Solutions of the Strong CP problem without axions

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9th Symposium on Prospects in the Physics of Discrete Symmetries
Ljubljana
3rd December 2024

in collaboration with:
Matteo Parricciatu (Rome III), Alessandro Strumia and Arsenii Titov (Pisa)
& Robert Ziegler [2411.08101] [2305.08908, 2406.01689]

the strong CP problem

$$\mathcal{L}_{QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{4g_3^2}\mathcal{G}_{\mu\nu}^{a}\mathcal{G}^{a\mu\nu} + \frac{\theta_{QCD}}{32\pi^2}\mathcal{G}_{\mu\nu}^{a}\tilde{\mathcal{G}}^{a\mu\nu}$$

$$\bar{\theta} = \theta_{QCD} + \arg\det m$$

$$d_n \approx 1.2\times10^{-16}\,\bar{\theta}\; e\cdot cm$$

$$|\bar{\theta}| \lesssim 10^{-10}$$ & $$\delta_{CKM} \approx \mathcal{O}(1)$$

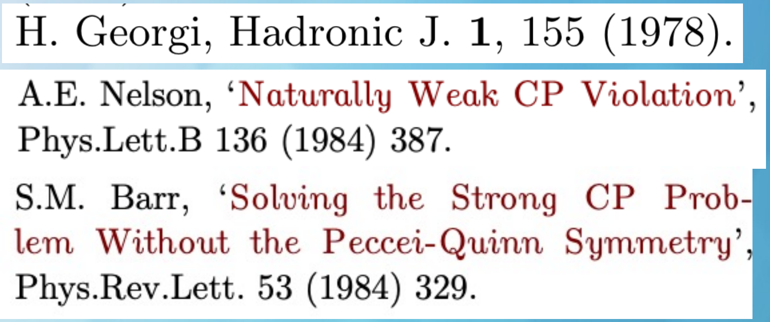
solutions

1. $\bar{\theta}$ promoted to a field, the axion, pseudoGB of a global, anomalous $U(1)_{PQ}$ symmetry
   VEV dynamically relaxed to zero by QCD dynamics

2. CP symmetry of the UV theory

an old idea, needs an extension of the SM

- extra Higgs doublets
- heavy VL quarks and several complex spin-zero fields



H. Georgi, Hadronic J. **1**, 155 (1978).

A.E. Nelson, 'Naturally Weak CP Violation', Phys.Lett.B 136 (1984) 387.

S.M. Barr, 'Solving the Strong CP Problem Without the Peccei-Quinn Symmetry', Phys.Rev.Lett. 53 (1984) 329.

- CP spontaneously broken such that
- $|\bar{\theta}| \lesssim 10^{-10}$ & $\delta_{CKM} \approx \mathcal{O}(1)$
- our solution has no heavy VL quarks in its minimal version, it requires

- supersymmetry
- a new gauge-singlet complex field z

$\mathcal{N}$=1 SUSY CP invariant theories

z are gauge-invariant chiral superfields
φ are the MSSM superfields

gauge kinetic function

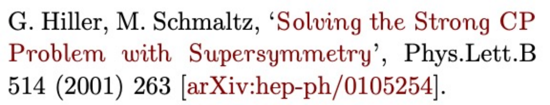
$$f_3 = \frac{1}{g_3^2} - i\,\frac{\theta_{QCD}}{8\pi^2}$$

$$\mathcal{L} = \int d^2\theta d^2\overline{\theta}\, K(z, e^{2V}\varphi, \bar{\tau}, \bar{\varphi}) + \left[\int d^2\theta\, w(z,\varphi) + \frac{1}{16}\int d^2\theta\, f\, WW + h.c\right]$$

kinetic terms

Yukawa couplings $\mathcal{Y}(z)$

$$\arg\det m(z) = \arg\det Y(z)\upsilon$$


G. Hiller, M. Schmaltz, 'Solving the Strong CP Problem with Supersymmetry', Phys.Lett.B 514 (2001) 263 [arXiv:hep-ph/0105254].

$$\bar{\theta} = -8\pi^2 Im\, f + \arg\det Y(z)\upsilon$$

no dependence on K

$\bar{\theta}$ from holomorphic data


A note on the predictions of models with modular flavor symmetries

Mu-Chun Chen (UC, Irvine), Saúl Ramos-Sánchez (Mexico U. and Munich, Tech. U.), Michael Ratz (UC, Irvine)
15, 2019)

Published in: *Phys.Lett.B* 801 (2020) 135153 • e-Print: 1909.06910 [hep-ph]

our solution relies on starting with a CP invariant theory and choosing

$$f_\alpha = c_\alpha \qquad (\alpha = 1,2,3)$$

$$\det Y_A(z) = c_A \qquad (A = U,D,E)$$

real constants by CP

➡ $$\bar{\theta} = -8\pi^2 Im\, f + \arg\det Y(z)v = 0$$

our solution relies on starting with a CP invariant theory and choosing

$$f_\alpha = c_\alpha \qquad (\alpha = 1,2,3)$$

$$\det Y_A(z) = c_A \qquad (A = U, D, E)$$

real constants
by CP

in each $A(=U, D, E)$ sectors

$$Y_A(z) = \begin{pmatrix} 0 & 0 & c_{A13} \\ 0 & c_{A22} & \cdot \\ c_{A31} & \cdot & \cdot \end{pmatrix}$$

$$\det Y_A(z) = -\, c_{A13}\, c_{A13}\, c_{A13}$$

c_{A3}, c_{A13}, c_{A13} real constants

our solution relies on starting with a CP invariant theory and choosing

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⇒ $$\det Y_A(z) = -\, c_{A13}\; c_{A13}\; c_{A13}$$

c_{A3}, c_{A13}, c_{A13} real constants

CKM phase generated by this block

$$Y_A(z) = \begin{pmatrix} 0 & 0 & c_{A13} \\ 0 & c_{A22} & c_{A23}(z) \\ c_{A31} & c_{A32}(z) & c_{A33}(z) \end{pmatrix}$$

⇐ picks up a nontrivial
phase from the z - VEV

this pattern can be generated by two simple requirements

1. assign a weight/charge to each field

field	D_i^c	Q_i	z_a	$\cdots$
weight	$k_{D_i^c}$	k_{Q_i}	k_{z_a}	$\cdots$

2 restrictions

$$k_{z_a} > 0$$

$$\sum_i (k_{D_i^c} + k_{Q_i}) = 0$$

example

$$k_{D_i^c} = (-1,0,+1)$$

$$k_{Q_i} = (-1,0,+1)$$

$$k_{z_1} = +1$$

$$k_{z_2} = +2$$

2. require $Y_{D_{ij}}(z)$ to be a polynomial in z_a of weight $(k_{D_i^c}+k_{Q_j})$

in previous example $(k_{D_i^c}+k_{Q_j}) =$

$$\begin{array}{cc} & \overbrace{\begin{matrix}-1 & \quad 0 \quad & +1\end{matrix}}^{k_{Q_j}} \\ k_{D_i^c}\left\{\begin{matrix}-1\\0\\+1\end{matrix}\right. & \begin{pmatrix}-2 & -1 & 0\\ -1 & 0 & +1\\ 0 & +1 & +2\end{pmatrix}\end{array}$$

$$Y_{D_{ij}}(z) = \begin{pmatrix} 0 & 0 & c_{D13} \\ 0 & c_{D22} & c_{D23}\, z_1 \\ c_{D31} & c_{D31}\, z_1 & c_{D33}\, z_1^2 + c'_{D33}\, z_2 \end{pmatrix}$$

only a mathematical trick?
more about this
in a moment…

pheno properties
of the multiplet z
(assume only one)

FF & Robert Ziegler [2411.08101]

A light relic: the CPon

assume all extra d.o.f. are very heavy, except for

ξ, one spin-zero component in Z

free parameters

m_ξ	mass
Λ	non-renormalizable couplings

$$\mathscr{L}_F = i\bar{\Psi}_A\gamma^\mu D_\mu\Psi_A - \bar{\Psi}_A\;\hat{m}_A\Psi_A - \frac{\xi}{\Lambda}\bar{\Psi}_A\left(y_S^A + i\;y_P^A\gamma_5\right)\Psi_A + \dots \qquad (A = U, D, E)$$

after EW symmetry breaking

$$\widehat{m}_A \quad \leftrightarrow \quad Y_A(z)v$$

$$y_{S,P}^A \quad \leftrightarrow \quad \frac{dY_A(z)}{dz}v \approx \widehat{m}_A$$

- no direct coupling to photons/gluons
- ξ has a CP-violating interactions <-> CPon
- $$tr\left(\widehat{m}_A^{-1}\,y_{S,P}^A\right) = 0 \leftrightarrow tr\left[Y_A(z)^{-1}\frac{dY_A(z)}{dz}\right] = 0$$

$$\det Y_A(z) = c_A$$

[valid also in a general class of Kähler metrics]

ξ as a Dark Matter candidate

if ξ sufficiently long-lived, it provides a DM candidate

For any mass m_ξ above the decay threshold into electron-positron, the lifetime is shorter than $10^{24}\ sec$, excluded by CMB data for a DM candidate

