Long-distance contributions

from lattice QCD & QED

Antonin Portelli (The University of Edinburgh) / 06 December 2024 / DISCRETE 2024, Ljubljana, Slovenia

- Isospin-breaking and electromagnetic corrections to hadronic interactions
- Non-local matrix elements: $K \to \pi \ell^+ \ell^-$ decays
- Multi-hadrons interactions
- (Not $g - 2$)

Lattice field theory

- Strong interactions described by **Quantum Chromodynamics (QCD)**
- In a discrete and Euclidean space-time, **QCD becomes equivalent to a statistical system**
  KG Wilson, Phys Rev D 10(8) (1974)
- Physical observables can be evaluated **through Monte-Carlo simulations**

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathrm{D}U\, O[U]\, \det(M[U])\, e^{-S[U]}$$

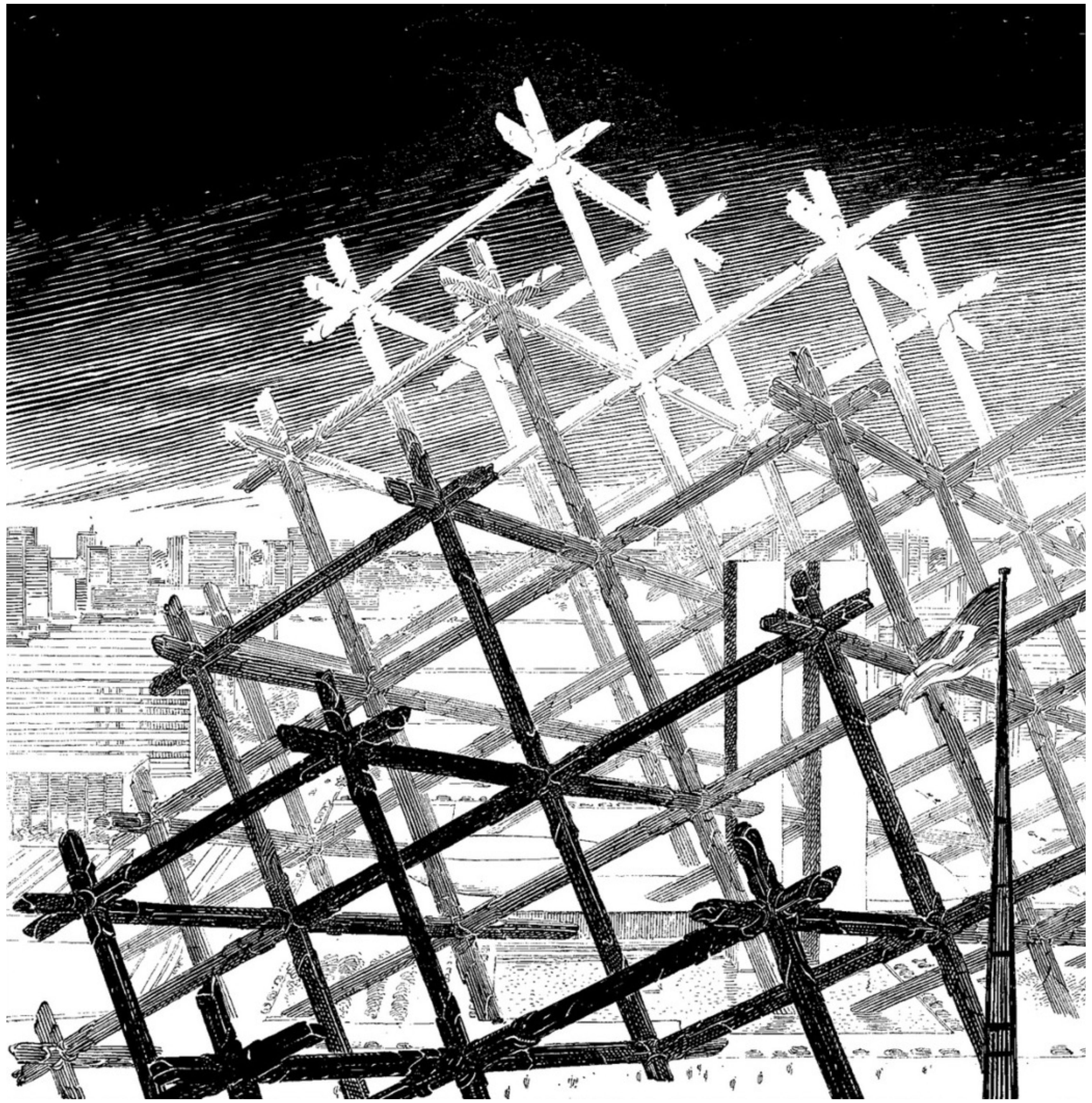


Image credit: Schuiten & Peeters, 1985, Casterman

Lattice field theory

- Lattice simulations have **billions of degrees of freedom**
- **They can potentially describe any strongly bound quantum field theory from first principles**
- Predictive capacity is directly bounded by
  - **available supercomputing power**
  - **algorithmic research progress**
  - **understanding of Euclidean field theory**

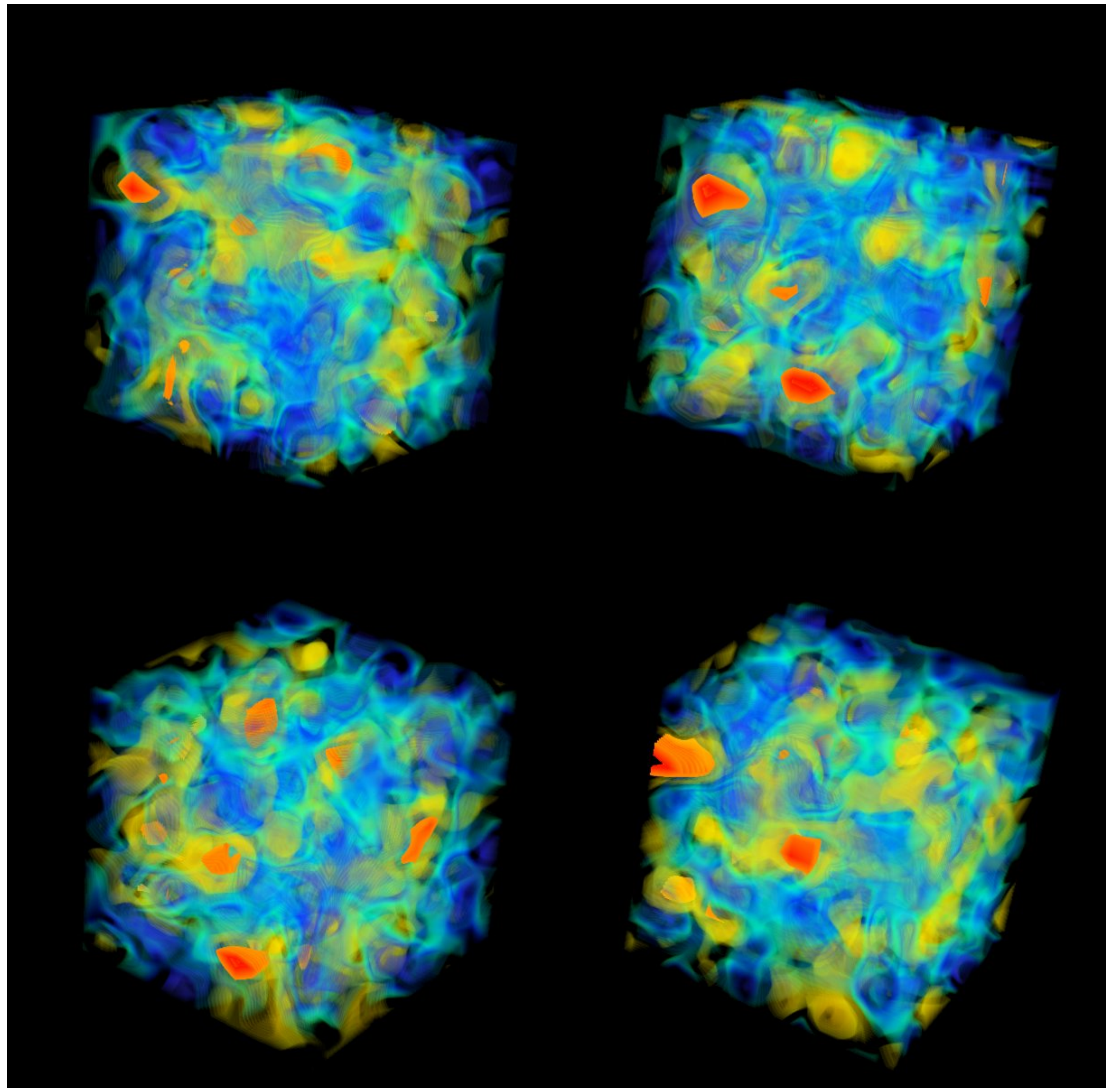


Image credit: University of Southampton

Isospin-breaking corrections to hadronic interactions

Image credit: Bergische Universität Wuppertal

Isospin-breaking (IB) corrections

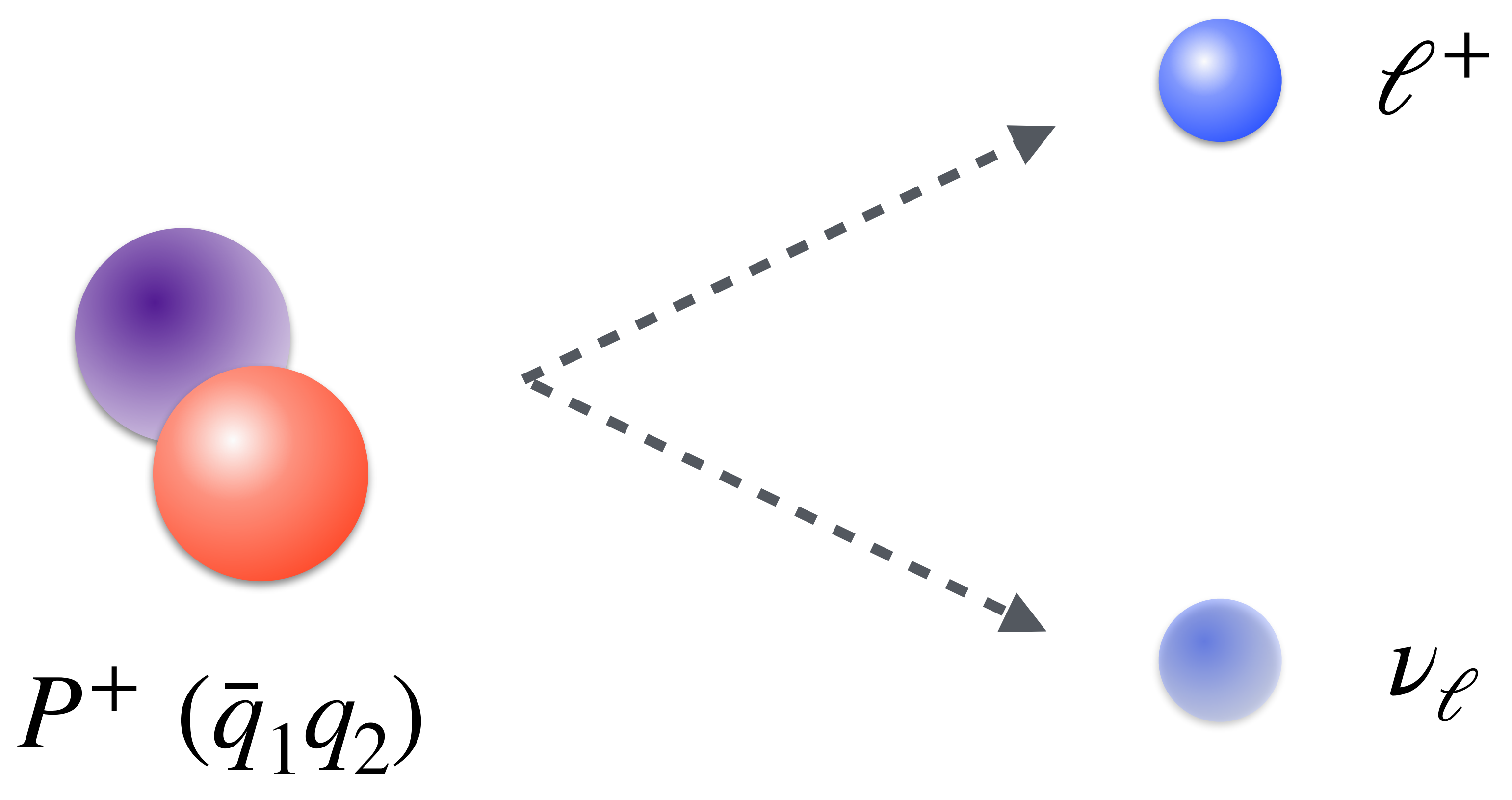
Beyond isospin symmetry

- Isospin symmetry assumed in most lattice calculations
- Violations generally expected to be $\mathcal{O}(1\%)$ of hadronic observables
- This is **highly relevant for** searches for new physics through precision measurements ($g-2$ & weak decays)
- **Main challenge** for lattice QCD: **adding QED**

IB corrections to weak decays

CKM matrix elements from leptonic decays

- Leptonic meson decay:
  quark pair W-boson annihilation
- Rate proportional to $|V_{q_1q_2}|^2$
- Allows to determine CKM matrix element from experimental rate…
- …but **needs a high-precision description of the hadronic dynamics**

