Generalized Symmetries in Particle Physics

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The notion of "global symmetry" has been generalized in various directions in the last 10 years

This has had a huge impact on formal QFT research from 2014-present

What can we learn from generalized symmetries about particle physics?

This will not be a comprehensive review - sorry for leaving many interesting things out.

My aims are:

1. To introduce the various kinds of generalized symmetry studied in formal theory
2. To select examples that illustrate things people have tried to do with generalized symmetries in particle physics (so far...)

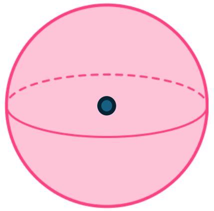
Some reviews / lecture notes:

More pheno: Reece, 2304.08512; Brennan, Hong, 2306.00912

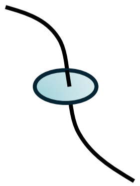
More formal: Schafer-Nameki, 2305.18296; Shao, 2308.00747 ;Bhardwaj et al, 2307.07547; Iqbal, 2407.20815

Plan

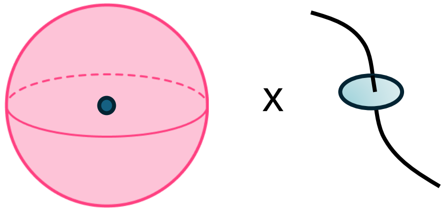
A. Ordinary symmetries via "topological defects"



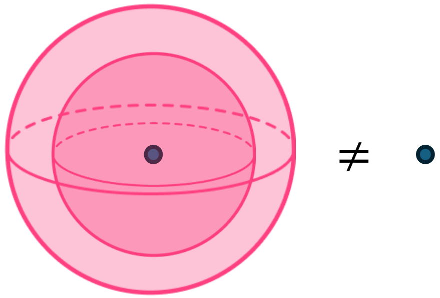
B. Generalization 1. Higher-form symmetries



C. Generalization 1b. Higher-group symmetries



D. Generalization 2. Non-invertible symmetries



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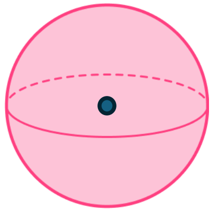
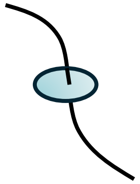


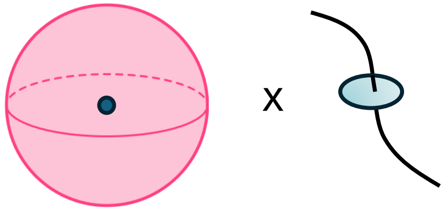
B. Generalization 1. Higher-form symmetries

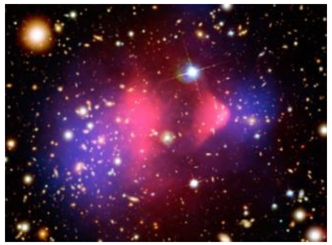
- Applications: axion quality from 5d; discrete gauge symmetries



C. Generalization 1b. Higher-group symmetries

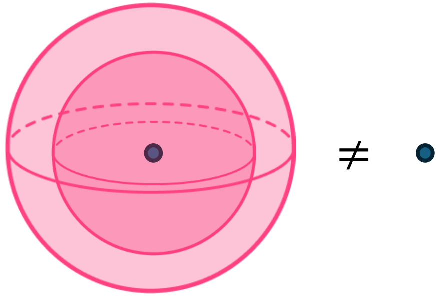
- Application: topological portal to dark sector

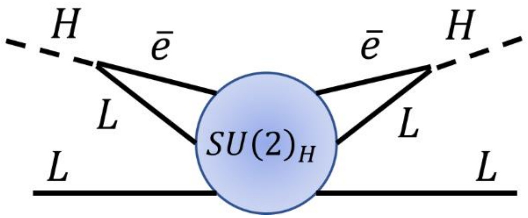




D. Generalization 2. Non-invertible symmetries

- Application: tiny ν masses; new flavour symmetries





1. Symmetries as Topological Operators

Gaiotto, Kapustin, Seiberg, Willett, 1412.5148

'Ordinary Symmetries'

We are used to deriving symmetries **from an action**

- Example: $L = \partial\phi\partial\phi^\dagger - V(\phi\phi^\dagger)$, symmetry $\phi(x) \to e^{i\alpha}\phi(x)$, $\alpha \in 2\pi\mathbb{R}/\mathbb{Z}$

‘Ordinary Symmetries’

We are used to deriving symmetries **from an action**

Conserved Current

- For continuous symmetries, then derive conserved current from L using **Noether’s theorem**
- $j^\mu = \delta\phi \cdot \frac{\partial L}{\partial(\partial_\mu \phi)}$ satisfies $\partial_\mu j^\mu = 0$ on EOMs
- Example: $j^\mu = i\left[\left(\partial^\mu \phi^\dagger\right)\phi - \phi^\dagger(\partial^\mu \phi)\right]$

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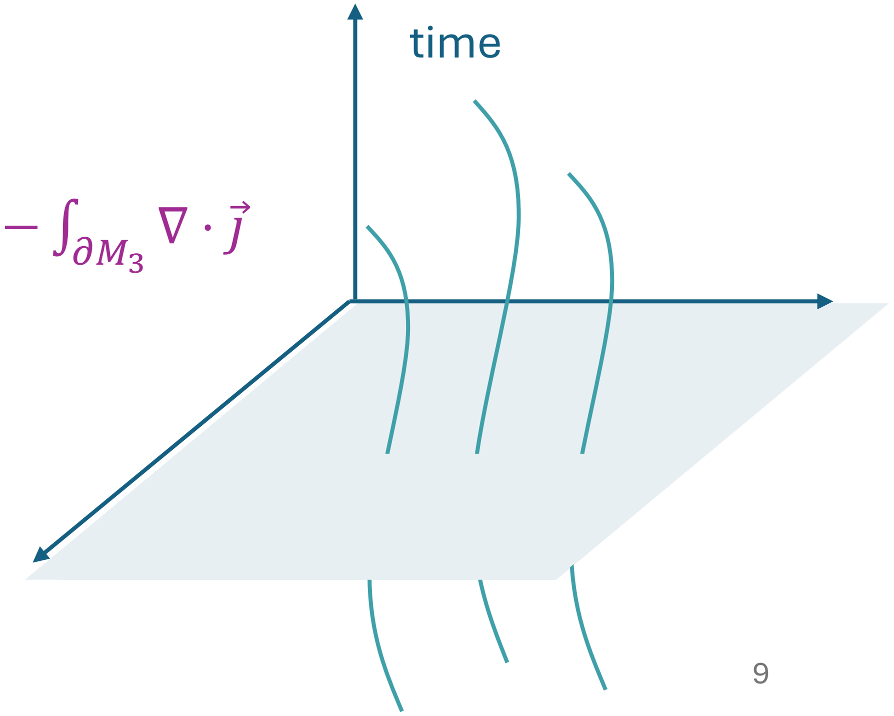
Conserved Charge

- From j^μ we construct a conserved charge by integrating
- $Q = \int_{\text{Spatial } M_3} j^0$, conserved up to bdy term: $\dfrac{dQ}{dt} = \int_{M_3} j^0 = -\int_{\partial M_3} \nabla \cdot \vec{j}$