$m_\xi \leq 1\ MeV$ ξ can only decay into two photons (or neutrinos)

$$\Gamma(\xi \to \gamma\gamma) \propto \frac{1}{16\,\pi^2}\,\frac{\alpha^2\, {m_\xi}^3}{\Lambda^2}\,\frac{{m_\xi}^4}{m_e^4}$$

at 1-loop

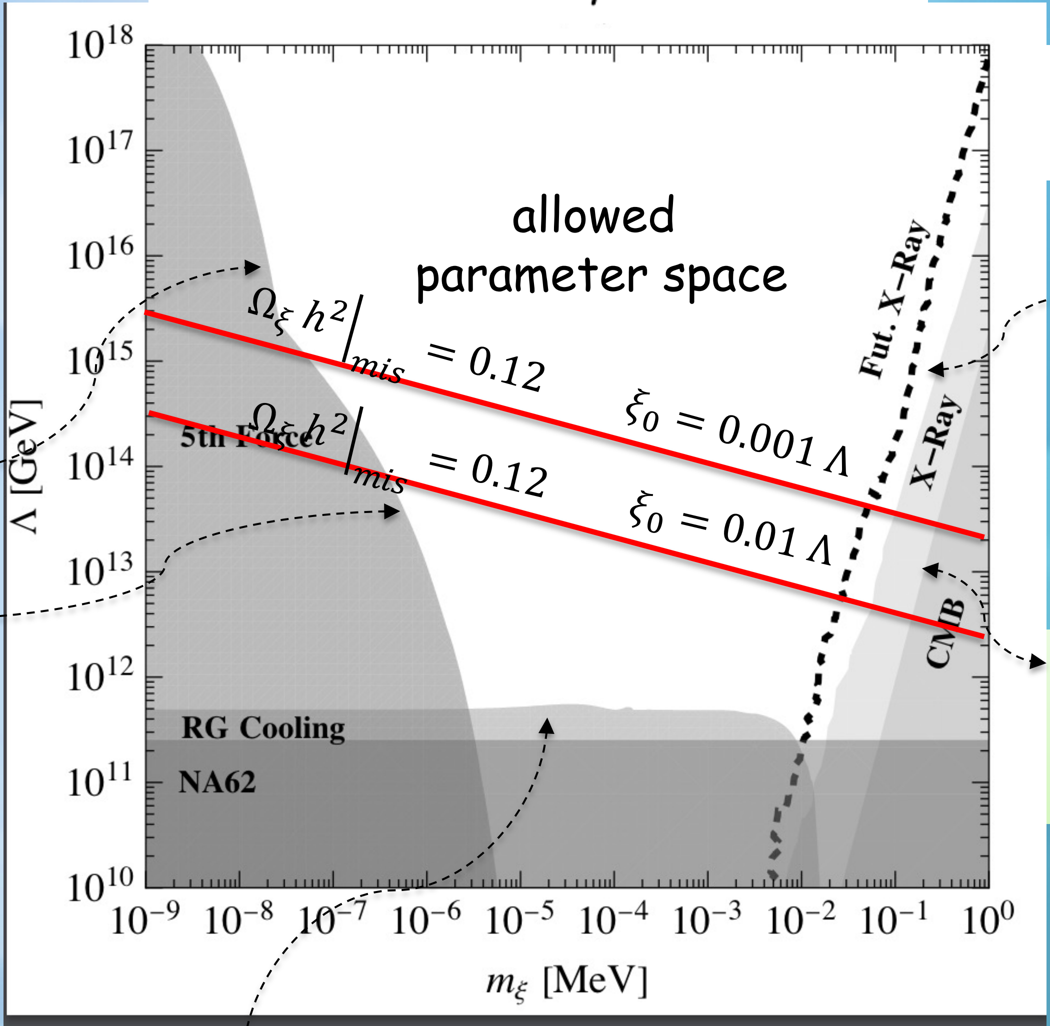
suppression $\frac{{m_\xi}^4}{m_e^4}$

from $tr\left(\widehat{m}_A^{-1}\, y_{S,P}^A\right) = 0$

$$\alpha \propto \left(g_{\xi NN}\right)^2 \approx \left(\frac{2}{27}\frac{m_N}{\Lambda}\right)^2$$

$$r = 1/M$$

$$\delta V_{ISL}(r) = -\frac{G m_1 m_2}{r}\alpha e^{-r/\lambda} .$$



$$\Gamma(\xi \rightarrow \gamma\gamma) \propto \frac{\alpha^2\, m_\xi{}^3}{\Lambda^2}\frac{m_\xi{}^4}{m_e^4}$$

limits from
Inverse
Square
Law
of gravity

X-rays diffuse
emissions
from DM decay
in galaxy clusters

CMB distortion from
energy of DM decay
or annihilation

stellar energy loss in
Red Giants and White Dwarfs

$$\frac{Y_{See}}{\Lambda} < 7\times10^{-16}$$

[here $Y_{See} \approx m_e$]

where do the rules come from?

1. $Y_{D\,ij}(z)$ a polynomial in z_a of weight $(k_{D_i^c}+k_{Q_j})$

2. $k_{z_a} > 0$

$$\sum_i (k_{D_i^c}+k_{Q_i}) = 0$$

1. $Y_{D_{ij}}(z)$ a polynomial in z_a of weight $(k_{D_i^c}+k_{Q_j})$

modular invariance

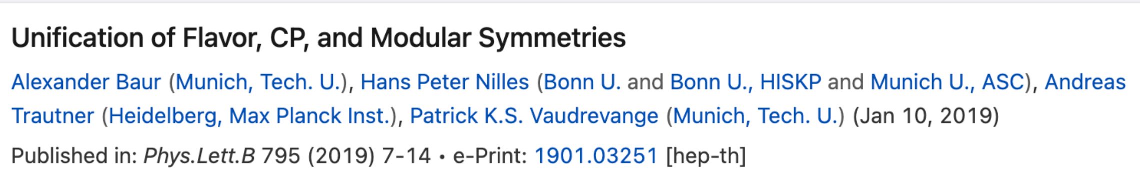
2. $k_{z_a} > 0$

$$\sum_i (k_{D_i^c}+k_{Q_i}) = 0$$

no modular forms of negative weight

absence of mixed modular-gauge anomalies

- string-theory motivated



Unification of Flavor, CP, and Modular Symmetries

Alexander Baur (Munich, Tech. U.), Hans Peter Nilles (Bonn U. and Bonn U., HISKP and Munich U., ASC), Andreas Trautner (Heidelberg, Max Planck Inst.), Patrick K.S. Vaudrevange (Munich, Tech. U.) (Jan 10, 2019)

Published in: *Phys.Lett.B* 795 (2019) 7-14 • e-Print: 1901.03251 [hep-th]

ST has no free parameters. Yukawa couplings are field-dependent quantities
in most ST compactifications 4D CP invariance
modular invariance is a key aspect of most ST compactifications

(other realizations are also possible)

CP & modular-invariance

- CP ↔ real coupling constants $f_3 = \frac{1}{g_3^2}$ $(\theta_{QCD} = 0)$

- modular invariance $SL(2, Z)$

Antonio Marrone talk on 4/12

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}$$

a, b, c, d integers
$ad - bc = 1$

$$Im(\tau) > 0$$

$$\varphi \rightarrow (c\,\tau + d)^{-k_\varphi}\ \varphi$$ matter multiplets

$$V \rightarrow V$$ vector multiplets

- $$w(\tau, \varphi) = U_i^c\, Y_{ij}^u(\tau)\, Q_j H_u + D_i^c\, Y_{ij}^d(\tau)\, Q_j H_d + E_i^c\, Y_{ij}^e(\tau)\, L_j H_d + \cdots$$

$$Y_{ij}^q(\tau) \rightarrow (c\tau + d)^{k_{ij}^q}\, Y_{ij}^q(\tau)$$

$$k_{ij}^u = k_{Q_j} + k_{U_i^c} + k_{H_u}$$

$$k_{ij}^d = k_{Q_j} + k_{D_i^c} + k_{H_d}$$

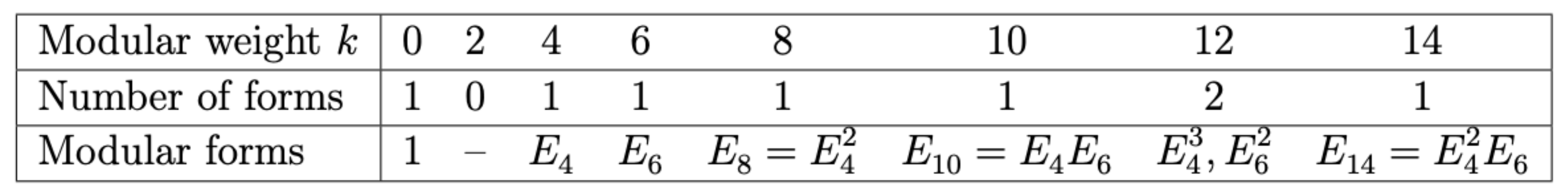
assuming no singularities: $Y_{ij}^q(\tau)$ are modular forms of weight k_{ij}^q

$k^q_{ij} < 0$: no modular forms

$k^q_{ij} = 0$: modular forms are constants

$k^q_{ij} > 0$: modular forms polynomials in $E_4(\tau), E_6(\tau)$

correspond to the z_a fields



Modular weight k	0	2	4	6	8	10	12	14
Number of forms	1	0	1	1	1	1	2	1
Modular forms	1	–	E_4	E_6	$E_8 = E_4^2$	$E_{10} = E_4 E_6$	E_4^3, E_6^2	$E_{14} = E_4^2 E_6$

$$\det Y(\tau) \to (c\tau + d)^{k_{\det}} \quad \det Y(\tau)$$

$$k_{\det} \quad = \sum_{i=1}^{3} \left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c}\right) + 3k_{H_u} + 3k_{H_d}$$

simplest solution for absence of gauge-modular anomalies

$$k_{H_u} + k_{H_d} = 0$$

$$k_{Q_i} = k_{U_i^c} = k_{D_i^c} = k_{L_i} = k_{E_i^c} = (-k, 0, k)$$

$k_{\det} \quad = 0$ → $\det Y(\tau)$ & f real constants

Example $k_{Q_i} = k_{U_i^c} = k_{D_i^c} = (-6{,}0,+6)$

$$Y_q(\tau) = \begin{pmatrix} 0 & 0 & c^q_{13} \\ 0 & c^q_{22} & c^q_{23}E_6 \\ c^q_{31} & c^q_{32}E_6 & c^q_{33}E_4^3 + c^{q\prime}_{33}E_6^2 \end{pmatrix}$$