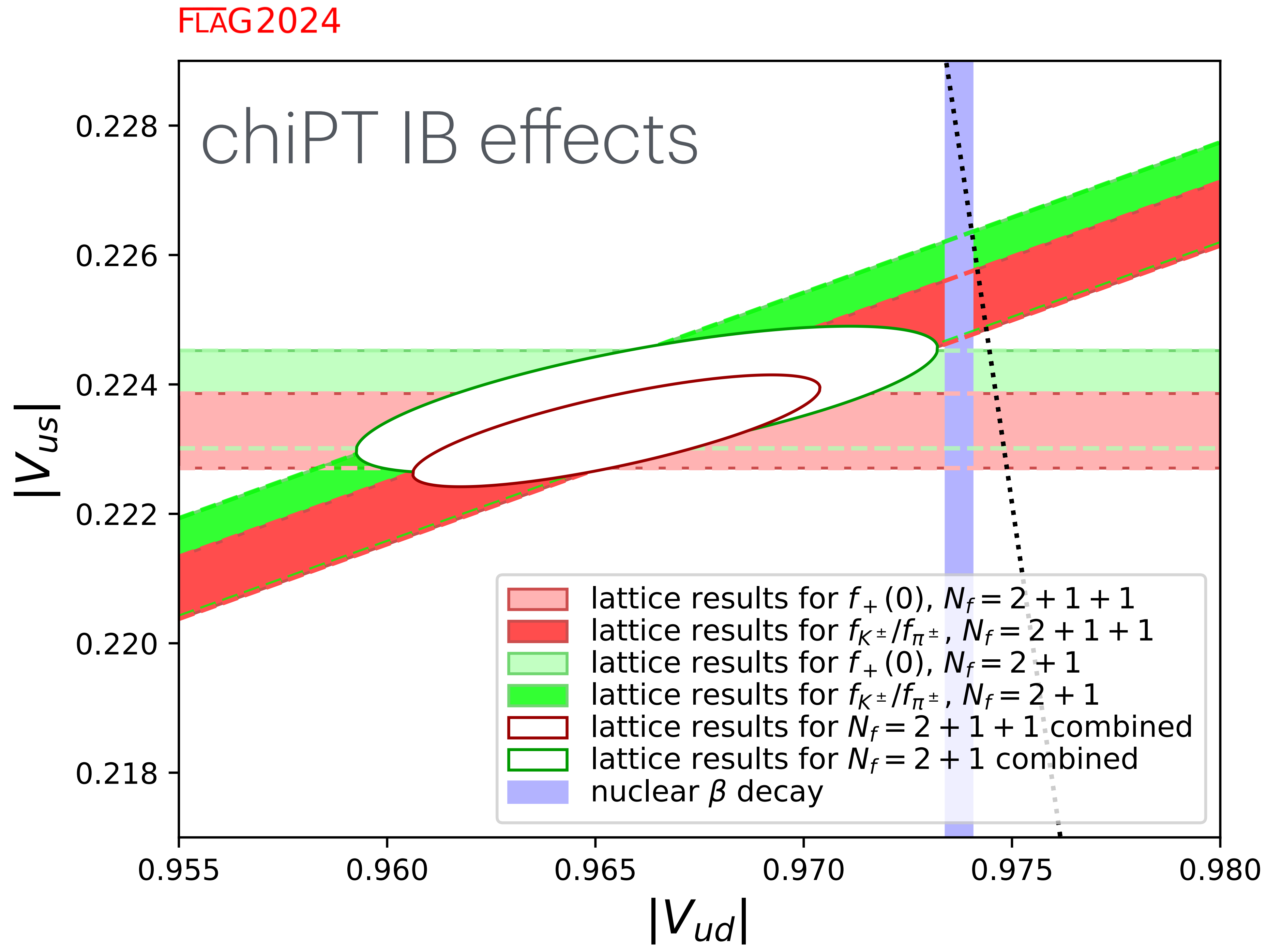


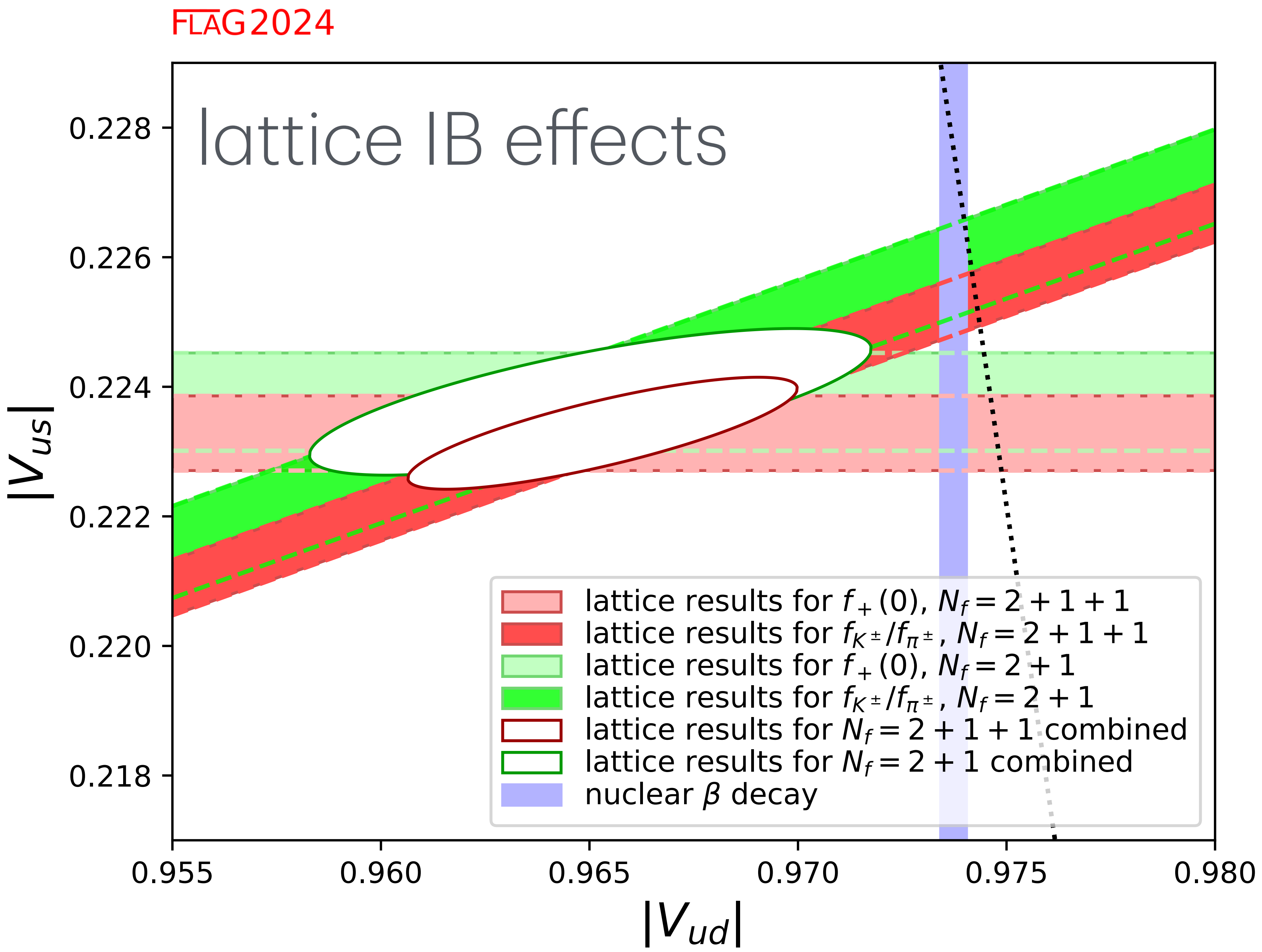
$$\Gamma(P^+ \to \ell^+ \nu_\ell[\gamma]) = \frac{G_F^2}{8\pi} f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 |V_{q_1q_2}|^2 (1 + \delta R_{\mathrm{P}})$$

Image credit: Matteo Di Carlo

IB corrections to weak decays

CKM first row





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- Neutron lifetime under scrutiny, but **IB corrections to leptonic decays also relevant**

IB corrections to weak decays

CKM charm and bottom coefficients

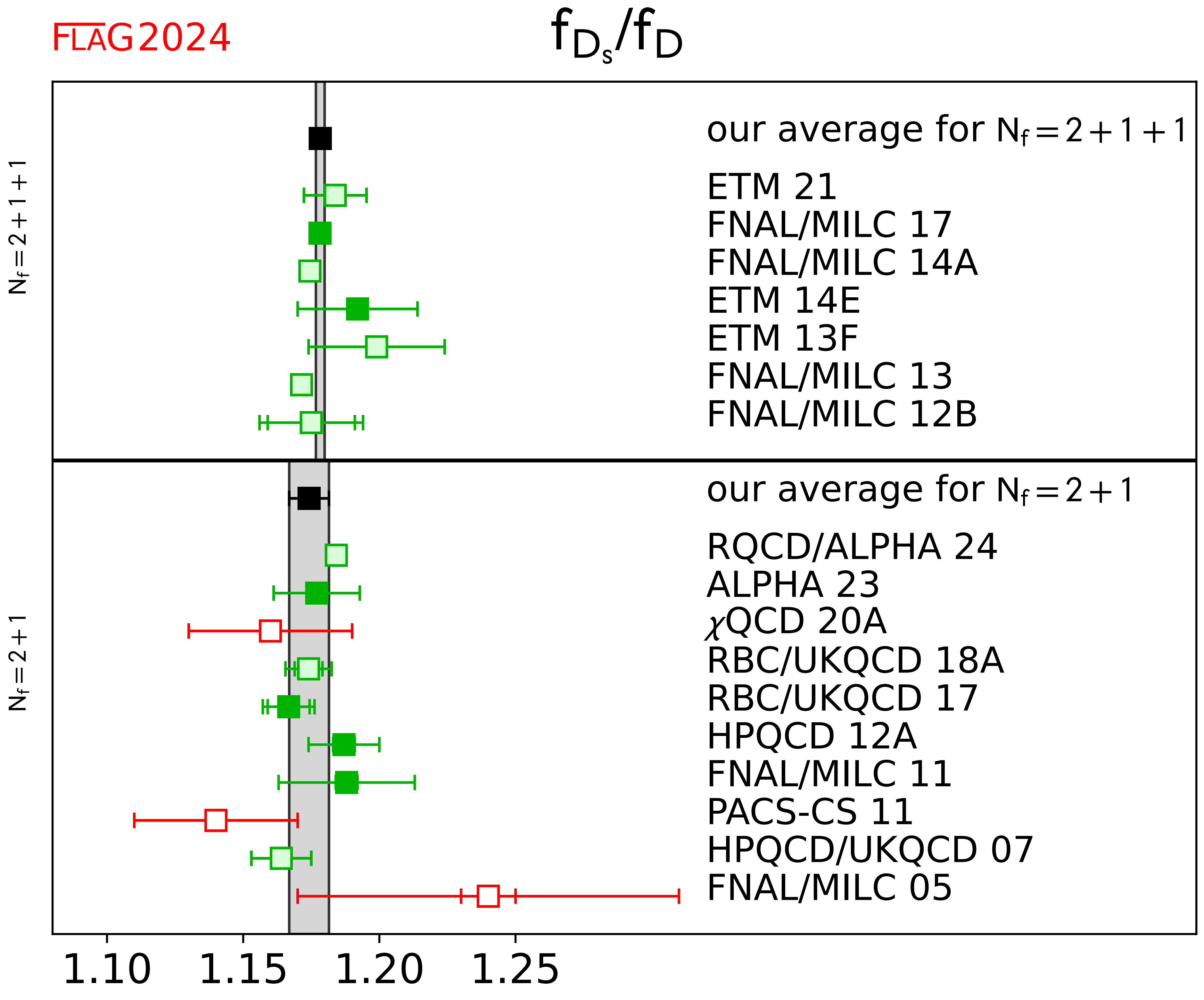

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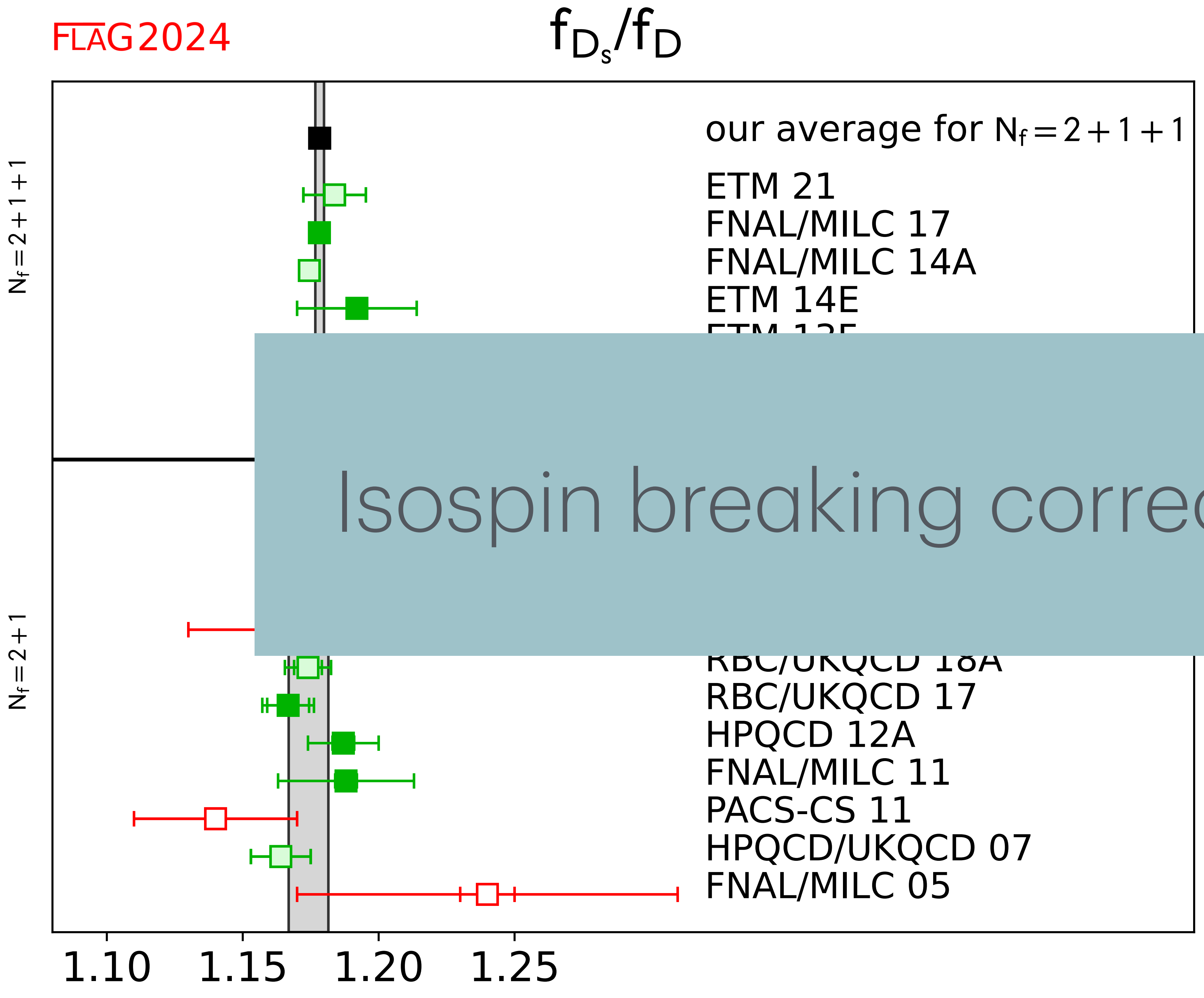
Table 30. Lattice inputs for decay constants $f_{B_{(s)}}$ and bag parameters $B_{B_{(s)}}$ in the SM. The current average of $f_{B_{(s)}}$ for $N_f = 2+1$ and $2+1+1$ are obtained from Refs. [150,213–216] and Refs. [212,217], respectively. The average of $B_{B_{(s)}}$ is obtained from Refs. [148,150,151]. $f_{B_{(s)}}\sqrt{B_{B_{(s)}}}$ is in units of MeV.

