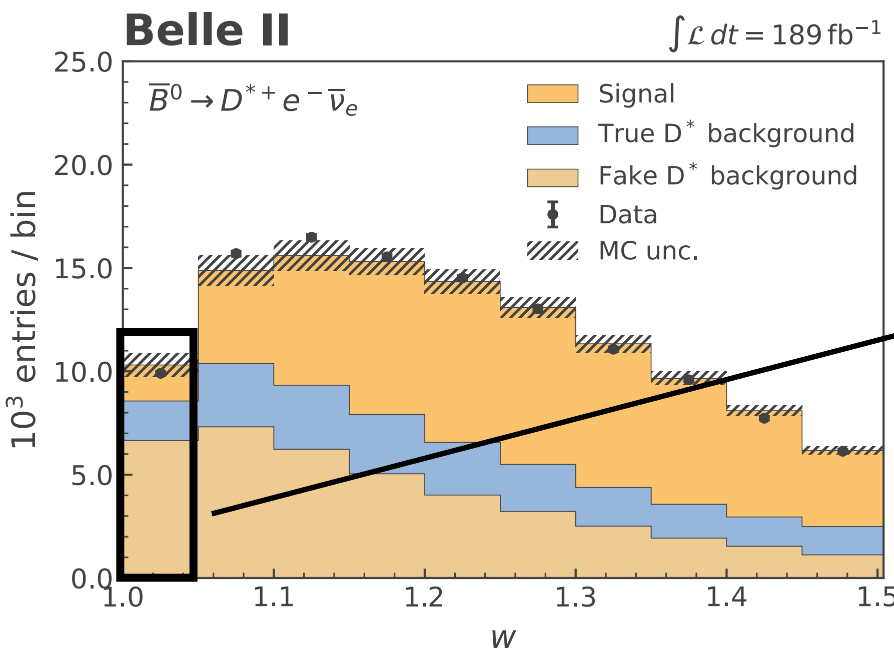


Determine **1D projections**:

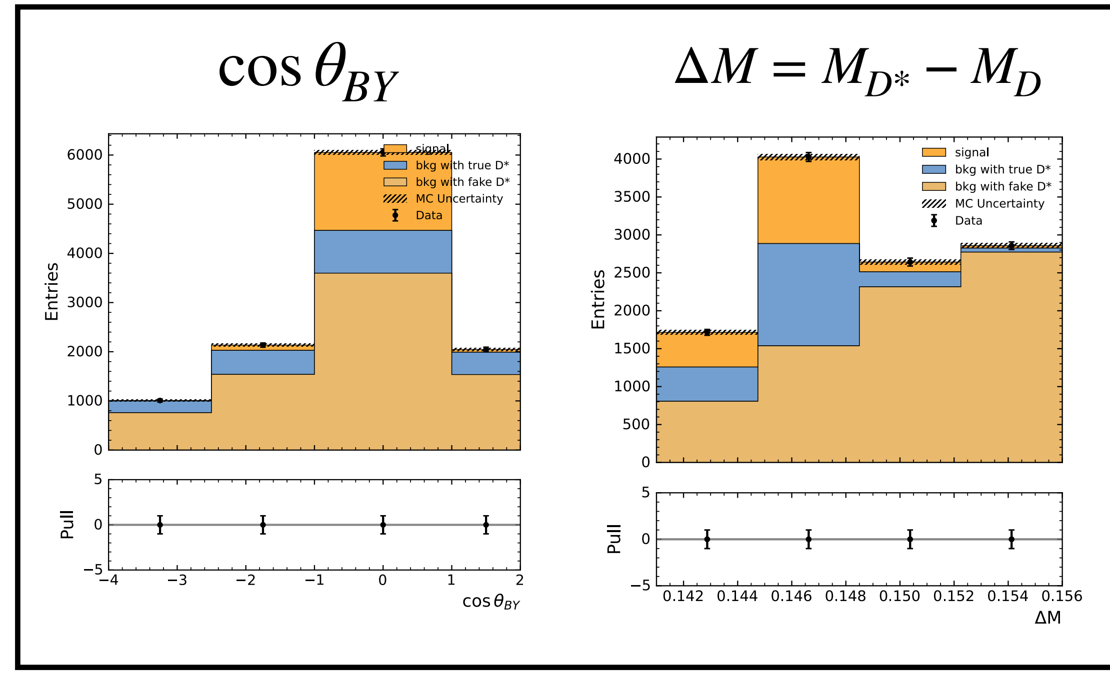


Recoil parameter: $$w = \frac{m_B^2 - m_{D*}^2 - q^2}{2 m_B m_{D*}^2}$$

Determine **1D projections**:

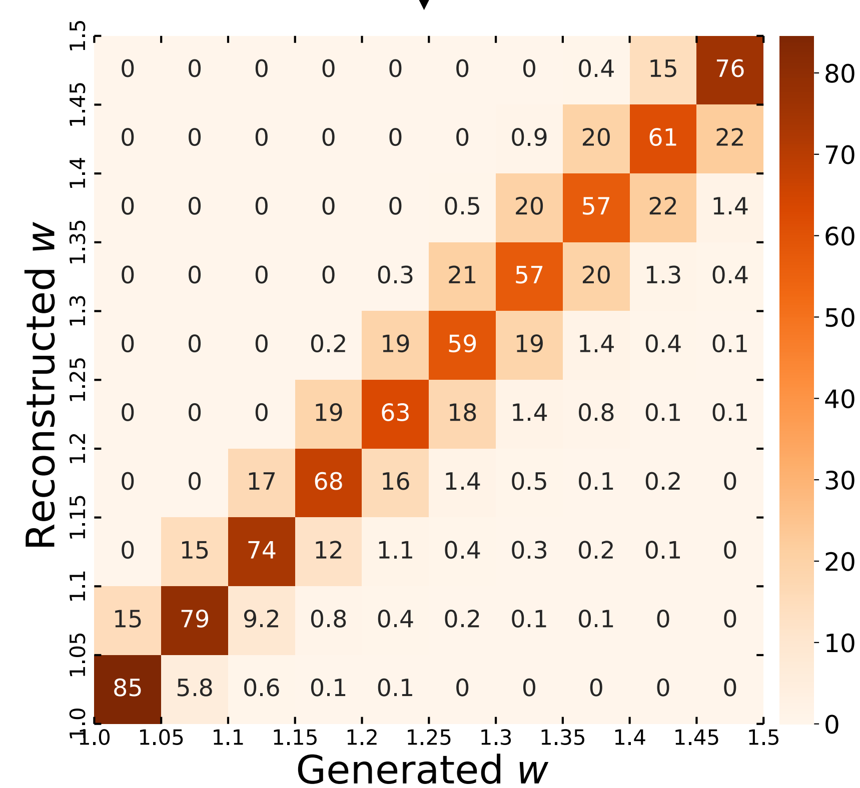


Fit



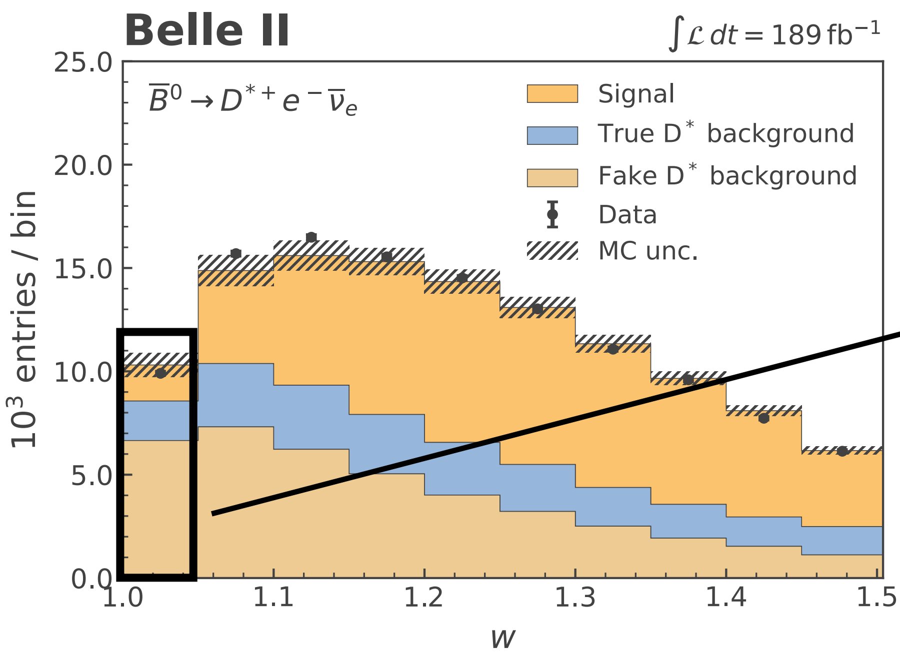
Correct for migration effects:

“Reco”

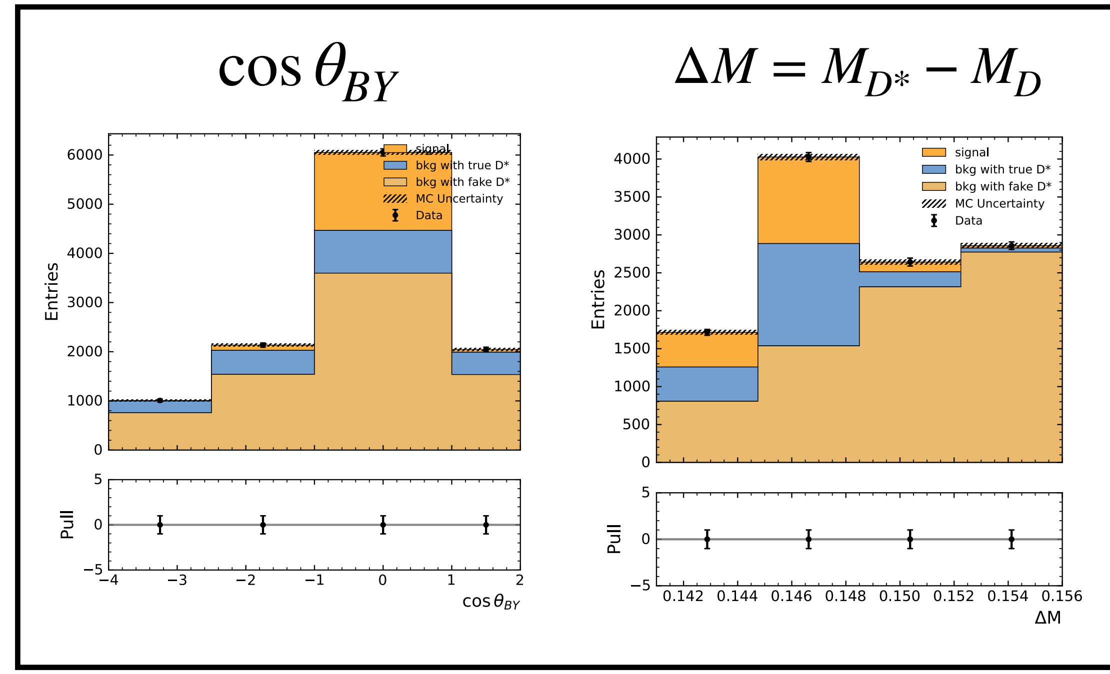


“True”

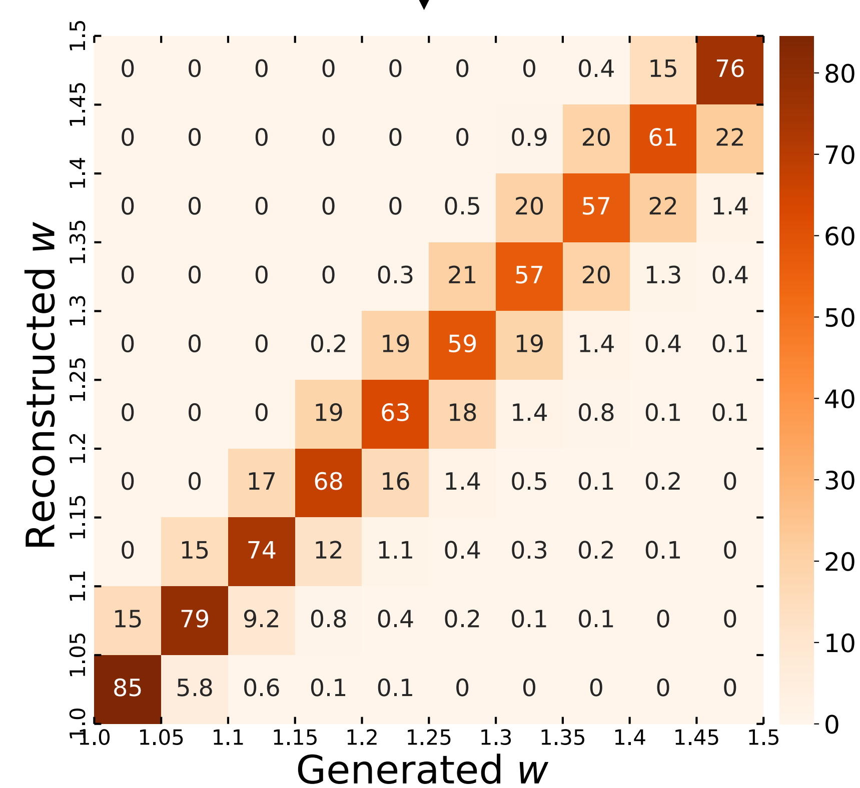
Determine **1D projections**:



Fit



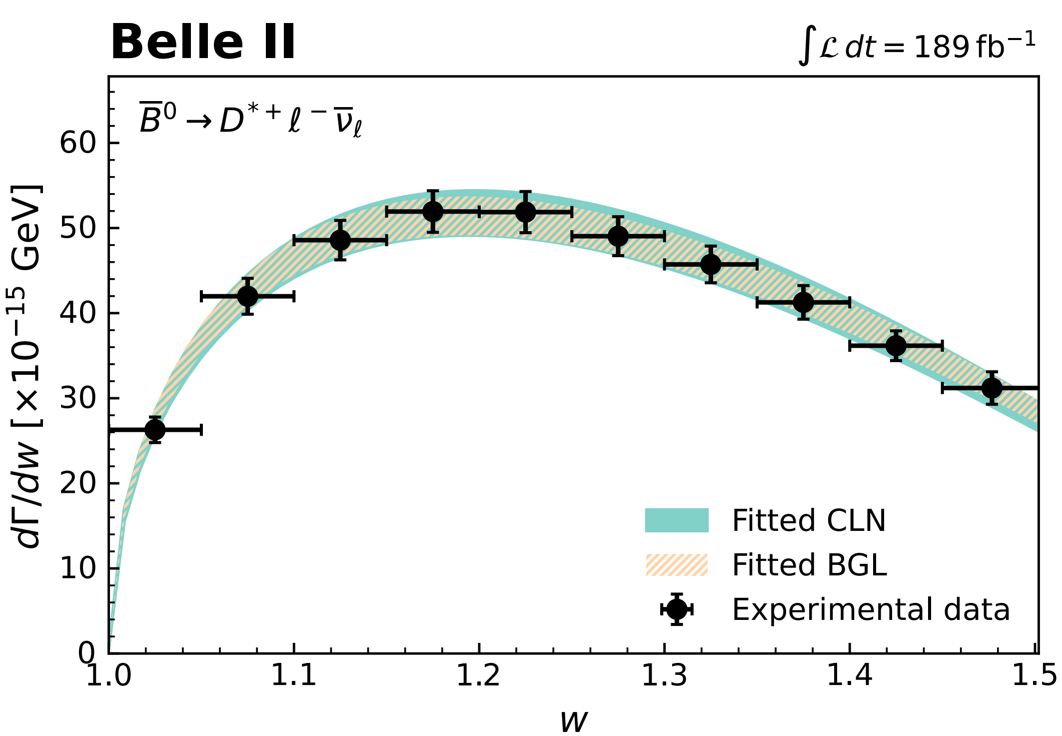
Correct for migration effects:

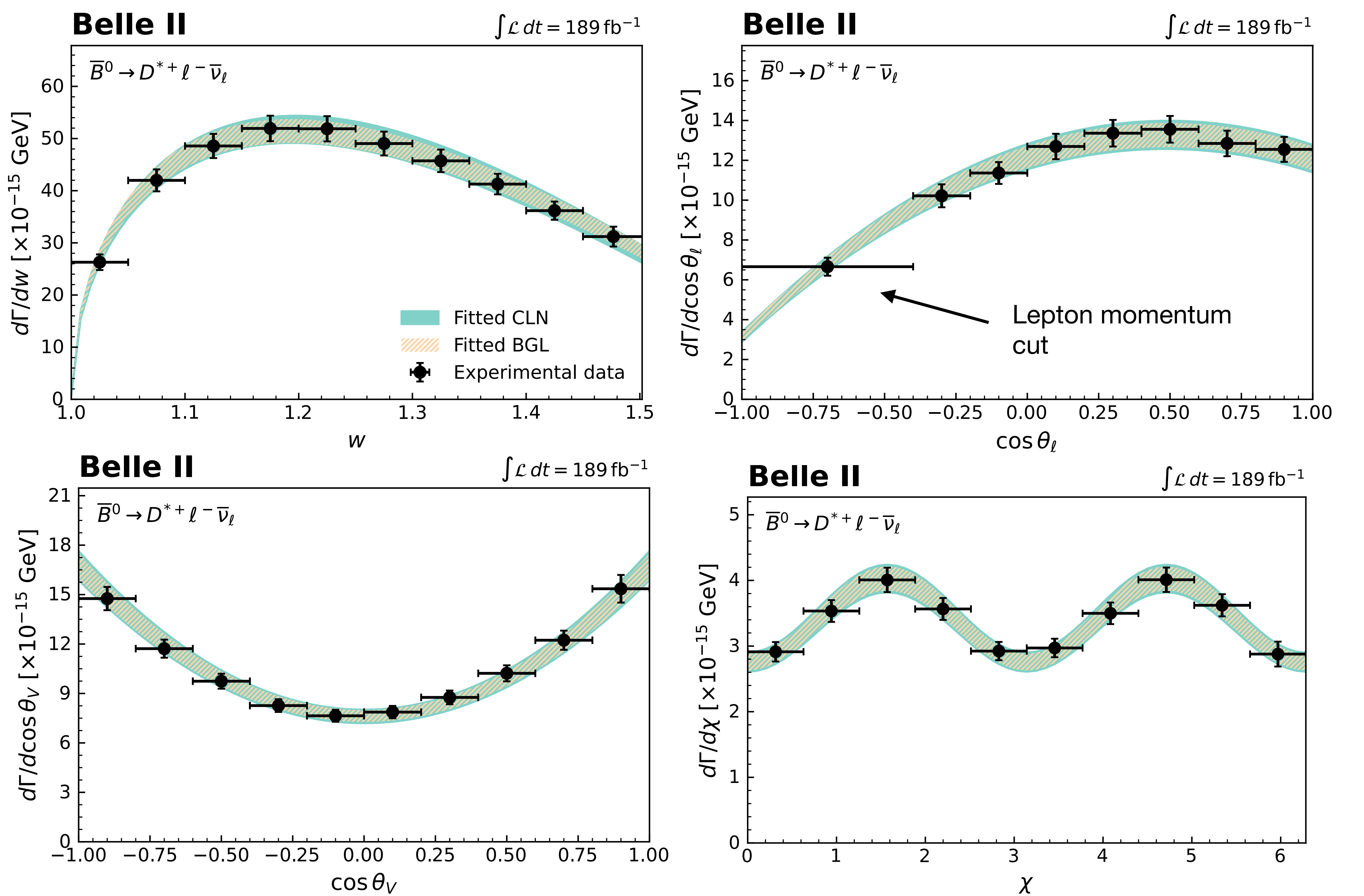


“Reco”

“True”

Correct for acceptance & efficiency





$$|V_{cb}|_{\mathrm{CLN}} = (40.2 \pm 0.3 \pm 0.9 \pm 0.6) \times 10^{-3}\,,$$

$$|V_{cb}|_{\mathrm{BGL}} = (40.6 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}.$$