$\tan\beta = 10 \quad \tau = 0.125 + i$

$$c^u_{ij} \approx 10^{-3} \begin{pmatrix} 0 & 0 & 1.56 \\ 0 & -1.86 & 0.87 \\ 1.29 & 4.14 & 3.51, 1.40 \end{pmatrix}, \qquad c^d_{ij} \approx 10^{-3} \begin{pmatrix} 0 & 0 & 1.55 \\ 0 & -2.59 & 4.59 \\ 0.378 & 0.710 & 0.734, 1.76 \end{pmatrix}$$

reproduce quark masses, mixing angles and CKM phase

$\delta_{CKM} \neq 0$ ⟺ $Im\,\det[Y_u^{+}Y_u, Y_d^{+}Y_d] \neq 0$ non-holomorphic

Leptons: $k_{L_i} = k_{E_i^c} = (-6{,}0,+6)$

$$c^e_{ij} = 10^{-3} \begin{pmatrix} 0 & 0 & 1.29 \\ 0 & 5.95 & 0.35 \\ -2.56 & 1.47 & 1.01, 1.32 \end{pmatrix}, \qquad c^\nu_{ij} = \frac{1}{10^{16}\,\mathrm{GeV}} \begin{pmatrix} 0 & 0 & 3.4 \\ 0 & 7.1 & 1.2 \\ 3.4 & 1.2 & 0.19, 0.95 \end{pmatrix}$$

deviations from $\bar{\theta} = 0$

SUSY unbroken

no corrections from K
no corrections from nonrenormalizable operators: $SL(2,\mathbb{Z})$
no corrections from additional moduli/singlets under reasonable assumptions

SUSY breaking corrections

potentially big if soft terms violate flavour in a generic way

minimized if $\Lambda_{CP} \gg \Lambda_{SUSY}$ (as e.g. in gauge mediation)

and soft breaking terms respect the flavour structure of the SM

$$\bar{\theta} \lesssim \frac{M_t^4 M_b^4 M_c^2 M_s^2}{v^{12}} J_{\rm CP} \tan^6\beta \sim 10^{-28} \tan^6\beta.$$

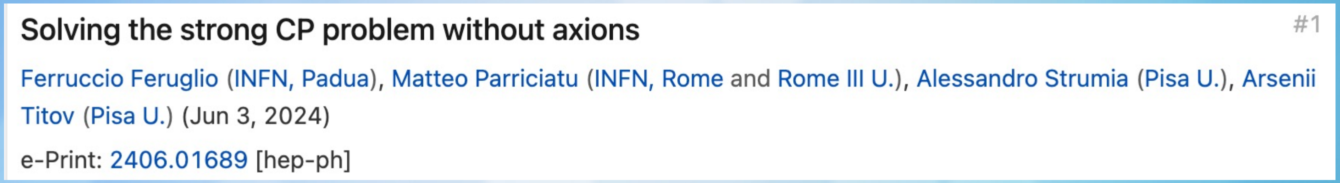
SM corrections

negligible: $\bar{\theta} \leq 10^{-18}$ at four loops

J.R. Ellis, M.K. Gaillard, 'Strong and Weak CP Violation', Nucl.Phys.B 150 (1979) 141.

I.B. Khriplovich, 'Quark Electric Dipole Moment and Induced θ Term in the Kobayashi-Maskawa Model', Phys.Lett.B 173 (1986) 193.

variants



#1

Solving the strong CP problem without axions

Ferruccio Feruglio (INFN, Padua), Matteo Parriciatu (INFN, Rome and Rome III U.), Alessandro Strumia (Pisa U.), Arsenii Titov (Pisa U.) (Jun 3, 2024)

e-Print: 2406.01689 [hep-ph]

- higher levels, smaller weight

modular forms associated with subgroups of $SL(2,Z)$

$$k_{Q_i} = k_{U_i^c} = k_{D_i^c} = (-1,0+1)\ or\ (-2,0+2)$$

perhaps easier to occur in string theory

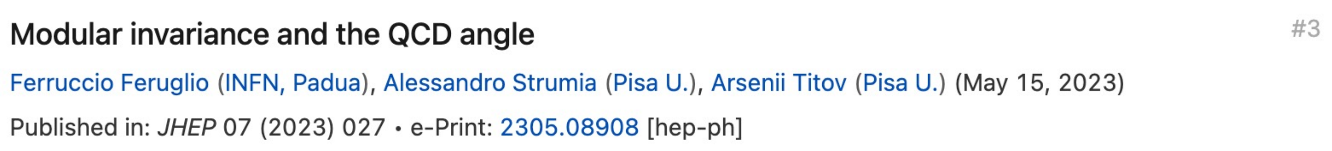
- with heavy vector-like quarks

anomaly of IR theory canceled by a nontrivial gauge kinetic function

$$f_{IR} = f_{UV} - \frac{1}{8\pi^2}\log\det Y_{Heavy}(\tau)$$

many more viable patterns of quark mass matrices

- can be extended to supergravity



#3

Modular invariance and the QCD angle

Ferruccio Feruglio (INFN, Padua), Alessandro Strumia (Pisa U.), Arsenii Titov (Pisa U.) (May 15, 2023)

Published in: *JHEP* 07 (2023) 027 • e-Print: 2305.08908 [hep-ph]

Conclusion

spontaneous CP violation as a solution of the strong CP problem
- supersymmetry
- one extra gauge-invariant supermultiplet

CP is a symmetry of the UV theory
Yukawa couplings are field-dependent
possibly shaped by modular invariance

string theory

if one spin-zero component of the
extra multiplet remains light
it can play the role of DM

CP-violating photo-reluctant ALP = the CPon
likely to be tested in a variety of experiments [ISL, stellar cooling, X-ray excess in globular clusters, flavour-violating meson decays,…]


THANK
YOU!

back-up slides

Ingredients

1. CP in the UV

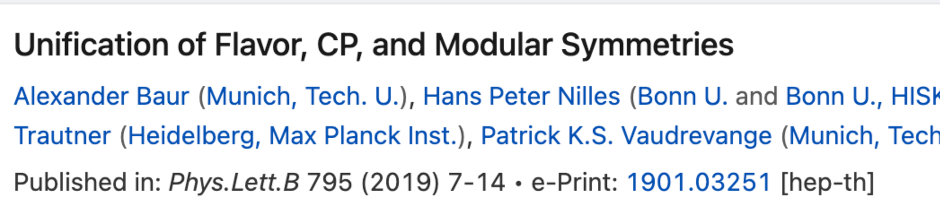
2. Yukawa couplings are field-dependent quantities

3. the vacuum has a redundant description: vacua related by $SL(2,\mathbb{Z})$ are equivalent

4. CP and $SL(2,\mathbb{Z})$ are unified in a gauge flavour symmetry

5. absence of anomalies

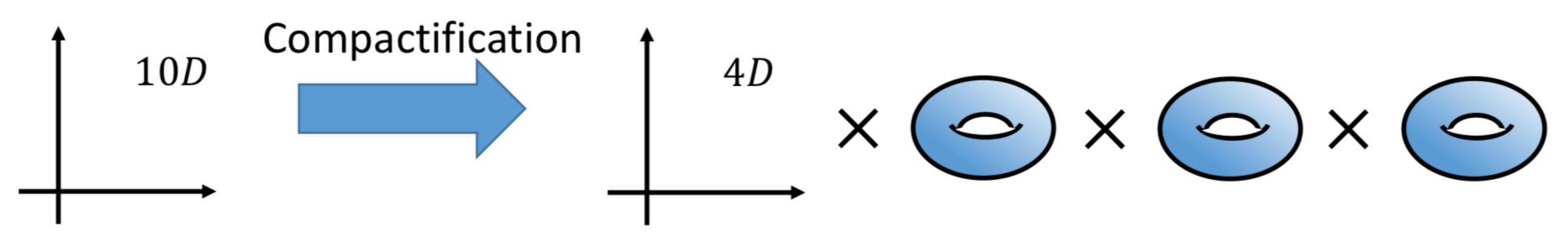
6. no singularities in the UV theory

Ingredients	String Theory
1. CP in the UV	the four-dimensional CP symmetry is a gauge symmetry in most string theory compactifications.
2. Yukawa couplings are field-dependent quantities	string theory has no free parameters an Yukawa couplings are set by moduli VEVs
3. the vacuum has a redundant description: vacua related by $SL(2,\mathbb{Z})$ are equivalent	modular invariance is a key ingredient of string theory compactifications
4. CP and $SL(2,\mathbb{Z})$ are unified in a gauge flavour symmetry	 **Unification of Flavor, CP, and Modular Symmetries** Alexander Baur (Munich, Tech. U.), Hans Peter Nilles (Bonn U. and Bonn U., HISK Trautner (Heidelberg, Max Planck Inst.), Patrick K.S. Vaudrevange (Munich, Tech Published in: *Phys.Lett.B* 795 (2019) 7-14 • e-Print: 1901.03251 [hep-th]
5. absence of anomalies	mandatory in string theory
6. no singularities in the UV theory	string theory is free of singularities. These arise in the IR when some UV modes become massless

