

# Malaphoric $Z'$ model

by

Ben Allanach, University of Cambridge

$b \rightarrow s\ell^+\ell^-$

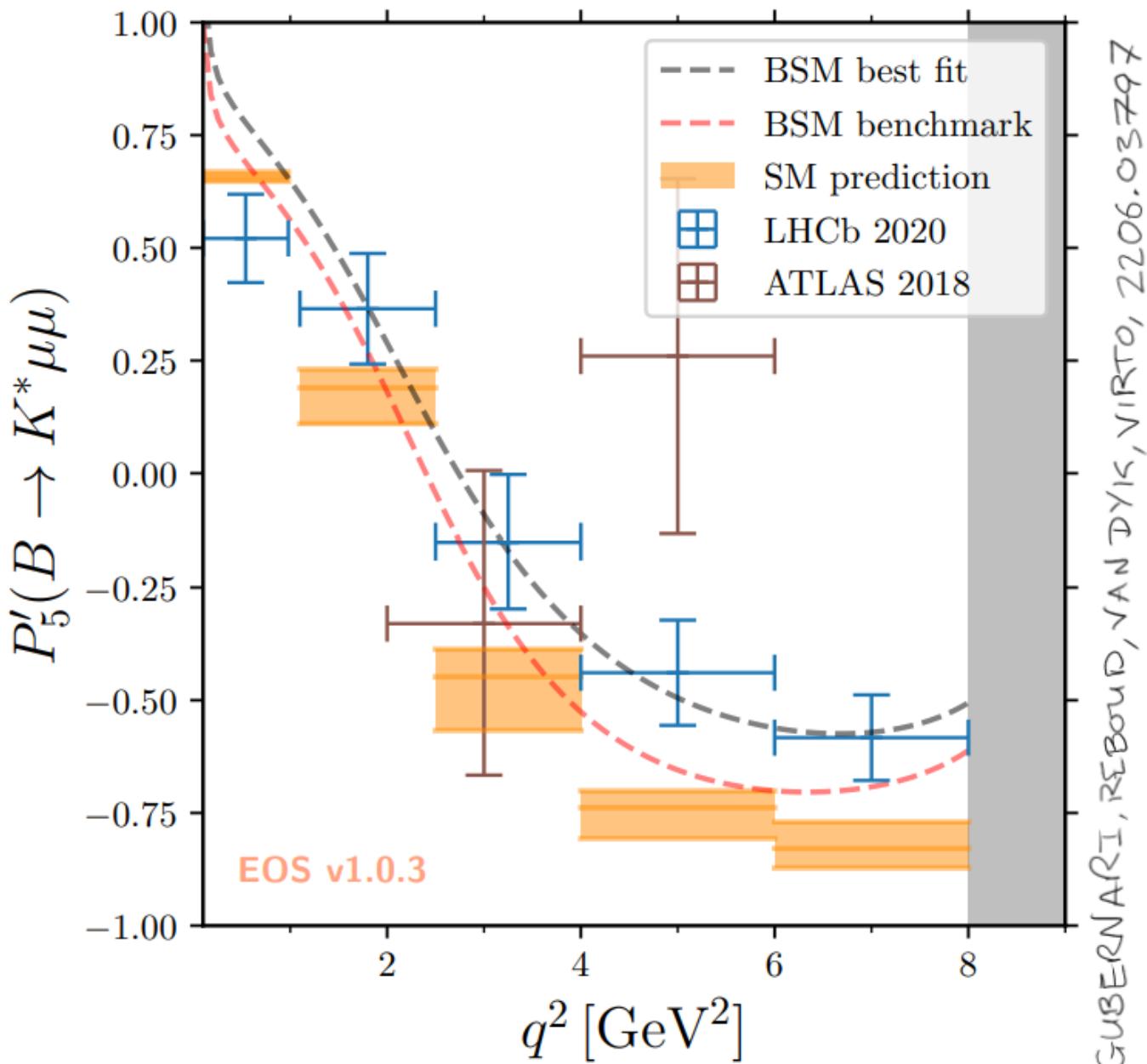
$B_3 - L_2$  model

Kinetic Mixing

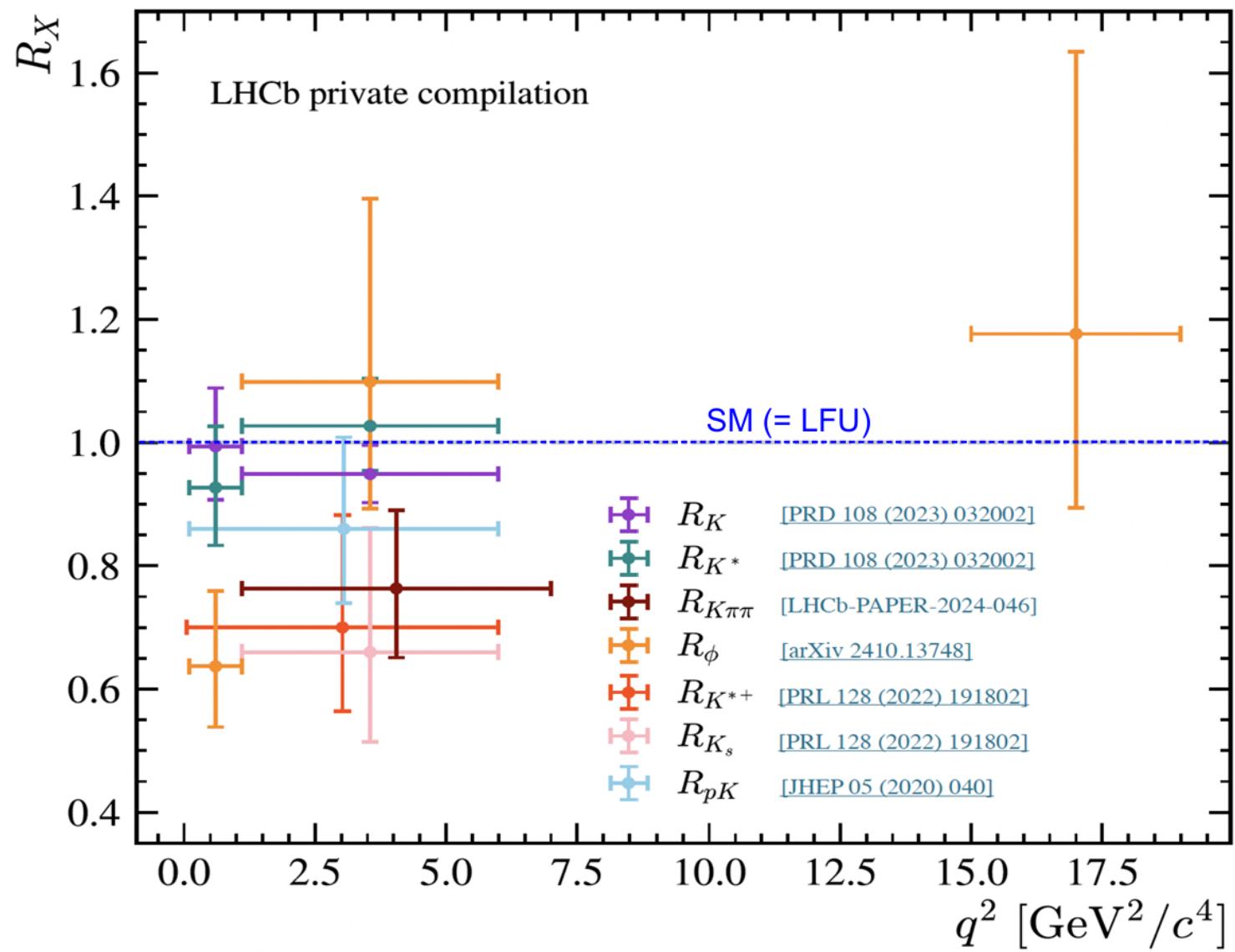
With: Davighi, Gubernari, Mullin



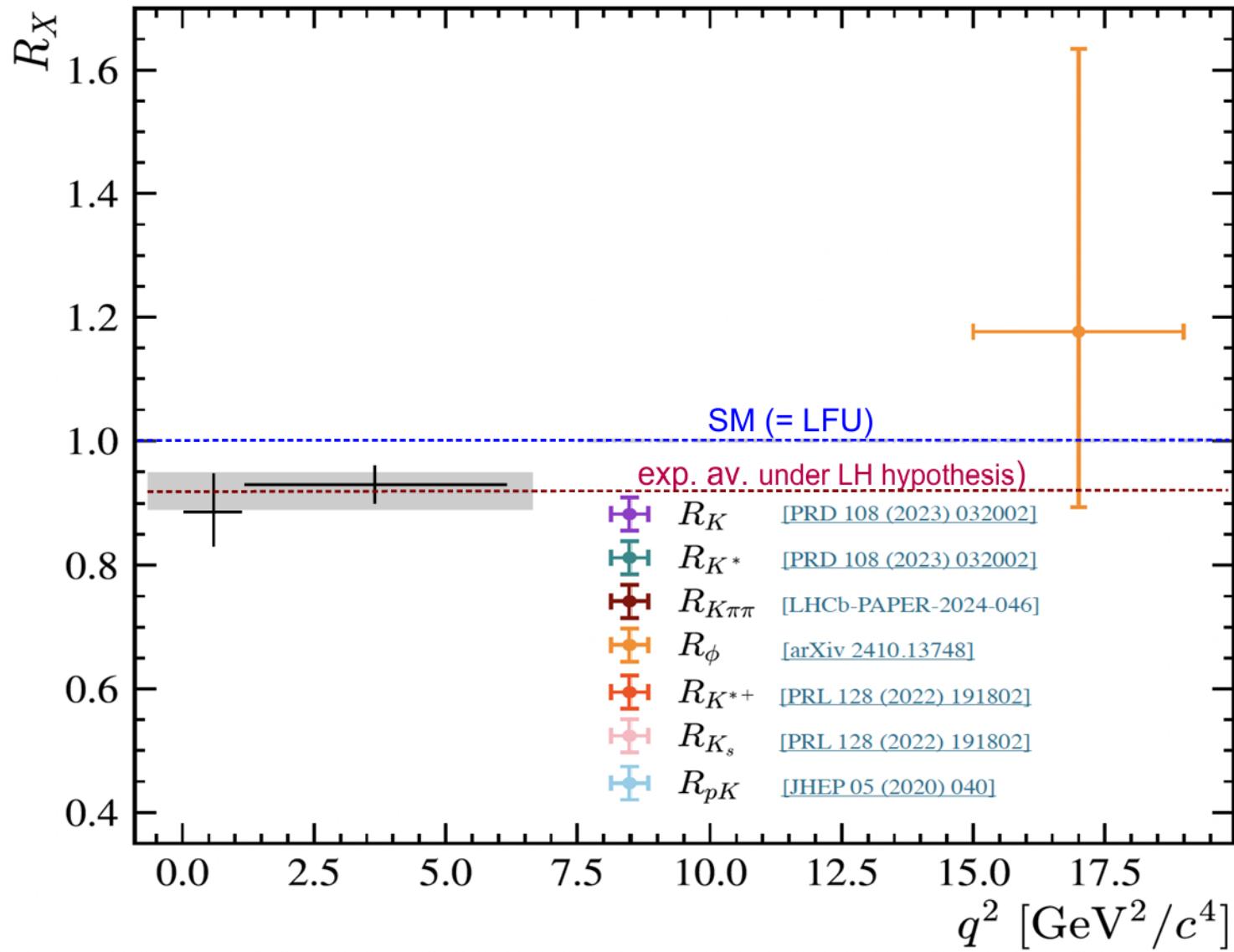
$$P'_5 = S_5 / \sqrt{F_L(1 - F_L)}$$



$$R_X = BR(B \rightarrow X\mu^+\mu^-)/BR(B \rightarrow Xe^+e^-)$$



# Thanks to G Isidori



# $B_3 - L_2$ model

SM-singlet scalar ‘flavon’  $\theta$

Additional  $U(1)_X$  gauge symmetry broken by  
 $\langle \theta \rangle \sim \text{TeV} \Rightarrow M_X = g_X X_\theta \langle \theta \rangle \sim \mathcal{O}(\text{TeV})$

SM+ $3\nu_R$  fermion content

**Zero** charges for first two generations of quark

$X = B_3 - L_2$  postdicts some small CKM<sup>1</sup>;  
 $X_\mu \leftrightarrow$  propagating  $Z'$

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<sup>1</sup>Bonilla et al, 1705.00915;  
2009.02197 (*simplified EFT*)

Alonso et al 1705.03858,

BCA

# Flavour

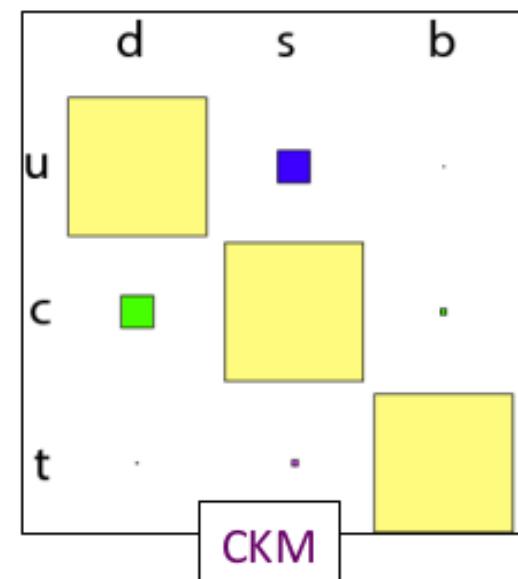
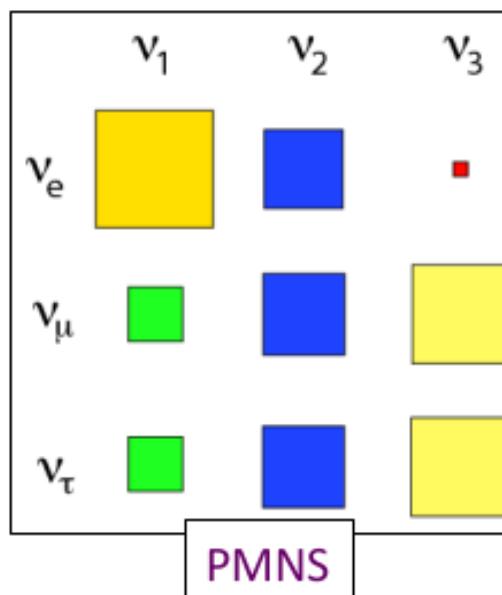
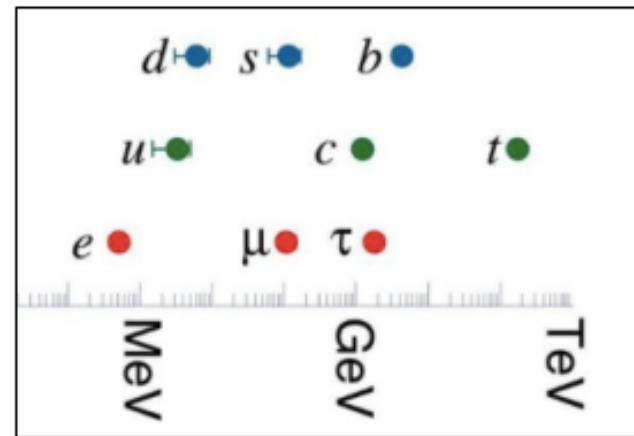
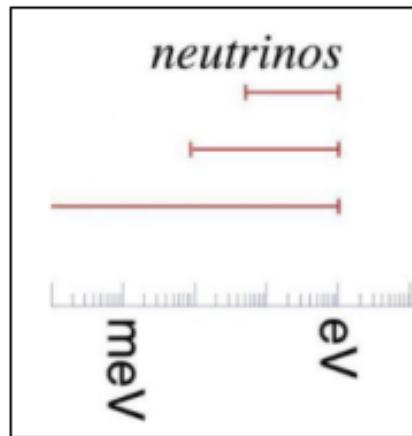


up

charm

top

# Family Structure



# Some Family Structure

$$Y_u \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad Y_d \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix},$$

Postdicts CKM angles  $|V_{cb}|$ ,  $|V_{ub}|$ ,  $|V_{ts}|$ ,  
 $|V_{td}|$  to be small

# A simple limiting case

$$V_{u_R} = V_{d_R} = V_{e_L} = V_{e_R} = 1$$

$$V_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{sb} & -\sin \theta_{sb} \\ 0 & \sin \theta_{sb} & \cos \theta_{sb} \end{pmatrix}.$$

$$\Rightarrow V_{u_L} = V_{d_L} V_{CKM}^\dagger \text{ and } V_{\nu_L} = V_{e_L} U_{PMNS}^\dagger.$$

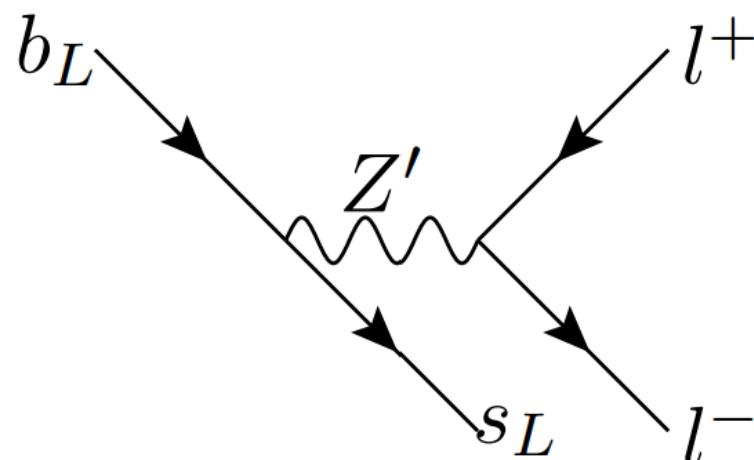
$$\begin{aligned}
\mathcal{L}_{X\psi} &= g_X \left( \overline{\mathbf{u}_L} \Lambda_\xi^{(u_L)} X \mathbf{u}_L + \overline{\mathbf{u}_R} \Lambda_\xi^{(u_R)} X \mathbf{u}_R \right. \\
&\quad + \overline{\mathbf{d}_L} \Lambda_\xi^{(d_L)} X \mathbf{d}_L + \overline{\mathbf{d}_R} \Lambda_\xi^{(d_R)} X \mathbf{d}_R \\
&\quad - \overline{\mathbf{e}_L} \Lambda_\Xi^{(e_L)} X \mathbf{e}_L - \overline{\mathbf{e}_R} \Lambda_\Xi^{(e_R)} X \mathbf{e}_R \\
&\quad \left. - \overline{\boldsymbol{\nu}_L} \Lambda_\Xi^{(\nu_L)} X \boldsymbol{\nu}_L - \overline{\boldsymbol{\nu}_R} \Lambda_\Xi^{(\nu_R)} X \boldsymbol{\nu}_R \right), \\
\Lambda_\xi^{(I)}_{[\Xi]} &\equiv V_I^\dagger \xi V_I, \quad \xi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \Xi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\end{aligned}$$

**X couplings**,  $I \in \{u_L, d_L, e_L, \nu_L, u_R, d_R, e_R\}$

# Important $X$ Couplings

$$g_{Z'} \left[ (\overline{d}_L \ \overline{s}_L \ \overline{b}_L) \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sin^2 \theta_{sb} & \frac{1}{2} \sin 2\theta_{sb} \\ 0 & \frac{1}{2} \sin 2\theta_{sb} & \cos^2 \theta_{sb} \end{pmatrix} X \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \right.$$

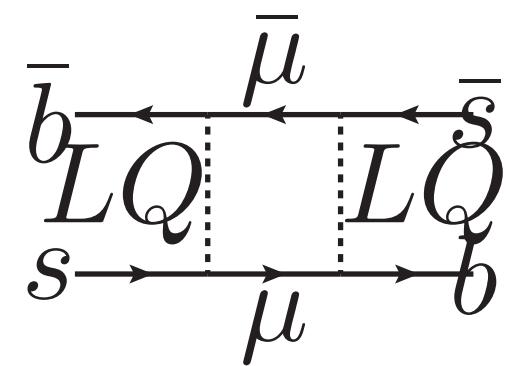
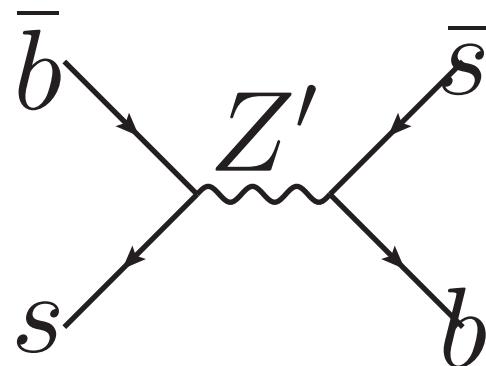
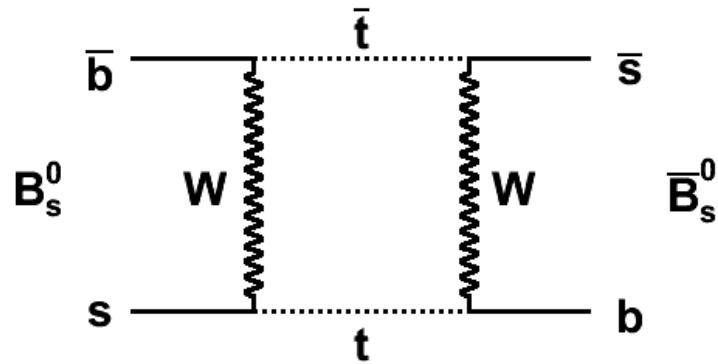
$$\left. - (\overline{e} \ \overline{\mu} \ \overline{\tau}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix} X \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \right]$$



- LFU Violating?  $C_9 \neq 0$

# $B_s - \bar{B}_s$ Mixing

Measurement agrees with SM.



$$g_{sb} = \frac{g_X}{2} \sin 2\theta_{sb} \lesssim \frac{M_{Z'}}{194 \text{ TeV}} \text{ but uncertain}$$

from QCD sum rules and lattice<sup>2</sup>.

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<sup>2</sup>King, Lenz, Rauh, arXiv:1904.00940

# smelli

Aebischer, Kumar, Stangl, Straub, 1810.07698:

Input: SMEFT coefficients  $C_i/\Lambda^2$ .