← **BGL truncation** order determined using **Nested Hypothesis Test**

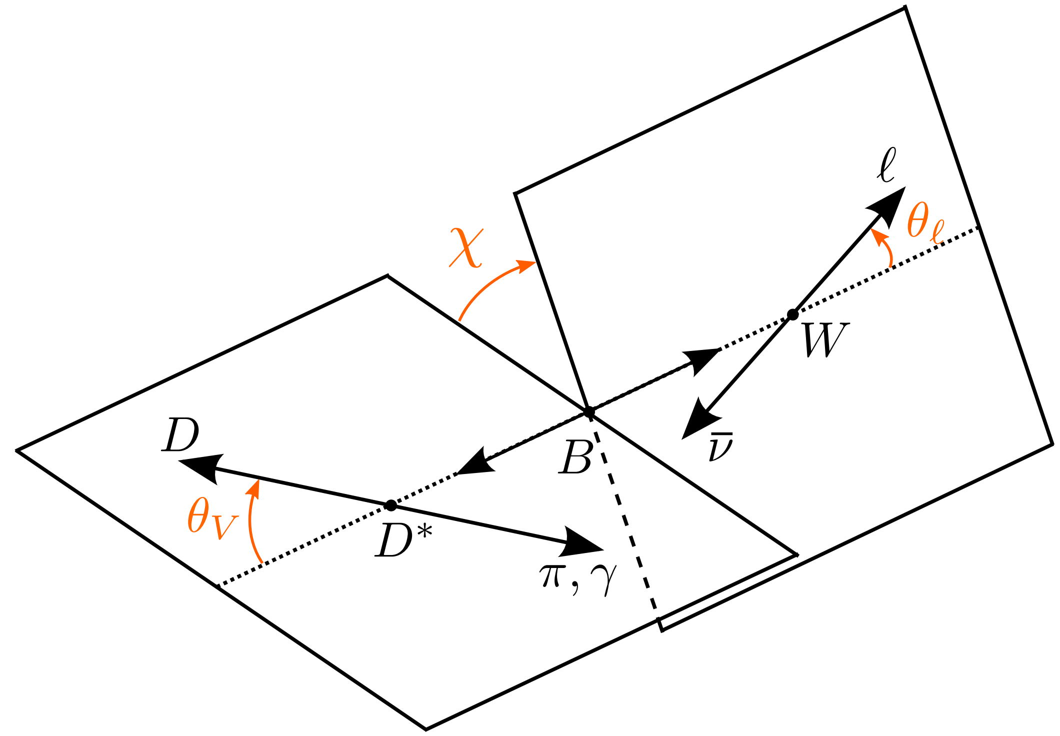
(n_a, n_b, n_c)	$\lvert V_{cb}\rvert \times 10^3$	$\rho_{\max}$	χ^2	Ndf	p-value
$(1,1,2)$	40.2 ± 1.1	0.28	40.5	32	14%
$(2,1,2)$	40.1 ± 1.1	0.97	38.6	31	16%
$(1,2,2)$	**40.6 ± 1.2**	**0.57**	**39.1**	**31**	**15%**
$(1,1,3)$	40.1 ± 1.1	0.97	40	31	13%
$(2,2,2)$	40.2 ± 1.3	0.99	38.6	30	13%
$(1,3,2)$	39.8 ± 1.3	0.98	37.6	30	16%
$(1,2,3)$	40.5 ± 1.2	0.97	39	30	13%

5. Test of **light-lepton universality** in **angular asymmetries** of hadronically tagged $B^0 \to D^{*-}\{e^+, \mu^+\}\,\nu$ decays at Belle II, [Phys.Rev.Lett. 131 (2023) 18, 181801, arXiv:2308.02023]

Construct **asymmetries**:

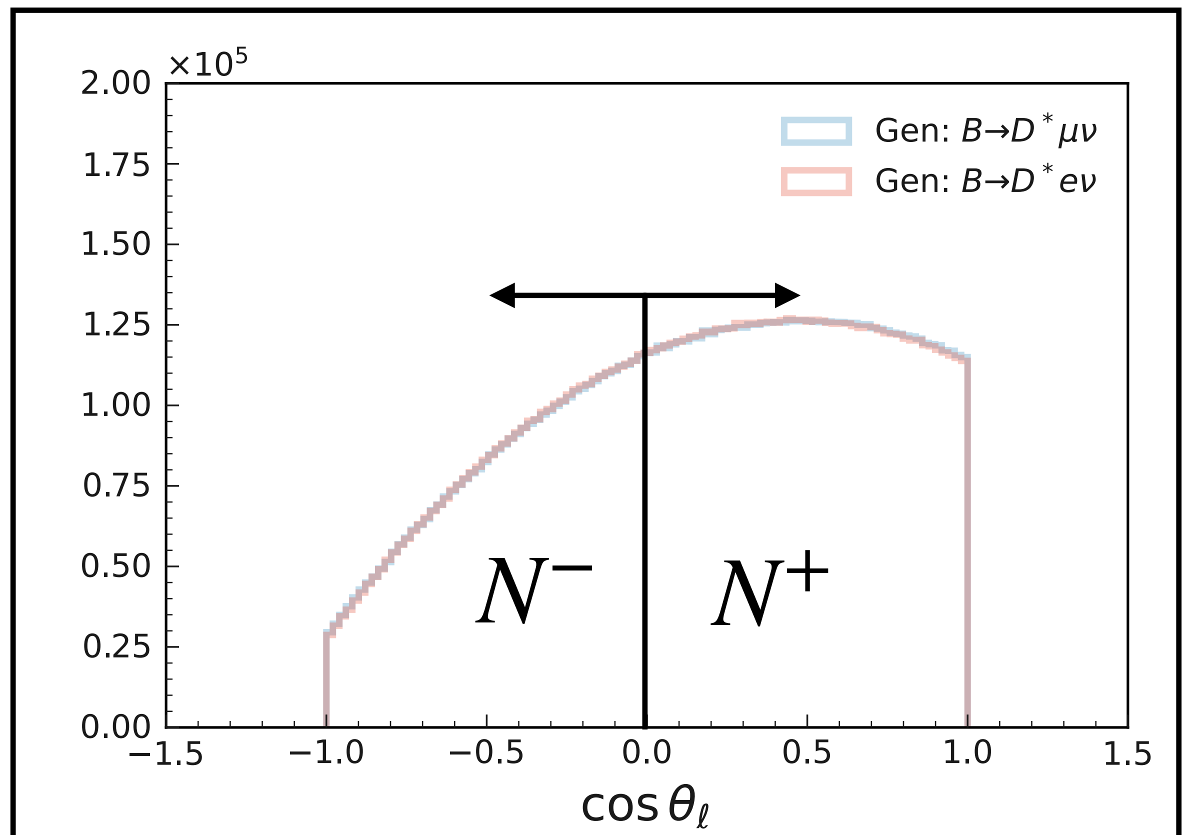
$$\mathcal{A}(w) = \left(\frac{\mathrm{d}\Gamma}{\mathrm{d}w}\right)^{-1}\left[\int_0^1 - \int_{-1}^0\right] dX \frac{\mathrm{d}\Gamma}{\mathrm{d}w\mathrm{d}X},$$

$$A_{\mathrm{FB}} : \mathrm{d}X \to \mathrm{d}(\cos\theta_l)$$
$$S_3 : \mathrm{d}X \to \mathrm{d}(\cos 2\chi)$$
$$S_5 : \mathrm{d}X \to \mathrm{d}(\cos\chi\cos\theta_V)$$
$$S_7 : \mathrm{d}X \to \mathrm{d}(\sin\chi\cos\theta_V)$$
$$S_9 : \mathrm{d}X \to \mathrm{d}(\sin 2\chi)$$



E.g. forward-backward asymmetry in $\cos\theta_\ell$

$$A_{\mathrm{FB}} = \frac{N^+ - N^-}{N^+ + N^+}$$



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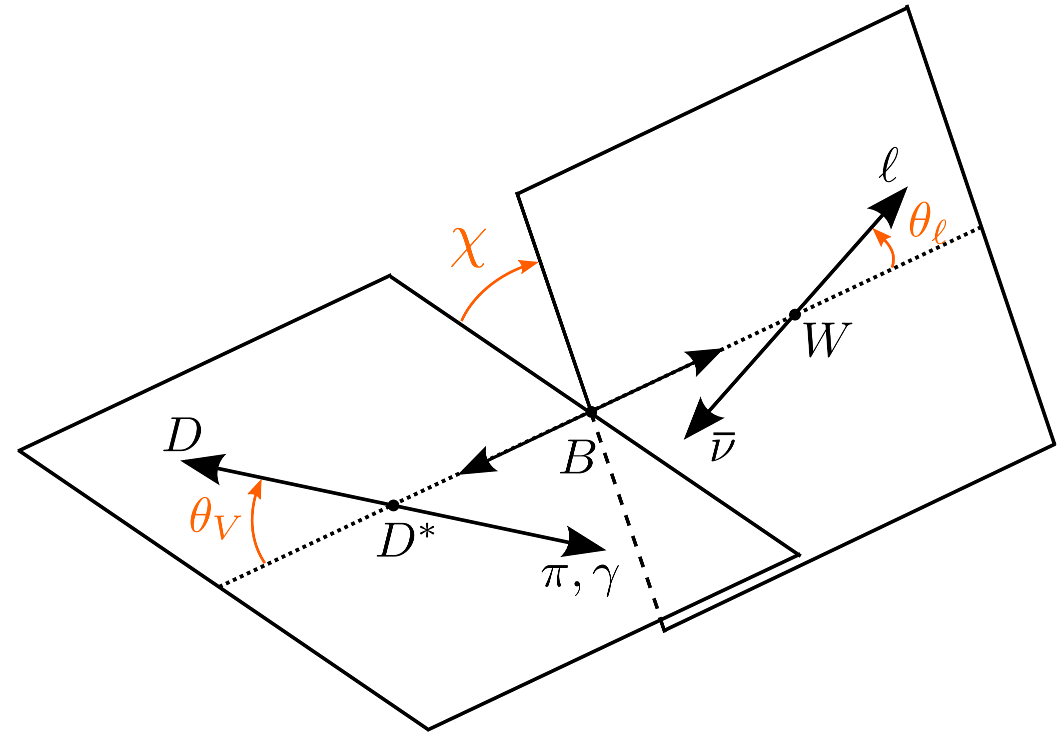
$$\begin{aligned} A_{\mathrm{FB}} &: \mathrm{d}X \to \mathrm{d}(\cos\theta_l) \\ S_3 &: \mathrm{d}X \to \mathrm{d}(\cos 2\chi) \\ S_5 &: \mathrm{d}X \to \mathrm{d}(\cos\chi\cos\theta_V) \\ S_7 &: \mathrm{d}X \to \mathrm{d}(\sin\chi\cos\theta_V) \\ S_9 &: \mathrm{d}X \to \mathrm{d}(\sin 2\chi) \end{aligned}$$



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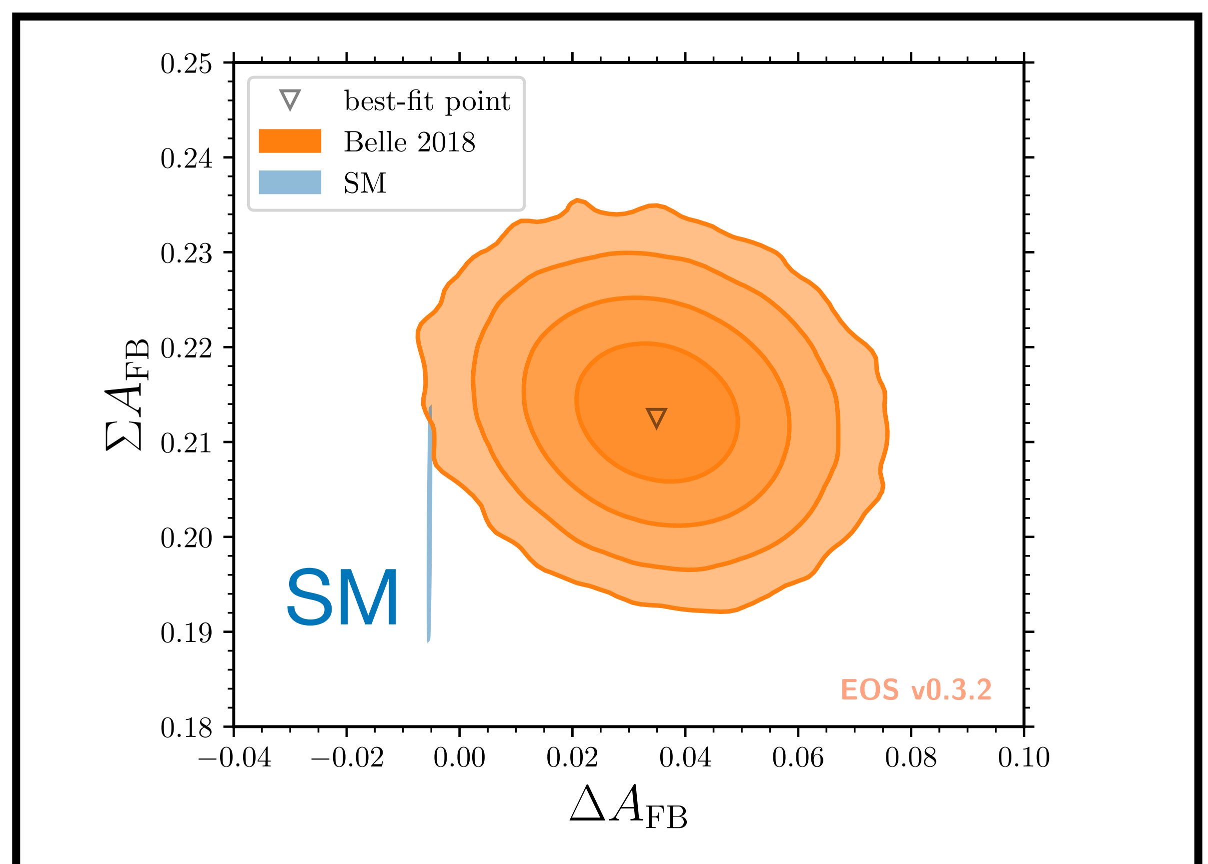
Bobeth et al. [*Eur.Phys.J.C* 81 (2021) 11, 984]



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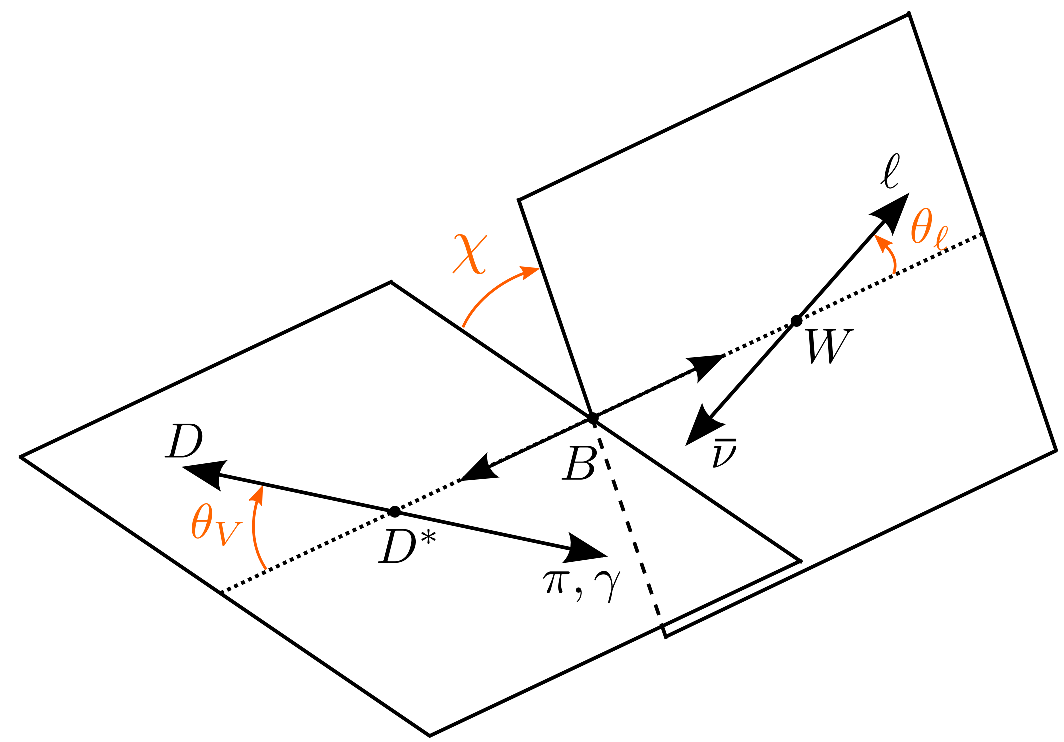
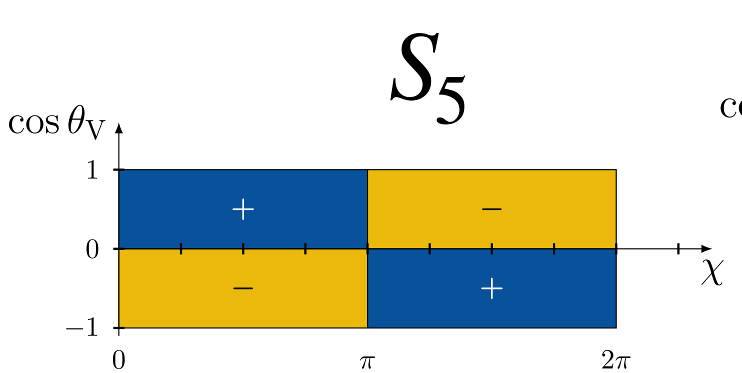
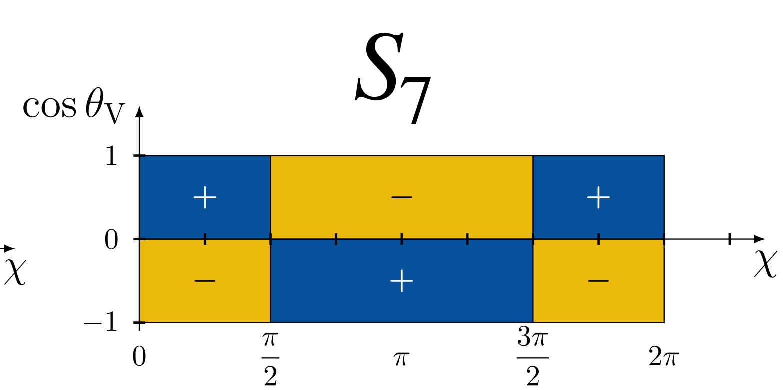
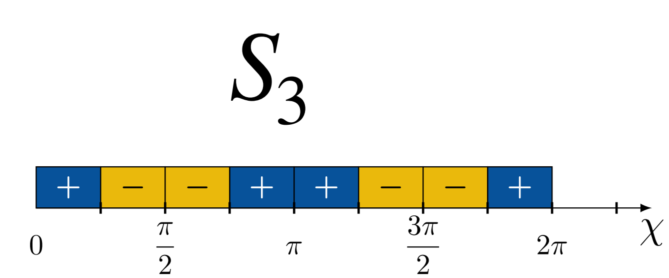
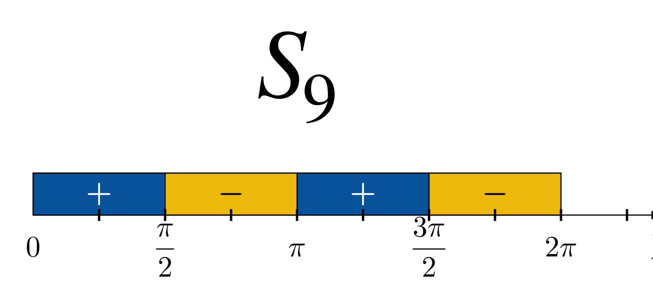
$$A_{\mathrm{FB}} : \mathrm{d}X \to \mathrm{d}(\cos\theta_l)$$
$$S_3 : \mathrm{d}X \to \mathrm{d}(\cos 2\chi)$$
$$S_5 : \mathrm{d}X \to \mathrm{d}(\cos\chi\cos\theta_V)$$
$$S_7 : \mathrm{d}X \to \mathrm{d}(\sin\chi\cos\theta_V)$$
$$S_9 : \mathrm{d}X \to \mathrm{d}(\sin 2\chi)$$



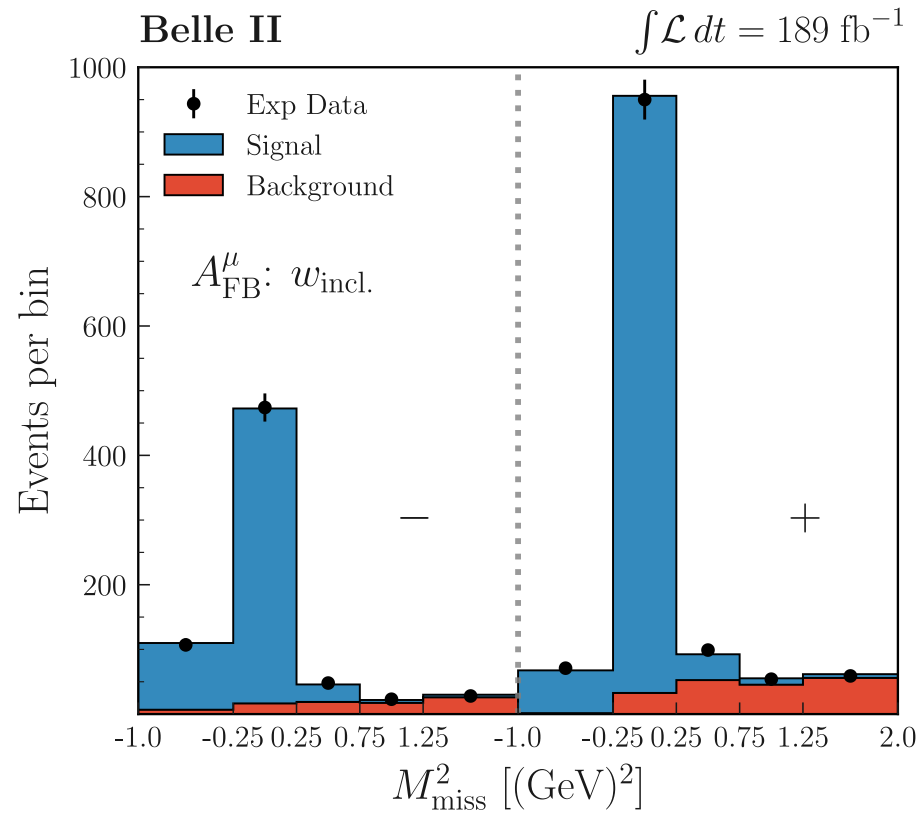






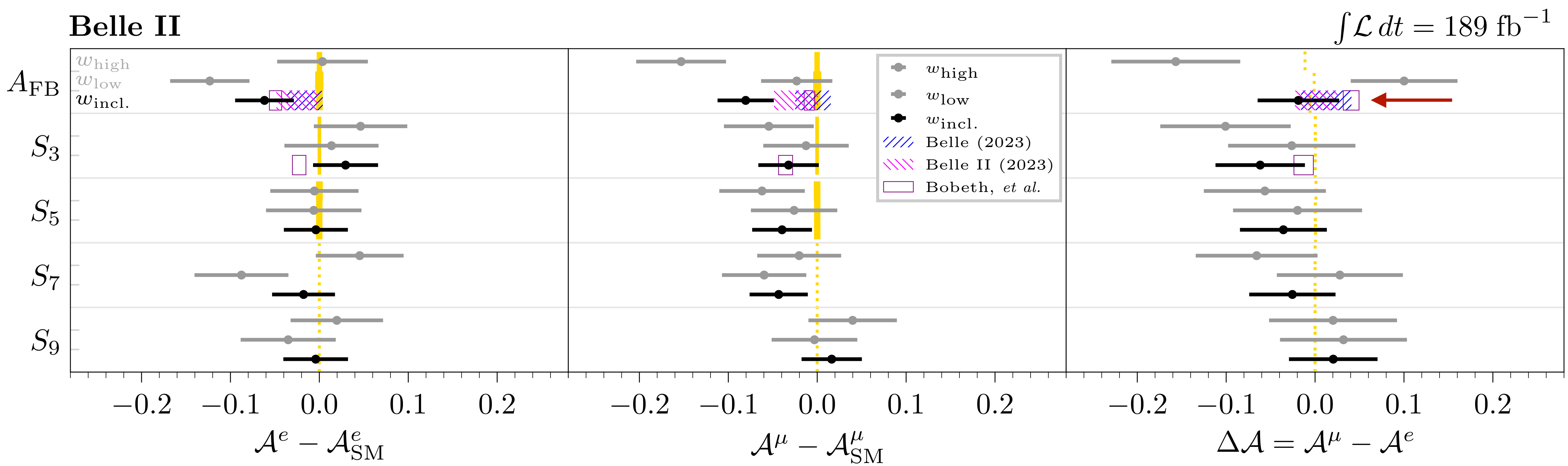


5. Test of **light-lepton universality** in **angular asymmetries** of hadronically tagged $B^0 \to D^{*-}\{e^+, \mu^+\}\,\nu$ decays at Belle II, [Phys.Rev.Lett. 131 (2023) 18, 181801, arXiv:2308.02023]



Can also split these **asymmetries** further into w **bins** :

$$w \in [1, w_{max}]$$
$$w \in [1, 1.275]$$
$$w \in [1.275, w_{max}]$$



6.

