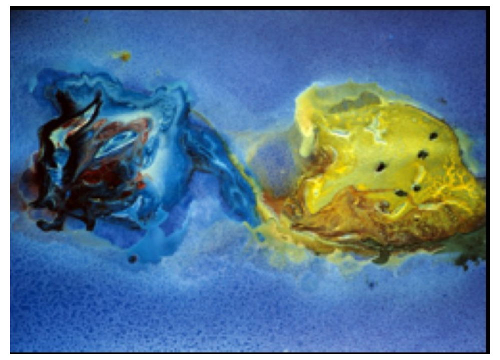
One Born-Oppenheimer EFT to rule them all: hybrids, tetraquarks, pentaquarks, quarkonium and doubly heavy baryons

Nora Brambilla

TUMQCD LATTICE Collaboration

TUM-IAS Focus Group Physics with Effective Field Theories



Quark Confinement and the Hadron Spectrum since 1994

Munich Data Science Institute

The exotic XYZ states represent a revolution in particle physics:
besides quarkonium we have for the first time tetraquarks, pentaquarks, hybrids

They have the potential to give us information on the fundamental strong force

They represent a big challenge for theory

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Focus of the talk

We introduce a QCD derived nonrelativistic effective field theory: the Born Oppenheimer EFT (BOEFT) that can address in the same framework quarkonium, tetraquarks, pentaquarks, hybrids and doubly heavy baryons

The BOEFT is based on symmetries and factorization

- It allows for QCD perturbative calculations at short distance
- It factorizes long distance in few flavour independent correlators to be calculated on the lattice

Factorization allows for model independent predictions

Examples of application includes:
quarkonium, tetraquarks
the X(3872)and the Tcc with insight in their nature
hybrids

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Novel tools to bridge perturbative methods with lattice QCD are key to this program, as well as the combination between different EFTs

Based on:

N.B. , A Mohapatra, T. Scirpa, A. Vairo, 'The nature of X(3827) and Tcc' 2411.14306

-> obtains equations for all cases

M. Berwein, N.B. , A. Mohapatra, A. Vairo, 2408.04719, in press on PRD, Editor's suggestion

M. Berwein, N.B. , J. Tarrus, A. Vairo, 1510.04299 -> establishes BOEFT

N.B. , G. Krein, J. Tarrus, A. Vairo, 1707.09647 -> generalizes to all cases

N.B. , W.K. Lai, J. Segovia, J. Tarrus, A. Vairo, 1805.07713

N.B. , W.K. Lai, J. Segovia, J. Tarrus 1908.11699 -> spin corrections (hybrids)

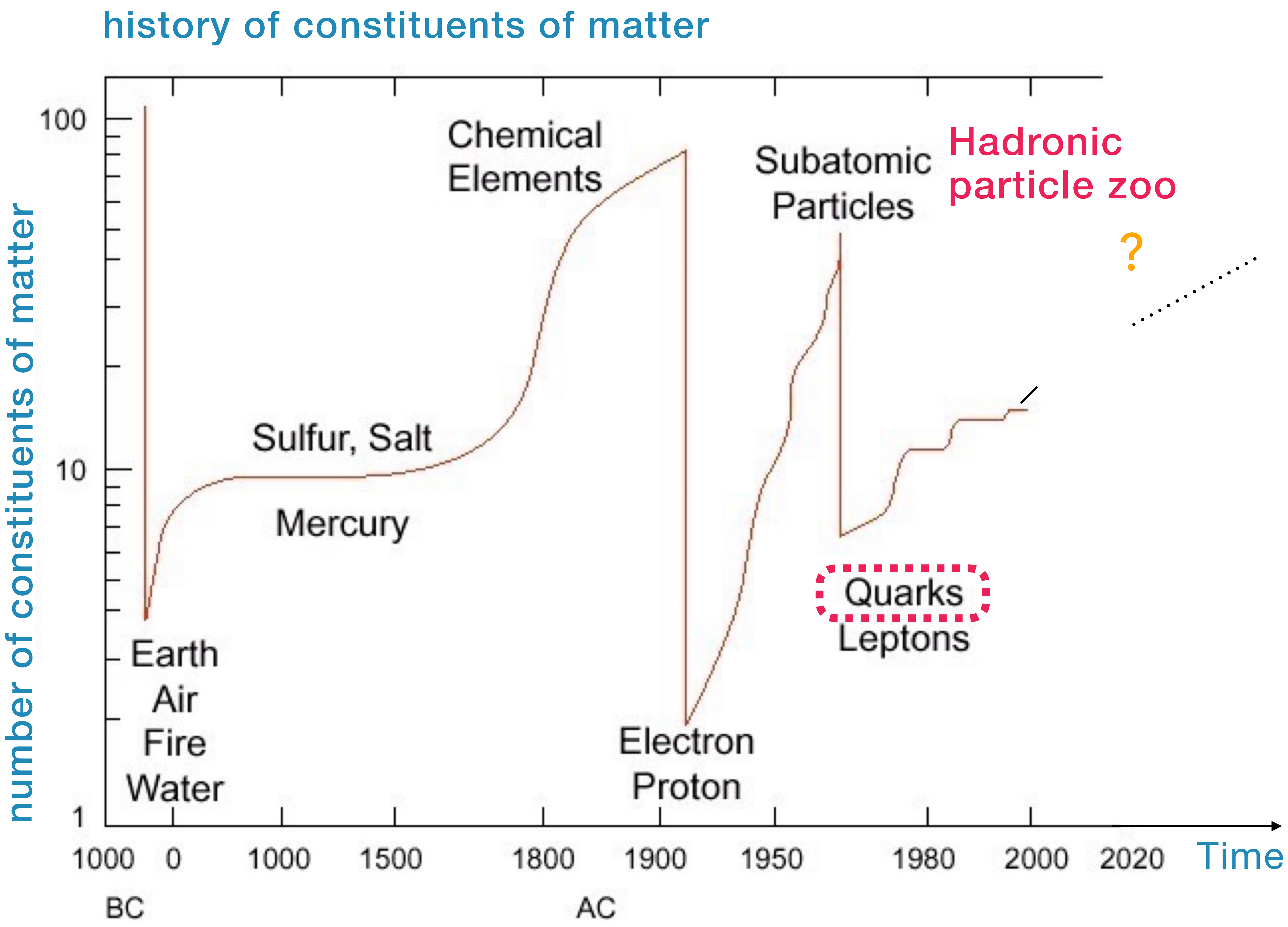
N.B. , W.K. Lai, A. Mohapatra, A. Vairo 2212.09187 -> semi-inclusive decays (hybrids)

N.B. , J. Soto, A. Pineda, A. Vairo hep-ph/9907240

N.B. , J. Soto, A. Pineda, A. Vairo hep-ph/0410047 -> quarkonium strongly coupled pNRQCD

N.B. , J. Soto, A. Pineda, A. Vairo hep-ph/0410047 the XYZ are a revolution in particle physics

Constituents of matter and fundamental forces



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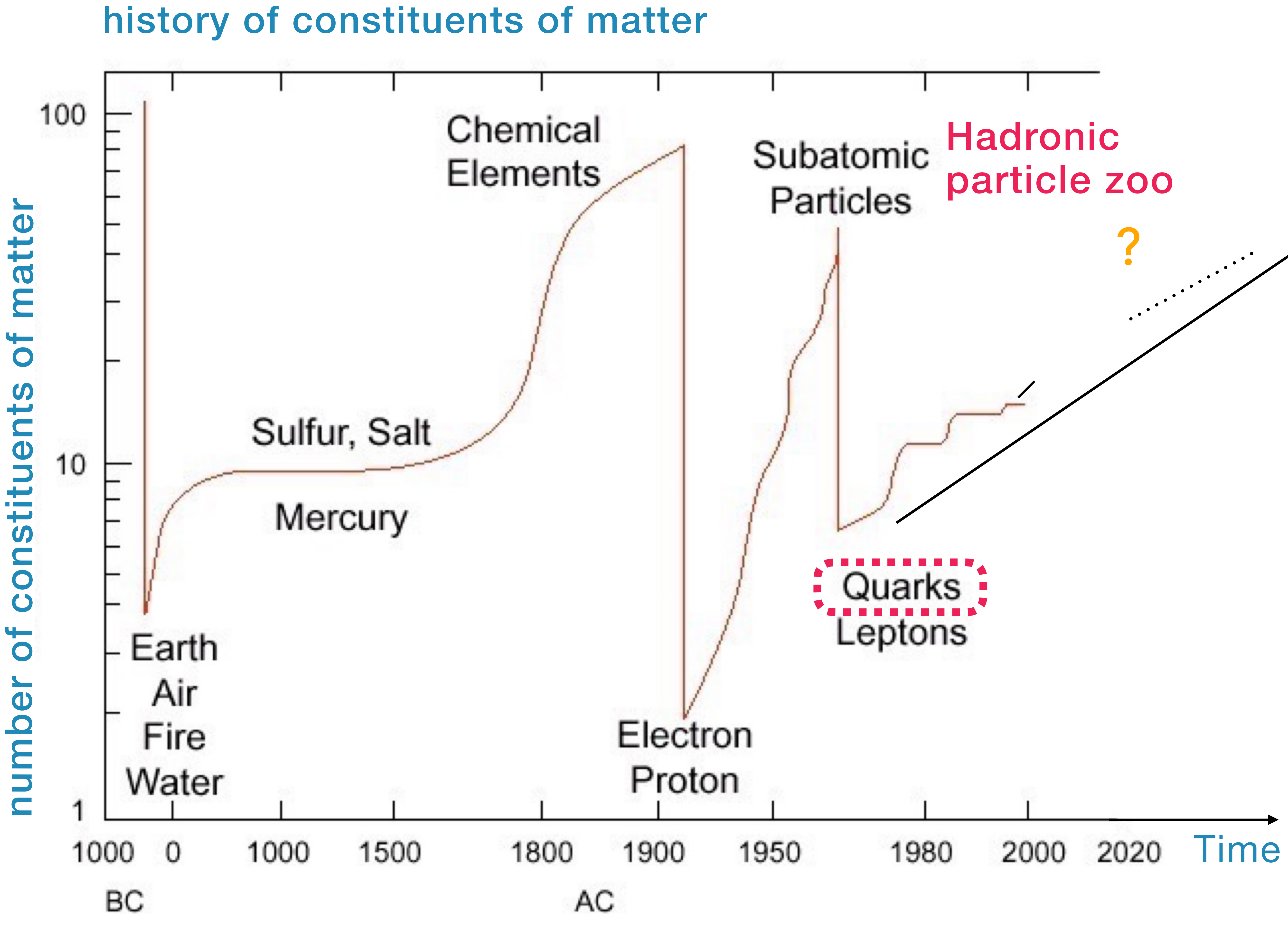
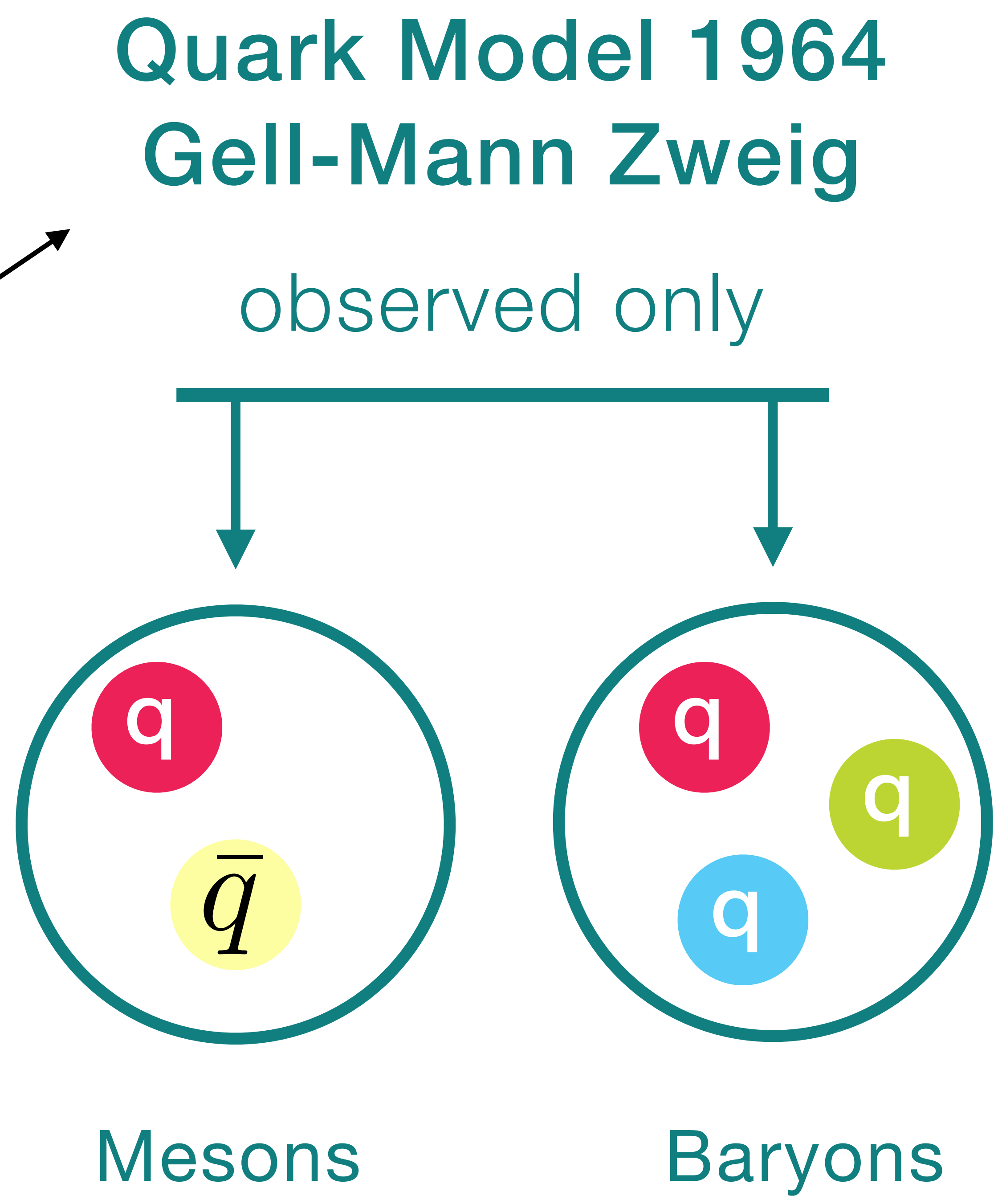




$qq\bar{q}\bar{q}$ $qqqq\bar{q}\dots$

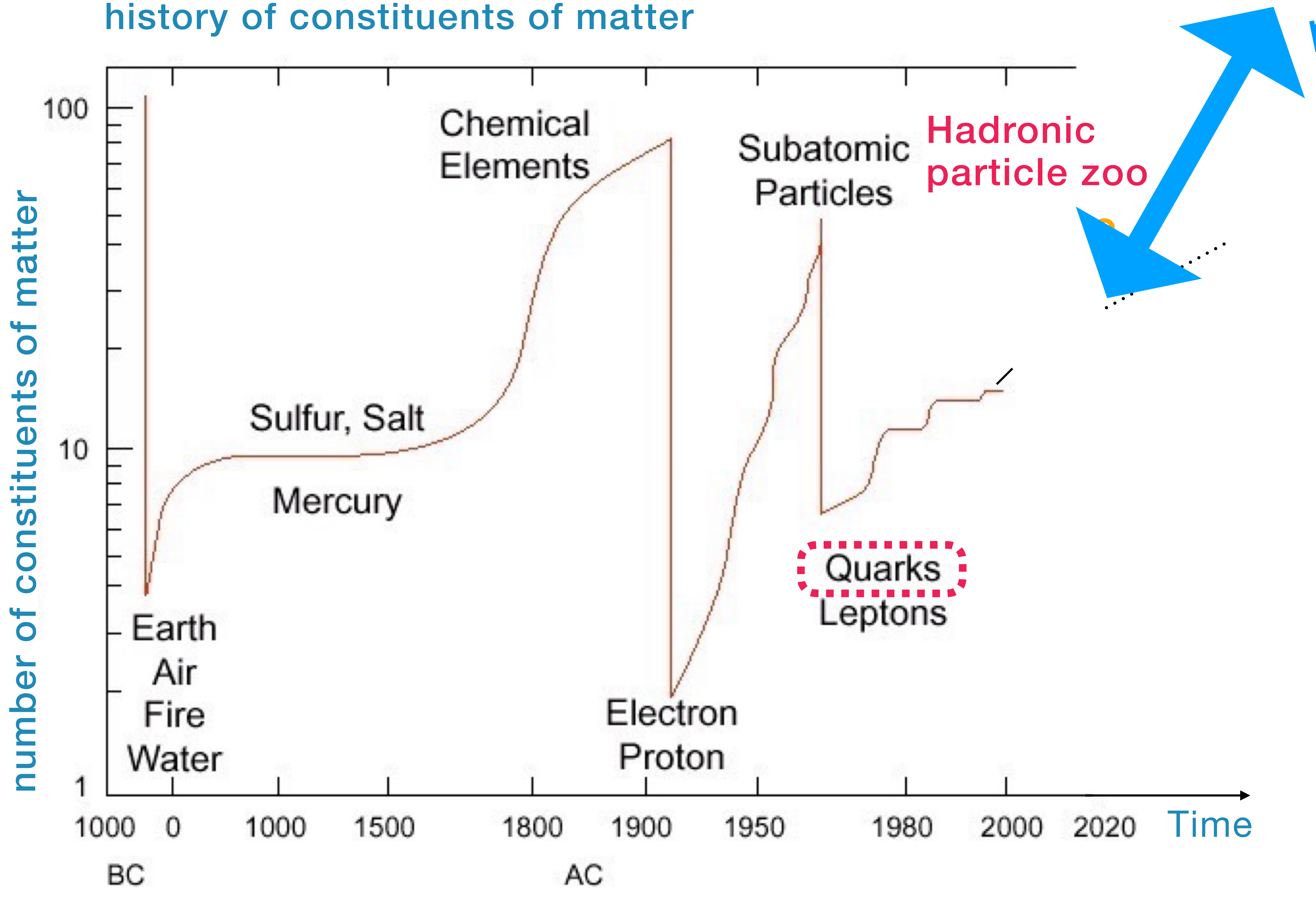
possible but not observed

same in QCD plus hybrids

Constituents of matter and fundamental forces

Beyond the standard model of particle physics

history of constituents of matter



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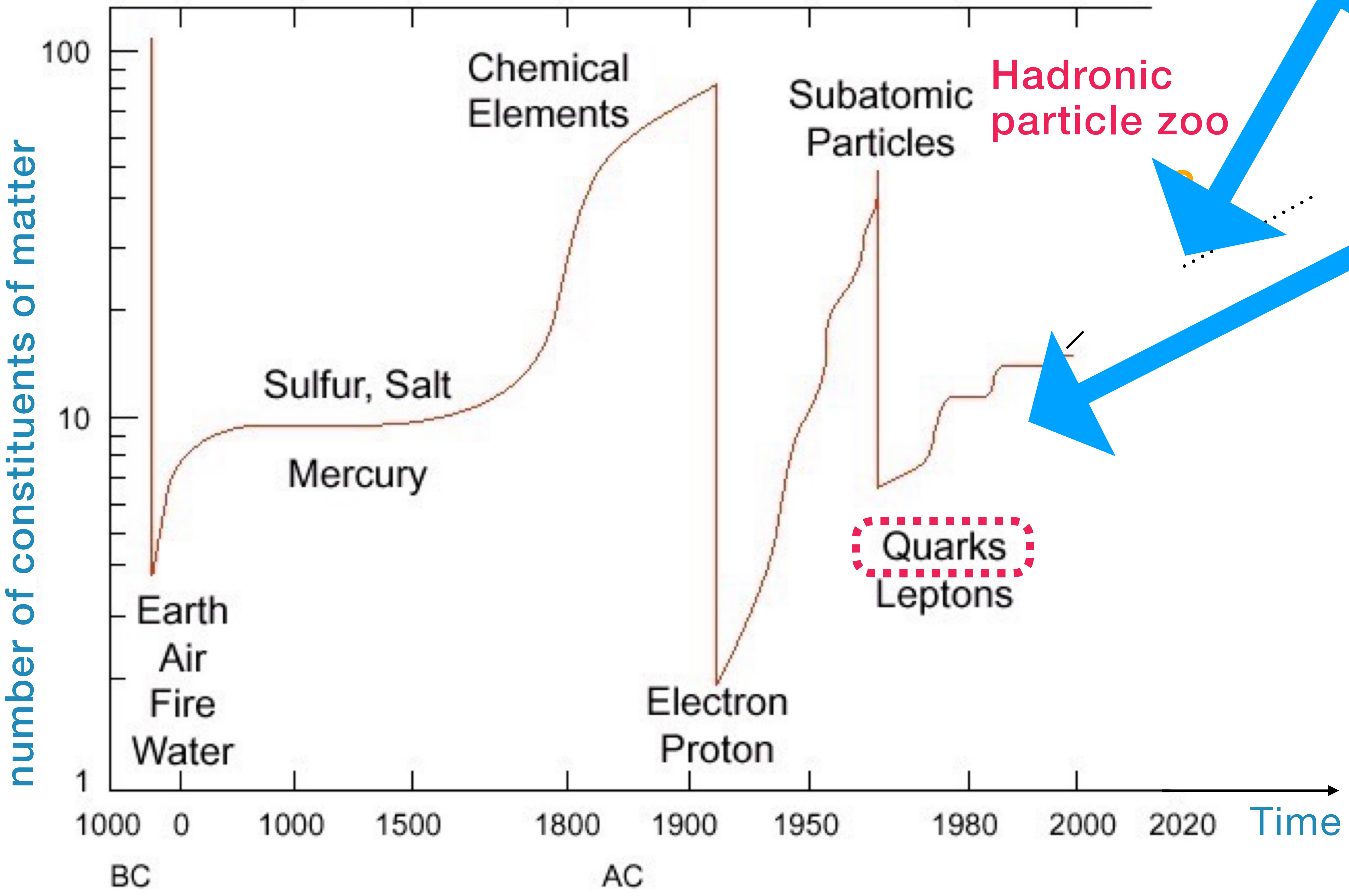


Beyond the standard quark model

With the XYZ exotic states discovery, observed in the sector with two heavy quarks

THE ECONOMIC TIMES

Scientists at CERN observe three "exotic" particles for first time

JARY 26, 2016 | 3 MIN READ

ysicists May Have Discovered a New "Tetraquark" Particle

Data from the DZero experiment shows evidence of a particle containing four different types of qu

WIRED

'Impossible' Particle Adds a Piece to the Strong Force Puzzle

The unexpected discovery of the double-charm tetraquark gives physicists fresh insight into the strongest of nature's fundamental forces.

HARD SCIENCE — JULY 16, 2022

Tetraquarks and pentaquarks: "Unnatural" forms of exotic matter have been found

Scientists have found three new examples of a very exotic form of matter made of quarks. They can yield insights into the early Universe.

CORRIERE DELLA SERA

Nuova straordinaria particella scoperta al Cern: il pentaquark

Consentirà di saperne di più sulla «forza forte» che tiene unite le particelle nel nucleo
i e sui componenti della materia

INDIA TODAY

Mysterious 'X' particles that formed moments after the big bang found in Large Hadron Collider

Le Monde

of unknown nature discovere

sector with tw

Les surprises du tétraquark, « collage » de particules élémentaires

La découverte d'une nouvelle particule à la structure particulièrement stable pourrait permettre aux chercheurs de vérifier leurs théories sur l'interaction forte.

BBC

Pentaquarks: scientists find new "exotic" configurations of quarks

Scientists have found new ways in which quarks, the tiniest particles known to humankind, group together.

ZEIT ONLINE

Cern-Forscher entdecken neues Teilchen

Die Physiker am Kernforschungszentrum in Genf haben die Existenz des Pentaquark-Teilchens nachgewiesen. Bislang war es nur in theoretischen ... beschrieben worden.