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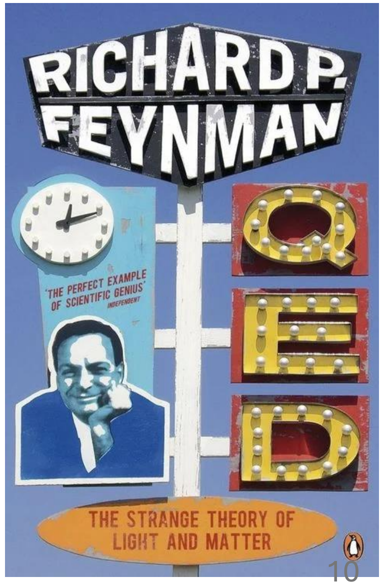
Gauging

- Introduce gauge field A_μ, and couple via $S = \int_{M_4} A_\mu j^\mu$
- Our Abelian Example: $\phi \to e^{i\alpha(x)}\phi,\ A_\mu \to A_\mu + \partial_\mu \alpha(x)$



'Ordinary Symmetries' via Differential Forms

Recall: differential forms are totally antisymmetric covariant tensors

We are used to deriving symmetries **from an action**

Conserved Current

- For continuous symmetries, then derive conserved current from L using **Noether's theorem**
- $j^\mu = \delta\phi \cdot \dfrac{\partial L}{\partial(\partial_\mu \phi)}$ satisfies $\partial_\mu j^\mu = 0$ on EOMs
- Defines a 1-form $j = j_\mu dx^\mu$, equivalently a 3-form $\star j = \epsilon_{\mu\nu\rho\sigma} j^\mu dx^\mu dx^\nu dx^\rho$ s.t. $d \star j = 0$

Conserved Charge

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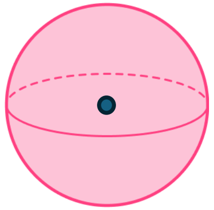
- $Q(M_3) = \int_{M_3} \star j$ on *any* closed M_3. "Conserved" $\Leftrightarrow$ "is topological", $Q(M_3 + \partial Y_4) = Q(M_3)$

i.e. value is independent of small wiggles of M_3



Gauging

- Introduce gauge field A_μ, and couple via $S = \int_{M_4} A_\mu j^\mu$
- Gauge field is a 1-form $A = A_\mu dx^\mu$, coupling $S = \int_{M_4} A \wedge \star j$

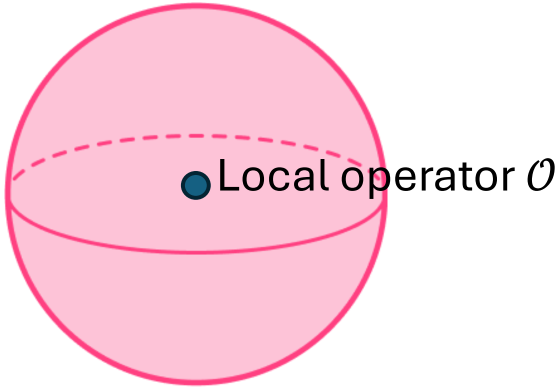
Symmetry Defect Operators

Gaiotto, Kapustin, Seiberg, Willett, 1412.5148

- The charge operator $Q(M_3)$, obtained from infinitesimal Noether procedure, lives in $\mathrm{Lie}\,(G)$
- Exponentiate it to get group elements:

$$U_{g=e^{i\alpha}}(M_3) \coloneqq \exp i\alpha Q(M_3) = \exp i\alpha \int_{M_3} \star\, j$$

Action on operators



$$U_g(M_3)\; O(x) = e^{i\alpha q_O}\; O(x)$$

If x inside M_3

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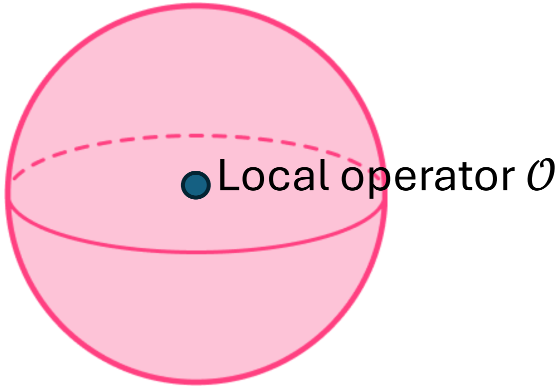
Key properties

1. $U_g(M_3)$ are all topological (“wiggle-independent”) b/c $d \star j = 0$
2. The algebra of these topological operators is a **group**
3. The $U_g(M_3)$ act on local ops: linking between 3-mfd and point.

Action on operators



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Symmetry Defect Operators

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Suggests of an abstract, action-free definition of symmetry directly in terms of defect operators

Global Symmetries = Topological Operators

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2. The algebra of these topological operators is a **group**
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- For continuous symmetries, the key **topological** property was guaranteed by the existence of a current ($\star\, j$) = a closed, differential form-valued operator

- But naturally **works for discrete symmetries too** (where cannot use Noether b/c no cts current); directly define $U_g(M_3)$ for e.g. $g \in \mathbb{Z}_N$

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3. ~~The $U_g(M_3)$ act on local operators~~

Both properties 2. and 3. can be **generalized!**

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Generalized Symmetries

With this "topological defect" picture for symmetries, we can generalize in different directions:

1. Higher-form symmetries — *Link symmetries with extended objects!*
2. Higher-group symmetries
3. Non-invertible symmetries — *Symmetries can form a richer algebra than a group!*

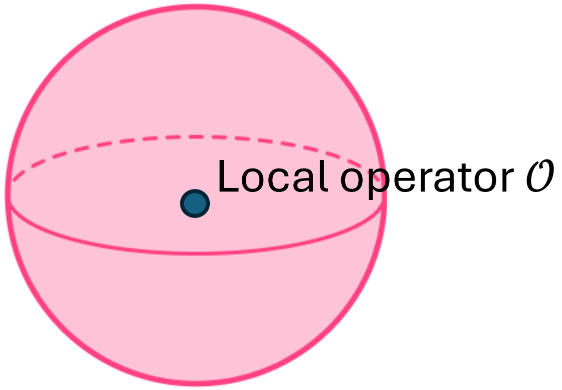
Generalization 1.
Higher Form Symmetries

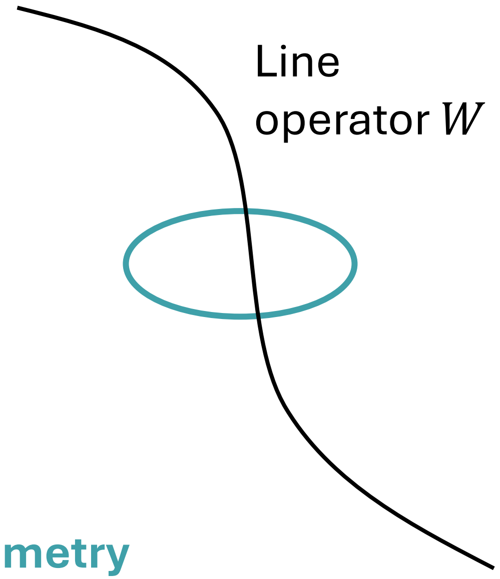
Gaiotto, Kapustin, Seiberg, Willett, 1412.5148

From 0-Form to 1-Form Symmetries

Ordinary (henceforth "0-form") symmetry: charged objects = local operators (0-dimensional)

Generalize to 1-form symmetry: charged objects = line operators (1-dimensional)





Ordinary "0-form" symmetry

- Top ops $U_g(M^{d-1})$ link points (0d)
- Current $J = \star\, j^{(1)}$ is a closed $d-1$ form (if cts.)
- Background g. field is a 1-form $A \mapsto A + d\alpha$
- Minimal coupling $S = \int_{M_4} A^{(1)} \wedge \star\, j^{(1)}$

1-form symmetry

- Top ops $U_g(M^{d-2})$ links with **lines** (1d)
- Current $J = \star\, j^{(2)}$ is a closed $d-2$ form (if cts.)
- Background g. field is a 2-form $B \mapsto B + d\Lambda^{(1)}$
- Minimal coupling $S = \int_{M_4} B^{(2)} \wedge \star\, j^{(2)}$

From 0-Form to Higher-Form Symmetries

- Higher p-form symmetry: charged objects = extended p-dimensional operators
- Current that we integrate is $J = \star j$, where $j = j_{\mu_1 \ldots \mu_{p+1}}$ is a $d - (p+1)$ form (if cts.)