Measurement of **Angular Coefficients** of $B \to D^* \ell \bar{\nu}_\ell$, Implications on $|V_{cb}|$ and Tests of Lepton Flavor Universality, [Phys.Rev.Lett. 133 (2024) 13, 131801, arXiv: 2310.20286]

Full angular information of $B \to D^* \ell \bar{\nu}_\ell$ can be encoded into **12 coefficients** :

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_V\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\chi} = \frac{G_F^2\,|V_{cb}|^2\,m_B^3}{2\pi^4}$$

$$\times \Big\{ J_{1s}\sin^2\theta_V + J_{1c}\cos^2\theta_V$$
$$+ (J_{2s}\sin^2\theta_V + J_{2c}\cos^2\theta_V)\cos 2\theta_\ell$$
$$+ J_3\sin^2\theta_V\sin^2\theta_\ell\cos 2\chi$$
$$+ J_4\sin 2\theta_V\sin 2\theta_\ell\cos\chi + J_5\sin 2\theta_V\sin\theta_\ell\cos\chi$$
$$+ (J_{6s}\sin^2\theta_V + J_{6c}\cos^2\theta_V)\cos\theta_\ell$$
$$+ J_7\sin 2\theta_V\sin\theta_\ell\sin\chi + J_8\sin 2\theta_V\sin 2\theta_\ell\sin\chi$$
$$+ J_9\sin^2\theta_V\sin^2\theta_\ell\sin 2\chi \Big\}.$$

→ Each of these coefficients is a function of $q^2 \sim w$

↓

With some smart folding, one can “easily” determine them

Based on the ideas of:
JHEP 05 (2013) 043
JHEP 05 (2013) 137
Phys. Rev. D 90, 094003 (2014)
http://cds.cern.ch/record/1605179

8 Coefficients relevant in massless limit & SM

6.

Measurement of **Angular Coefficients** of $B \to D^* \ell \bar{\nu}_\ell$, Implications on $|V_{cb}|$ and Tests of Lepton Flavor Universality, [Phys.Rev.Lett. 133 (2024) 13, 131801, arXiv: 2310.20286]

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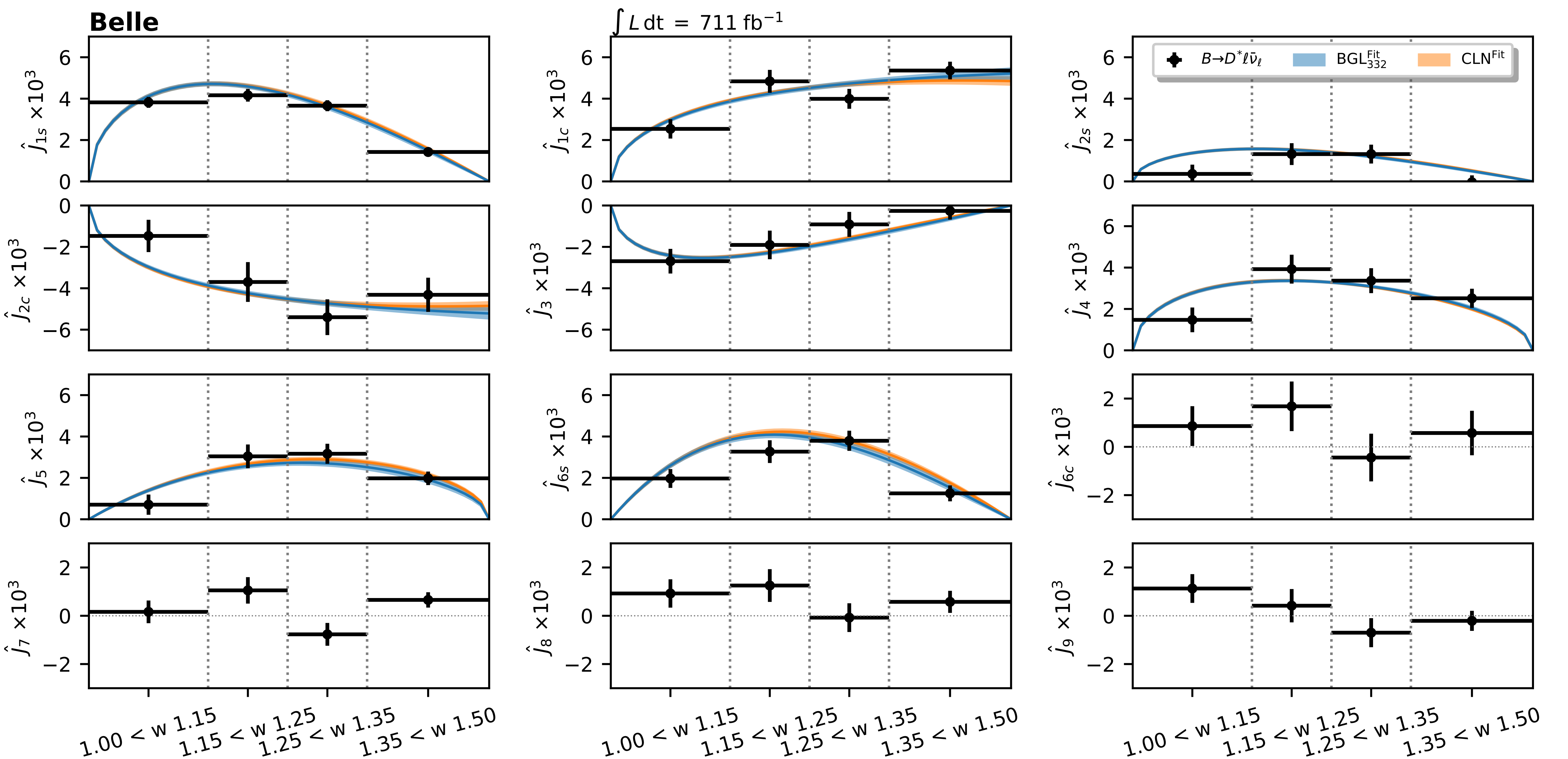
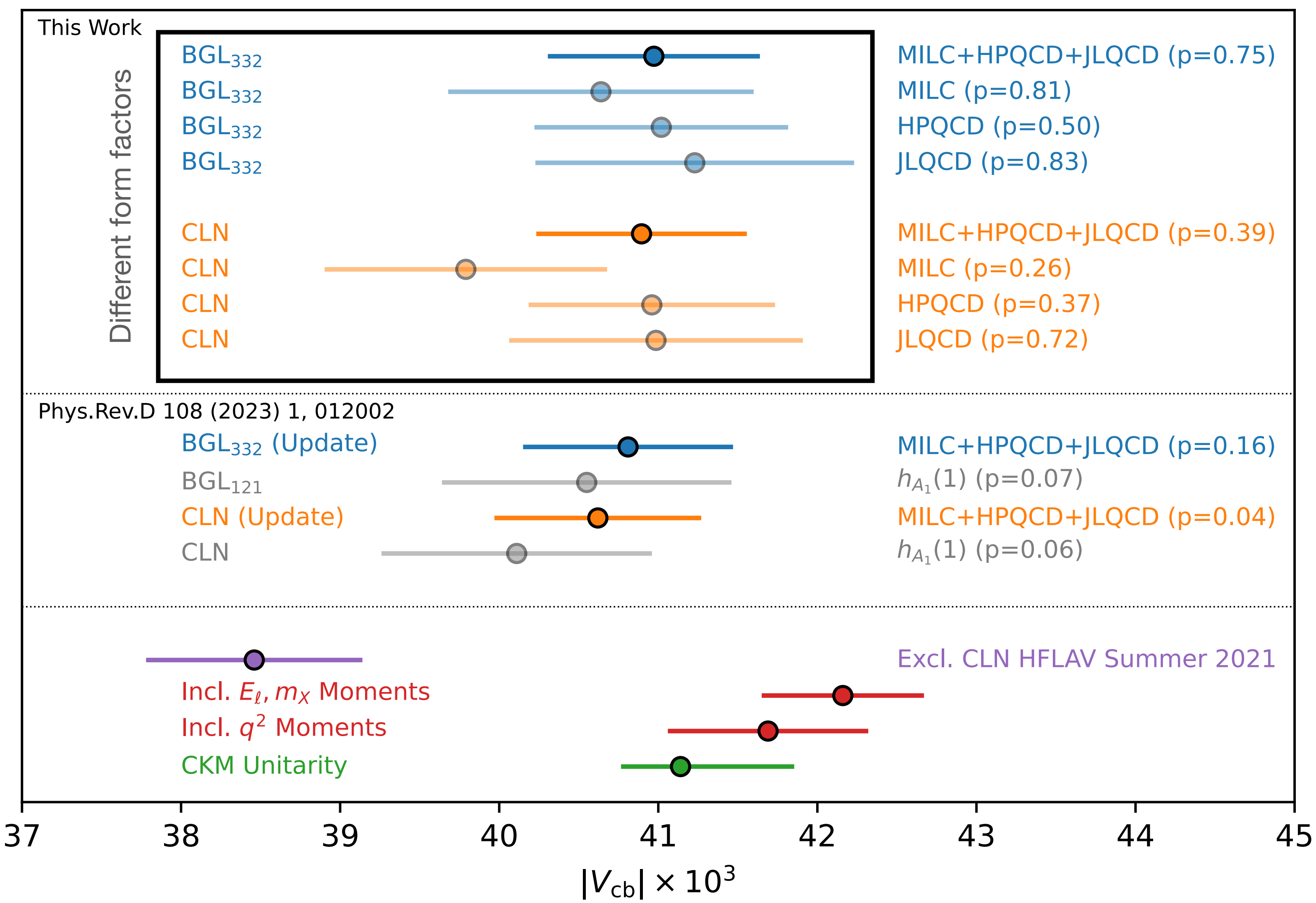


6. Measurement of **Angular Coefficients** of $B \to D^* \ell \bar{\nu}_\ell$, Implications on $|V_{cb}|$ and Tests of Lepton Flavor Universality, [Phys.Rev.Lett. 133 (2024) 13, 131801, arXiv: 2310.20286]

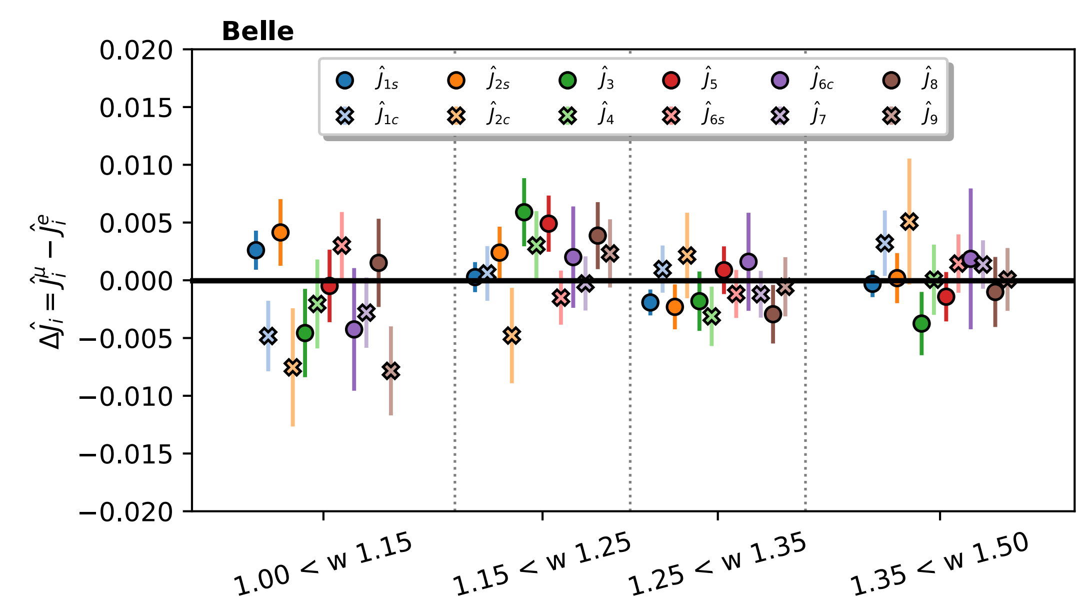
Full angular information of $B \to D^* \ell \bar{\nu}_\ell$ can be encoded into **12 coefficients** :

Can be analyzed to determine $|V_{cb}|$ & test **Lepton Flavor Universality**

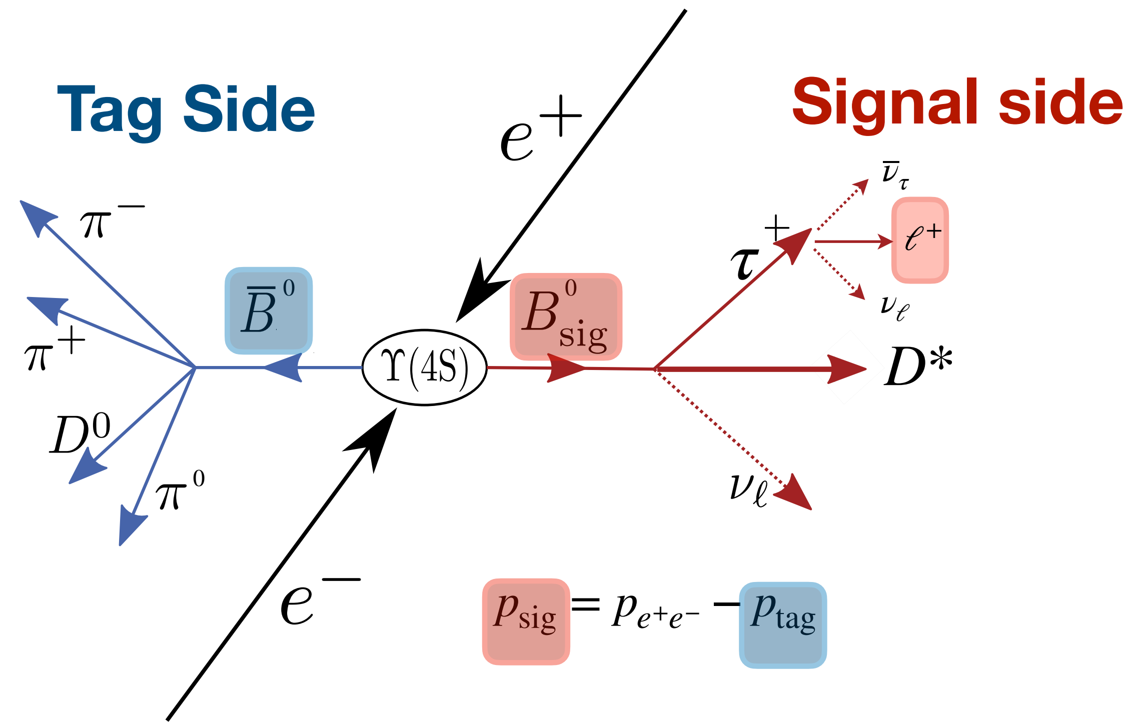
Different Lattice Input



e vs . μ



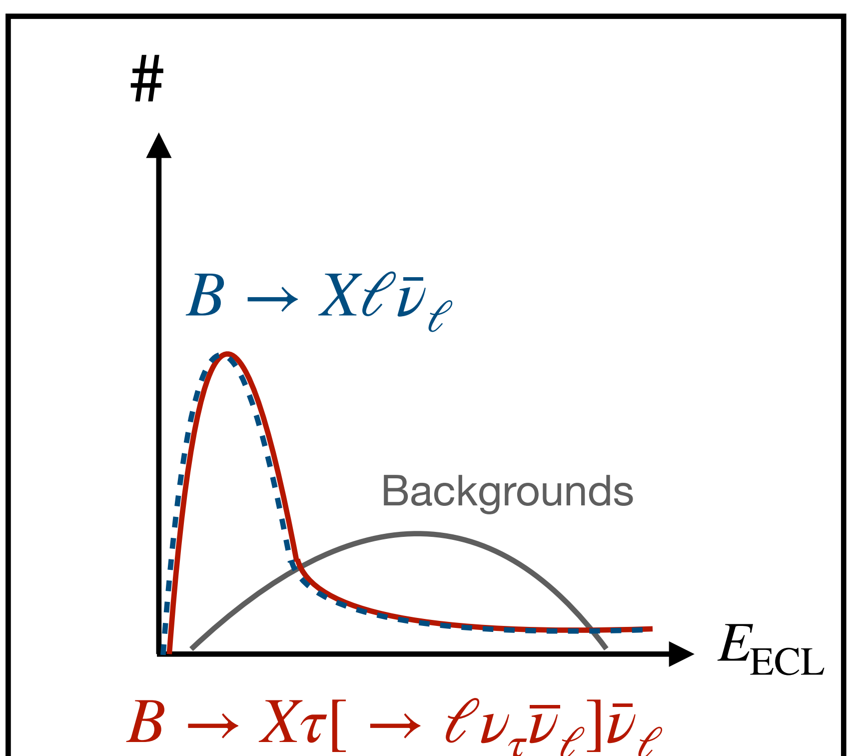
7. A test of **lepton flavor universality** with a measurement of $\mathscr{R}(D^*)$ using had. B tagging at the Belle II experiment, [Phys.Rev.D 110 (2024) 7, 072020, 2401.02840]

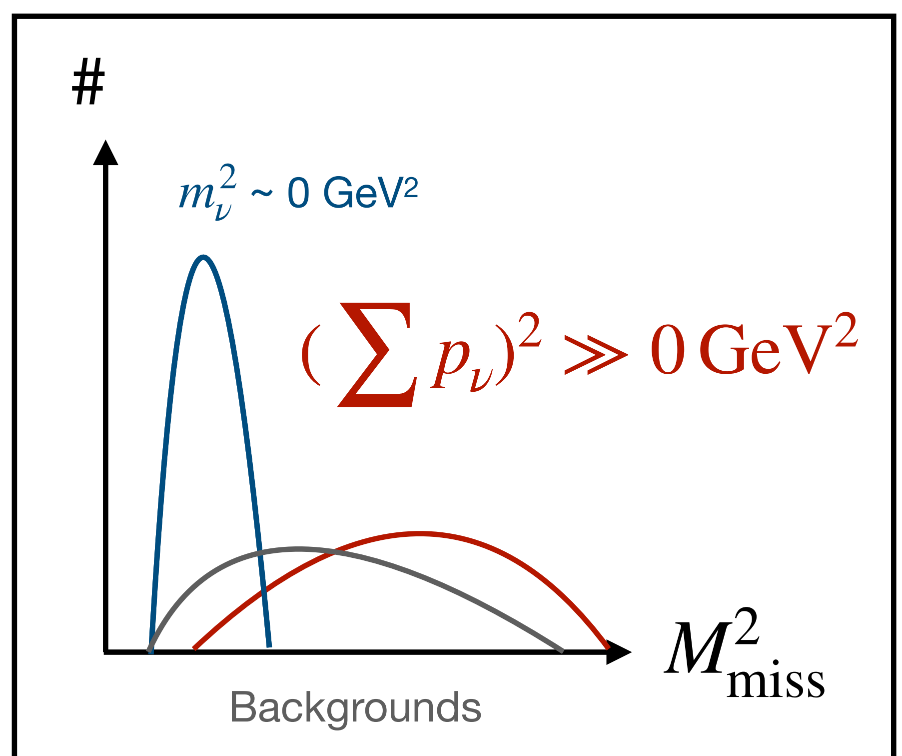


Key variables:

$E_{\rm ECL}$: $M^2_{\rm miss} = \left(p_{B_{\rm sig}} - p_X - p_\ell\right)^2$

Unassigned neutral energy depositions in calorimeter





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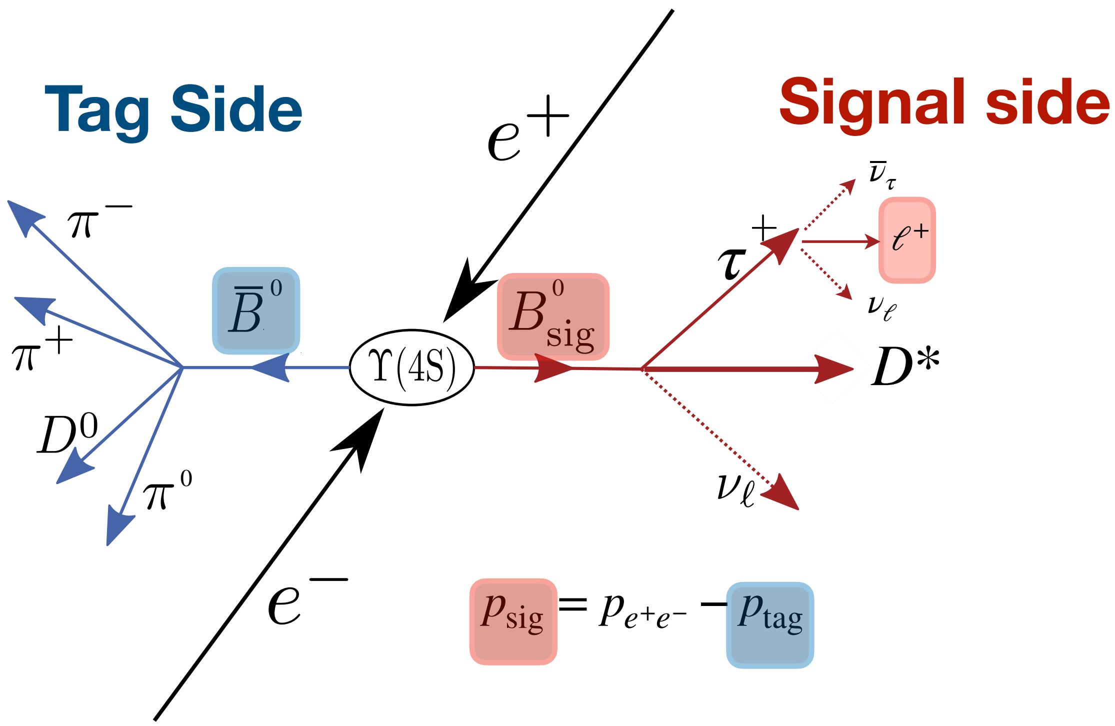
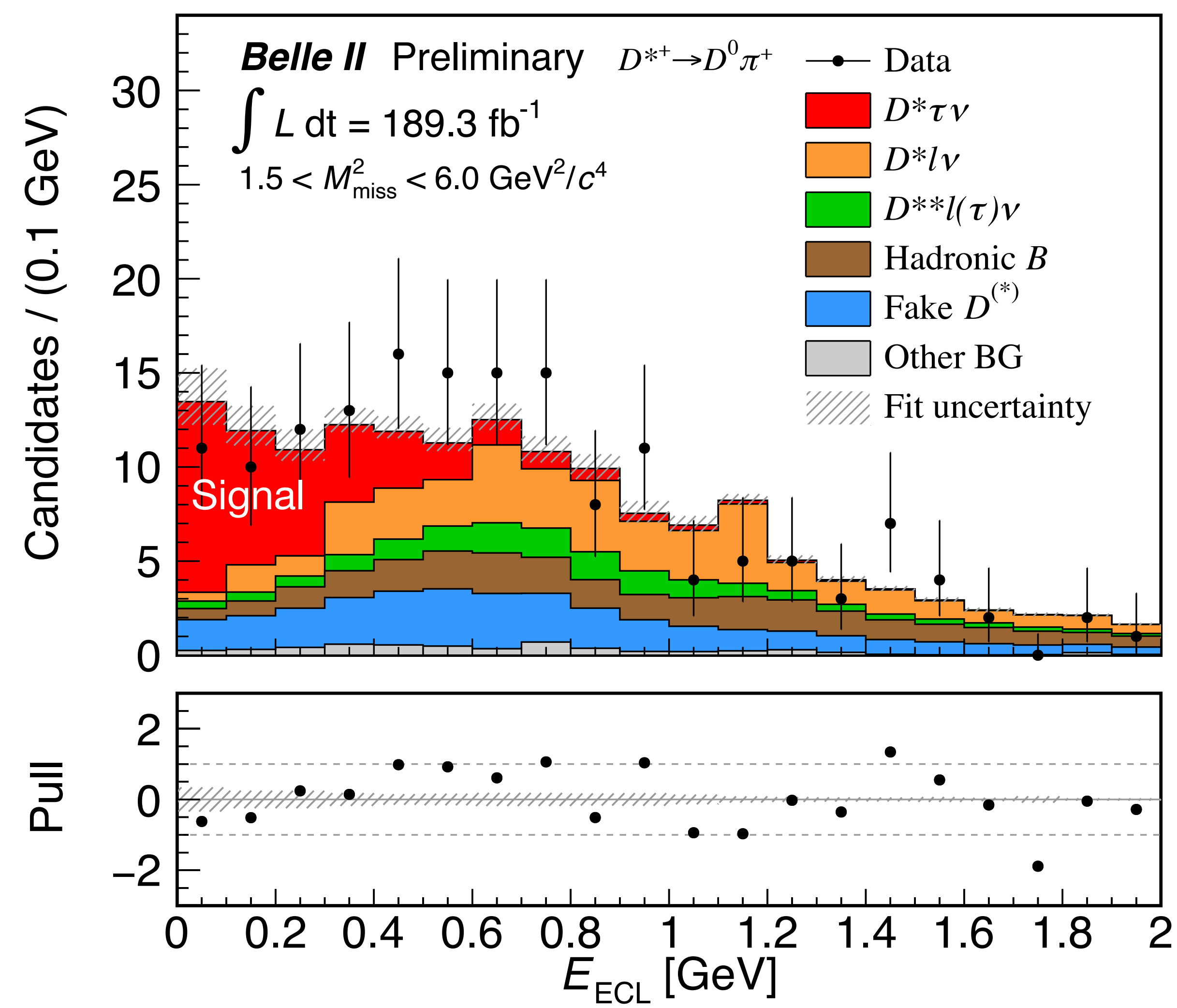


Key variables:

$$E_{ECL} \quad : \quad M^2_{miss} = \left(p_{B_{sig}} - p_X - p_\ell\right)^2$$

Signal enhanced region of $M^2_{miss} \in [1.5, 6]\,\mathrm{GeV}^2$



2D likelihood fit:

SM expectation: 0.249 +/- 0.002

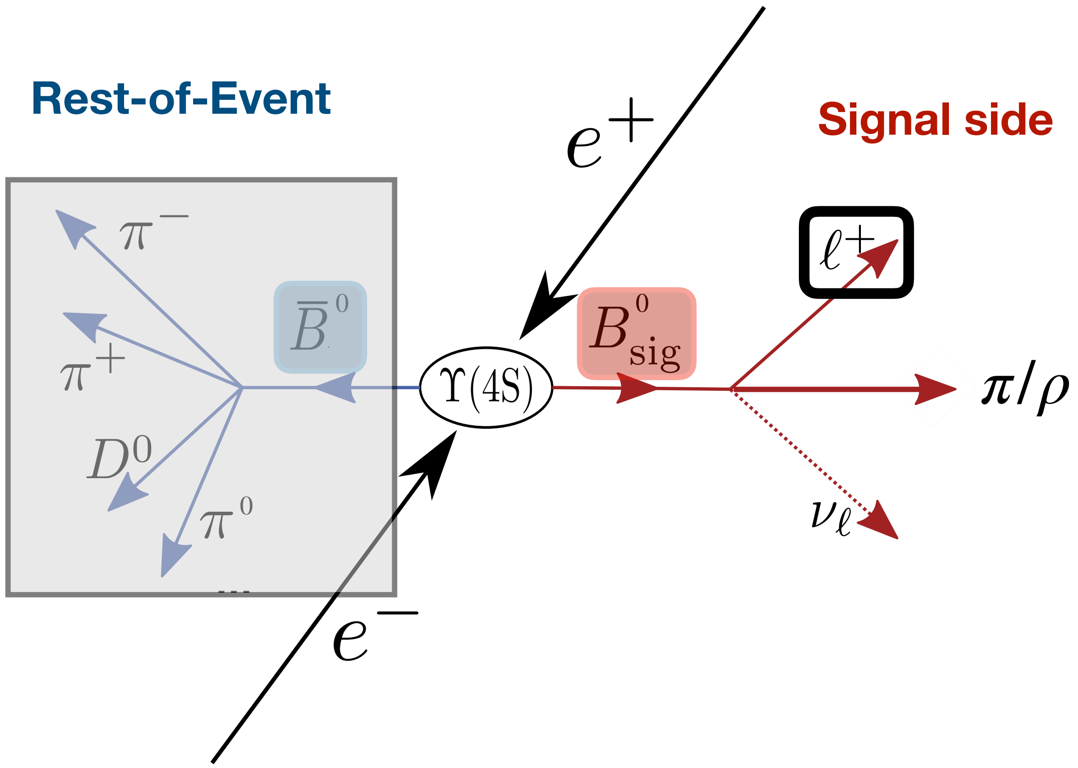
$$R(D^*) = 0.262\ ^{+0.041}_{-0.039}(\mathrm{stat})\ ^{+0.035}_{-0.032}(\mathrm{syst}),$$

Dominant Systematic Errors

Source	Uncertainty
PDF shapes	$^{+9.1\%}_{-8.3\%}$
Simulation sample size	$^{+7.5\%}_{-7.5\%}$
$\overline{B} \to D^{**}\ell^-\overline{\nu}_\ell$ branching fractions	$^{+4.8\%}_{-3.5\%}$
Fixed backgrounds	$^{+2.7\%}_{-2.3\%}$
Hadronic B decay branching fractions	$^{+2.1\%}_{-2.1\%}$
Reconstruction efficiency	$^{+2.0\%}_{-2.0\%}$
Kernel density estimation	$^{+2.0\%}_{-0.8\%}$

8.

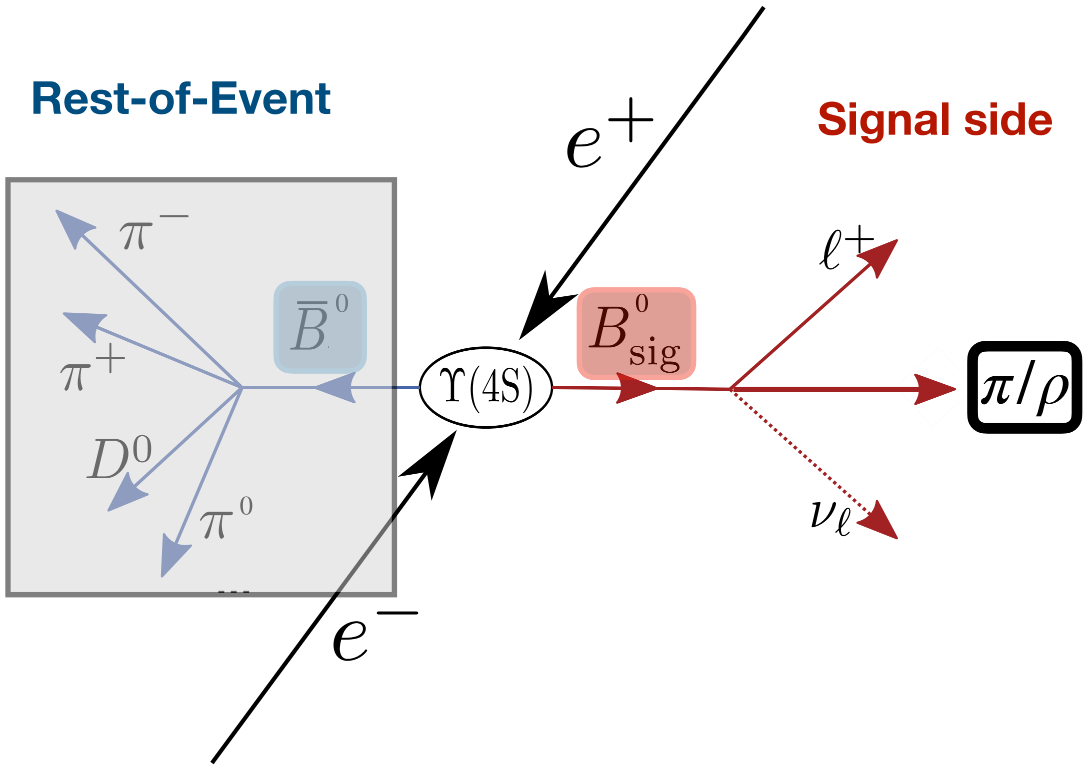
Determination of $|V_{ub}|$ from simultaneous measurements of untagged $B^0 \to \pi^- \ell \bar{\nu}_\ell$ and $B^+ \to \rho^0 \ell \bar{\nu}_\ell$ decays [Submitted to PRD, arXiv:2407.17403]



First reconstruct lepton: **expect fairly** energetic lepton (> 1 GeV) and below kinematic limit (< 2.85 GeV)

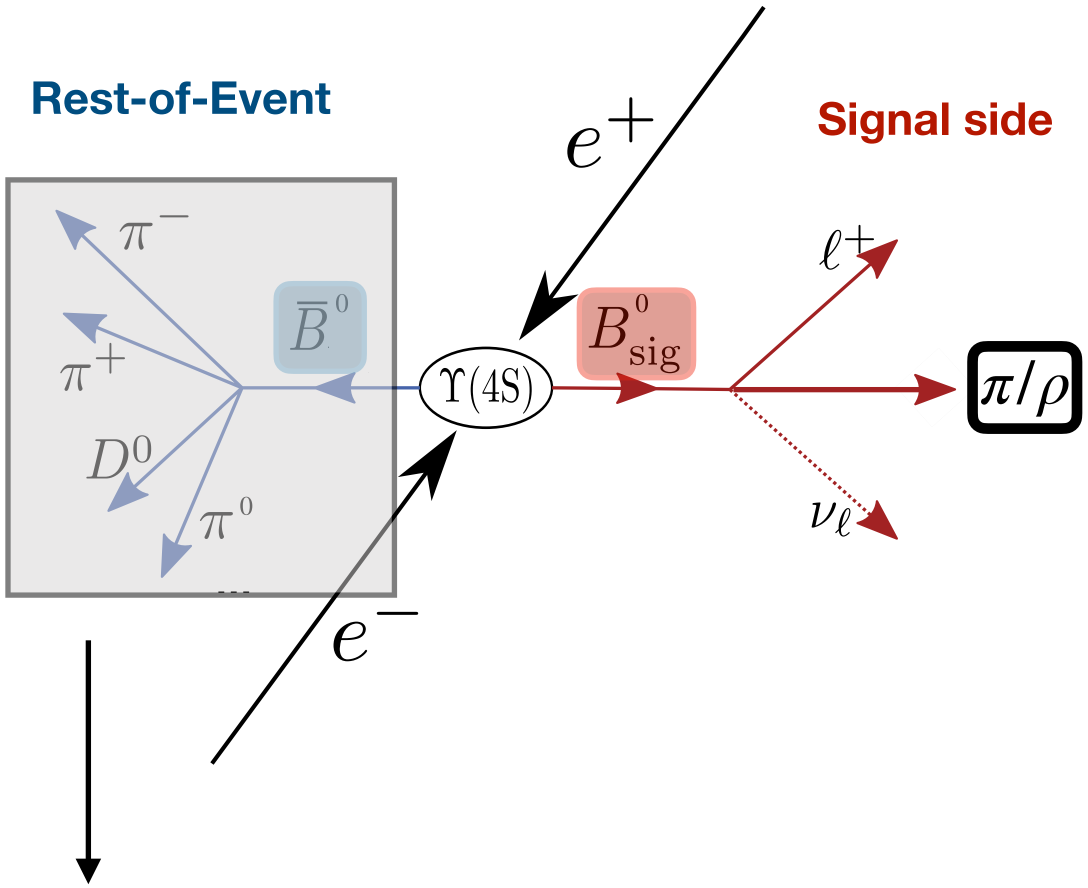
8.

Determination of $\left|V_{ub}\right|$ from simultaneous measurements of untagged $B^0 \to \pi^- \ell \bar{\nu}_\ell$ and $B^+ \to \rho^0 \ell \bar{\nu}_\ell$ decays [Submitted to PRD, arXiv:2407.17403]



Reconstruct tracks consistent with pions and construct ρ^0 candidates

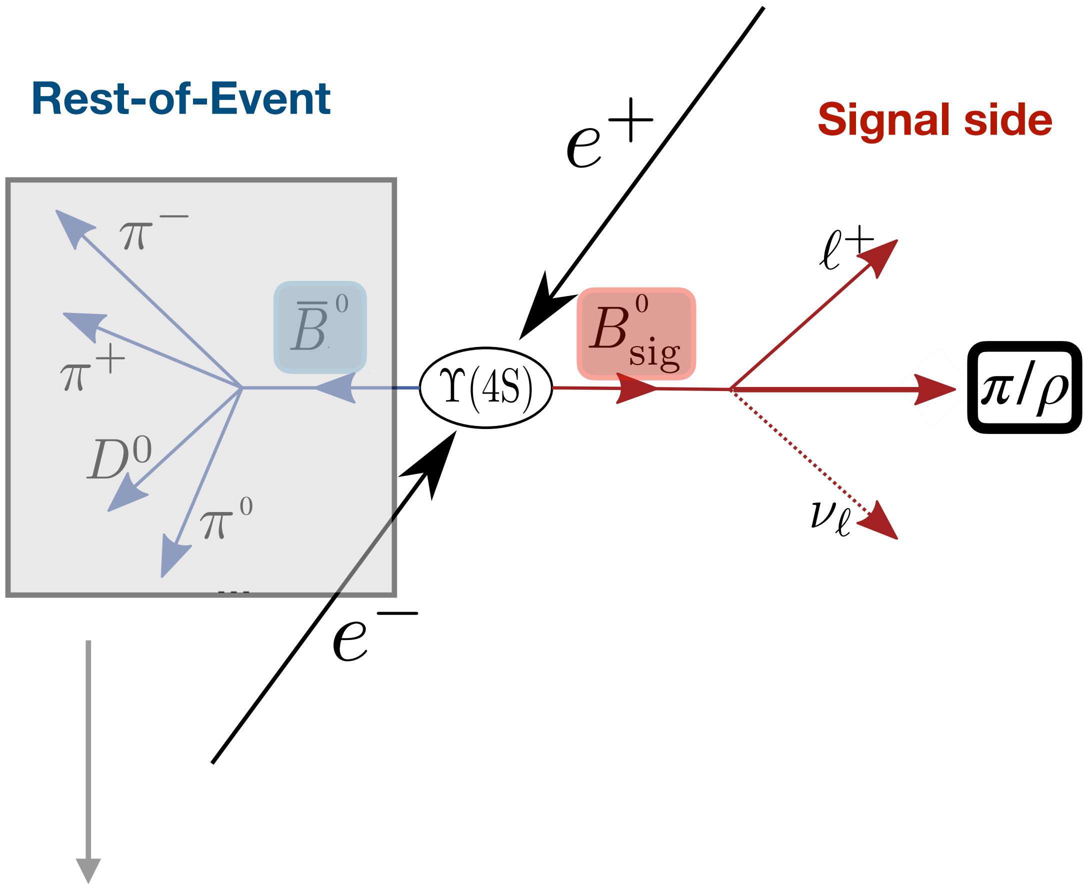
8. Determination of $|V_{ub}|$ from simultaneous measurements of untagged $B^0 \to \pi^- \ell \bar{\nu}_\ell$ and $B^+ \to \rho^0 \ell \bar{\nu}_\ell$ decays [Submitted to PRD, arXiv:2407.17403]



Reconstruct **Rest-of-Event** particles to estimate neutrino kinematics & reconstruct $q^2 = \left(p_{B_{\rm sig}} - p_{\pi/\rho}\right)^2$

8.