LHCb discovers longest-lived exotic matter yet

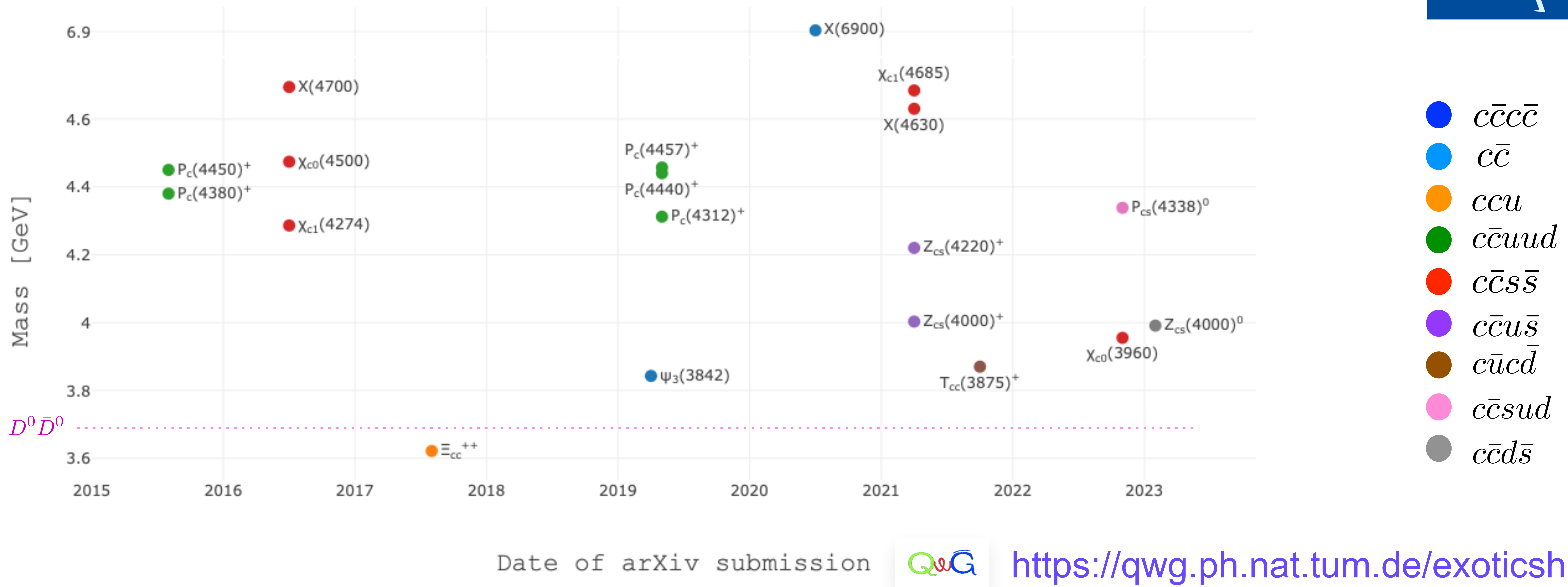
08/04/21 | By Sarah Charley

The newly discovered tetraquark provides a unique window into the interactions of the particles that make up atoms.

symmetry

The present revolutions

XYZ Revolution: Discoveries of Exotic States in the Sector With Two Heavy Quarks: Beyond the Standard Quark Model!



https://qwg.ph.nat.tum.de/exoticshub/

INTERPLAY AMONG MANY EXPERIMENTS:

UPCOMING EXPERIMENTS: STCF

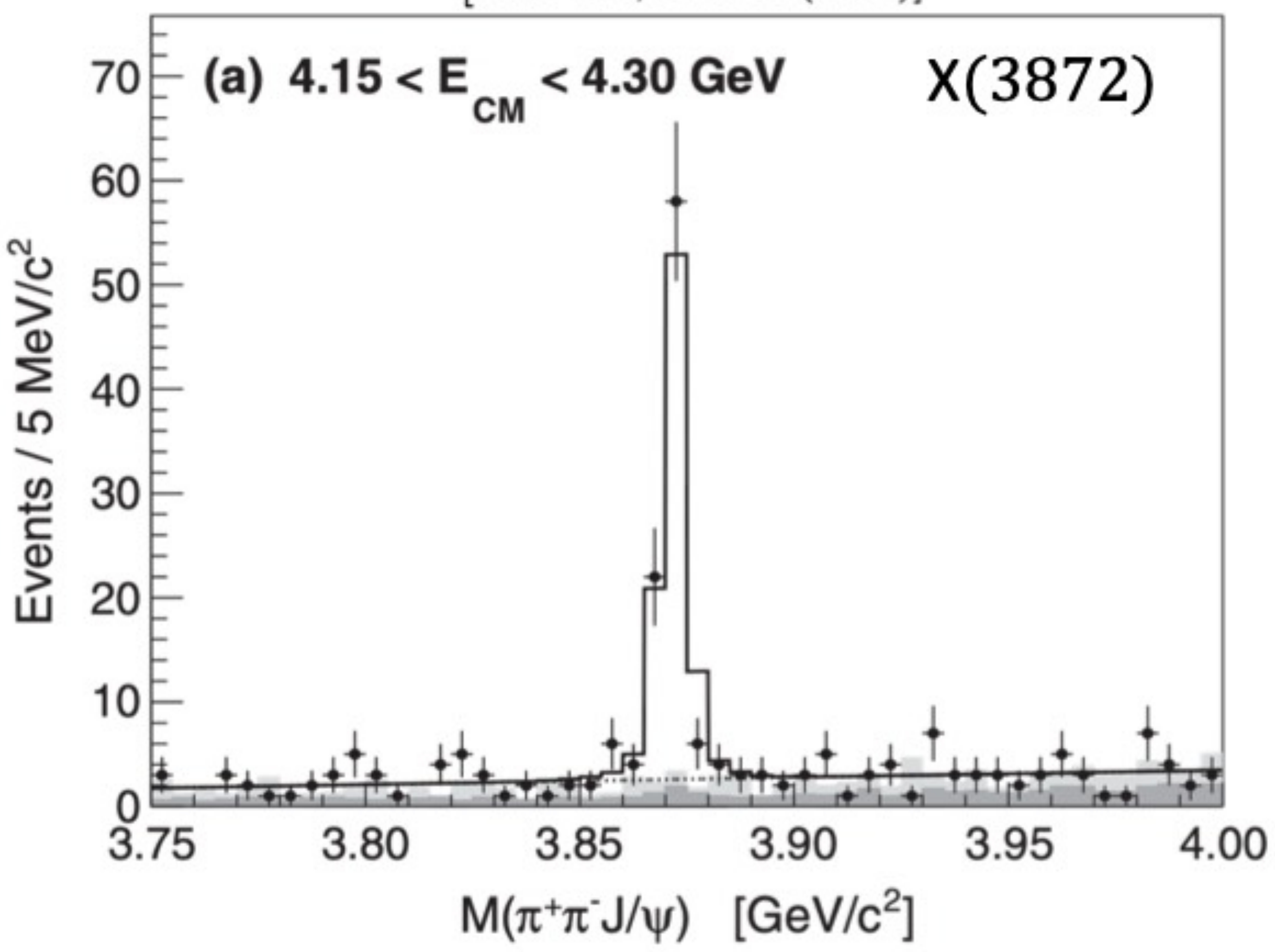
Electron ion

X(3872) aka $\chi_{c1}(3872)$

Some surprisingly narrow states even if above/at strong decay thresholds

$e^+e^- \to \gamma X(3872);\quad X(3872) \to \pi^+\pi^- J/\psi$

[PRL 122, 232002 (2019)]



$$J^{PC} = 1^{++} \qquad I = 0$$

Observed in e+e-, B decays, hadroproduction (large cross section 30nb)

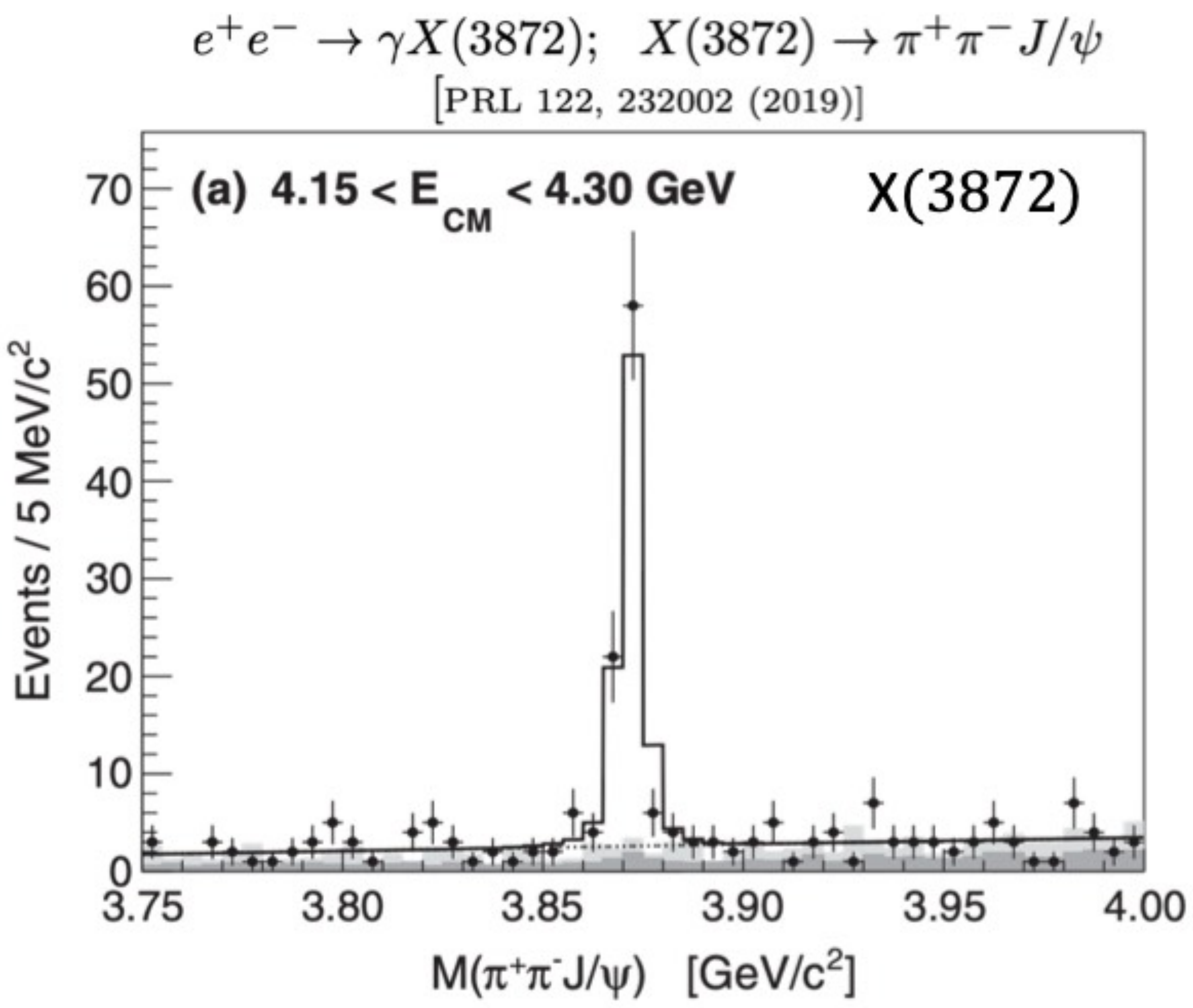
Compositeness, radiative decays, production suggest the presence of a compact component

$M_{X(3872)} - M_{D^0 D^{*0}} = 0.01 \pm 0.14 MeV$ <-within 100 KeV of the threshold (molecule?)

width of 1 MeV! very small binding energy

X(3872) aka

Some surprisingly narrow states even if above/at strong decay thresholds



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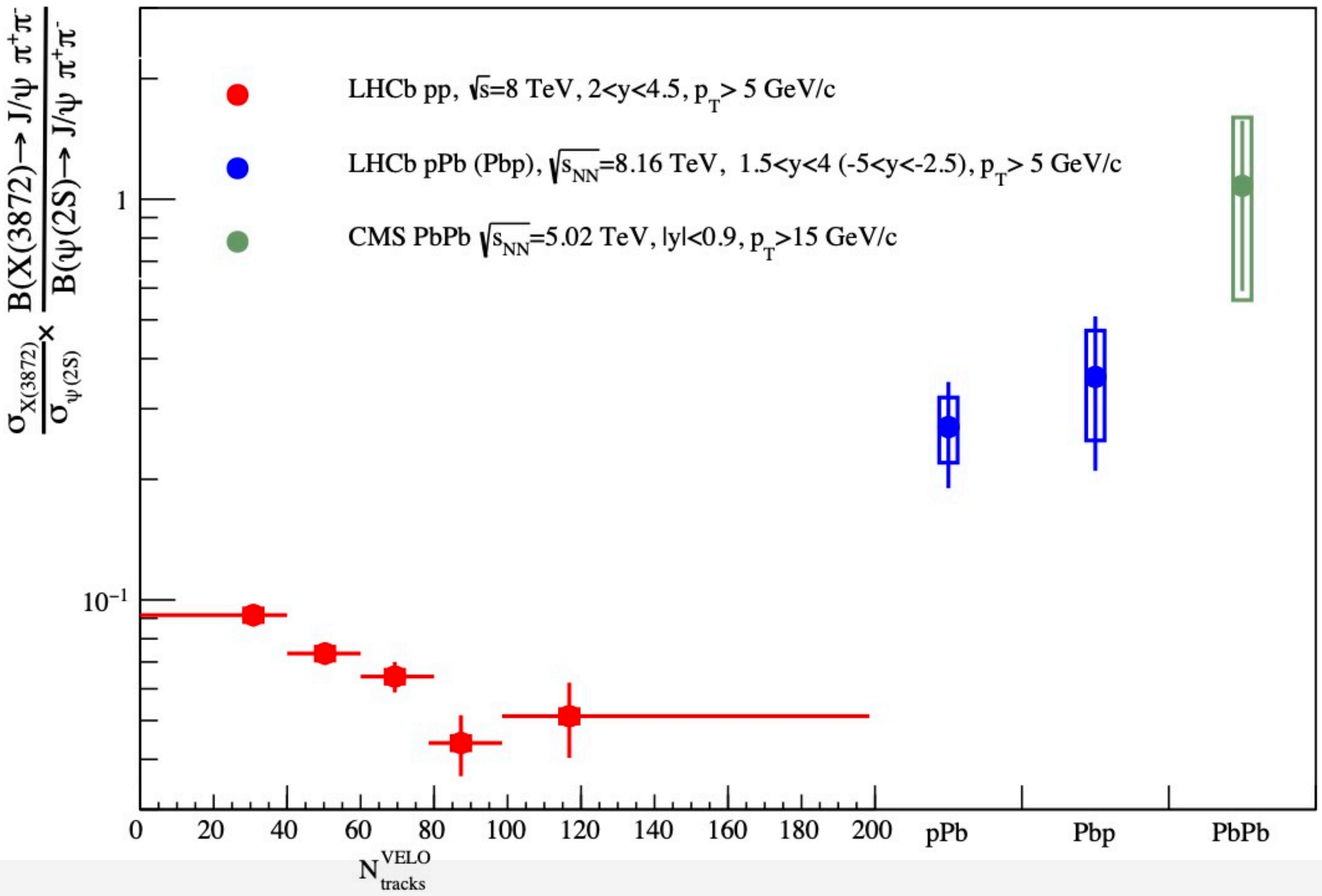
Produced in heavy ions where the deconfined strongly coupled QCD medium (Quark Gluon Plasma-QGP) is formed

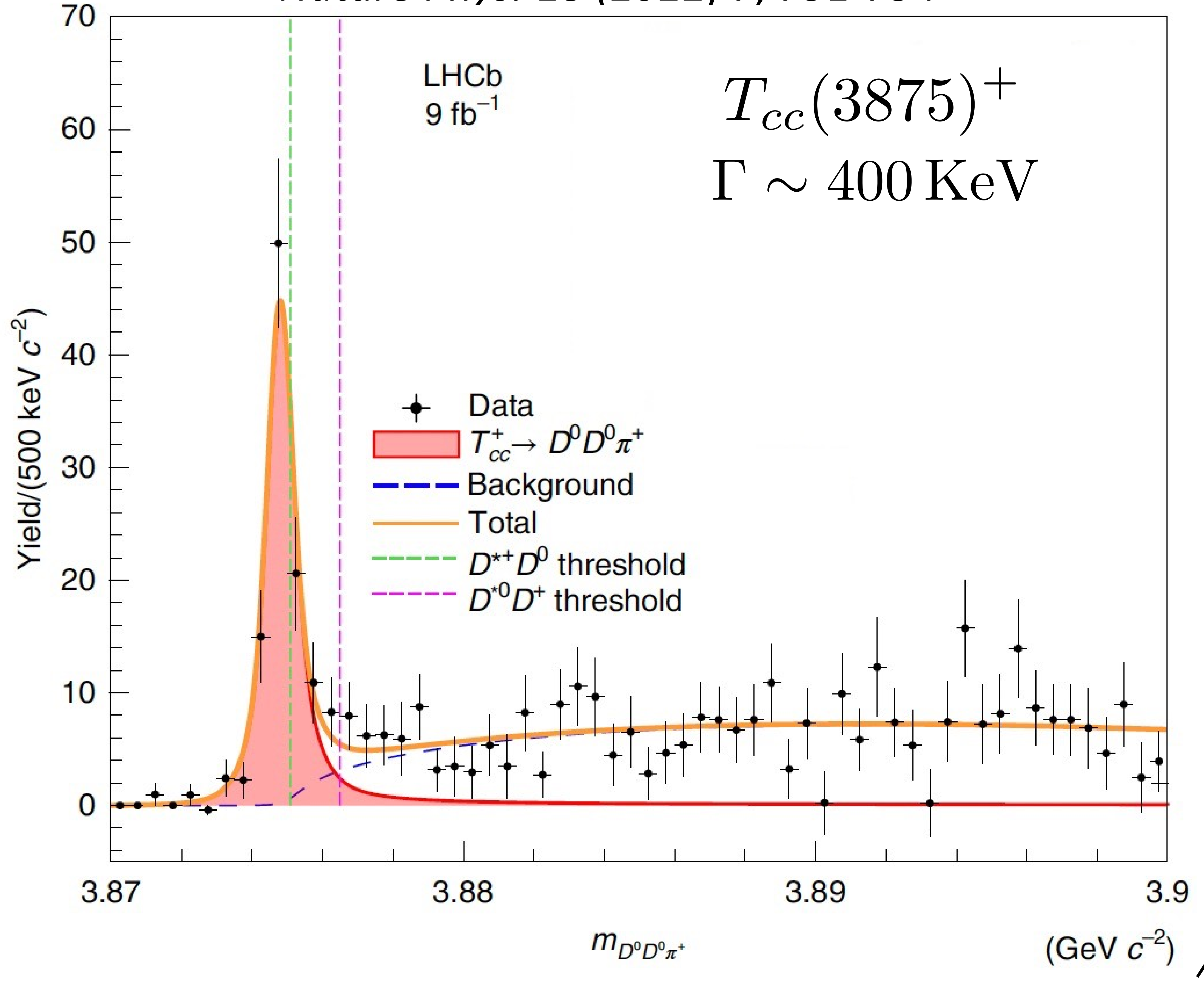


New perspectives for XYZ studies!

$T_{cc}^{+}(3875)$

The longest lived exotic matter ever found!

Nature Phys. 18 (2022) 7, 751-754



$J^P = 1^+$ $\quad$ $I = 0$

<-within 300 KeV of the threshold (molecule?)

<-width of 48 KeV!

$$M_{T_{cc}(3875)^+} - (M_{D^{*+}} + M_{D^0}) = -0.27 \pm 0.06 MeV$$

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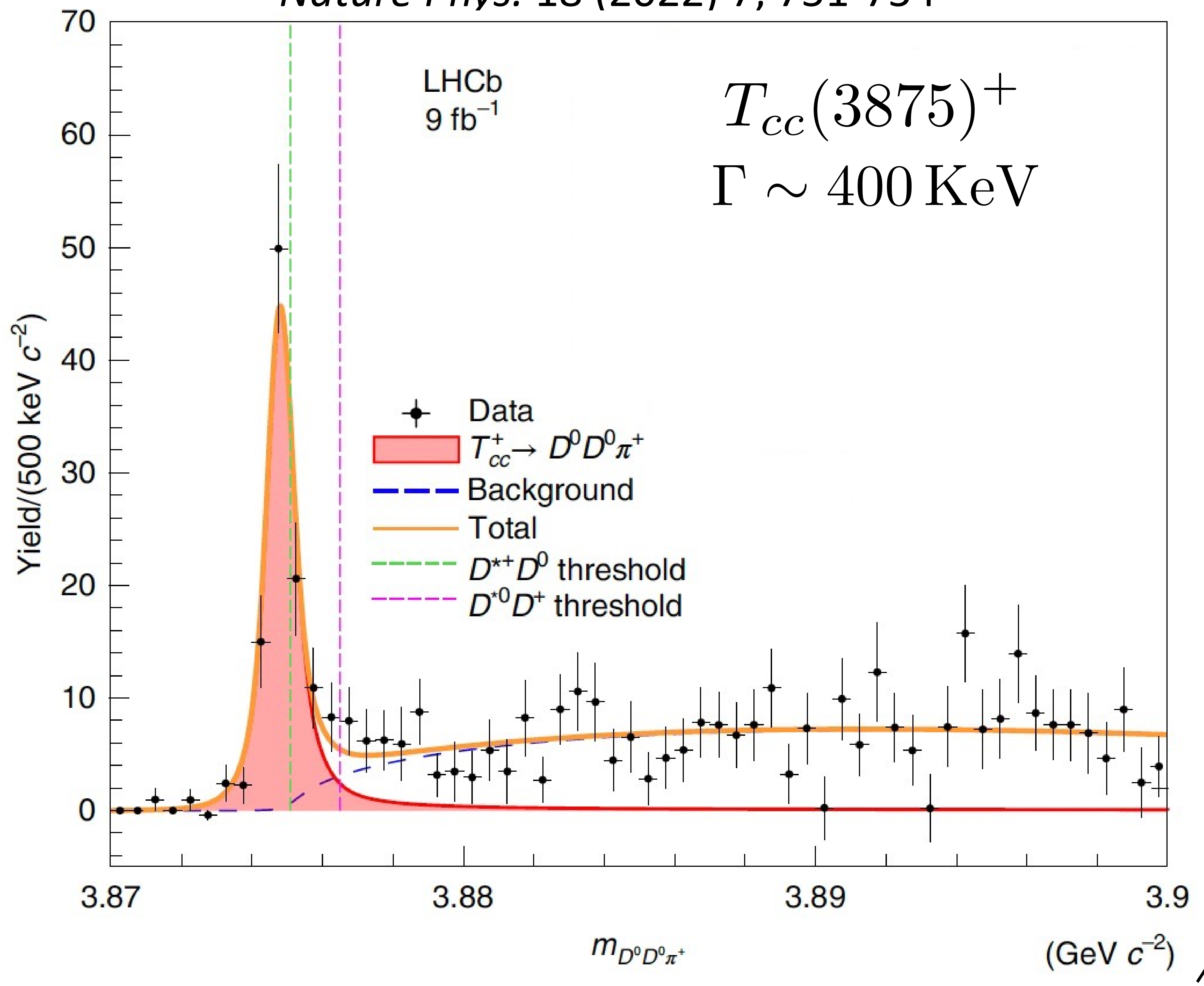

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XYZs not merely composite particles, have unique properties can give us information about the strong force

—>Novel strongly correlated exotics systems

The present revolution: XYZ a great theoretical challenge

Close/above threshold new degrees of freedom like glue and light quarks are nonperturbative part in the binding.

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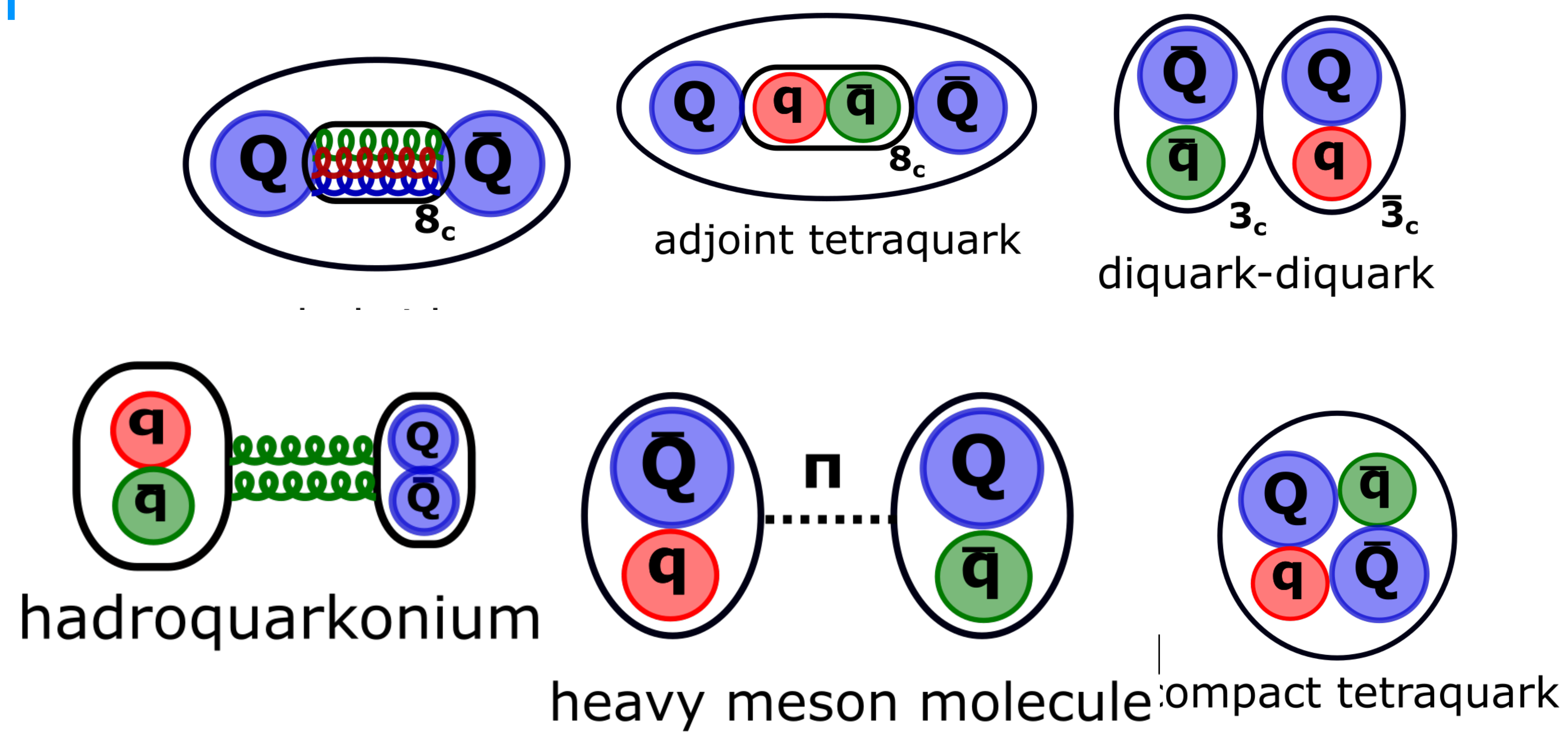
- Models assume some special degrees of freedom and a model interaction



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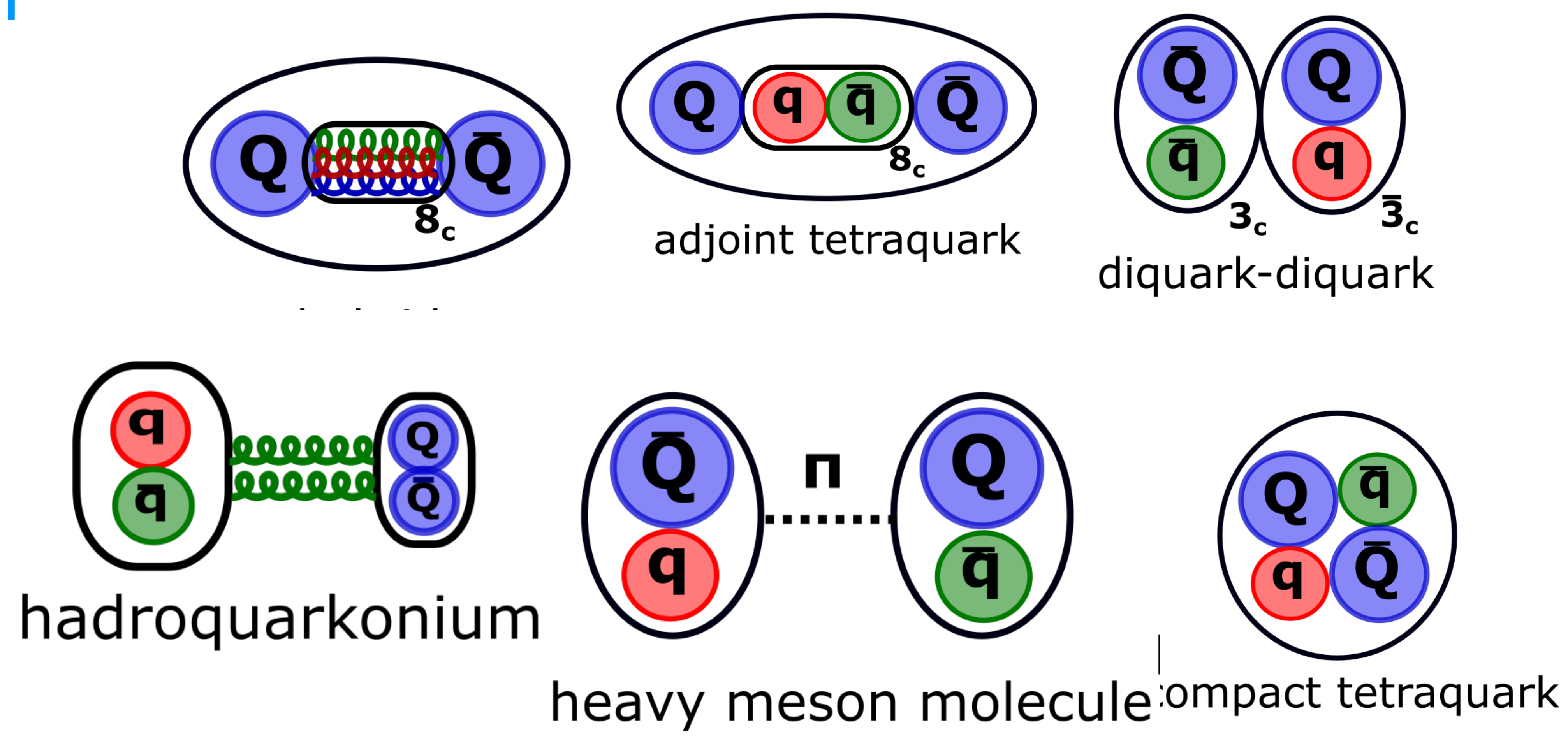

- On the nature of the X(3872) two models in particular compete: molecule versus tetraquark