[Being a "form" means $j_{\mu_1 \ldots \mu_{p+1}}$ is totally antisymmetric in exchanging indices]

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- E.g. for a $U(1)$-valued p-form symmetry, defect operator is

$$U_{g=e^{i\alpha}}\left(M_{d-(p+1)}\right) = \exp\left(i\alpha \int_{M_{d-(p+1)}} \star\, j\right)$$

  which acts on p-dimensional extended operators (the objects which can carry charge)

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- The defect operator is "topological" i.e. is a symmetry iff $dJ = d\star j = 0$. In components

$$\partial_\mu j^{\mu\mu_1 \dots \mu_p} = 0$$

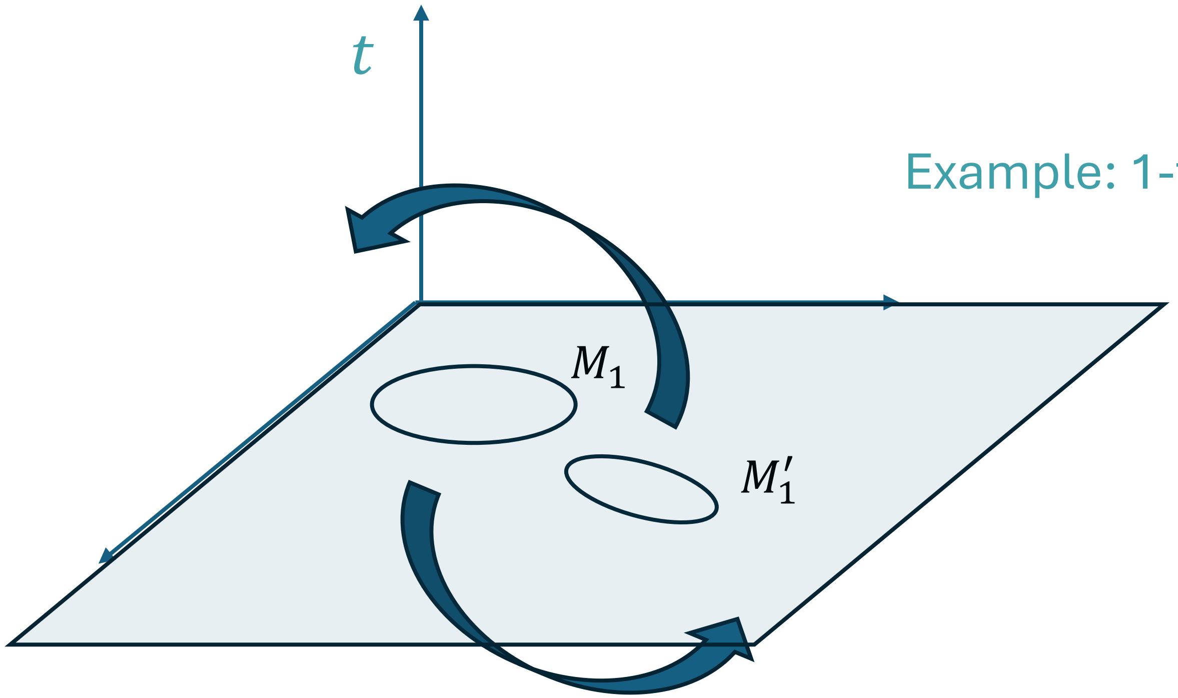
[just $\partial_\mu j^\mu = 0$ in familiar 0-form case]

Higher-Form Symmetries are Abelian

- Any surfaces $M_{k\leq d-2}$ and $M'_{k\leq d-2}$ can be moved around eachother

  [here we think in the Hamiltonian picture, with time-like foliation…]

- $\therefore$ no well-defined ordering $\therefore$ $U_g(M_{k\leq d-2})$ and $U_{g'}(M'_{k\leq d-2})$ must commute!



Example: 1-form symmetries in 3d

Higher form symmetries are **not** deduced from variation of the action!

(Lagrangian density is a **local operator** – its variation naturally yields a 1-form)

So, how do we find them?

Higher-form symmetries: how to find them?

Continuous higher form symmetry

- Need to identify co-closed p-form operators j, i.e. satisfying $d \star j = 0$, for $p > 1$

Example: Maxwell theory (free photons) in 4d, $S = \int da \wedge\star\, da$, has two 1-form symmetries!

Maxwell has two closed 2-form operators, $j^{(e)} = f = da$ and $j^{(m)} = \star f$ $\quad [\,(\star f)^{\rho\sigma} = \epsilon^{\mu\nu\rho\sigma} f_{\mu\nu}\,]$

Conservation is closure condition:

$d \star j^{(e)} = d \star f = 0$ b/c no charged matter $\qquad$ $d \star j^{(m)} = df = 0$ by Bianchi id

Topological operators:

$$U_g^{(e)}(M_2) = \exp\left(i\alpha \int_{M_2} \star f\right) = \exp\left(i\alpha \int_{M_2} \mathbf{E}\cdot \mathbf{dS}\right), \qquad U_g^{(m)}(M_2) = \exp\left(i\alpha \int_{M_2} f\right) = \exp\left(i\alpha \int_{M_2} \mathbf{B}\cdot \mathbf{dS}\right)$$

These act on (i.e. link with) Wilson line operators $L_q(\gamma) = e^{i\oint_\gamma A}$, i.e. worldlines of non-dynamical heavy charge, or ‘t Hooft lines (magnetic version)

$$U_{e^{i\alpha}}^{(e)}(M_2) \cdot L_q(\gamma) = e^{i\alpha\,\mathrm{Link}(M_2,\gamma)} L_q(\gamma)$$

Higher-form symmetries: how to find them?

Continuous higher form symmetry

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Non-Example: Yang-Mills in 4d, $\boldsymbol{S = \int \mathrm{Tr}\, f \wedge\star f}$

For $U(1)$, Maxwell eqn $d \star f = 0$ and Bianchi $df = 0$

For $SU(N)$, Yang-Mills equation $Df \sim (d + a\wedge)f = 0$, no closed 2-form! Likewise $D \star f = 0$

So Yang-Mills, in contrast to Maxwell theory, does not have continuous 1-form symmetries

Higher-form symmetries: how to find them?