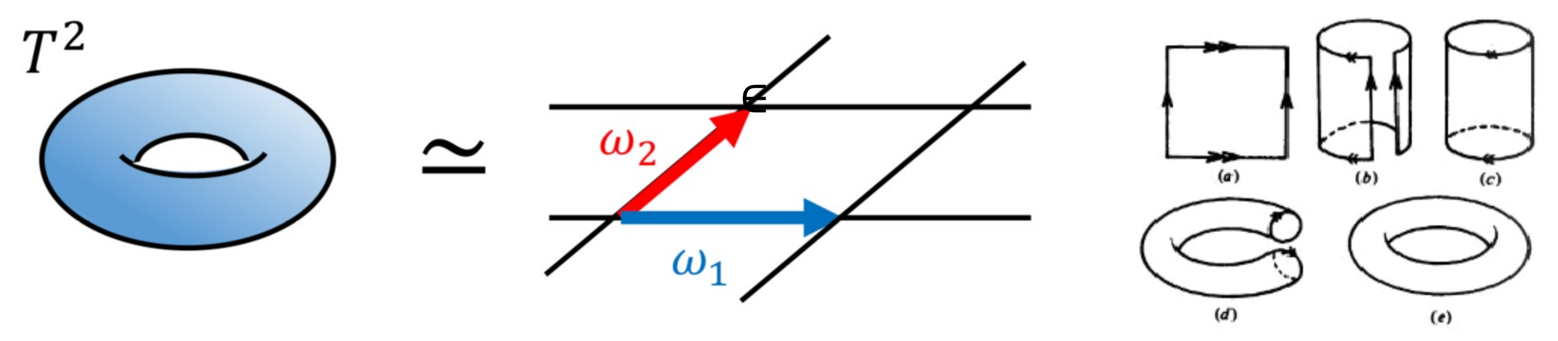
modular invariance

[see H.P. Nilles talk]

string theory in d=10 need 6 compact dimensions



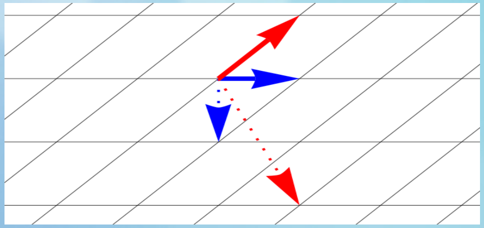
simplest compactification: 3 copies of a torus T^2



tori parametrized by

$$\mathcal{M} = \left\{\tau = \frac{\omega_2}{\omega_1} \quad Im(\tau) > 0\right\}$$

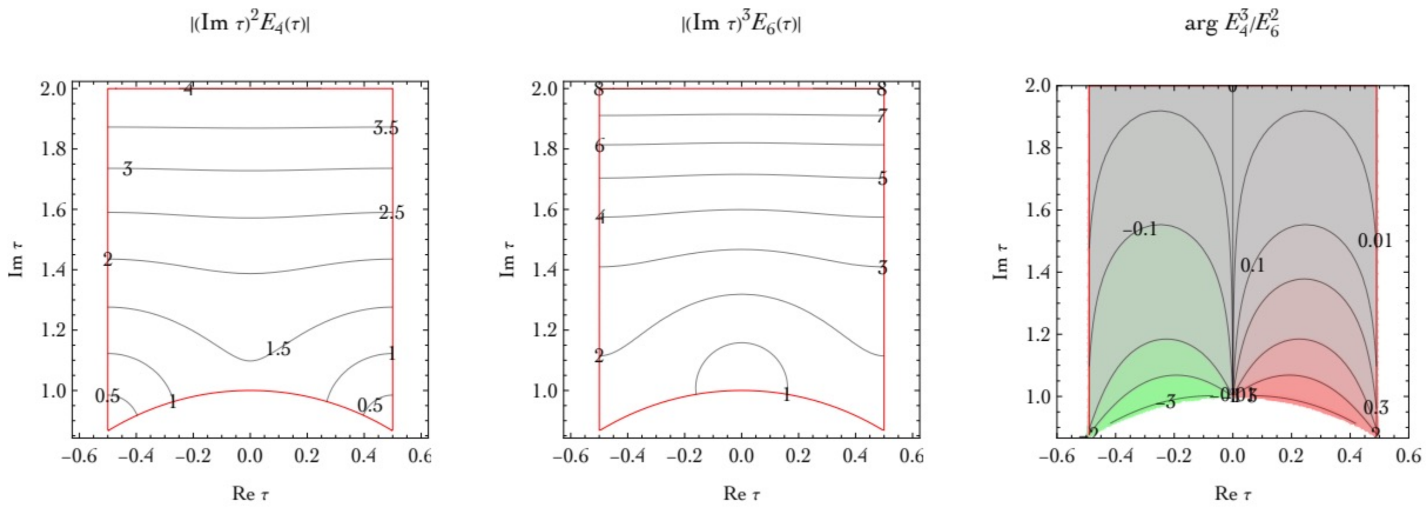
lattice left invariant by modular transformations:



$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \in SL(2, Z)$$

a, b, c, d integers

$ad - bc = 1$

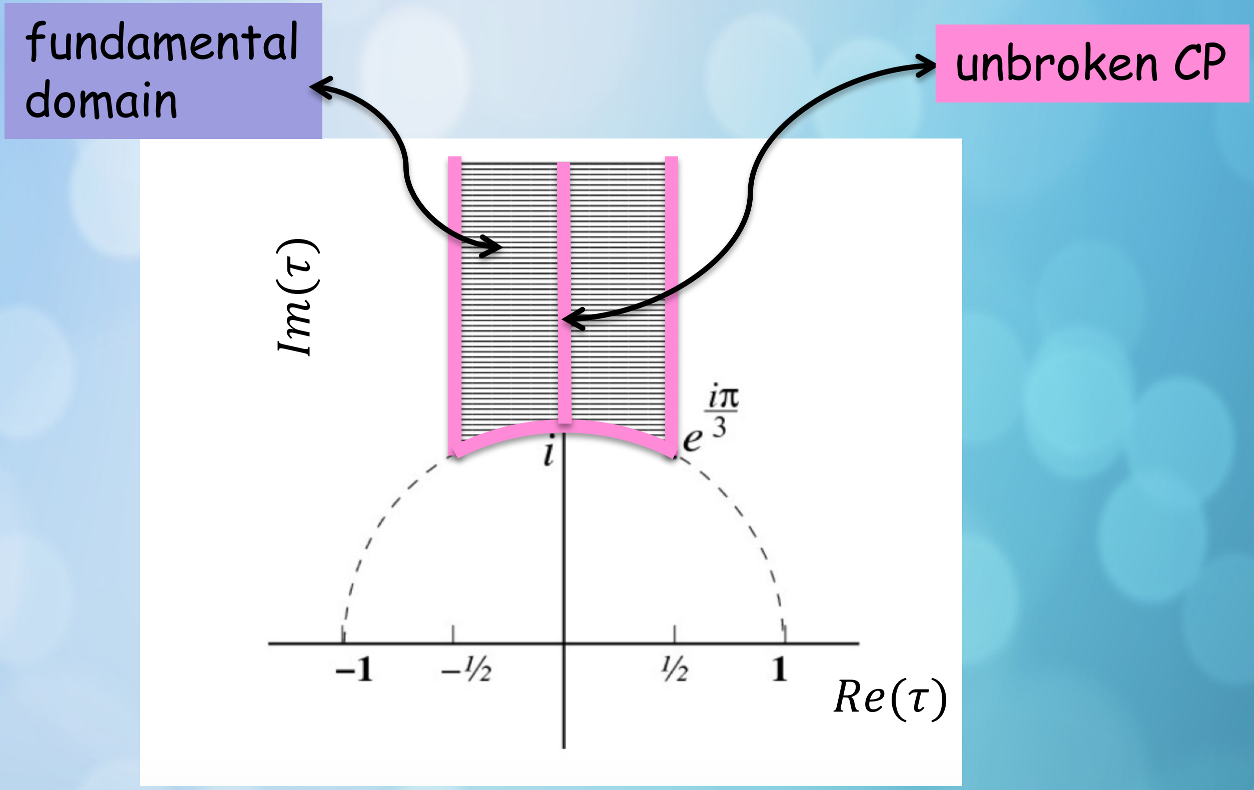


$|(\mathrm{Im}\ \tau)^2 E_4(\tau)|$

$|(\mathrm{Im}\ \tau)^3 E_6(\tau)|$

$\arg E_4^3/E_6^2$

- τ promoted to a field. Through a gauge choice we can restrict τ to the fundamental domain



- CP

$$\tau \rightarrow -\tau^*$$

[up to modular transformations]

[Novichkov, Penedo, Petcov and Titov 1905.11970
Baur, Nilles, Trautner and Vaudrevange, 1901.03251]

axion solution

🟩 $\bar{\theta}$ dynamically relaxed to zero by the axion, would-be GB of a global, anomalous $U(1)_{PQ}$ symmetry

🟩 provides a candidate for DM

🟩 many axion candidates in e.g. superstring theories

🟥 axion quality problem

minimum of $V(a)$ should be at $a = 0$

$$V(a) = V_{QCD}(a) - M^4 e^{-S} \cos(\frac{a}{f_a} + \delta)$$

$$M = M_P$$
$$\delta = \mathcal{O}(1)$$

➡ $S \geq 200$

🟥 axion undetected, so far

Nelson-Barr solution | our solution

CP ia a symmetry of the UV,
SB to get $\bar{\theta} = 0$ & $\delta_{CKM} = \mathcal{O}(1)$

CP ➡ $\theta_{QCD} = 0$

Nelson-Barr solution

- heavy vector-like quark sector

$$\begin{array}{cc} & \begin{array}{c:c} Q & q \end{array} \\ \hline m = & \left(\begin{array}{c:c} \mu & \lambda_a \eta_a \\ 0 & y\, v \end{array}\right) \end{array}$$

- CP spontaneously broken by $\langle \eta_a \rangle$ complex

[one is not enough]