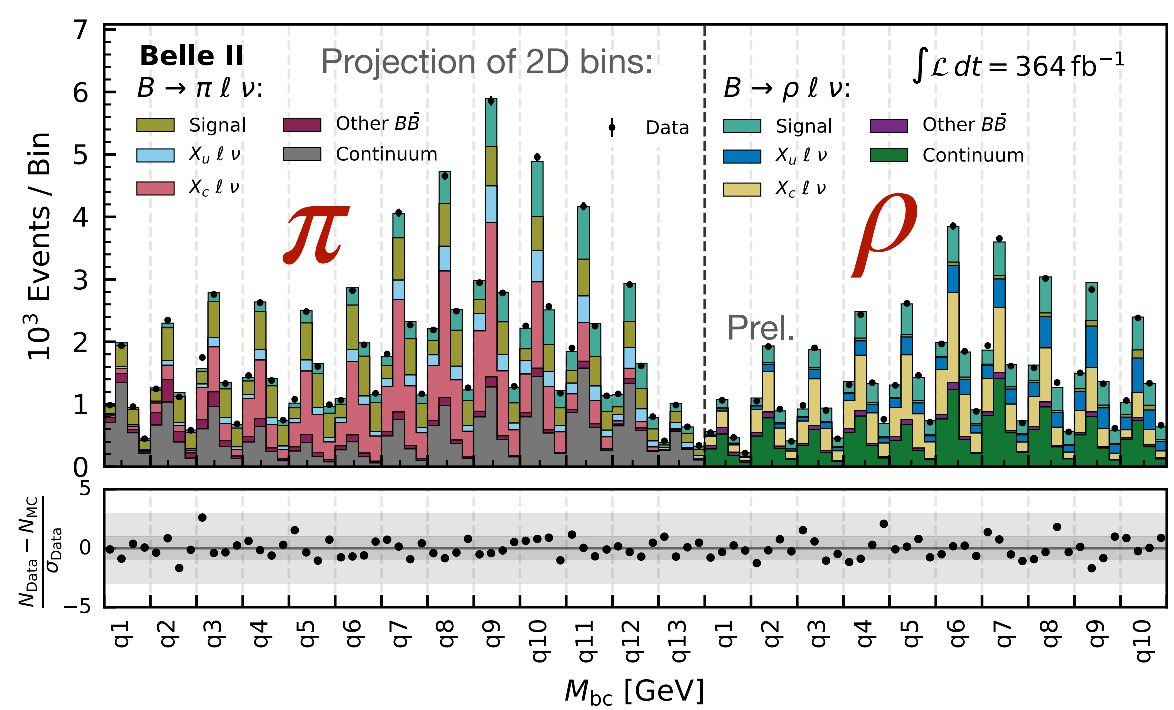
Determination of $|V_{ub}|$ from simultaneous measurements of untagged $B^0 \to \pi^- \ell \bar{\nu}_\ell$ and $B^+ \to \rho^0 \ell \bar{\nu}_\ell$ decays [Submitted to PRD, arXiv:2407.17403]



$$\Delta E = E_B^* - E_{\rm beam}^* = E_B^* - \frac{\sqrt{s}}{2},$$

$$M_{\rm bc} = \sqrt{E_{\rm beam}^{*2} - |\vec{p}_B^{\,*}|^2} = \sqrt{\left(\frac{\sqrt{s}}{2}\right)^2 - |\vec{p}_B^{\,*}|^2}$$

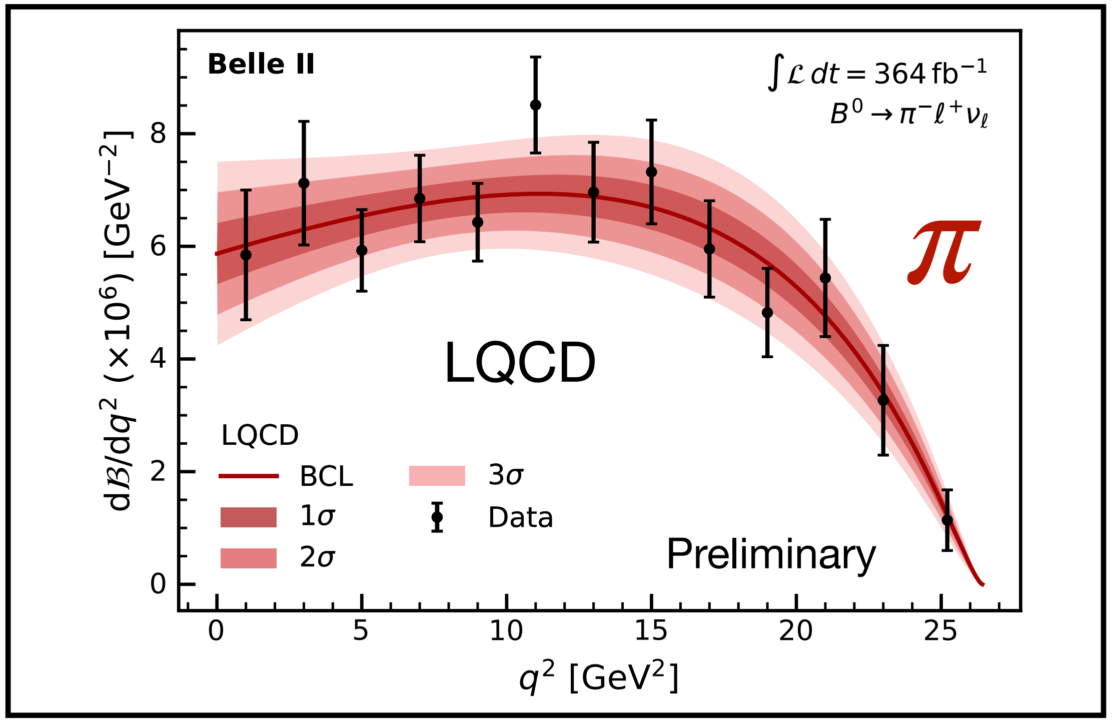
Use 2D fit to $M_{\rm bc} : \Delta E$ to determine **simultaneously** π & ρ signal yields & use forward folding to account for detector effects etc.

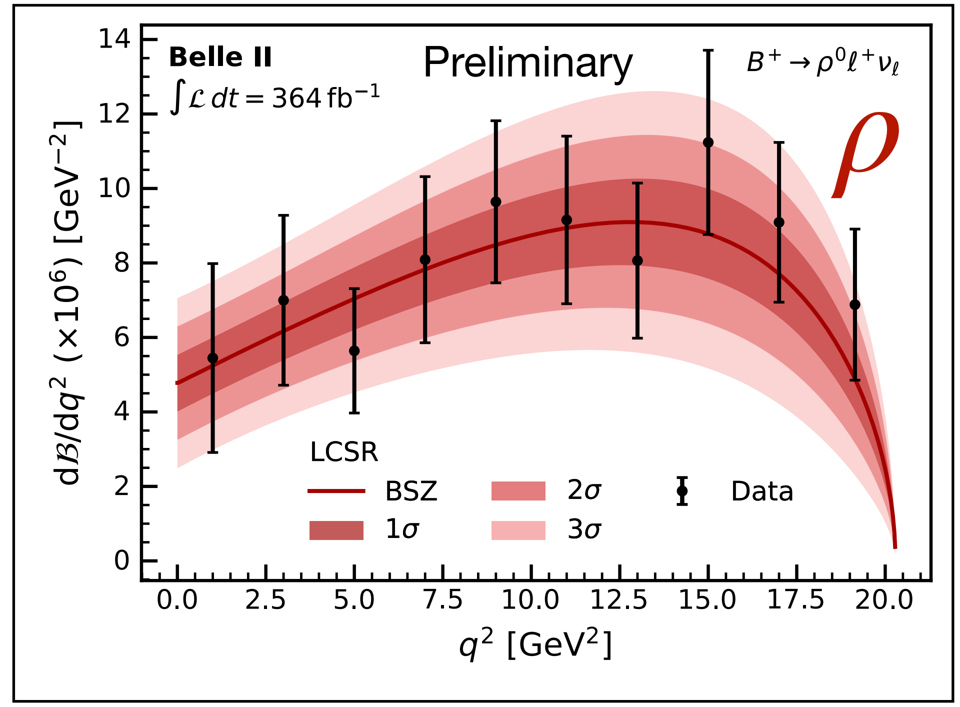


8.

Determination of $|V_{ub}|$ from simultaneous measurements of untagged $B^0 \to \pi^- \ell \bar{\nu}_\ell$ and $B^+ \to \rho^0 \ell \bar{\nu}_\ell$ decays [Submitted to PRD, arXiv:2407.17403]

Measured differential branching fractions as a function of q^2:



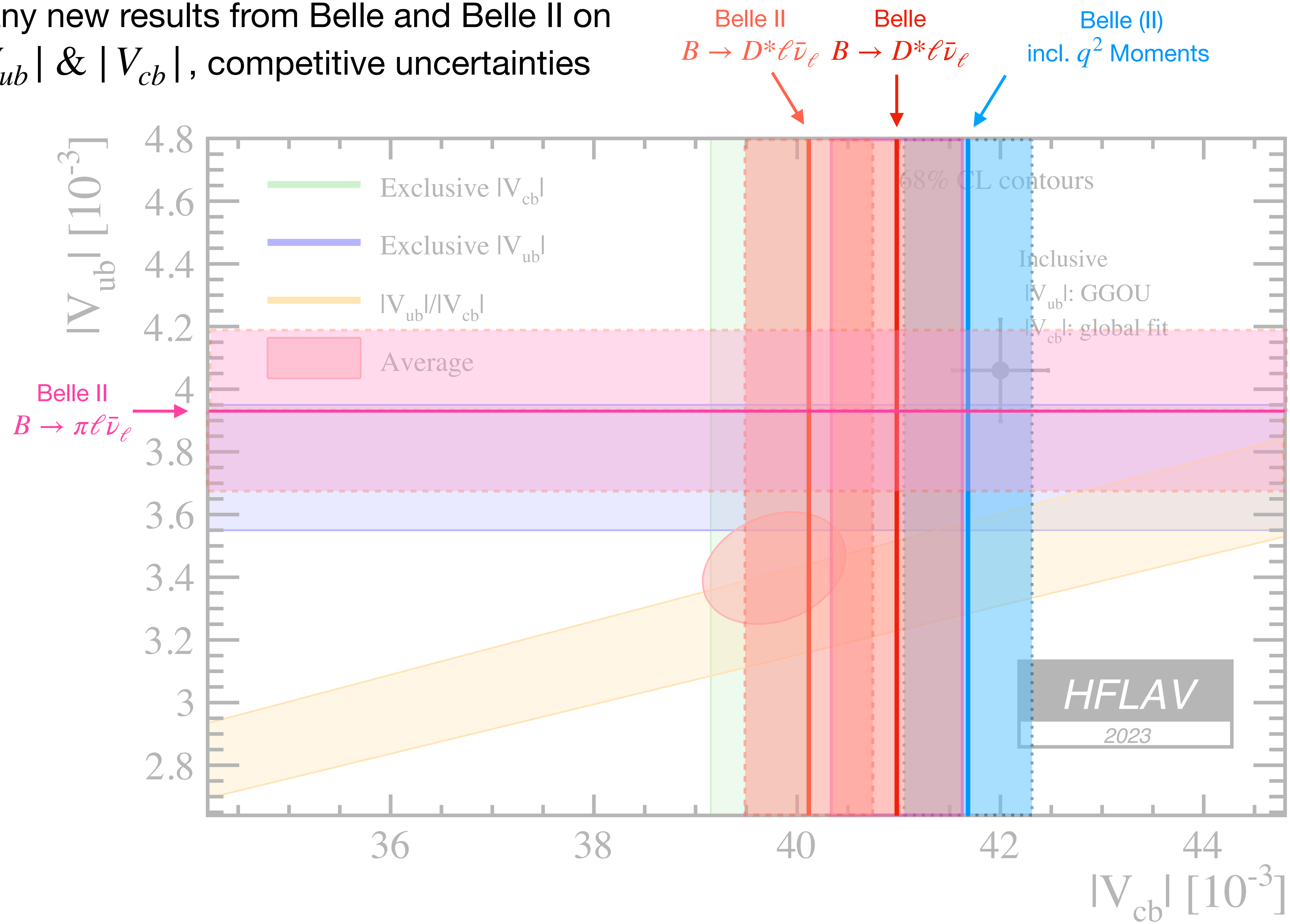


Fit results with Lattice QCD (LQCD) and/or light cone sum rules (LCSR):

$B^0 \to \pi^+ l^- \bar{\nu}_l$:	$\lvert V_{ub}\rvert_{LQCD} = (3.93 \pm 0.09_{stat} \pm 0.13_{syst} \pm 0.19_{theo}) \times 10^{-3}$ $\lvert V_{ub}\rvert_{+LCSR} = (3.73 \pm 0.07_{stat} \pm 0.07_{syst} \pm 0.16_{theo}) \times 10^{-3}$
$B^- \to \rho^0 l^- \bar{\nu}_l$:	$\lvert V_{ub}\rvert_{LCSR} = (3.19 \pm 0.12_{stat} \pm 0.17_{syst} \pm 0.26_{theo}) \times 10^{-3}$

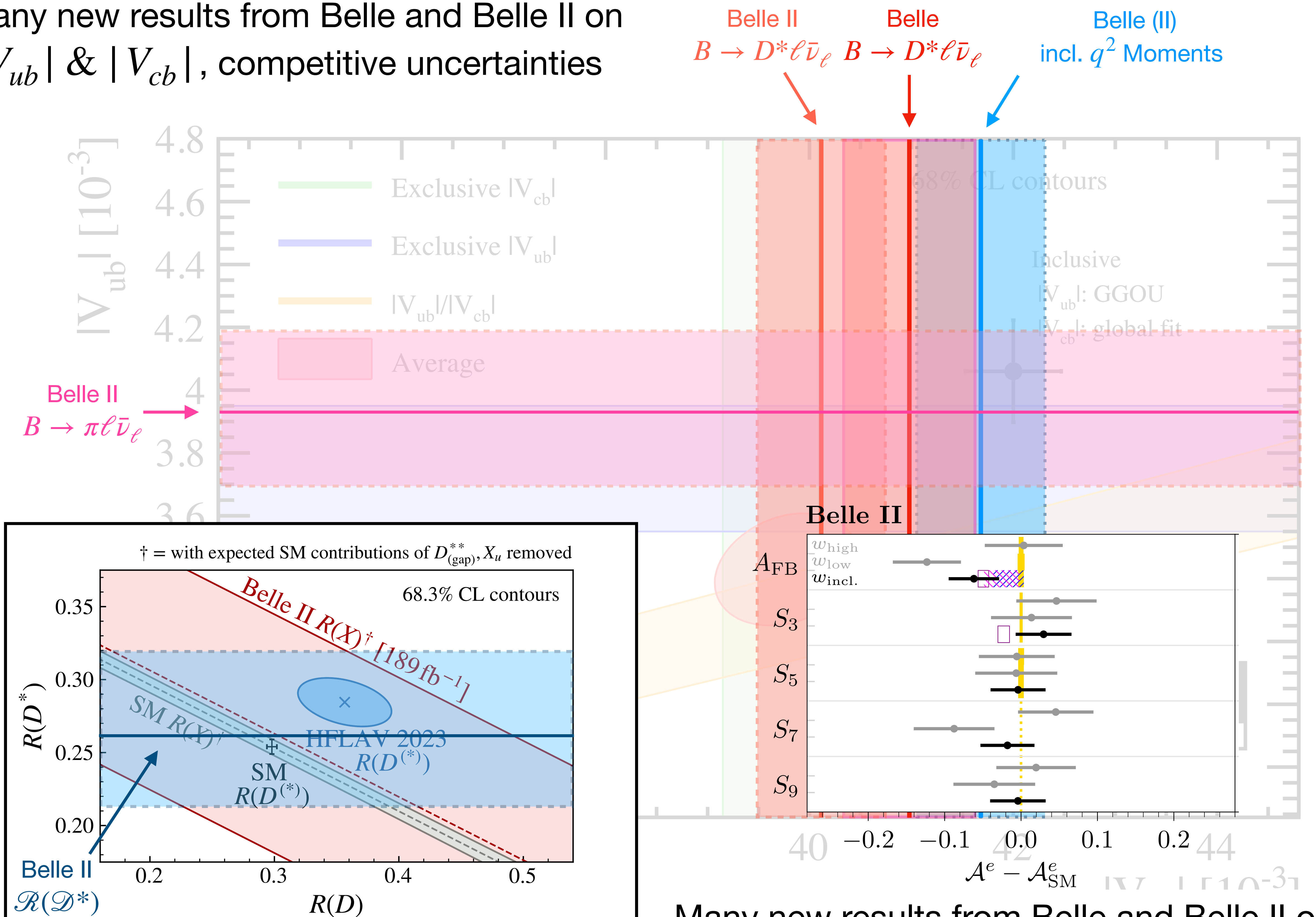
Summary & Conclusion

Many new results from Belle and Belle II on $|V_{ub}|$ & $|V_{cb}|$, competitive uncertainties



Summary & Conclusion

Many new results from Belle and Belle II on $|V_{ub}|$ & $|V_{cb}|$, competitive uncertainties



Many new results from Belle and Belle II on LFU!

More Information

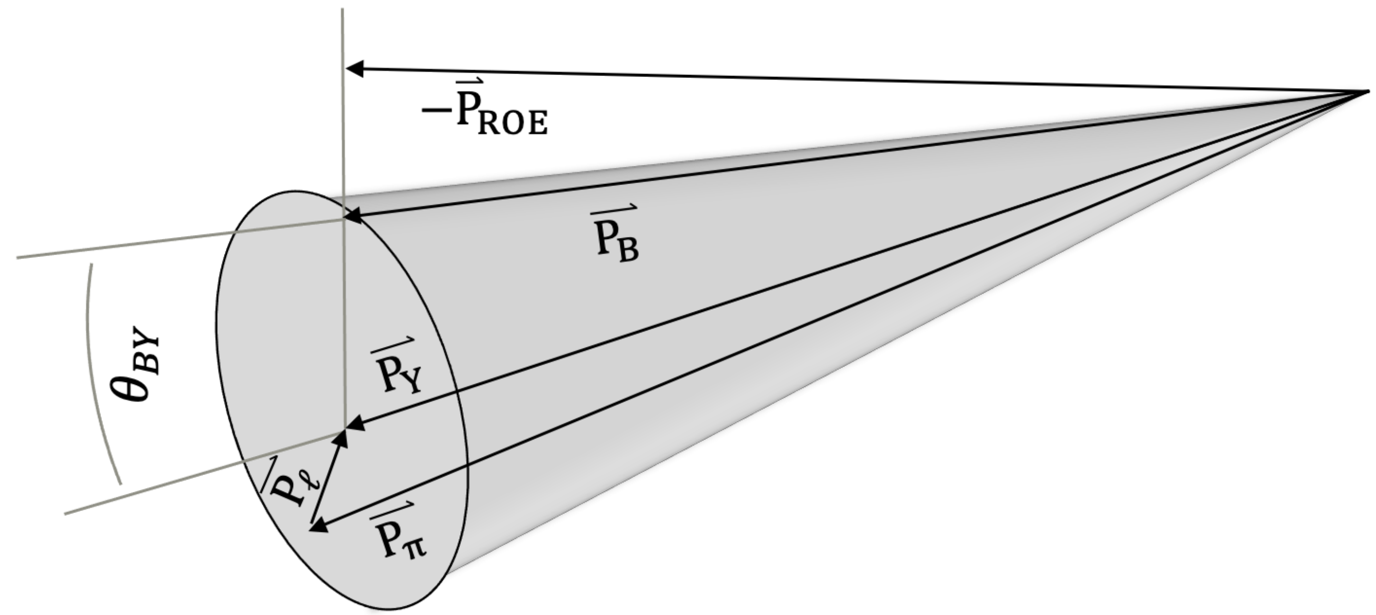
Florian Bernlochner (florian.bernlochner@uni-bonn.de)

q^2 Reconstruction Methods

Can exploit that the B meson lies on a **cone**, whose opening angle is fully determined by properties of visible particles:

$$\cos\theta_{BY} = \frac{2E_B E_Y - m_B^2 - m_Y^2}{2|\mathbf{p}_B||\mathbf{p}_Y|}$$

$Y = \pi\ell / \rho\ell$



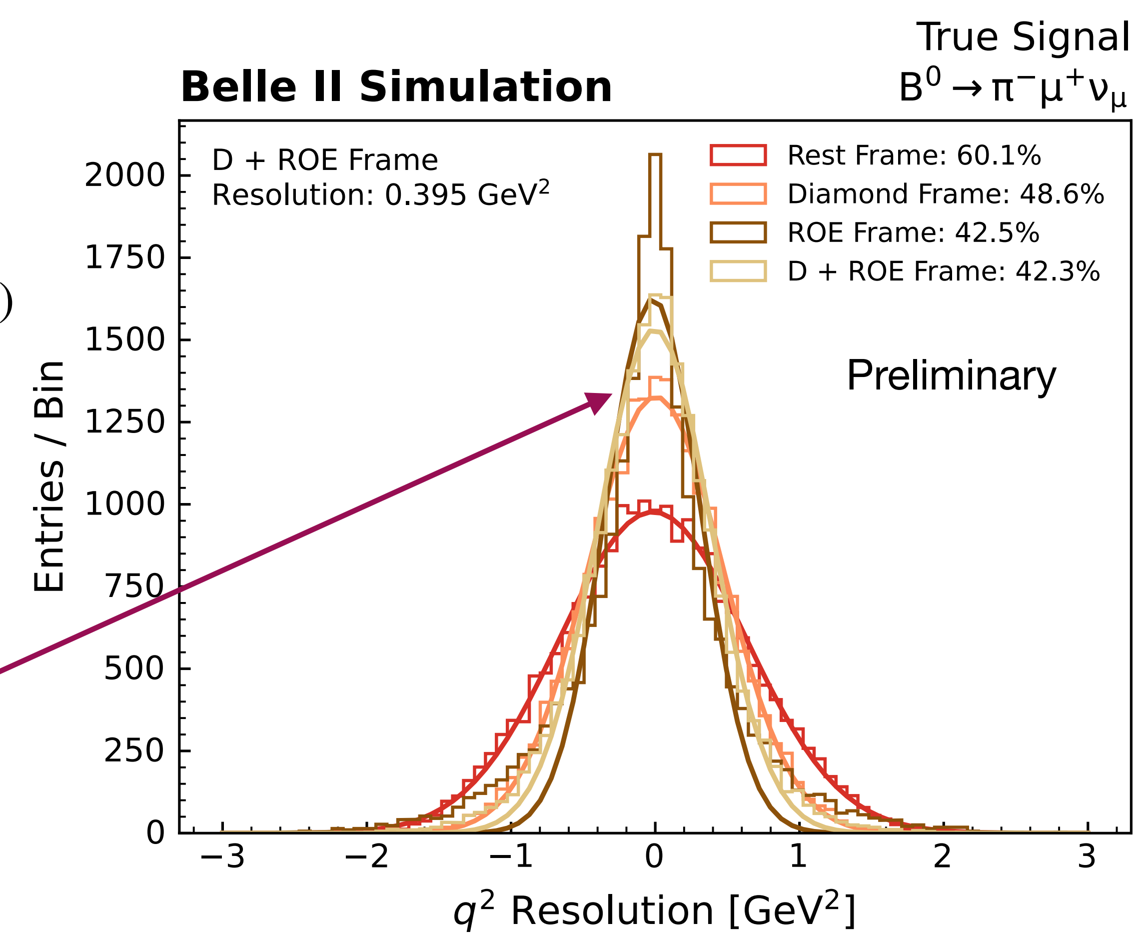
Can use this to estimate B meson direction building a weighted average on the cone

$$(E^B, p_B^x, p_B^y, p_B^z) = (\sqrt{s}/2, |\mathbf{p}_B|\sin\theta_{BY}\cos\phi, |\mathbf{p}_B|\sin\theta_{BY}\sin\phi, |\mathbf{p}_B|\cos\theta_{BY})$$

with weights according to $w_i = \sin^2\theta_i$ with θ denoting the polar angle

(following the angular distribution of $\Upsilon(4S) \to B\bar{B}$)