Discrete higher form symmetry

- Not even a current! Instead, look directly for the charged objects it might act on

Example: $\boldsymbol{SU(N)}$ Yang-Mills in 4d $\boldsymbol{S = \int \mathrm{Tr}\, f \wedge \star f}$

$\mathbb{Z}_N$ "centre" 1-form symmetry acts on Wilson lines that cannot be screened by local operators

- The 1-form symmetry distinguishes global form of otherwise identical gauge groups, e.g.

$$G = SU(N)\text{: 1-form symmetry is } \mathbb{Z}_N^{(e)}, \qquad G = \frac{SU(N)}{\mathbb{Z}_N}\text{: 1-form symmetry is } \mathbb{Z}_N^{(m)}$$

- Similarly, there are **4 different versions of SM gauge group** with different 1-form symmetries

c.f. Tong, 1705.01853 etc

- Application: 4d $SU(2N)$ Yang-Mills at $\theta = \pi$ has a mixed anomaly involving $\mathbb{Z}_N^{(e)}$ 1-form symmetry and parity. Used to prove the YM vacuum at $\theta = \pi$ is non-trivial.

Gaiotto, Kapustin, Komargodski, Seiberg, 1703.00501

Higher-form symmetries: how to break them?

Harder than for ordinary symmetries!

- They do not "see" local operators in the Lagrangian, so cannot break via "small irrelevant operators" as for 0-form accidental symmetries

Instead, we break them by **introducing new degrees of freedom**

E.g. going from Maxwell → QED

$$d \star j^{(e)} = d \star f = \rho_e(x) \neq 0 \text{ anymore!}$$

This breaks the $U(1)^e$ 1-form symmetry

[but not $U(1)^m$]

Axion Quality from 1-Form Symmetry

Craig, Kongsore 2408.10295

- Spacetime with one extra compact dimension $M_5 = \mathbb{R}^{1,3} \times S^1_R$
- With extra 5d $U(1)$ gauge field C, action $S = \int_{M_5} -\frac{1}{2g^2} dC \wedge\star (dC) + \frac{N}{8\pi^2} C \wedge \mathrm{Tr}\,(G \wedge G)$
- Axion is the zero-mode of 5d g.f. along the circle i.e. it is a Wilson line of the extra-dim theory

$$a = \int_{S^1} C$$

Naturally periodic $a \sim a + 2\pi f$, where $f_a = 1/g\sqrt{2\pi R}$

- Remember 1-form symmetries act on e.g. Wilson lines $L_q(\gamma) = e^{i\oint_\gamma C}$!
- 5d electric 1-form symmetry for $C \longrightarrow$ shift symmetry for a (+ 4d electric 1-form symmetry)

$$U^{(e)}_{e^{i\alpha}}(M_{3=5-2}) \cdot L_q(\gamma) = e^{i\alpha \mathrm{Link}(M_2,\gamma)} L_q(\gamma), \quad \xrightarrow{\text{dim reduction}} \quad a \mapsto a + \alpha f_a$$

New insight:

Axion potential generated by *explicit breaking of the 5d 1-form symmetry*.

Higher-form symmetries are **harder to break**! Tightly controlled axion potential & quality

Axion Quality from 1-Form Symmetry

Craig, Kongsore 2408.10295

New insight:
Can classify all the ways of generating axion potential (from 5d) via the 1-form symmetry

A Symmetry Breaking Scorecard

	Current Equation	Remnant Symmetry	Potential
Electrically Charged Matter	$\mathrm{d}J_e = j_{\text{matter}}$	$\mathbb{Z}_q^{(1)}$	$V \simeq \frac{(m_{5D}R)^2}{(2\pi R)^4} e^{-m_{5d}R}\cos(q\theta)$
Gauging Two-Form Magnetic	$\mathrm{d}J_e = \frac{M}{2\pi}\mathrm{d}K$	$\mathbb{Z}_M^{(1)}$	$m_\theta = \frac{M}{2\pi} g_4\, e_{K,4}$
Gauging One-Form Electric	$kJ'_e = \frac{1}{e_B^2}\mathrm{d}\star\mathrm{d}B$	$U(1)^{(1)}$	$m_{\tilde{A}} = k\, e_{B,4}\, f$
ABJ Term	$\mathrm{d}J_e = \frac{N}{8\pi^2}\mathrm{Tr}\left[G\wedge G\right]$	$\mathbb{Z}_N^{(1)}$	$V \simeq -\Lambda_{\text{QCD}}^4\cos(N\theta)$

From Marius Kongsore's BSM Forum @ CERN, 26/9/2024

Some speculation

While this 5d axion story is in a sense already known, it is an example of a general statement:

p-form symmetry in $(4+p)$-dimensions → 0-form symmetry in 4d.

[very well known in string theory]

Other PP applications? E.g. can we apply to the hierarchy problem?

Ack! Remember, higher-form symmetries must be Abelian…

More higher-form speculation

Work in progress with Markus Dierigl

Discrete gauge symmetries in 4d (e.g. to explain neutrino mass and mixings) generically lead to **higher-form discrete global** symmetries

Why?

- They predict extended operators e.g. strings which cannot be shrunk away because they carry a quantized topological charge. A higher-form symmetry measures this charge.

So what?

- What happens when I throw such strings into a black hole? Cannot radiate the topological charge!
- This is the "**swampland cobordism conjecture**" McNamara, Vafa, 1909.10355
- Must break the global higher-form symmetry!
- We saw this was hard, but that it can be done via **dynamical extended objects**!

E.g. cosmic strings, monopoles, …

String theory examples:
Montero, Vafa, 2008.11729

More higher-form speculation

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Discrete gauge symmetries in 4d (e.g. to explain neutrino mass and mixings) generically lead to **higher-form discrete global** symmetries

- These "unshrinkable" objects are classified by "**bordism groups**":

2-form symmetry 1-form symmetry

JD, Gripaios, Lohitsiri, 2207.10700

p	1	2	3	4	5	6
$\tilde{\Omega}_p^{\mathrm{Spin}}(BS_3)$	$\mathbb{Z}/2$	$\mathbb{Z}/2$	$\mathbb{Z}/8 \times \mathbb{Z}/3$	0	0	0
$\tilde{\Omega}_p^{\mathrm{Spin}}(BA_4)$	$\mathbb{Z}/3$	$\mathbb{Z}/2$	$\mathbb{Z}/12$	0	$\mathbb{Z}/9$	$\mathbb{Z}/2$
$\tilde{\Omega}_p^{\mathrm{Spin}}(BQ_8)$	$(\mathbb{Z}/2)^2$	$(\mathbb{Z}/2)^2$	$\mathbb{Z}/8 \times (\mathbb{Z}/4)^2$	$\mathbb{Z}/2$	$(\mathbb{Z}/2)^2$	$\mathbb{Z}/2$
$\tilde{\Omega}_p^{\mathrm{Spin}}(B\mathrm{SL}(2,\mathbb{F}_3))$	$\mathbb{Z}/3$	0	$\mathbb{Z}/8 \times \mathbb{Z}/3$	$\mathbb{Z}/2$	$(\mathbb{Z}/2)^2 \times \mathbb{Z}/9$	$\mathbb{Z}/2$
$\tilde{\Omega}_p^{\mathrm{Spin}}(BD_{2n}),\ n$ odd	$\mathbb{Z}/2$	$\mathbb{Z}/2$	$\mathbb{Z}/8 \times \mathbb{Z}/n$	0	0	0
$\tilde{\Omega}_p^{\mathrm{Spin}}(BD_{2n}),\ n = 2^{k+1}$	$(\mathbb{Z}/2)^2$	$(\mathbb{Z}/2)^3$	$(\mathbb{Z}/8)^2 \times \mathbb{Z}/(2n)$	$(\mathbb{Z}/2)^2$	0	$\mathbb{Z}/2$

Predict strings, monopoles!