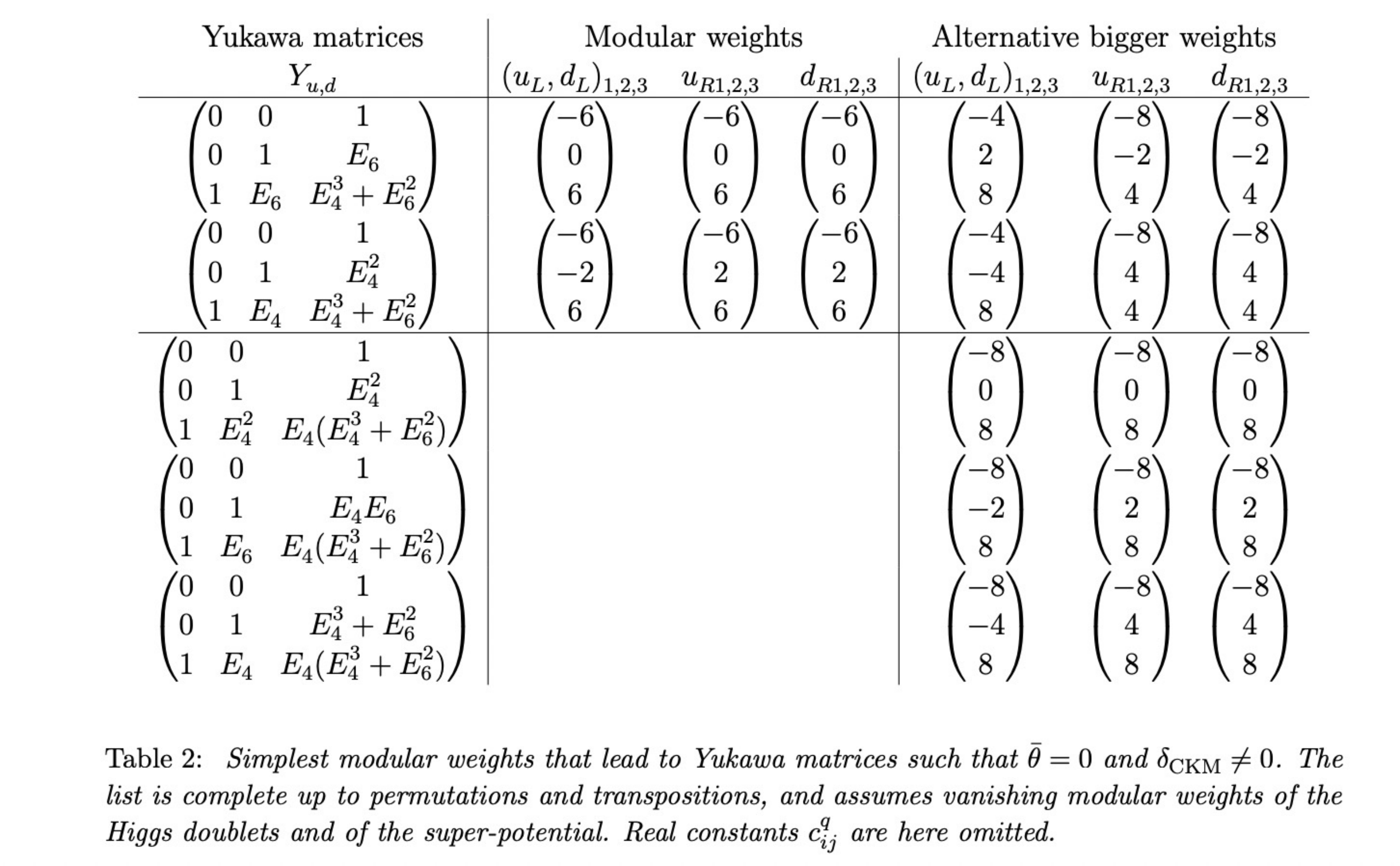
- $\mu \approx \lambda_a \eta_a$ [tuning]

our solution

no extra matter

CP spontaneously broken by τ alone

no tuning



Yukawa matrices	Modular weights			Alternative bigger weights		
$Y_{u,d}$	$(u_L, d_L)_{1,2,3}$	$u_{R1,2,3}$	$d_{R1,2,3}$	$(u_L, d_L)_{1,2,3}$	$u_{R1,2,3}$	$d_{R1,2,3}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_6 \\ 1 & E_6 & E_4^3 + E_6^2 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ 2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ -2 \\ 4 \end{pmatrix}$	$\begin{pmatrix} -8 \\ -2 \\ 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^2 \\ 1 & E_4 & E_4^3 + E_6^2 \end{pmatrix}$	$\begin{pmatrix} -6 \\ -2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^2 \\ 1 & E_4^2 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4 E_6 \\ 1 & E_6 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ -2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 2 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^3 + E_6^2 \\ 1 & E_4 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ -4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 8 \end{pmatrix}$

Table 2: *Simplest modular weights that lead to Yukawa matrices such that $\bar{\theta} = 0$ and $\delta_{\rm CKM} \neq 0$. The list is complete up to permutations and transpositions, and assumes vanishing modular weights of the Higgs doublets and of the super-potential. Real constants c_{ij}^q are here omitted.*

ξ and flavour-violating decays

ξ in this mass range can be produced in flavour-violating meson/lepton decays in a wide class of models

$$y^A_{S,P} \approx \begin{pmatrix} m_{A1} & \sqrt{m_{A1}m_{A2}} & \sqrt{m_{A1}m_{A3}} \\ \sqrt{m_{A1}m_{A2}} & m_{A2} & \sqrt{m_{A2}m_{A3}} \\ \sqrt{m_{A1}m_{A3}} & \sqrt{m_{A2}m_{A3}} & m_{A3} \end{pmatrix}$$

- $$\Gamma(K^+ \to \pi^+\xi) \approx \frac{m_K^3}{16\,\pi\, m_s^2}\frac{\left|y^D_{12}\right|^2}{\Lambda^2}$$

$y^D_{S12} \approx 10\ MeV$

[NA62]

$BR(K^+ \to \pi^+\xi) \leq 5{\times}10^{-11}$ ➡ $\Lambda > 10^{11}\ GeV$

- $$BR(\mu^+ \to e^+\xi) = \frac{6\pi^2}{G_F^2 m_\mu^4}\frac{\left(y^E_{S12}\right)^2 + (y^E_{P12})^2}{\Lambda^2}$$

$y^E_{S,P12} \approx 10\ MeV$

[TRIUMF]

$BR(\mu^+ \to e^+\xi) \leq 2.5{\times}10^{-6}$ ➡ $\Lambda > 10^8\ GeV$

fixed point

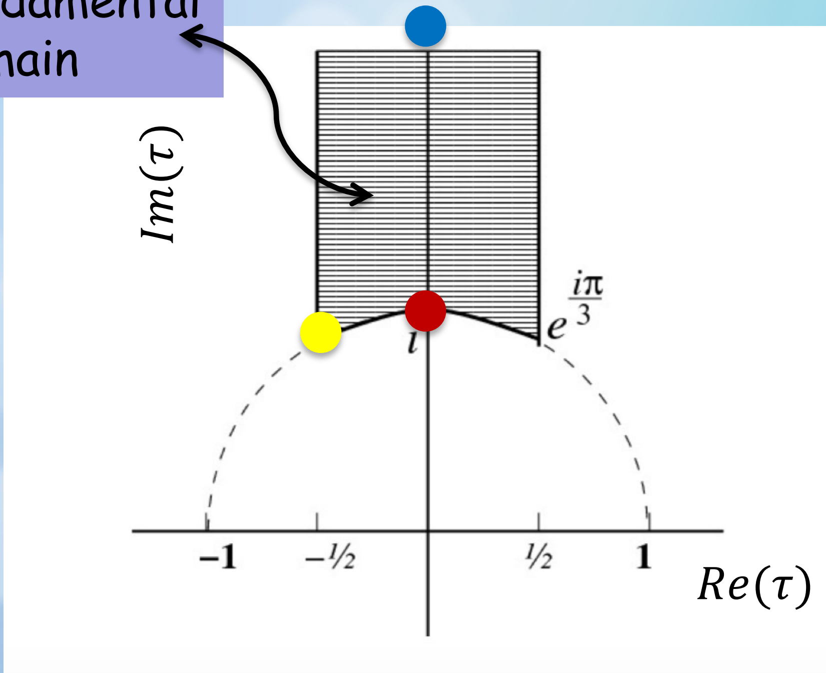
● $\tau = i$ $\quad \tau \xrightarrow{S} -\frac{1}{\tau}$ $\quad \mathbb{Z}_4^S$

● $\tau = e^{i\,2\pi/3}$ $\quad \tau \xrightarrow{ST} -\frac{1}{\tau+1}$ $\quad \mathbb{Z}_2^{ST} \times \mathbb{Z}_2^{S^2}$

● $\tau = i\,\infty$ $\quad \tau \xrightarrow{T} \tau + 1$ $\quad \mathbb{Z}^T \times \mathbb{Z}_2^{S^2}$

residual symmetry

fundamental domain



modular invariance completely broken everywhere but at three fixed points

$SL(2,Z)$ generated by

$$S : \tau \to -\frac{1}{\tau} \quad , \qquad T : \tau \to \tau + 1$$

heavy quarks and singularities

- heavy quarks not needed, but they can exist in the UV

  example

  $$k_\varphi = (\underbrace{-6,-2,0}_{\text{chiral}},\underbrace{+2,+6}_{\text{heavy vector-like quark}})$$

  $$k_{H_u} + k_{H_d} = 0$$

- UV theory $$\bar{\theta} = -8\pi^2 Im\, f_{UV} + \arg\det Y_{UV} = 0$$

- IR theory has an anomalous field content, anomaly cancelled by:

  $$f_{IR} = f_{UV} - \frac{1}{8\pi^2}\log\det Y_{Heavy}(\tau)$$

$$\bar{\theta} = -8\pi^2 Im\, f_{IR}(\tau) + \arg\det Y_{Light}(\tau) =$$

$$= +\arg\det Y_{Heavy}(\tau) + \arg\det Y_{Light}(\tau)$$

$$= \arg\det Y_{UV} = 0$$

- $Y_{Light}(\tau)$ is singular at τ values such that $\det Y_{Heavy}(\tau) = 0$

$\mathcal{N} = 1$ supergravity

$$K = -h^2 \log(-i\tau + i\tau^+) + \cdots$$

corrections of $\mathcal{O}(k_W)$?