One can also **combine** both estimates

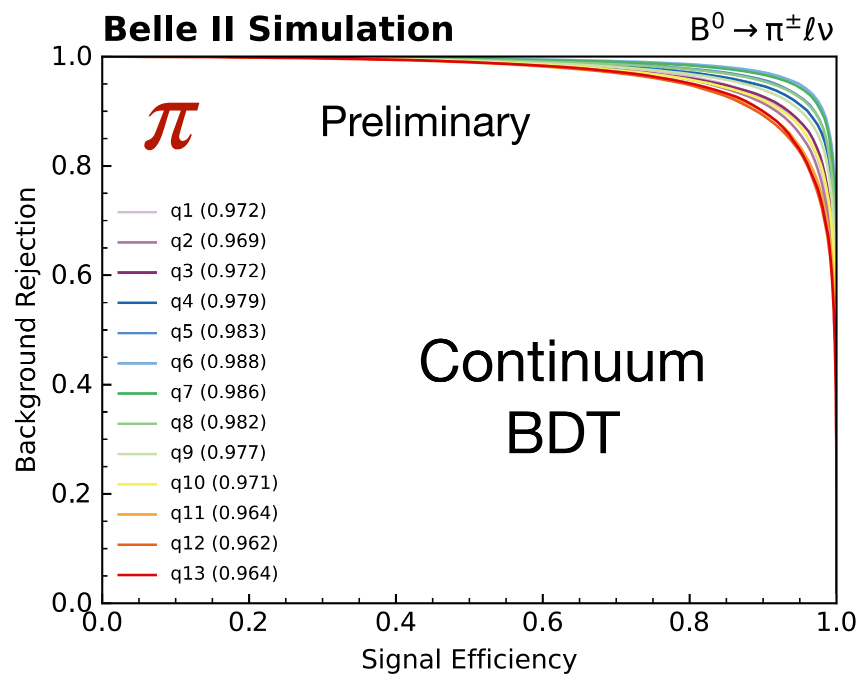


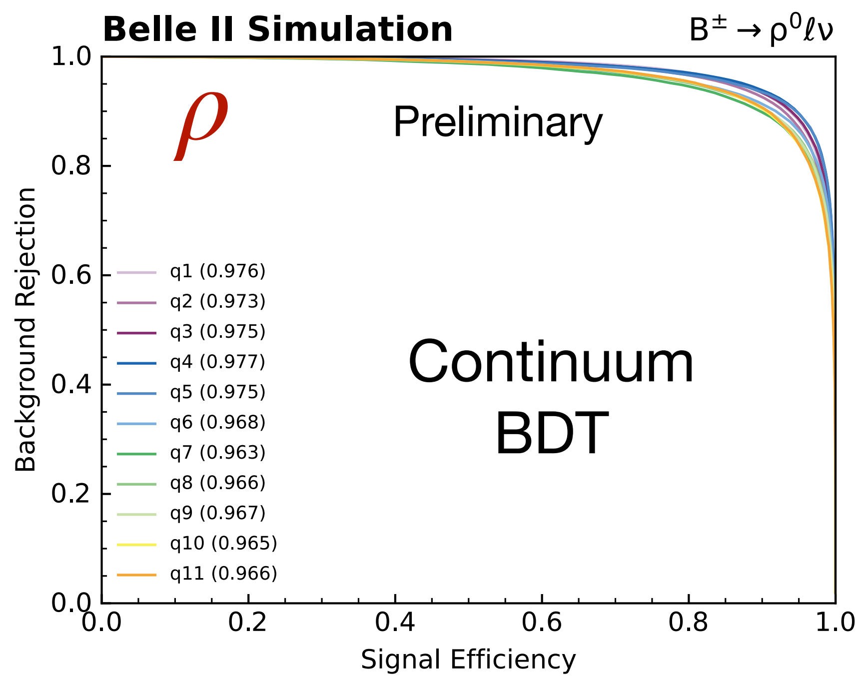
Multivariate Sledgehammer

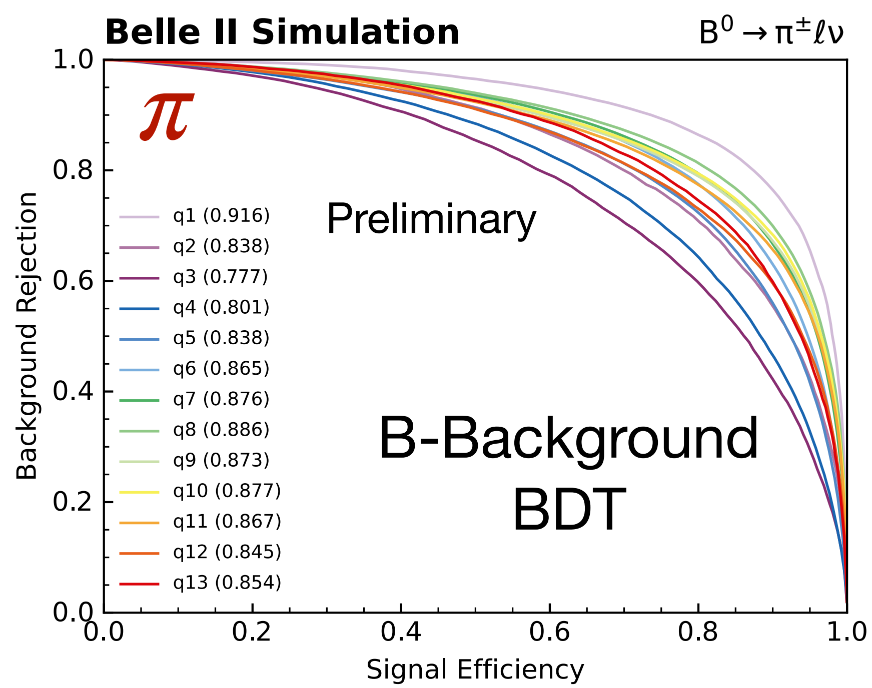
ROC curves:

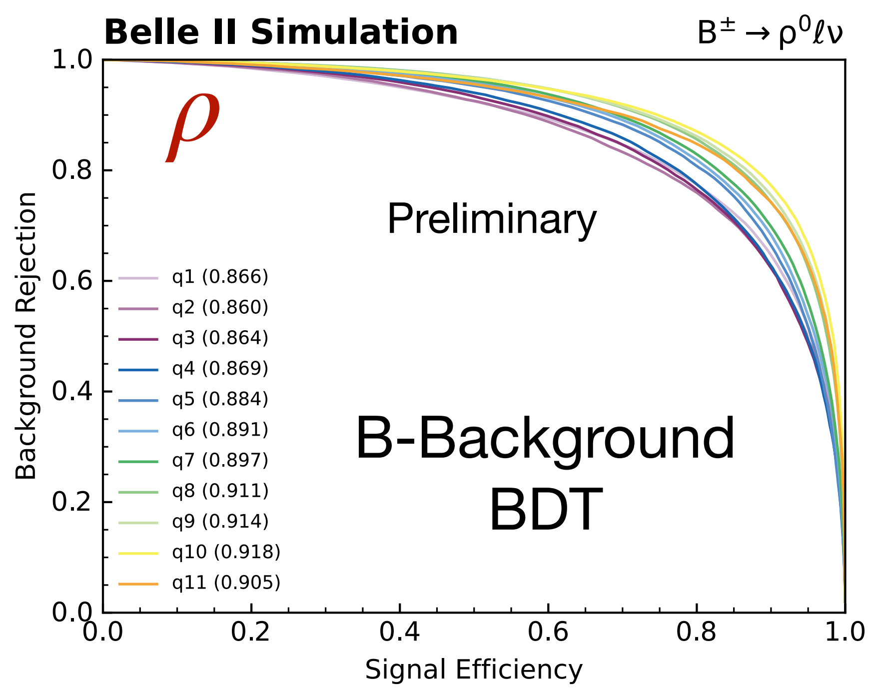
Optimize selection
For each bin
maximizing FOM

$$\frac{N_{\text{sig}}}{\sqrt{N_{\text{sig}} + N_{\text{bkg}}}},$$







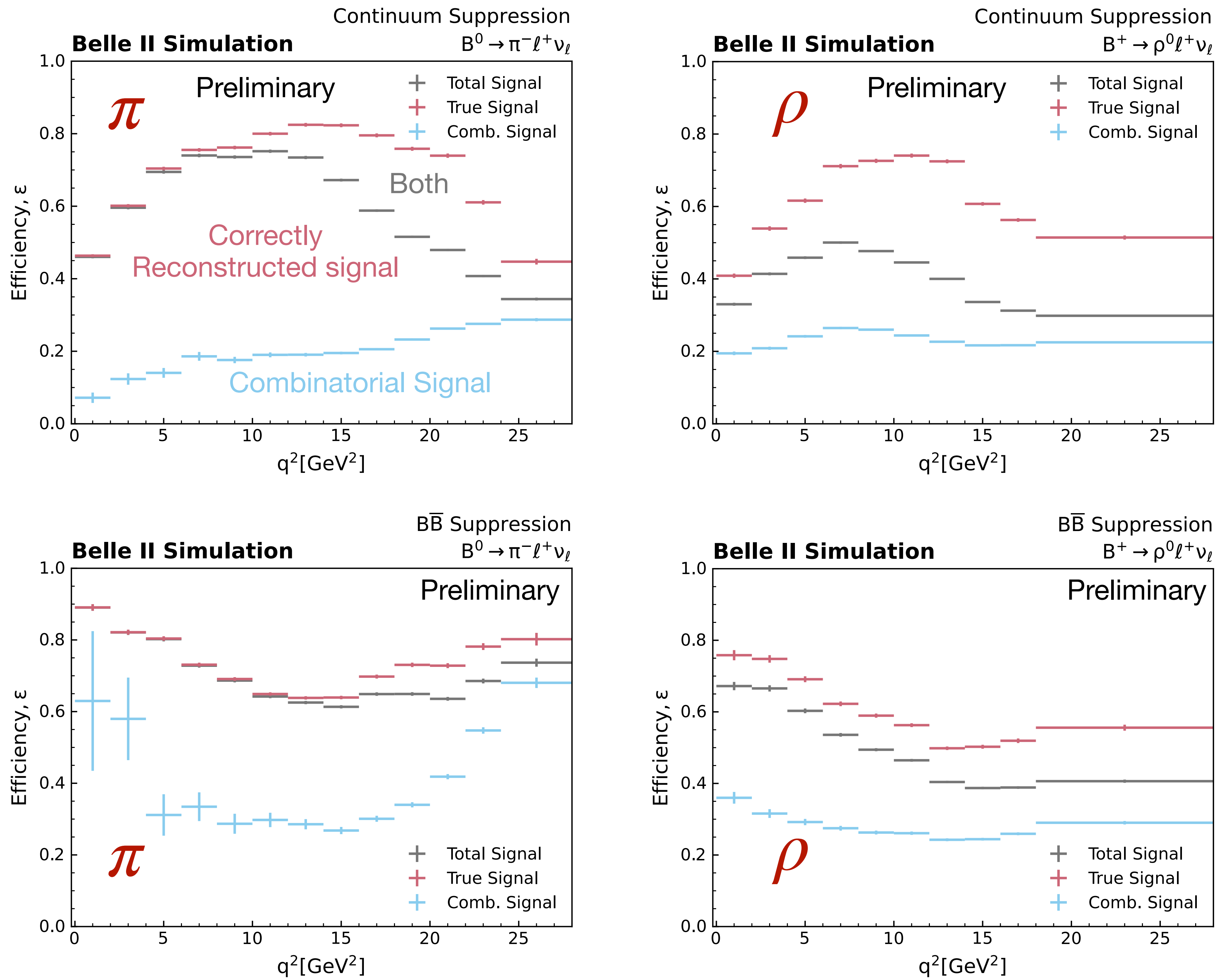


Multivariate Sledgehammer

ROC curves:

Optimize selection
For each bin
maximizing FOM

$$\frac{N_{\rm sig}}{\sqrt{N_{\rm sig}+N_{\rm bkg}}},$$



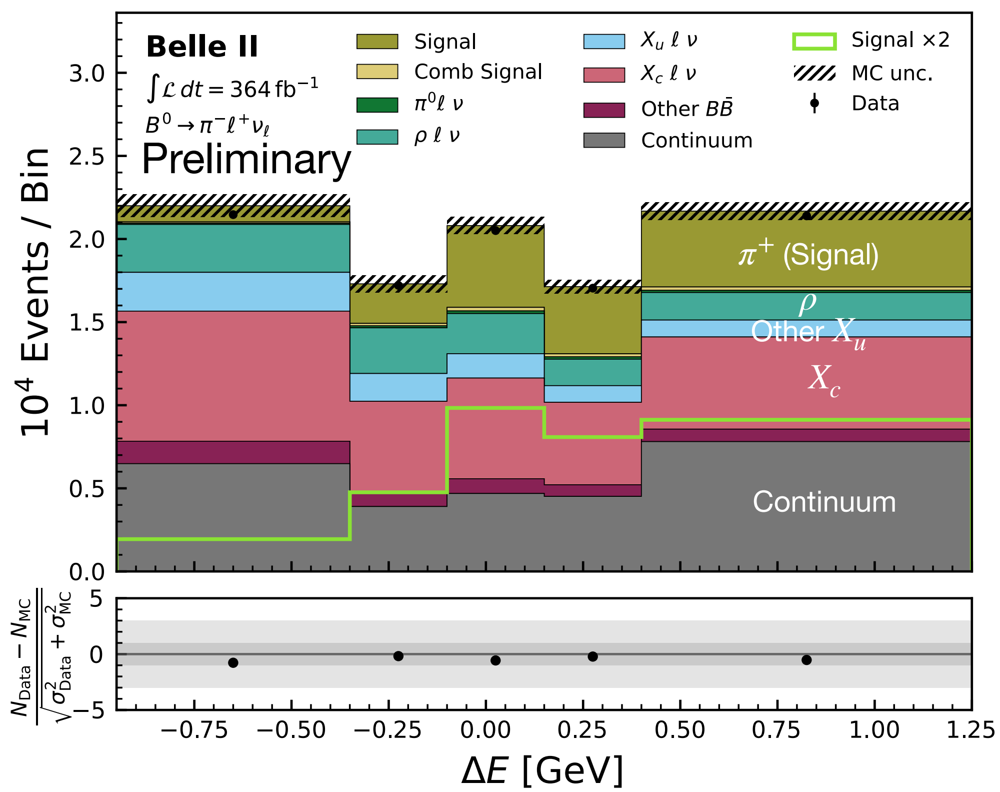
$$0 \simeq \boxed{\Delta E = E_B^* - E_{\mathrm{beam}}^* = E_B^* - \frac{\sqrt{s}}{2},}$$

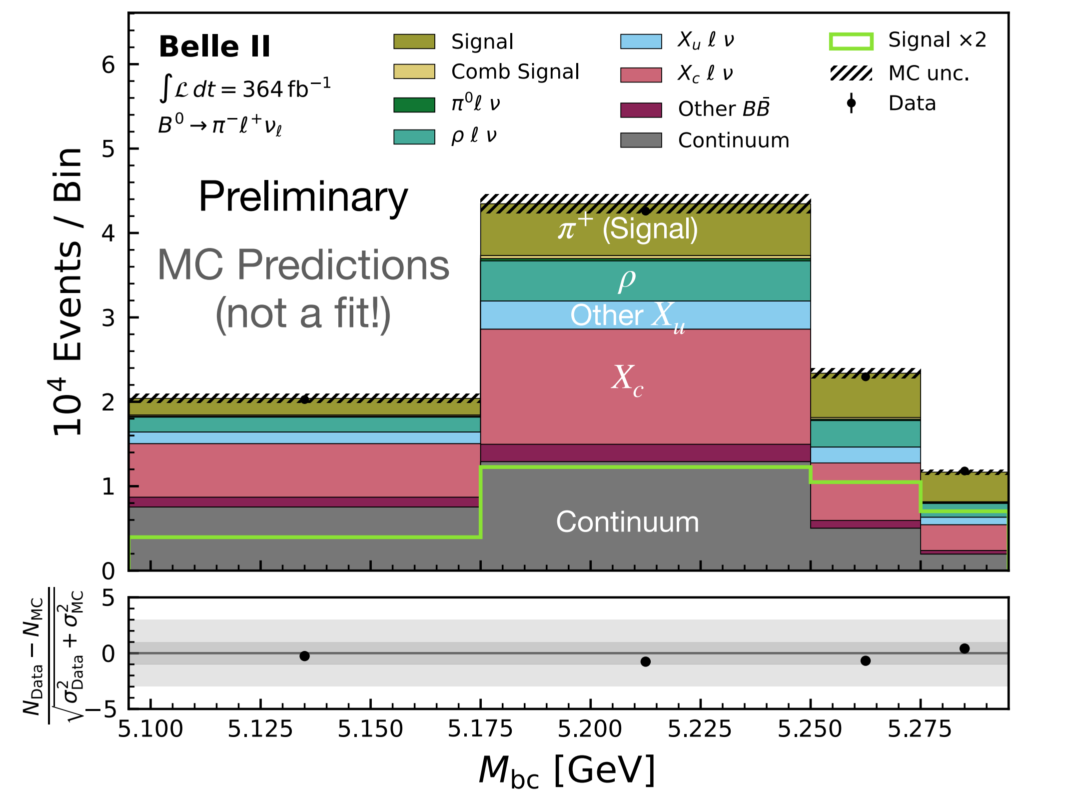
$$E_B^* = E_\ell^* + E_{\pi/\rho}^* + E_{\mathrm{miss}}^*$$

$$\boxed{M_{\mathrm{bc}} = \sqrt{E_{\mathrm{beam}}^{*2} - |\vec{p}_B^{\,*}|^2} = \sqrt{\left(\frac{\sqrt{s}}{2}\right)^2 - |\vec{p}_B^{\,*}|^2}} \simeq m_B$$

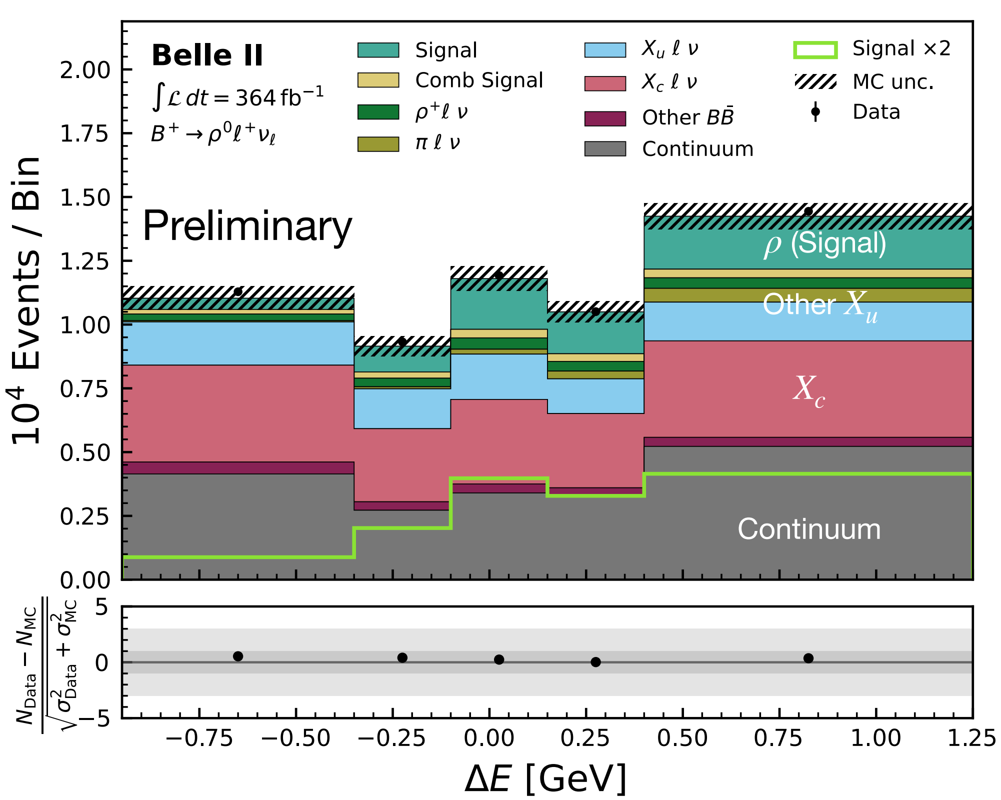
$$\vec{p}_B^{\,*} = \vec{p}_\ell^{\,*} + \vec{p}_{\pi/\rho}^{\,*} + \vec{p}_{\mathrm{miss}}^{\,*}$$