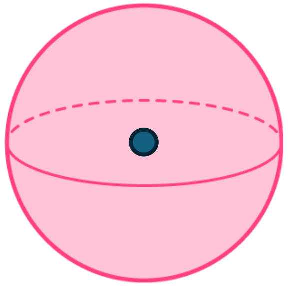
Non-trivial global anomalies

Generalization 1b.
Higher **Group** Symmetries

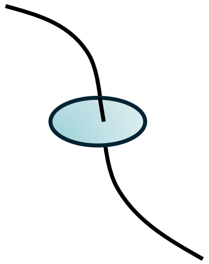
Kapustin, Thorngren, 1309.4721; Sharpe, 1508.04770; Cordova, Dumitrescu, Intriligator, 2009.00138

Higher Group Symmetries

- Higher form symmetries of different degrees can mix to form what is known as a “higher-group” structure in mathematics (described by *higher-bundles* with connection)
- Simplest case is **2-group symmetry**:



FUSE!



= **2-group** generalized symmetry

0-form symmetry

1-form symmetry

- 2-group connection consists of a pair of gauge fields with intertwined g. transformation:

1-form g. field: $A \mapsto A + d\alpha$

2-form g. field: $B \mapsto B + d\Lambda^{(1)} + \frac{n}{2\pi}\alpha dA$

$n \in \mathbb{Z}$, called the “Postnikov class”, that classifies the particular 2-group symmetry we have

Higher Group Symmetry is Quantized!

- There is a **2-group current algebra** analogous to the familiar anomalous current algebra:

$$\left\langle \partial^\mu j_\mu^{L,A}(x) j_\nu^{L,B}(y) - \delta(x-y) f^{ABC} j_\nu^C(y) \right\rangle \sim \left\langle n\, \delta^{AB} \partial^\rho \delta(x-y) j_{\rho\nu}^{(2)}(y) \right\rangle$$

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From anomaly matching to symmetry matching!

- 2-group class $n \in \mathbb{Z}$ ∴ cannot change continuously under any deformation, including RG
- … so, like an anomaly, it must match from UV to IR!
- This gives 2-group more power than 1-form and 0-form separately: cannot break one the 1-form symmetry without explicitly breaking the 0-form “flavour” symmetry at the same scale

[like non-abelian current algebra]

This is the “**2-group emergence theorem**” of Cordova, Dumitrescu, Intriligator 1802.04790

Used in Cordova, Koren 2212.13193 to study GUT embeddings of SM

Topological Portal to Dark Sector

JD, Greljo, Selimović, 2401.09528

We propose a **new portal** to the dark sector, that is a **topological effective interaction**:

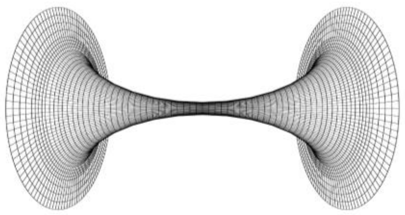
- Invariant under $SU(3)_L \times SU(3)_R \times SU(2)_D$ global symmetry; scale $E \sim 100\text{s MeV} \div \text{GeV}$
- $n \in \mathbb{Z}$ for consistency of EFT; c.f. Dirac monopole

$$S = n \int_{X_5} \mathrm{Tr}\,(g^{-1}dg)^3 \wedge \mathrm{Vol}_2$$

$$\rightarrow \frac{n}{16\pi^2 f_{D^2}} \int_{M_4} \left[\frac{1}{f_\pi^3} f_{abc}\, d\pi^a d\pi^b d\pi^c + \frac{1}{f_\pi}\left(\pi_0 + \frac{\eta}{\sqrt{3}}\right) F\right] \wedge d\chi_1 \wedge d\chi_2$$

Visible Sector

π, K, η, γ



Dark Sector

χ = DM pions on $S^2 = \frac{SU(2)}{U(1)}$

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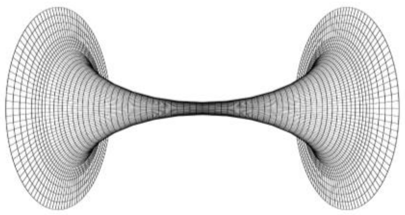


Unique QCD-like coset featuring topological portal!

		Portal			SIMP
p	1	2	3	4	5
$H^p(SU(2))$	0	0	$\mathbb{R}$	–	–
$H^p(SU(n)),\ n \geq 3$	0	0	$\mathbb{R}$	0	$\mathbb{R}$
$H^p(SU(2)/SO(2))$	0	$\mathbb{R}$	–	–	–
$H^p(SU(3)/SO(3))$	0	0	0	0	$\mathbb{R}$
$H^p(SU(4)/SO(4))$	0	0	0	$\mathbb{R}$	$\mathbb{R}$
$H^p(SU(n)/SO(n)),\ n \geq 5$	0	0	0	0	$\mathbb{R}$
$H^p(SU(2n)/Sp(2n)),\ n \geq 2$	0	0	0	0	$\mathbb{R}$

Topological Portal to Dark Sector

JD, Greljo, Selimović, 2401.09528

$$S = n \int_{M_4} \left[f_{abc}\, d\pi^a d\pi^b d\pi^c + \left(\pi_0 + \frac{\eta}{\sqrt{3}}\right) F \right] \wedge d\chi_1 \wedge d\chi_2$$

The topological portals
The 2 $\rightarrow$ 2 is more relevant

Contrast to SIMP DM:

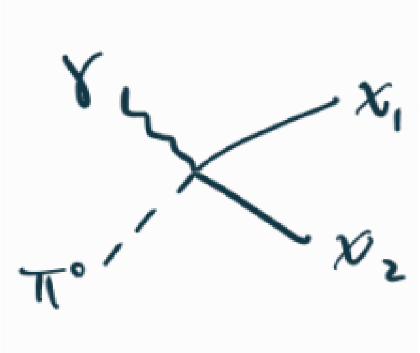
- WZW involving 5 dark pions gives 3 $\rightarrow$ 2 purely within DS
- additional portal needed for thermalisation

See Josef Pradler's talk

Topological Portal to Dark Sector

JD, Greljo, Selimović, 2401.09528

$$S = n \int_{M_4} \left(\pi_0 + \frac{\eta}{\sqrt{3}} \right) F \wedge d\chi_1 \wedge d\chi_2$$



- With indices: $\frac{n}{16\pi^2 f_\pi f_D^2} \epsilon^{\mu\nu\rho\sigma} e\pi^0 F_{\mu\nu} \partial_\rho \chi_1 \partial_\sigma \chi_2$
- Can explain DM relic abundance via $\pi_0 \gamma \leftrightarrow \chi_1 \chi_2$ freeze-out after QCD phase transition, for m_χ **up to few GeV**
- If small mass splitting $m_{\chi_2} - m_{\chi_1} > m_{\pi_0}$ then $\chi_2 \to \chi_1 \gamma \pi_0$, leaving χ_1 as relic DM

Remember differential forms are **antisymmetric**!