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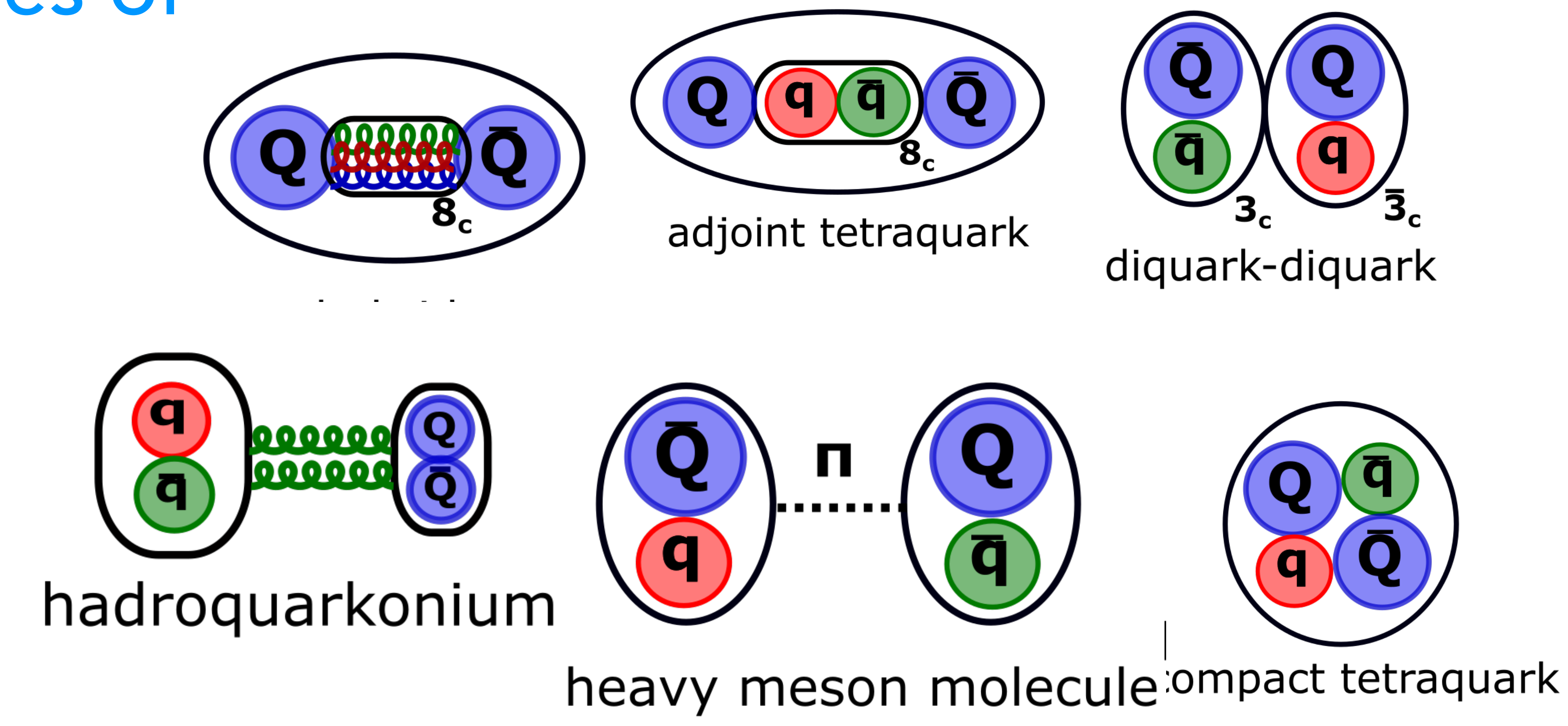
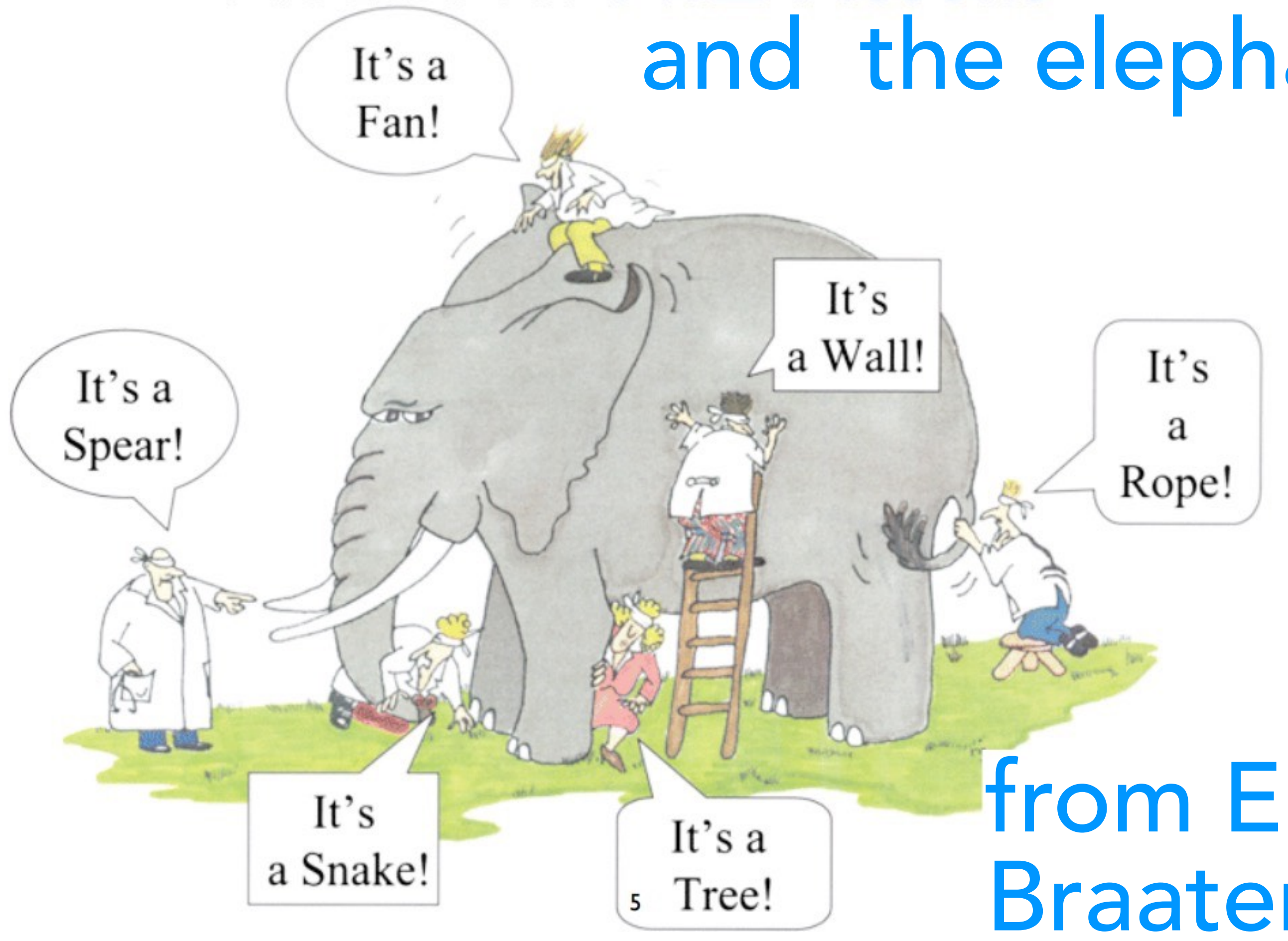


the blind men and the elephant



from E. Braaten

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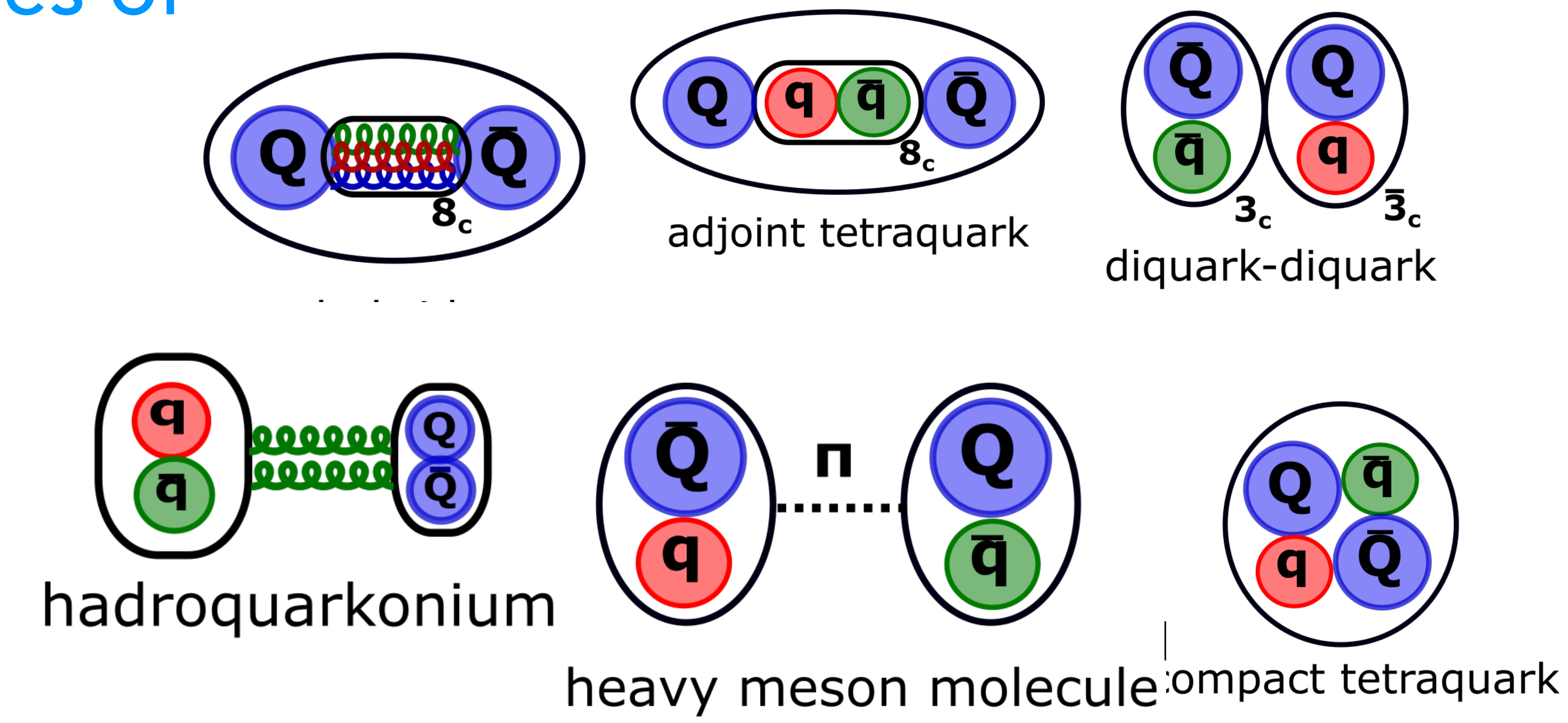

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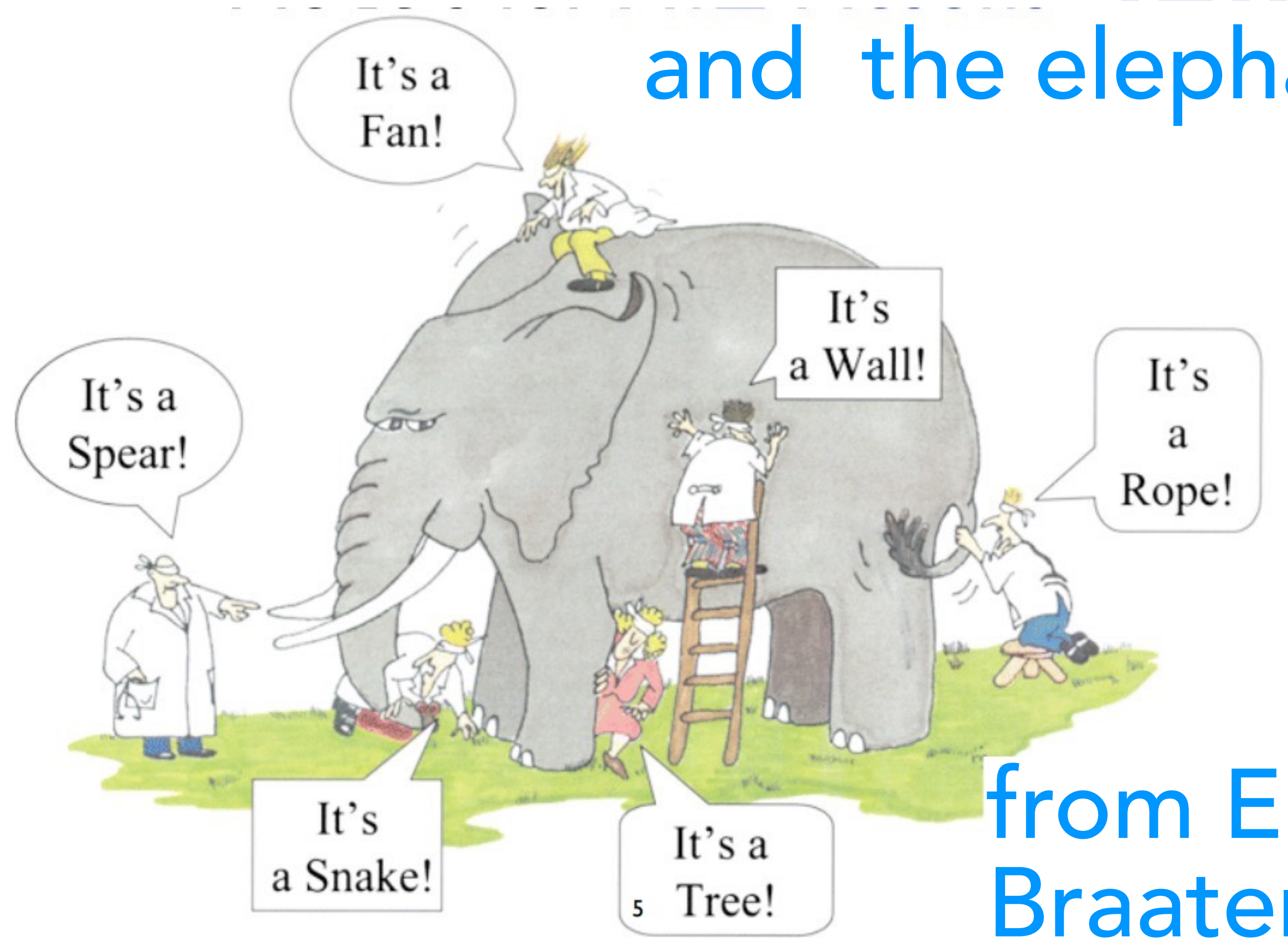

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- Lattice calculation of exotics masses are limited by the large number of open decay modes and they are not Immediately suited for production and in medium studies

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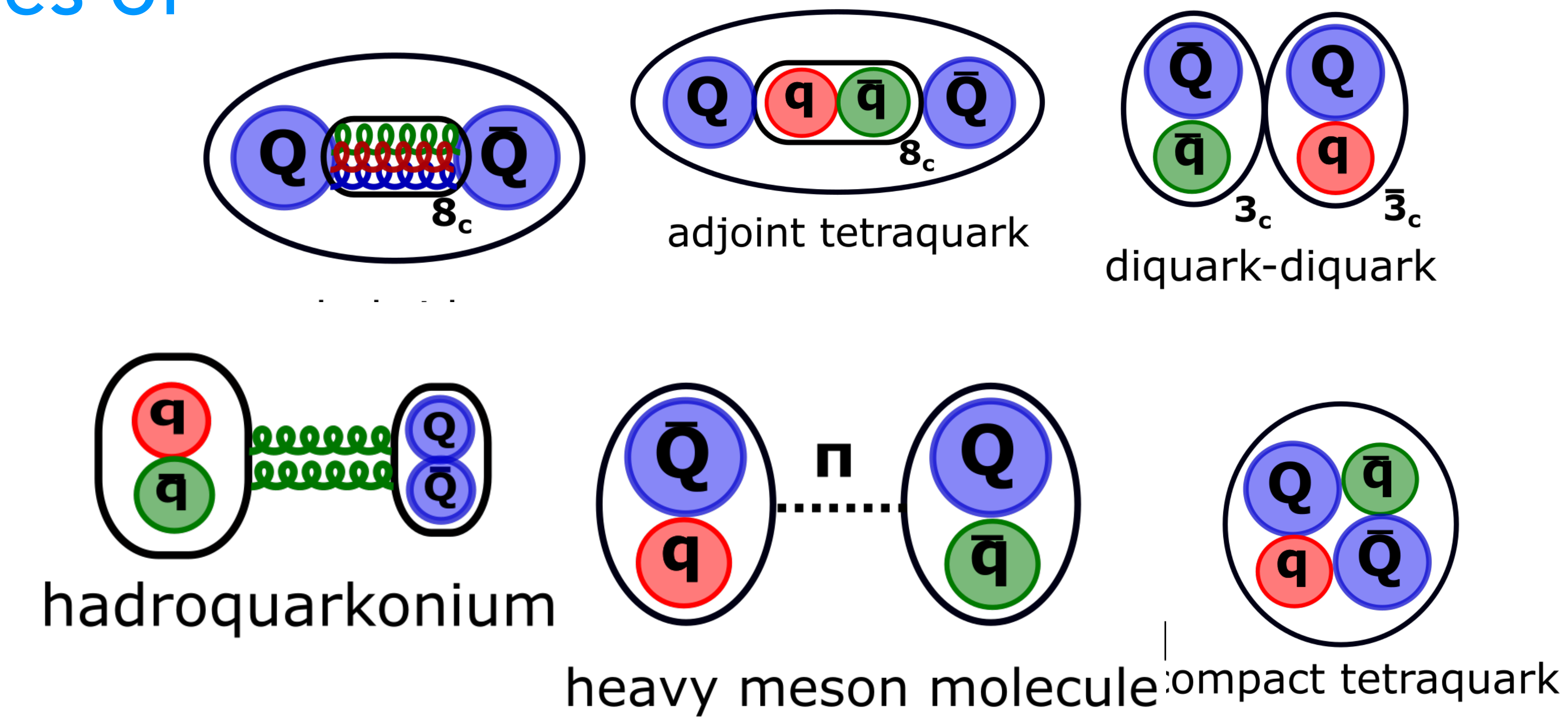

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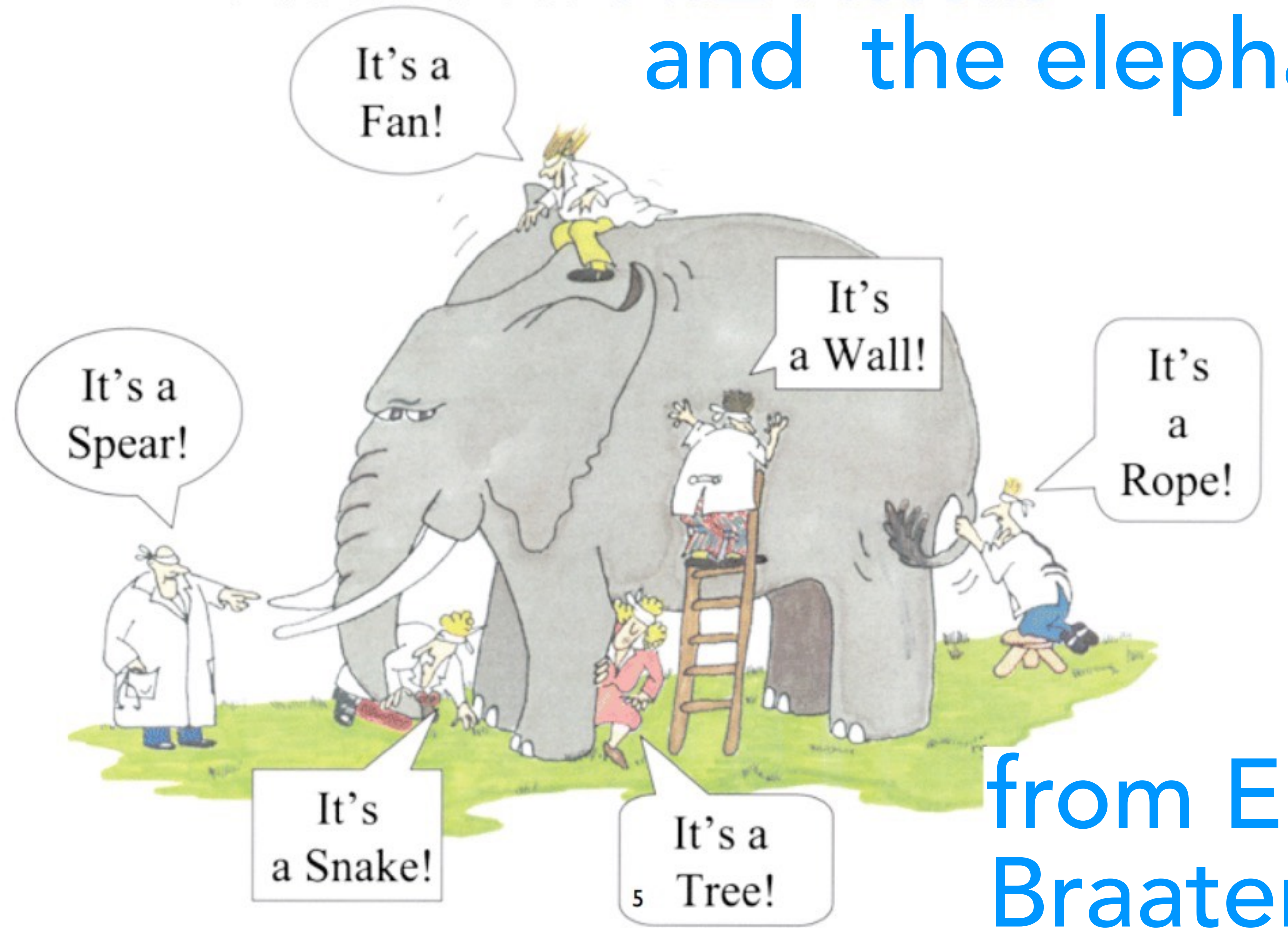

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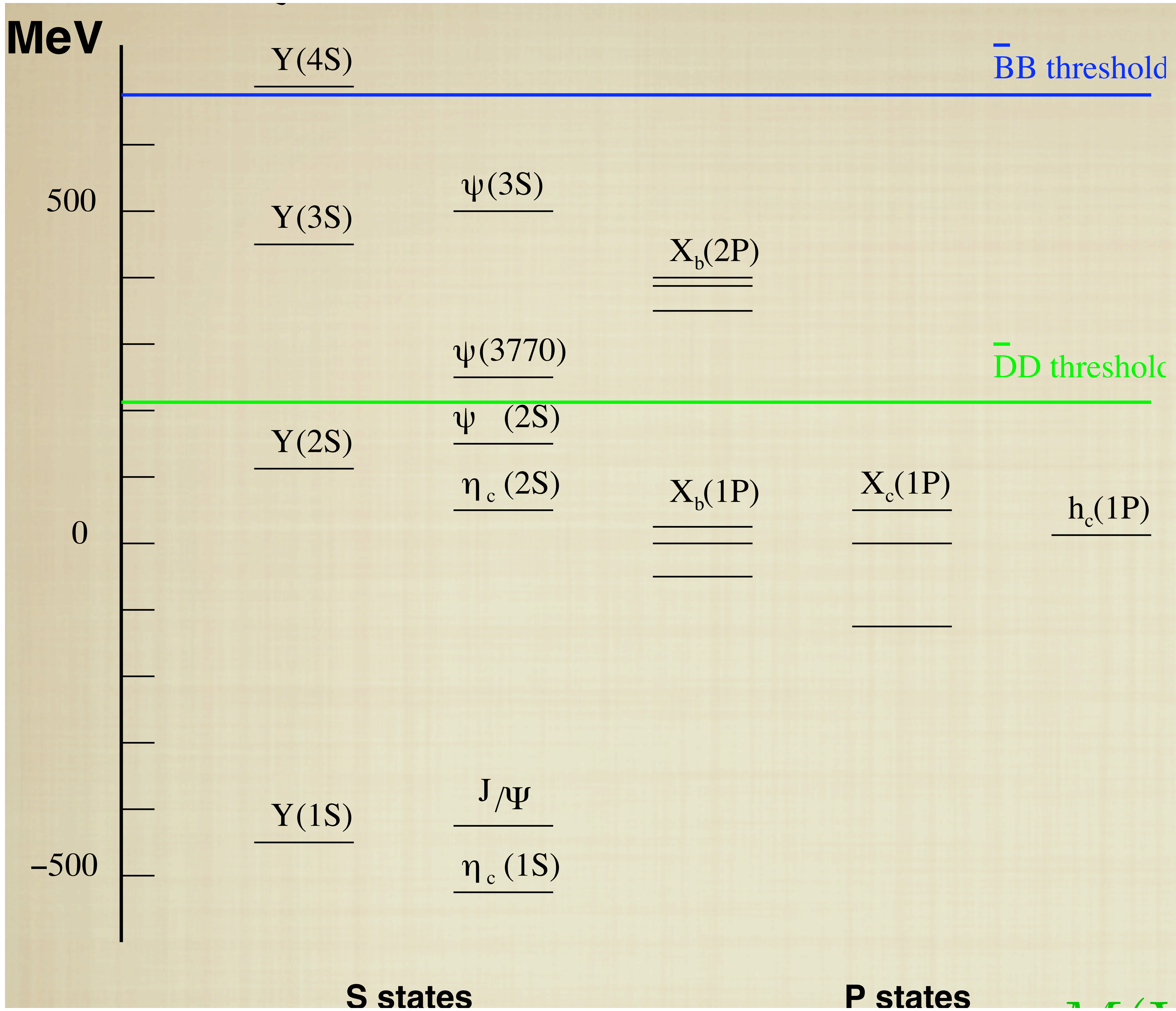
A flexible approach rooted in QCD that can address all properties of XYZ, spectra, transitions, production, propagation in medium is needed allowing also to study the nature of the QCD force

Nonrelativistic EFTs simplify the problem for a multiscale system,

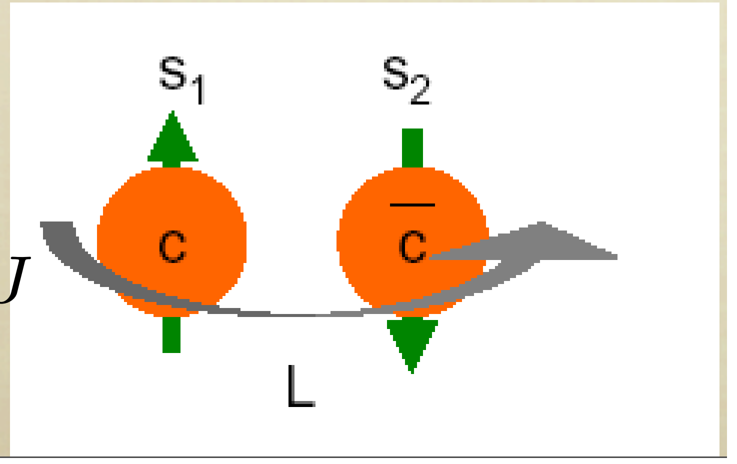
-> make the expansion in the scales explicit at the Lagrangian level