Output:  $\chi^2$

Hundreds of  $B$ —observables  
31 EWPOs

# Kinetic Mixing

BCA, Gubernari 2409.06804

This will induce a **family independent** component to the  $Z'$  couplings

$$J^\mu = g_X \sum_{\psi'} X_{\psi'} \overline{\psi'} \gamma^\mu \psi',$$

$$j_\mu = ig' Y_H [H^\dagger D_\mu H - (D_\mu H)^\dagger H] + g' \sum_{\psi'} Y_\psi \overline{\psi'} \gamma_\mu \psi'.$$

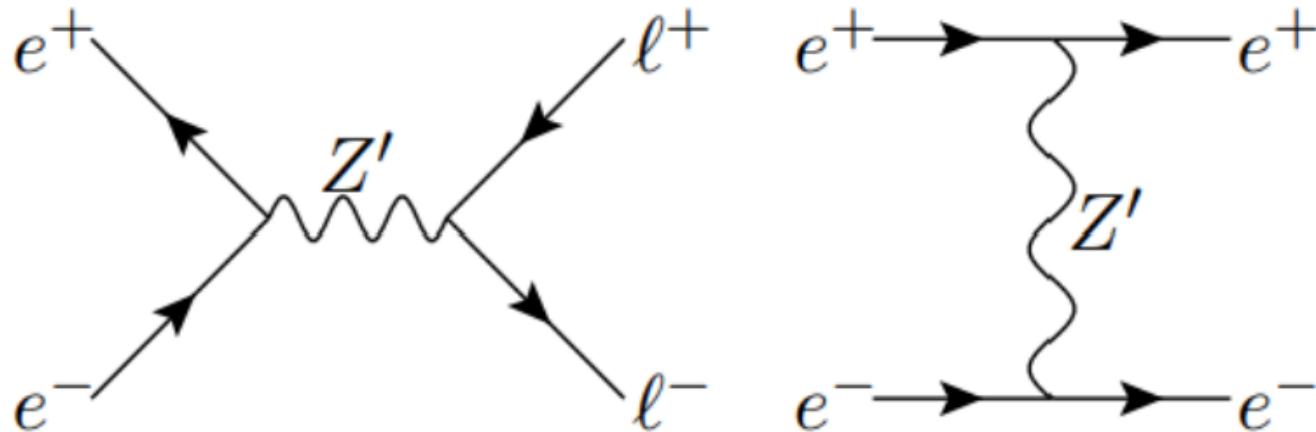
$$\mathcal{L}_{XB} = -\frac{\epsilon}{2} B_{\mu\nu} X^{\mu\nu} - X_\mu J^\mu - B_\mu j^\mu.$$

# Integrate out heavy $X_\mu$

$$\begin{aligned}\mathcal{L}_6 &= -\frac{1}{2M_X^2} J_\mu J^\mu - \frac{\epsilon}{M_X^2} (\partial_\nu B^{\mu\nu}) J_\mu \\ &\quad - \frac{\epsilon^2}{2M_X^2} (\partial_\nu B^{\mu\nu}) (\partial^\rho B_{\mu\rho}) \\ &= -\frac{1}{2M_X^2} (J_\mu - \epsilon j_\mu) (J^\mu - \epsilon j^\mu)\end{aligned}$$

# LEP constraints

BCA, Mullin, 2306.08669



SMEFT contributions:  $C_{ee}^{11ii}$ ,  $C_{ll}^{11ii}$ ,  $C_{le}^{1ii1}$

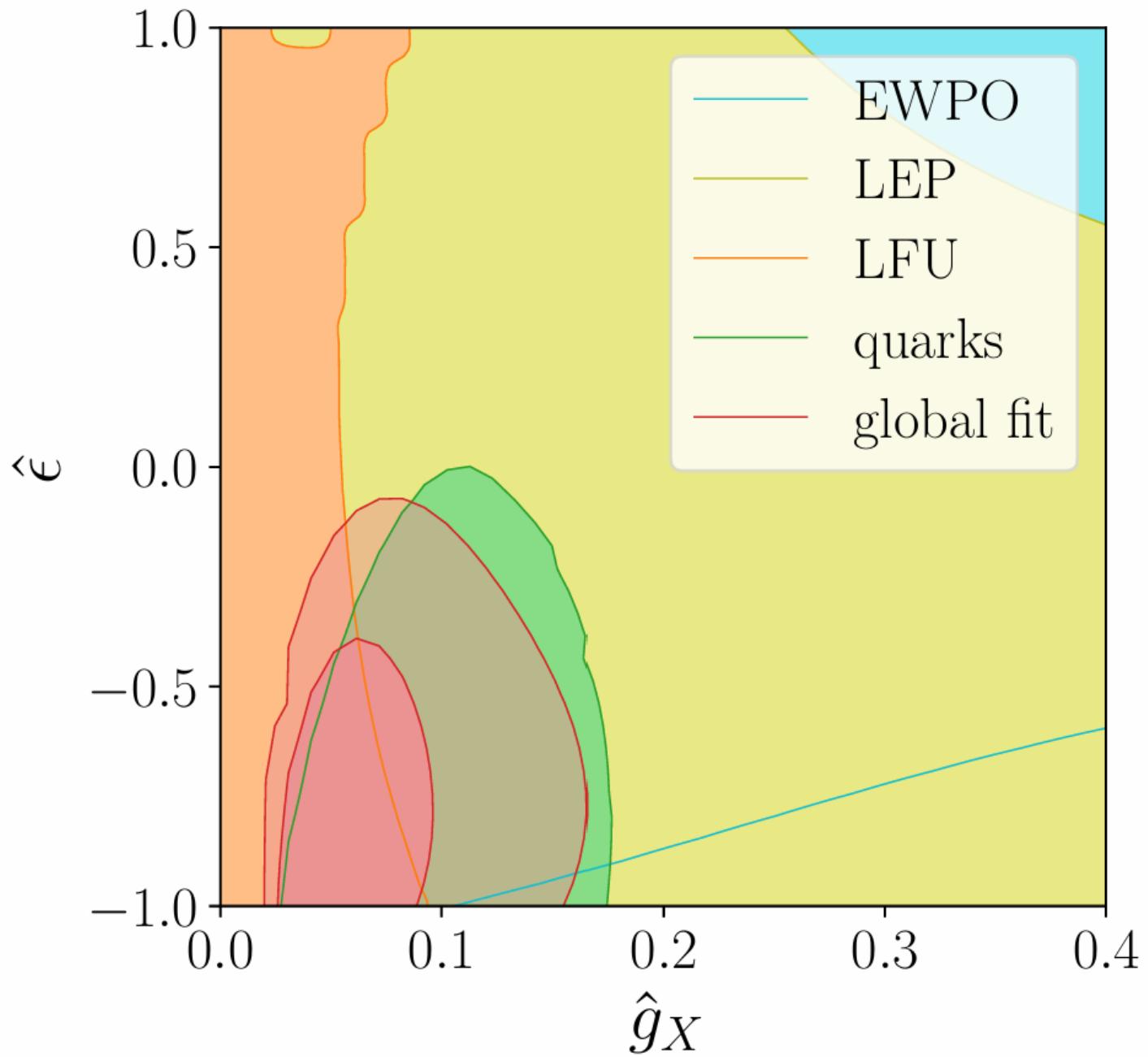
Code into `flavio` (cf Falkowski, Mimouni 1511.07434): 148 LEP2 bins,  
 $\sigma$ ,  $A_{FB}$

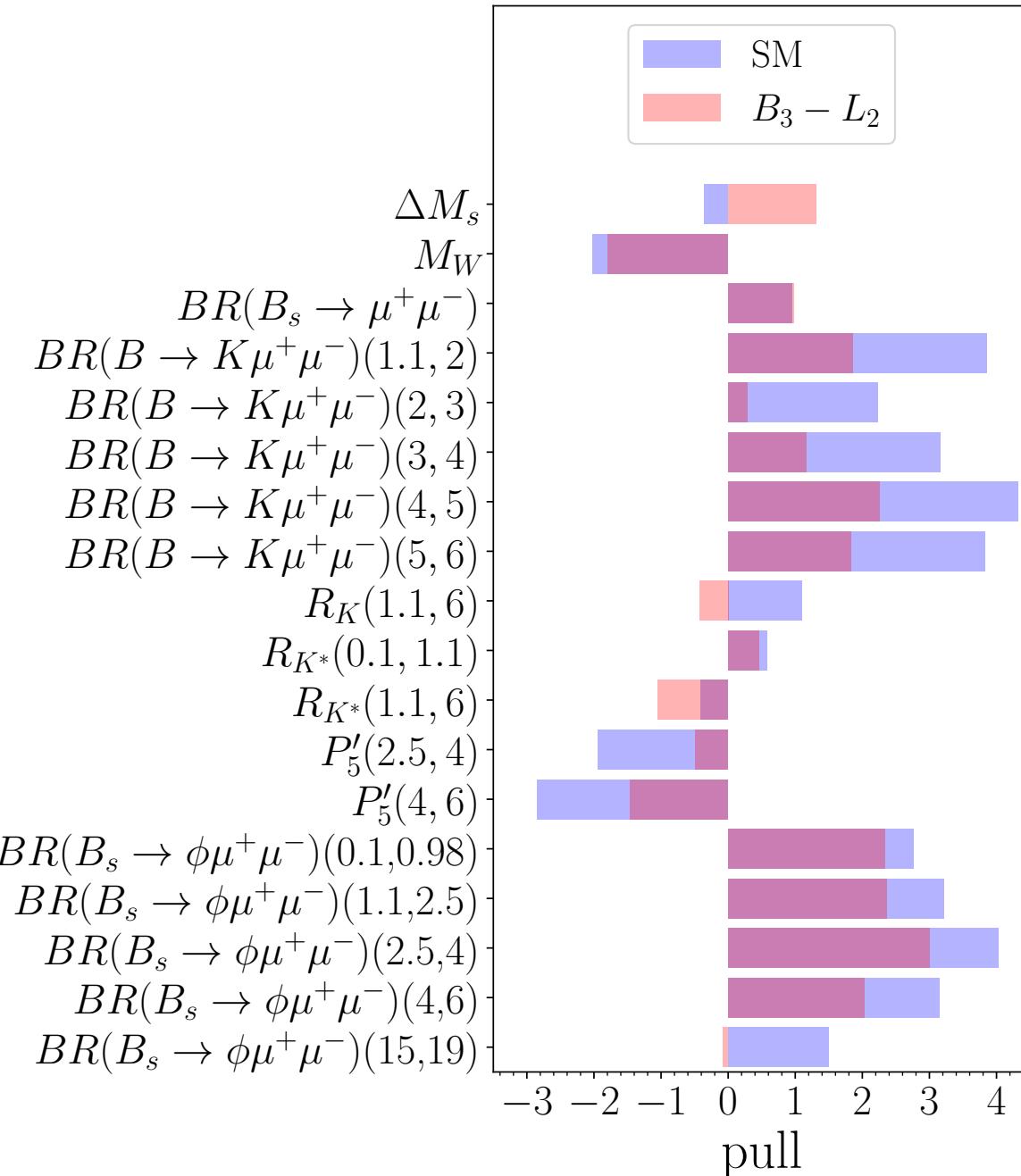
# Global Fit

Observables: 306 quarks, 31 EWPOs, 148 LEP2, 24 LFU

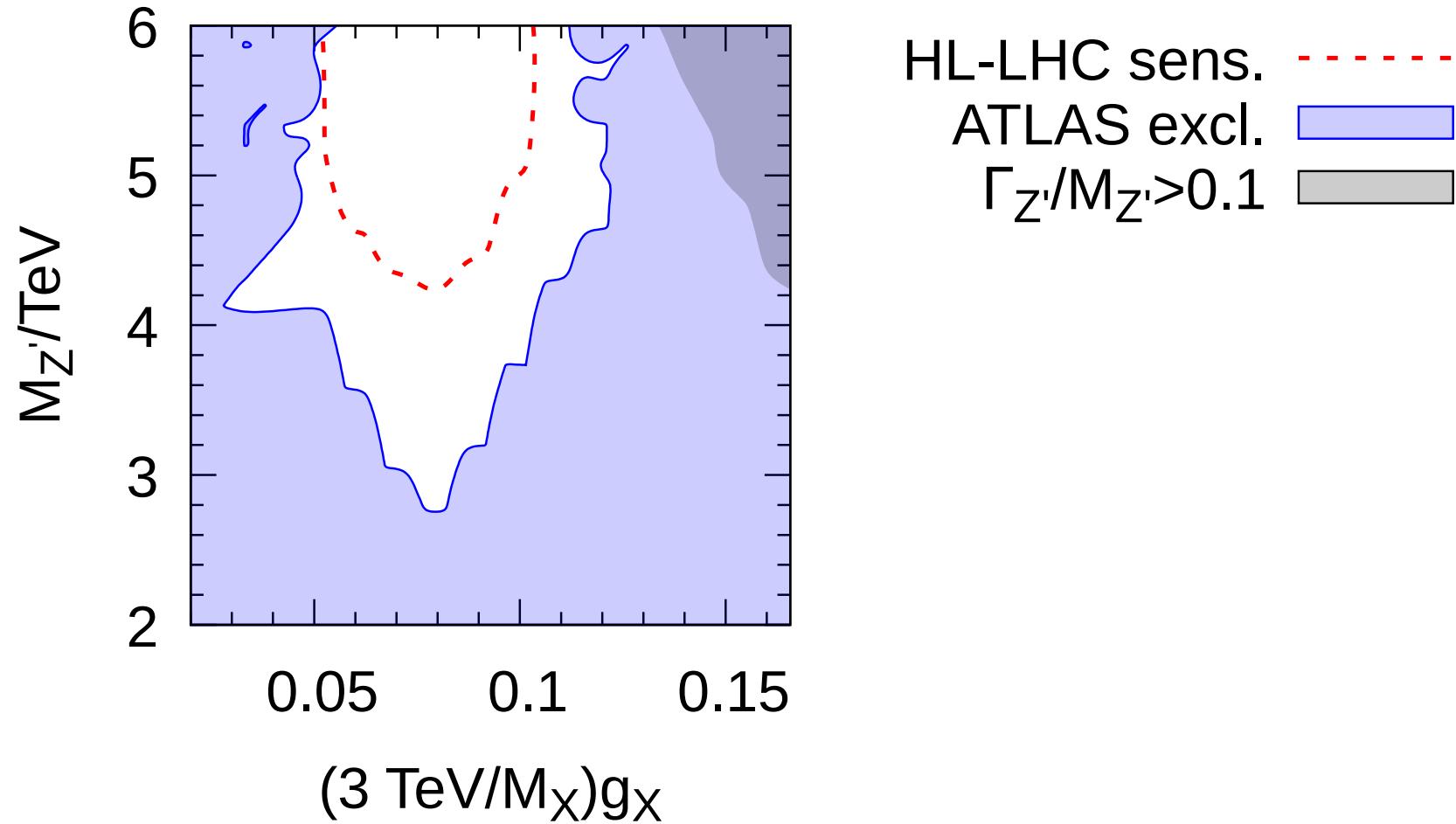
$$\hat{\epsilon} := \epsilon \frac{3 \text{ TeV}}{M_X}, \quad \hat{g}_X := g_X \frac{3 \text{ TeV}}{M_X}.$$

| $\hat{\epsilon}$ | $\hat{g}_X$ | $\theta_{sb}$ | $\Delta\chi^2_{\text{quarks}}$ | $\Delta\chi^2_{\text{EWPO}}$ | $\Delta\chi^2_{\text{LEP2}}$ | $\Delta\chi^2_{\text{LFU}}$ | $\Delta\chi^2_{\text{global}}$ |
|------------------|-------------|---------------|--------------------------------|------------------------------|------------------------------|-----------------------------|--------------------------------|
| 0                | 0.082       | -0.11         | 36.2                           | 0.0                          | 0.00                         | -3.8                        | <b>32.8</b>                    |
| -0.86            | 0.048       | -0.19         | 40.1                           | -0.4                         | -0.02                        | 0.8                         | <b>40.1</b>                    |





# BCA, 2412.01956



# Epilogue

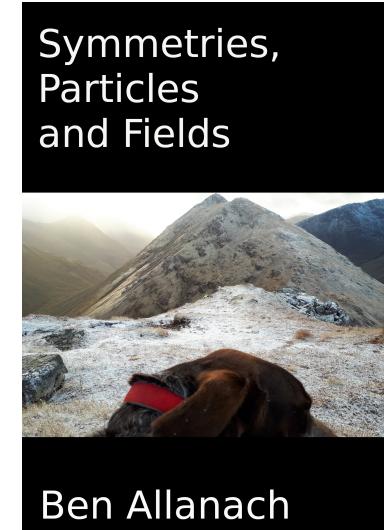
Calculated SMEFT coefficients for kinetically-mixed  $Z'$  models cf Dawson, Forslund, Schnubel 2404.01375

Improves fit of  $B_3 - L_2$  model.

Links to my [music](#), [book \(18€\)](#) and [Quantum Selves art](#):



CHOOSE A MUSIC SERVICE



Ben Allanach



# UV Completion

The  $B_3 - L_2$  model with kinetic mixing is approximately physically equivalent to an unmixed model with

$$X := B_3 - L_2 + \alpha Y$$

where  $\alpha \in \mathbb{Q}$  is chosen appropriately.

# $l^+l^-$ ATLAS 13 TeV 139 fb $^{-1}$

e.g. 2 track-based isolated  $\mu$ ,  $p_T > 30$  GeV with reconstructed vertex.<sup>3</sup> Only keep pair with highest ( $|p_{T_1}| + |p_{T_2}|$ ).

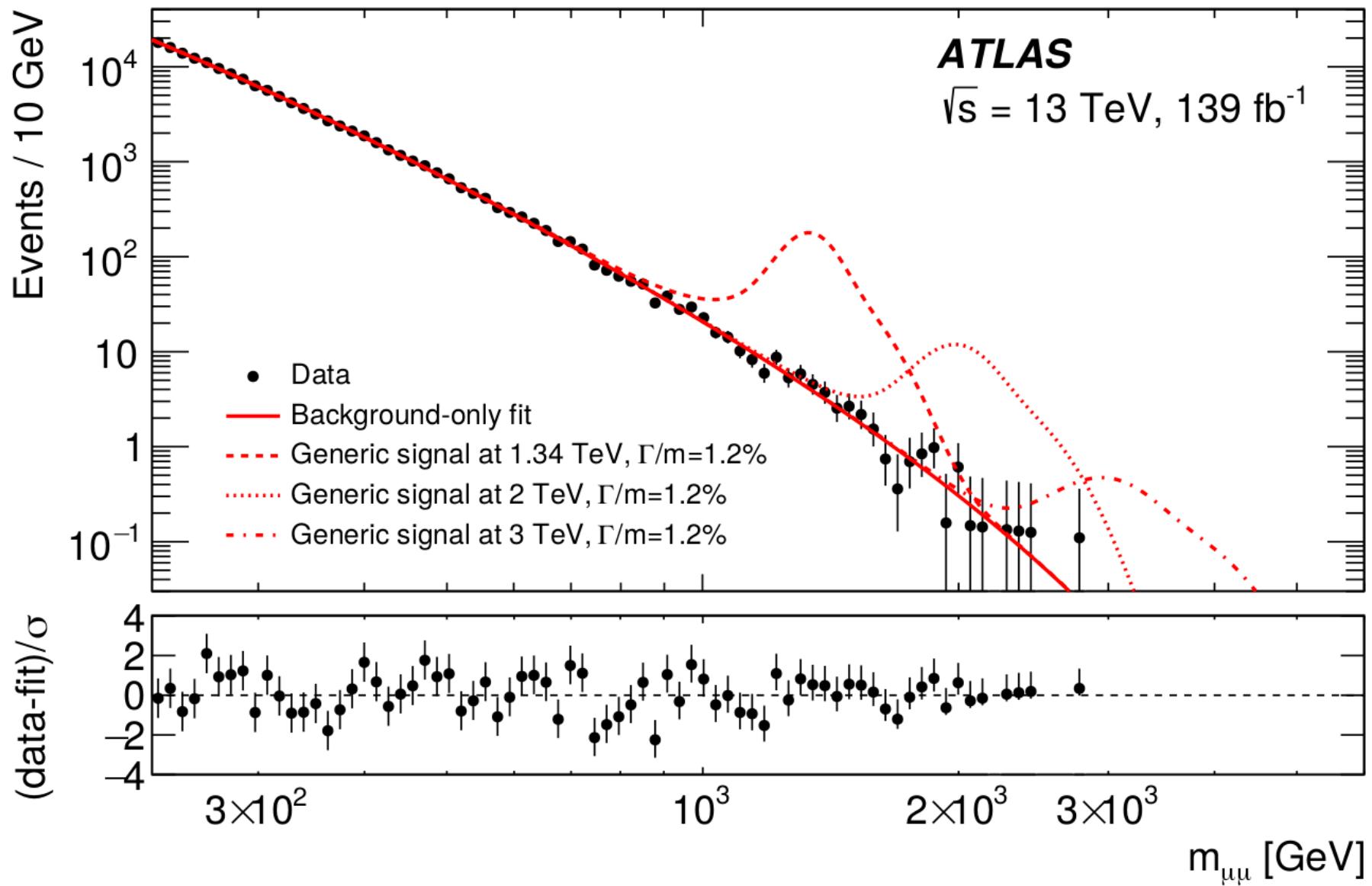
$$m_{\mu_1\mu_2}^2 = (p_1^\mu + p_2^\mu) (p_{1\mu} + p_{2\mu})$$

CMS also has a similar analysis<sup>4</sup>

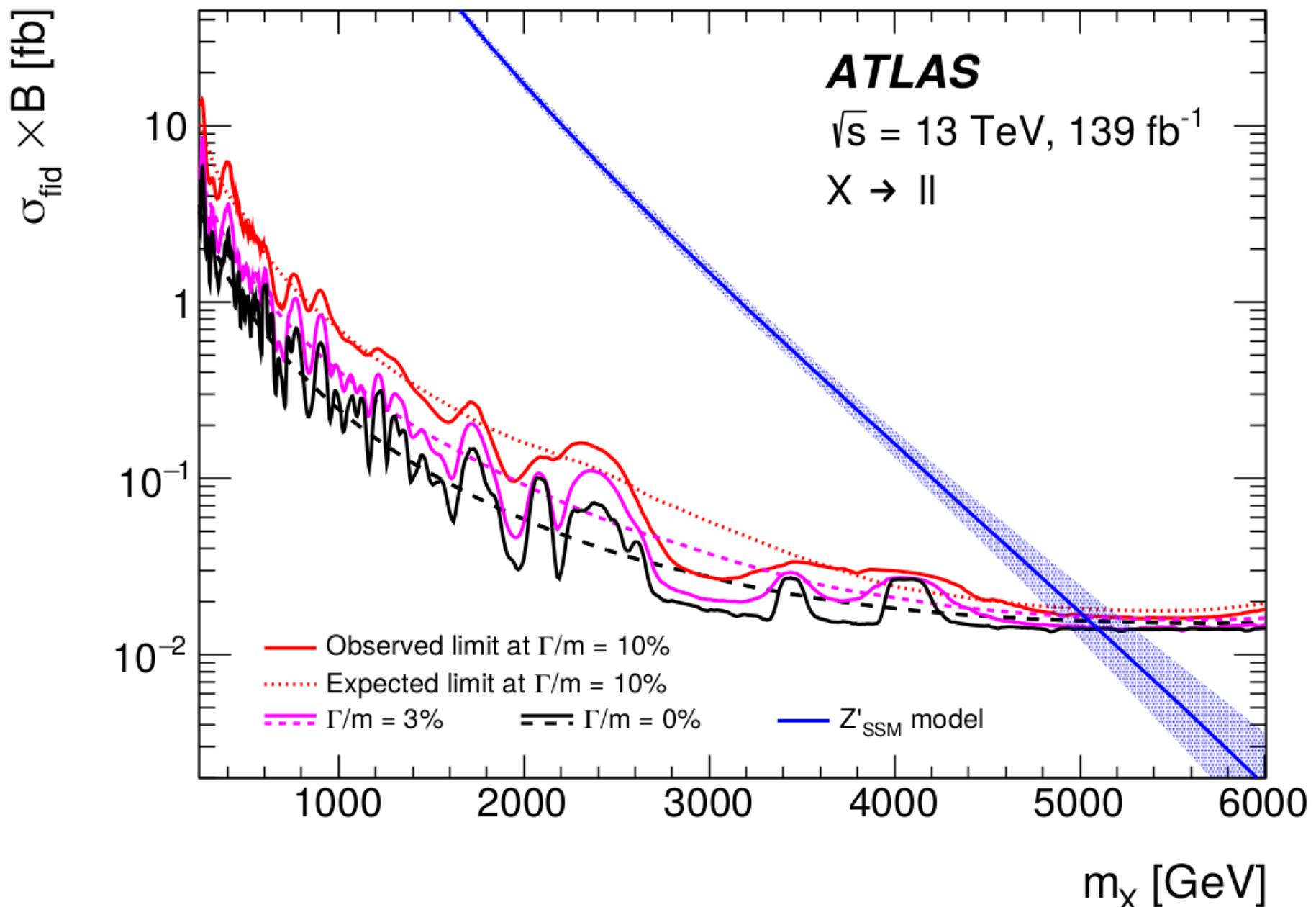
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<sup>3</sup>ATLAS, 1903.06248