N_f	Input	f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
2+1+1	Current	188(3)	227(4)	1.203(0.007)
	5 yr w/o EM	188(1.5)	227(2.0)	1.203(0.0035)
	5 yr with EM	188(2.4)	227(3.0)	1.203(0.013)
	10 yr w/o EM	188(0.60)	227(0.80)	1.203(0.0014)
	10 yr with EM	188(2.0)	227(2.4)	1.203(0.012)
2+1	Current	192.0(4.3)	228.4(3.7)	1.201(0.016)
	5 yr w/o EM	192.0(2.2)	228.4(1.9)	1.201(0.0080)
	5 yr with EM	192.0(2.9)	228.4(2.9)	1.201(0.014)
	10 yr w/o EM	192.0(0.86)	228.4(0.74)	1.201(0.0032)
	10 yr with EM	192.0(2.1)	228.4(2.4)	1.201(0.012)
N_f	Input	$f_B\sqrt{B_B}$	$f_{B_s}\sqrt{B_{B_s}}$	ξ
2+1	Current	225(9)	274(8)	1.206(0.017)
	5 yr w/o EM	225(4.5)	274(4.0)	1.206(0.0085)
	5 yr with EM	225(5.0)	274(4.8)	1.206(0.015)
	10 yr w/o EM	225(1.8)	274(1.6)	1.206(0.0034)
	10 yr with EM	225(2.9)	274(3.2)	1.206(0.013)
N_f	Input	B_B	B_{B_s}	B_{B_s}/B_B
2+1	Current	1.30(0.09)	1.35(0.06)	1.032(0.036)
	5 yr w/o EM	1.30(0.045)	1.35(0.030)	1.032(0.018)
	5 yr with EM	1.30(0.047)	1.35(0.033)	1.032(0.021)
	10 yr w/o EM	1.30(0.018)	1.35(0.012)	1.032(0.0072)
	10 yr with EM	1.30(0.022)	1.35(0.018)	1.032(0.013)

Belle II Physics Book (2019)

IB corrections to weak decays

CKM charm and bottom coefficients



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Isospin breaking corrections critically needed in all cases

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	[illegible]	[illegible]	[illegible]	[illegible]0032)
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IB corrections to weak decays

First physical *K* & *π* leptonic decay calculation

- First calculation at the physical point of **IB corrections to K & π leptonic decay rate ratio**
  P Boyle, **AP**, *et al.* JHEP 02 (2023)

$$\delta R_{K\pi} = -\,0.0086(3)_{\text{stat.}}\left({}^{+11}_{-4}\right)_{\text{fit}}(5)_{\text{disc.}}(5)_{\text{quench.}}(39)_{\text{vol.}}$$

- Largely based on the RM123S formalism
  N Carrasco, *et al.* PRD 91(7) (2015)

- Still **uncontrolled systematics**
  FV effects, QED quenching, continuum limit

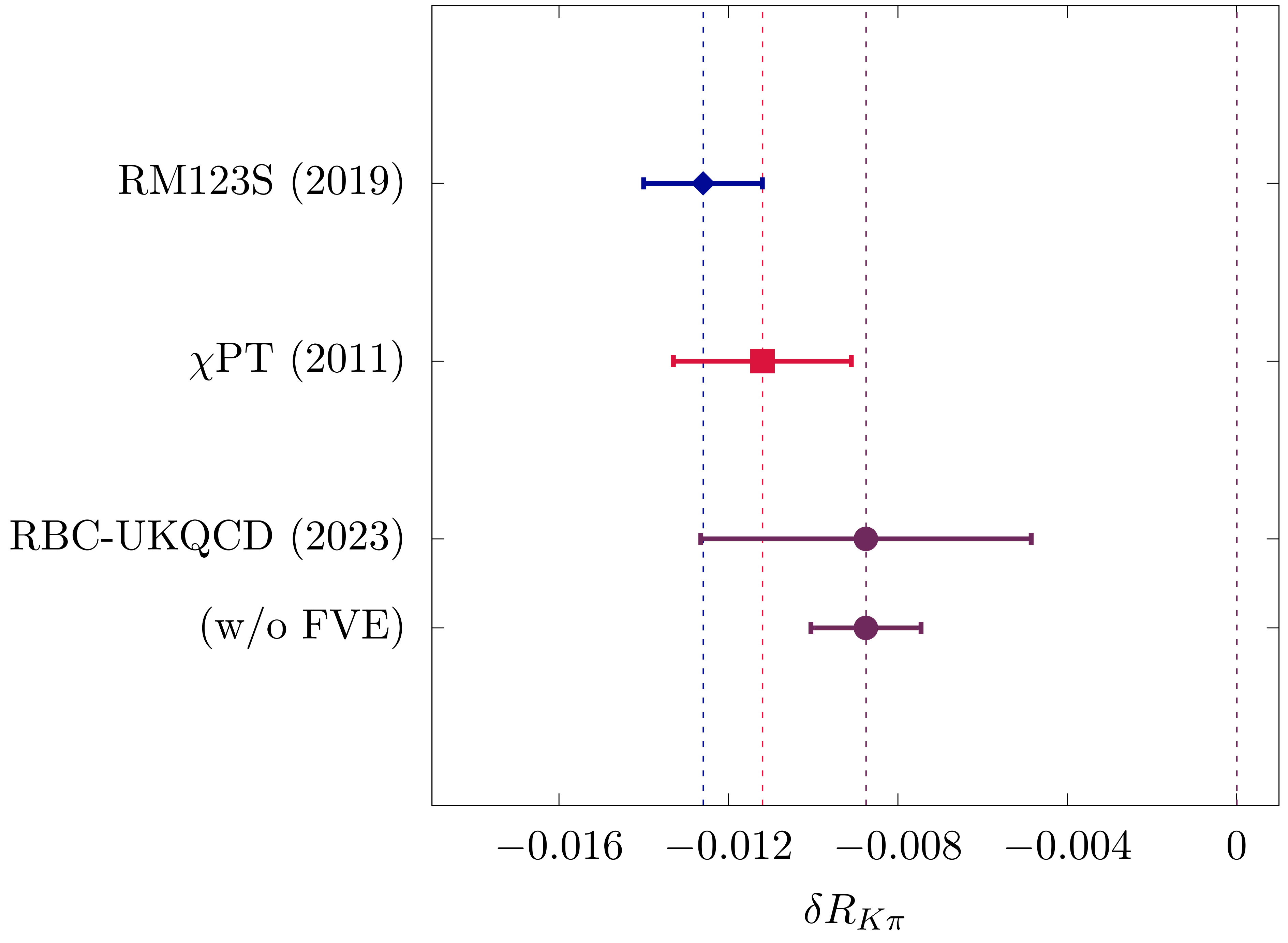


Image credit: Matteo Di Carlo

Finite-volume QED

Zero-mode singularities

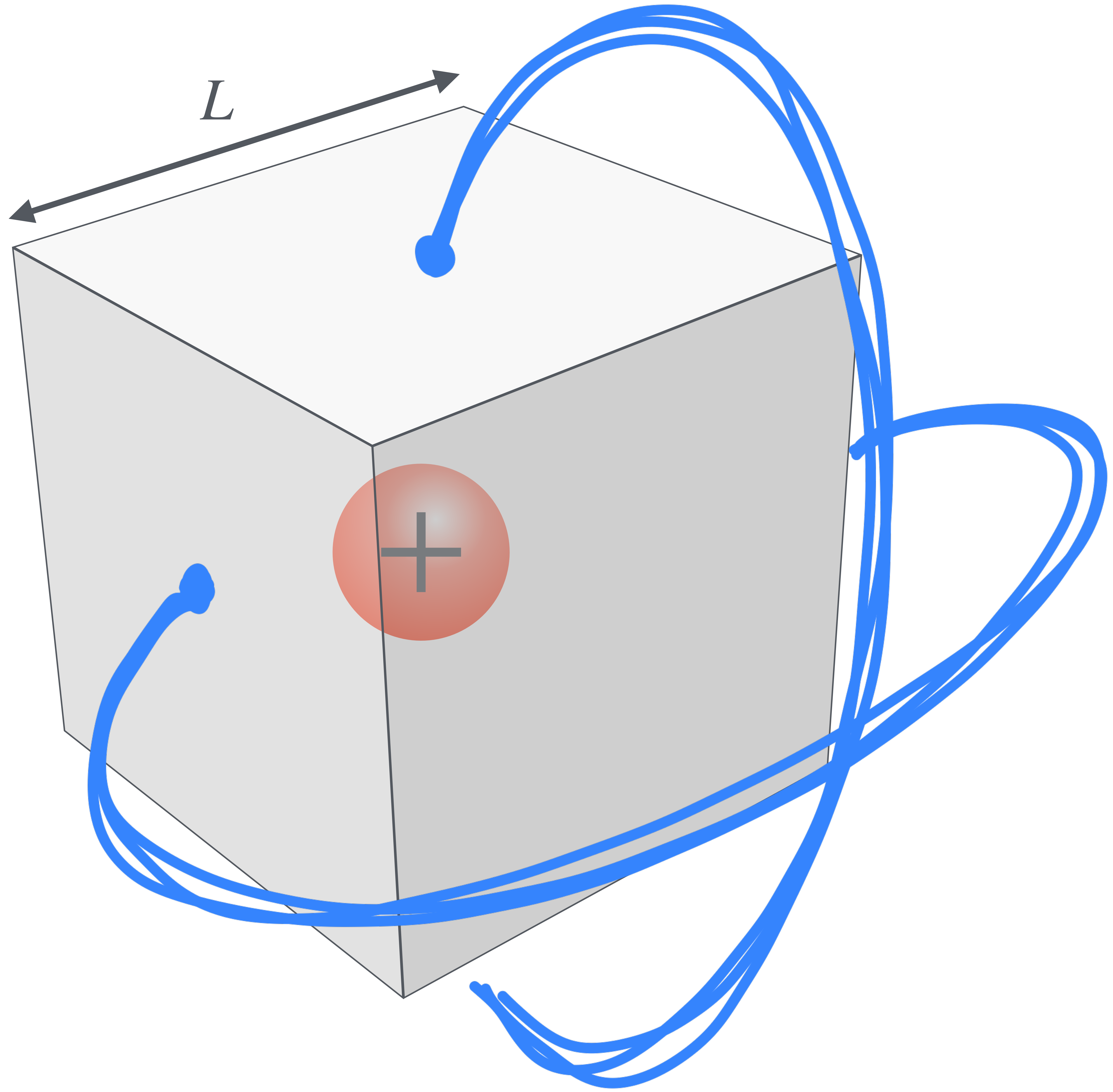
- Periodic boundary conditions

  $\Longrightarrow$ **EM field feedback loop**

- **Large finite-volume (FV) effects expected**

- In reality, it is worse than that:
  Feedback loop diverges

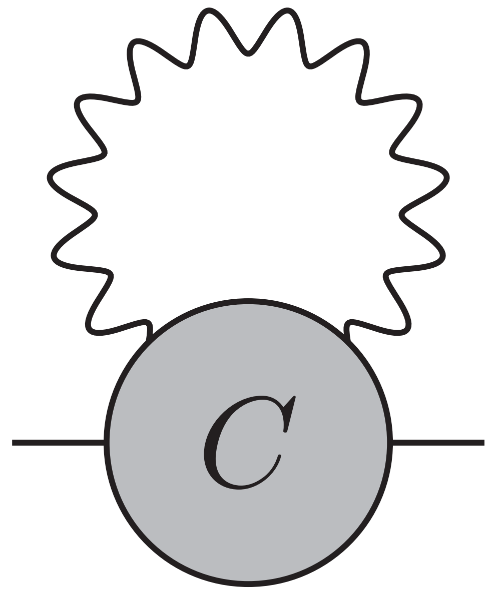
  (think about a lattice of Coulomb potentials)