- So there is no corresponding "elastic channel" involving $\chi_1 \chi_1 \to \mathrm{SM}$
- Naturally explains why we haven't seen DM in direct/indirect detection

The Topological Portal Encodes 2-group Symmetry

JD, Lohitsiri, 2407.20340

QCD

$$S_{\mathrm{WZW}} = n \int_{M_4} \pi_0 F \wedge F$$

QCD + dark pions

$$S_{\mathrm{WZW}} = n \int_{M_4} \pi_0 F \wedge d\chi_1 \wedge d\chi_2$$

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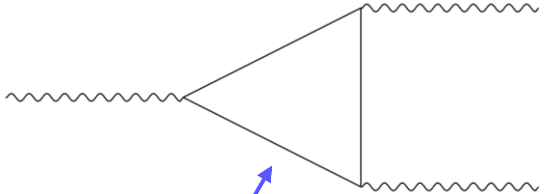
Matches anomalies in $SU(3)_L \times SU(3)_R$



Quarks in the UV theory

QCD + dark pions

$$S_{\text{WZW}} = n \int_{M_4} \pi_0 F \wedge d\chi_1 \wedge d\chi_2$$

But **no mixed anomaly** between $SU(3)$ and $SU(2)_D$

Quantized **WZW term without anomalies**??

(NEW!)

UV ??

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JD, Lohitsiri, 2407.20340

QCD

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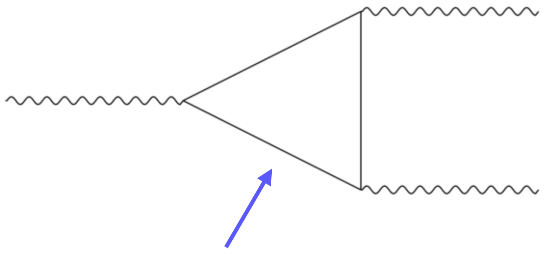


QCD + dark pions

$$S_{\text{WZW}} = n \int_{M_4} \pi_0 F \wedge d\chi_1 \wedge d\chi_2$$

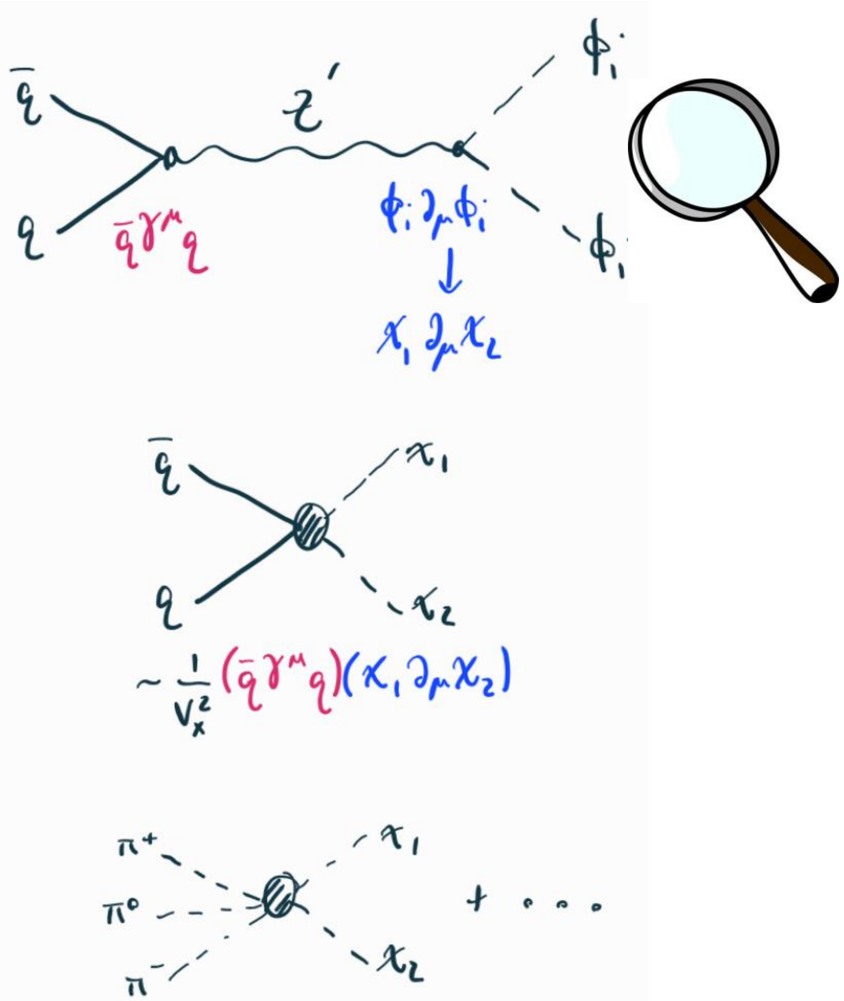
Generalized symmetries solve the puzzle:
There **is** a **2-group symmetry** to match!

Quick way:

- 1-form symmetry, $j^{(2)} = \star\, d\chi_1 \wedge d\chi_2$
- Minimal coupling $S_{\text{coup}} = \int_{M_4} B^{(2)} \wedge \star\, j^{(2)}$
- 2-group background gauge transformation
  $$\pi_0 \to \pi_0 + d\alpha \qquad B \mapsto B + n\alpha F$$
- $S_{\text{WZW}} + S_{\text{coup}}$ is invariant ∴ 2-group symmetry

The Topological Portal Encodes 2-group Symmetry

JD, Lohitsiri, 2407.20340



QCD + dark pions

$$S_{\mathrm{WZW}} = n \int_{M_4} \pi_0 F \wedge d\chi_1 \wedge d\chi_2$$

2-group matching then **guides us to the UV**:

- Non-abelian dark gauge group ruled out!
- Abelian dark gauge field **does work,** with very particular UV couplings
- 2-group, and top portal coefficient, is **quantized** so EFT matching is **tree-level exact!** (New!)

[Reminiscent of anomaly matching in 2d Schwinger model across bosonization duality]

Is this just a theoretical mechanism for dark matter, or can it be tested?

We already saw there is no signal in (in)direct detection experiments.

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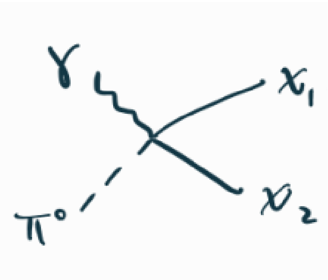
We already saw there is no signal in (in)direct detection experiments.