<sup>4</sup>CMS, 2103.02708

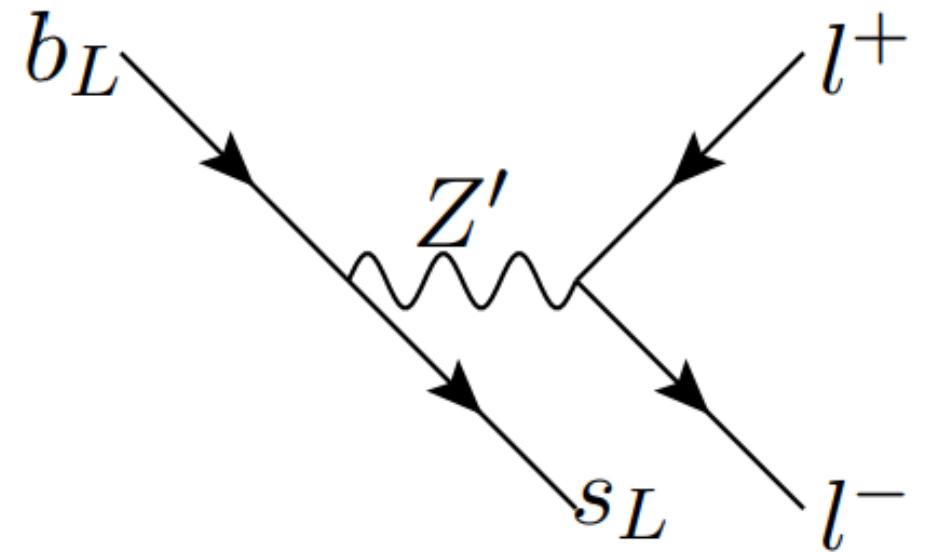
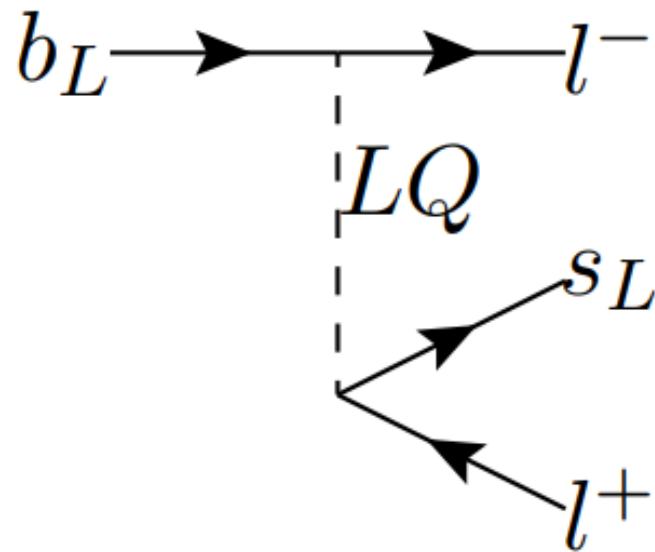


# ATLAS $l^+l^-$ limits





# Tree-level Explanations



Interferes with Standard Model process

smelli

Aebischer, Kumar, Stangl, Straub  
1810.07698:

Input: SMEFT coefficients  $C_i/\Lambda^2$ .

Output:  $\chi^2$

Hundreds of  $B$ –observables

31 EWPOs

# SMEFT

Parameterises heavy new physics effects

$$\mathcal{L} = \mathcal{L}_{4D} + \sum_{d=5} \sum_i \frac{C_i}{\Lambda^{d-4}} \mathcal{O}.$$

*Assumptions:*

All BSM fields have mass scale  $\Lambda \gg$  scale of observables.

Higgs doublet linearly realises EWSB

# Important term

2499  $d = 6$  terms

$$\mathcal{L} = \dots + \frac{(C_{lq}^{(1)})^{2223}}{\Lambda^2} (\bar{L}_2 \gamma_\alpha L_2) (\bar{Q}_2 \gamma^\alpha Q_3)$$

mediates  $b \rightarrow s\mu^+\mu^-$  transitions.

Here, from integrating  $Z'$  out:

$$\frac{(C_{lq}^{(1)})^{2223}}{\Lambda^2} = \frac{-3 \sin 2\theta_{sb} g_X^2}{M_X}$$



# SMEFT WCs/ $(g_X^2/M_X^2)$

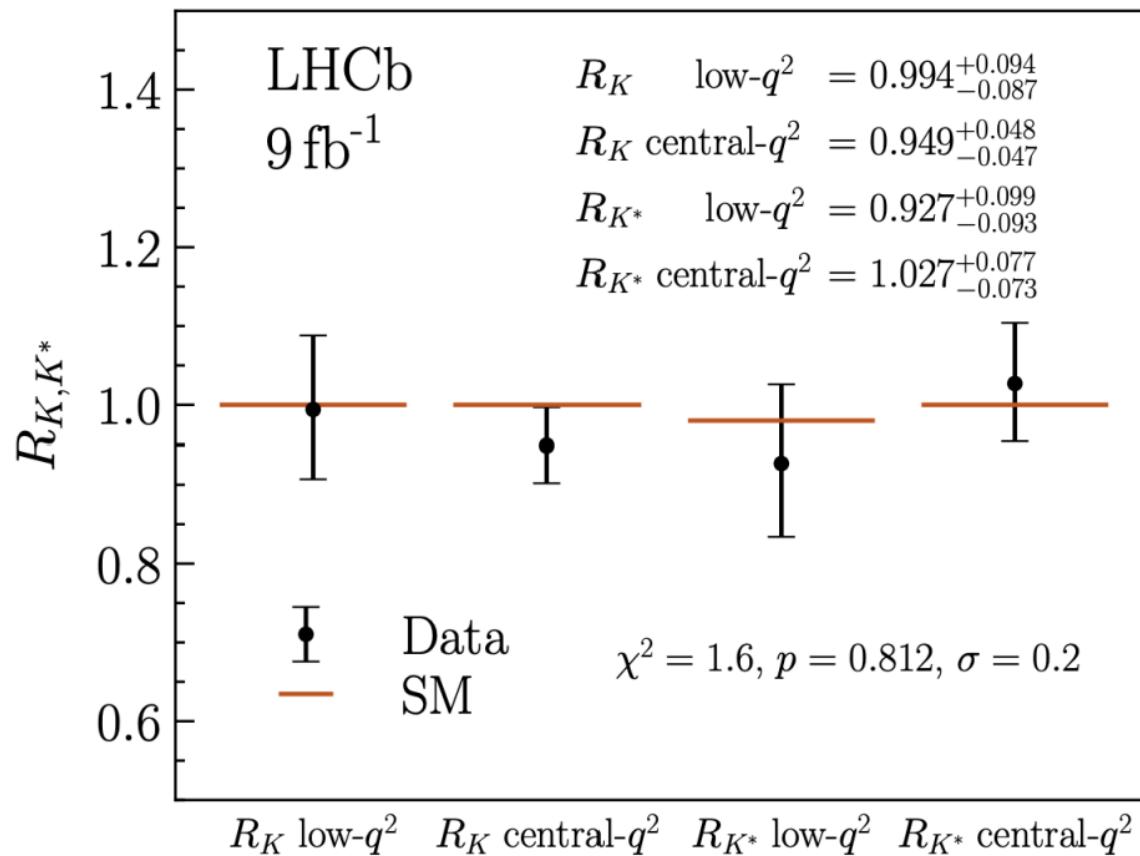
BCA, Davighi, 2211.11766

| WC                      | value  | WC                      | value                           |
|-------------------------|--|-------------------------|---------------------------------|
| $C_{ll}^{2222}$         | $-\frac{9}{2}$   | $(C_{lq}^{(1)})^{22ij}$ | $3\Lambda_{\Xi}^{(d_L)}{}_{ij}$ |
| $(C_{qq}^{(1)})^{ijkl}$ | $\Lambda_{\Xi}^{(d_L)}{}_{ij}\Lambda_{\Xi}^{(d_L)}{}_{kl}\frac{\delta_{ik}\delta_{jl}-2}{2}$ | $C_{ee}^{2222}$         | $-\frac{9}{2}$                  |
| $C_{uu}^{3333}$         | $-\frac{1}{2}$   | $C_{dd}^{3333}$         | $-\frac{1}{2}$                  |
| $C_{eu}^{2233}$         | 3  | $C_{ed}^{2233}$         | 3                               |
| $(C_{ud}^{(1)})^{3333}$ | -1   | $C_{le}^{2222}$         | -9                              |
| $C_{lu}^{2233}$         | 3  | $C_{ld}^{2233}$         | 3                               |
| $C_{qe}^{ij22}$         | $3\Lambda_{\Xi}^{(d_L)}{}_{ij}$  | $(C_{qu}^{(1)})^{ij33}$ | $-\Lambda_{\Xi}^{(d_L)}{}_{ij}$ |
| $(C_{qd}^{(1)})^{ij33}$ | $-\Lambda_{\Xi}^{(d_L)}{}_{ij}$  |                         |                                 |

| wilson | flavio | smelli >

# output

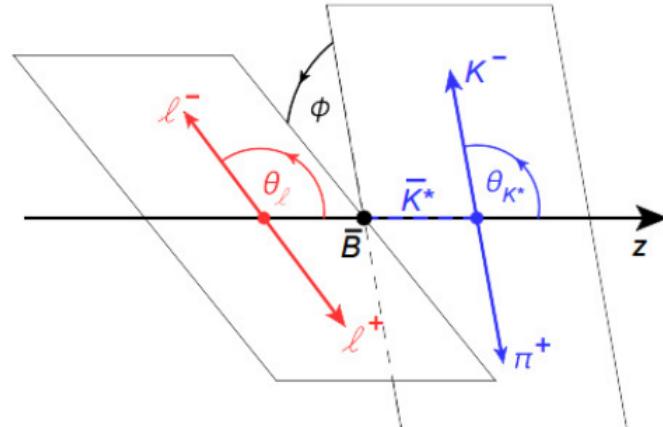
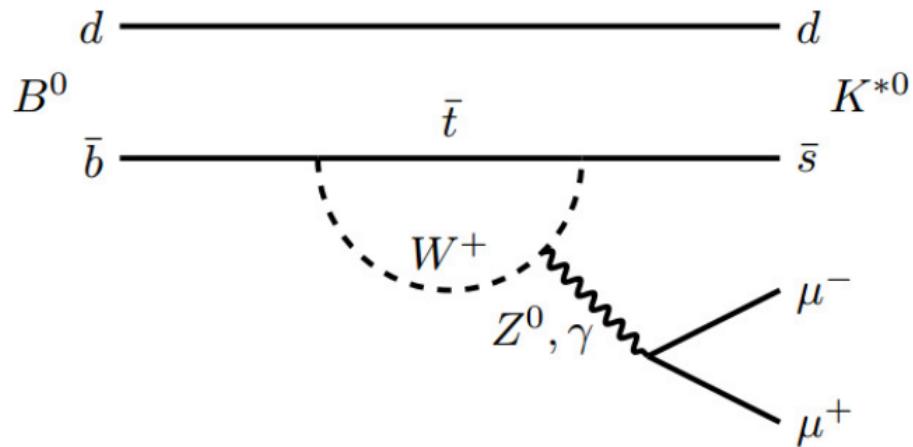
# LHCb 2212.09152



$$R_X(q^2) = \frac{BR(B \rightarrow X \mu^+ \mu^-)}{BR(B \rightarrow X e^+ e^-)}(q^2)$$



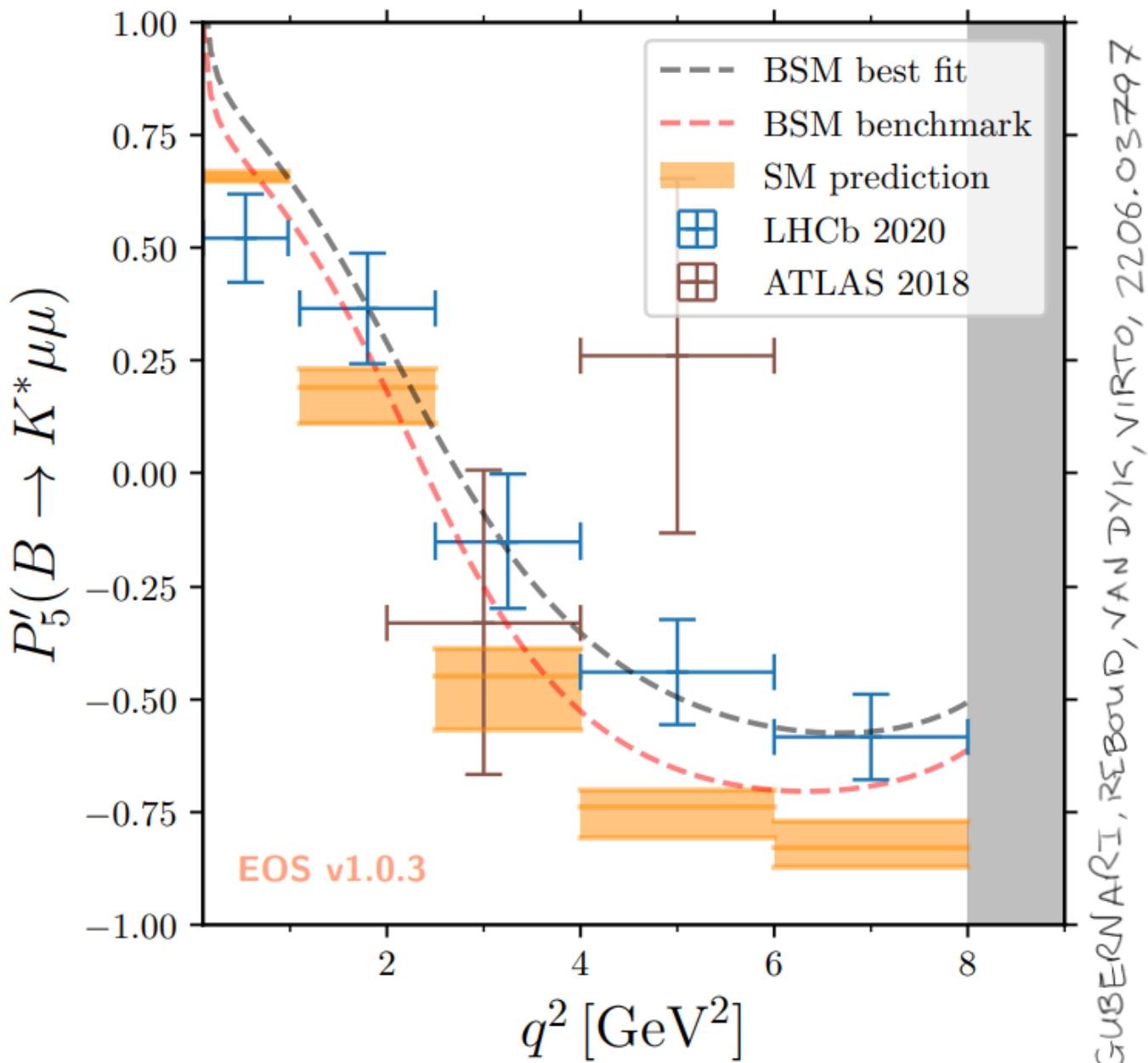
$$B^0 \rightarrow K^{*0} (\rightarrow K^+ \pi^-) \mu^+ \mu^-$$



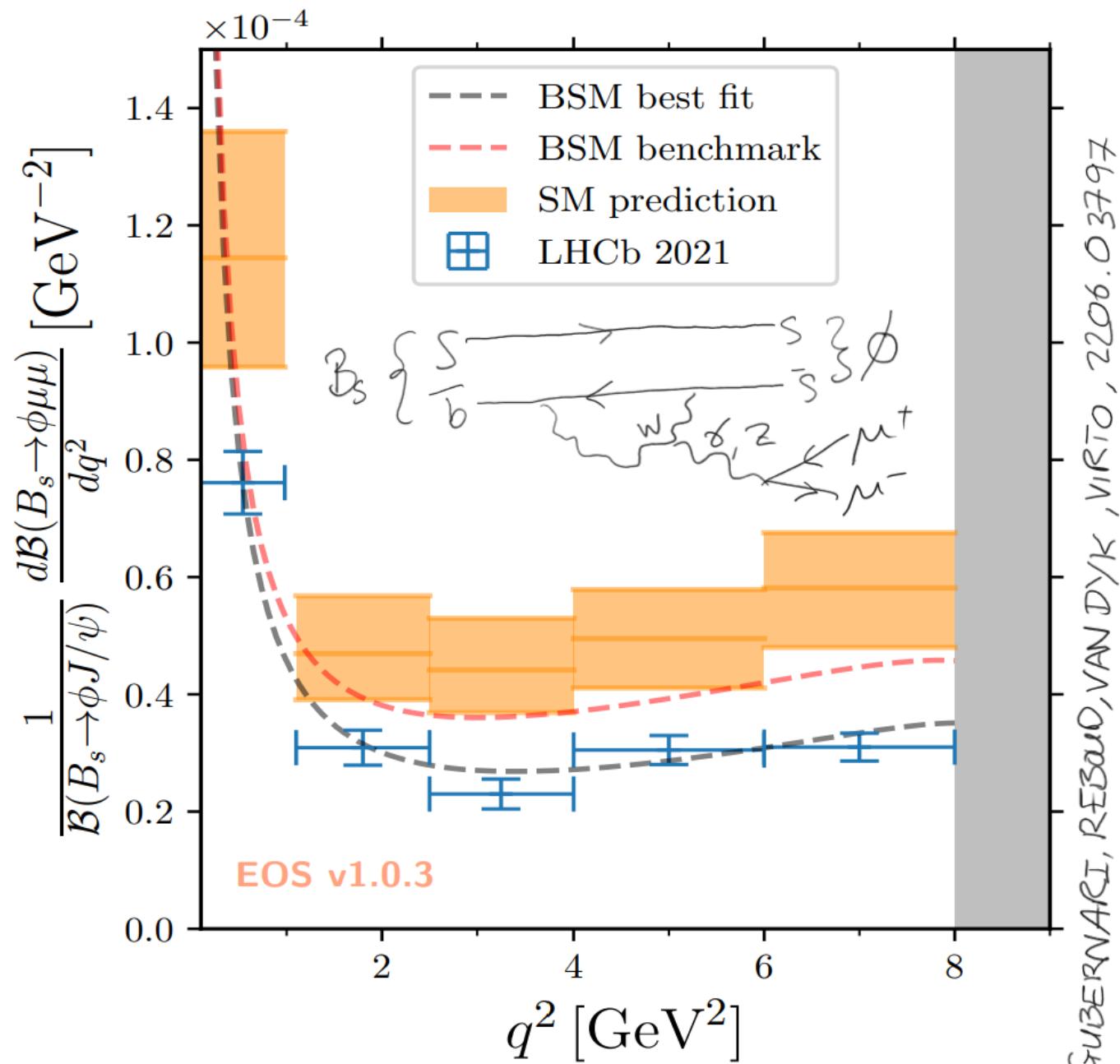
Decay fully described by three helicity angles  $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$  and  $q^2 = m_{\mu\mu}^2$

$$\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^3(\Gamma + \bar{\Gamma})}{d\vec{\Omega}} = \frac{9}{32\pi} \left[ \frac{3}{4}(1 - \textcolor{blue}{F}_L) \sin^2 \theta_K + \textcolor{blue}{F}_L \cos^2 \theta_K + \frac{1}{4}(1 - \textcolor{blue}{F}_L) \sin^2 \theta_K \cos 2\theta_\ell - \textcolor{blue}{F}_L \cos^2 \theta_K \cos 2\theta_\ell + \textcolor{blue}{S}_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + \textcolor{blue}{S}_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi + \textcolor{blue}{S}_5 \sin 2\theta_K \sin \theta_\ell \cos \phi + \frac{4}{3}\textcolor{blue}{A}_{\text{FB}} \sin^2 \theta_K \cos \theta_\ell + \textcolor{blue}{S}_7 \sin 2\theta_K \sin \theta_\ell \sin \phi + \textcolor{blue}{S}_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi + \textcolor{blue}{S}_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right]$$