$$k_W = \frac{h^2}{M_{Pl}^2} \to 0$$

back to the rigid case

- K and w no more independent

$$\mathcal{G} = \frac{K}{M_{Pl}^2} + \log\left|\frac{w}{M_{Pl}^3}\right|^2$$

$$w(\tau) \to (c\tau + d)^{-k_W}\, w(\tau)$$

$$k_W > 0$$

no negative weight modular forms, $w(\tau)$ singular somewhere

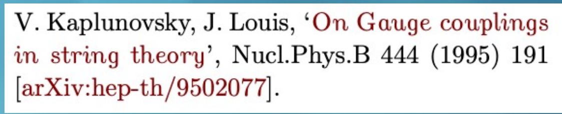
- modular-QCD anomaly modified into

$$\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c} - 2k_W\right) + 3k_W$$

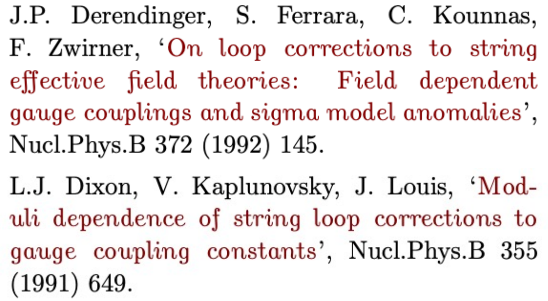
quarks

gluino

can be rotated away
if gluino is massless



V. Kaplunovsky, J. Louis, ‘On Gauge couplings in string theory’, Nucl.Phys.B 444 (1995) 191 [arXiv:hep-th/9502077].



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L.J. Dixon, V. Kaplunovsky, J. Louis, ‘Moduli dependence of string loop corrections to gauge coupling constants’, Nucl.Phys.B 355 (1991) 649.

spontaneously broken supergravity

- $\bar{\theta} = -8\pi^2 Im\, f + \arg\det M_{quark} + 3\arg M_3 = 0$

$\arg\det M_{quark} = 0$ ⇐ $\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c} - 2k_W\right) = k_{H_u} + k_{H_d} = 0$

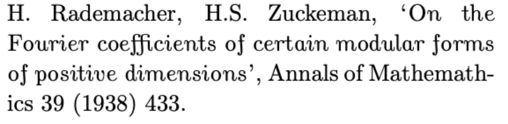
$\arg M_3 = -\arg w$ ⇐ if no other phases from SUSY breaking $M_3 = \frac{1}{2} e^{\frac{K}{2M_{Pl}^2}} K^{i\bar{j}} D_{\bar{j}} w^{+} f_i$

- assume unique singularity at $\tau = i\infty$

$$w(\tau) = \cdots + c_0 M_{Pl}^3\, \eta(\tau)^{-2k_W}$$

$$f = \cdots + 3\frac{k_W}{4\pi^2}\log\eta(\tau)$$

$\eta(\tau)$ Dedekind eta function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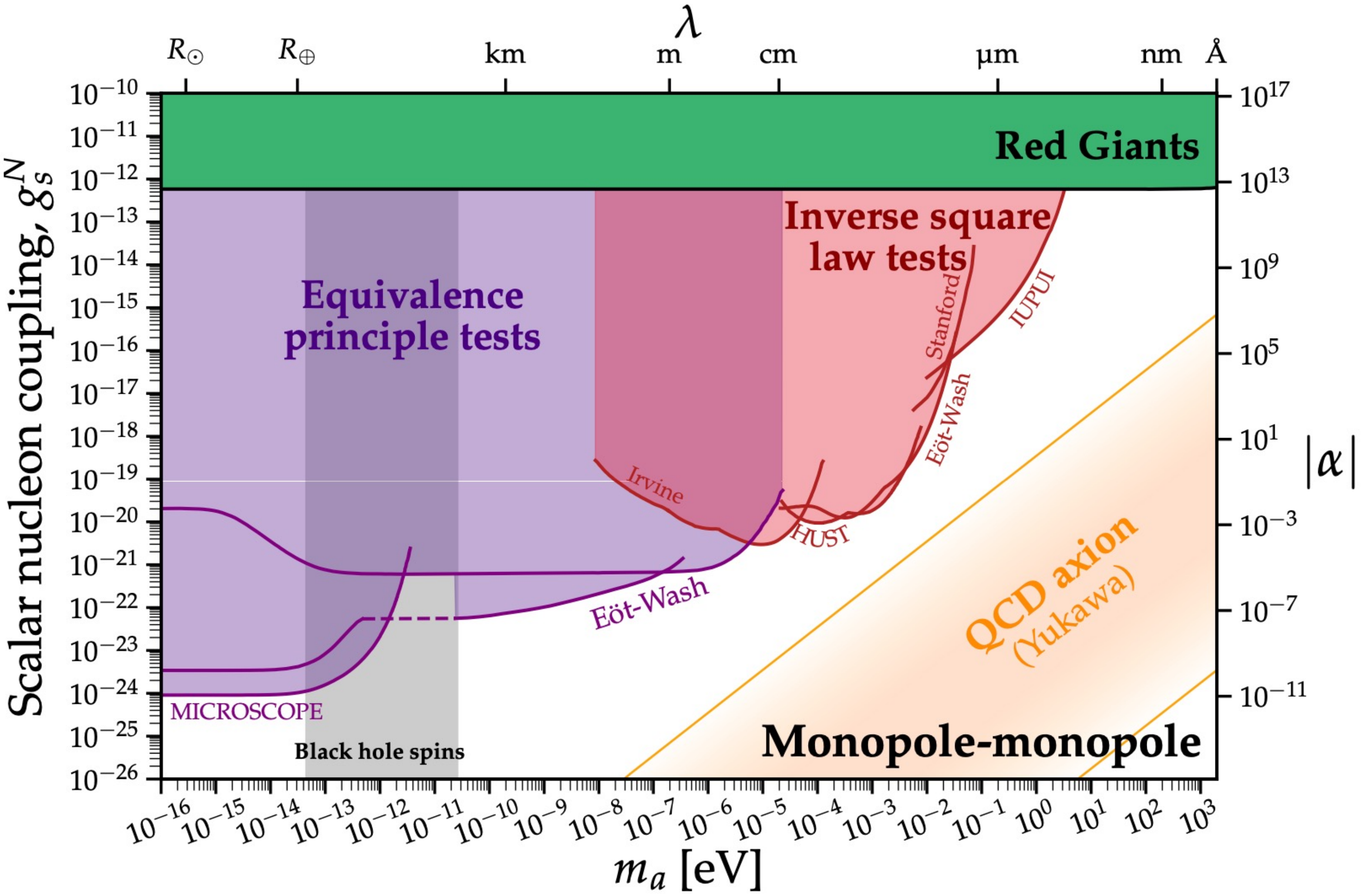
cancels the gluino anomaly

$$\bar{\theta} = -8\pi^2 Im\, f + 3\arg M_3 = 0$$


λ
$R_\odot$
$R_\oplus$
km
m
cm
μm
nm
Å
Scalar nucleon coupling, g_s^N
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
10^{-26}
10^{17}
10^{13}
10^{9}
10^{5}
10^{1}
10^{-3}
10^{-7}
10^{-11}
$|\alpha|$
Red Giants
Inverse square law tests
Equivalence principle tests
Stanford
IUPUI
Eöt-Wash
Irvine
HUST
Eöt-Wash
MICROSCOPE
Black hole spins
QCD axion
(Yukawa)
Monopole-monopole
10^{-16}
10^{-15}
10^{-14}
10^{-13}
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
m_a [eV]

Stellar cooling bounds on new light particles: plasma mixing effects

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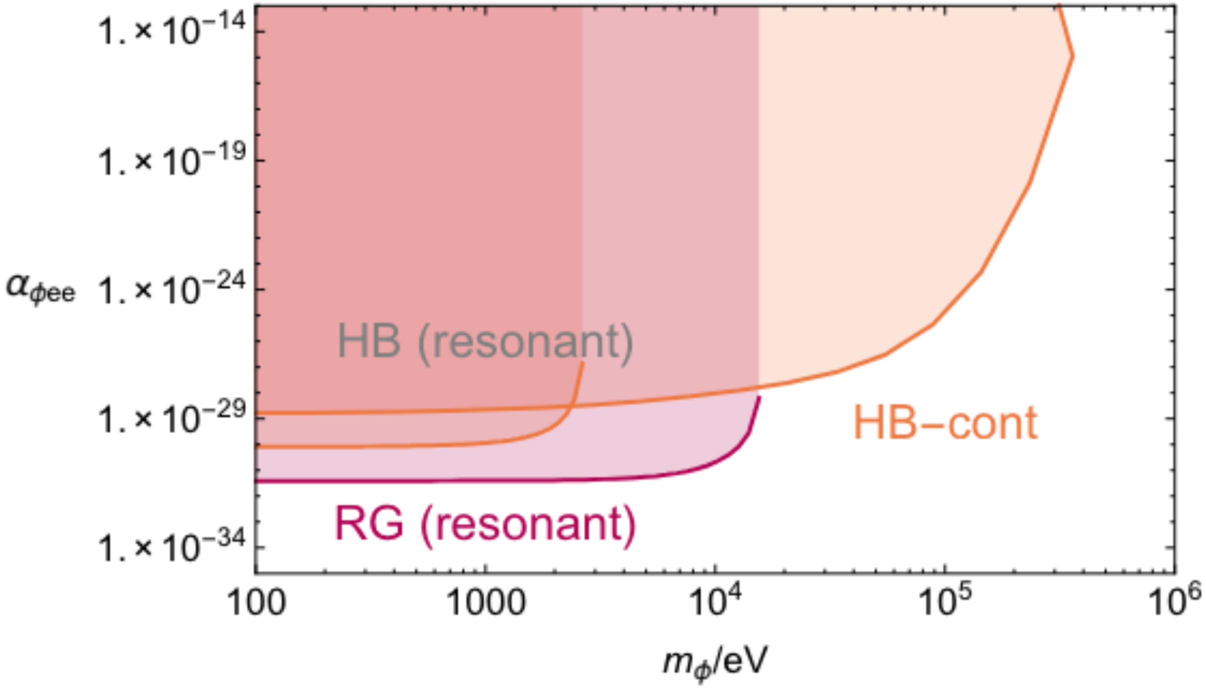
Edward Hardy,[a] **Robert Lasenby**[b]