But

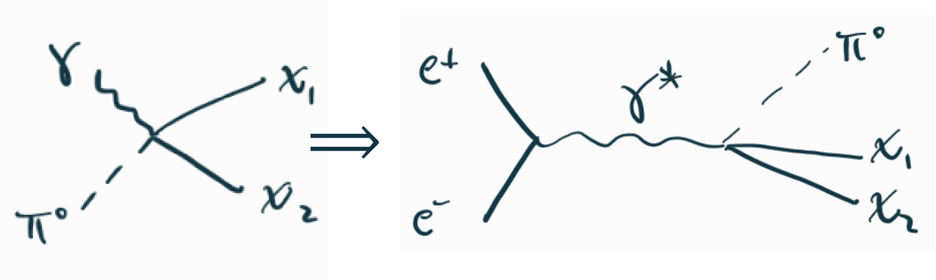


$\Longrightarrow$

and fitting relic abundance tells us $m_\chi \sim 3\ \mathrm{GeV}$…

A Novel Phenomenology

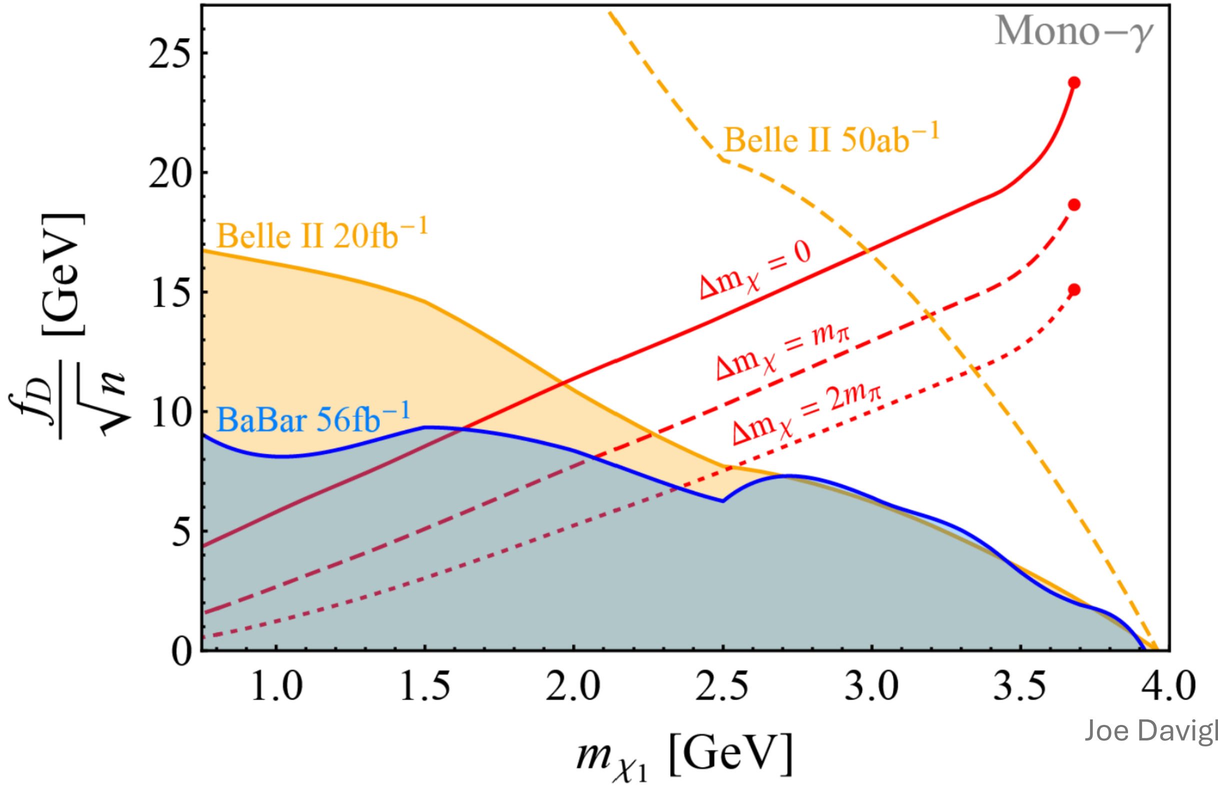
JD, Greljo, Selimović
2401.09528



$m_\chi \sim$ 3 GeV

Δm_χ	$\lesssim 1.7 m_{\pi^0}$	$\gtrsim 1.7 m_{\pi^0}$
Signature	$\pi^0 + \not{E}_T$	$\pi^0 + \not{E}_T + \mathrm{DV}(\pi^0 \gamma \not{E}_T)$

Final state π^0 reconstructed as photon if boosted to few GeV, so can recast $\gamma + \mathrm{Inv}$ searches for signature 1



Monophoton recast demonstrates **tremendous prospects at Belle II**

There is no data relevant to the DV region (currently veto-ed in these mono-photon searches)

If observe a signal, there is a definite prediction

$$\frac{\sigma(e^+e^- \to \eta \chi_1 \chi_2)}{\sigma(e^+e^- \to \pi_0 \chi_1 \chi_2)} = \frac{1}{\sqrt{3}}$$

smoking gun!

To be studied: complementary high-E stuff at LHC
(UV completion via 2-group is essential)

Generalization 2.
Non-Invertible Symmetries

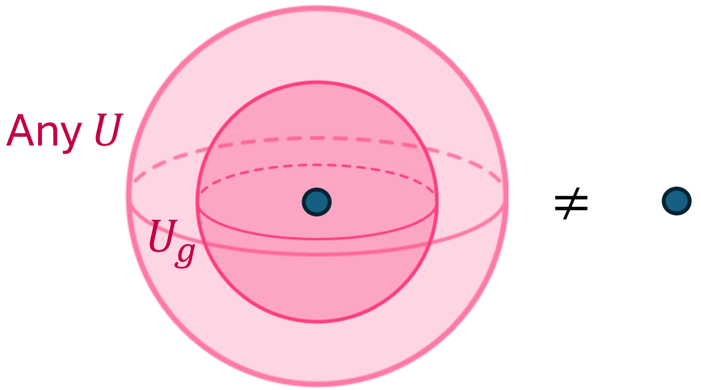
Origins in many old examples in condensed matter, CFT, e.g. Kramers-Wannier duality

Non-invertible Symmetries

- Generalize group multiplication law $U_g(M_3)U_{g'}(M_3) = U_{gg'}(M_3)$ to a "fusion algebra":

$$U_a(M_3)U_b(M_3) = \sum_c N_{ab}^c\, U_c(M_3)$$

- Each U_a need not have an inverse

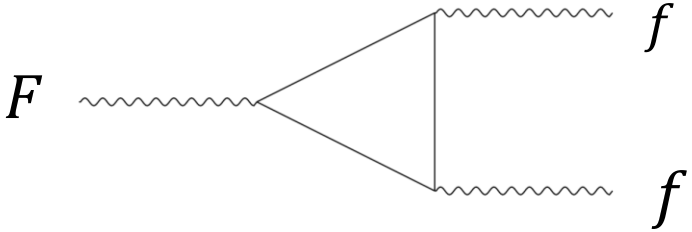


- Mathematically, these fusion categories are rigorous only in low dimensions

4d NIS from ABJ anomalies

Consider 0-form global $U(1)$ symmetry with Noether current j s.t. there is an ABJ anomaly

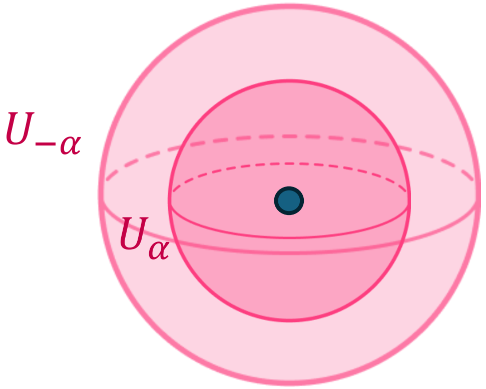
$$d \star j = \frac{1}{16\pi^2} f \wedge f$$



Naively, we lose topological property of our defect operators $U_\alpha(M_3) = e^{i\alpha \int_{M_3} \star j}$ b/c $d \star j \neq 0$

But, for $\alpha = p/q \in \mathbb{Q}$, can fix up by subtracting a *fractional Chern—Simons theory* on M_3

[I won't explain what this means, sorry!]



$$= \quad Z_{\text{fractional CS}}(M_3) \neq 1$$

Upshot: ∃ topological gauge invariant operators (a.k.a. symmetries) for each rational angle $\alpha = p/q$, but without inverses.