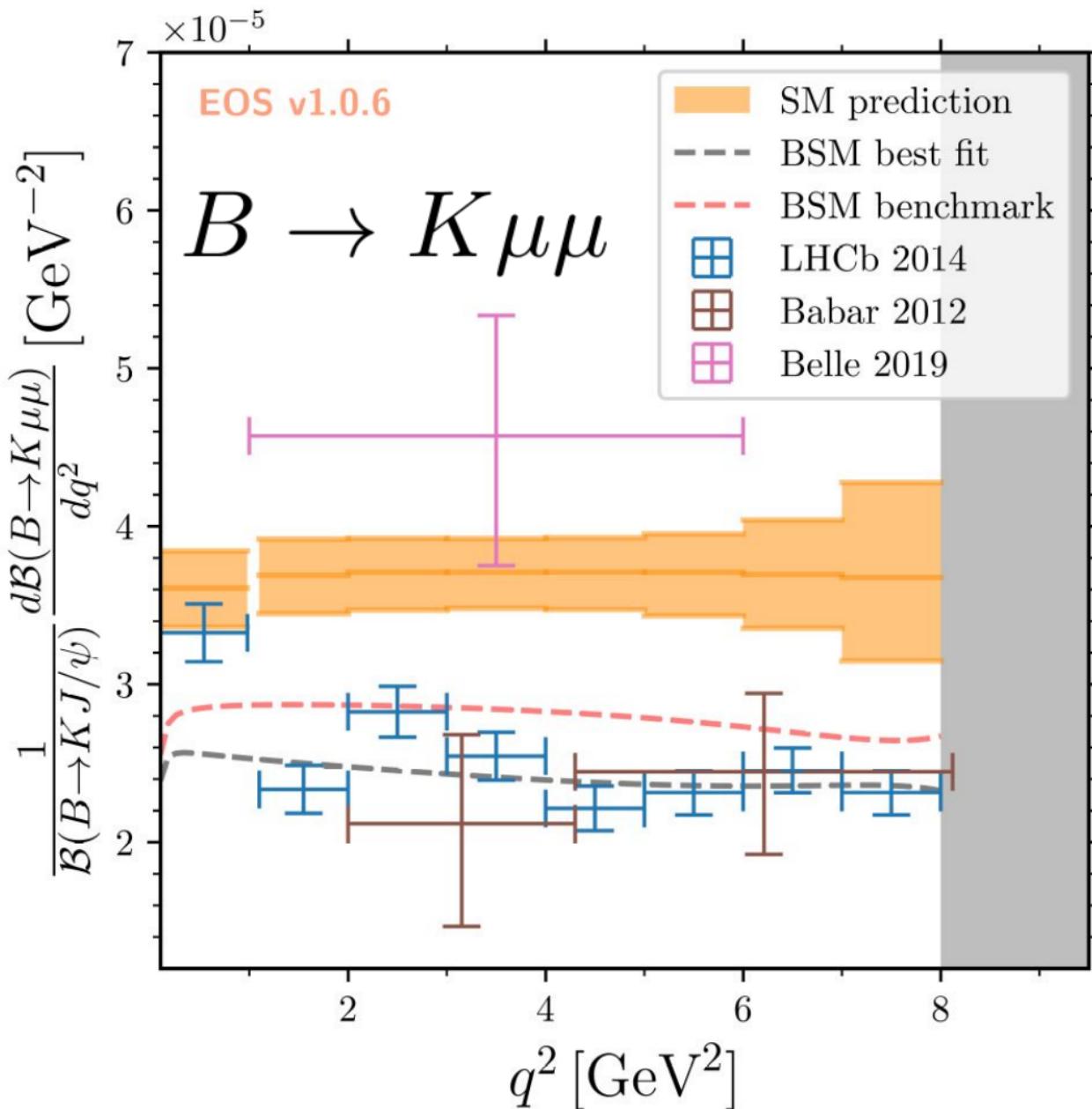
$$P'_5 = S_5 / \sqrt{F_L(1 - F_L)}$$



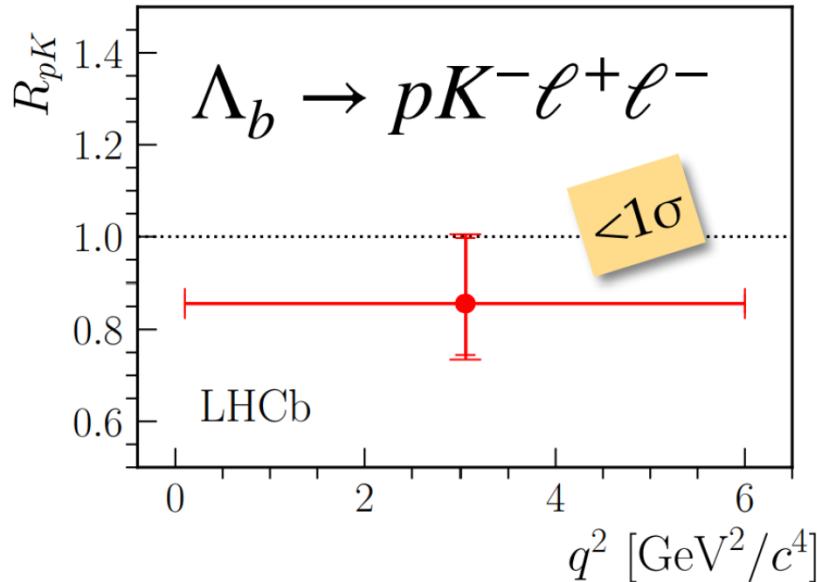
$$B_s \rightarrow \phi \mu^+ \mu^- : \phi = (s\bar{s})$$



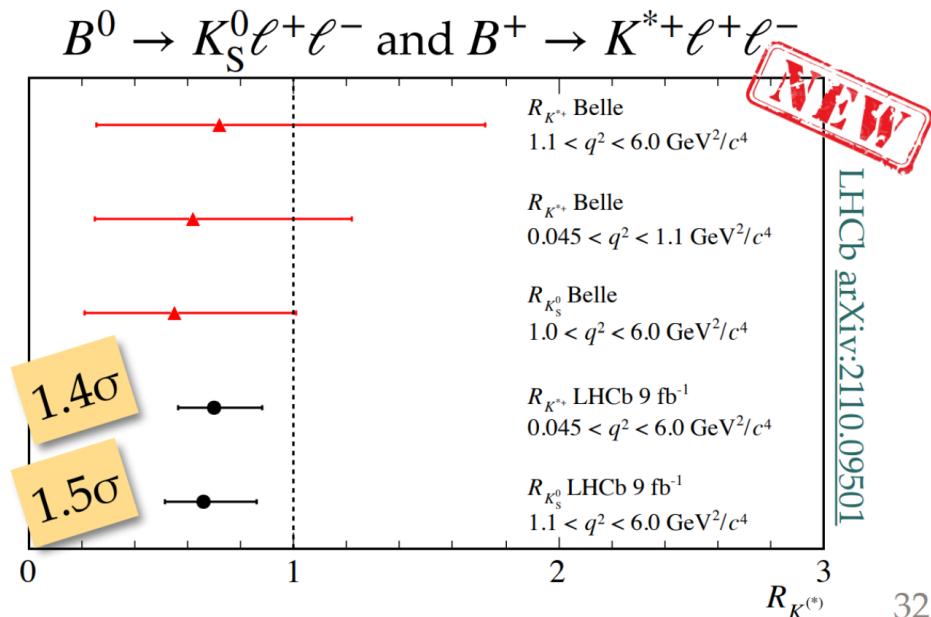
# $BR(B \rightarrow K\mu^+\mu^-)$



# Other LFU



LHCb, JHEP 05 (2020) 040



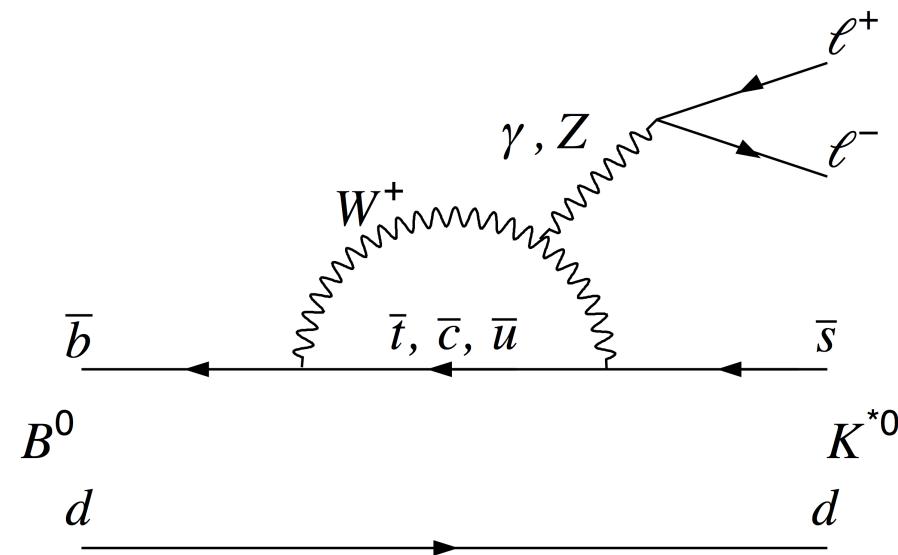
$$B_s \rightarrow \phi \ell^+ \ell^- ,$$

$$B \rightarrow \pi \ell^+ \ell^- ,$$

$$B \rightarrow K \pi^+ \pi^- \ell^+ \ell^- , \dots \text{to come}$$

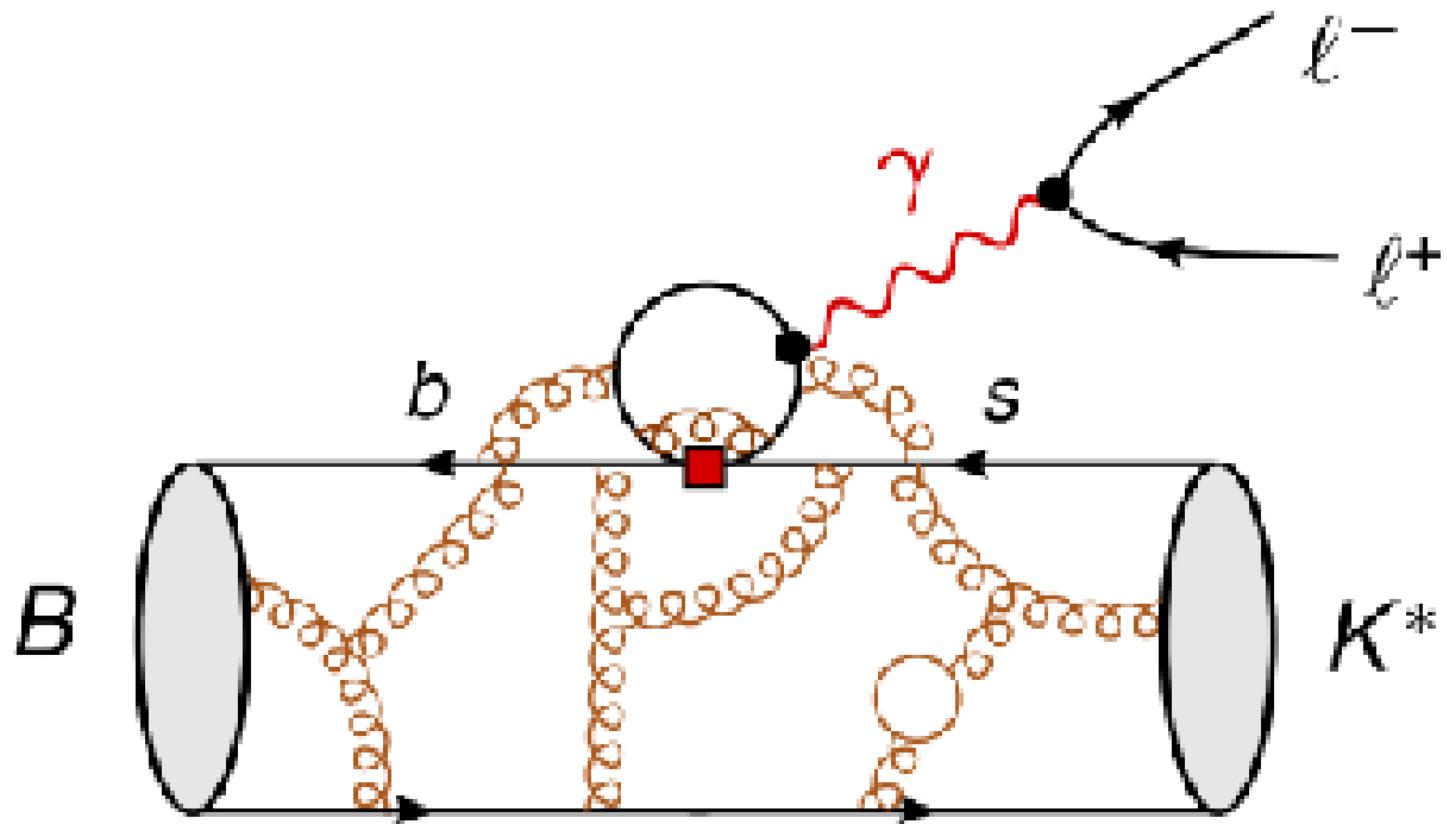
# $b \rightarrow sl^+l^-$ in Standard Model

BR  $\sim \mathcal{O}(10^{-6})$ : loop+EW+CKM



$$R_{K^*} = \frac{BR(B \rightarrow K^* \mu^+ \mu^-)}{BR(B \rightarrow K^* e^+ e^-)} = 1.00$$

# Form Factors



# Predicting $B \rightarrow M\ell^+\ell^-$ : FFs

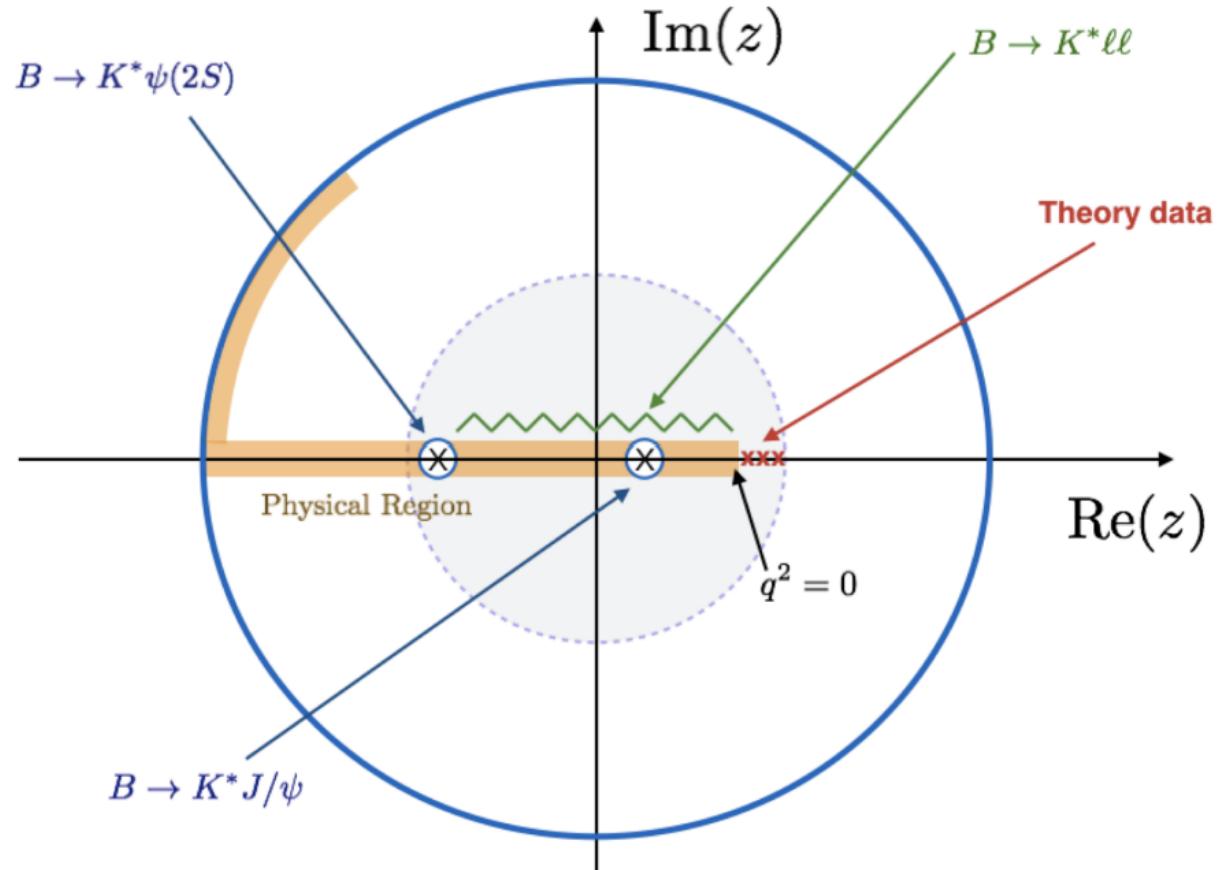
$$\mathcal{A} = \text{local} + \text{non-local}$$

local: interpolate lattice at high  $q^2 = m_{ll}^2$  and LCSR at low  $q^2$ .

non-local: no lattice. Most use QCD factorisation: perturbative charm loop+ad-hoc

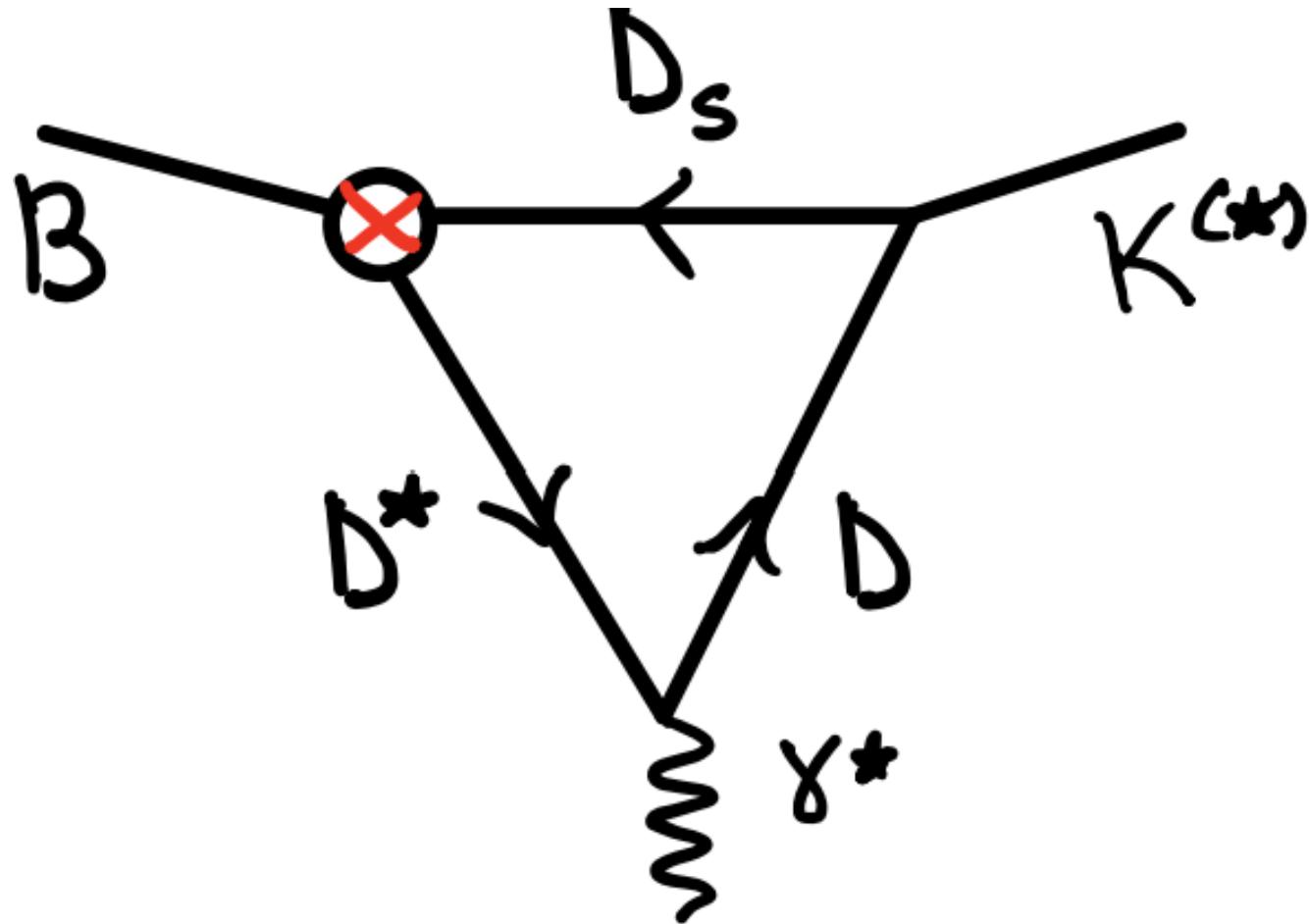
EOS approach: interpolate  $q^2 < 0$  LCOPE and measurements of BRs/angular dists at  $q^2 = M_{J/\psi}^2$ .

$$q^2 \rightarrow z(q^2), |z| < 1$$



$$C_9^{LD} \propto \sum_n a_n z^n \quad 1707.07305 \text{ truncation} \quad 2205.03797$$

# Caveat Emptor



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2212.10516

# Backup

# Ultra-violet completion?

This model is *equivalent* to

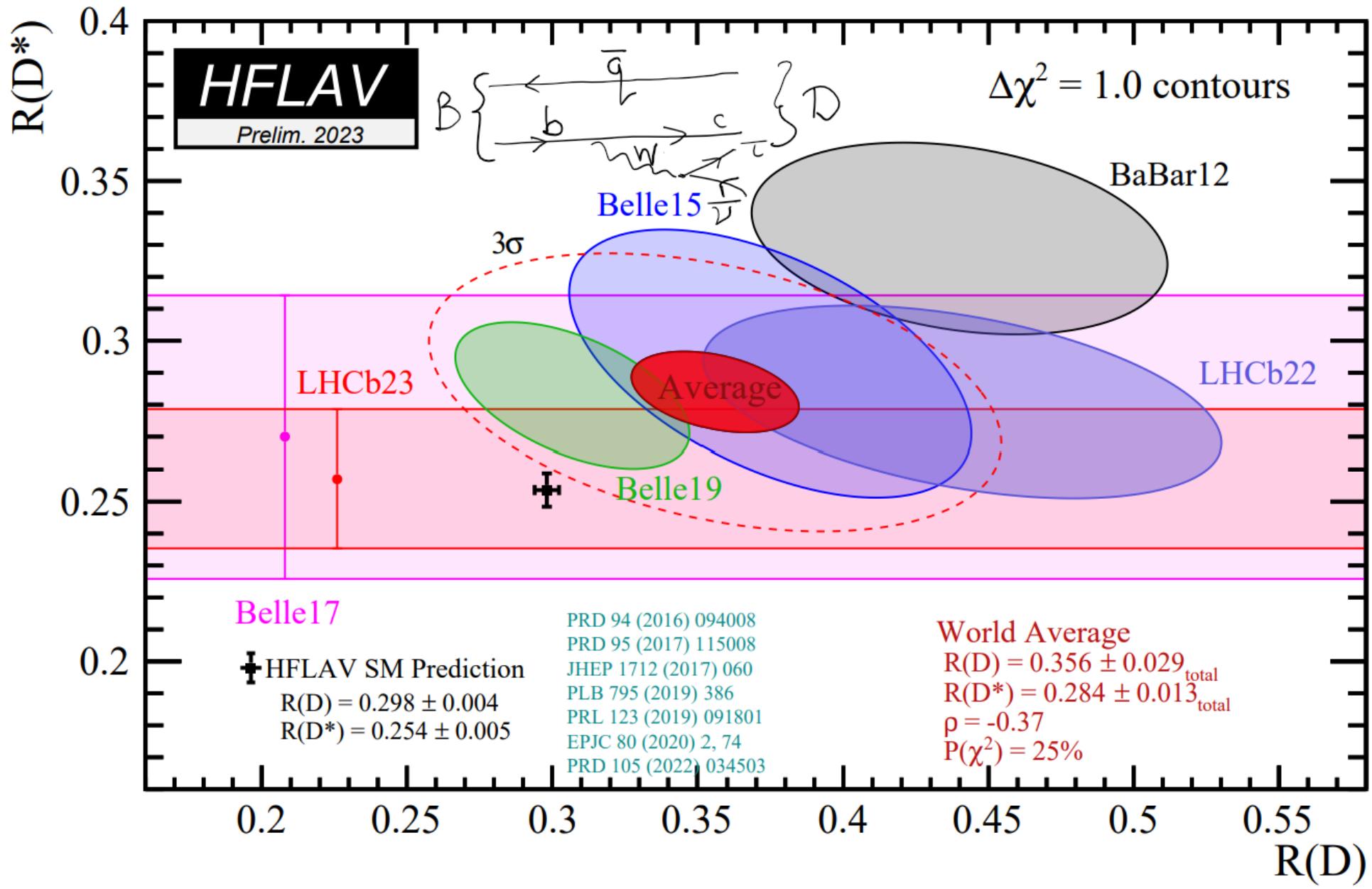
$$SU(3) \times SU(2) \times U(1)_Y \times U(1)_{X_1}$$

*without* kinetic mixing and

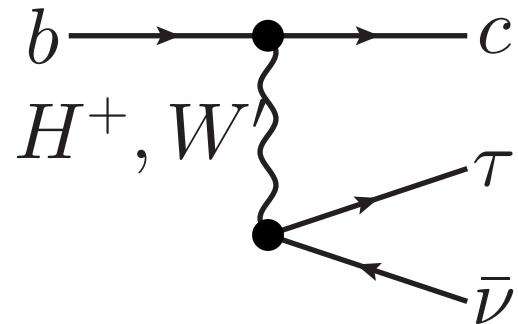
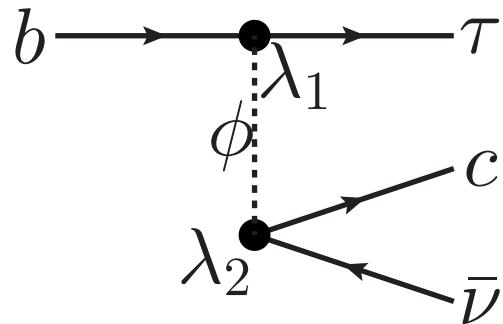
$$X_1 := B_3 - L_2 + \alpha Y,$$

where  $\alpha \in \mathbb{Q}$ .