Quarkonium scales



Levels normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$



$$M(\Upsilon(1S)) = 9.460\,\text{GeV}$$
$$M(J/\psi) = 3.097\,\text{GeV}$$

THE MASS SCALE IS PERTURBATIVE

$$m_Q \gg \Lambda_{\text{QCD}}$$

$$m_b \simeq 5\,\text{GeV};\, m_c \simeq 1.5\,\text{GeV}$$

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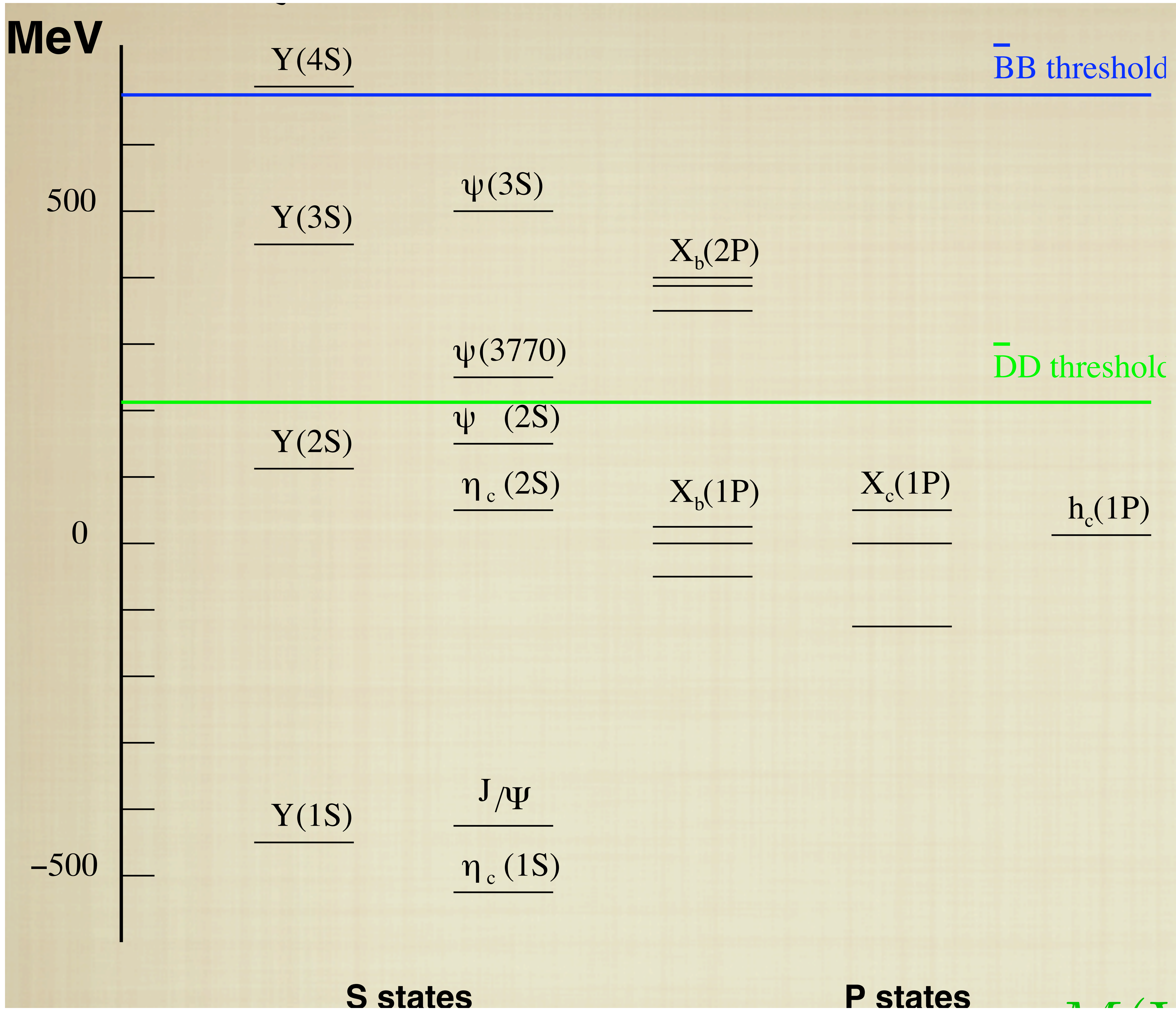

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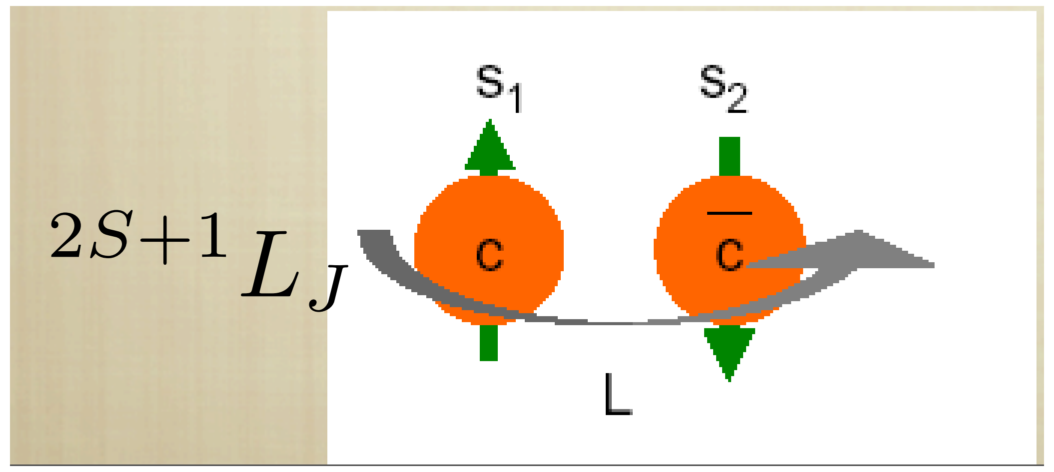

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The system is nonrelativistic(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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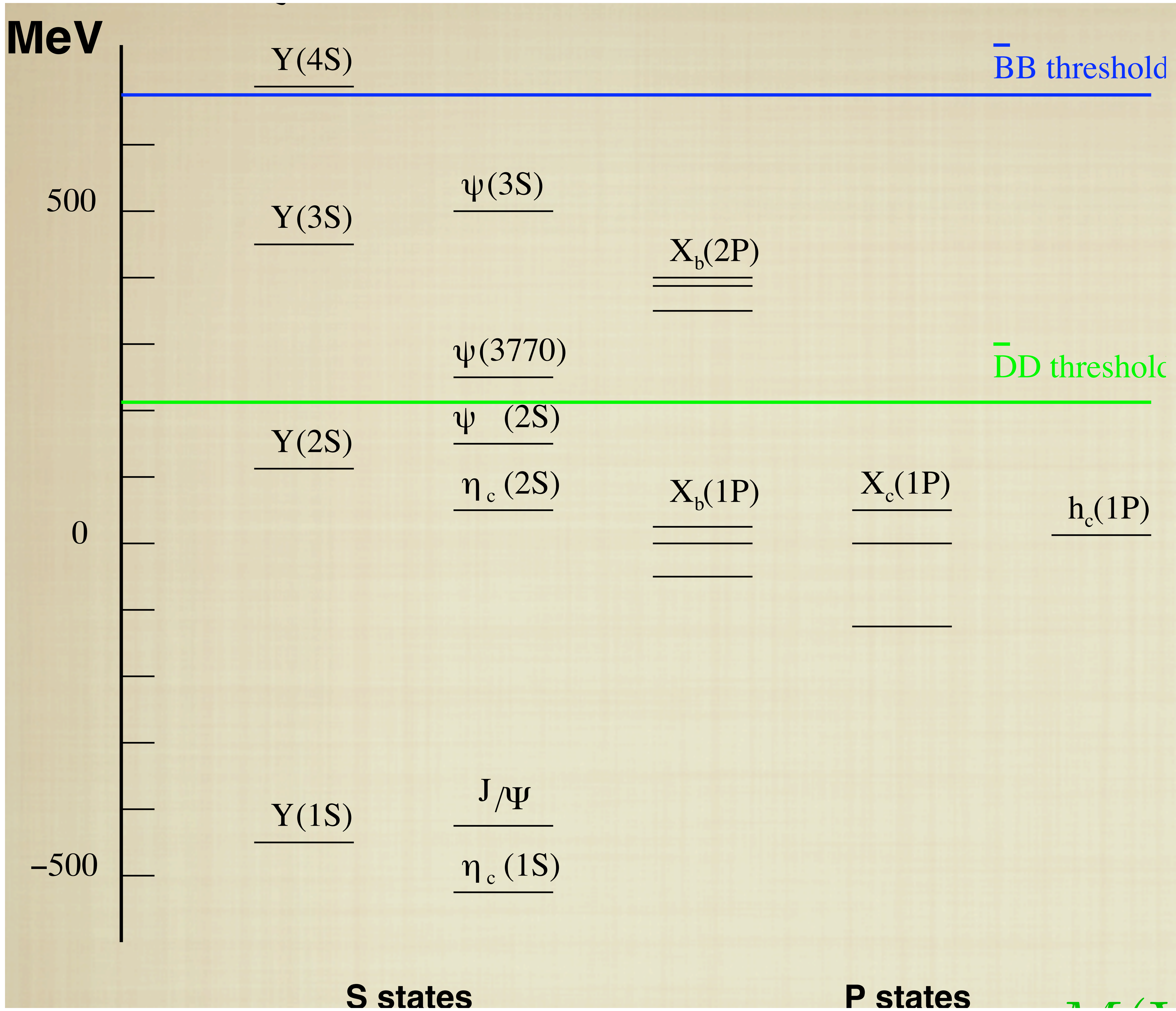

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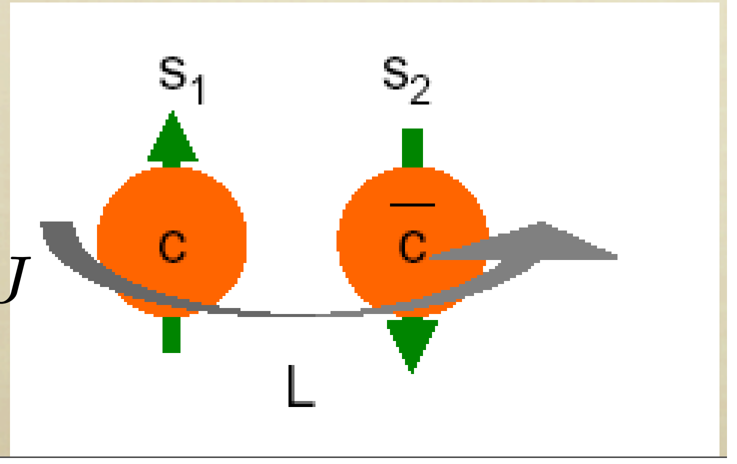

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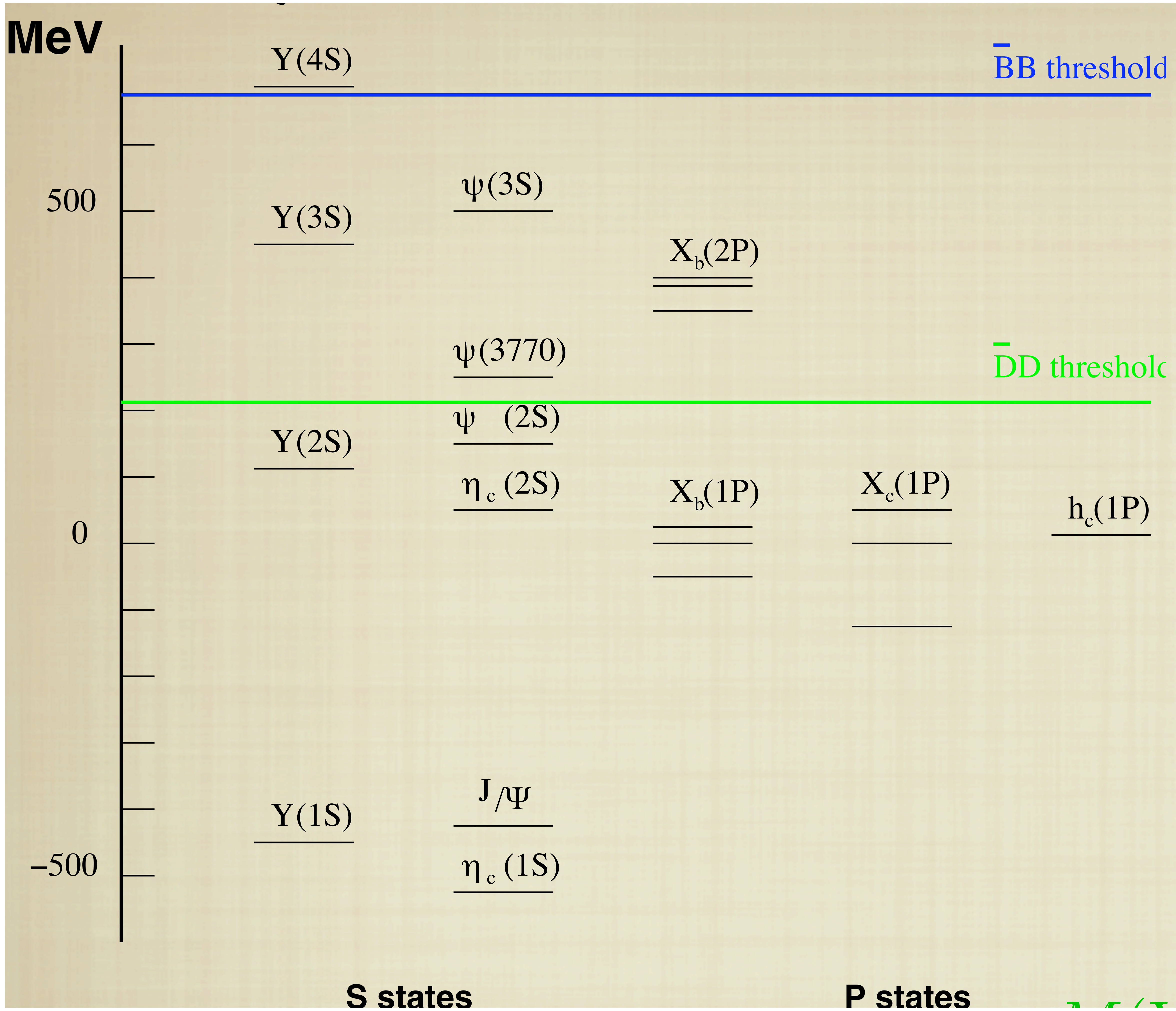

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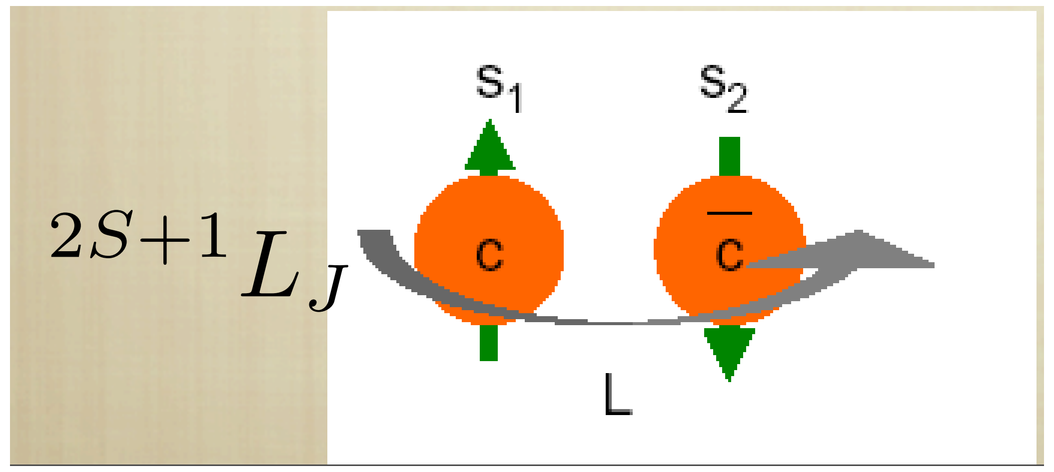

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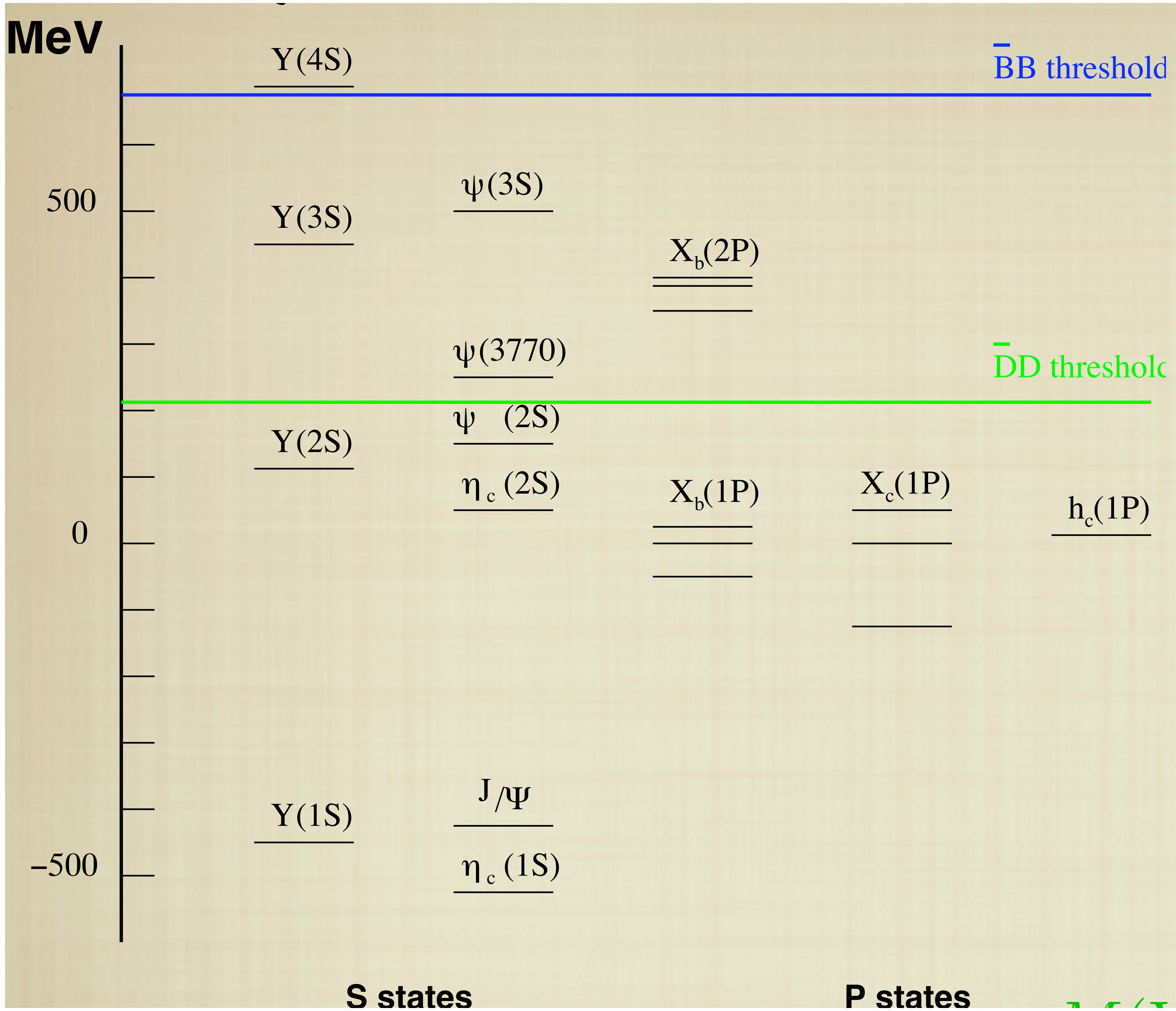

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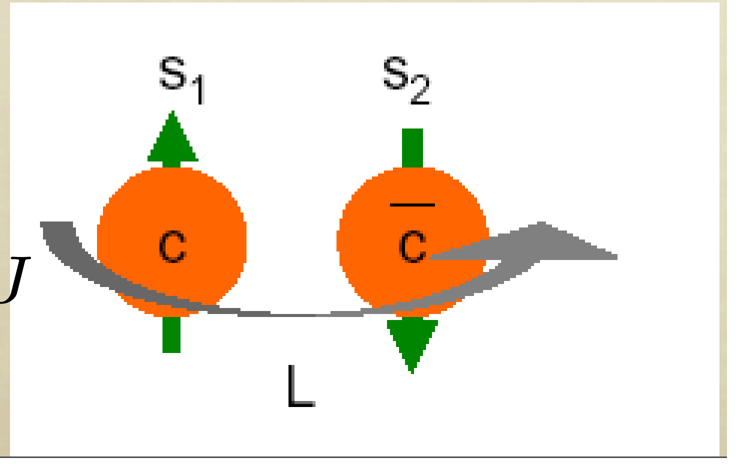

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r^-1 \sim Lambda_QCD

Strongly coupled pNRQCD and Born Oppenheimer EFT

A nonperturbative problem: construct a pNREFT description on the basis of scale separations and symmetries

Two heavy quarks with large mass $m \gg \Lambda_{\rm QCD}$ and residual scale separation $\Lambda_{\rm QCD} \gg E$

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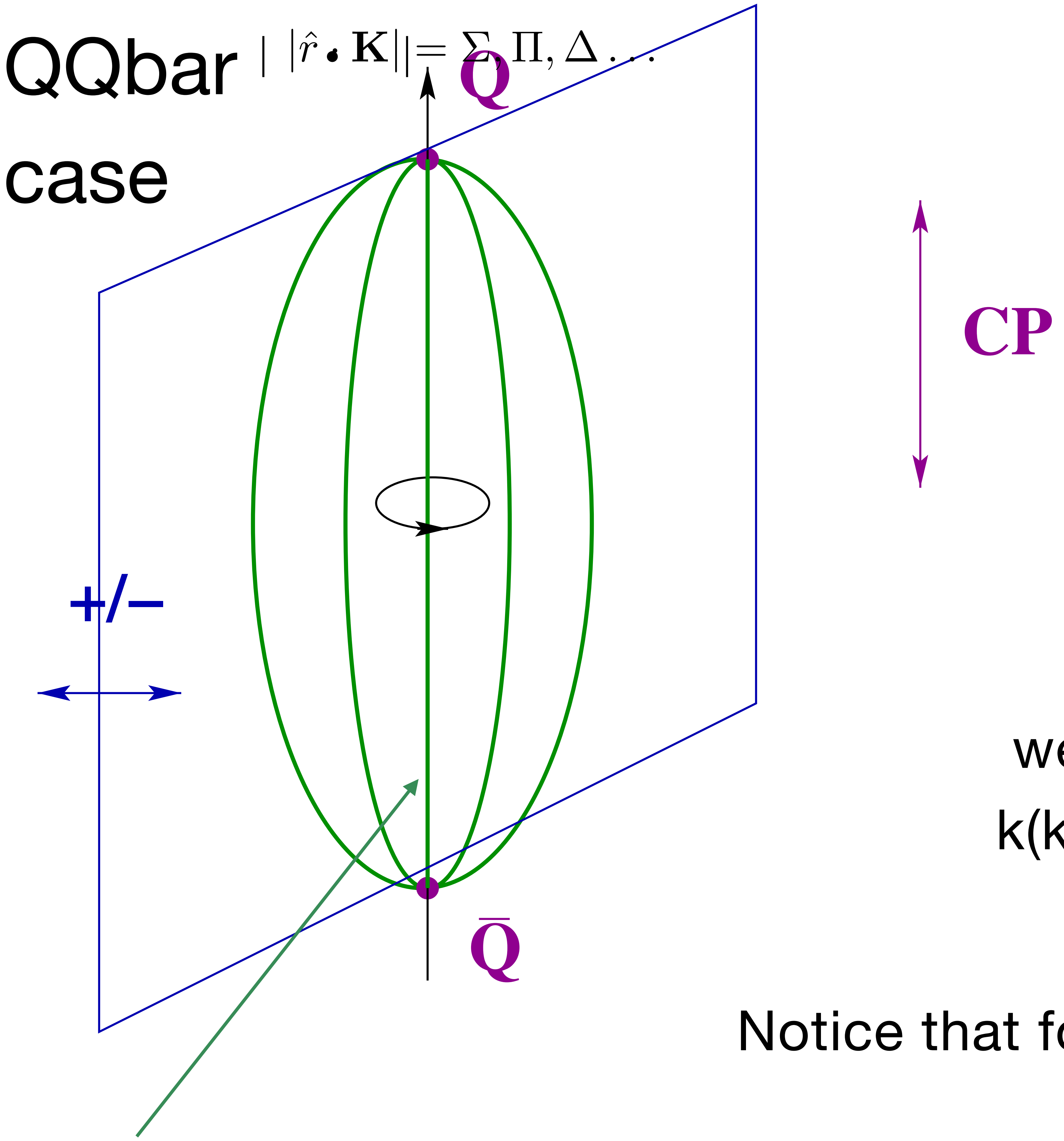
QQbar case



Nonperturbative light degrees of freedom
glue and light quarks

NRQCD static energies identified by the quantum number of $D_{\infty h}$

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \ (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g), -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

$$\Lambda_\eta^\sigma$$

we label the Light Degrees of Freedom (LDF) by $\kappa = \{k^{PC}, f\}$ where k(k+1) is the eigenvalue of the K^2 and we add the flavour quantum number

Notice that for r->0 the cylindrical symmetric becomes spherical (O(3) X C)

$\mathbf{r} = Q\bar{Q}$ distance $\quad \mathbf{R} =$ center of mass

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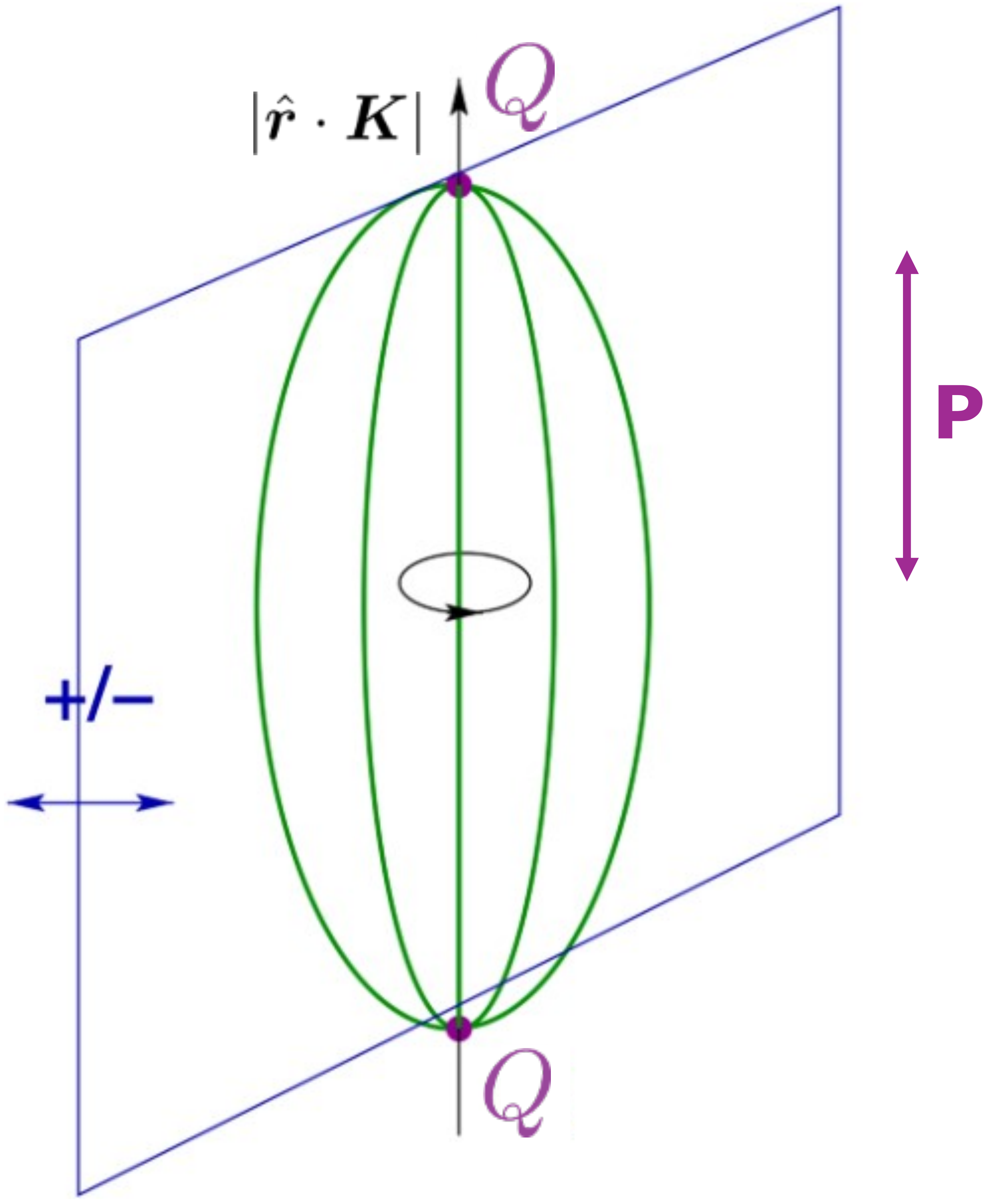

produce a hierarchy of NRQCD static energies identified by the quantum number of $D_{\infty h}$