Finite-volume QED

Zero-mode singularities, quantum field theory

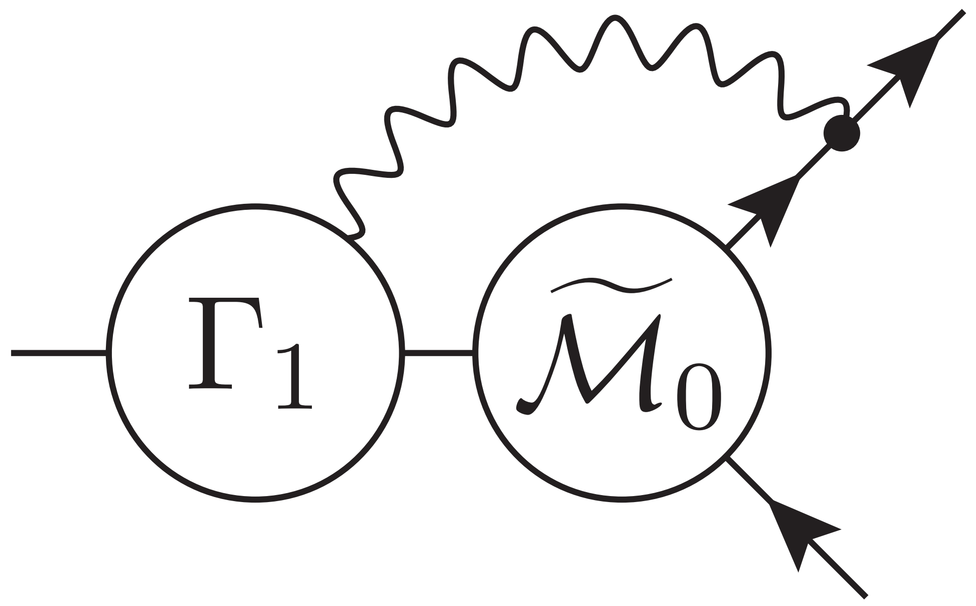
- One-loop QED amplitude



$$\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\,\frac{f(\mathbf{k})}{\mathbf{k}^2} \longmapsto \frac{1}{L^3}\sum_{\mathbf{k}}\frac{f(\mathbf{k})}{\mathbf{k}^2}\,, \quad \text{with} \quad \mathbf{k} = \frac{2\pi}{L}\mathbf{n}$$

maybe divergent **IR divergences**	undefined because of $f(\mathbf{0})/0$ term

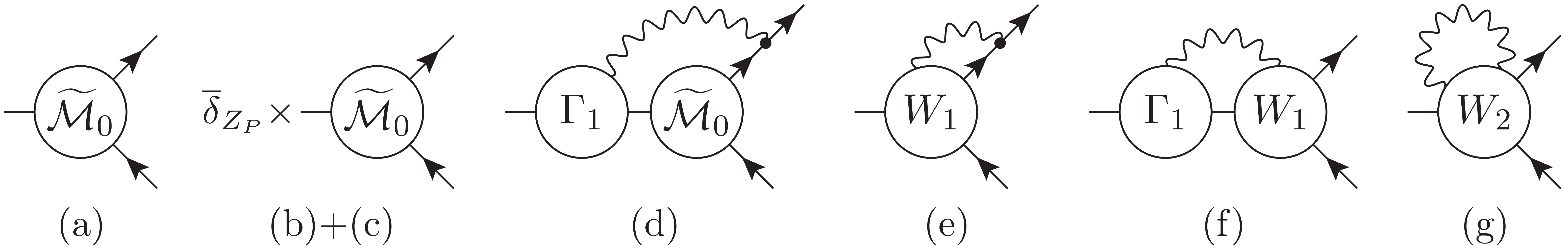
- Regularisation or change of BC required



- $\mathrm{QED_L}$: remove all 3D zero-modes $\mathbf{k} = \mathbf{0}$.
  Non-local modification of QED

Finite-volume QED

Leptonic decays

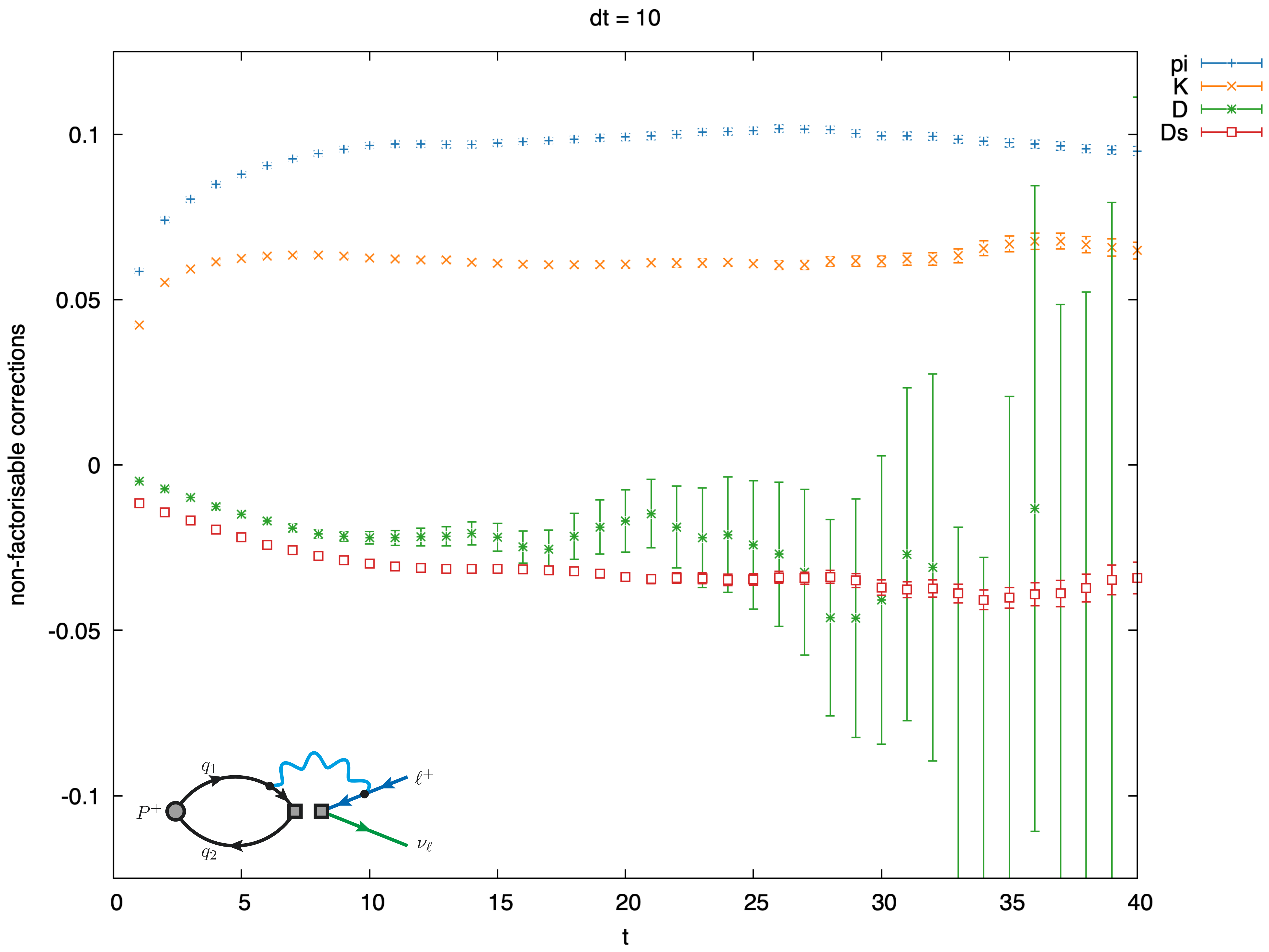


- Known, universal $\log(ML)$, $1/L$ finite-size effects
  B Lucini, *et al.* PRD 95(3) (2017)
- Known structure-dependent $1/L^2$ finite-size effects
  M Di Carlo, *et al.* PRD 105(1) (2022)
- Unknown structure-dependent $1/L^3$, **potentially large**: source of large uncertainty in P Boyle, **AP**, *et al.* JHEP 02 (2023)

IB corrections to weak decays

Future progress

- First calculation for D & D_s decays
- **Collaboration with KEK** on B & B_s decays
- Continuum limit for K & π decays
- Improving FV QED / removing $1/L^3$ terms
  - Z Davoudi, **AP**, *et al.* Phys Rev D 99(3) (2019)
  - M Di Carlo, PoS Lattice 2023 (2024)
  - M Di Carlo, **AP**, *et al.* in preparation (2025)



$K \to \pi \ell^+ \ell^-$ decays

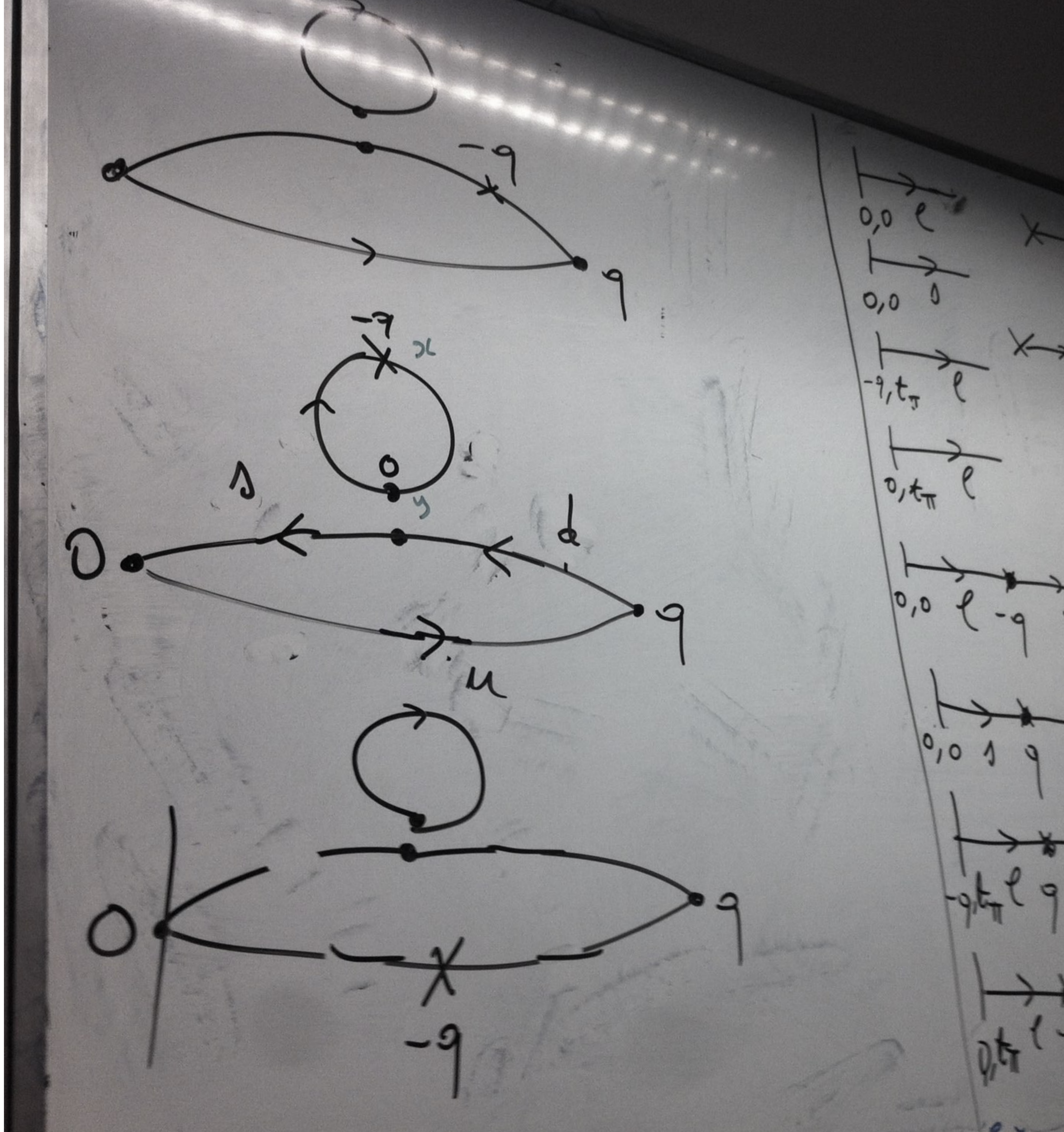
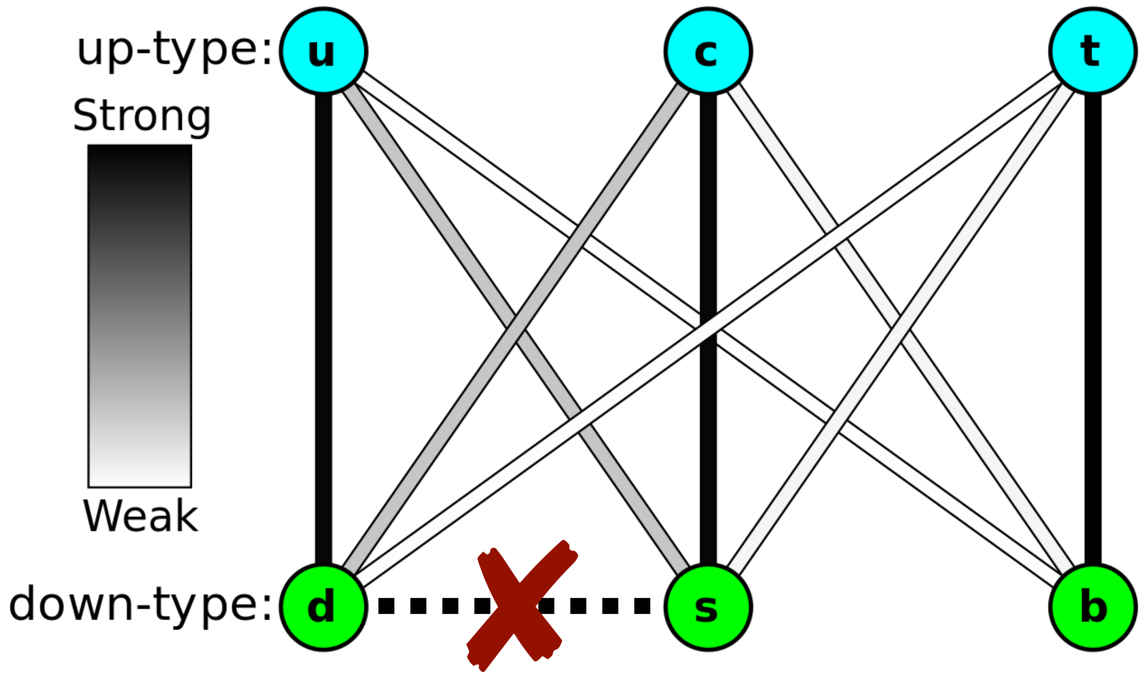
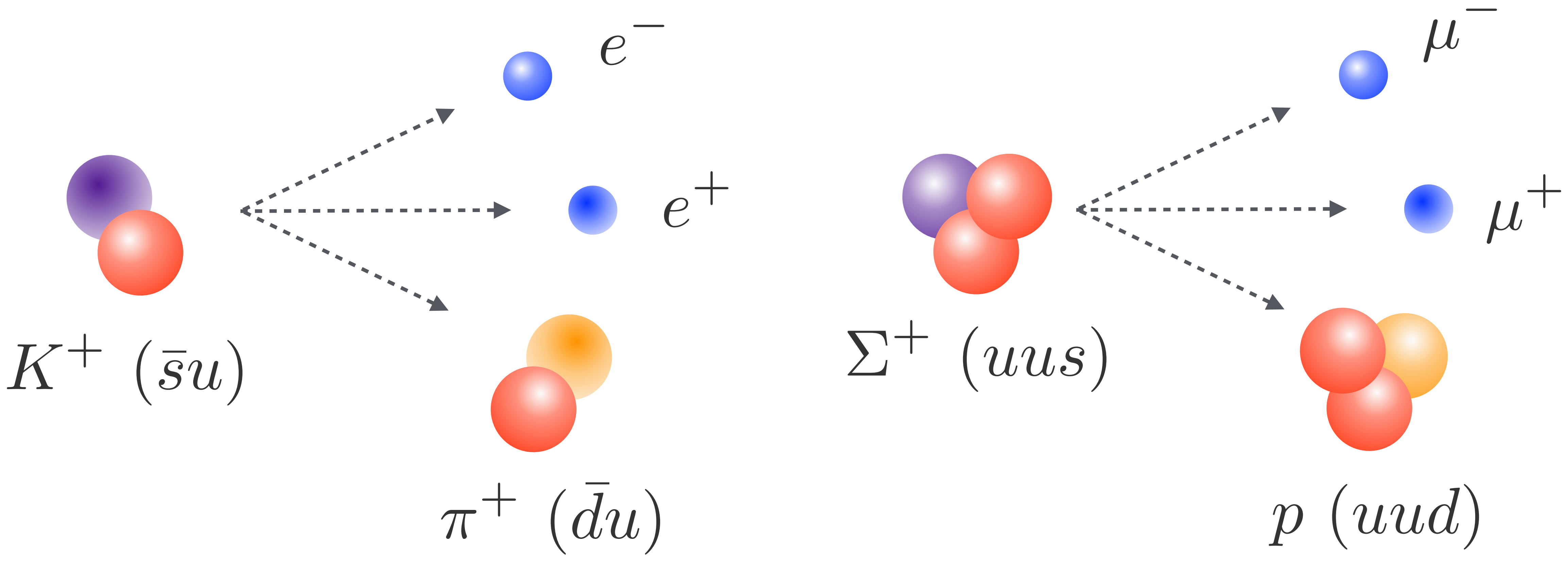