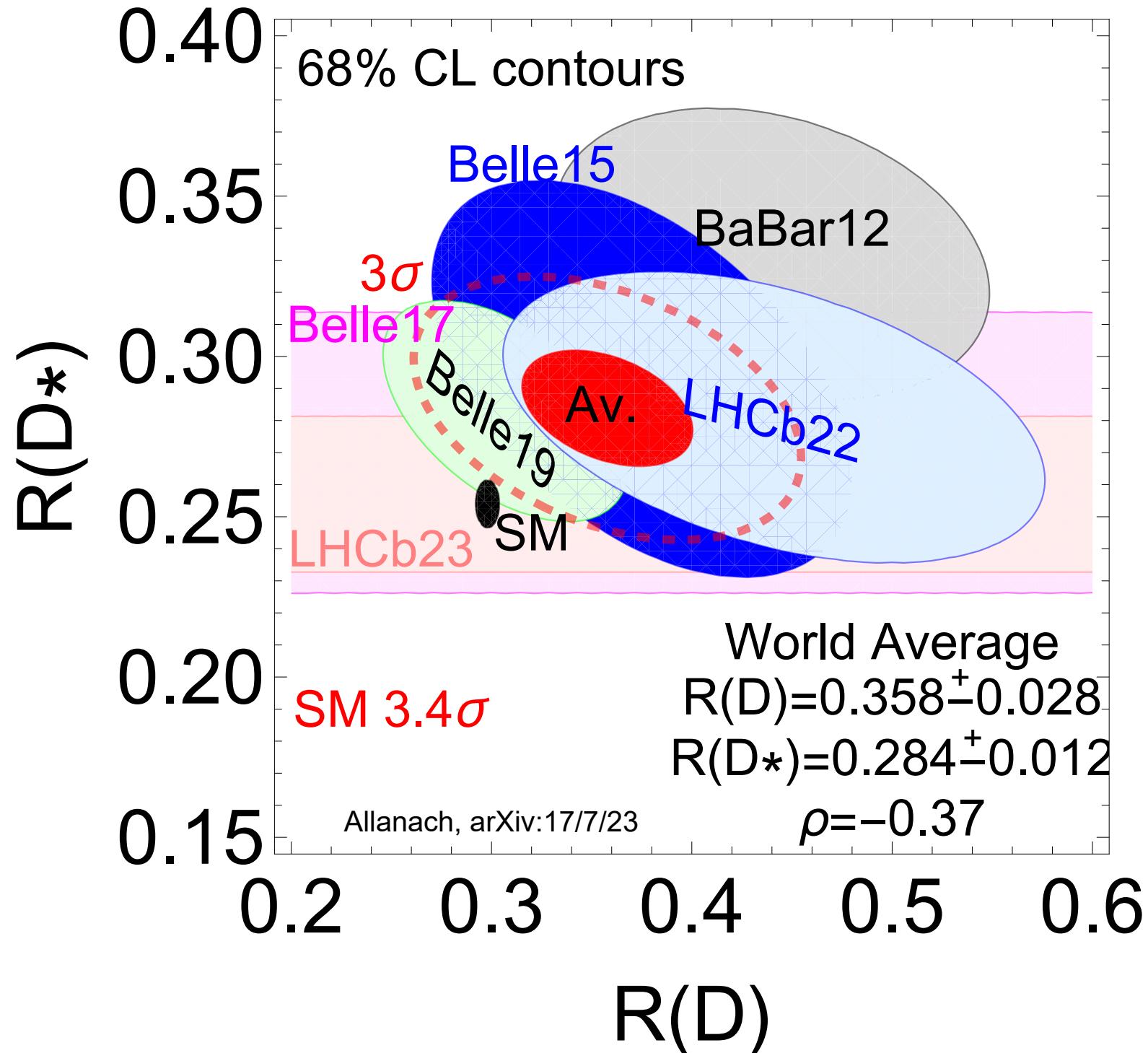
$$R_{D^{(*)}} = BR(B \rightarrow D^{(*)}\tau\nu) / BR(B \rightarrow D^{(*)}\ell\nu_\ell)$$



# $R_{D^{(*)}}$ : BSM Explanations



$$\mathcal{L}_{WET} = -\frac{2\lambda_1\lambda_2}{M^2} (\bar{c}\gamma^\mu P_L \nu) (\bar{\tau}\gamma_\mu P_L b) + H.c.$$



# 2022 Measurement

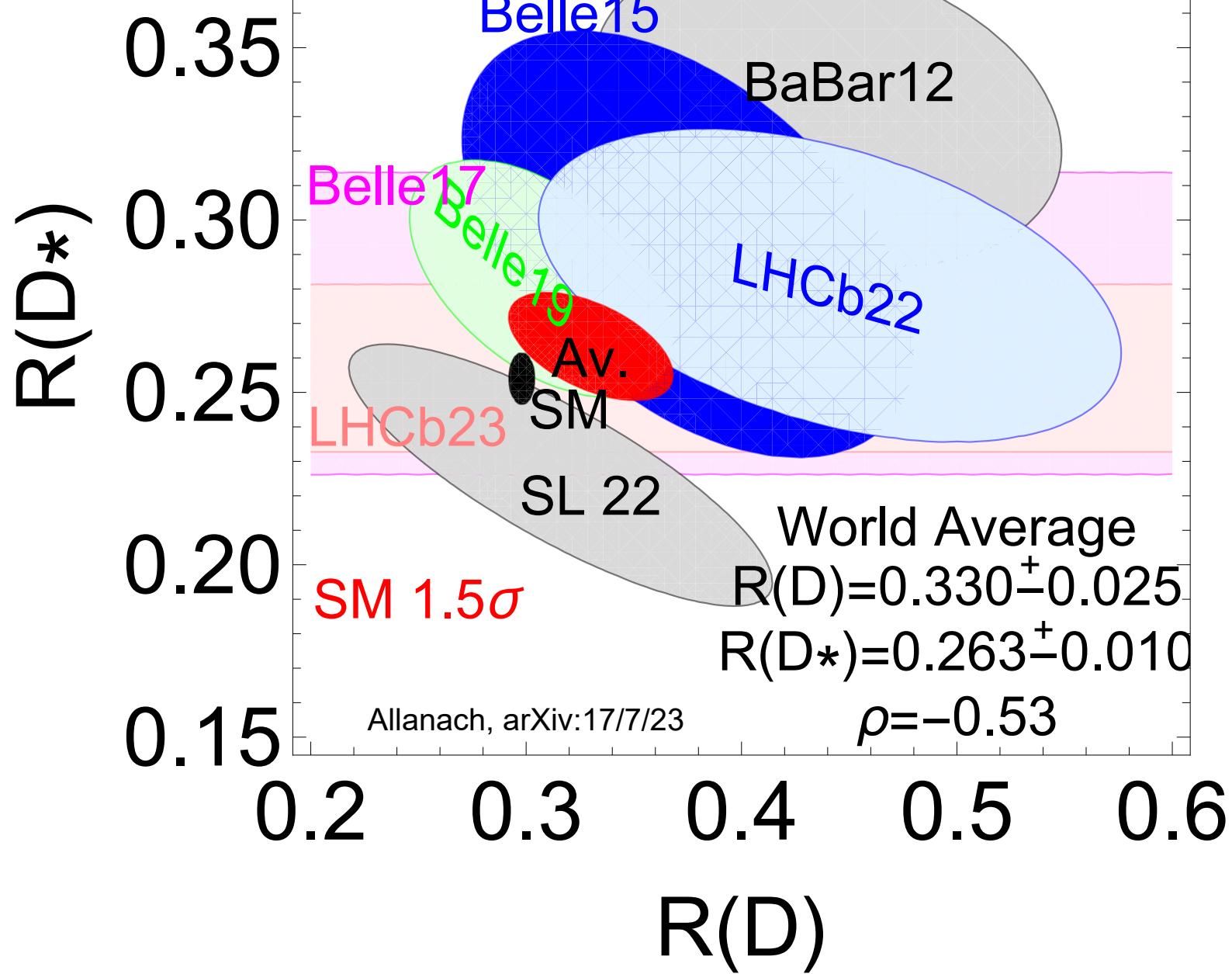
Using BaBar data (not official BaBar analysis)  
and *semi-leptonic* tag: (2012 used *hadronic*)

$$R(D) = 0.316 \pm 0.062 \pm 0.019$$

$$R(D^*) = 0.226 \pm 0.022 \pm 0.012$$

$$\rho = -0.82$$

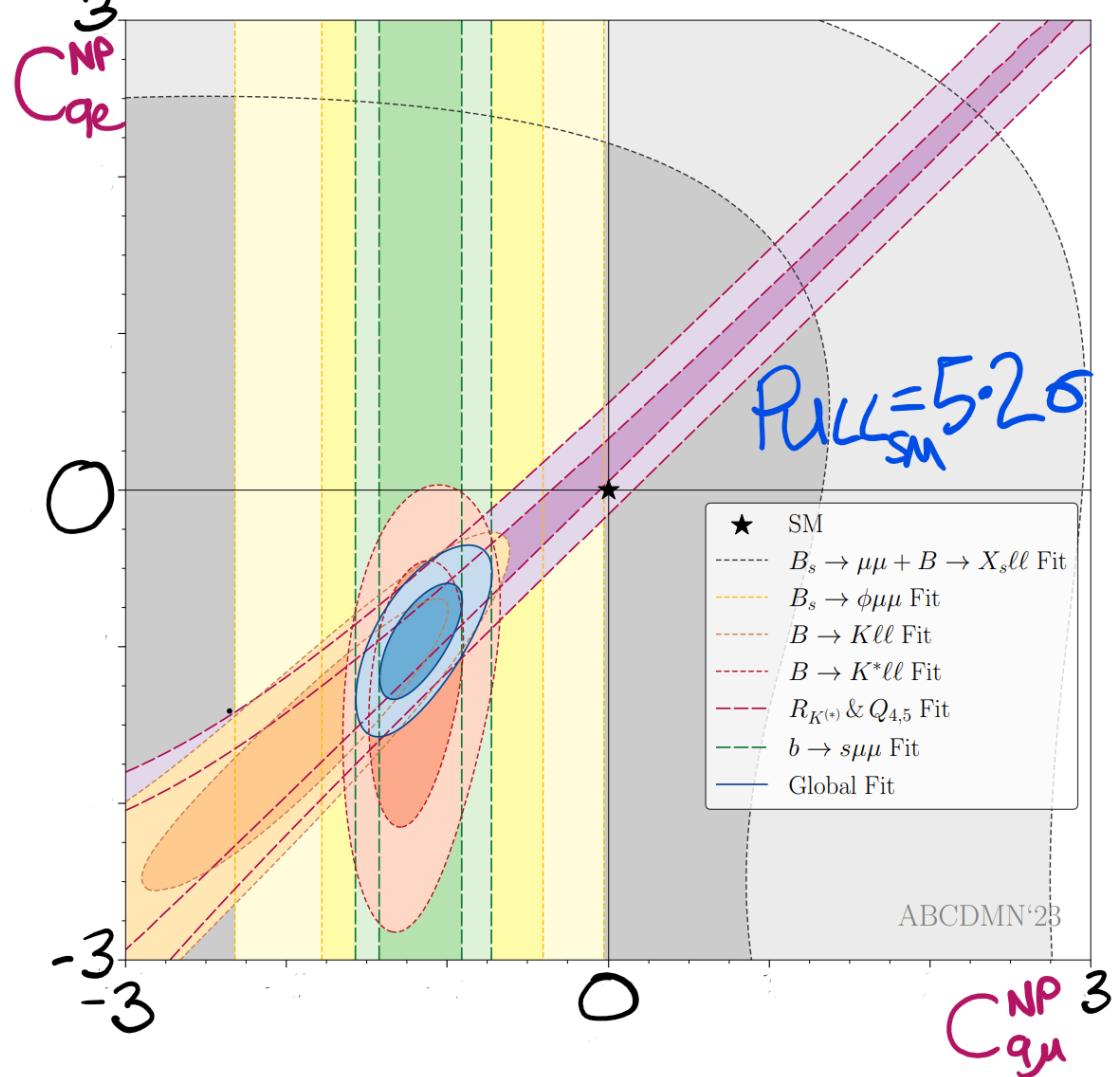
Yunxuan Li, *Search for Beyond Standard Model Physics at BaBar*, (2022), Caltech Ph.D. thesis  
<https://resolver.caltech.edu/CaltechTHESIS:05232022-144829107>

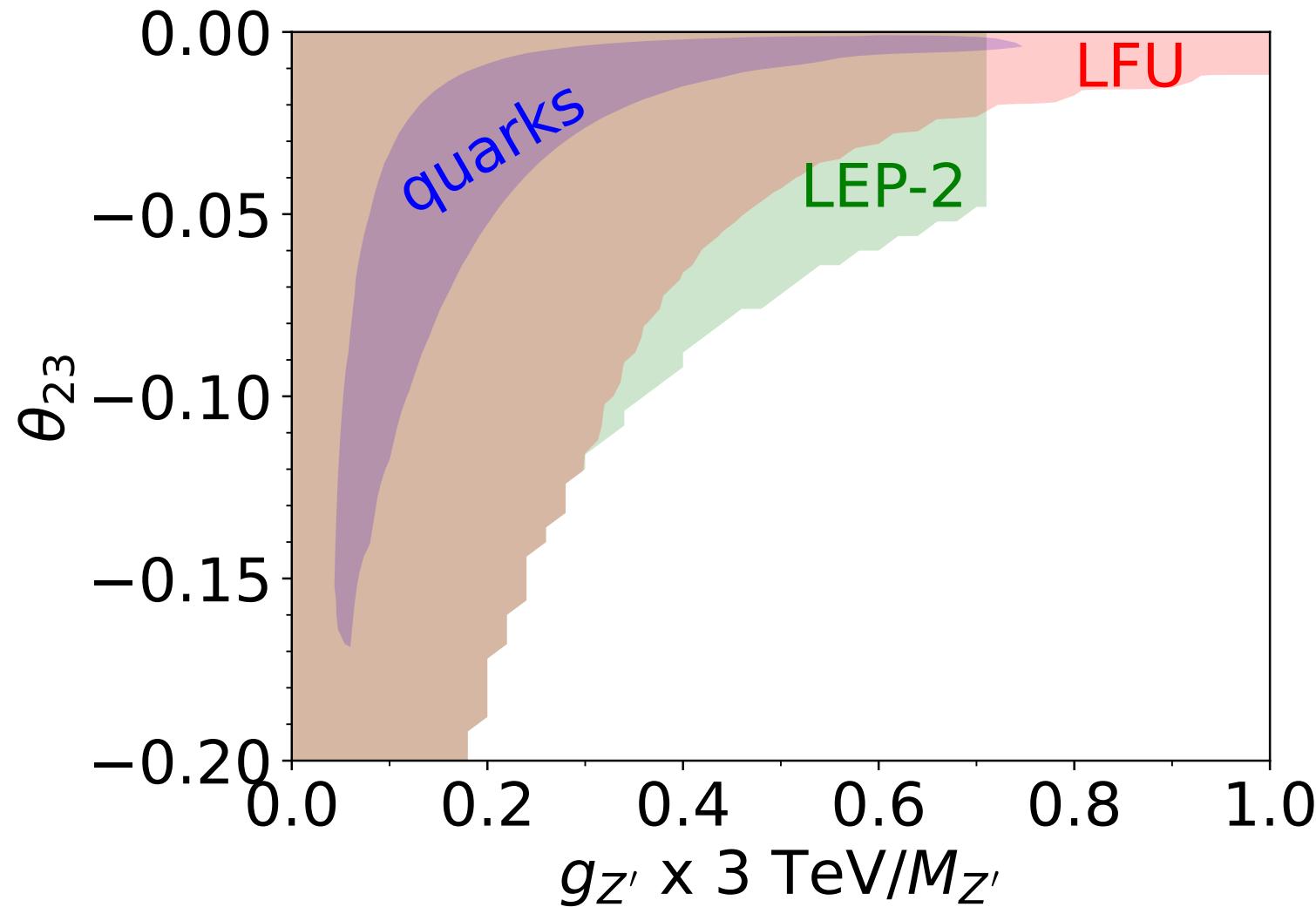


# $\mu/e$ Neutral Current Fits

Alguero et al, 2304.07330

$$\mathcal{L} = N[C_{9\mu}^{NP}(\bar{b}_L \gamma^\alpha s_L)(\bar{\mu} \gamma_\alpha \mu) + C_{9e}^{NP}(\bar{b}_L \gamma^\alpha s_L)(\bar{e} \gamma_\alpha e)] + H.c.$$





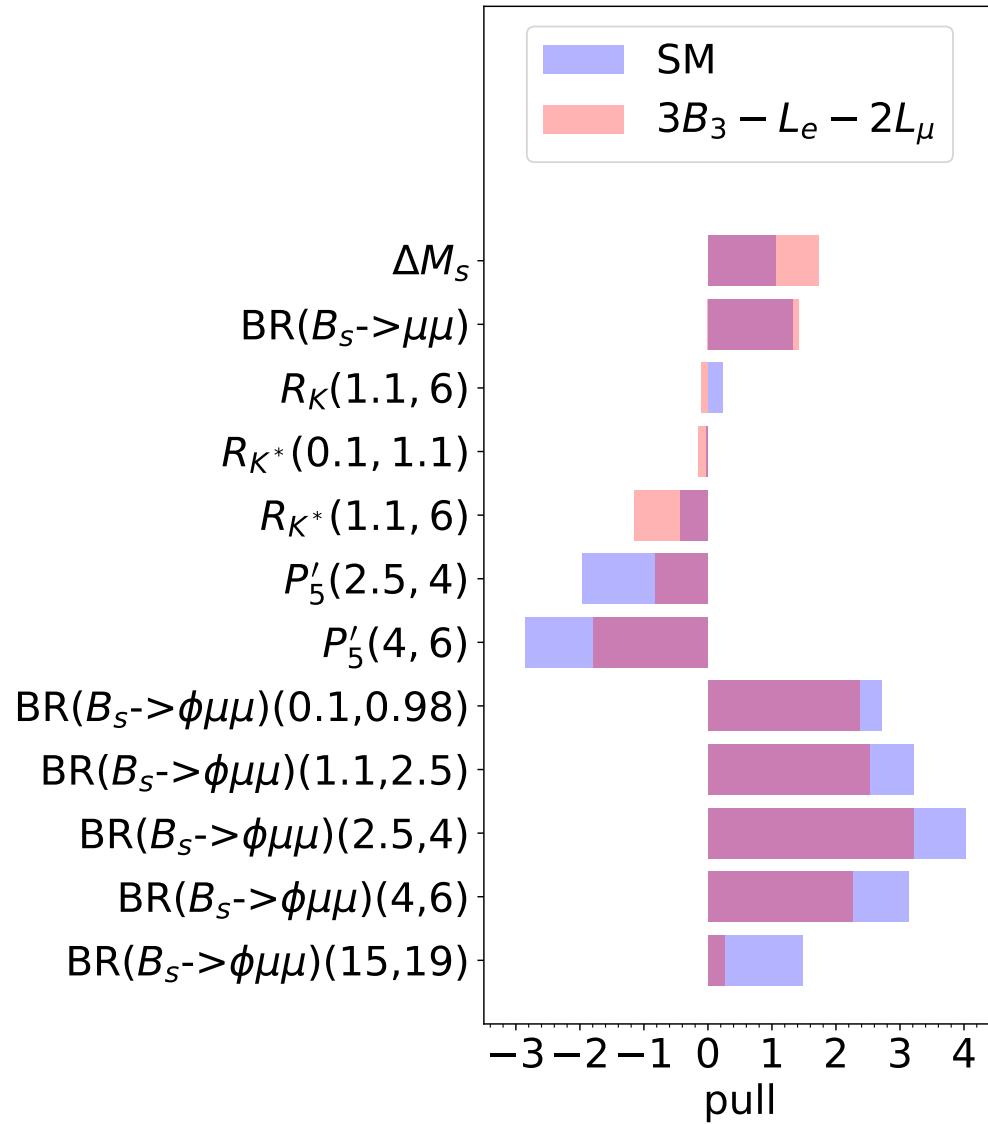
BCA, Mullin, 2306.08669

# $3B_3 - L_e - 2L_\mu$ model

|        | $\chi^2 - \chi^2_{SM}$ | p-value | measurement         | pull |
|--------|------------------------|---------|---------------------|------|
| LFU    | -0.2                   | .85     | $R_{K^*}(0.1, 1.1)$ | -0.1 |
| LEP    | -0.4                   | .58     | $R_{K^*}(1.1, 6)$   | -1.1 |
| quarks | -14.7                  | .10     | $R_K(0.1, 1.1)$     | -0.3 |
| global | -15.3                  | .28     | $R_K(1.1, 6)$       | -0.1 |

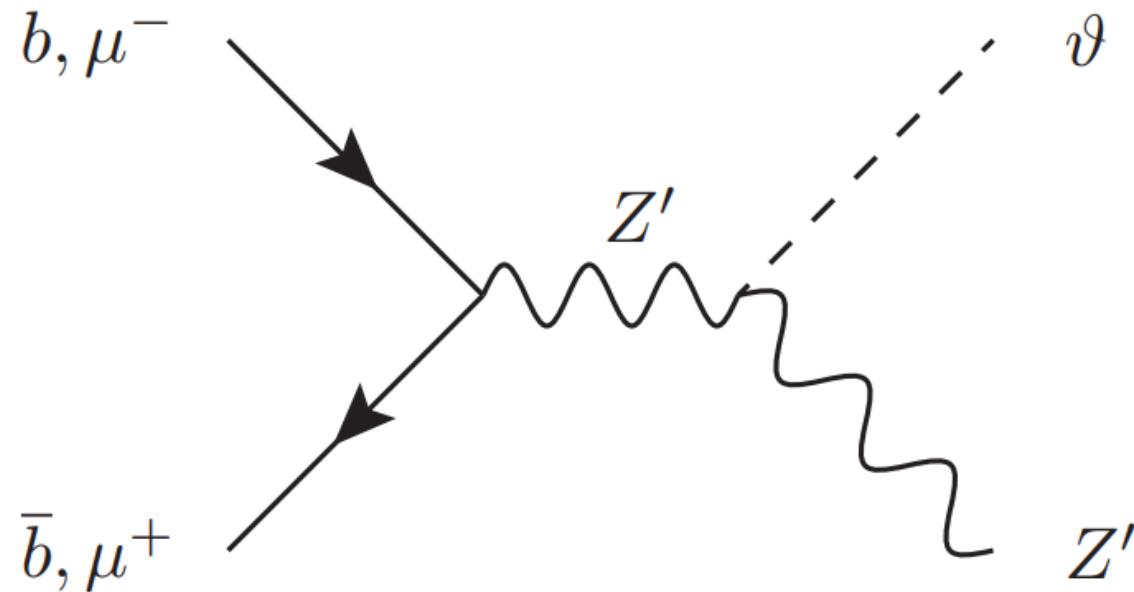
$g_{Z'} = 0.2, \theta_{sb} = -0.03$  best-fit