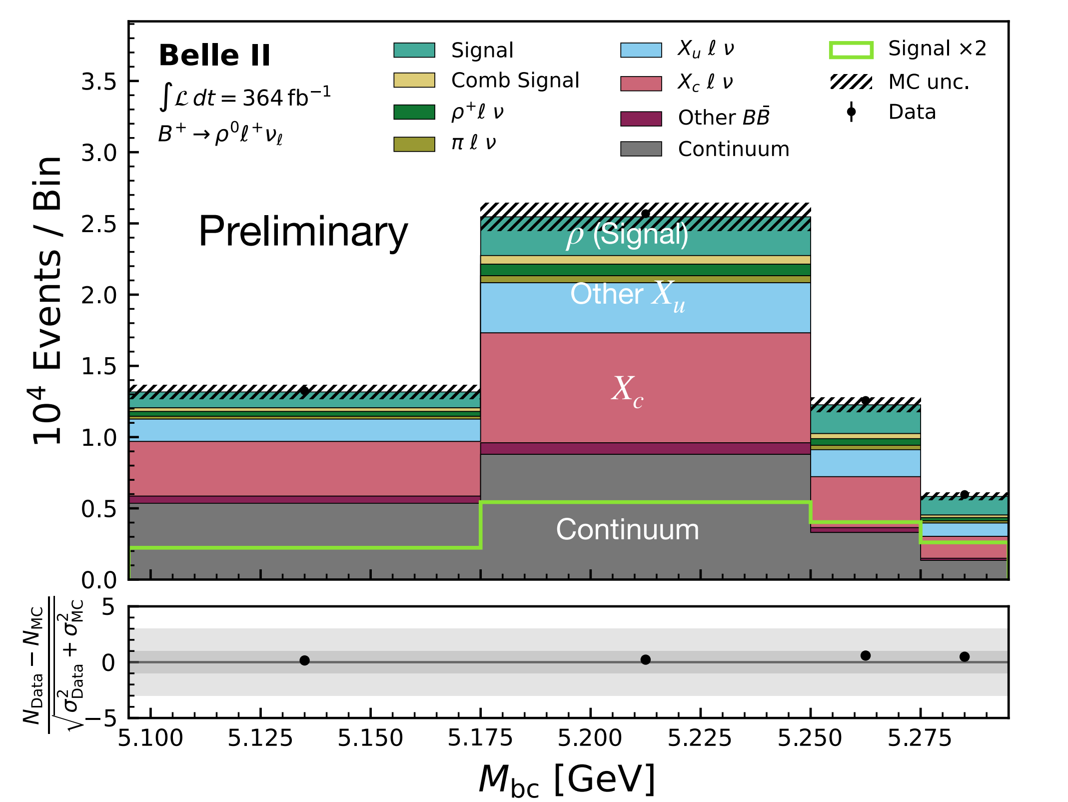
π





ρ





Going Hybrid : MC for $B \to X_u \ell \bar{\nu}_\ell$

Exclusive make-up of $B \to X_u \ell \bar{\nu}_\ell$:

$\mathcal{B}$	Value B^+	Value B^0
$B \to \pi\, \ell^+\, \nu_\ell$ a,e	$(7.8 \pm 0.3) \times 10^{-5}$	$(1.5 \pm 0.06) \times 10^{-4}$
$B \to \eta\, \ell^+\, \nu_\ell$ b,e	$(3.9 \pm 0.5) \times 10^{-5}$	-
$B \to \eta'\, \ell^+\, \nu_\ell$ b,e	$(2.3 \pm 0.8) \times 10^{-5}$	-
$B \to \omega\, \ell^+\, \nu_\ell$ c,e	$(1.2 \pm 0.1) \times 10^{-4}$	-
$B \to \rho\, \ell^+\, \nu_\ell$ c,e	$(1.6 \pm 0.1) \times 10^{-4}$	$(2.9 \pm 0.2) \times 10^{-4}$
$B \to X_u\, \ell^+\, \nu_\ell$ d,e	$(2.2 \pm 0.3) \times 10^{-3}$	$(2.0 \pm 0.3) \times 10^{-3}$

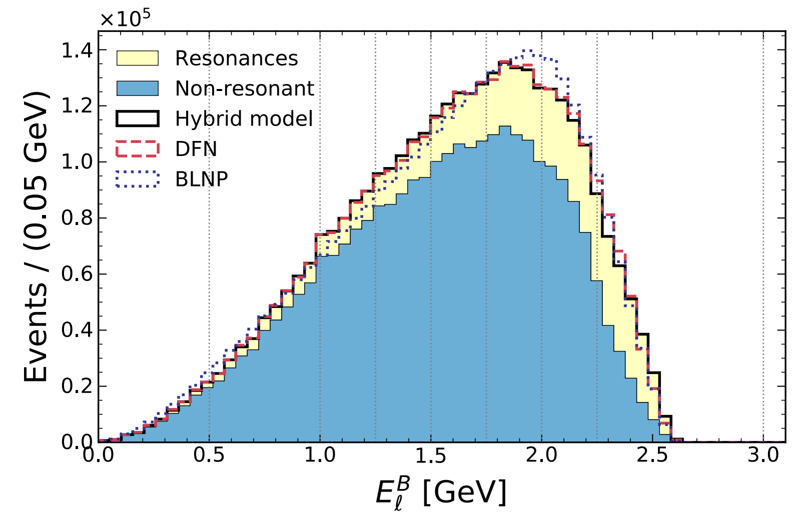
Hybrid = Combining exclusive & inclusive predictions

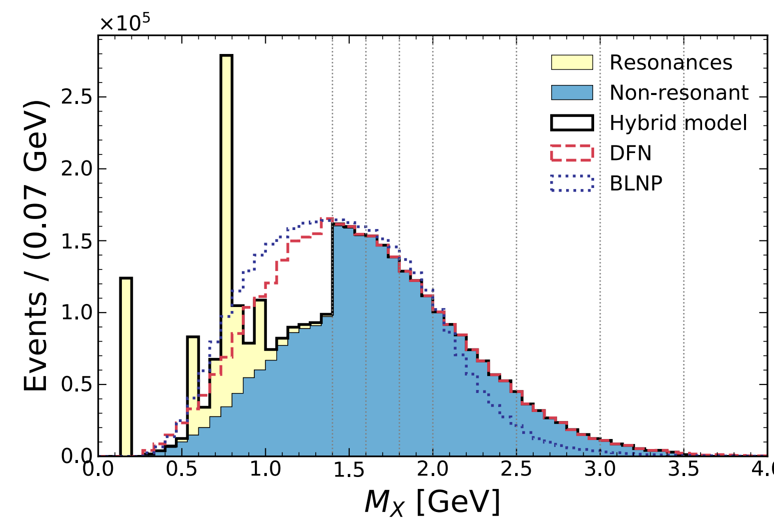
$$\Delta\mathcal{B}^{\mathrm{incl}}_{ijk} = \Delta\mathcal{B}^{\mathrm{excl}}_{ijk} + w_{ijk} \times \Delta\mathcal{B}^{\mathrm{incl}}_{ijk} \,,$$

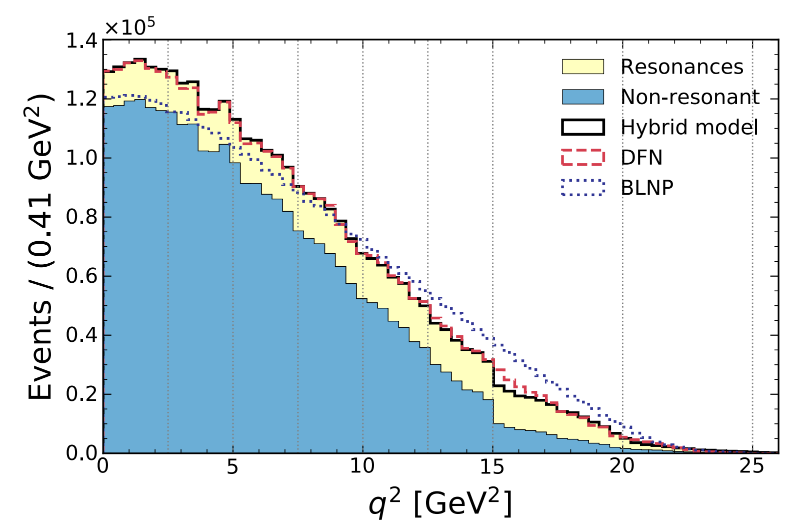
$$q^2 = [0, 2.5, 5, 7.5, 10, 12.5, 15, 20, 25]\,\mathrm{GeV}^2 \,,$$

$$E_\ell^B = [0, 0.5, 1, 1.25, 1.5, 1.75, 2, 2.25, 3]\,\mathrm{GeV} \,,$$

$$M_X = [0, 1.4, 1.6, 1.8, 2, 2.5, 3, 3.5]\,\mathrm{GeV} \,.$$



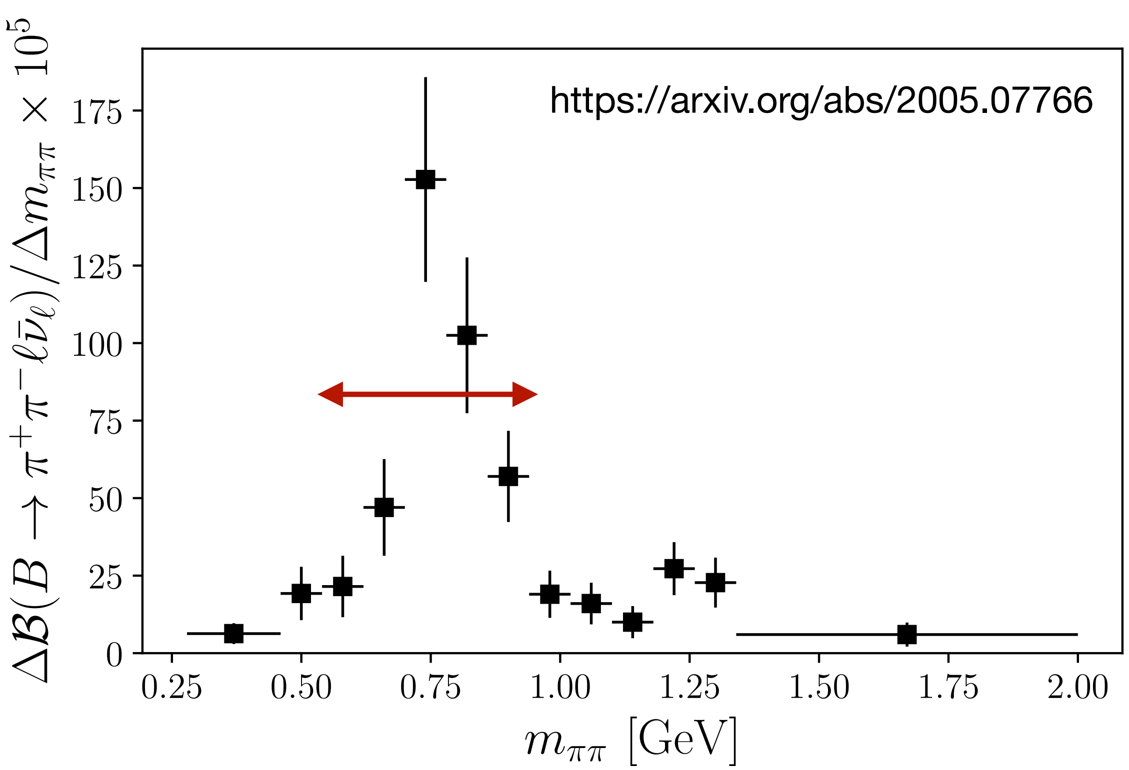


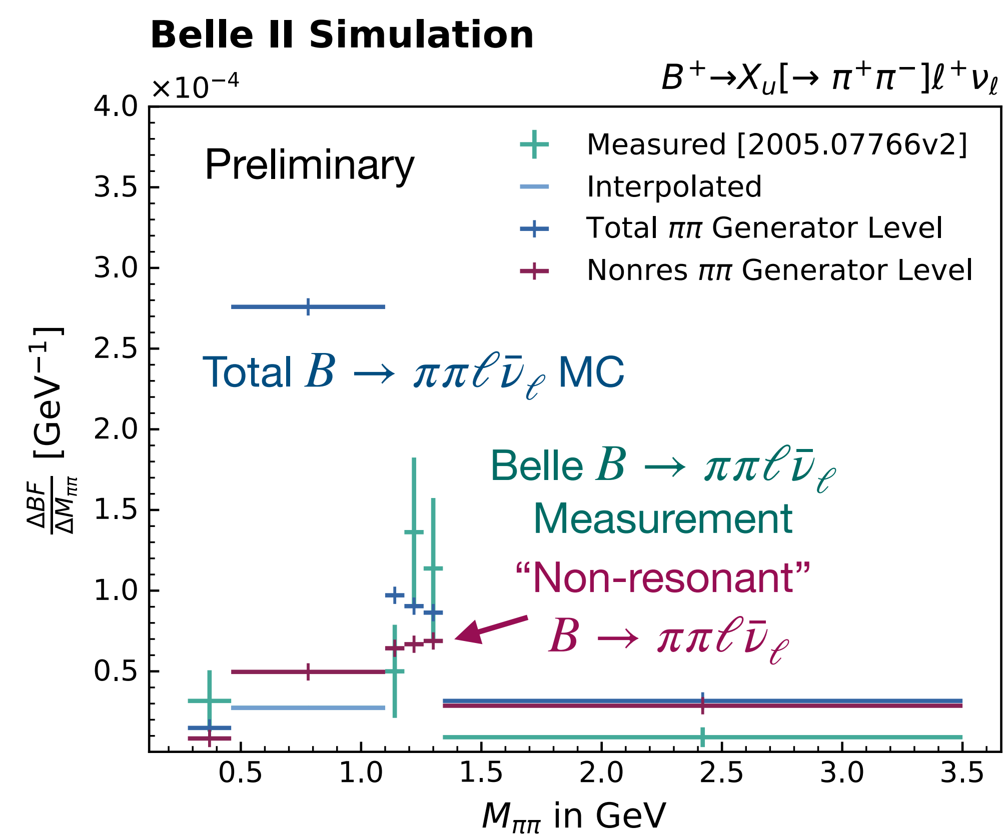


A word on "Non-resonant" $B \to \pi\pi\ell\bar{\nu}_\ell$ Background

"Non-resonant" $B \to \pi\pi\ell\bar{\nu}_\ell$ = everything that is not $B \to \rho\ell\bar{\nu}_\ell$

Use Belle measurement of $B \to \pi\pi\ell\bar{\nu}_\ell$ to estimate contamination using a simple linear model; fit distribution with **coarse** binning and vary within uncertainties.





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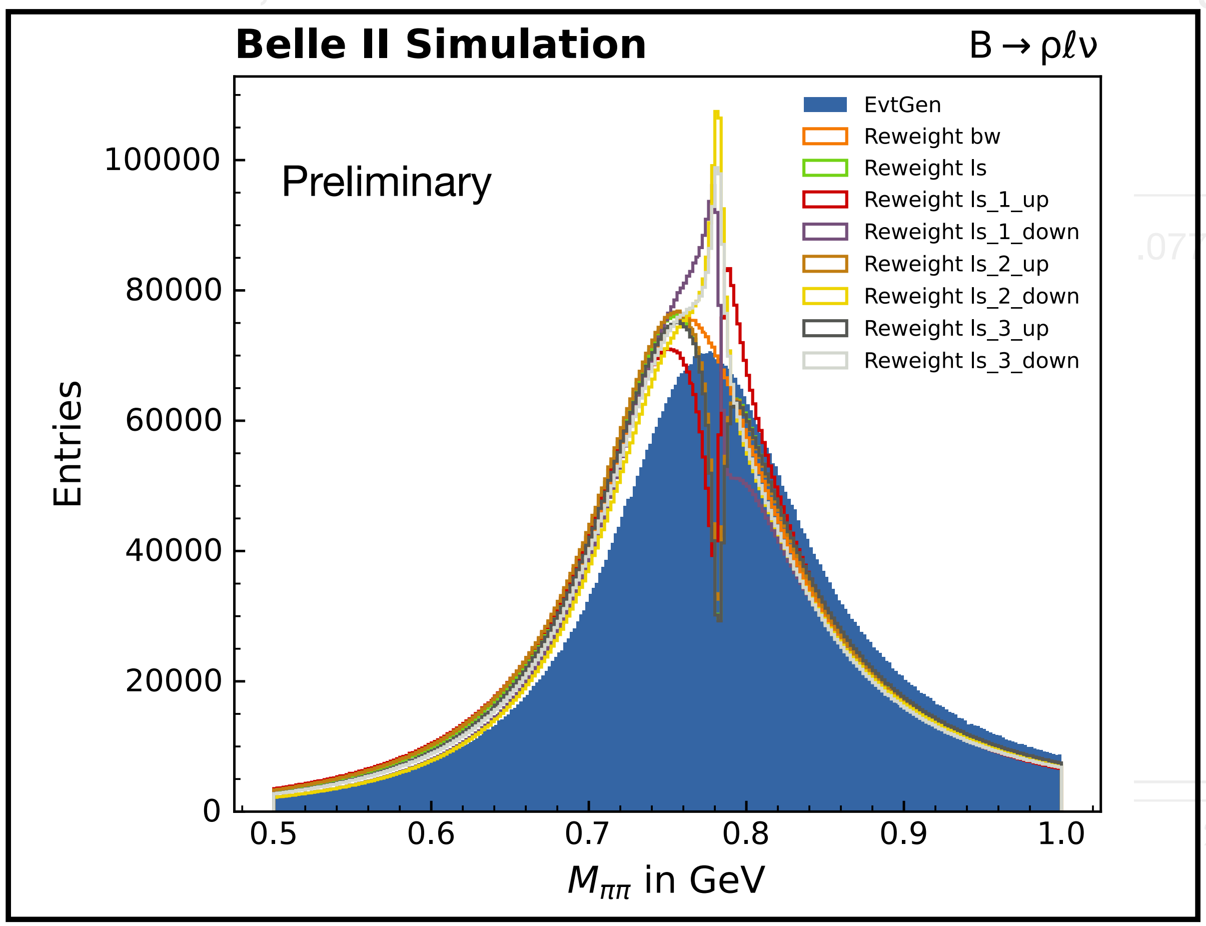
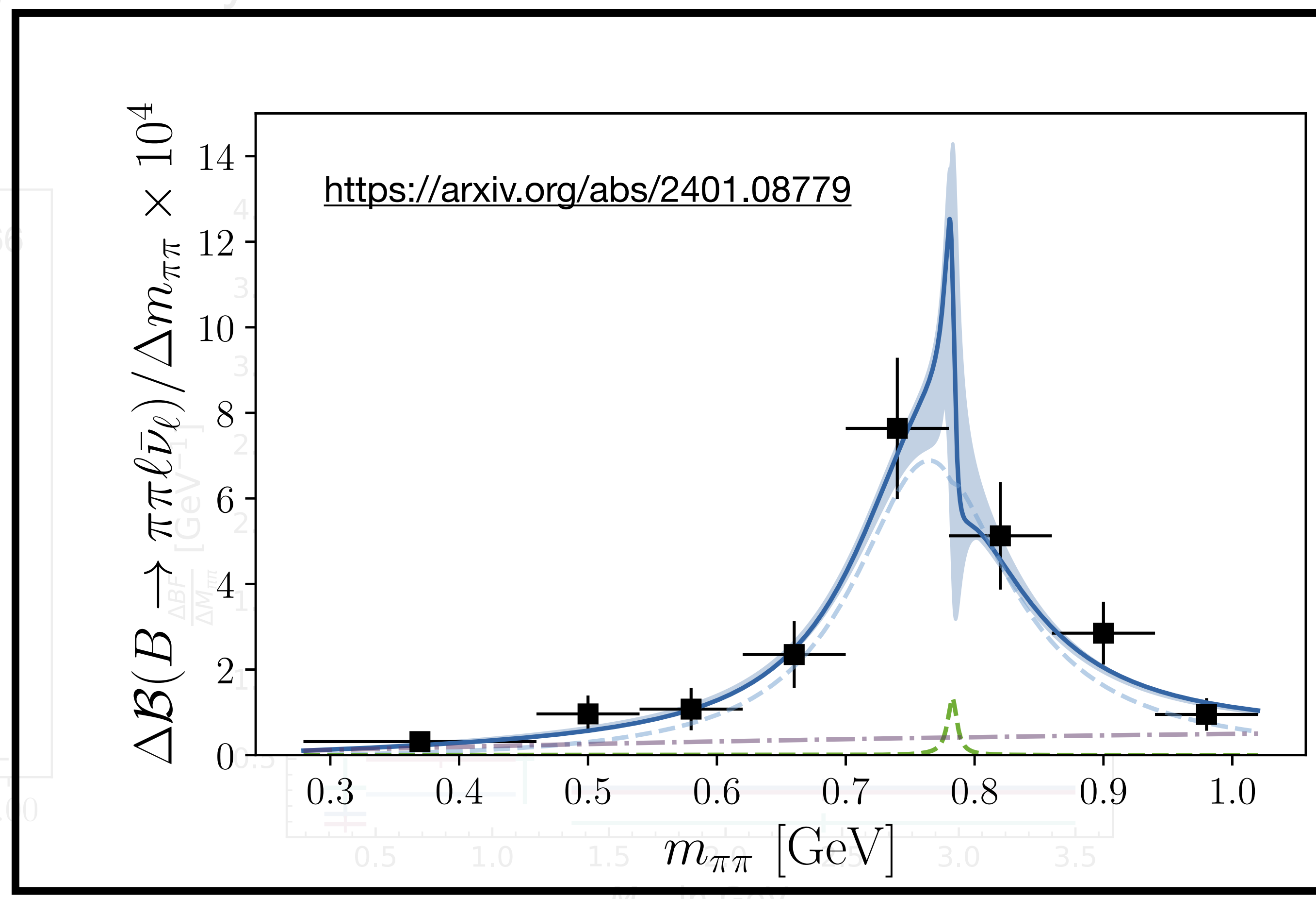
“Non-resonant” $B \to \pi\pi\ell\bar{\nu}_\ell$ = everything that is not $B \to \rho\ell\bar{\nu}_\ell$

Also correct for chosen ρ mass in the simulation; add uncertainties due to ρ-ω interference from fit to Belle $B \to \pi\pi\ell\bar{\nu}_\ell$ spectrum from https://arxiv.org/abs/2401.08779

Use Belle measurement of $B \to \pi\pi\ell\bar{\nu}_\ell$ to estimate contamination using a simple linear model; fit distribution with **coarse** binning and vary within uncertainties.