Image credit: AP, Southampton 2013

Rare $s \to d$ decays





- A promising avenue for new physics is to study **flavour-changing neutral current** decays
- Those are forbidden at leading order in the SM, **sensitivity to new physics is increased**
- They generally feature **long-distance multi-hadron corrections**

NH Christ, **AP**, *et al.* Phys Rev D 92(9) (2015)

PA Boyle, **AP**, *et al.* Phys Rev D Lett 107(1) (2023)

F Erben, **AP**, *et al.* JHEP 04 (2024)

PA Boyle, **AP**, *et al.* arXiv:2406.19193 (2024)

Amplitude parameterisation

- $K^c \to \pi^c \gamma^*$ amplitude ($c \in \{0,+\}$)

$$\mathscr{A}^c_\mu(q^2) = \int \mathrm{d}^4x \, \langle \pi^c(\mathbf{p}) \,|\, \mathrm{T}[J_\mu(0) H_W(x)] \,|\, K^c(\mathbf{k}) \rangle$$

$$= -i \frac{G_F}{(4\pi)^2} [q^2 (k+p)_\mu - (M_K^2 - M_\pi^2) q_\mu] \, V_c(z)$$

$$q = p - k$$

$$z = q^2 / M_K^2$$

- Low-energy parameterisation

$$V_c(z) = a_c + b_c \, z + V_c^{\pi\pi}(z)$$

Analytical continuation issues

- **In Euclidean space-time, states below initial energy generate large contamination of Euclidean the time integral**

- Happens potentially for $K \to \pi, \pi\pi, \pi\pi\pi \to \pi\gamma^*$

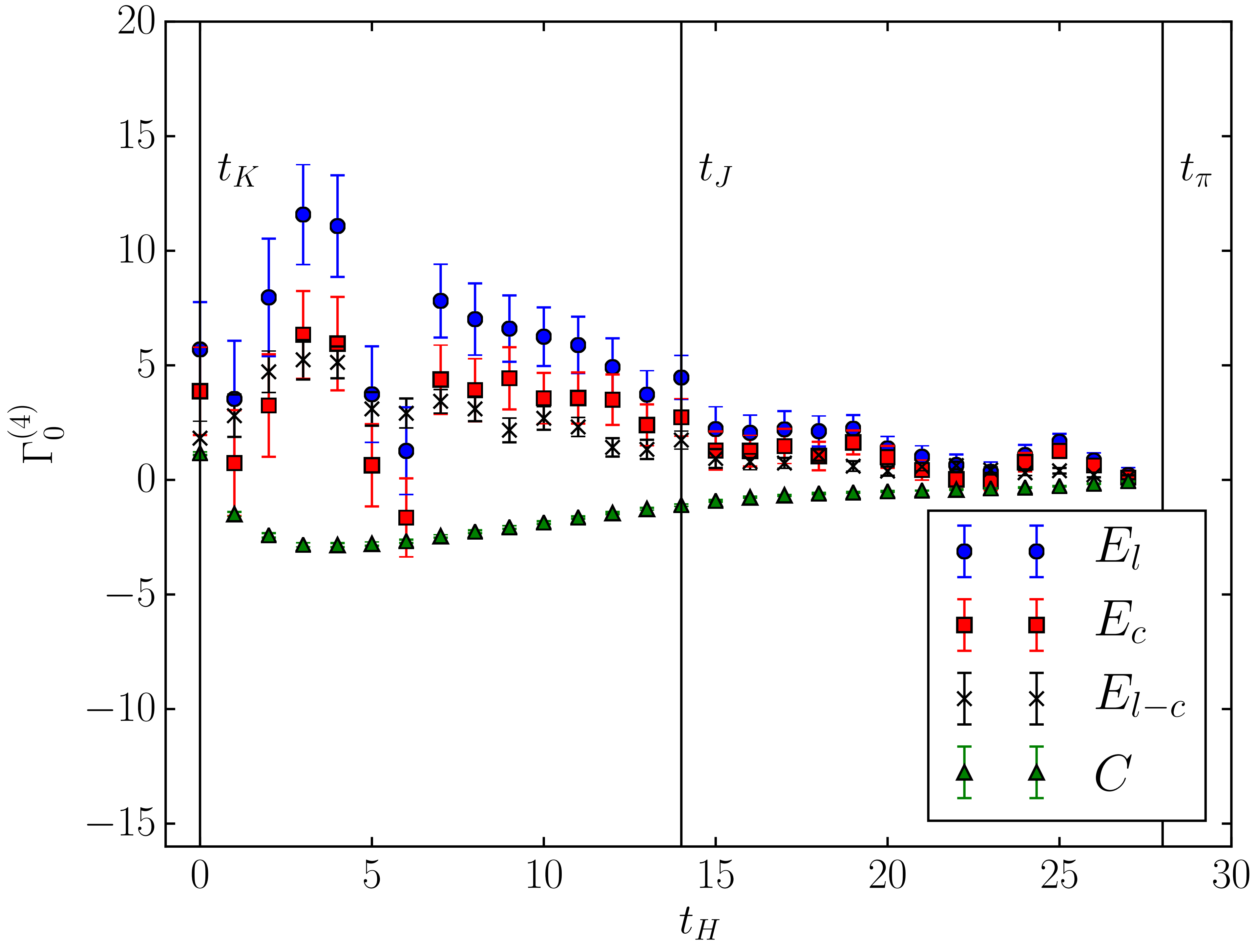
- $\pi\pi$ forbidden in $K \to \pi\ell^+\ell^-$ (allowed in $K \to \pi\nu\bar{\nu}$)

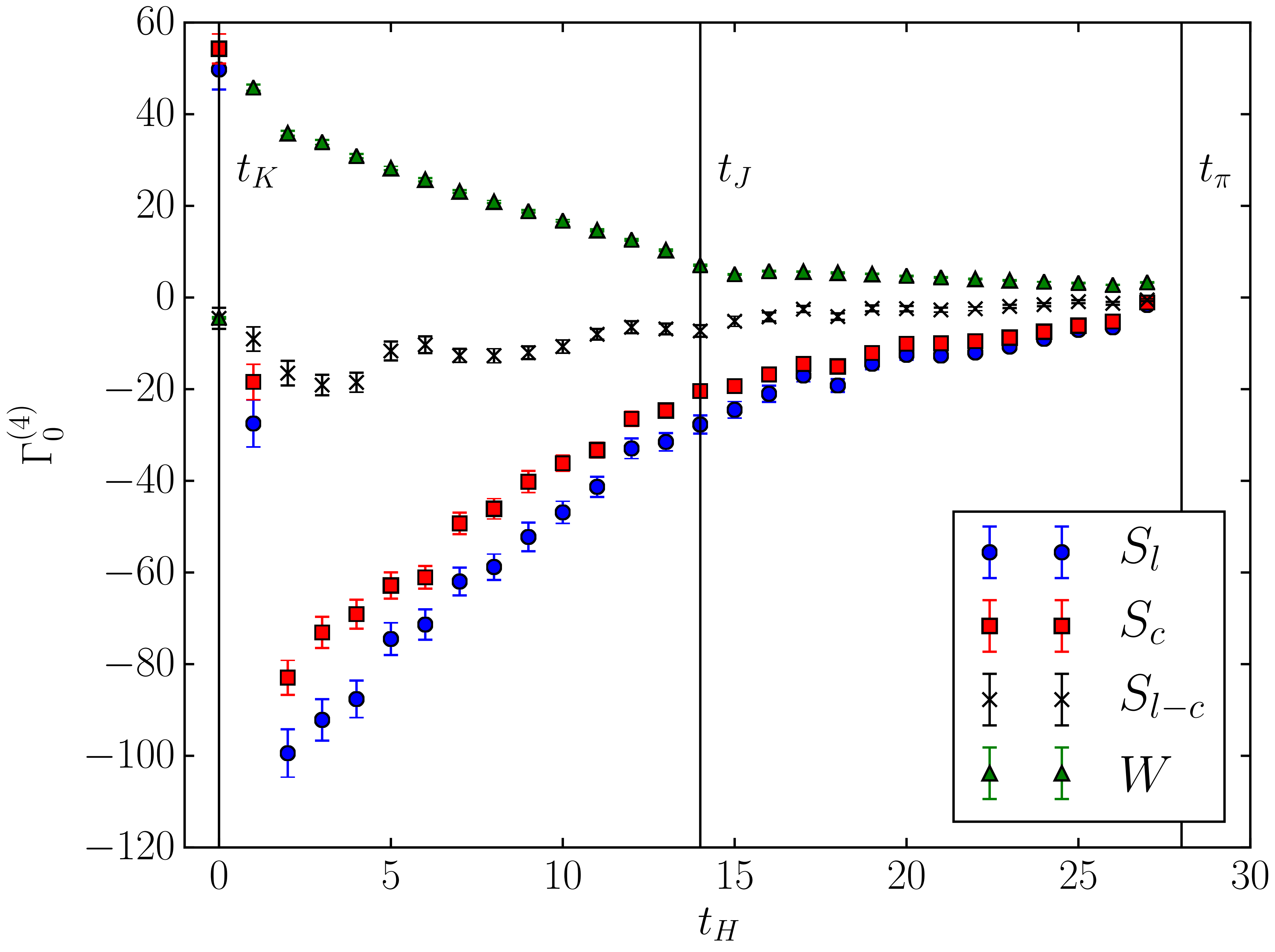
- Several subtraction strategies possible
  NH Christ, AP, et al., PRD 92(9) 094512 (2015)