BCA, Mullin, 2306.08669



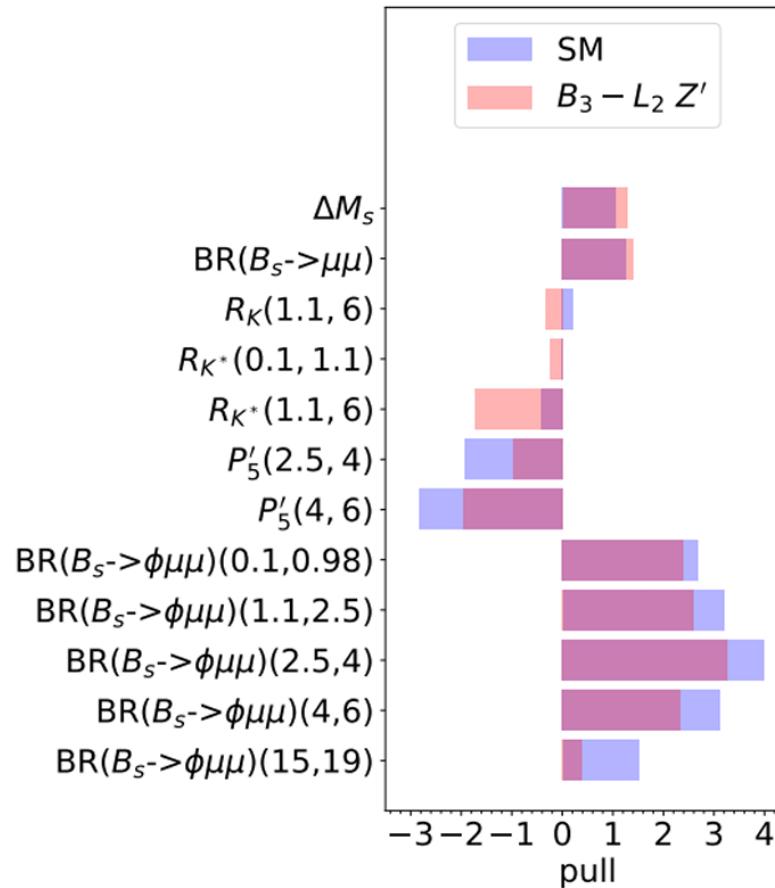
# Flavonstrahlung

Models of  $Z'$  ilk possess  $\mathcal{L} = \lambda H H^\dagger \theta \theta^\dagger \Rightarrow$  a *flavonstrahlung* signature:

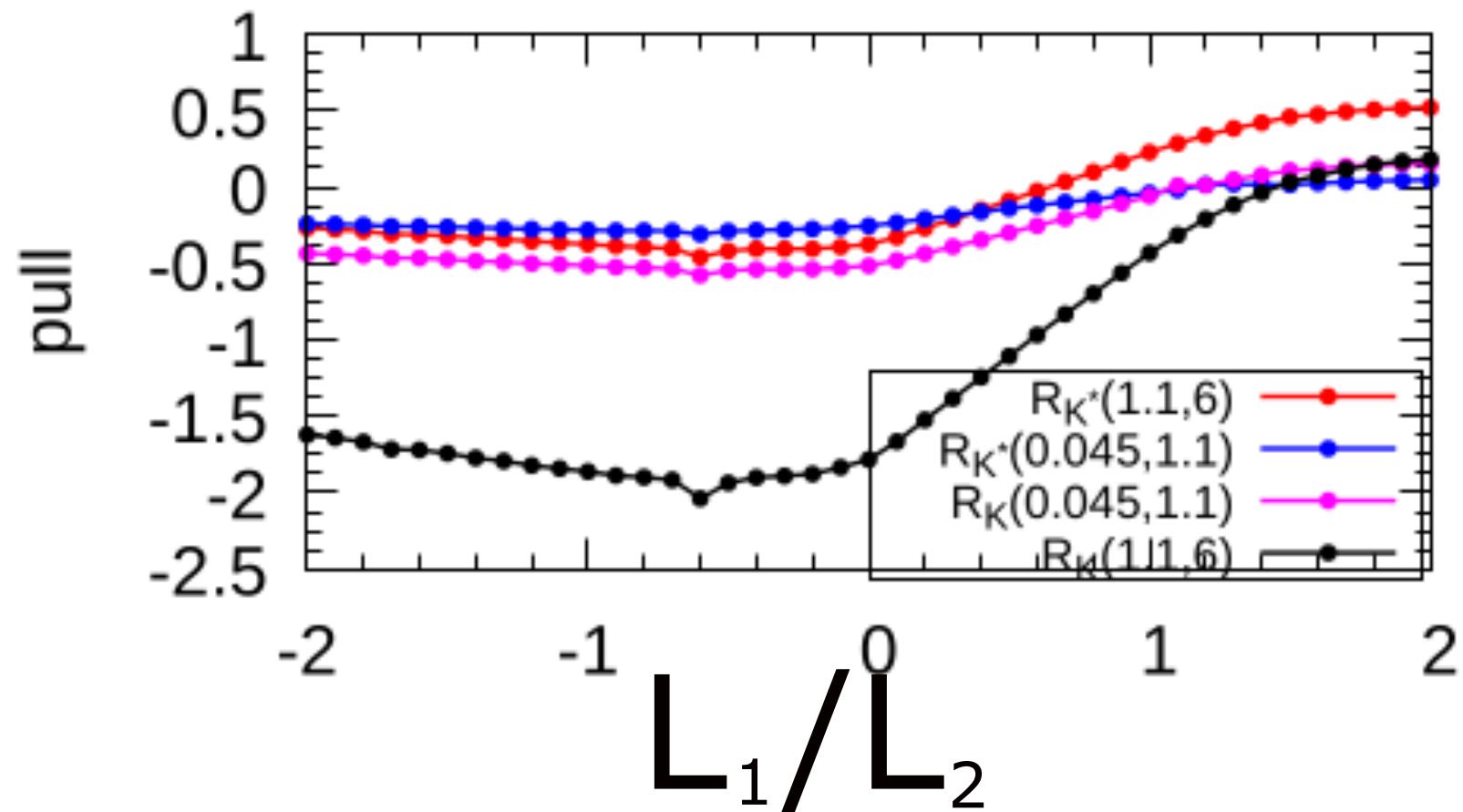


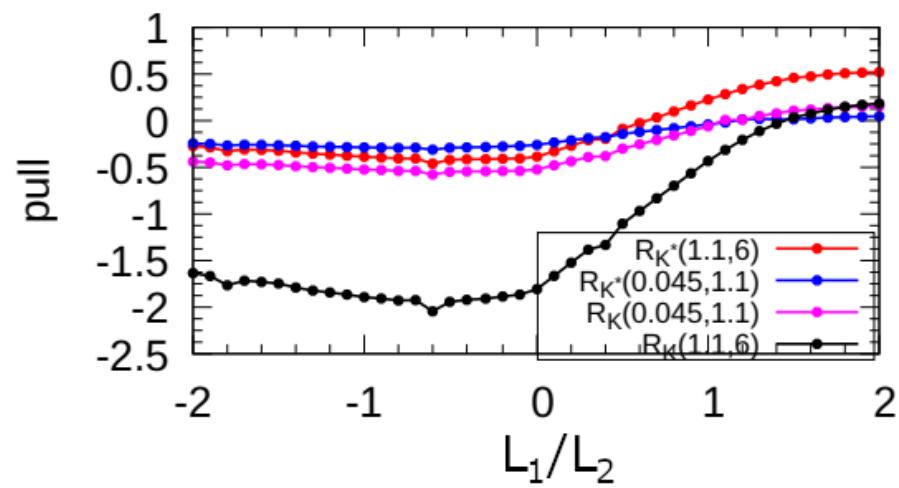
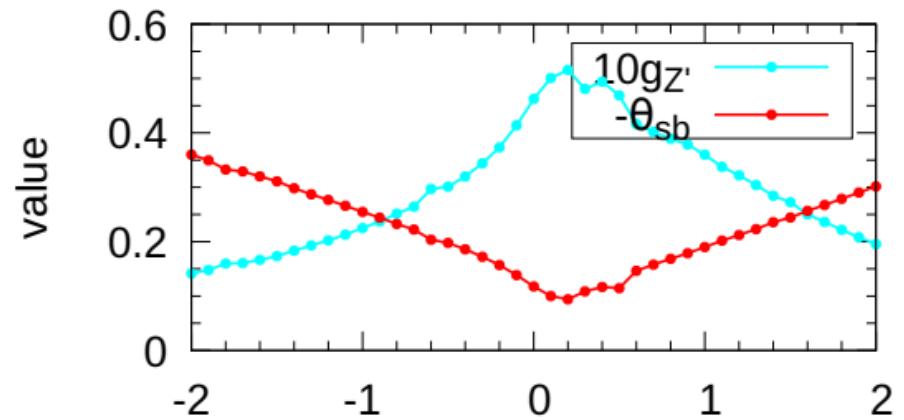
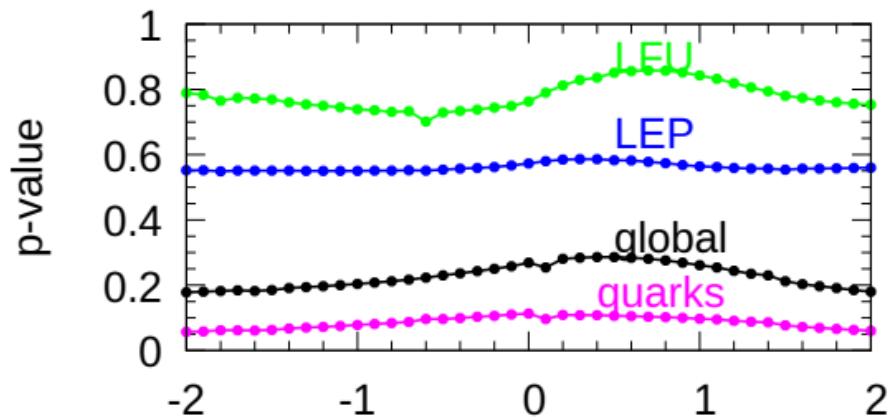
BCA, 2009.02197; BCA, Loisa, 2212.07440

# Pull=(theory-exp)/error



BCA , Davighi , 2211.11766

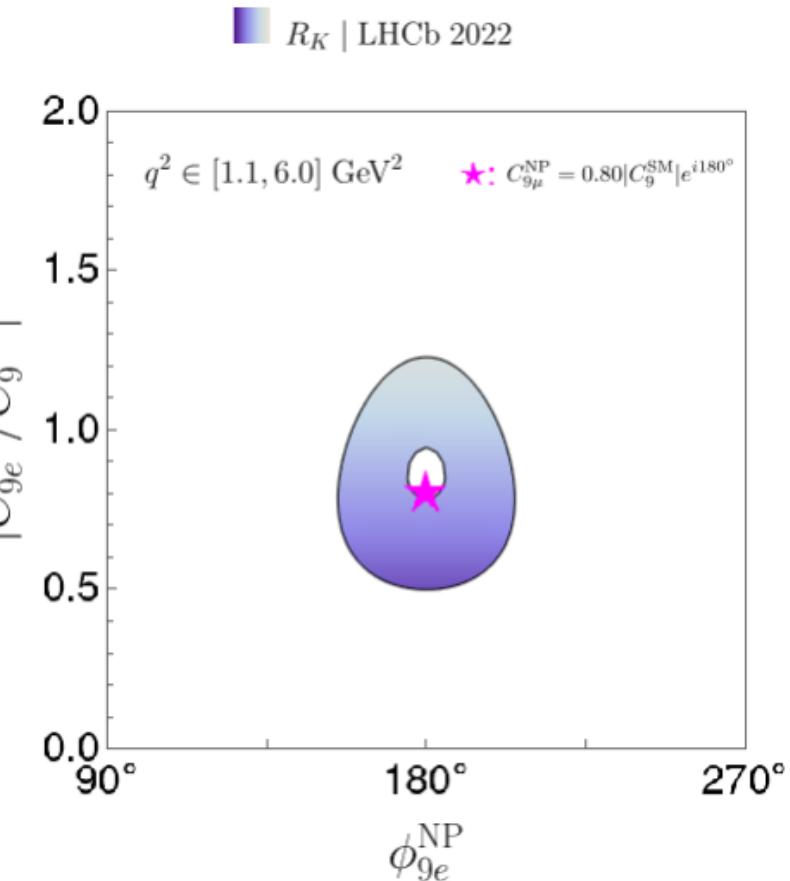
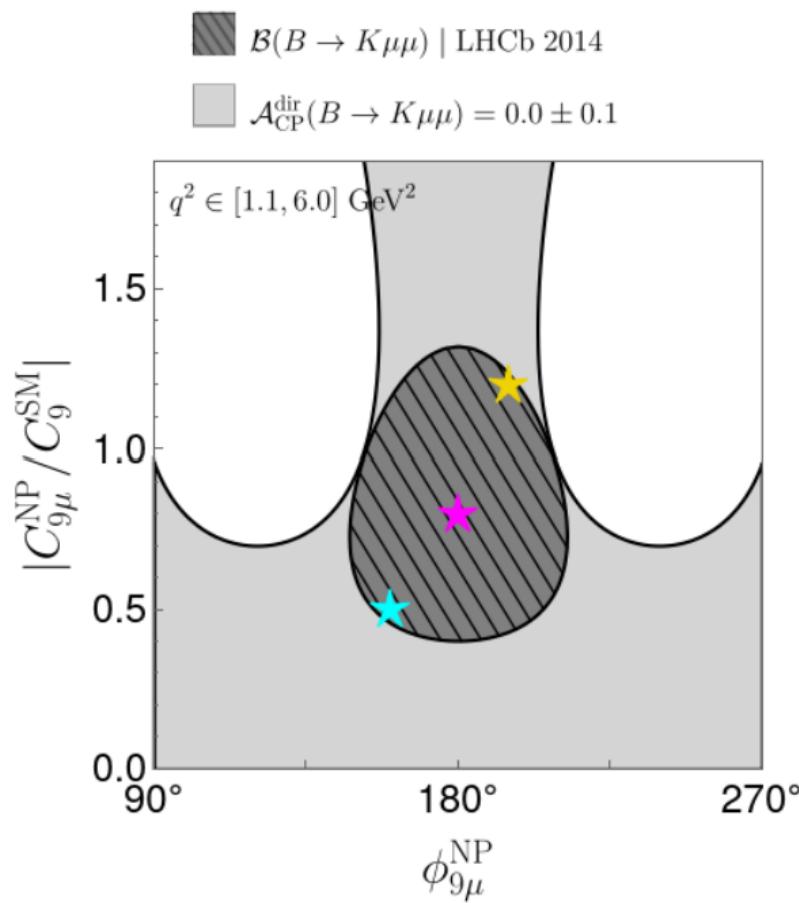




# $e \neq \mu$ allowed

Fleischer, Malami, Rehult, Keri Vos, 2303.08764;  $C_{9\ell}^{NP} = |C_{9\ell}^{NP}|e^{i\phi_{9\ell}^{NP}}$

$$\mathcal{L} = N(\bar{b}_L \gamma^\alpha s_L) [C_{9\mu}^{NP} (\bar{\mu} \gamma_\alpha \mu) + C_{9e}^{NP} (\bar{e} \gamma_\alpha e)] + H.c.$$



# Anomaly cancellation

Need to pick  $X$  charges for fermions consistent with QFT anomaly cancellation.

$$X = 3B_3 -$$

$$(X_e L_e + X_\mu L_\mu + [3 - X_e - X_\mu] L_\tau)$$

works (proof in 2306.08669).

# Trident Neutrino Process

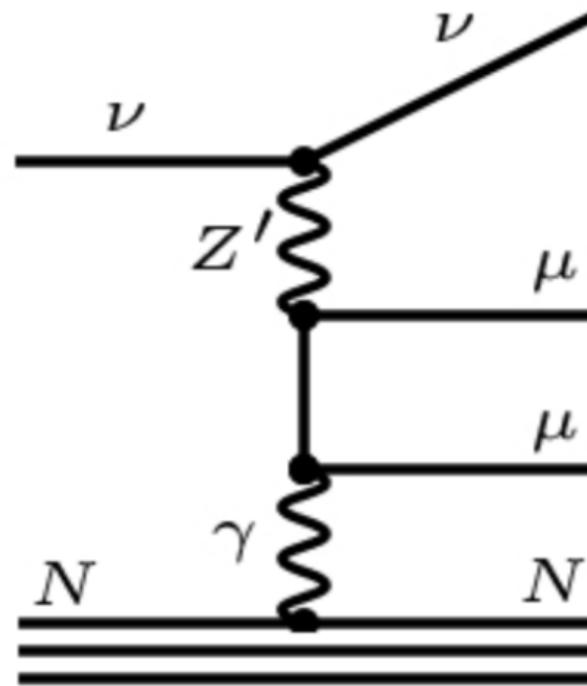
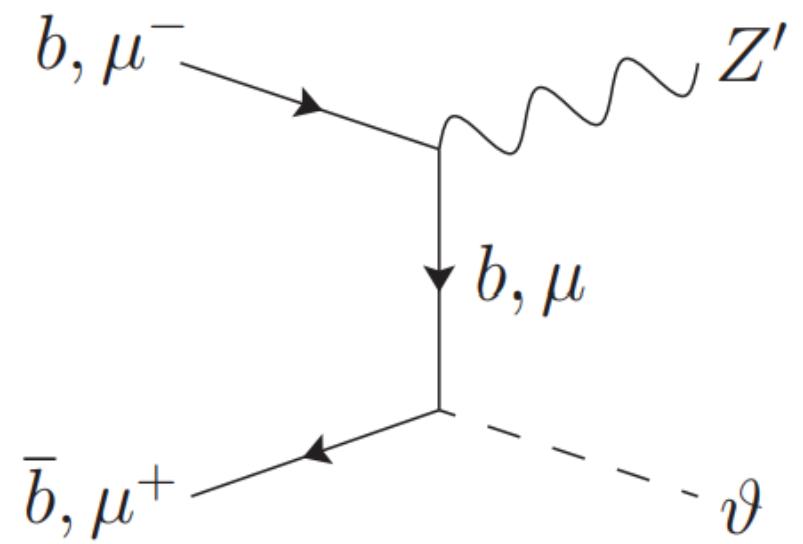
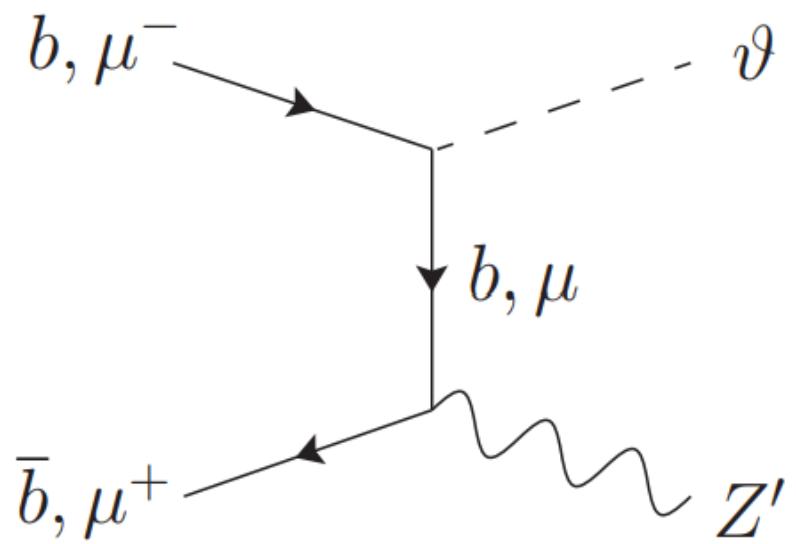
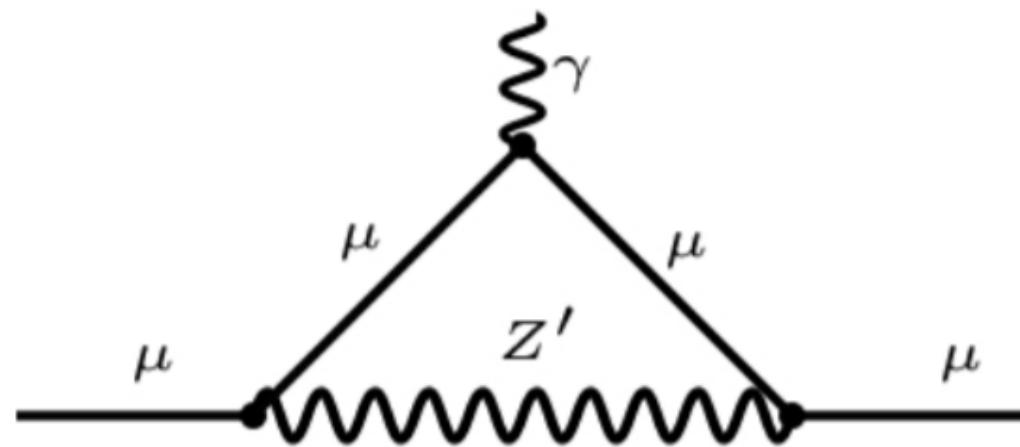
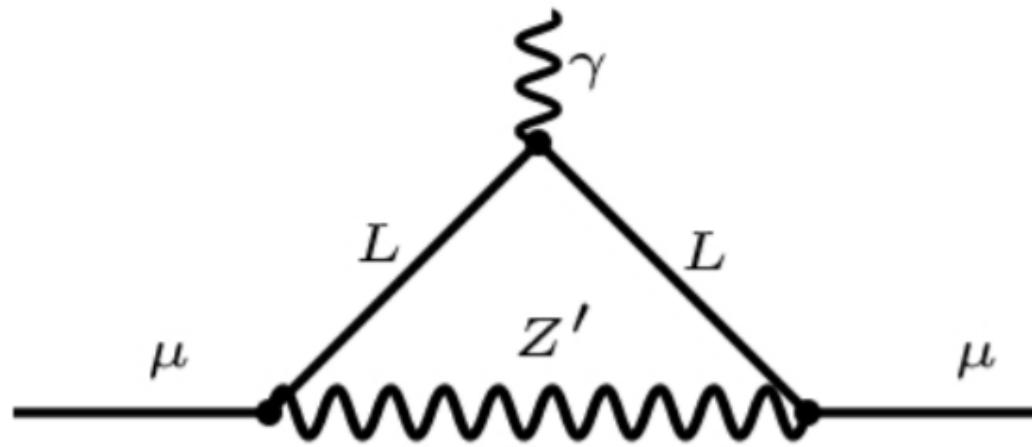


FIG. 10. Neutrino trident process that leads to constraints on the  $Z^\mu$  coupling strength to neutrinos-muons, namely  $M_{Z'}/g_{\nu\mu} \gtrsim 750$  GeV.