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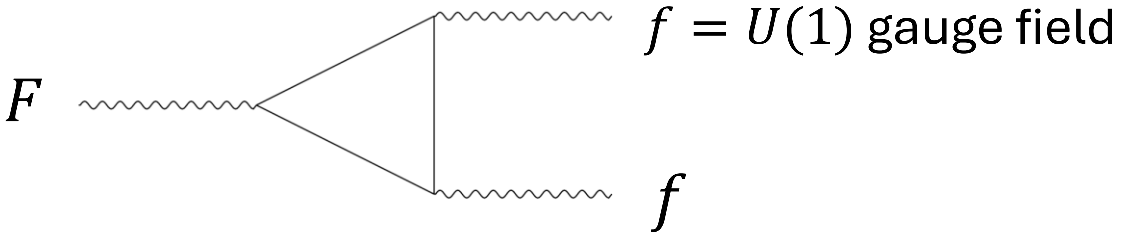


- These "remnant symmetries" remain after the ABJ breaks $U(1)$
  - Protects e.g. $U(1)_B$ in SM, despite the mixed anomaly with hypercharge!
- There is no analogous NIS remaining after an ABJ anomaly with a *non*-abelian f
- Simpler to see this distinction via selection rules on correlators:

$$\langle O \rangle \sim \int Da D\psi D\bar{\psi} e^{-S} O \to \int Da D\psi D\bar{\psi} \left(1 + i\alpha \underbrace{\int_{S^4} f \wedge f}_{=\,0 \text{ for abelian } f}\right) e^{-S} O = \langle O \rangle$$

[but $\neq 0$ if non-abelian]

- The action on local ops (therefore scattering observables) is just like an invertible symmetry

Tiny Neutrino Masses from NIS

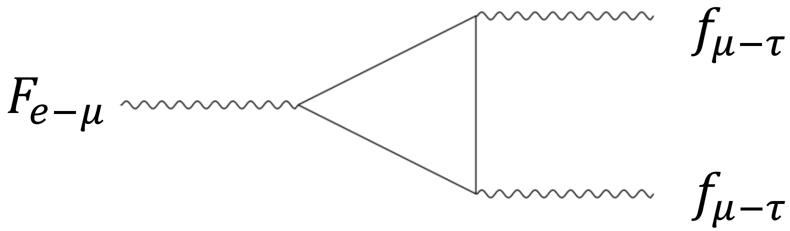
Córdova, Hong, Koren, Ohmori, 2211.07639

- This NIS can be emergent in the IR, upon breaking non-abelian $G \to U(1)$ with ABJ!
- Then effects breaking the NIS come from UV instantons ⇒ exponentially suppressed

Example: gauge lepton-flavoured symmetries for tiny neutrinos masses

UV $G = SU(3)_L$

IR $G = U(1)_{L_\mu - L_\tau}$



Neutrino Dirac mass gets generated by $SU(3)_H$ instantons

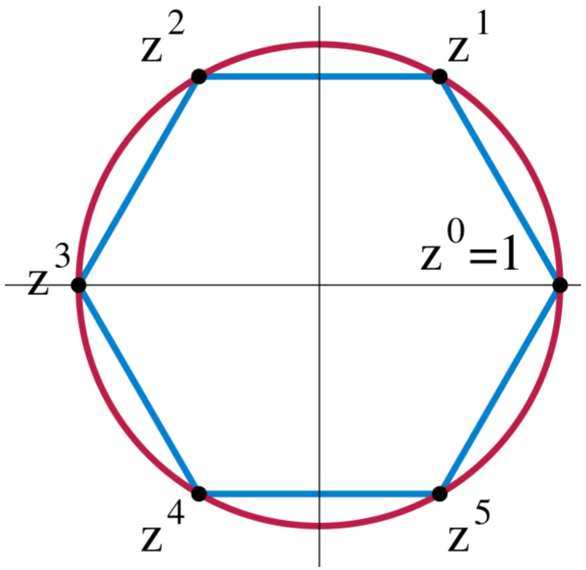
$$y_\nu = y_\tau e^{-\frac{8\pi^2}{g_H^2}} \ll y_\tau$$

Discrete NIS for Flavour

Kobayashi, Otsuka, 2408.13984
Kobayashi, Otsuka, Tanimoto, 2409.05270

- Type IIB string theory compactified on $T^2 \times T^2 \times T^2$
- Magnetic flux through torus has two consequences:
  1. Breaks $U(1)$ translations down to discrete $\mathbb{Z}_N$ subgroups
  2. Gives chiral fermion zero modes (index theorem), acted on by the $\mathbb{Z}_N$s
- Last step: gauge $\mathbb{Z}_2$ reflection: turns the $\mathbb{Z}_N$s into **non-invertible symmetries**!

Very similar in spirit to modular symmetry approach
See Ferruccio Feruglio's talk



Discrete $\mathbb{Z}_N$, gauge $\mathbb{Z}_2$ reflection
Charges q and $N-q$ are identified

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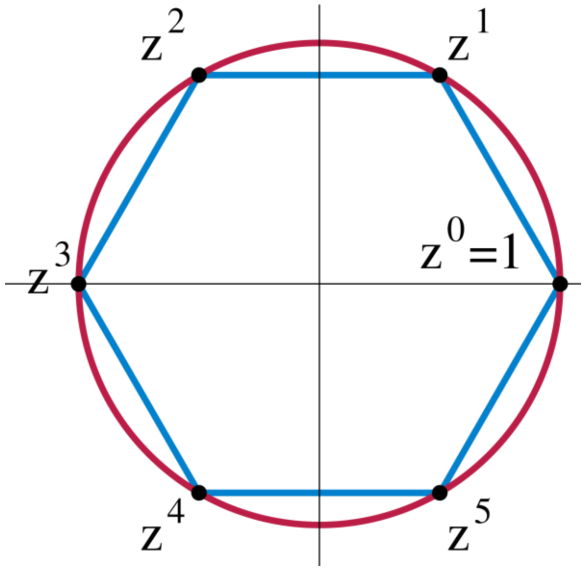

Discrete $\mathbb{Z}_N$, gauge $\mathbb{Z}_2$ reflection (not a subgroup)
Charges q and $N - q$ are identified

Selection rules for different $\mathbb{Z}_N$ give different "nearest-neighbour interaction" Yukawa textures.

A **new playground** for explaining quark and lepton mass and mixings

Example, $N = 5$

$$Y_D = \begin{pmatrix} a_D & a'_D & 0 \\ 0 & b_D & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR}, \qquad Y_U = \begin{pmatrix} a_U & 0 & 0 \\ 0 & b_U & c_U \\ 0 & c'_U & d_U \end{pmatrix}_{LR},$$

Conclusions

Generalized symmetries have already taught us a huge amount in Hep-TH and string theory

We are just beginning to find interesting examples of generalized symmetries in particle physics.

So far, lots of the PP applications are "reframings" of previously known phenomena e.g:

- ABJ anomalies ~ Non-invertible symmetry
- Operator-valued anomalies ~ 2-group symmetry

Dictionary for translating anomalies to new symmetries

- Instanton generated effects are small ~ protected by non-invertible symmetry
- Axion shift symmetry protected in 5d by gauge invariance ~ protected by 1-form symmetry

But there are also new BSM ideas coming out in 2024, e.g:

- Topological portals to dark matter manifest 2-group symmetry, exotic EFT matching
- Non-invertible symmetries for novel flavour textures

We want more applications of generalized symmetries that tell us something completely new!

Neutrino masses

Strong CP problem

Proton stability

ABJ anomalies

Non-invertible symmetry

Conjugacy class symmetries for the flavour puzzle

Global structure of SM gauge group

Axion quality problem

1-form symmetry

Defect predictions for discrete gauge theories

2-group symmetry

Unification patterns for SM gauge group

Topological Portals to Dark Matter

?

Subsystem symmetry

?

?

Not obvious how this can be applied in HEP