Systematic Uncertainties

Largest uncertainties:

Continuum modelling
simulated sample size
“non-resonant” $\pi\pi$ (for ρ) & X_u modelling
Physics constraints (N_{BB}, f_{+0}, isospin assumptions)
…

Depending on the bin systematically or statistically limited ; more data will help to reduce this further

ρ

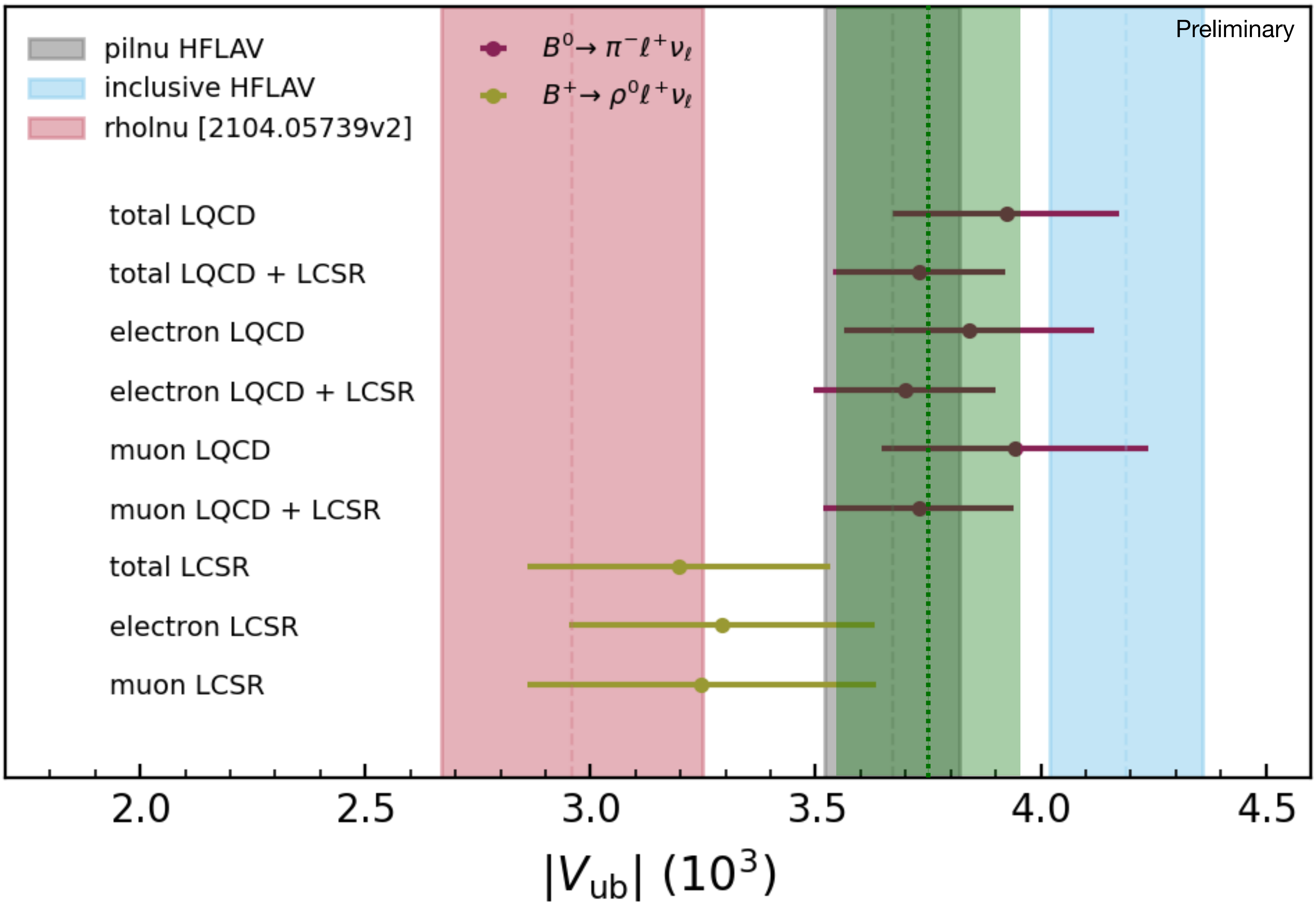
Source ($B^+ \to \rho^0 \ell^+ \nu_\ell$)	$q1$	$q2$	$q3$	$q4$	$q5$	$q6$	$q7$	$q8$	$q9$	$q10$
Detector effects	2.8	2.0	1.6	1.1	1.7	1.9	2.4	1.4	1.4	1.6
Beam energy	2.1	1.9	1.9	1.5	1.3	1.1	1.0	0.9	0.8	0.5
Simulated sample size	14.1	7.8	7.4	6.3	6.3	5.2	6.4	5.6	6.2	7.3
BDT efficiency	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
Physics constraints	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8
Signal model	0.7	0.2	0.2	0.2	0.3	0.4	0.5	0.3	1.8	2.4
ρ lineshape	1.7	1.6	2.0	1.0	1.9	1.8	1.4	0.9	1.6	1.7
Nonresonant $B \to \pi\pi\ell\nu_\ell$	5.6	6.3	6.7	8.6	9.3	10.7	10.1	7.0	7.8	11.8
DFN parameters	3.6	5.5	4.1	3.5	1.1	1.2	2.7	1.7	1.9	2.3
$B \to X_u\ell\nu_\ell$ model	1.7	3.0	3.8	5.0	5.8	6.1	6.3	1.9	7.2	12.4
$B \to X_c\ell\nu_\ell$ model	1.8	1.9	1.7	1.1	1.4	1.7	0.9	0.9	1.9	2.6
Continuum	31.5	24.3	17.0	19.6	13.2	14.8	16.0	16.6	15.2	18.7
Total systematic	35.6	27.5	21.0	23.5	18.8	20.5	21.6	19.4	20.2	27.0
Statistical	30.0	17.5	20.8	14.4	12.4	13.6	14.1	10.4	12.2	11.8
Total	46.6	32.6	29.6	27.6	22.6	24.6	25.8	22.0	23.6	29.5

π

Source ($B^0 \to \pi^- \ell^+ \nu_\ell$)	$q1$	$q2$	$q3$	$q4$	$q5$	$q6$	$q7$	$q8$	$q9$	$q10$	$q11$	$q12$	$q13$
Detector effects	2.0	0.9	1.1	1.0	1.0	1.1	1.1	1.0	0.9	1.2	2.3	4.1	5.8
Beam energy	0.6	0.8	0.7	0.8	0.7	0.6	0.6	0.6	0.5	0.5	0.5	0.6	0.7
Simulated sample size	4.7	3.8	3.3	3.2	3.2	2.9	3.8	3.7	4.0	4.5	5.9	8.0	13.6
BDT efficiency	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
Physics constraints	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9
Signal model	0.1	0.1	0.2	0.1	0.0	0.2	0.2	0.4	0.3	0.8	0.9	0.2	4.9
ρ lineshape	0.1	0.1	0.3	0.3	0.2	0.1	0.3	0.1	0.3	0.1	0.2	0.2	0.6
Nonresonant $B \to \pi\pi\ell\nu_\ell$	0.5	0.6	0.4	0.4	0.5	1.0	1.2	1.0	0.8	1.8	1.2	2.3	14.3
DFN parameters	0.8	0.4	1.5	1.6	1.4	1.7	1.2	0.1	0.7	1.2	2.9	3.5	3.7
$B \to X_u\ell\nu_\ell$ model	0.2	0.4	0.3	0.4	0.2	0.9	1.1	1.2	1.0	1.3	1.6	0.7	8.7
$B \to X_c\ell\nu_\ell$ model	1.4	2.0	1.7	1.3	1.3	1.4	1.8	1.6	1.3	1.4	1.1	0.5	1.7
Continuum	15.1	11.3	7.6	7.1	5.8	5.7	8.1	8.3	9.6	10.4	14.5	23.8	34.4
Total systematic	16.4	12.6	9.3	8.7	7.7	7.7	10.0	9.9	11.1	12.2	16.6	26.0	41.6
Statistical	11.0	8.8	7.9	7.0	7.5	6.4	7.9	7.7	9.1	10.7	9.6	14.6	22.6
Total	19.7	15.4	12.2	11.2	10.7	10.0	12.7	12.6	14.4	16.3	19.1	29.8	47.3

Summary

$$|V_{ub}| = (3.75 \pm 0.06_{\,\mathrm{exp}} \pm 0.19_{\,\mathrm{theo}}) \times 10^{-3}\,,$$



How can we measure these coefficients?

Step 1: bin up phase-space in $q^2 \sim w$ in however many bins you can afford

Step 2: Determine the # of signal events in specific phase-space regions

The coefficients are related to a weighted sun of events in a given q^2 bin

$$J_i = \frac{1}{N_i} \sum_{j=1}^{8} \sum_{k,l=1}^{4} \eta^{\chi}_{ij}\, \eta^{\theta_\ell}_{ik}\, \eta^{\theta_V}_{il} \left[\chi^i \otimes \theta^j_\ell \otimes \theta^k_V\right]$$

Normalization Factor

Weights

Phase space region

J_i	η_i^χ	$\eta_i^{\theta_\ell}$	$\eta_i^{\theta_V}$	normalization N_i
J_{1s}	$\{+\}$	$\{+,a,a,+\}$	$\{-,c,c,-\}$	$2\pi(1)2$
J_{1c}	$\{+\}$	$\{+,a,a,+\}$	$\{+,d,d,+\}$	$2\pi(1)(2/5)$
J_{2s}	$\{+\}$	$\{-,b,b,-\}$	$\{-,c,c,-\}$	$2\pi(-2/3)2$
J_{2c}	$\{+\}$	$\{-,b,b,-\}$	$\{+,d,d,+\}$	$2\pi(-2/3)(2/5)$
J_3	$\{+,-,-,+,+,-,-,+\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$
J_4	$\{+,+,-,-,-,-,+,+\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_5	$\{+,+,-,-,-,-,+,+\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_{6s}	$\{+\}$	$\{+,+,-,-\}$	$\{-,c,c,-\}$	$2\pi(1)2$
J_{6c}	$\{+\}$	$\{+,+,-,-\}$	$\{+,d,d,+\}$	$2\pi(1)(2/5)$
J_7	$\{+,+,+,+,-,-,-,-\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_8	$\{+,+,+,+,-,-,-,-\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_9	$\{+,+,-,-,+,+,-,-\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$

$a = 1 - 1/\sqrt{2}, b = a\sqrt{2}, c = 2\sqrt{2} - 1, d = 1 - 4\sqrt{2}/5$

E.g. for J_3: **Split** χ into **2 Regions**

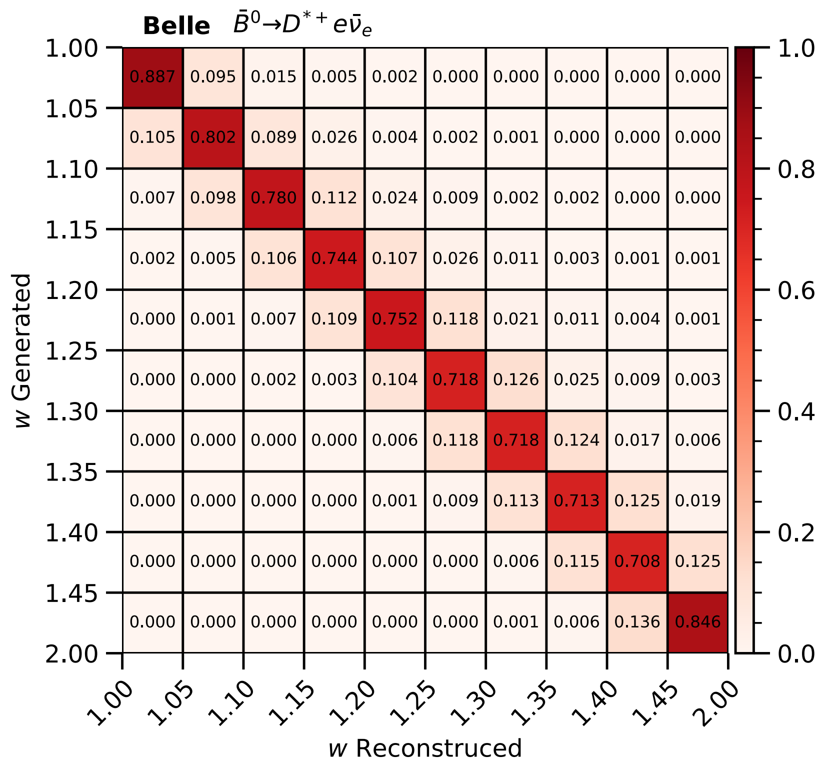
$'+' : \chi \in [0,\pi/4], [3/4\pi,5/4\pi], [7/4\pi,2\pi]$ $\quad \tilde{N}_+$

$'-' : \chi \in [\pi/4,3/4\pi], [5/4\pi,7/4\pi]$ $\quad \tilde{N}_-$

Step 3: Reverse Migration and Acceptance Effects

Resolution effects: events with a **given "true" value** of $\{q^2, \cos\theta_\ell, \cos\theta_V, \chi\}$ can **fall into different reconstructed** bins

E.g. w migration matrix



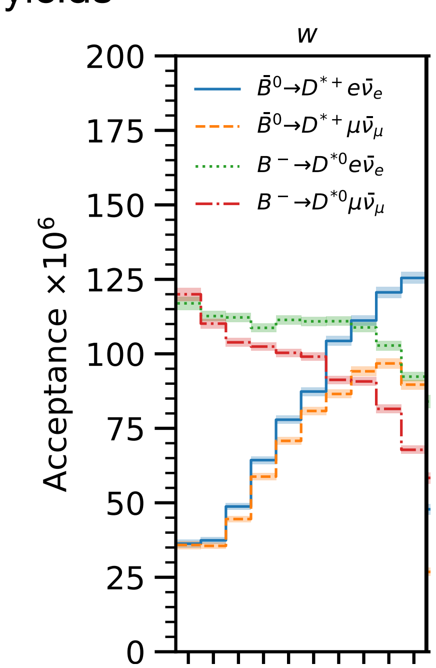
arXiv:2301.07529 **[hep-ex]**

$$\begin{pmatrix} \tilde{N}_+^{q_1^2} \\ \tilde{N}_-^{q_1^2} \\ \tilde{N}_+^{q_2^2} \\ \tilde{N}_-^{q_2^2} \\ \tilde{N}_+^{q_3^2} \\ \tilde{N}_-^{q_3^2} \\ \tilde{N}_+^{q_4^2} \\ \tilde{N}_-^{q_4^2} \\ \tilde{N}_+^{q_5^2} \\ \tilde{N}_-^{q_5^2} \end{pmatrix} = \mathscr{M} \begin{pmatrix} N_+^{q_1^2} \\ N_-^{q_1^2} \\ N_+^{q_2^2} \\ N_-^{q_2^2} \\ N_+^{q_3^2} \\ N_-^{q_3^2} \\ N_+^{q_4^2} \\ N_-^{q_4^2} \\ N_+^{q_5^2} \\ N_-^{q_5^2} \end{pmatrix}$$

← Unfolded yields

Migration matrix

Bkg subtracted yields



arXiv:2301.07529 **[hep-ex]**

Acceptance x Efficiency Corrections:

$$N_+^{q_i^2} \cdot e^{-1}_{\mathrm{eff},+,q_i^2} = n_+^{q_i^2}$$

$$N_-^{q_i^2} \cdot e^{-1}_{\mathrm{eff},-,q_i^2} = n_-^{q_i^2}$$

← Acceptance / Eff. corrected yields

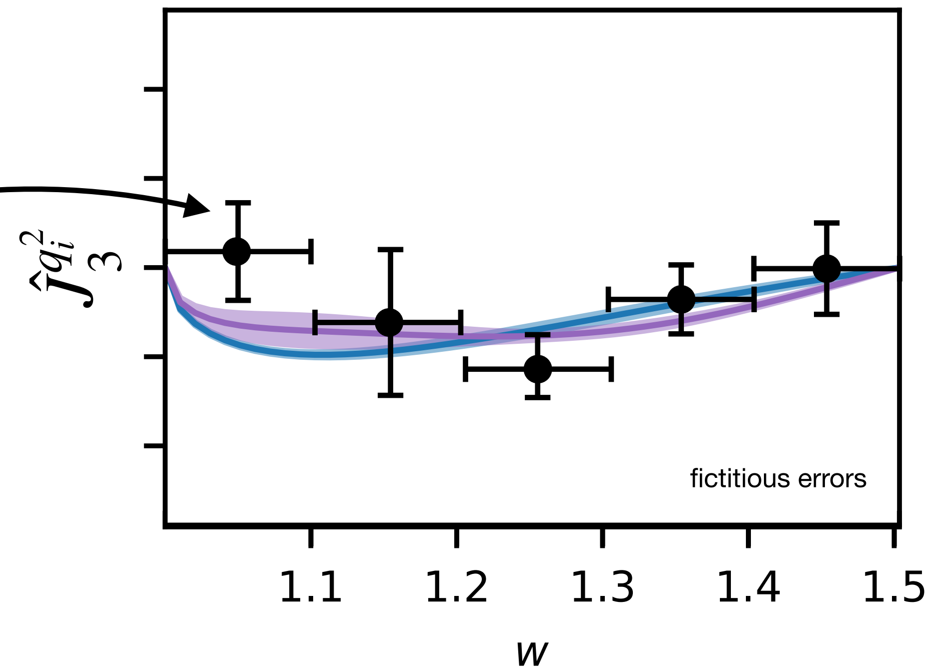
Unfolded yields

Step 4: Calculate J_i for a given w/q^2 bin

$$n_+^{q_i^2}, \quad n_-^{q_i^2} \rightarrow \hat{J}_3^{q_i^2} = \frac{1}{\Gamma} \times \frac{n_+^{q_i^2} - n_-^{q_i^2}}{4(4/3)^2}$$

Normalization

$$\Gamma = \frac{8}{9}\pi\left(3\sum_i J_{1c}^{q_i^2} + 6\sum_i J_{1s}^{q_i^2} - \sum_i J_{2c}^{q_i^2} - 2\sum_i J_{2s}^{q_i^2}\right)$$



More **involved** for the **other** coefficients: need full experimental covariance between all measured w/q^2 bins and coefficients (statistical overlap, systematics)

SM:

$$\{J_{1s}^{q_i^2}, J_{1c}^{q_i^2}, J_{2s}^{q_i^2}, J_{2c}^{q_i^2}, J_{3}^{q_i^2}, J_{4}^{q_i^2}, J_{5}^{q_i^2}, J_{6s}^{q_i^2}\}$$

e.g. **5 x 8** = **40 coefficients**

or full thing (SM + NP)
with **5 x 12** = **60 coefficients**

J_i	η_i^χ	$\eta_i^{\theta_\ell}$	$\eta_i^{\theta_V}$	normalization N_i
J_{1s}	$\{+\}$	$\{+,a,a,+\}$	$\{-,c,c,-\}$	$2\pi(1)2$
J_{1c}	$\{+\}$	$\{+,a,a,+\}$	$\{+,d,d,+\}$	$2\pi(1)(2/5)$
J_{2s}	$\{+\}$	$\{-,b,b,-\}$	$\{-,c,c,-\}$	$2\pi(-2/3)2$
J_{2c}	$\{+\}$	$\{-,b,b,-\}$	$\{+,d,d,+\}$	$2\pi(-2/3)(2/5)$
J_3	$\{+,-,-,+,+,-,-,+\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$
J_4	$\{+,+,-,-,-,-,+,+\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_5	$\{+,+,-,-,-,-,+,+\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_{6s}	$\{+\}$	$\{+,+,-,-\}$	$\{-,c,c,-\}$	$2\pi(1)2$
J_{6c}	$\{+\}$	$\{+,+,-,-\}$	$\{+,d,d,+\}$	$2\pi(1)(2/5)$
J_7	$\{+,+,+,+,-,-,-,-\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_8	$\{+,+,+,+,-,-,-,-\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_9	$\{+,+,-,-,+,+,-,-\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$

$$a = 1 - 1/\sqrt{2}, b = a\sqrt{2}, c = 2\sqrt{2} - 1, d = 1 - 4\sqrt{2}/5$$