# $t$ -channel



$$(g - 2)_\mu$$



# $H\vartheta$ potential

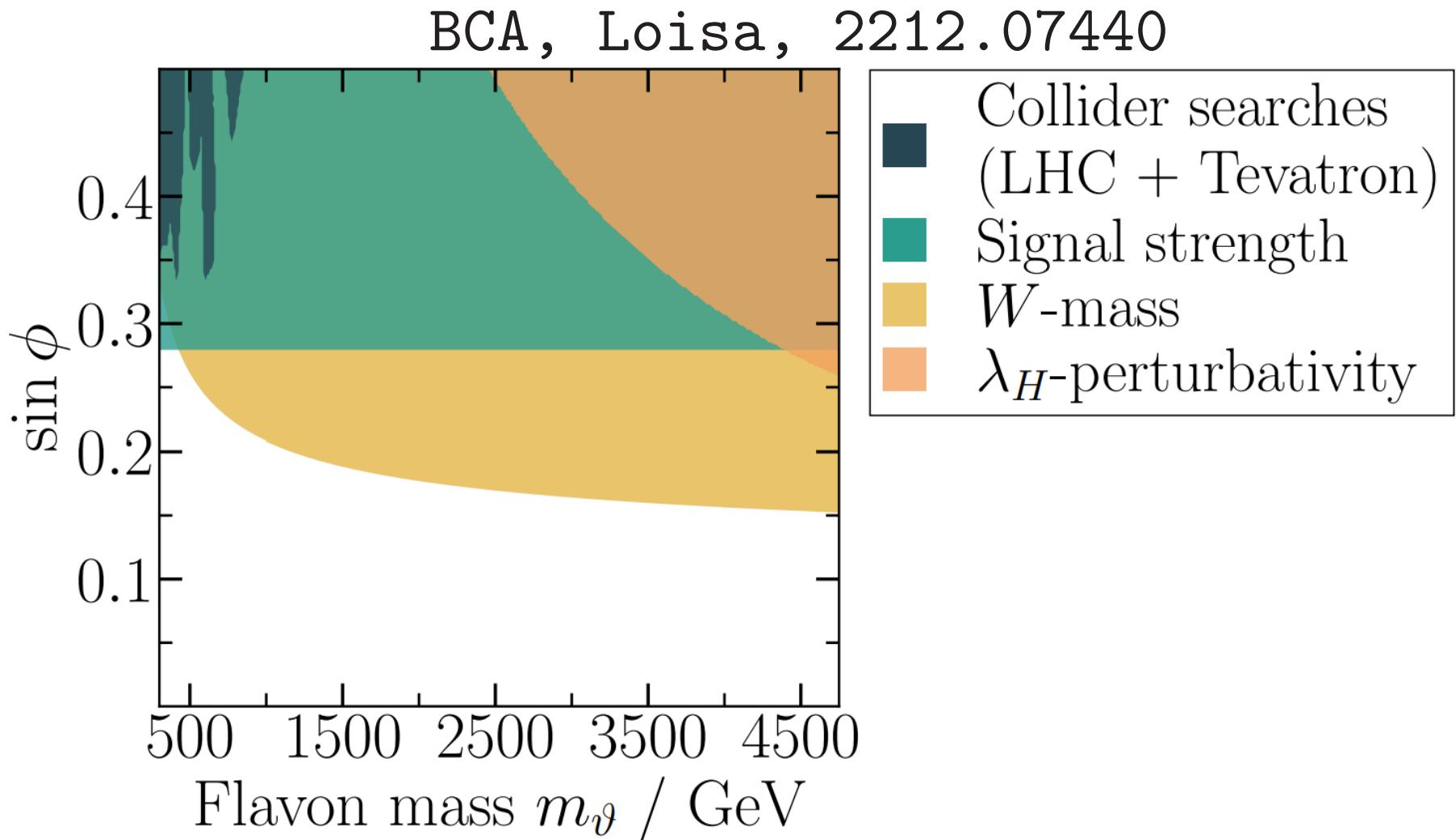
$$\begin{aligned} V &= -\mu^2 H^\dagger H + \lambda_H (H^\dagger H)^2 - \mu_\theta^2 \theta^* \theta + \\ &\quad \lambda_\theta (\theta^* \theta)^2 + \lambda_{\theta H} \theta^* \theta H^\dagger H \\ &= -\frac{1}{2} \begin{pmatrix} h' & \vartheta' \end{pmatrix} M^2 \begin{pmatrix} h' \\ \vartheta' \end{pmatrix} + \dots \\ M^2 &= \begin{pmatrix} 2\lambda_H v_H^2 & \lambda_{\theta H} v_H v_\theta \\ \lambda_{\theta H} v_H v_\theta & 2\lambda_\theta v_\theta^2 \end{pmatrix} \end{aligned}$$

# $H\vartheta$ mixing

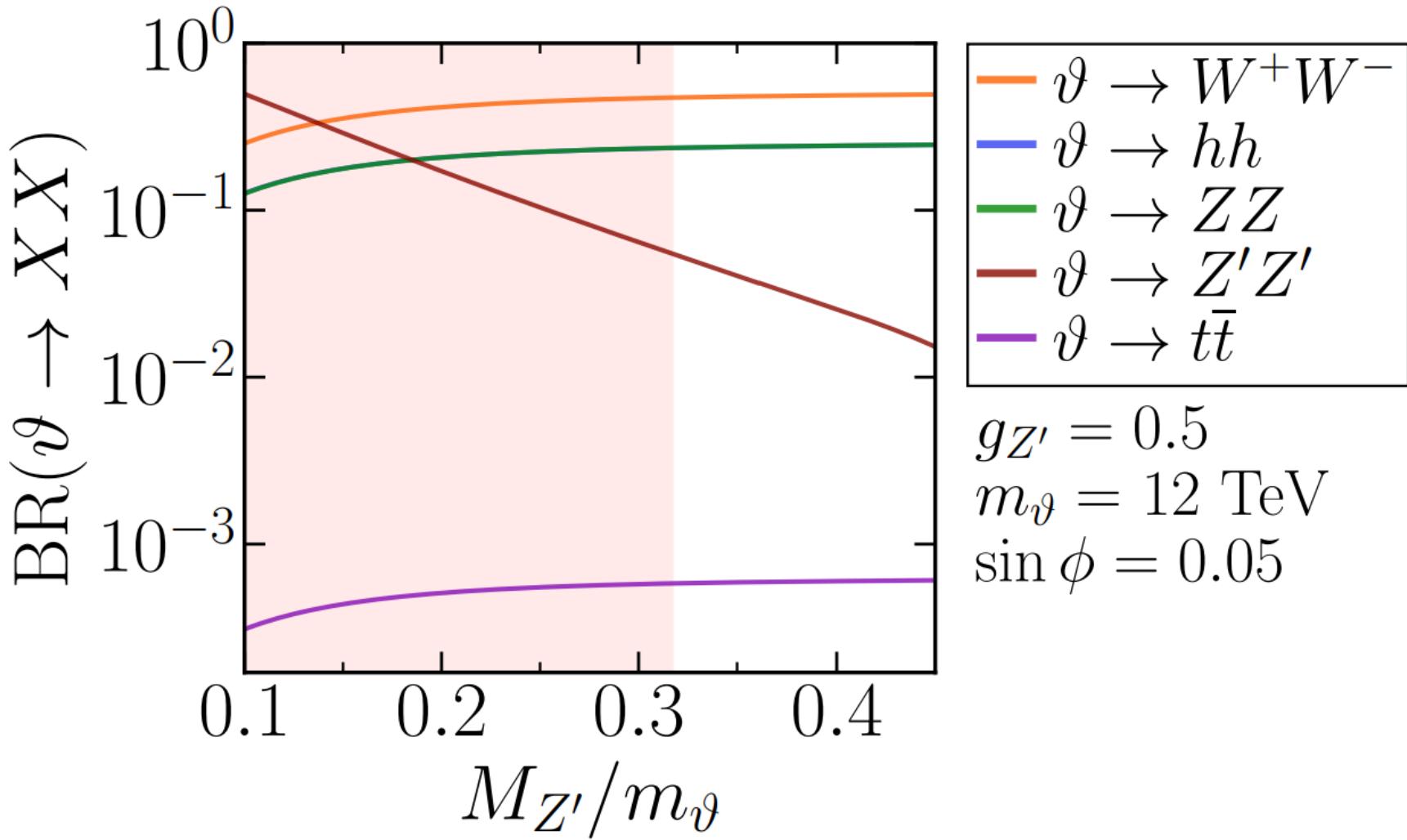
$$\begin{pmatrix} h \\ \vartheta \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} h' \\ \vartheta' \end{pmatrix}$$
$$\sin 2\phi = \frac{2\lambda_{\theta H} v_h v_\theta}{m_\vartheta^2 - m_h^2}. \quad (-9)$$

Three parameters:  $v_\theta = M_{Z'}/g_{Z'}$ ,  $m_\vartheta$  and  $\phi$ .