4-point correlators

Unphysical point

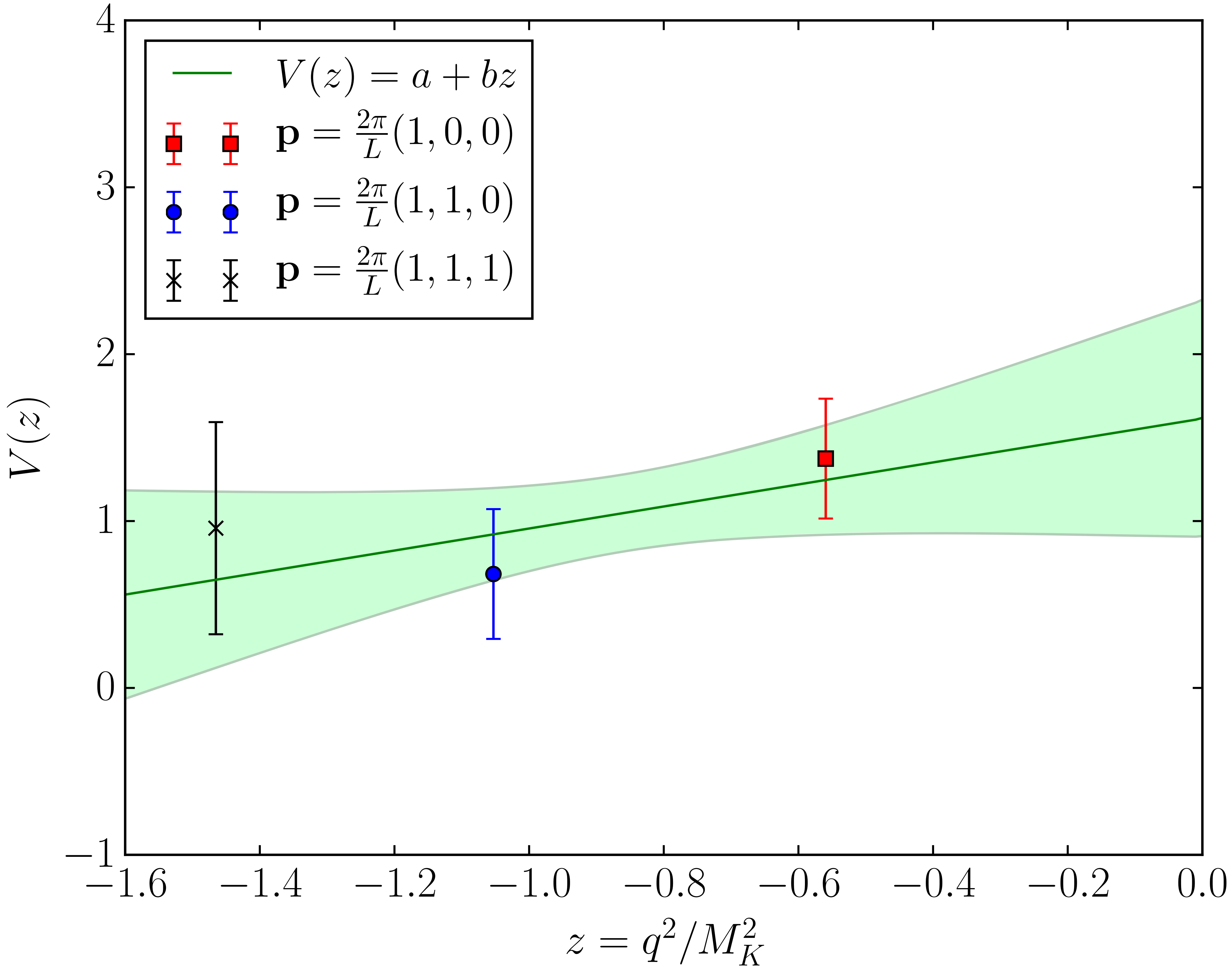
NH Christ, AP, et al., PRD 94(11) 114516 (2016)





Amplitude result

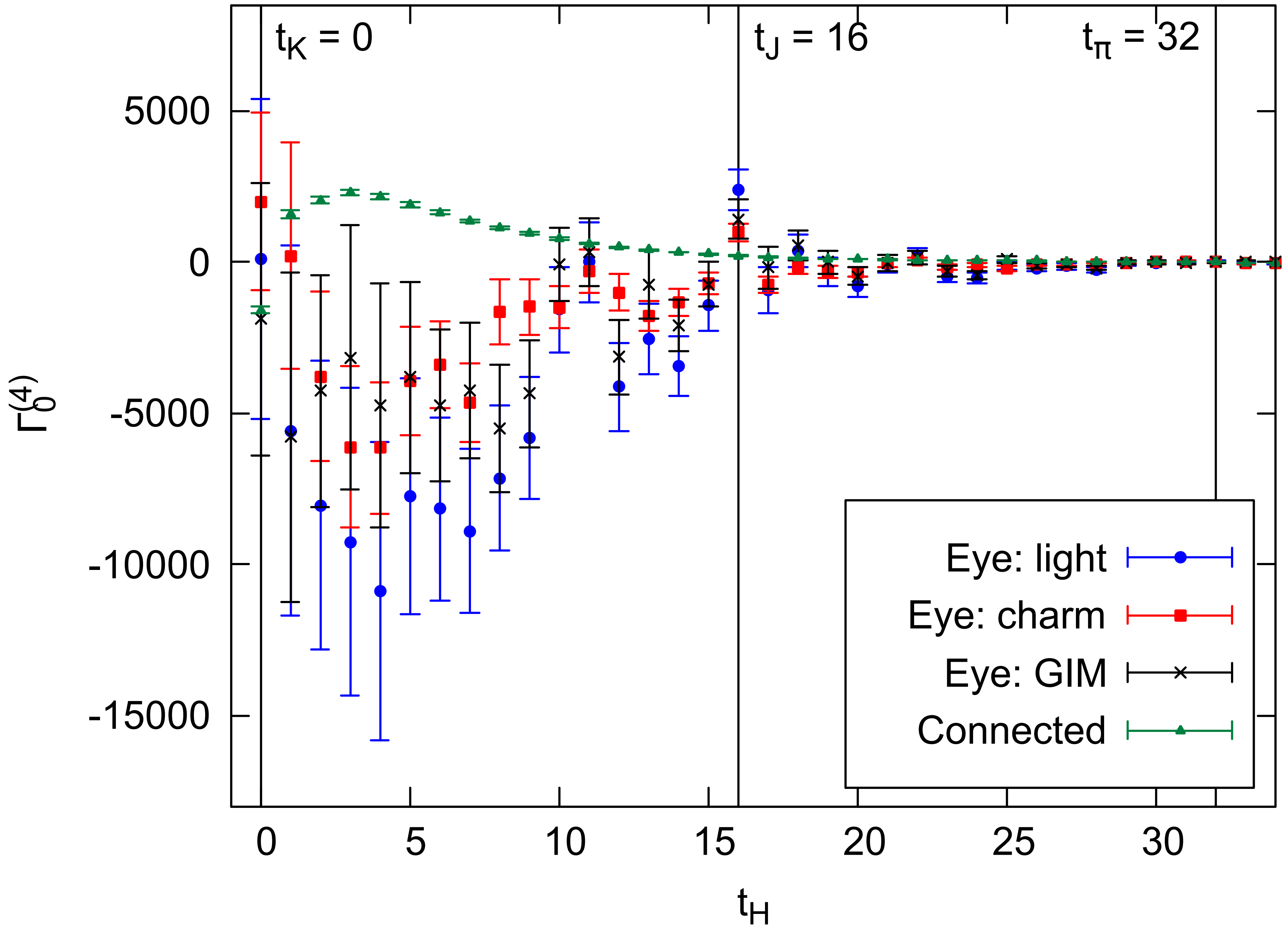
Unphysical point NH Christ, AP, et al., PRD 94(11) 114516 (2016)

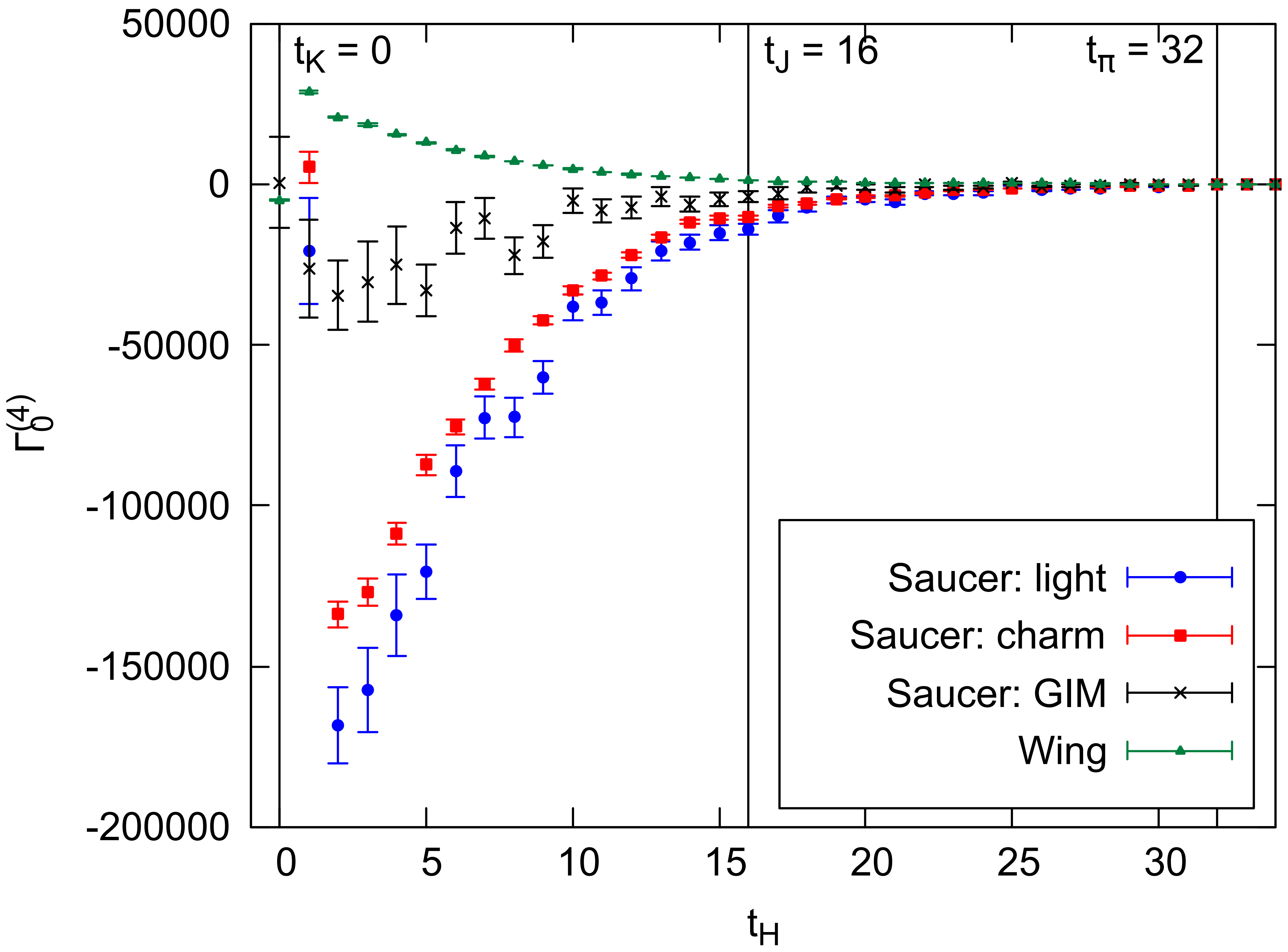


4-point correlators

Physical point

PA Boyle, AP, et al., PRD 107(1) L011503 (2022)