viable patterns of Yukawas

$$\begin{pmatrix} Y_{11}^{(0)} & Y_{13}^{(p_1-p_2)} & Y_{13}^{(p_1-p_3)} \\ 0 & Y_{22}^{(0)} & Y_{23}^{(p_2-p_3)} \\ 0 & 0 & Y_{33}^{(0)} \end{pmatrix}.$$

$$\begin{pmatrix} Y_{11}^{(0)} & Y_{12}^{(0)} & Y_{13}^{(p_1-p_3)} \\ Y_{21}^{(0)} & Y_{22}^{(0)} & Y_{23}^{(p_1-p_3)} \\ 0 & 0 & Y_{33}^{(0)} \end{pmatrix}.$$

$$\begin{pmatrix} Y_{11}^{(0)} & Y_{12}^{(0)} & Y_{13}^{(0)} \\ Y_{21}^{(0)} & Y_{22}^{(0)} & Y_{23}^{(0)} \\ Y_{31}^{(0)} & Y_{32}^{(0)} & Y_{33}^{(0)} \end{pmatrix}.$$

DM abundance from misalignment

High reheating temperature

$$\Omega_\xi h^2|^{\rm RD}_{\rm mis} \approx 0.12 \left(\frac{\Lambda\theta_0}{1.1\times10^{11}\,{\rm GeV}}\right)^2 \left(\frac{M}{\rm keV}\right)^{1/2}$$

Low reheating temperature & early matter domination

$$\Omega_\xi h^2|^{\rm EMD}_{\rm mis} \approx 0.12 \left(\frac{\Lambda\theta_0}{6.7\times10^{11}\,{\rm GeV}}\right)^2 \left(\frac{T_R}{18\,{\rm TeV}}\right)$$

DM abundance from freeze-in

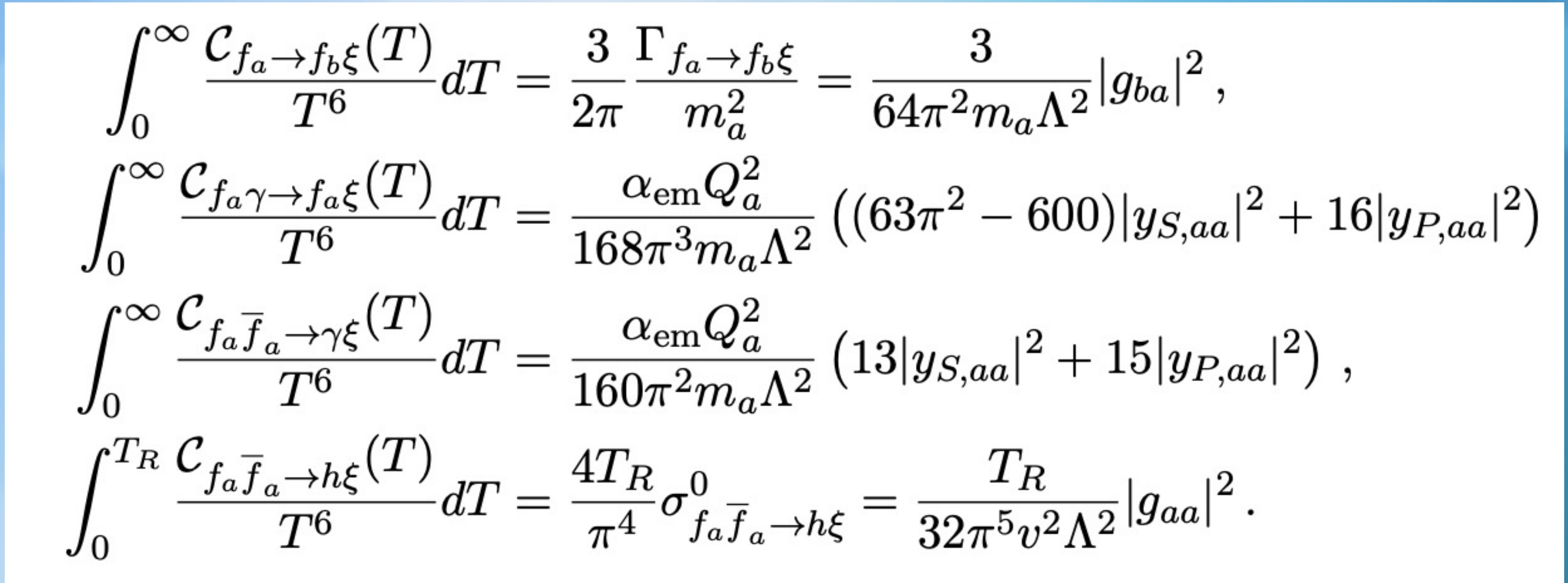
$$\Omega_\xi h^2 = \sum_i \frac{4.6 \times 10^{27}}{g_{*s}\sqrt{g_*}} M \int_0^{T_R} \frac{\mathcal{C}_i(T)}{T^6} dT$$

$$\mathcal{C}_{f_a \to f_b \xi} = \frac{T m_a^2}{\pi^2} K_1\left(\frac{m_a}{T}\right) \Gamma_{f_a \to f_b \xi}\,,$$

$$\mathcal{C}_{f_a \gamma \to f_a \xi} = \frac{T}{8\pi^4} \int_{m_a^2}^{\infty} \left(1 - \frac{m_a^2}{s}\right)^2 s^{3/2} \sigma_{f_a \gamma \to f_a \xi}(s) K_1\left(\frac{\sqrt{s}}{T}\right) ds$$

$$\mathcal{C}_{f_a \bar{f}_a \to \gamma \xi} = \frac{T}{8\pi^4} \int_{4m_a^2}^{\infty} \left(1 - \frac{4m_a^2}{s}\right) s^{3/2} \sigma_{f_a \bar{f}_a \to \gamma \xi}(s) K_1\left(\frac{\sqrt{s}}{T}\right) ds\,,$$

$$\mathcal{C}_{f_a \bar{f}_a \to h \xi} = 2\mathcal{C}_{f_a h \to f_a \xi} = \frac{T}{8\pi^4} \int_0^{\infty} s^{3/2} \sigma^0_{f_a \bar{f}_a \to h \xi} K_1\left(\frac{\sqrt{s}}{T}\right) ds\,,$$

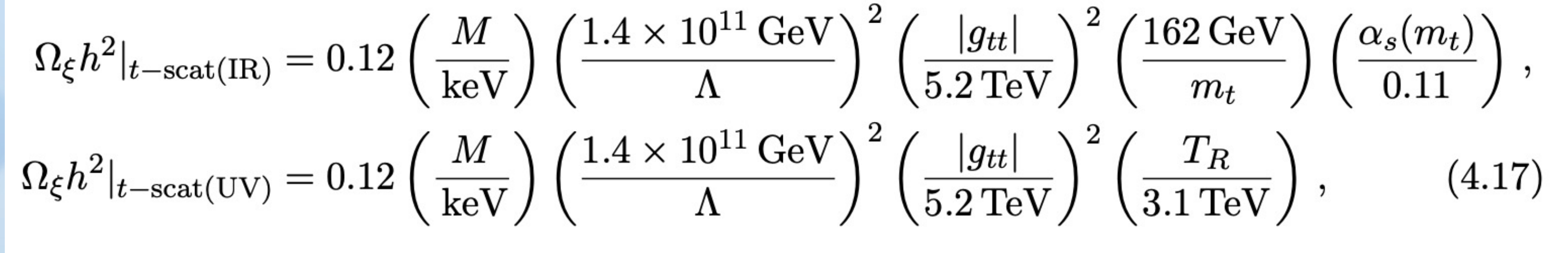


$$\int_0^{\infty} \frac{\mathcal{C}_{f_a \to f_b \xi}(T)}{T^6} dT = \frac{3}{2\pi} \frac{\Gamma_{f_a \to f_b \xi}}{m_a^2} = \frac{3}{64\pi^2 m_a \Lambda^2} |g_{ba}|^2 \,,$$

$$\int_0^{\infty} \frac{\mathcal{C}_{f_a \gamma \to f_a \xi}(T)}{T^6} dT = \frac{\alpha_{\rm em} Q_a^2}{168\pi^3 m_a \Lambda^2} \left((63\pi^2 - 600)|y_{S,aa}|^2 + 16|y_{P,aa}|^2\right)$$

$$\int_0^{\infty} \frac{\mathcal{C}_{f_a \bar{f}_a \to \gamma \xi}(T)}{T^6} dT = \frac{\alpha_{\rm em} Q_a^2}{160\pi^2 m_a \Lambda^2} \left(13|y_{S,aa}|^2 + 15|y_{P,aa}|^2\right) \,,$$