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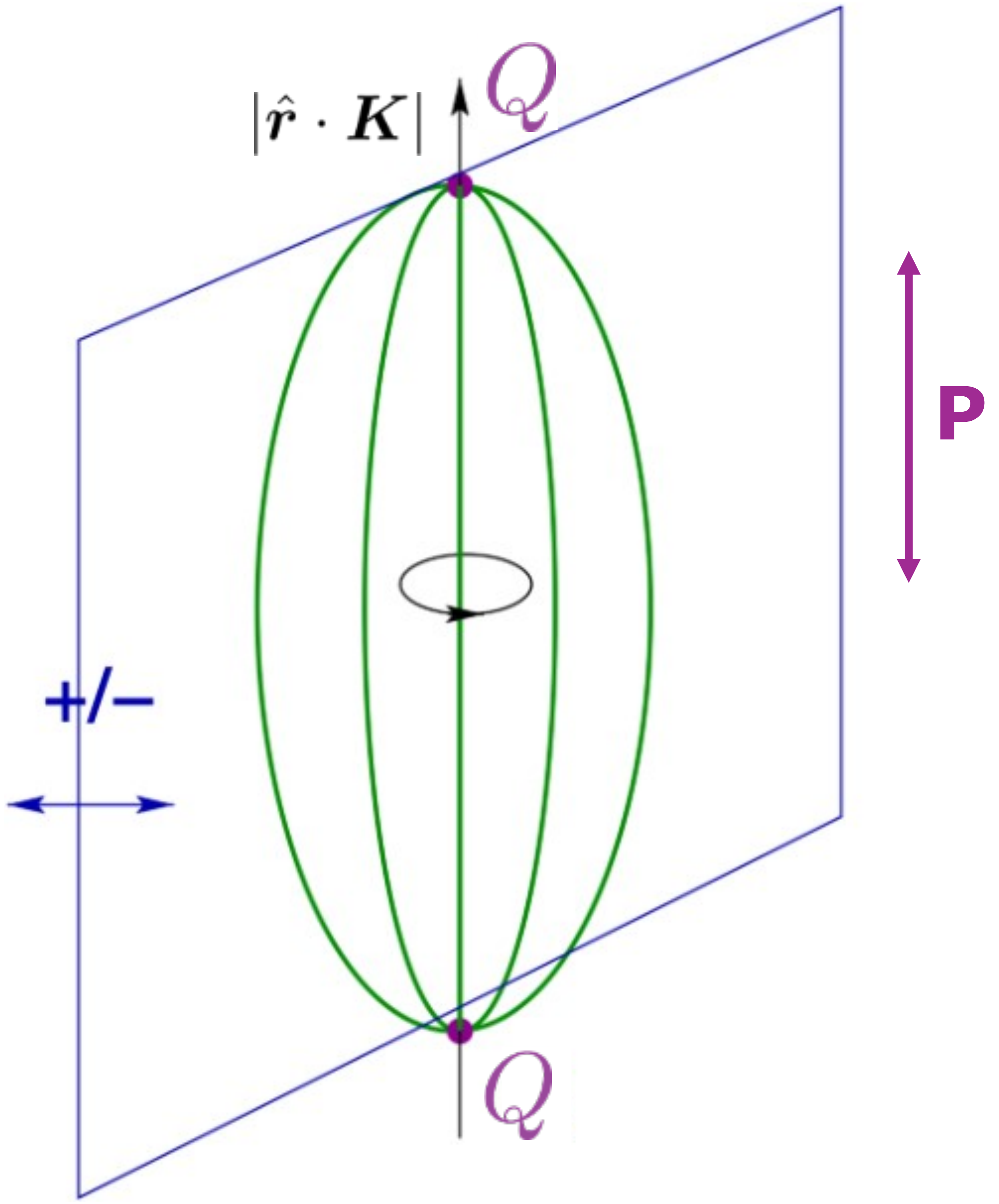

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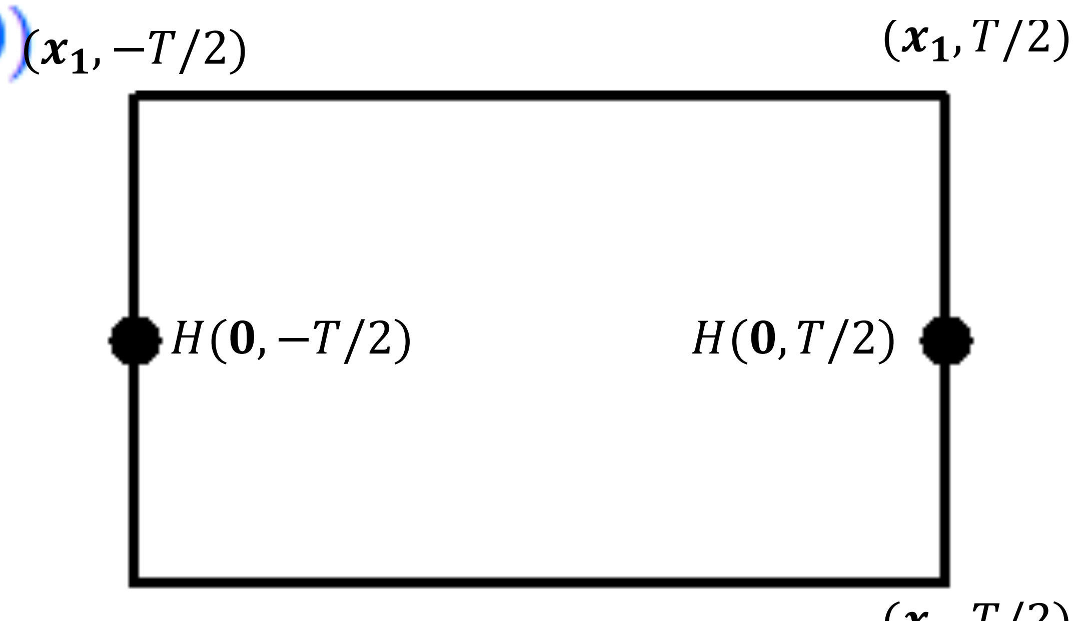
$$\Lambda_\eta^\sigma$$

$$\mathcal{H}^{(0)} = \int d^3\mathbf{x}\, \frac{1}{2} \left(\mathbf{\Pi}^a \mathbf{\Pi}^a + \mathbf{B}^a \mathbf{B}^a \right) - \sum^{n_f} \bar{q}\, i\mathbf{D} \cdot \boldsymbol{\gamma}\, q$$

$$\mathcal{H}^{(0)} |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2) |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1) \chi(\mathbf{x}_2) |n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$E_n^{(0)}(r) = \lim_{T \to \infty} \frac{i}{T} \log \langle X_n, T/2 | X_n, -T/2 \rangle$$



(b)

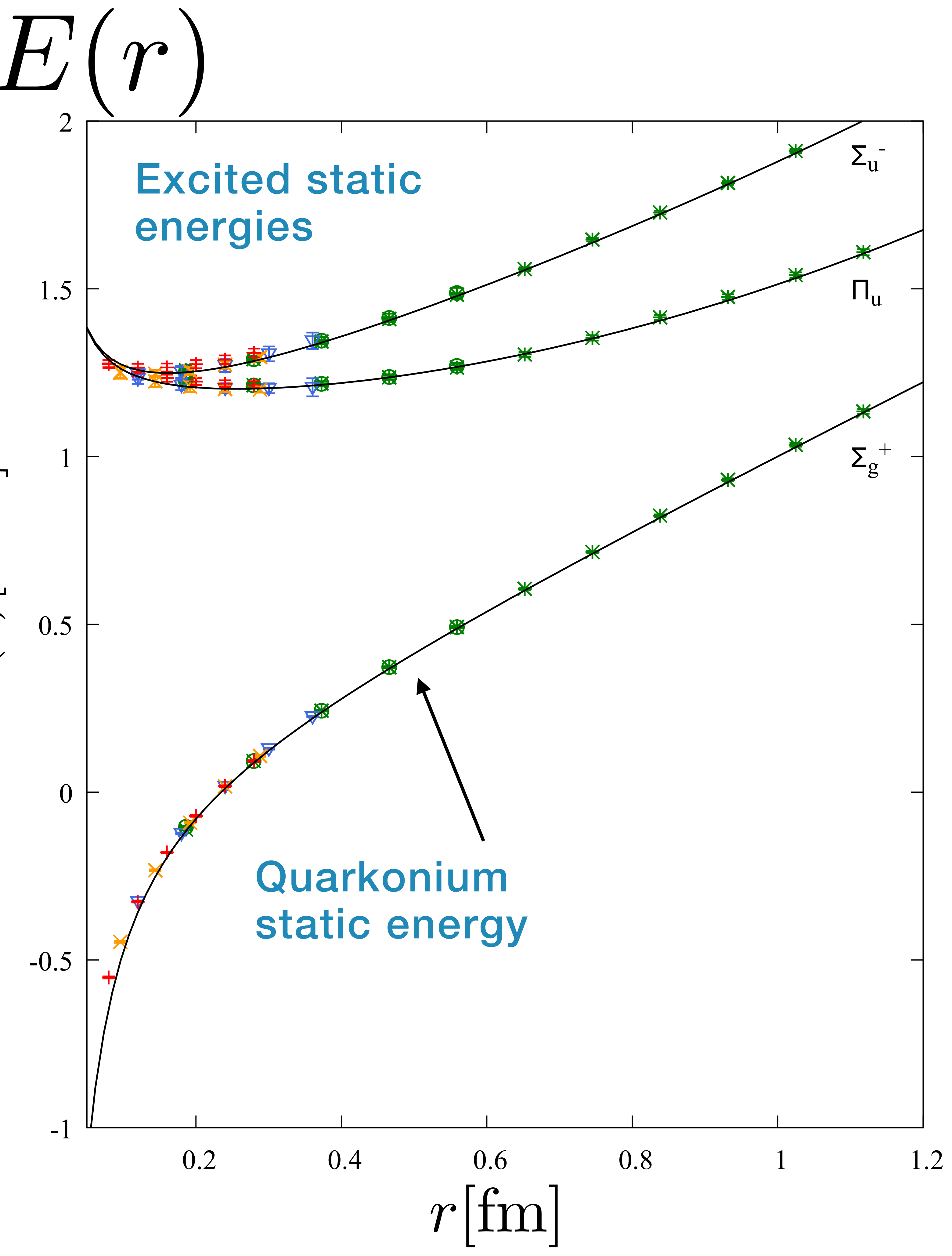
Phi =Wilson lines and H= gluonic and light quarks

These two cases contain quarkonium, hybrids, tetraquarks, pentaquarks and doubly heavy baryons

Strongly coupled pNRQCD and Born Oppenheimer EFT

A nonperturbative problem: construct a pNREFT description on the basis of scale separations and symmetries

Two heavy quarks with large mass $m \gg \Lambda_{\rm QCD}$ and residual scale separation $\Lambda_{\rm QCD} \gg E$



produce a hierarchy of NRQCD static energies identified by the quantum number of $D_{\infty h}$

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \; (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g), \; -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

$$\Lambda_\eta^\sigma$$

$$\mathcal{H}^{(0)} = \int d^3\mathbf{x}\,\frac{1}{2}\left(\mathbf{\Pi}^a\mathbf{\Pi}^a + \mathbf{B}^a\mathbf{B}^a\right) - \sum^{n_f} \bar{q}\, i\mathbf{D}\cdot\boldsymbol{\gamma}\, q$$

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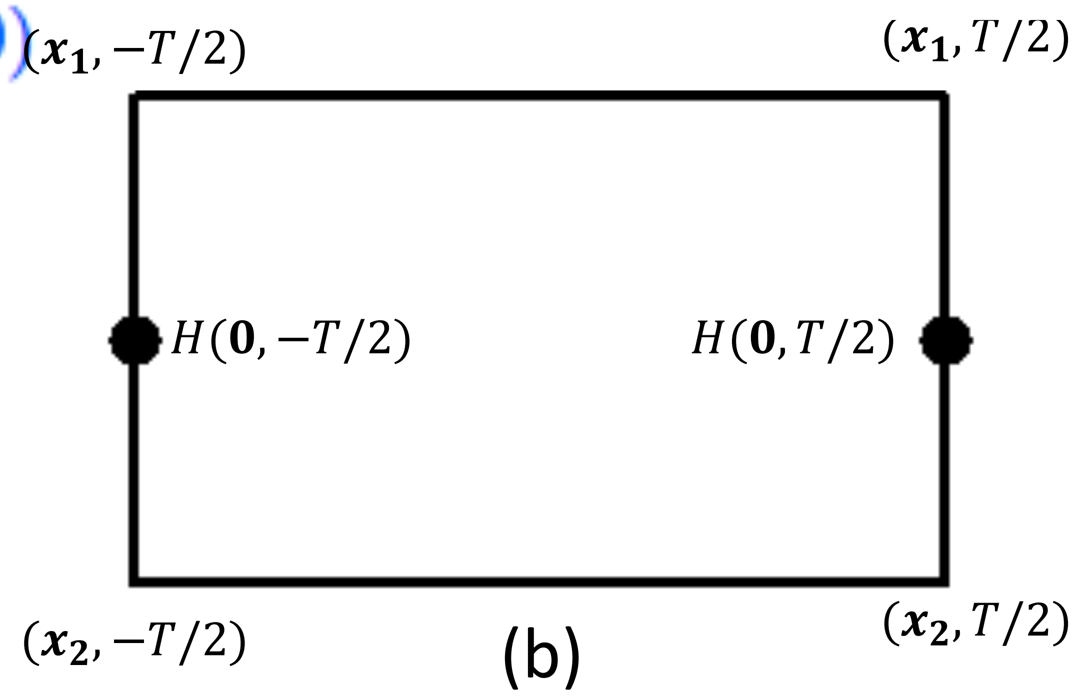

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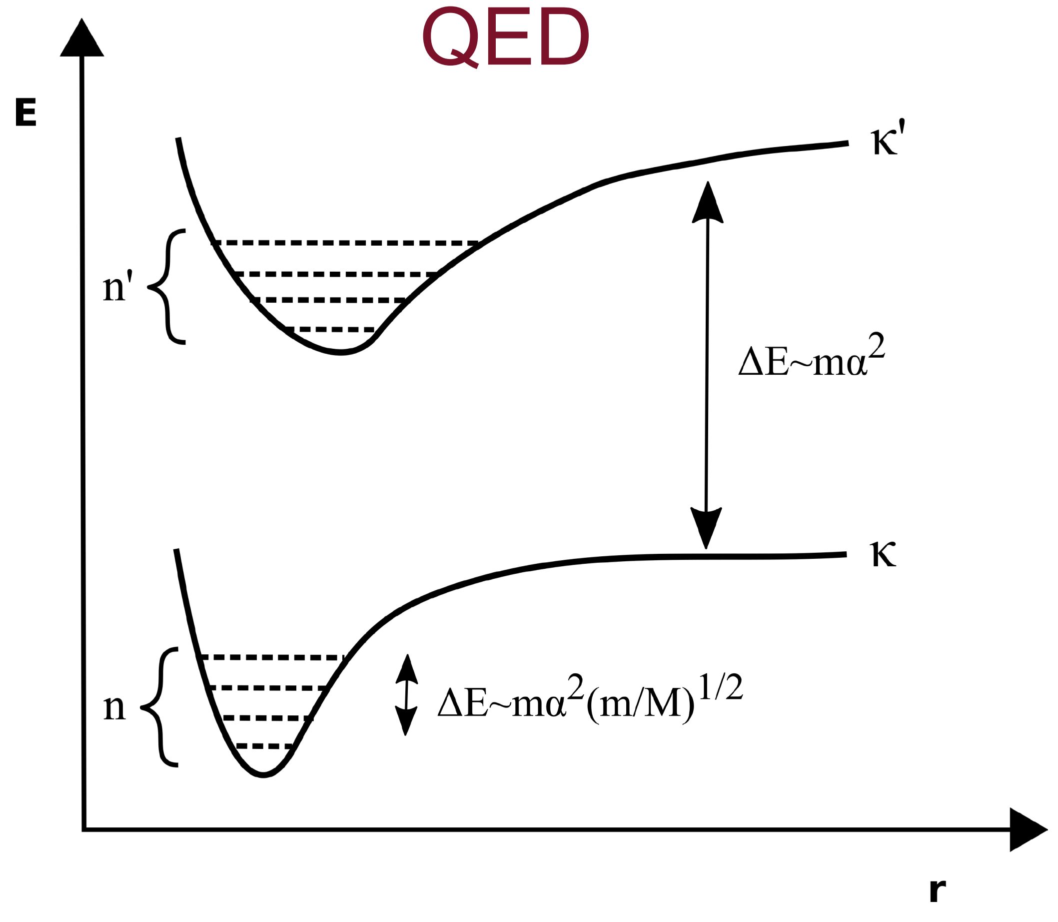
Born Oppenheimer physics



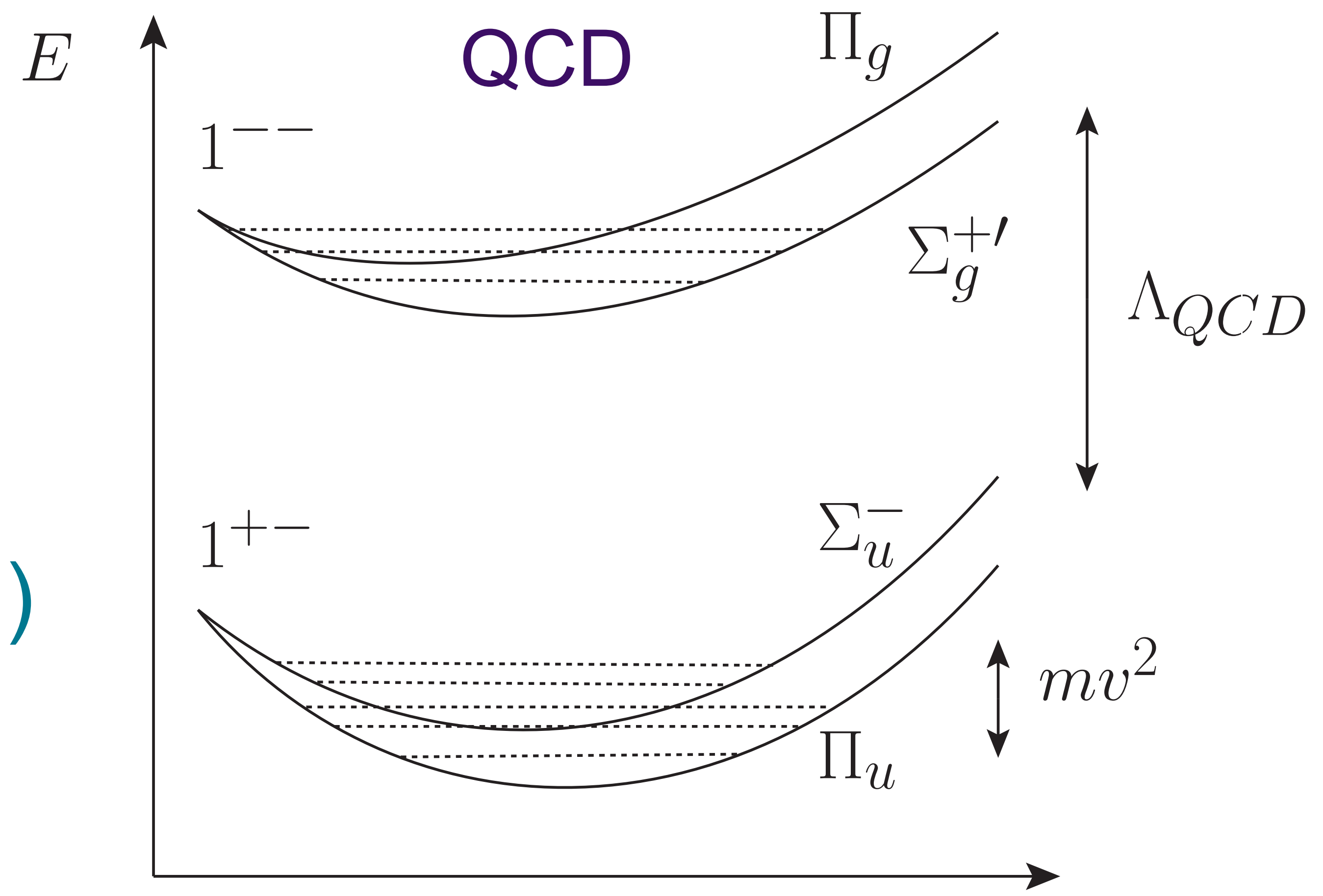
$$\Lambda_{QCD} > mv^2$$

fast (gluons, light quarks) and slow (heavy quarks)

like in molecular physics (fast-electrons, slow nuclei)

Braaten PRL 111 (2013) 162003

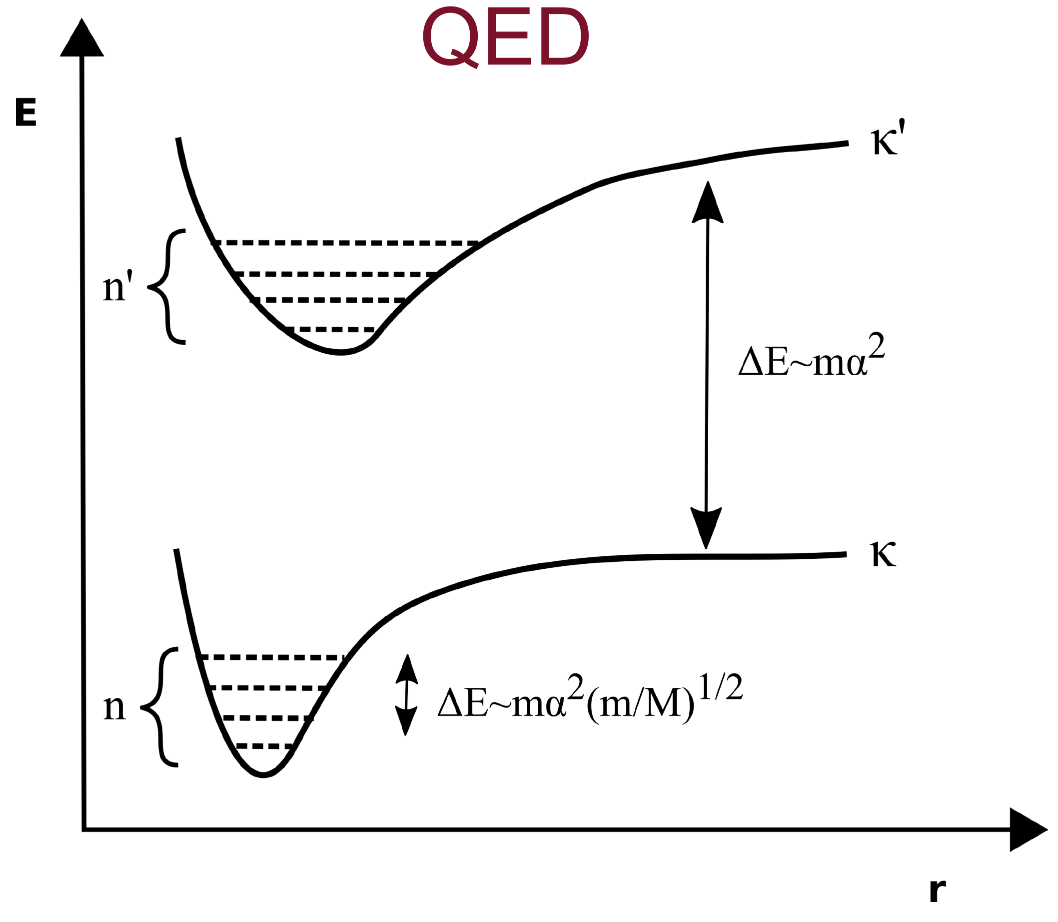
Braaten Langmack Smith PRD 90 (2014) 014044



Higher excitations
develop a gap of order Lambda_QCD

Introducing a finite mass m:

- The spectrum of the mv^2 fluctuations around the lowest static energy is the quarkonium spectrum
- The spectrum of the mv^2 fluctuations around the higher excitations is the exotic spectrum (hybrids and tetraquarks)



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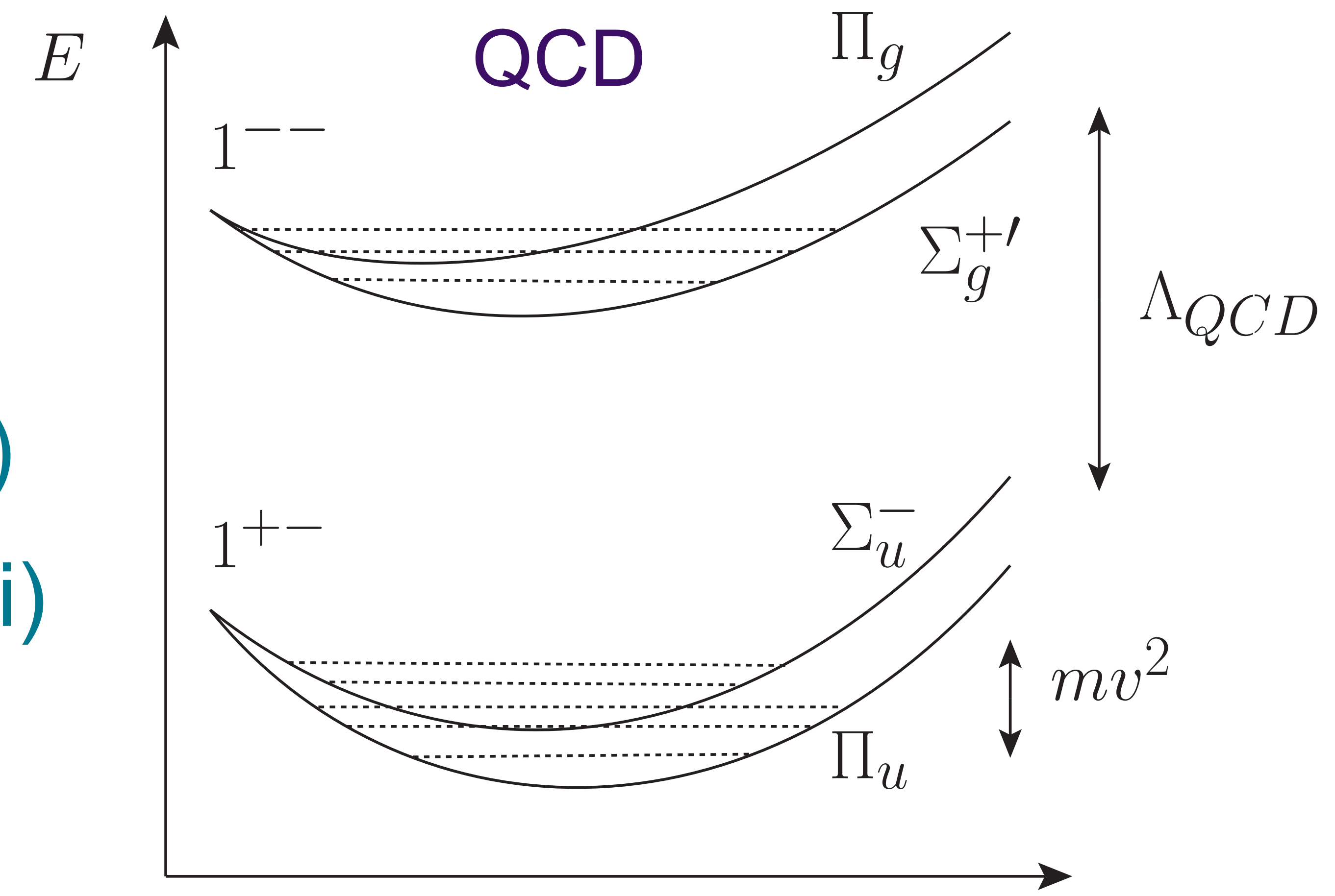