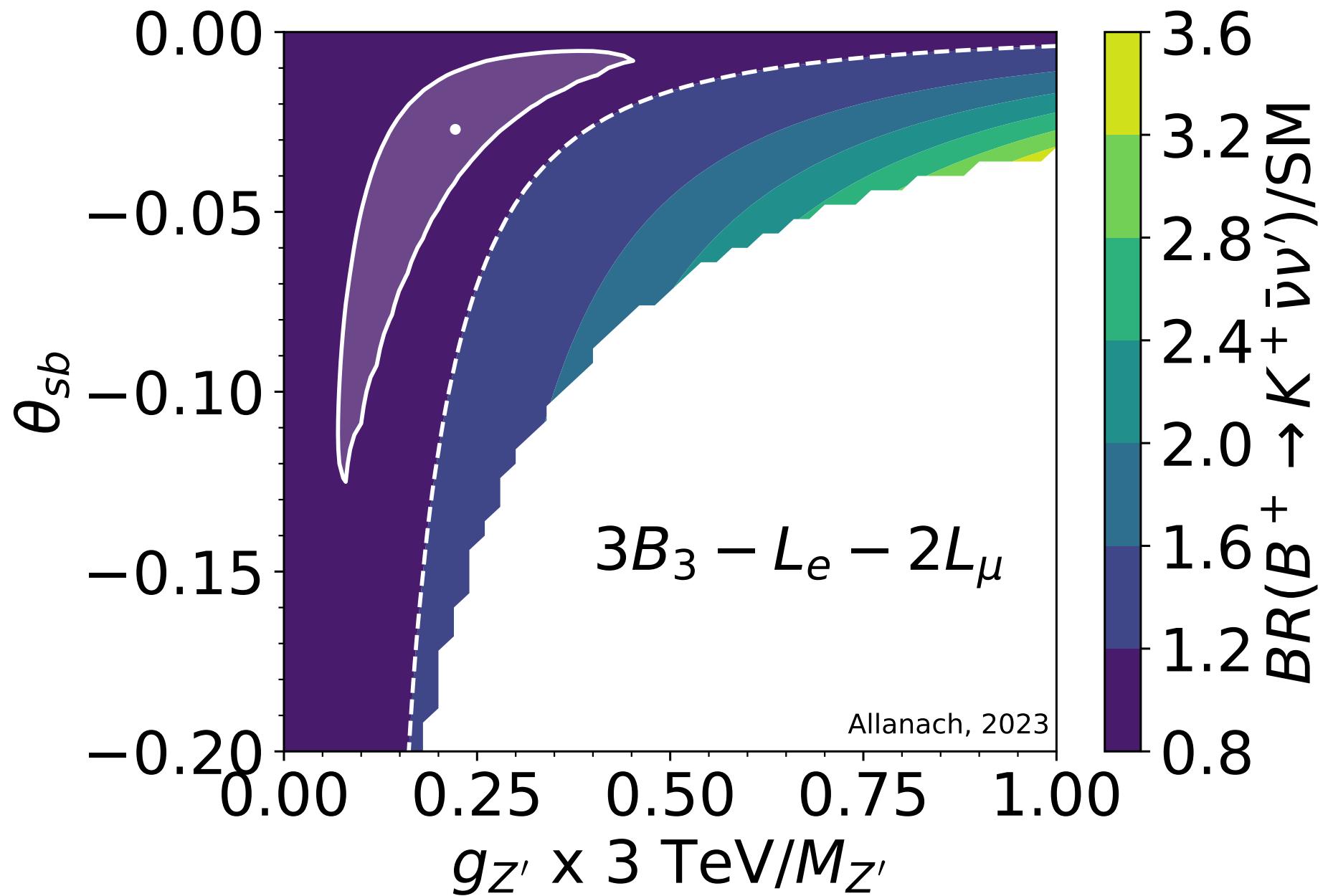
# Higgs Signal Strength



# $\vartheta$ BRs

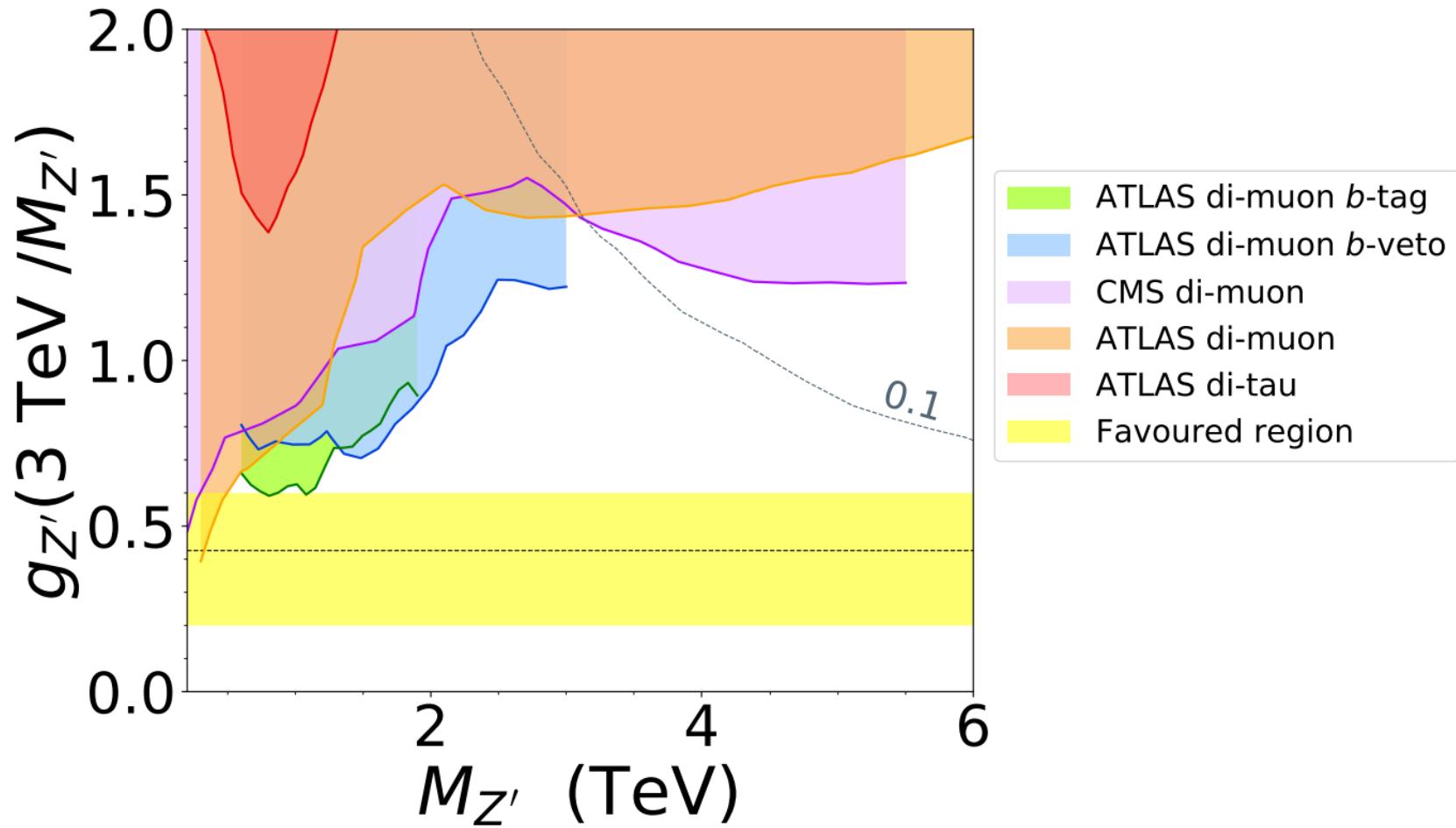




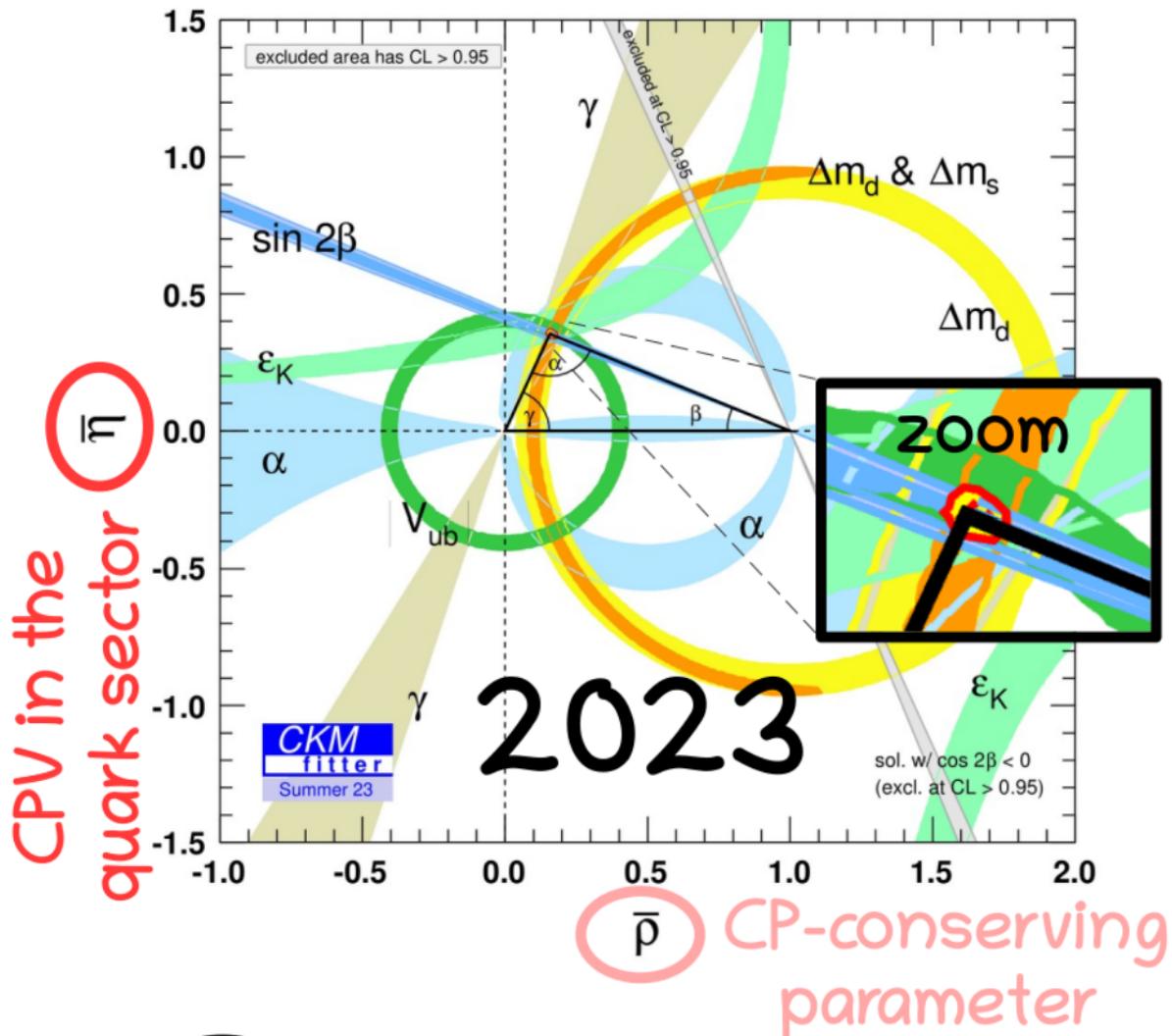


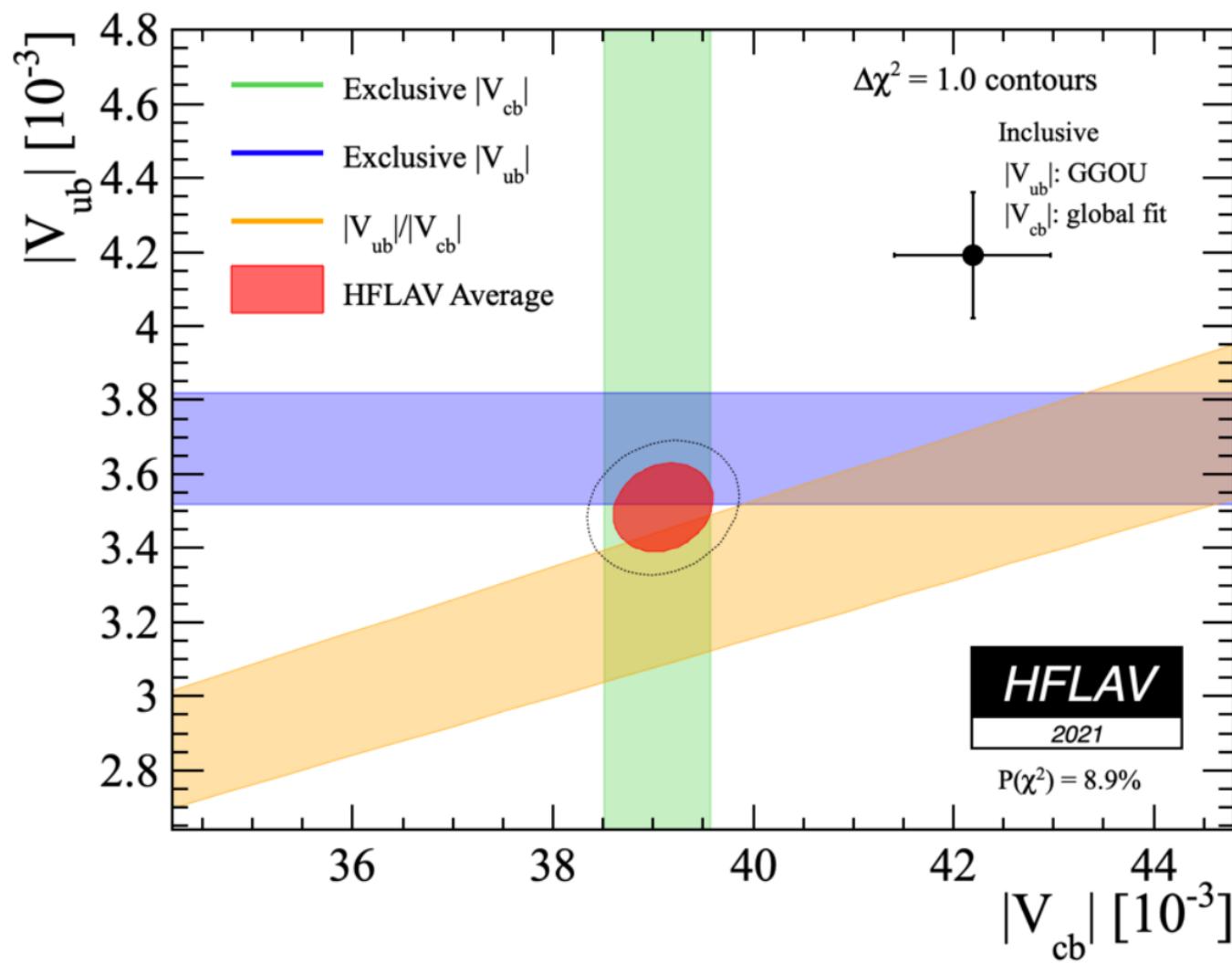


# $Z'$ Searches<sup>5</sup>



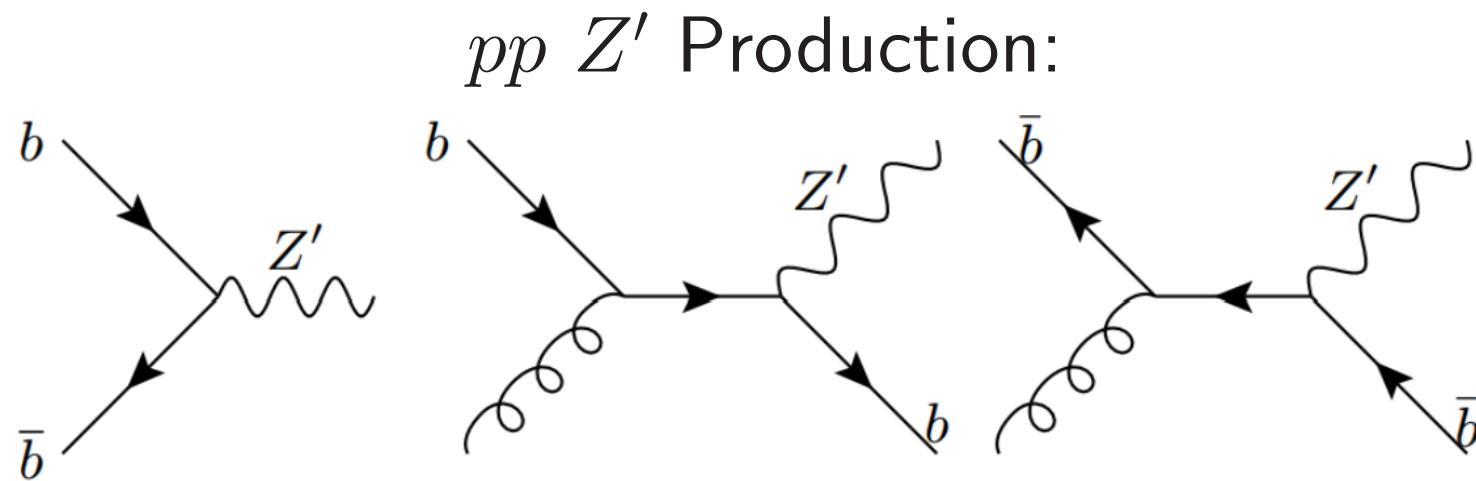
<sup>5</sup>BCA, Banks, 2111.06691





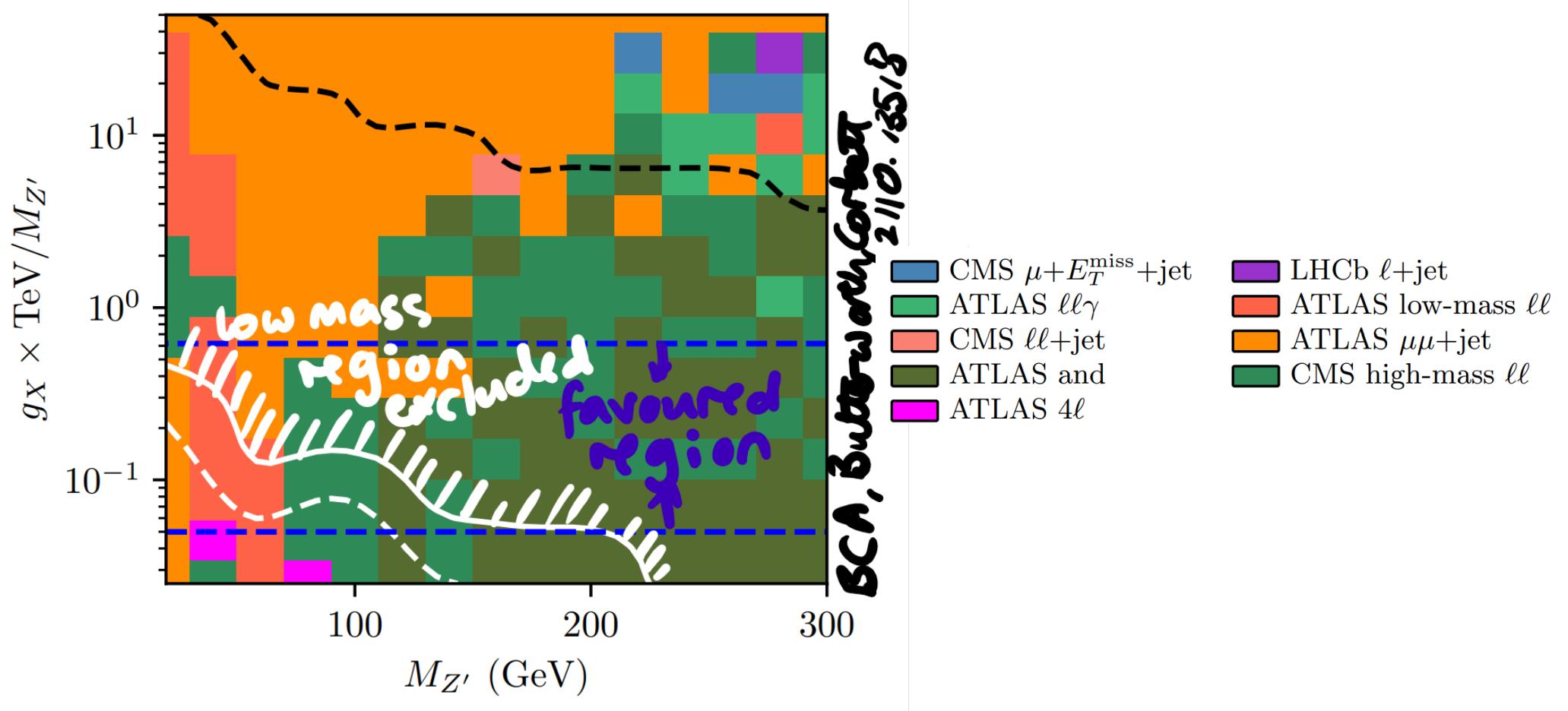
# $Z'$ Decay Modes

| Mode         | BR   | Mode       | BR   | Mode            | BR   |
|--------------|------|------------|------|-----------------|------|
| $t\bar{t}$   | 0.15 | $b\bar{b}$ | 0.15 | $\nu\bar{\nu}'$ | 0.23 |
| $\mu^+\mu^-$ | 0.37 | $e^+e^-$   | 0.09 |                 |      |



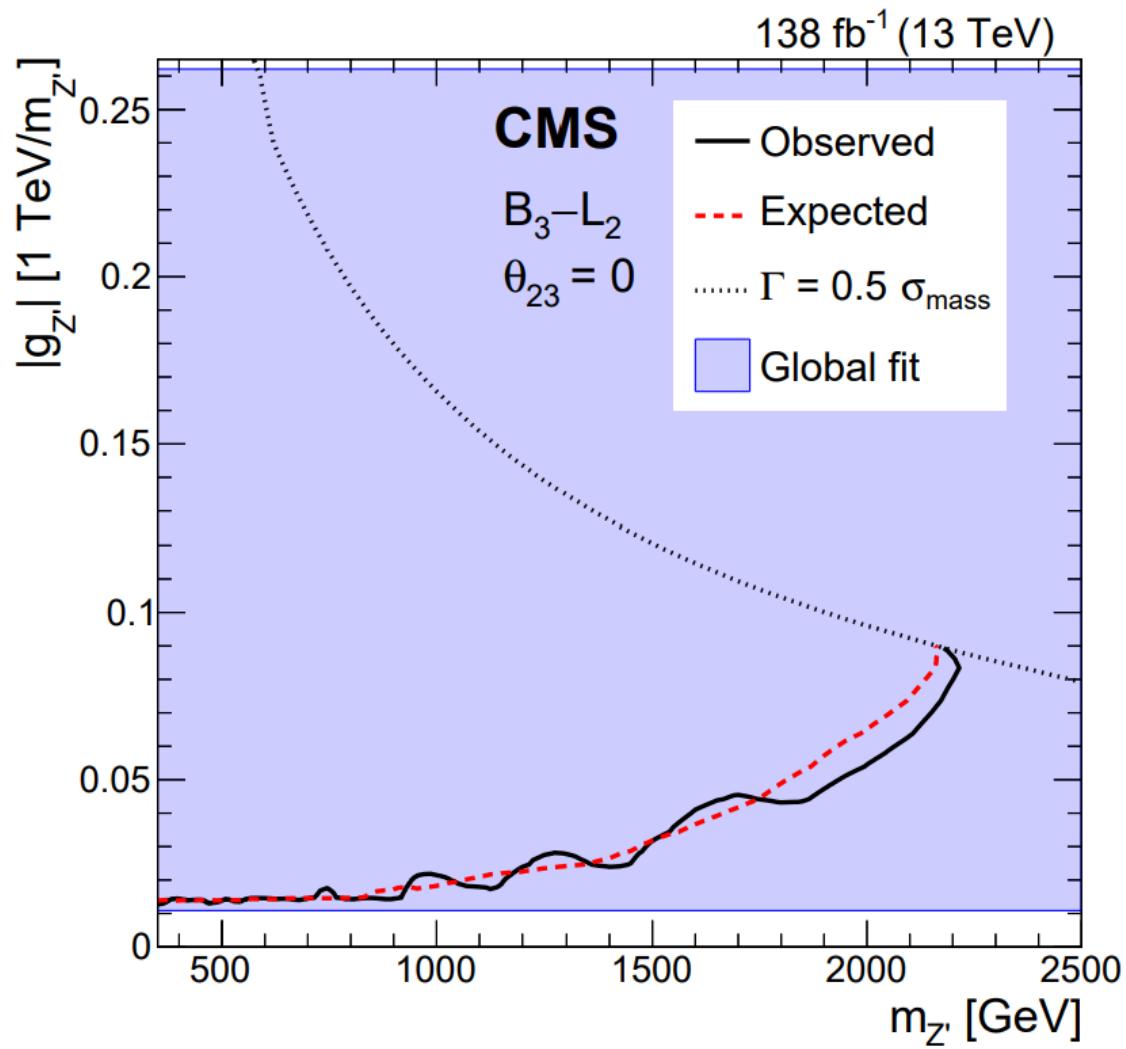
$$\sigma_{prod} \propto g_{Z'}^2 \cos^4 \theta_{sb} = g_{Z'}^2 (1 - 2\theta_{sb}^2 + \mathcal{O}(\theta_{sb}^4))$$

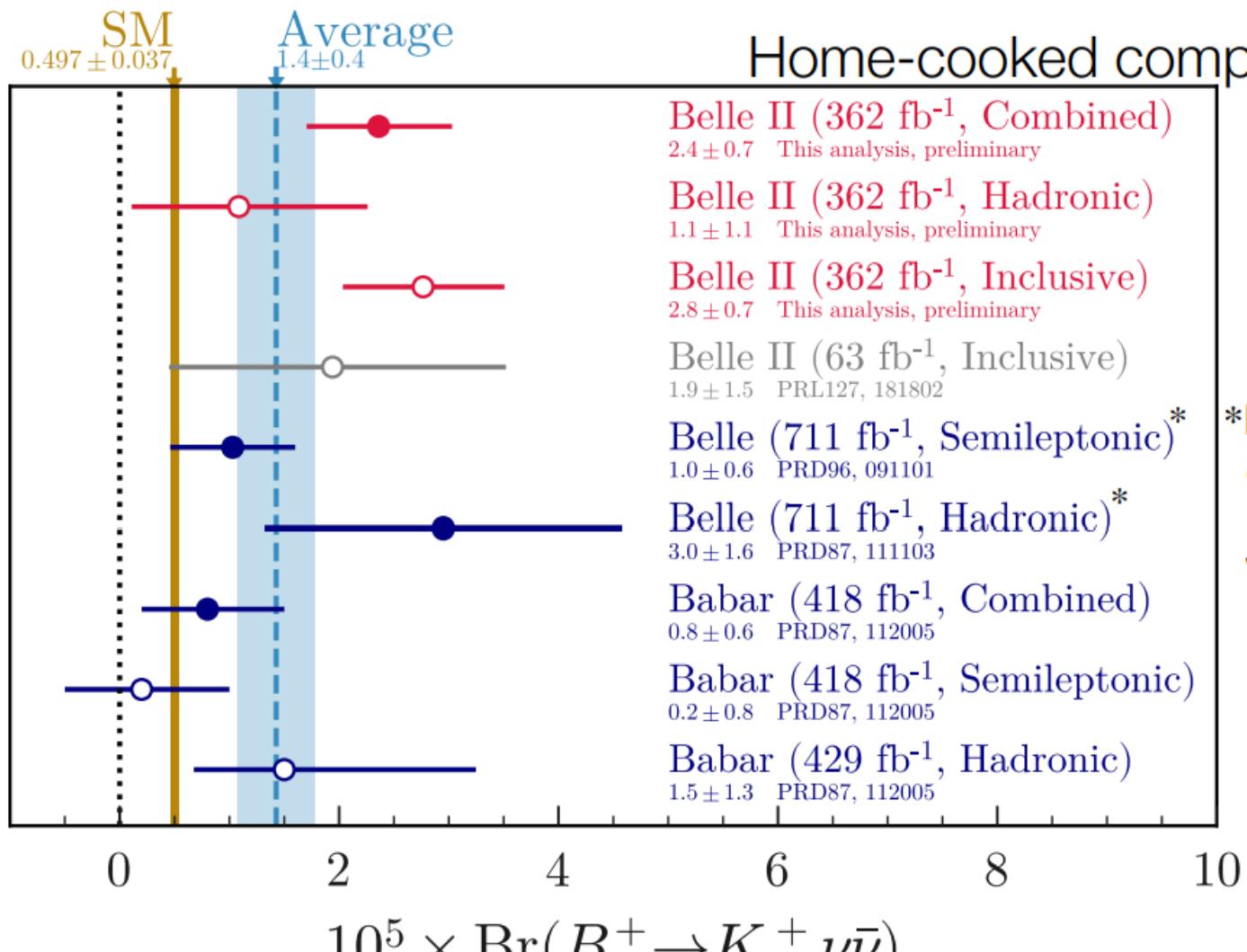
# $B_3 - L_2$ model's ${}^6Z'$



<sup>6</sup>Bonilla, Modak, Srivastava, Valle, 1705.00915; Alonso, Cox, Han, Yanagida 1705.03858

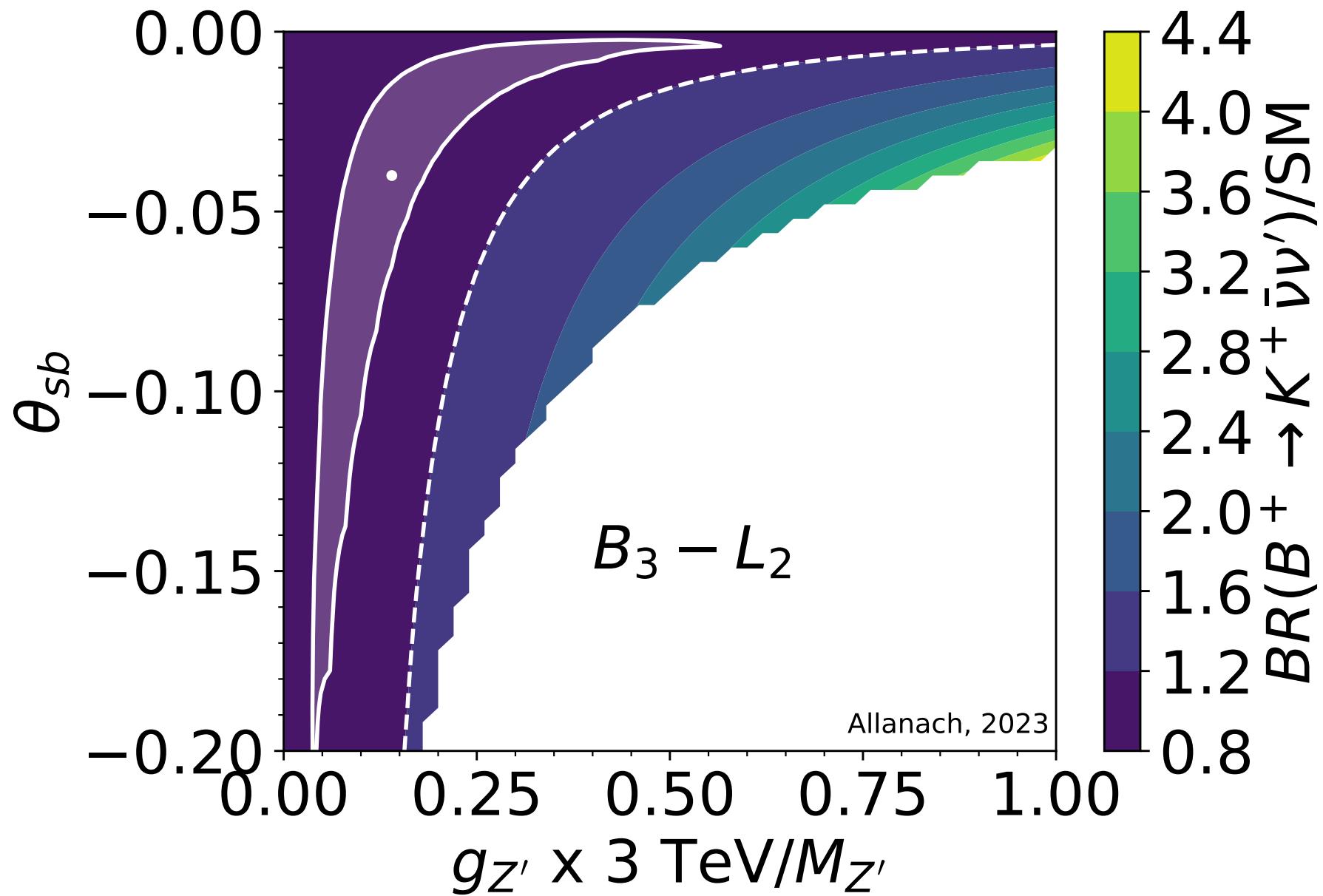
# CMS $\mu^+ \mu^- b$ 2307.08708



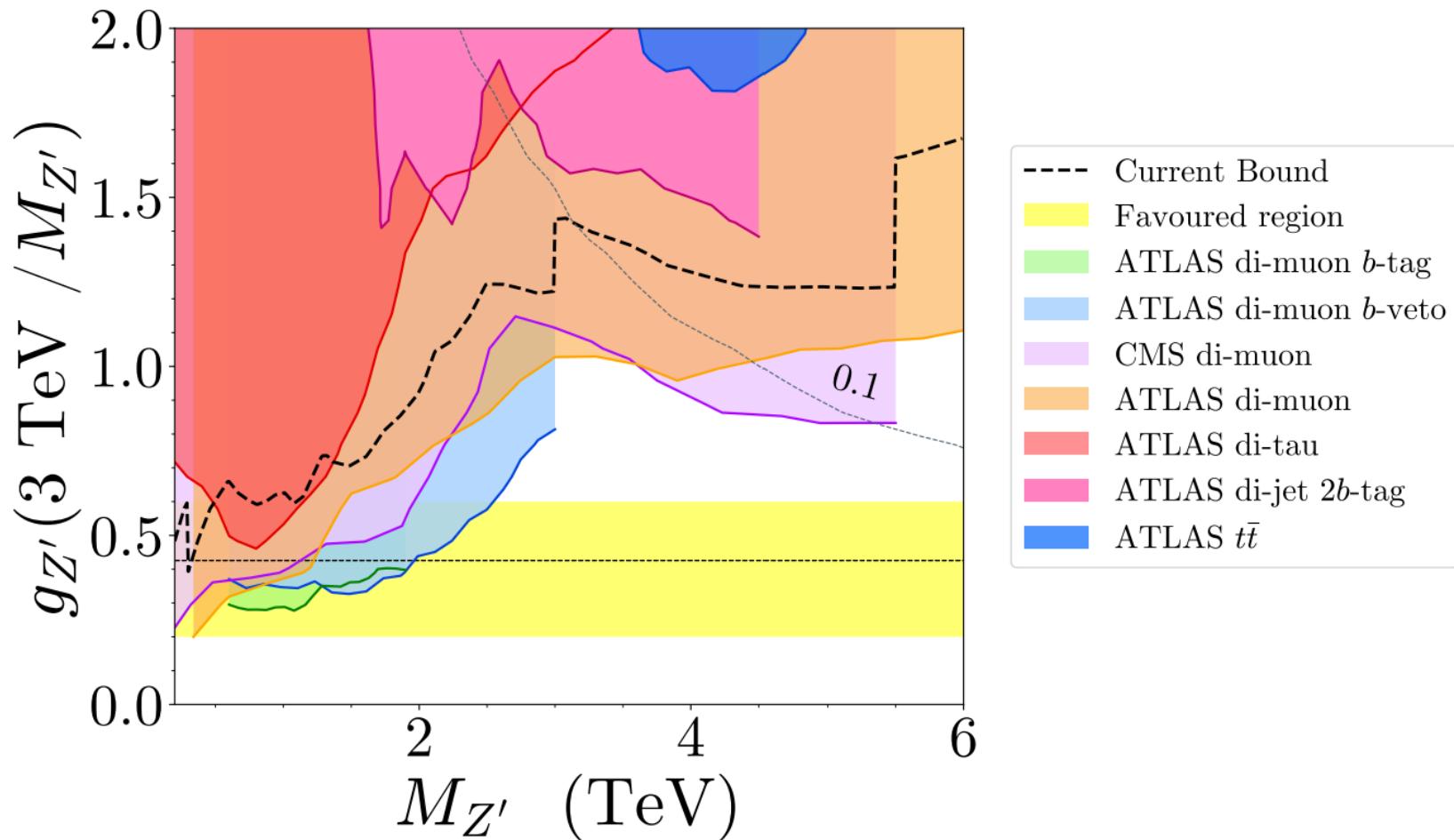


\*Belle reports only upper limits.  
We calculate BF ourselves

**Overall compatibility is good  $\chi^2/\text{ndf} = 4.3/4$**



# $Y_3$ HL-LHC sensitivity<sup>7</sup>



<sup>7</sup>BCA, Banks, 2111.06691

# Scalar LQ<sup>8</sup>: eg $S_3 \sim (\bar{3}, 3, 1/3)$