$$\int_0^{T_R} \frac{\mathcal{C}_{f_a \bar{f}_a \to h \xi}(T)}{T^6} dT = \frac{4T_R}{\pi^4} \sigma^0_{f_a \bar{f}_a \to h\xi} = \frac{T_R}{32\pi^5 v^2 \Lambda^2} |g_{aa}|^2 \,.$$



$$\Omega_\xi h^2|_{t-\text{scat(IR)}} = 0.12 \left(\frac{M}{\text{keV}}\right) \left(\frac{1.4 \times 10^{11}\,\text{GeV}}{\Lambda}\right)^2 \left(\frac{|g_{tt}|}{5.2\,\text{TeV}}\right)^2 \left(\frac{162\,\text{GeV}}{m_t}\right) \left(\frac{\alpha_s(m_t)}{0.11}\right) ,$$

$$\Omega_\xi h^2|_{t-\text{scat(UV)}} = 0.12 \left(\frac{M}{\text{keV}}\right) \left(\frac{1.4 \times 10^{11}\,\text{GeV}}{\Lambda}\right)^2 \left(\frac{|g_{tt}|}{5.2\,\text{TeV}}\right)^2 \left(\frac{T_R}{3.1\,\text{TeV}}\right) , \qquad (4.17)$$

total DM abundance

$$\Omega_\xi h^2|_{\rm tot}^{\rm NAT0} = \left(\frac{M}{\rm keV}\right)\left(\frac{10^{12}\,{\rm GeV}}{\Lambda}\right)^2\left(5.6\times10^{-6}+0.17\frac{T_R}{10^8\,{\rm GeV}}\right)+\Omega_\xi h^2|_{\rm mis}\,,$$

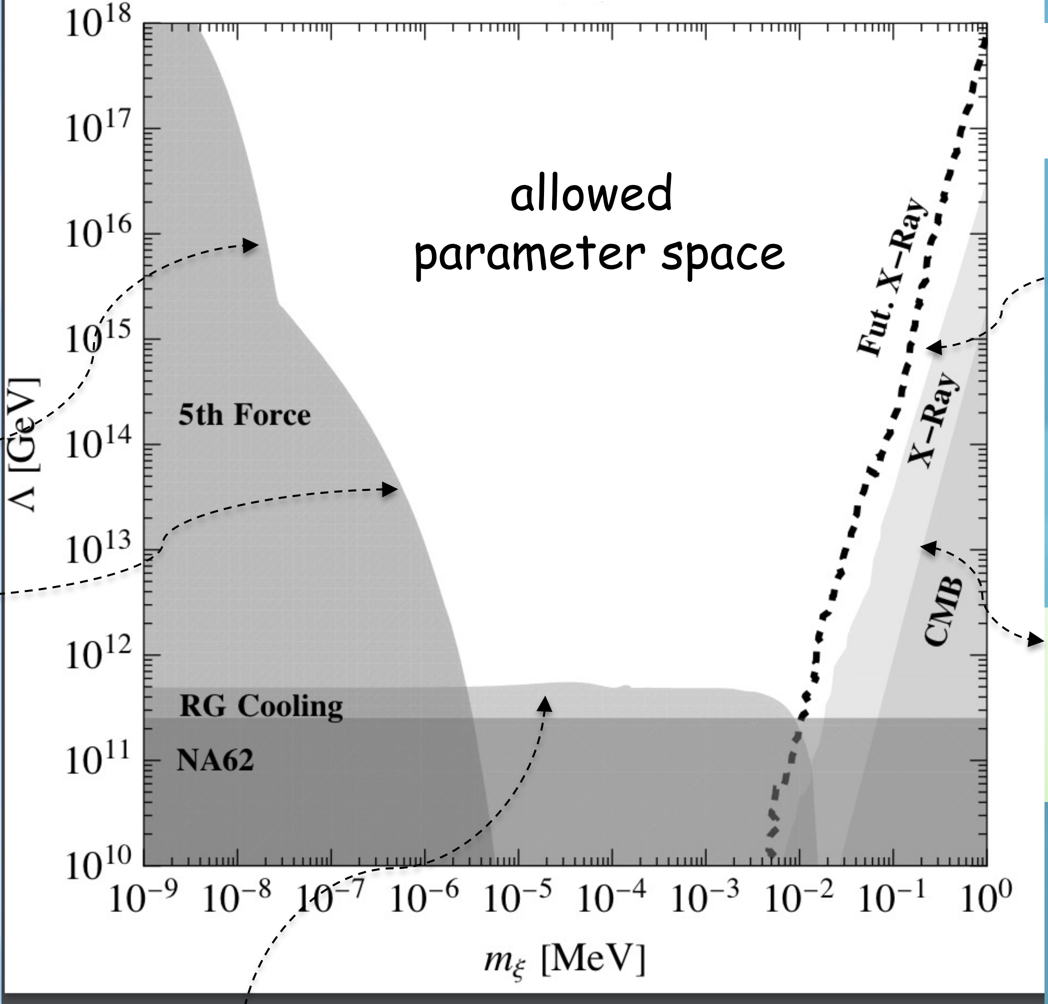
$$\Omega_\xi h^2|_{\rm tot}^{\rm FIT0} = \left(\frac{M}{\rm keV}\right)\left(\frac{10^{12}\,{\rm GeV}}{\Lambda}\right)^2\left(3.6\times10^{-2}+0.11\frac{T_R}{10^4\,{\rm GeV}}\right)+\Omega_\xi h^2|_{\rm mis}\,,$$

$$\Omega_\xi h^2|_{\rm mis} = 0.12\,\theta_0^2\begin{cases}\left(\frac{\Lambda}{1.1\times10^{11}\,{\rm GeV}}\right)^2\left(\frac{M}{\rm keV}\right)^{1/2} & T_R \geq 6.7\times10^5\,{\rm GeV}\sqrt{M/\,{\rm keV}}\\ \left(\frac{\Lambda}{6.7\times10^{11}\,{\rm GeV}}\right)^2\left(\frac{T_R}{18\,{\rm TeV}}\right) & T_R < 6.7\times10^5\,{\rm GeV}\sqrt{M/\,{\rm keV}}\end{cases}$$

$$\alpha \propto (g_{\xi NN})^2$$
$$\approx \left(\frac{2}{27}\frac{m_N}{\Lambda}\right)^2$$

$$r = 1/M$$

$$\delta V_{ISL}(r) = -\frac{G m_1 m_2}{r}\alpha e^{-r/\lambda} .$$



$$\Gamma(\xi \to \gamma\gamma) \propto \frac{\alpha^2\, m_\xi{}^3}{\Lambda^2}\frac{m_\xi{}^4}{m_e^4}$$

X-rays diffuse
emissions
from DM decay
in galaxy clusters

CMB distortion from
energy of DM decay
or annihilation

limits from
Inverse
Square
Law
of gravity

stellar energy loss
in Red Giants

$$\frac{Y_{See}}{\Lambda} < 7\times 10^{-16}$$

[here $Y_{See} \approx m_e$]

only a mathematical trick?
more about this
in a moment…

cancellation of modular anomalies

$$\psi_{can} \to \left(\frac{c\tau + d}{c\tau^{+} + d}\right)^{-\frac{k_\varphi}{2}} \psi_{can}$$

conditions for gauge-modular anomaly cancellation

$SU(3)$ $$\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c}\right) = 0$$

$SU(2)$ $$\sum_{i=1}^{3}(3k_{Q_i} + k_{L_i}) + k_{H_u} + k_{H_d} = 0$$

$U(1)$ $$\sum_{i=1}^{3}\left(k_{Q_i} + 8k_{U_i^c} + 2k_{D_i^c} + 3k_{L_i} + 6k_{E_i^c}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0$$

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$U(1)$ $$\sum_{i=1}^{3}\left(k_{Q_i} + 8k_{U_i^c} + 2k_{D_i^c} + 3k_{L_i} + 6k_{E_i^c}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0$$

simplest solution:

$$k_{H_u} + k_{H_d} = 0$$

$$k_{Q_i} = k_{U_i^c} = k_{D_i^c} = k_{L_i} = k_{E_i^c} = (-k, 0, k)$$

$$k_{\det} = 0$$

$\det Y(\tau)$ & f real constants

$$\bar{\theta} = -8\pi^2 Im\, f + \arg\det Y = 0$$

The **Chandra X-ray Observatory** is a space-based telescope designed to observe X-rays from high-energy regions of the universe, such as the remnants of exploded stars, clusters of galaxies, and matter around black holes. It was launched on **July 23, 1999**, aboard the Space Shuttle *Columbia* as part of NASA's Great Observatories program.

The **XMM-Newton Observatory**, also known as the **X-ray Multi-Mirror Mission (XMM)**, is a European Space Agency (ESA) space telescope designed to observe X-rays from high-energy sources in the universe. It was launched on **December 10, 1999**, aboard an Ariane 5 rocket. Named after **Sir Isaac Newton**, it is one of the most sensitive X-ray telescopes ever built.

The **Nuclear Spectroscopic Telescope Array (NuSTAR)** is a NASA space telescope designed to observe high-energy X-rays from some of the most energetic phenomena in the universe. It was launched on **June 13, 2012**, aboard a Pegasus XL rocket and is the first telescope to focus on the hard X-ray part of the spectrum (energies ranging from 3 to 79 keV).

The **INTEGRAL Telescope** (short for the **International Gamma-Ray Astrophysics Laboratory**) is a space-based observatory launched by the **European Space Agency (ESA)** on **October 17, 2002**. Its mission is to study the most energetic and exotic phenomena in the universe by observing gamma rays, X-rays, and visible light. INTEGRAL is one of the most sensitive gamma-ray telescopes ever launched and has significantly advanced our understanding of high-energy astrophysics. INTEGRAL is designed to detect radiation over a wide range of energies, from **3 keV (X-rays)** to **10 MeV (gamma rays)**.