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Nonperturbative matching to the pNREFT

systematically

$$\langle H|\mathcal{H}|H\rangle = \langle nljs|\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}|nljs\rangle$$

expand quantomechanically NRQCD states and energies in 1/m around the zero order and identify the QCD potentials

$|\underline{0};\mathbf{x}_1\mathbf{x}_2\rangle -> |(Q\bar{Q})_1\rangle \rightarrow$ Quarkonium Singlet

$E_0(r) -> V_0(r)$ pNRQCD

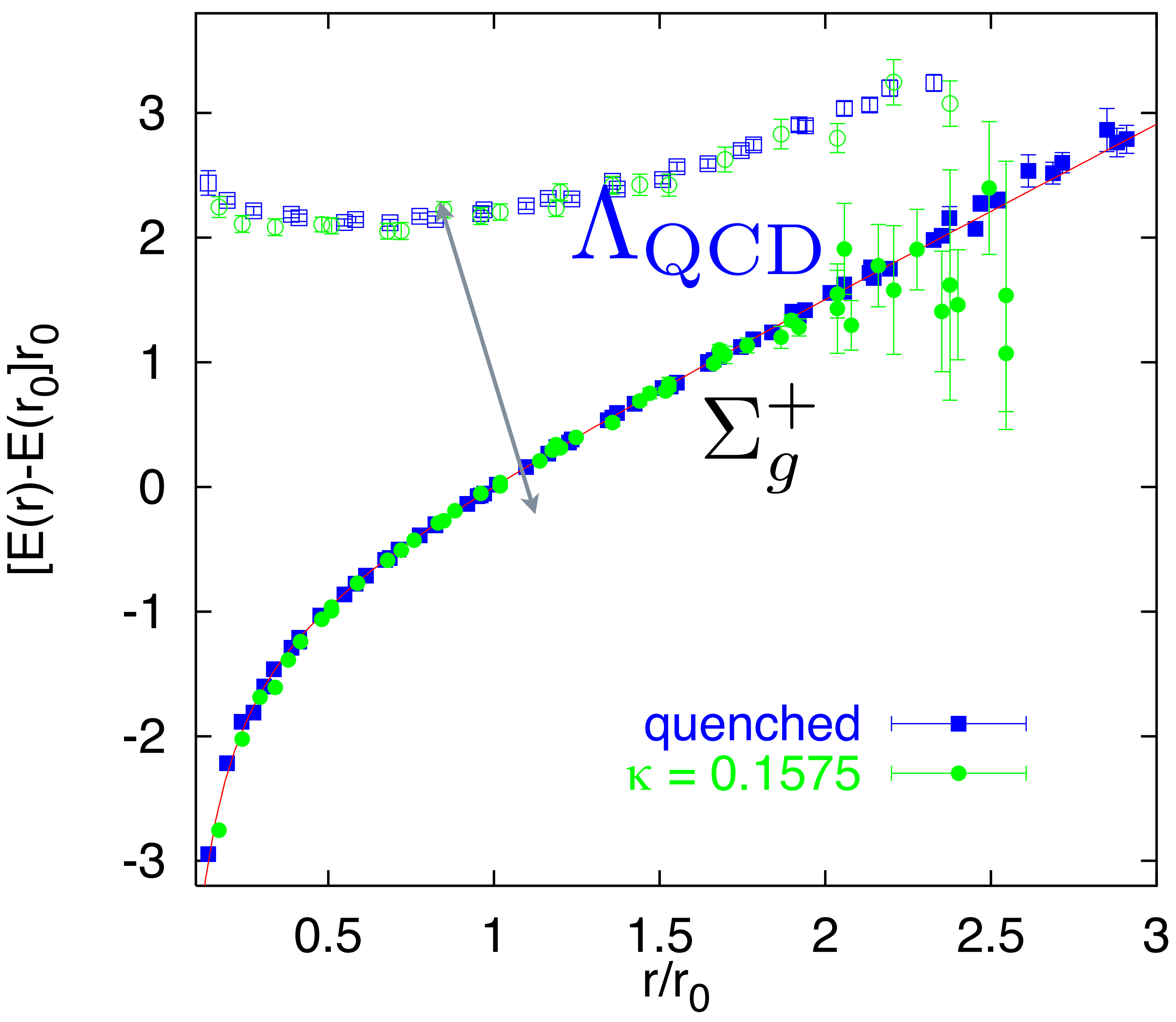
$|\underline{n} > 0;\mathbf{x}_1\mathbf{x}_2\rangle -> |(Q\bar{Q})g^{(n)}\rangle \rightarrow$ Higher Gluonic Excitations

$|Q\bar{Q}q\bar{q}\rangle$ Tetraquarks

$E_n^{(0)}(r) -> V_n^{(0)}(r)$ BOEFT

Strongly coupled pNRQCD for quarkonium

Σ_g^+ k^PC=0^++



Bali et al. 98

the potentials come from integrating out all scales up to mv^2

- gluonic excitations develop a gap Λ_{QCD} and are integrated out

$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

Brambilla Pineda Soto Vairo 00

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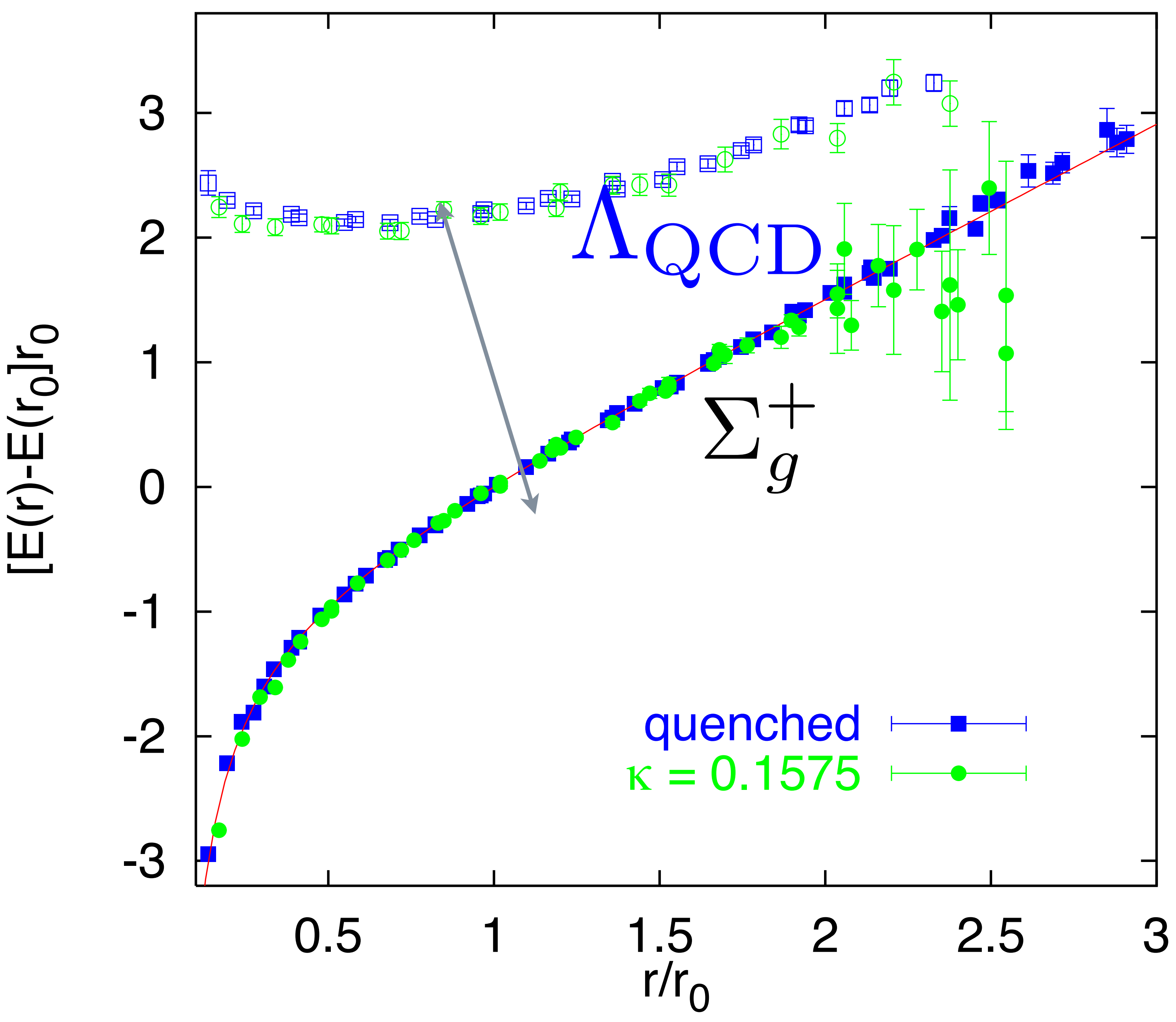

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$$\mathcal{L} = \mathrm{Tr}\left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S \right\} + \Delta\mathcal{L}(\text{US light quarks})$$

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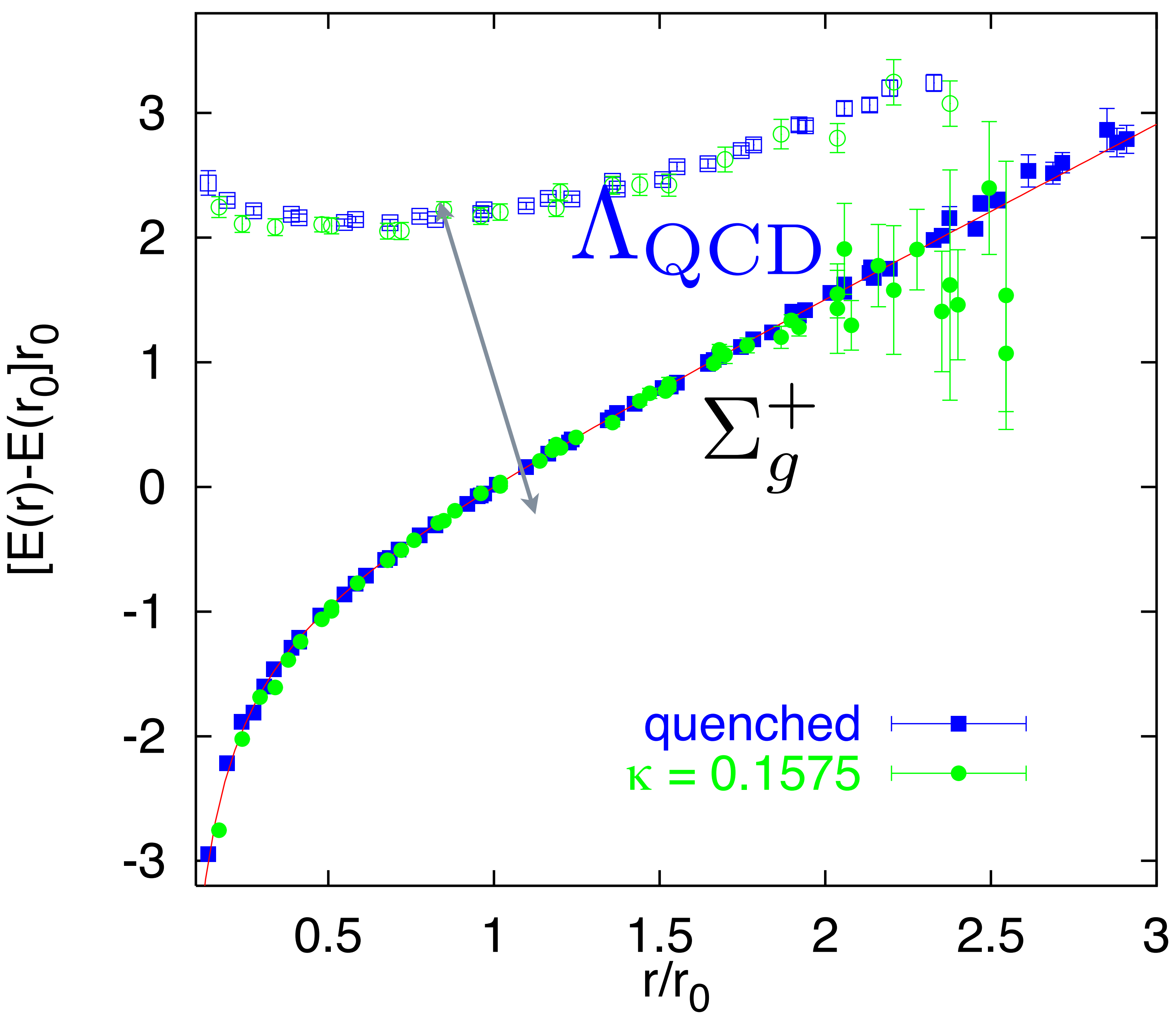

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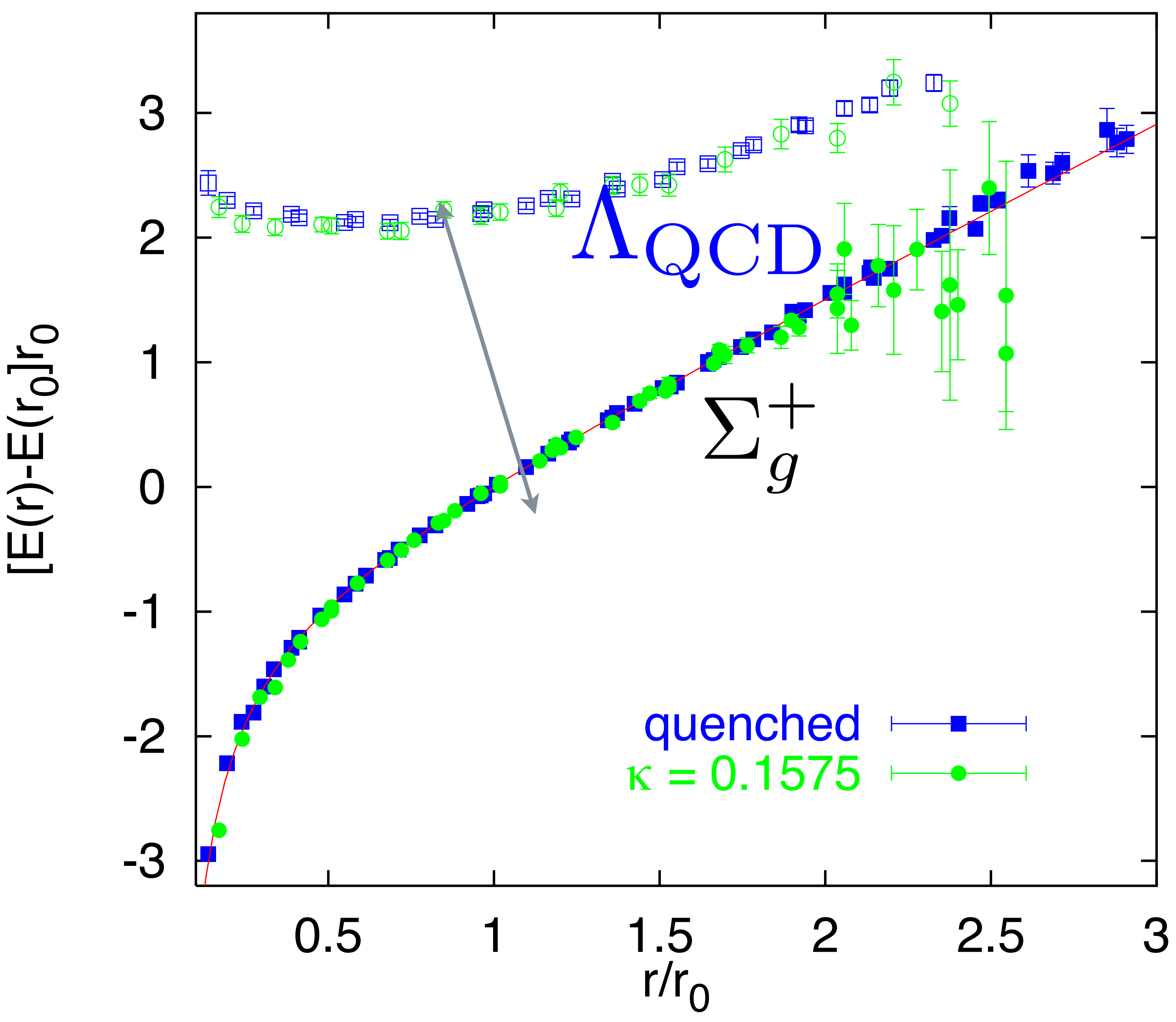

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Applications regard: Spectrum, decays, production at LHC, studies of confinement

The singlet potential has the general structure

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static — spin dependent — velocity dependent

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

$$W = \langle \exp\{ig \oint A^\mu dx_\mu\}\rangle$$

E_0(r)= $V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r\times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln \langle \square \rangle$

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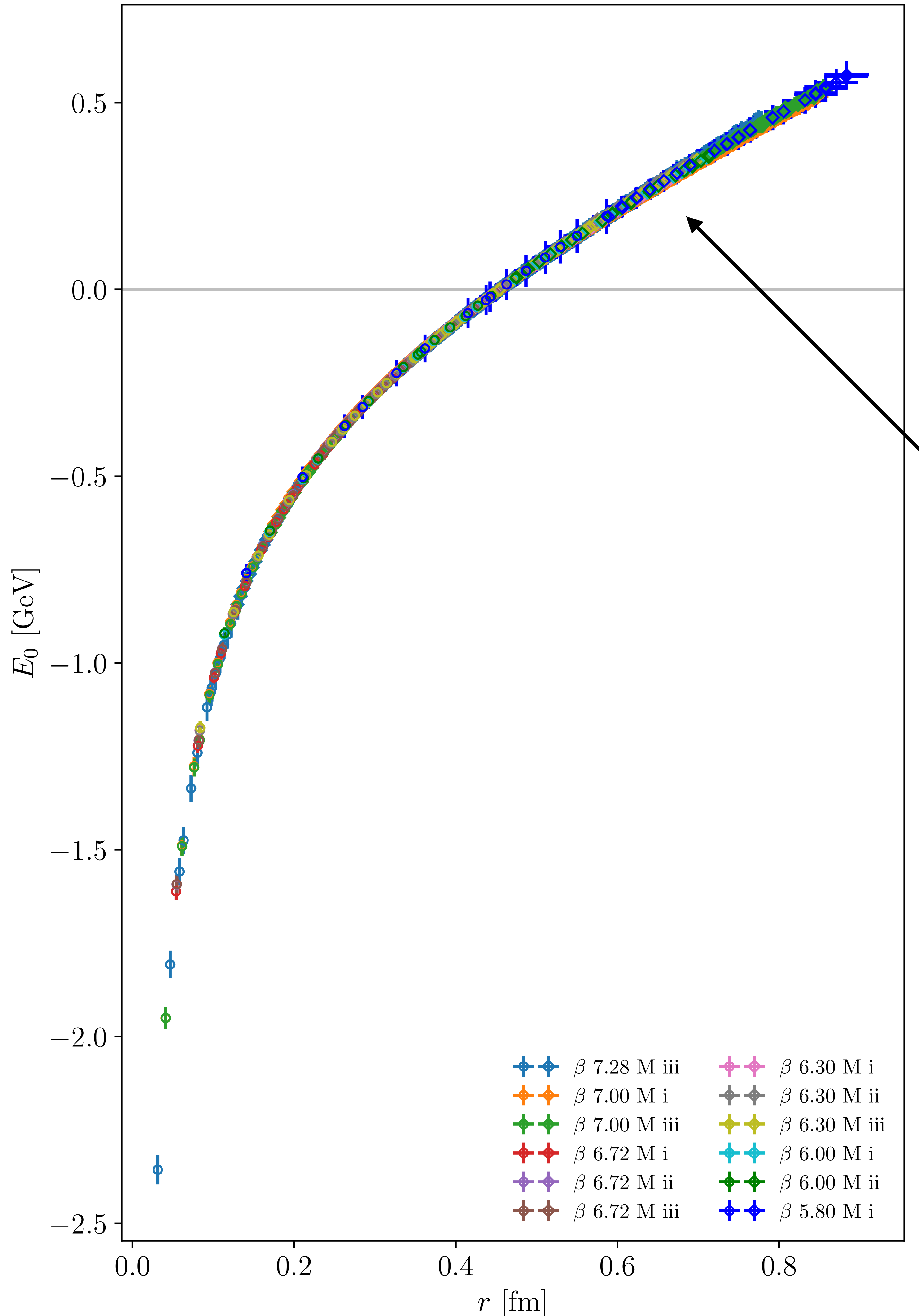
State of the art:

2+1+1 quarkonium

static energy E_0(r)

Σ_g^+



confinement

TUMQCD
N.B., Delgado, Kronfeld, Leino, Petreczky, Steinbeisser, Vairo, Weber *Phys.Rev.D* 107 (2023) 7, 074503 •

Behaviour of the static energy at short distance $r \ll \Lambda_{\rm QCD}^{-1}$

Multipole expansion in r is possible: color singlet and color octet degrees of freedom:

theory is weakly coupled pNRQCD

The gauge fields are multipole expanded:

$$A(R,r,t) = A(R,t) + \mathbf{r}\cdot\boldsymbol{\nabla}A(R,t) + \ldots$$

$$\mathcal{L}^{\rm pNRQCD} = \int d^3r\, {\rm Tr}\,\{S^\dagger(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \cdots)S + O^\dagger(iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \cdots)O +$$

LO in r

$$+V_A(S^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger\mathbf{r}\cdot g\mathbf{E}S) + \frac{V_B}{2}(O^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E})\} + \ldots$$

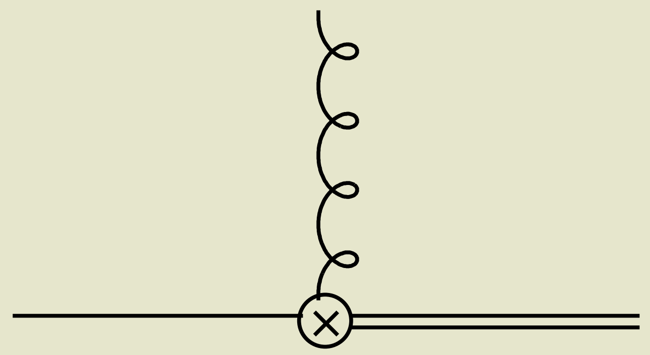
NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

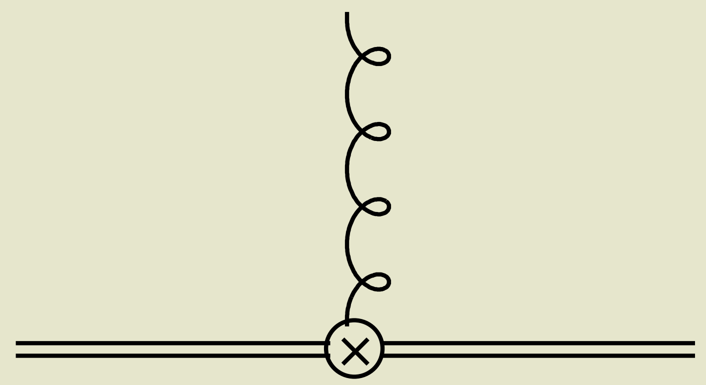
Feynman rules

$\theta(T)\, e^{-iTV_s}$

$\theta(T)\, e^{-iTV_o}\left(e^{-i\int dt\, A^{\rm adj}}\right)$



$O^\dagger\mathbf{r}\cdot g\mathbf{E}\,{\rm S}$



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LO in r

$$\left. + V_A(S^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger \mathbf{r}\cdot g\mathbf{E}S) + \frac{V_B}{2}(O^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E})\right\} + \ldots$$
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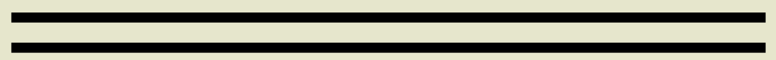

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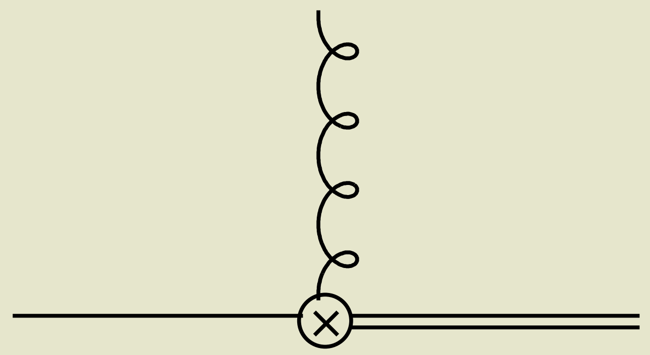

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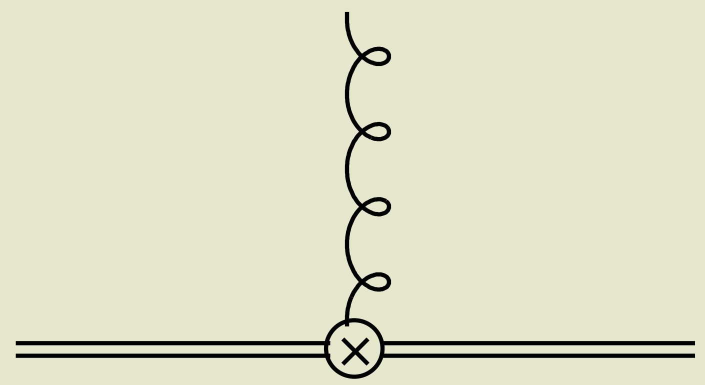
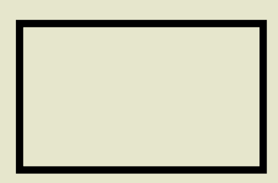
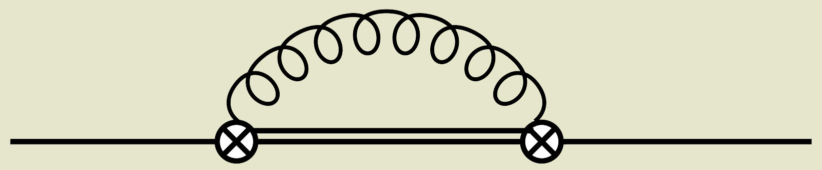