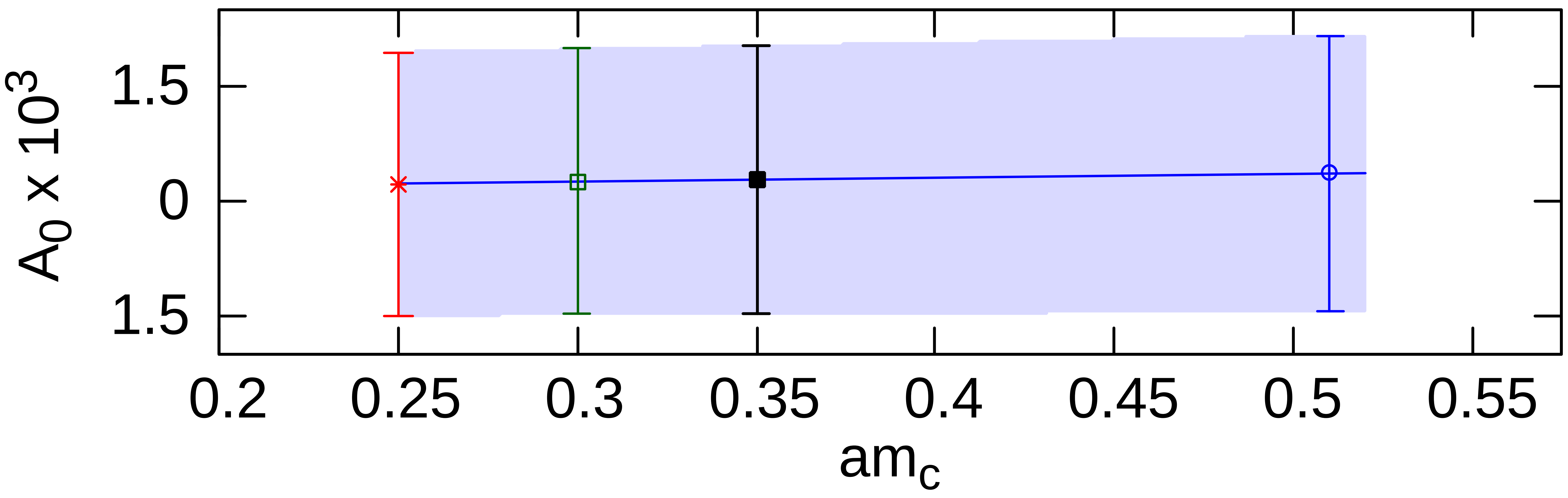




Amplitude result

Physical point

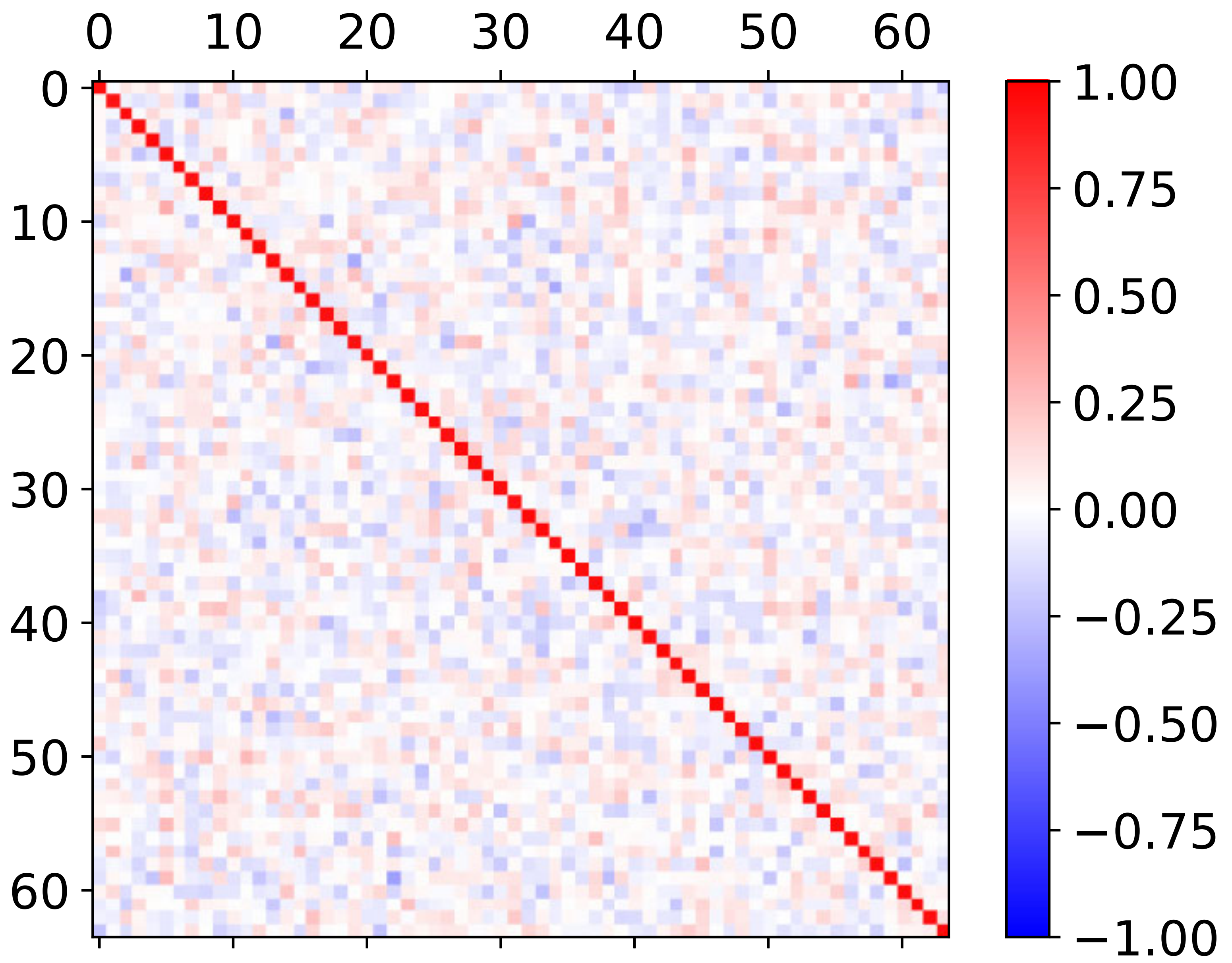
PA Boyle, AP, et al., PRD 107(1) L011503 (2022)



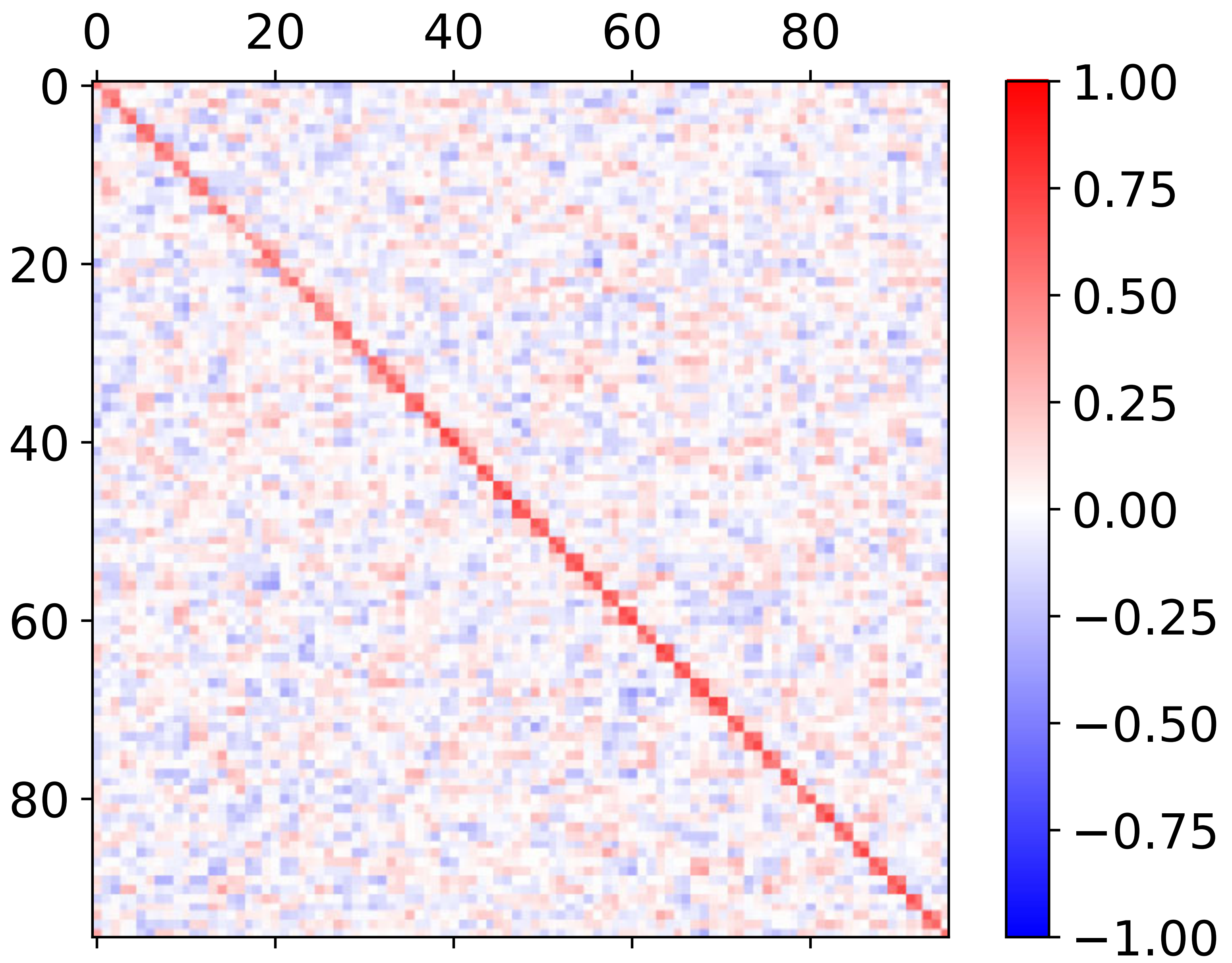
$$a_+ = -0.87(4.44)$$

A hint about the noise issue

- Correlations between up and charm loops is a **huge factor in GIM loops uncertainty**



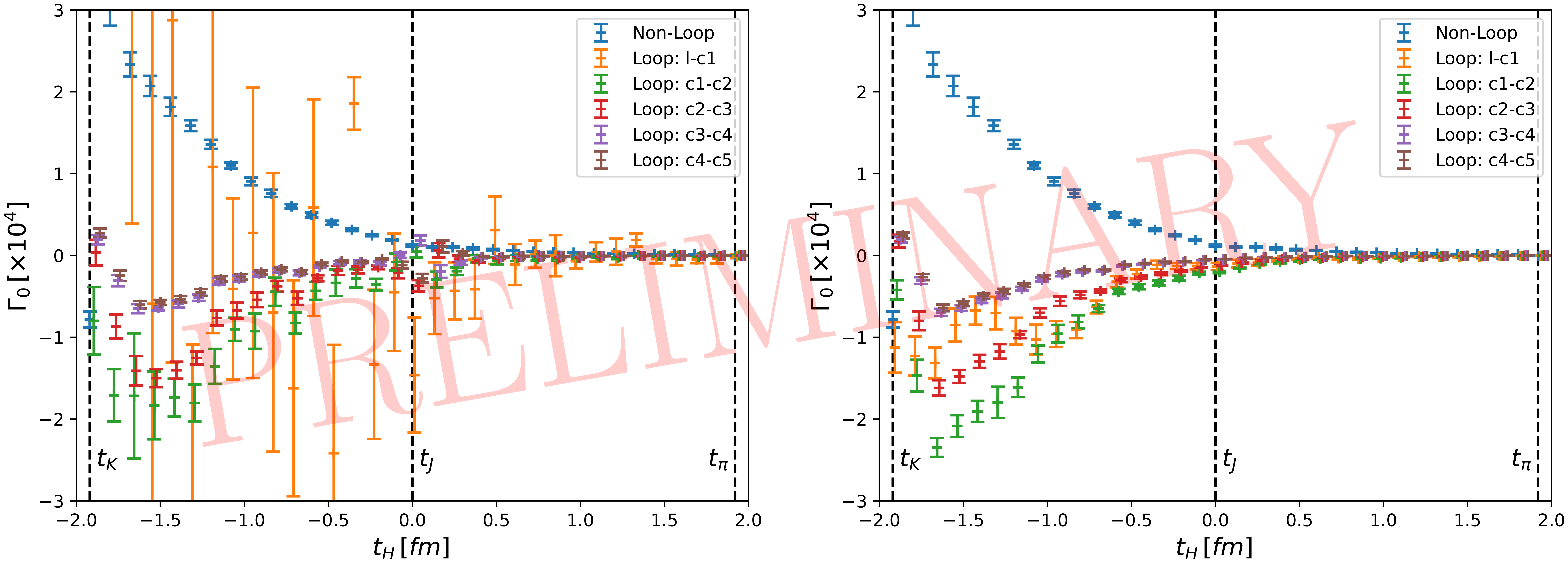
2016 unphysical data



2022 physical point data

Improved estimators

4-point physical correlators / work with R Hill and R Hodsgon



Standard estimator | Frequency-splitting estimator

M Bordone, et al., to appear soon (Kaon@J-PARC 2024 Summary)