$O^\dagger\{\mathbf{r}\cdot g\mathbf{E},\, O\}$

The NRQCD static energy E_0 is calculable in perturbation theory in pNRQCD

$$V^{(0)}(r,\mu') = \lim_{T\to\infty}\frac{i}{T}\ln\langle \square \rangle - \text{(diagram)} + \cdots$$





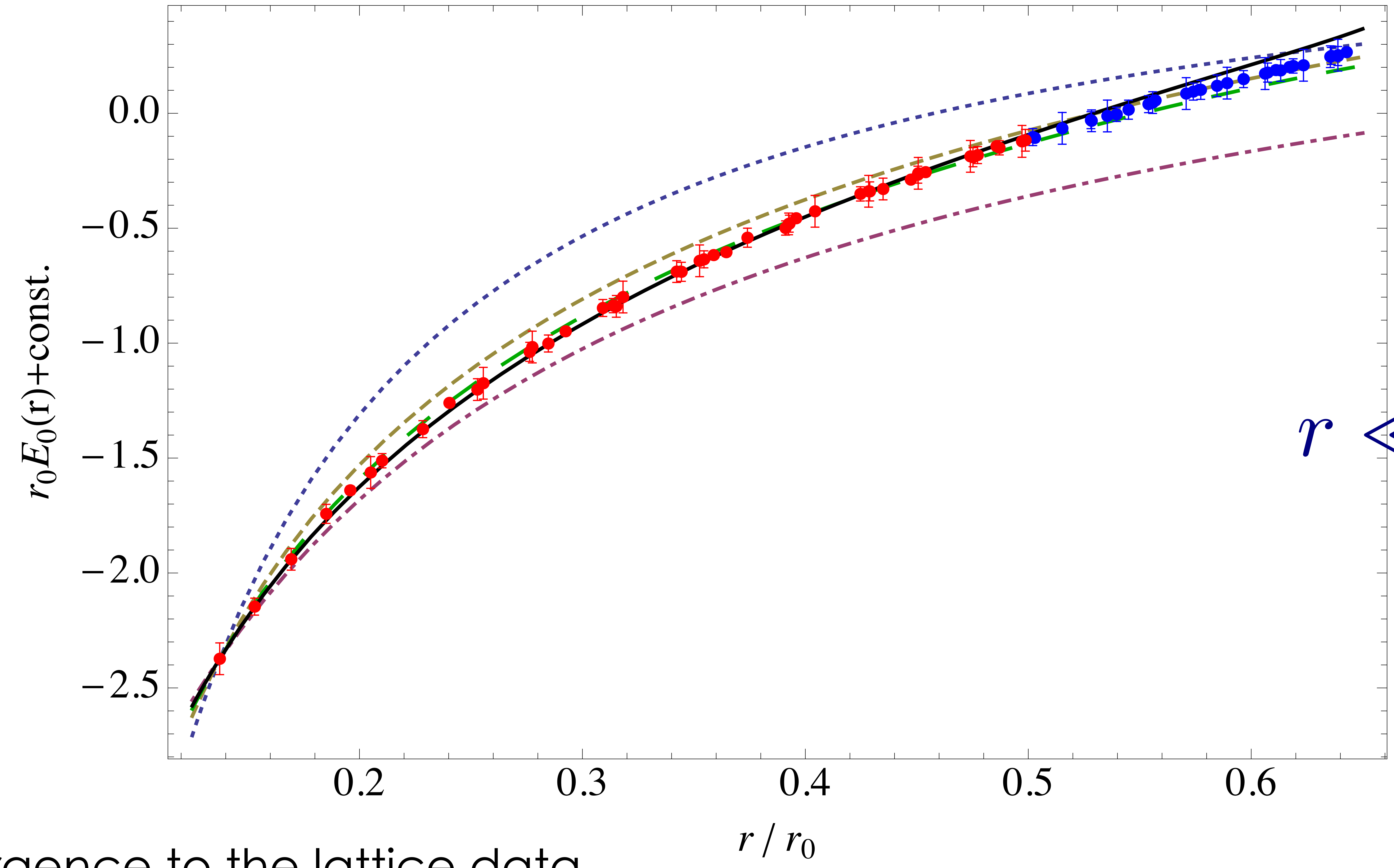
$$= E_0(r) + \frac{i}{N}\int_0^\infty dt\, e^{-it(V_o - V)}\langle {\rm Tr}\,\mathbf{r}\cdot g\mathbf{E}(t)\,\mathbf{r}\cdot g\mathbf{E}(0)\rangle(\mu') + \cdots$$

QQbar singlet static energy at NNNLL in pNRQCD in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014, with Weber 2019

Full control

at short distance!



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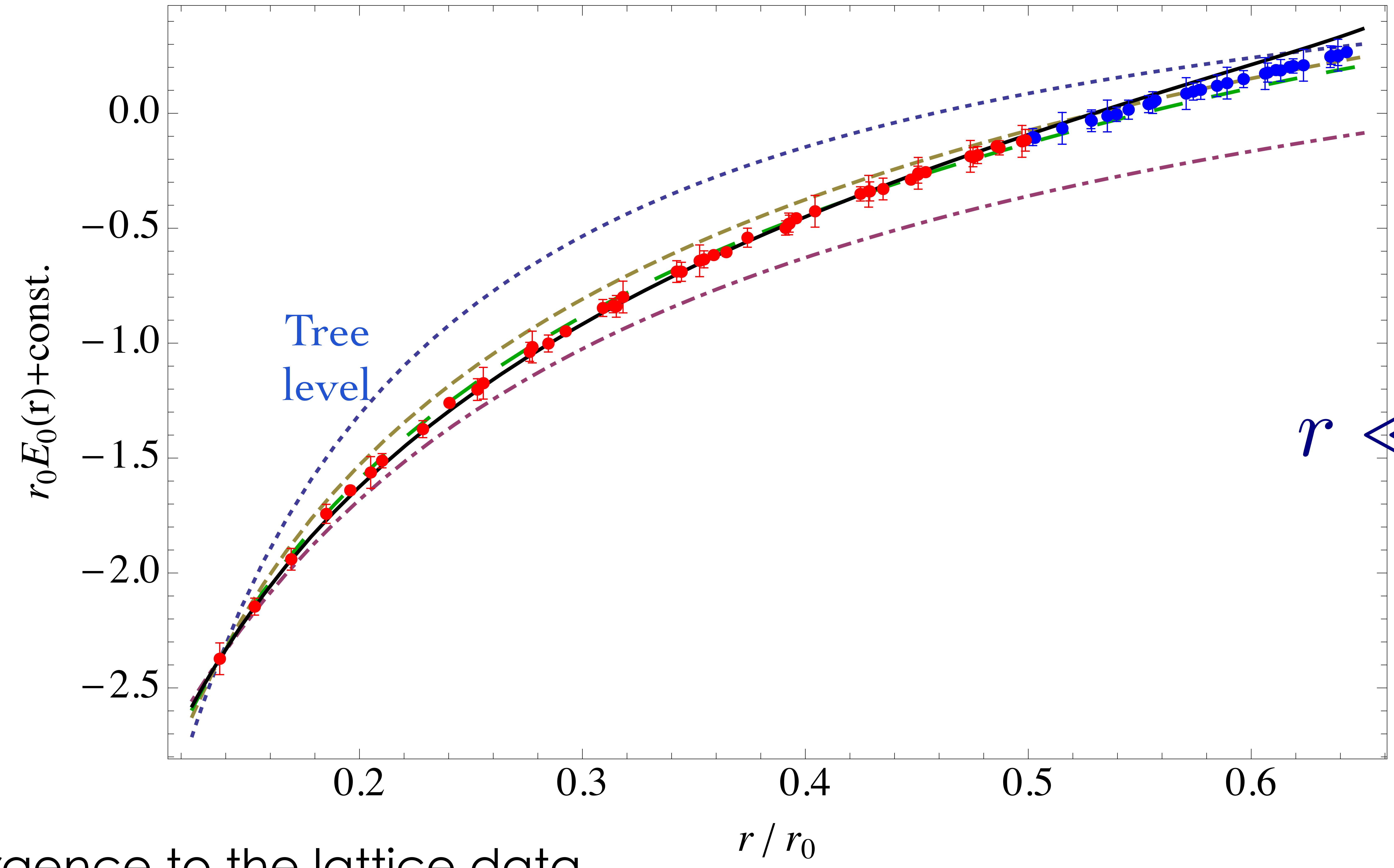

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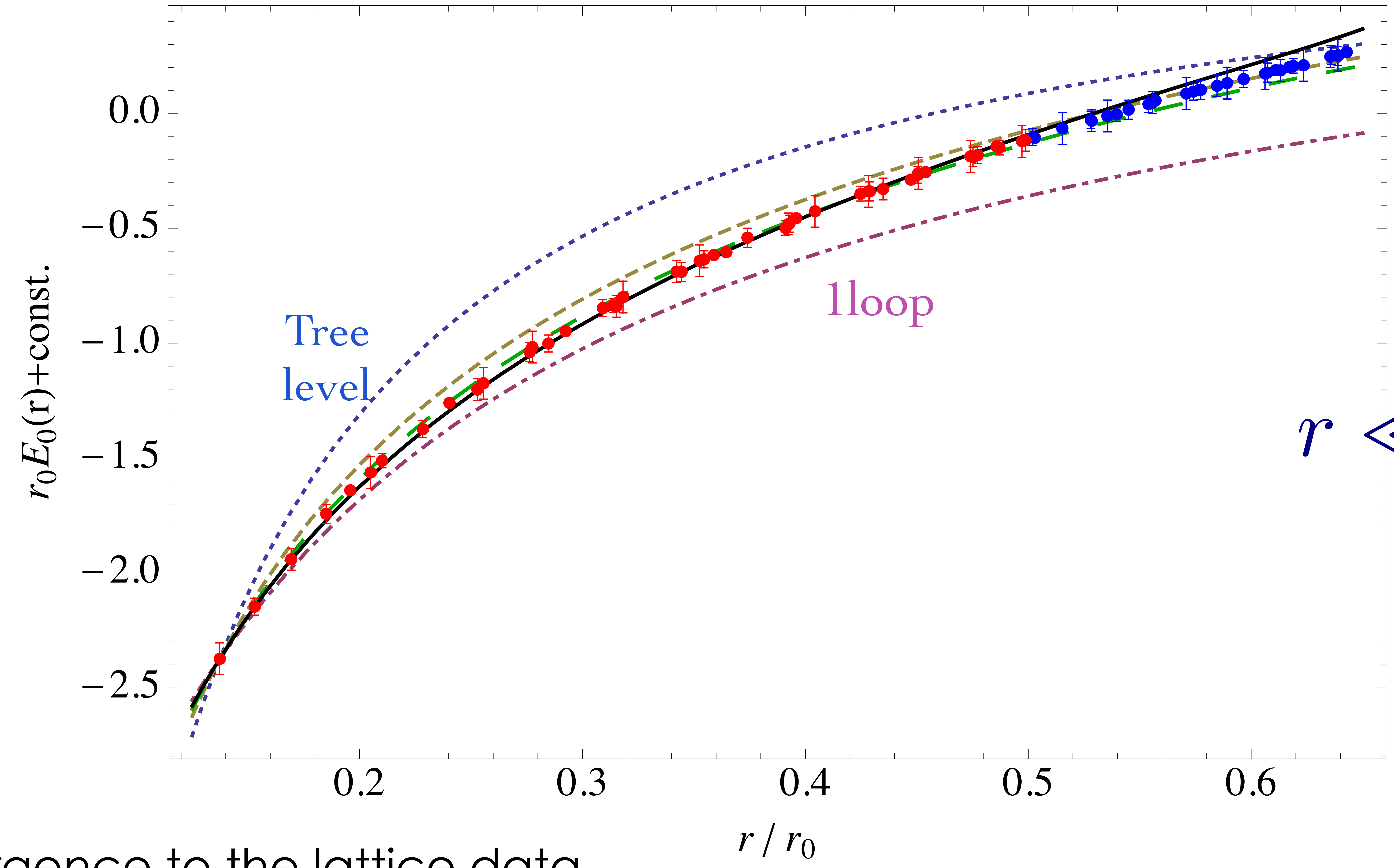

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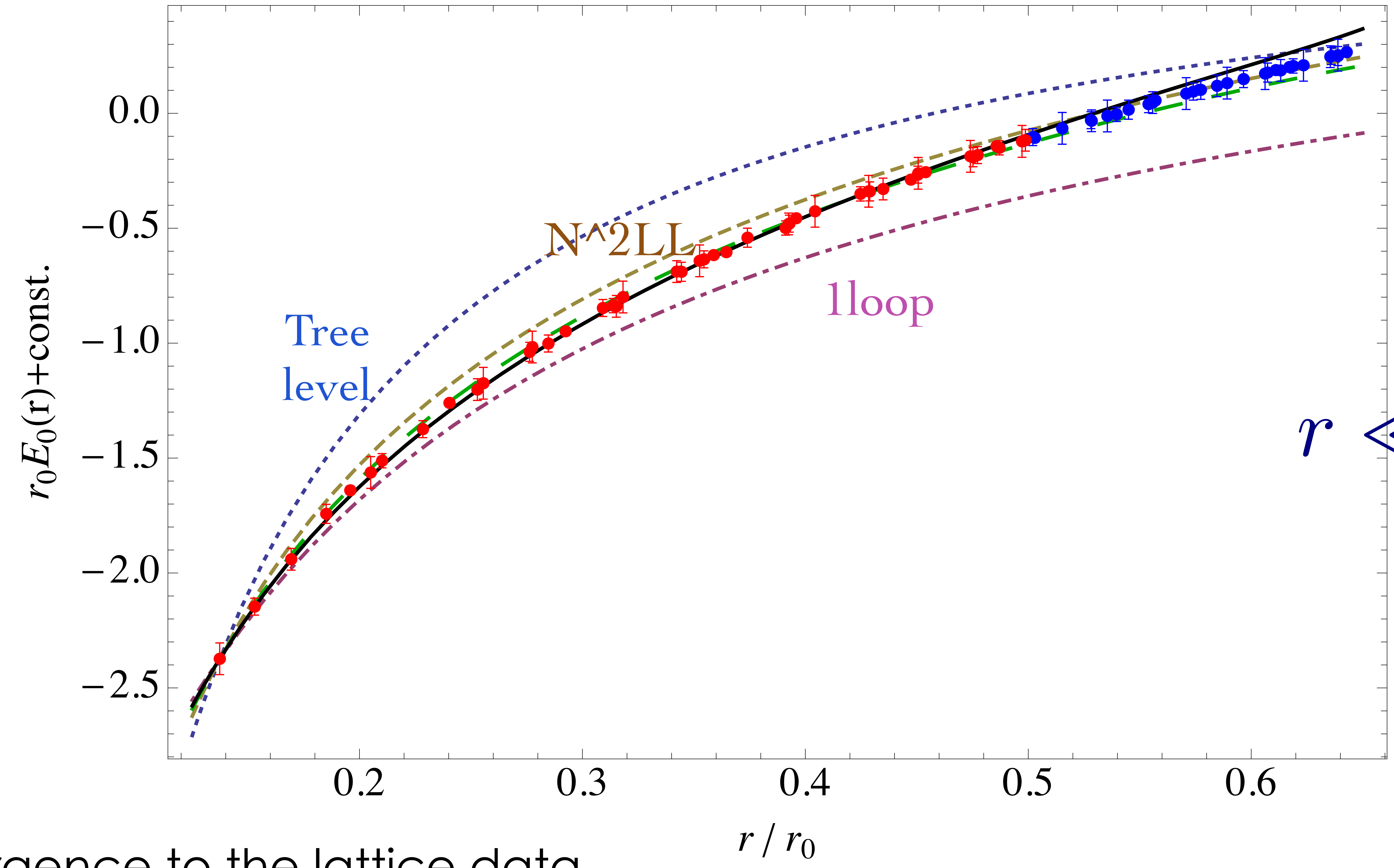

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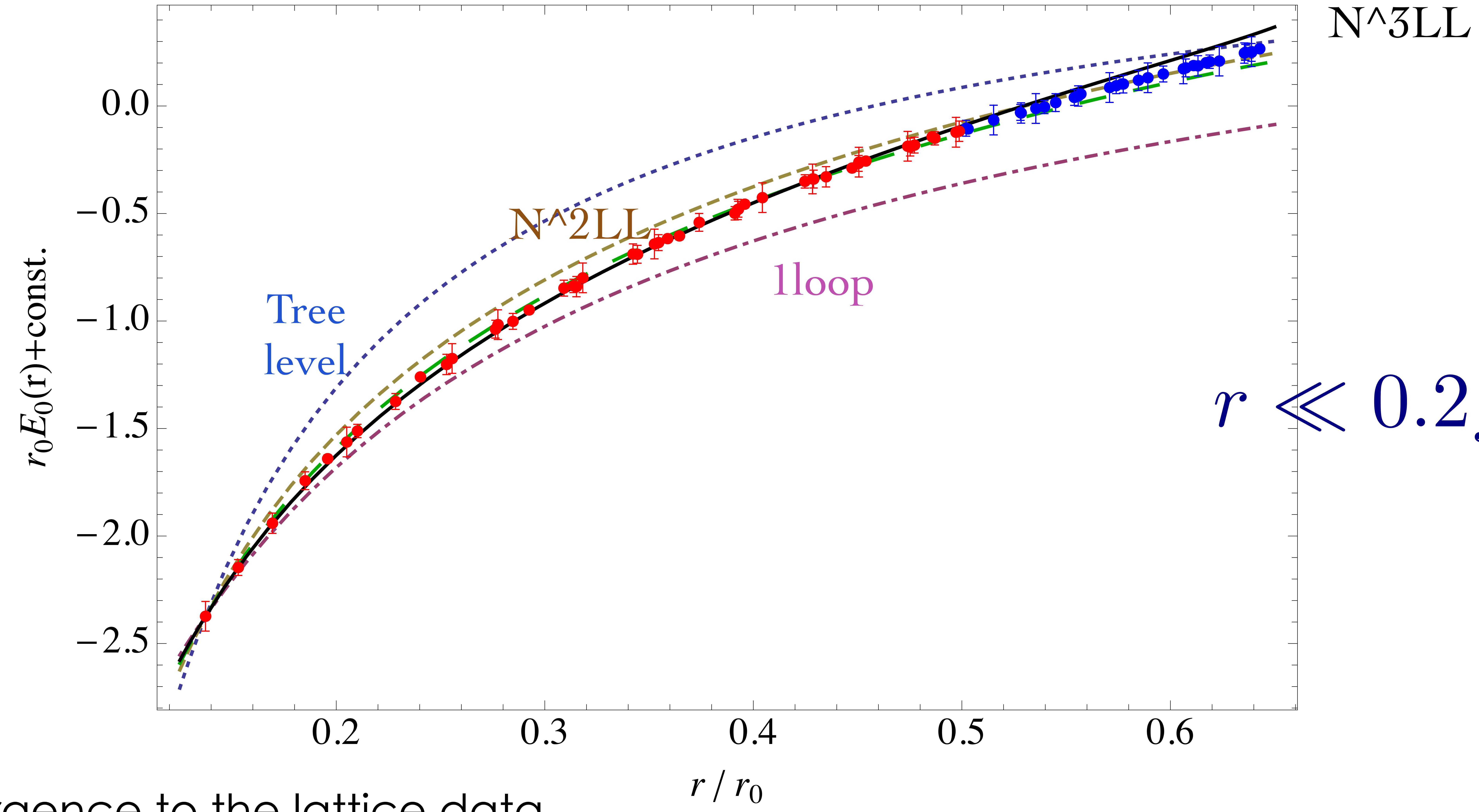

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$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t \left\langle \boxed{E(t)\;\; E(t)} \right\rangle$$

$$V_{SD}^{(2)} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \left\langle \boxed{i \;\; j}_{E(t),\,B} \right\rangle \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2) \quad |V_{LS}^{(2)}$$

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \left\langle \boxed{i \;\; j} \right\rangle - \frac{2c_F - 1}{2} \nabla^k V^{(0)} \right) \mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2) \quad |V_{LS}^{(1)}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\left\langle \boxed{i \;\; j} \right\rangle - \frac{\delta_{ij}}{3} \left\langle \boxed{\;\;} \right\rangle \right) \left(\mathbf{S}_1 \cdot \mathbf{S_2} - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S_2} \cdot \hat{\mathbf{r}}) \right) \quad |V_T$$

$$+\left(\frac{2}{3} c_F^2 i \int_0^\infty dt \left\langle \boxed{\;\;} \right\rangle - 4\left(d_{sv} + \frac{4}{3} d_{vv} \right) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S_2} \quad |V_S$$

gauge invariant wilson loops can be calculated also in QCD vacuum model and large N

Pineda Vairo PRD 63 (2001) 054007

Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

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- the potentials contain the contribution of the scale m inherited from NRQCD matching coefficients—> they cancel any QM divergences, good UV behaviour
- the flavour dependent part is extracted in the NRQCD matching coefficients
- the nonperturbative part is factorized and depends only on the glue —> only one lattice calculation to get the dynamics and the observables instead of an ab initio calculation of multiple Green functions

In the EFT it has been possible to calculate

quarkonium production and in nonequilibrium evolution in

medium which makes this treatment promising for XYZ

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This can be extended to the XYZ that are however a much more interesting case

BOEFT construction

- We consider all the NRQCD static energies in presence of glue and light quarks: QQbarg, QQbar qqbar, QQ qbar qbar, QQbar qqq, QQ qqqbar, QQq

- We define the NRQCD static energies via gauge-invariant correlator of appropriate interpolating operators

- We calculate the short distance behaviour: gluelumps, adjoint meson, triplet mesons, sextet mesons ….

- BO quantum number is conserved: BO static energies evolve in heavy-light static energies with the same quantum numbers: allows to understand the form of the fundamental strong force

- We consider separately NRQCD static energies separated by a gap Lambda_QCD

- We match the NRQCD static energies to the corresponding potentials in BOEFT

Mixing appears: at short distance between static energies with same k—> coupled Schr. Eqs.

at large distance between static energies with same BO numbers that get close in energy (avoided level crossing)—> couples Schr. Eqs.

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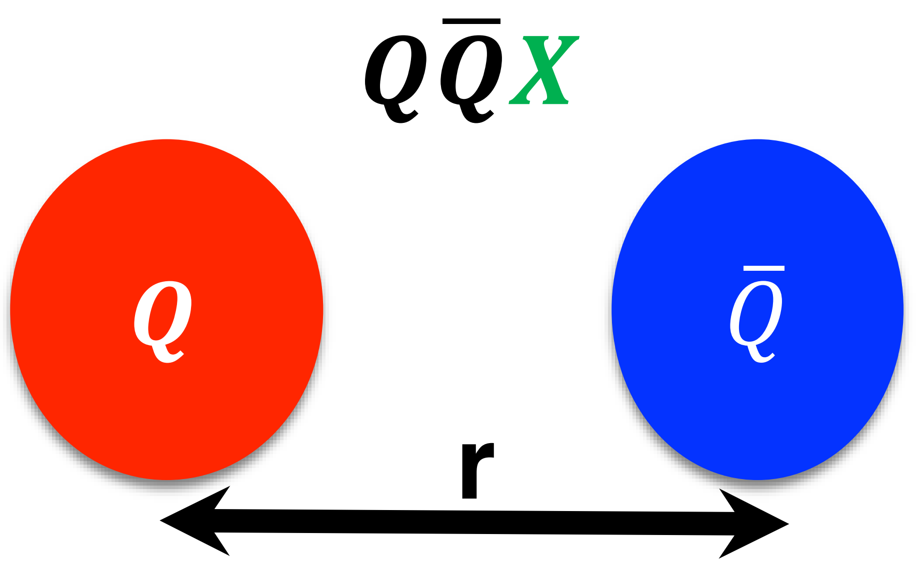
Notice that all these points are fixed by BOEFT symmetries

Exotic hadrons and corresponding NRQCD static energies



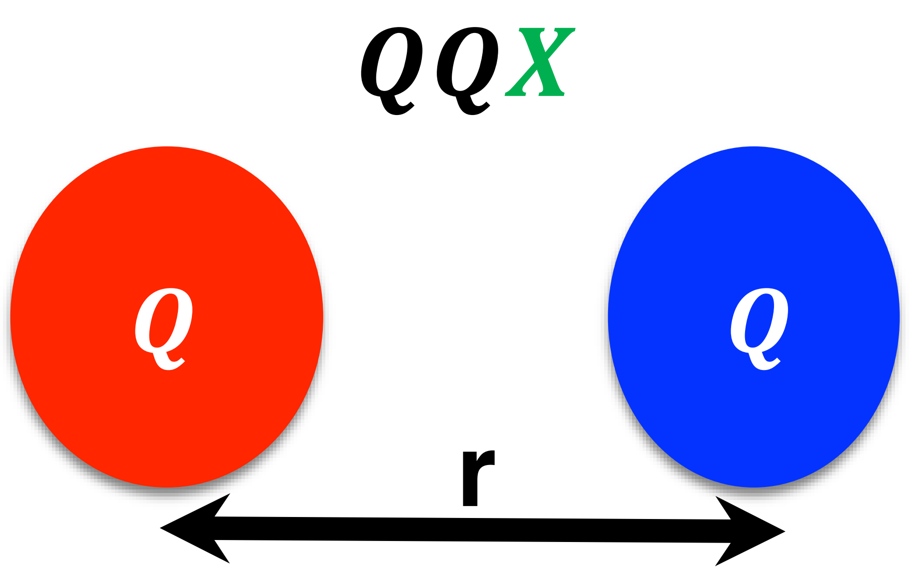
color: $3 \otimes \bar{3} = 1 \oplus 8$

X_8= gluon $\rightarrow$ **Hybrid**

$X_8 = q\bar{q} \rightarrow$ **Tetraquark**

$X_8 = qqq \rightarrow$ **Pentaquark**

Total angular momentum of **$Q\bar{Q}$X** or **QQX** :
$\mathbf{J} = \mathbf{L}_{\mathbf{Q\bar{Q}}} + \mathbf{K} + \mathbf{S}_{\mathbf{Q\bar{Q}}}$



color: $3 \otimes \bar{3} = \bar{3} \oplus 6$

X= q $\rightarrow$ **Double heavy baryon**

$X = \bar{q}\bar{q} \rightarrow$ **Tetraquark**

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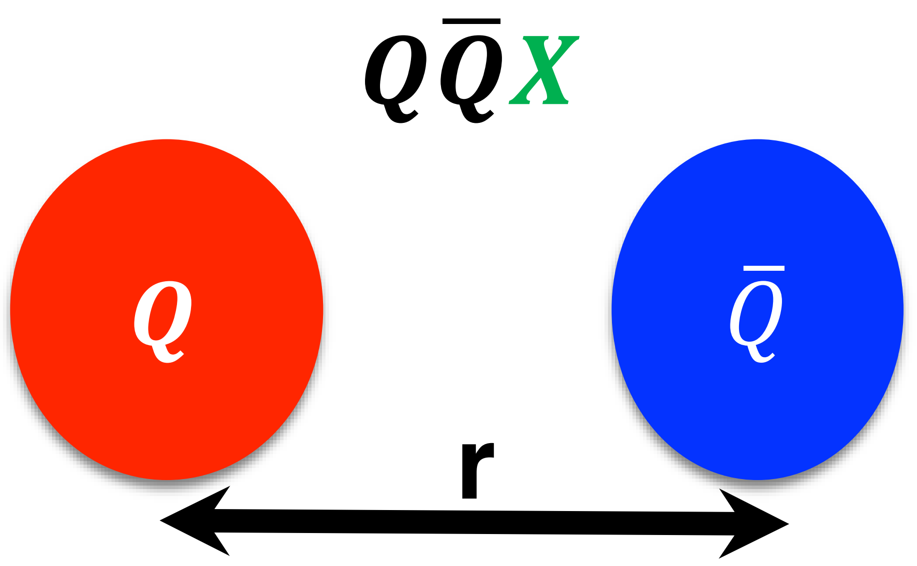

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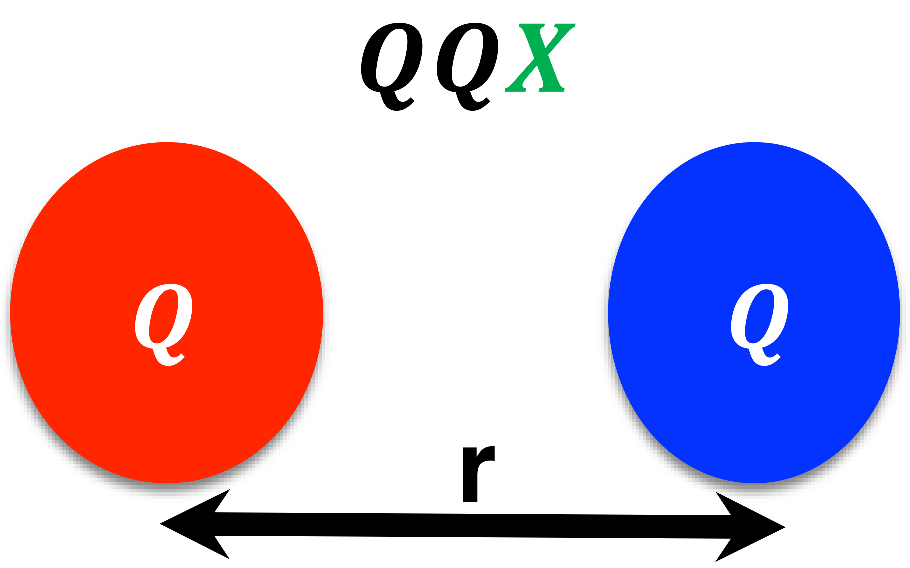

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$$E^{(0)}_{\kappa,|\lambda|}(r) = \lim_{T\to\infty} \frac{i}{T} \log\left[\langle \mathrm{vac}|\mathcal{O}_{\kappa,\lambda}(T/2,\, \boldsymbol{r},\, \boldsymbol{R})\, \mathcal{O}^{\dagger}_{\kappa,\lambda}(-T/2,\, \boldsymbol{r},\, \boldsymbol{R})|\mathrm{vac}\rangle\right]$$

$$\mathcal{O}_{\kappa,\lambda}(t,\boldsymbol{r}) = \chi^{\dagger}(t,\boldsymbol{r}/2)\,\phi(t;\boldsymbol{r}/2,\boldsymbol{0})\,P^{\alpha\dagger}_{\kappa,\lambda}H^{\alpha}_{\kappa}(t,\boldsymbol{0})\,\phi(t;\boldsymbol{0},-\boldsymbol{r}/2)\,\psi(t,-\boldsymbol{r}/2)$$

H^{α}_{κ} : LDF (gluon or light-quarks) operator characterizing X based on quantum # κ (isospin, color etc..)