Multi-hadron interactions

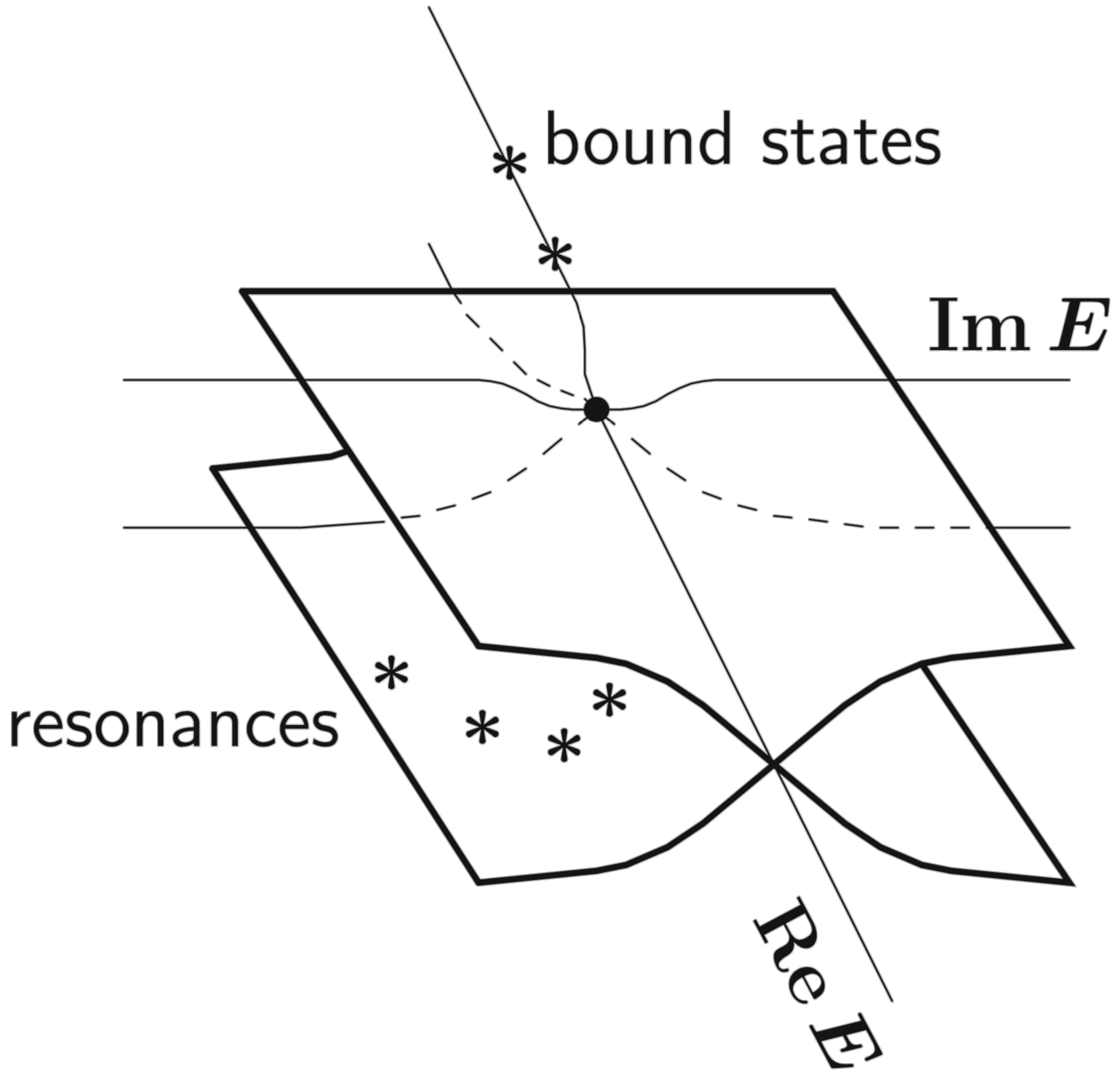


Image Credit: DOI 10.1007/s00601-012-0376-4

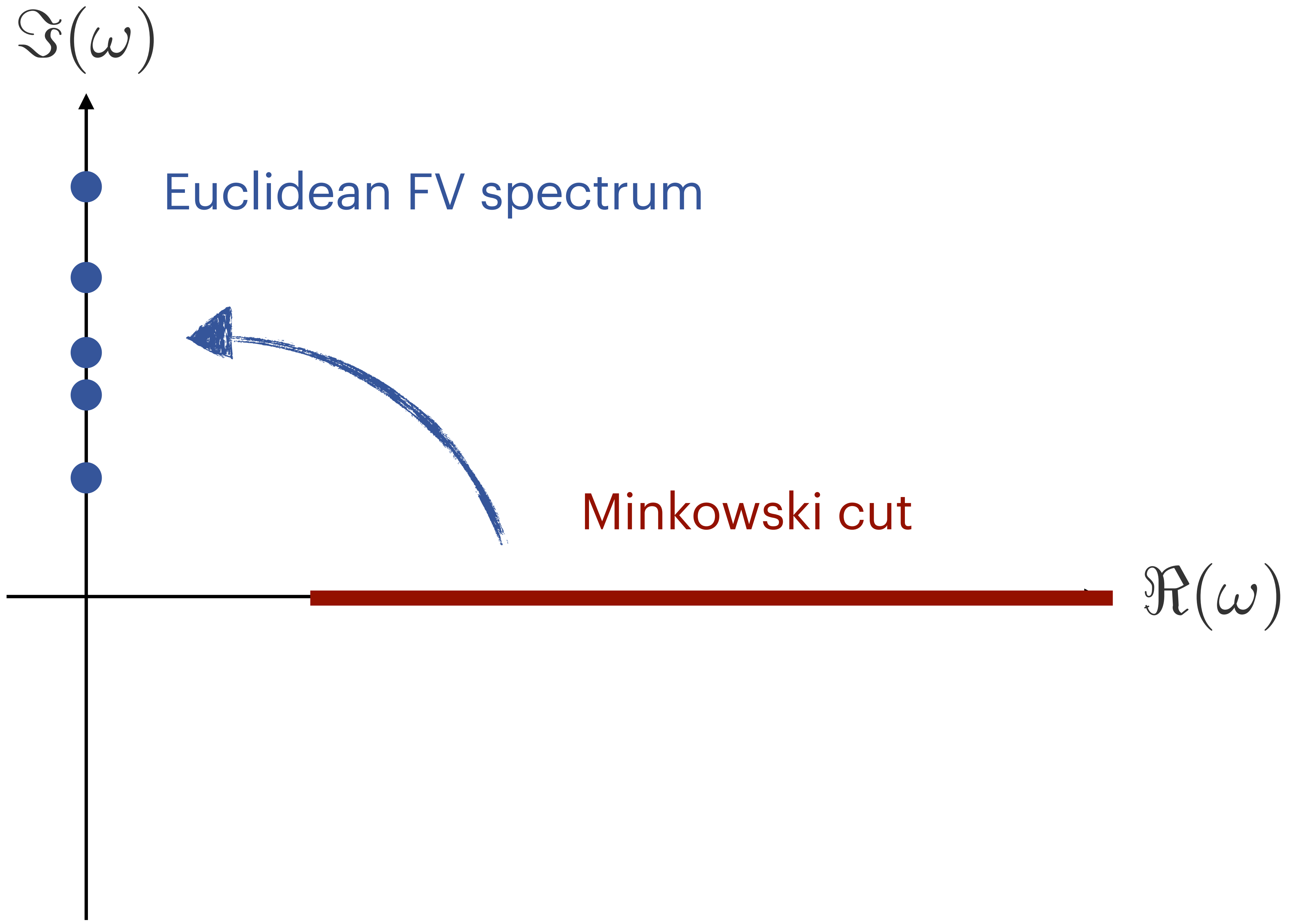
Multi-hadron interactions

General issue for lattice simulations

- There is a theorem saying that hadronic scattering amplitudes cannot, in principle, be extracted from Euclidean field theory
  L Maiani & M Test, Phys Lett B 245 (1990)

- This was circumvented by noticing that splittings between **discrete energy levels in a finite volume encode information about scattering amplitudes**

- This is often referred as *Lüscher formalism*
  M Lüscher, Commun. Math. Phys. 105(2) (1986)



Hadronic resonances

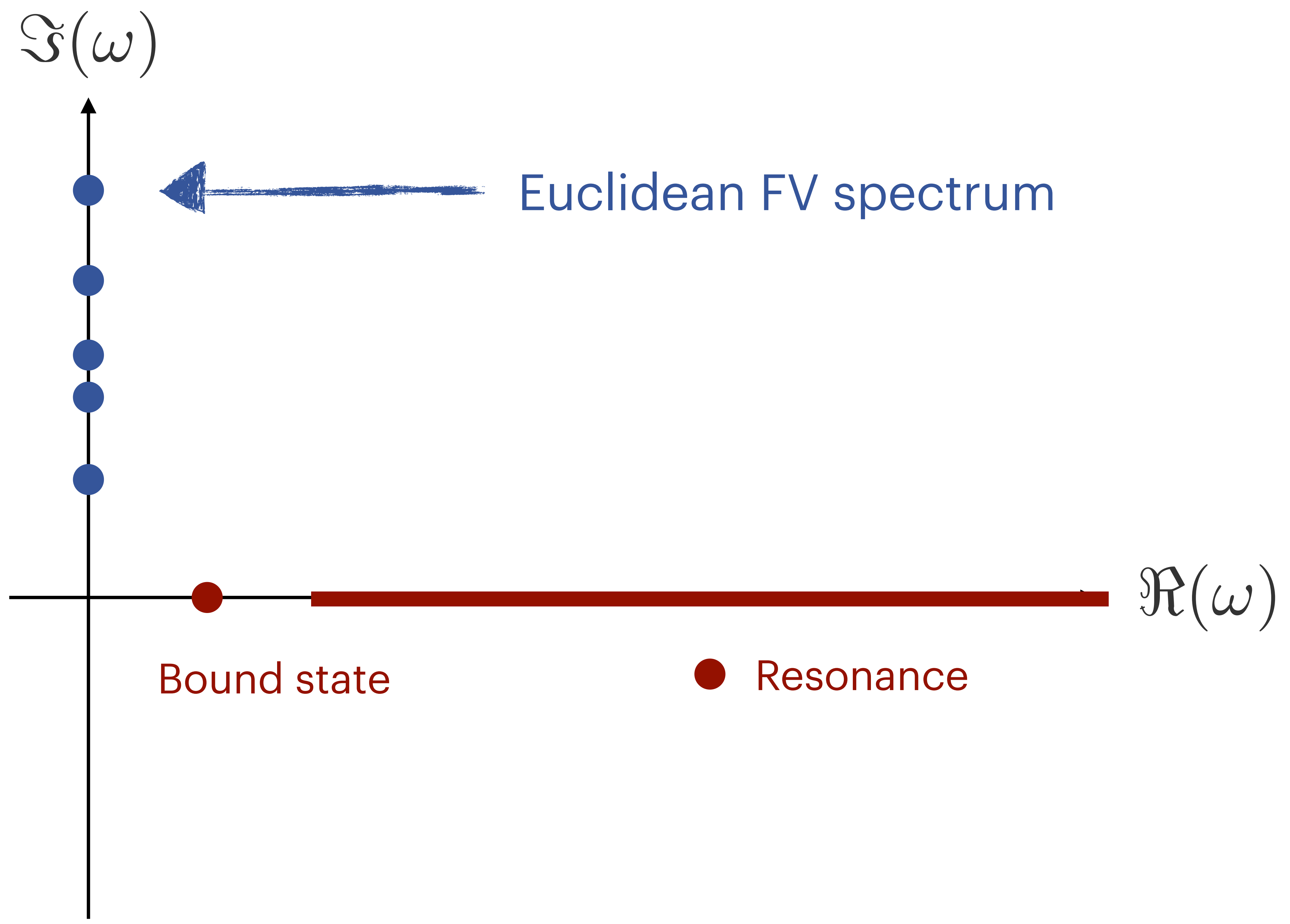
Lattice determination

1. Determine **Euclidean FV energy levels**

2. Relate to phase-shift model using
   Lüscher quantisation condition
   M Lüscher, Commun. Math. Phys. 105(2) (1986)

$$n\pi - \delta\left(\sqrt{\omega_n^2 - 4m^2}\right) = \phi\left(\frac{L}{2\pi}\sqrt{\omega_n^2 - 4m^2}\right)$$

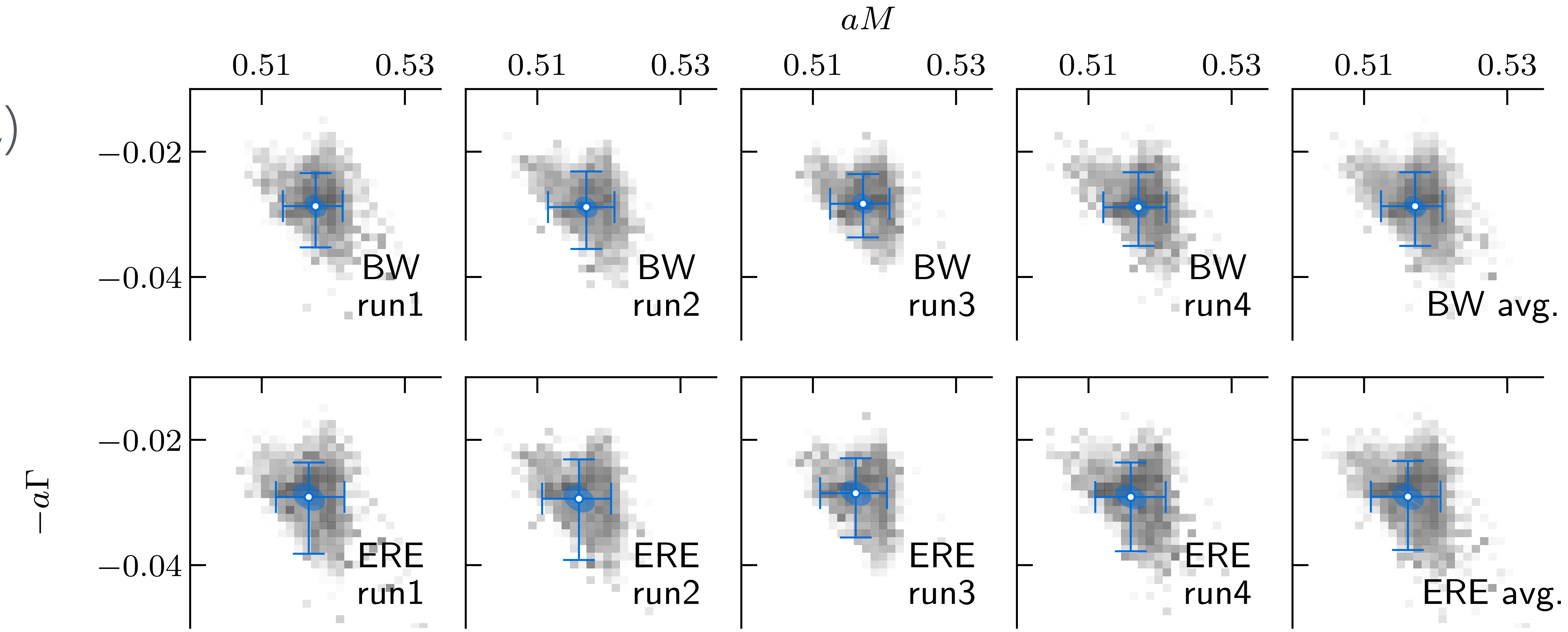
3. **Solve for amplitude poles**



Lattice description of resonances

First physical point ρ & K^* simulation

$K\pi\,(I = 1/2)$



- First determination of the ρ and K^* poles using **physical point 2+1 lattice simulations**
- First data-driven assessment of **analysis modelling and systematic errors**

PA Boyle, **AP**, *et al.* arXiv:2406.19193 (2024)

Image credit: Nelson Lachini

Conclusion

- Lattice QCD is entering the **era of physical, precise predictions** for
  - **Long-distance hadronic & electromagnetic** corrections to weak decays
  - Weak decays into **unstable states**
- More theoretical work on the way in key aspects e.g.
  - Treatment of **heavy quarks**
  - **Final state long-distance interactions**, particularly electromagnetic
- Hopefully crucial help for flavour physics measurements in future experiments

Thank you for your attention!