$P^{\alpha}_{\kappa,\lambda}$: Projection vectors for projecting onto cyclindrical symmetry $D_{\infty h}$ representations.

Tetraquark Lattice NRQCD static energies

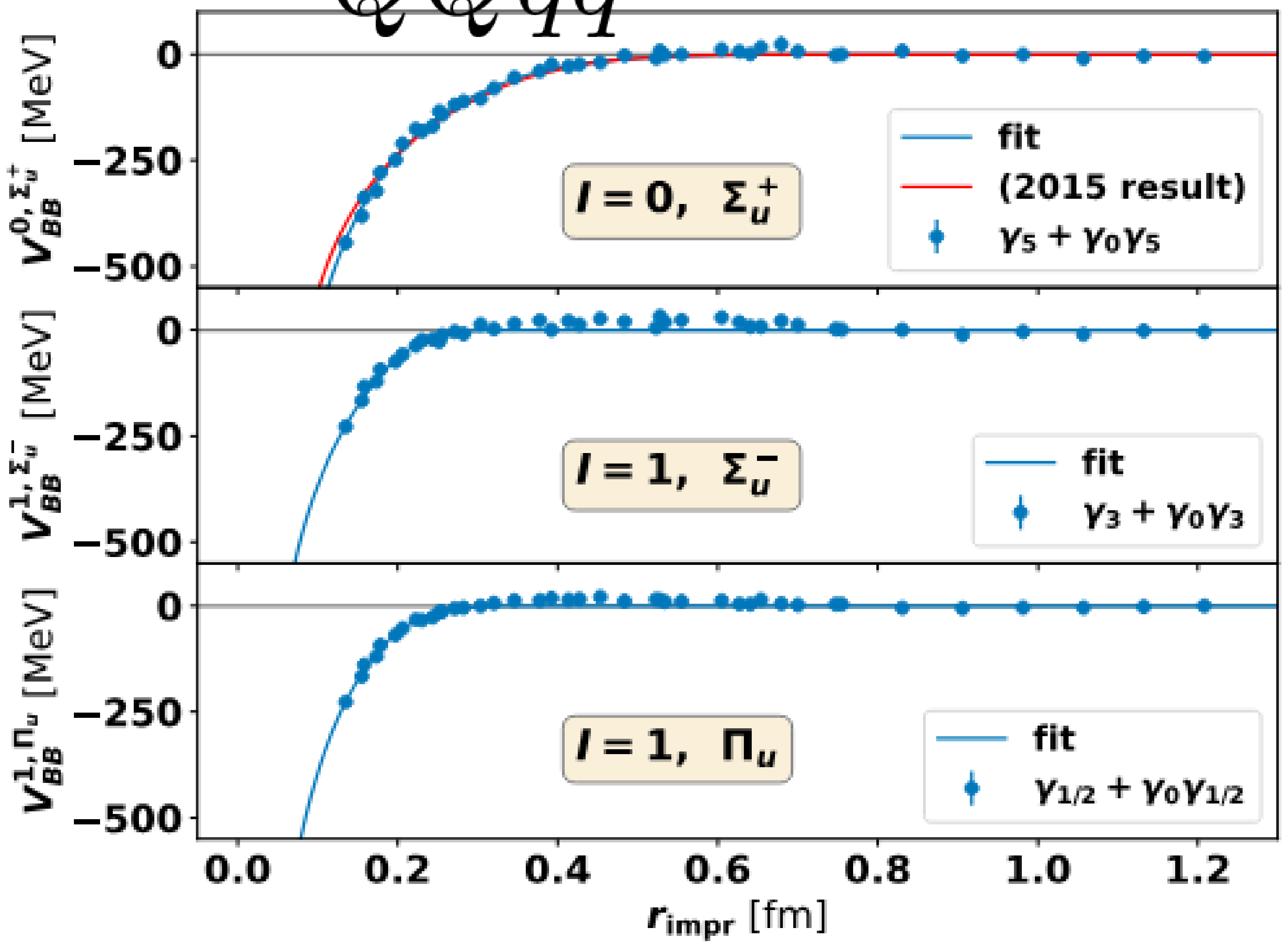
already calculated with other interpolators

Sadl, Prevlosek, 211014568

Liu Aoki Doi Hatsuda Ikea and Meng

Hal QCD method 2401.13917 heavy-light

$QQ\bar{q}\bar{q}$



Mueller et al, PoS LATTICE2023, 64 (2024)

Bicudo, Cichy, Peters, Wagner, Phys. Rev. D. 93, (2016)

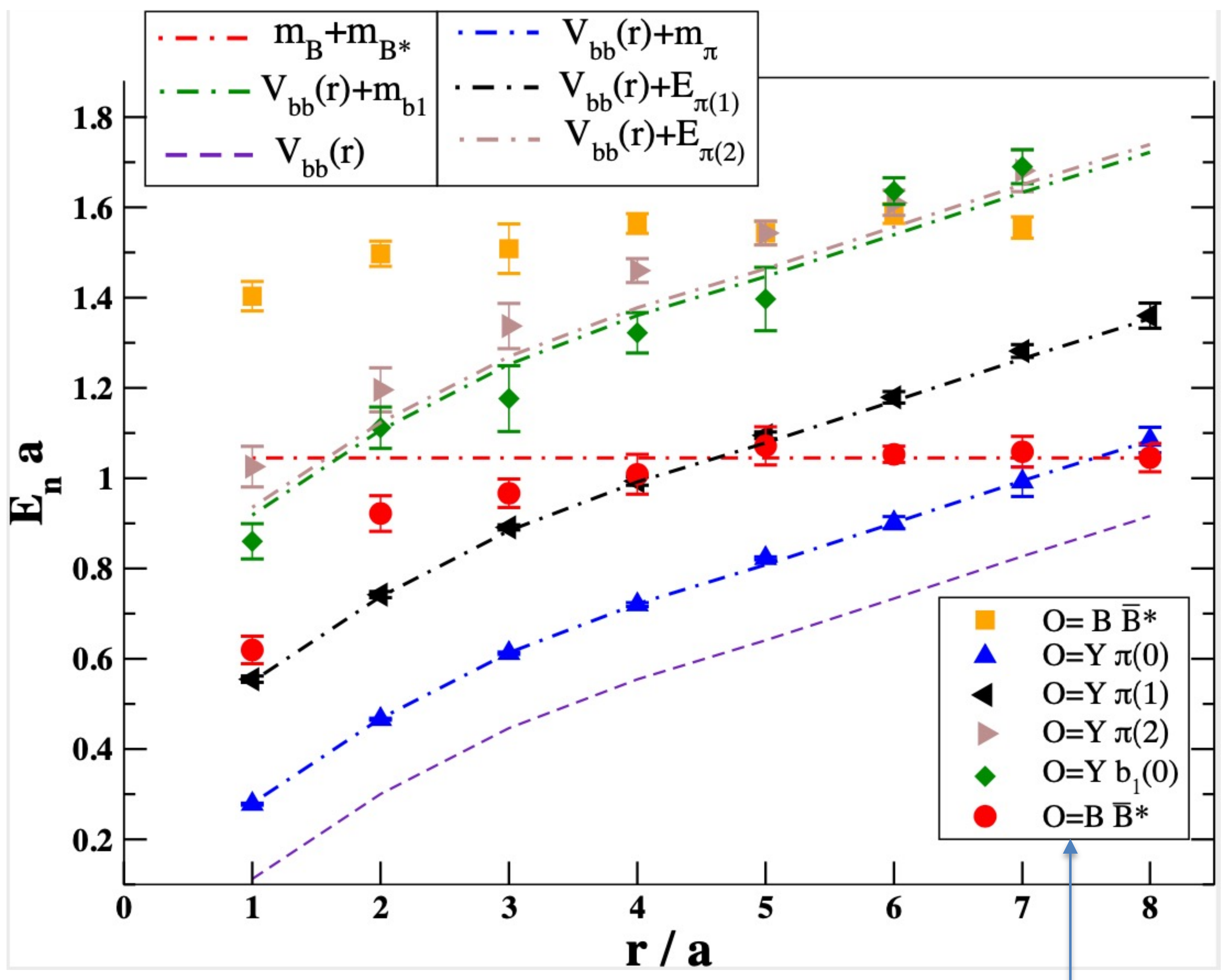
Bicudo, Marinkovic, Muller Wagner 2409.10786

BO heavy-light operators

$\bar{b}\,b\,\bar{d}\,u$

Z_b channe $\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$

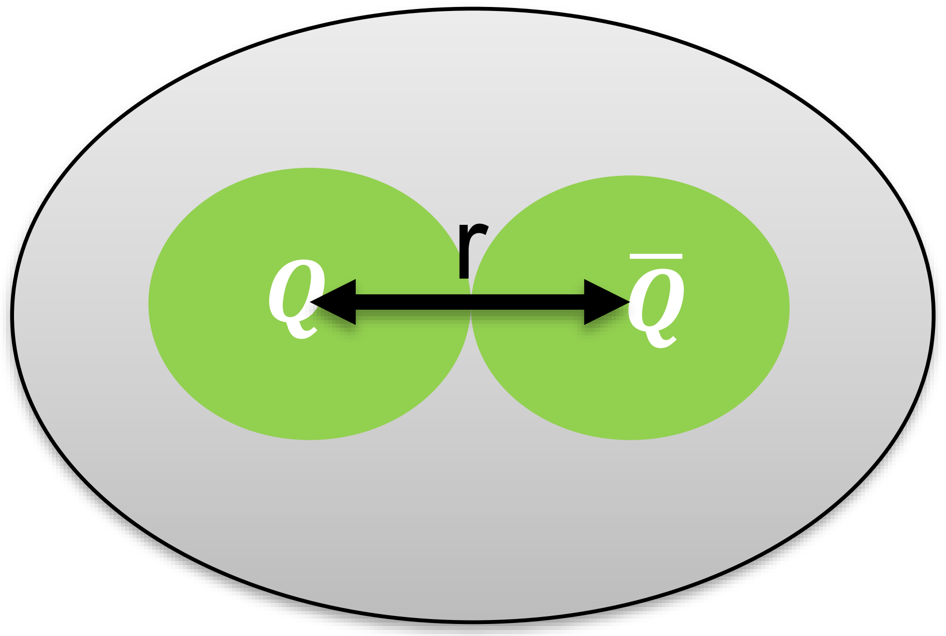
Eigen-energies $E_n(r)$: channel S_h=1, CP=-1, ε=-1



$m_B+m_{B^*}$

BOEFT: Short distance behaviour of the NRQCD static energies

LDF-quantum #: $\kappa = \{K^{PC}, f\}$



Short-distance ($r \to 0$)

$Q\bar{Q}$: $$E^{(0)}_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 + \cdots$$

$Q\bar{Q}X$: $$E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_o(r) + \Lambda_{H_\kappa} + b_{\Lambda_\eta^\sigma} r^2 + \cdots$$

QQX: $$E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_l(r) + \Lambda_{H_\kappa,l} + b_{\kappa\lambda,l}\, r^2 + \cdots \quad (l = T, \Sigma)$$

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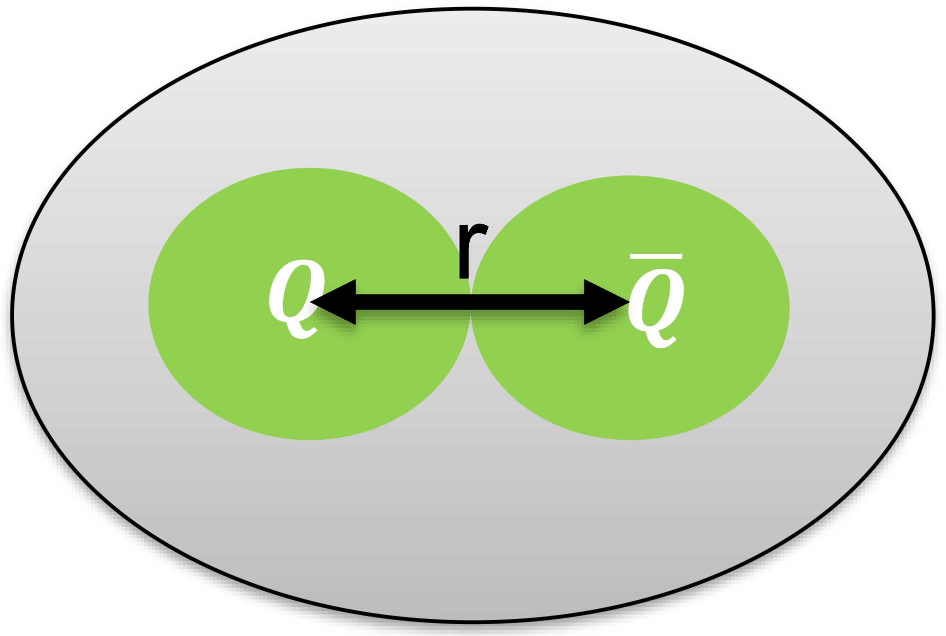


Short-distance ($r \to 0$)

$$V_s(r) = -\frac{4\alpha_s}{3r}, \; V_o(r) = \frac{\alpha_s}{6r}$$
$$V_T(r) = -\frac{2\alpha_s}{3r}, \; V_\Sigma(r) = \frac{\alpha_s}{3r}$$

$Q\bar{Q}$: $E^{(0)}_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 + \cdots$

$Q\bar{Q}X$: $E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_o(r) + \Lambda_{H_\kappa} + b_{\Lambda_\eta^\sigma} r^2 + \cdots$

QQX: $E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_l(r) + \Lambda_{H_\kappa,l} + b_{\kappa\lambda,l}\, r^2 + \cdots \; (l = T, \Sigma)$

$$\Lambda_{H_\kappa} = \lim_{T\to\infty} \frac{i}{T} \langle \text{vac}| H^a_\kappa(T/2, \boldsymbol{R})\, \phi^{ab}\,(T/2, -T/2)\, H^{a\dagger}_\kappa(-T/2, \boldsymbol{R})|\text{vac}\rangle$$

- Gluelump / adjoint meson or baryon mass for $Q\bar{Q}X$ states
- Triplet meson or baryon / Sextet meson or baryon mass for QQX states
- Λ_{H_κ} depends only on κ ->degeneration

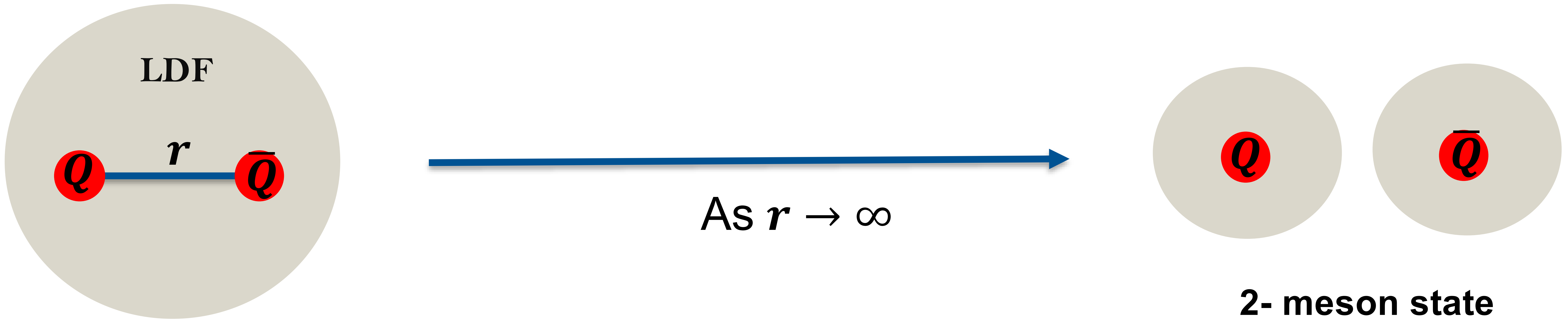
Most recent results on gluelump spectrum:

Herr, Schlosser, Wagner Phys. Rev. D 109 (2024)

Adjoint meson spectrum (1^{--} & 0^{-+}):

Foster, Michael (UKQCD) Phys. Rev. D 59 (1999)

BOEFT: Quarkonium and Tetraquarks evolve in heavy-light at large distance



Consider $Q\bar{Q}q\bar{q}$ system:

BO-quantum # Λ_η^σ for adjoint meson:

$Q\bar{Q}$ (color)	Light Spin κ^{PC}	Λ_η^σ $(D_{\infty h})$
Octet	0^{-+}	Σ_u^-
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

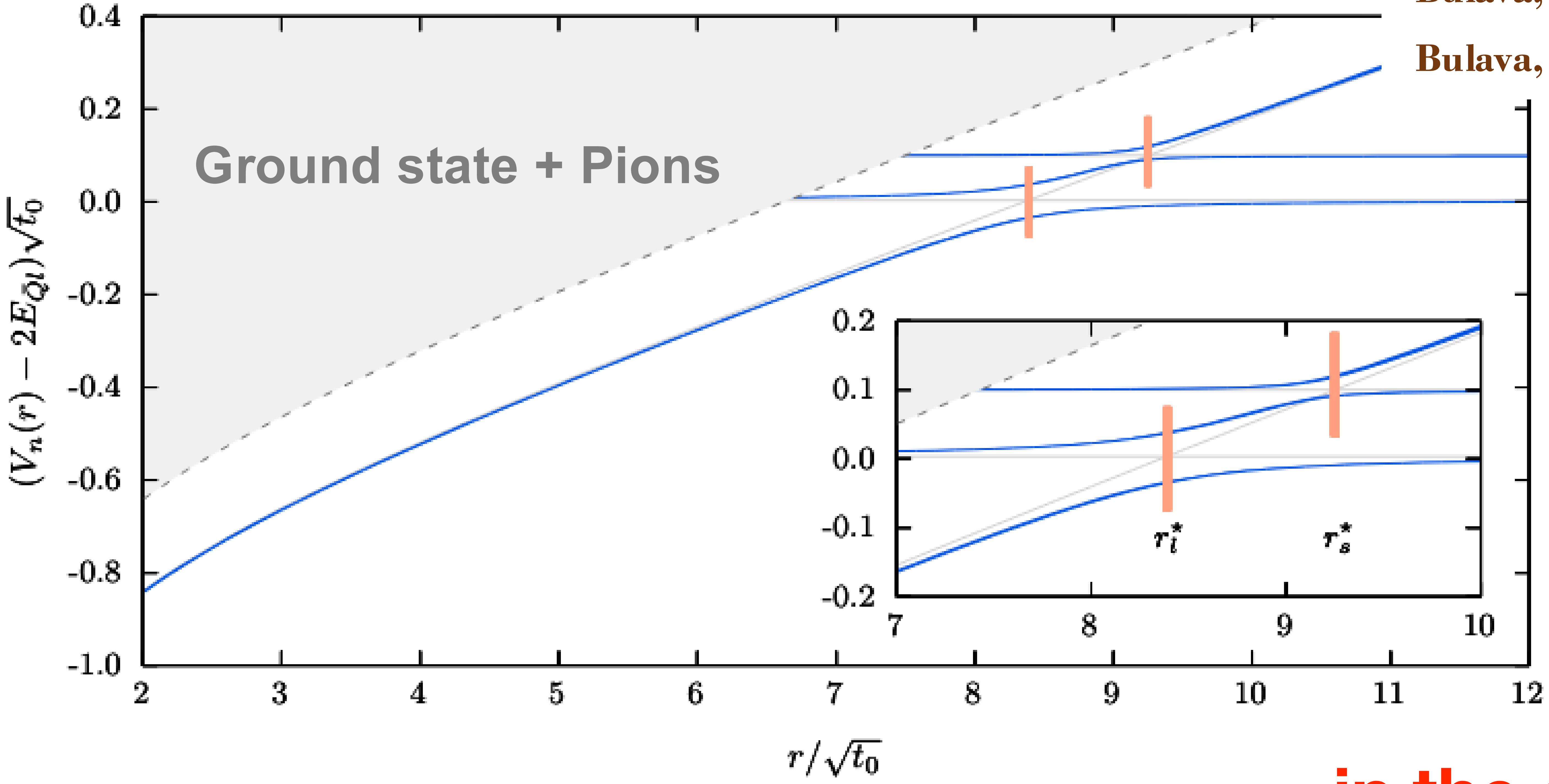
BO-quantum # Λ_η^σ for meson-antimeson

$K_{\bar{q}}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

} s-wave+s-wave
Ex. $\mathbf{D\bar{D}}$ threshold

Meson-antimeson have same BO-quantum # Λ_η^σ as of adjoint meson !!!

BOEFT: Avoided level crossing between quarkonium $1\Sigma_g$ and tetraquark $2\Sigma_g$



Bulava, Hoerz, Knechtli, Koch, Moir, Morningstar, Peardon, Phys. Lett. B. 793

Bulava, Knechtli, Koch, Morningstar, Peardon, Phys. Lett. B. 854 (2024)

avoided level crossing

In adiabatic repr

in the diabatic reps gives mixing

Model Hamiltonian for determining parameters:

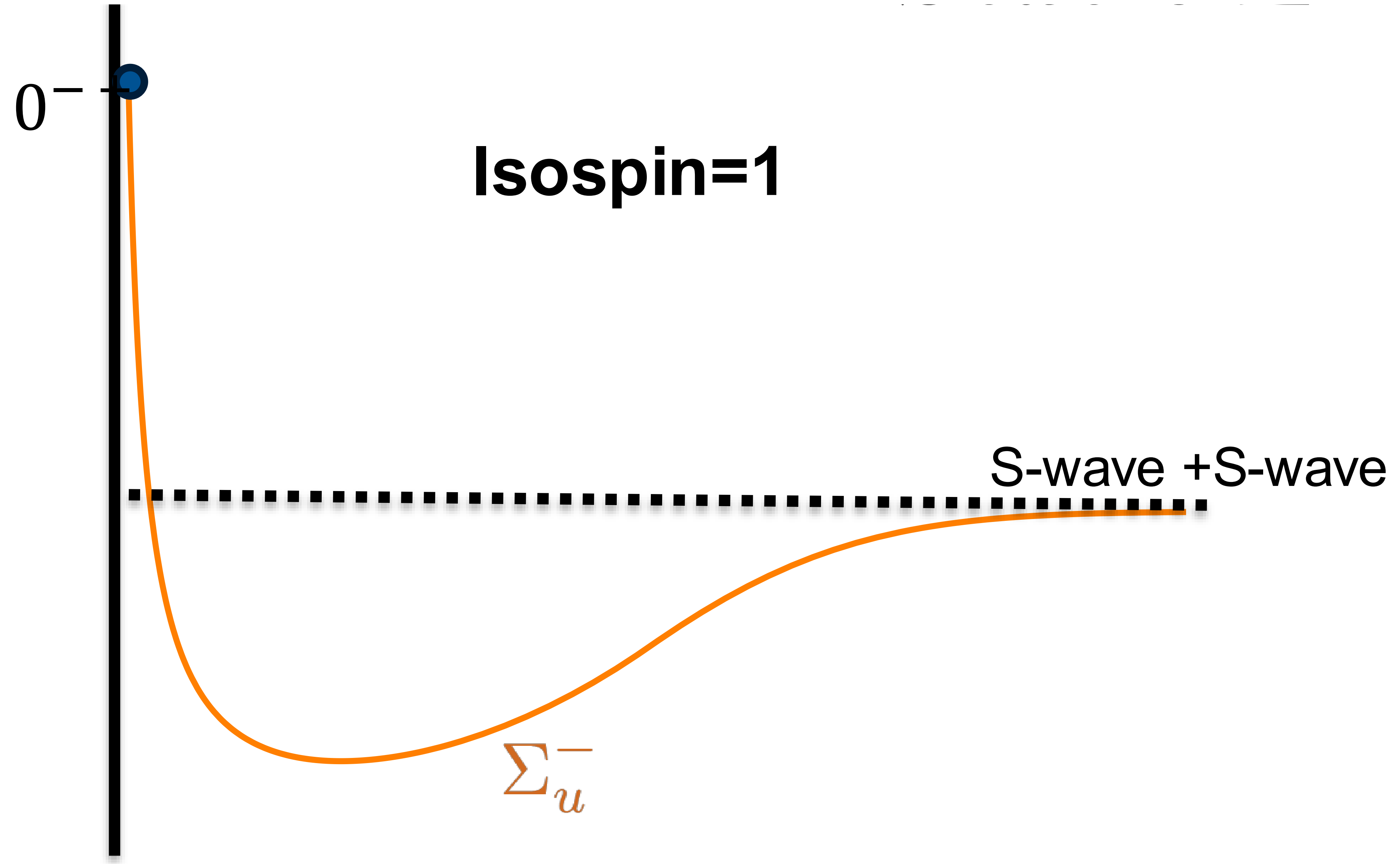
$$H(r) = \begin{pmatrix} \hat{V}(r) & \sqrt{2}g_l & g_s \\ \sqrt{2}g_l & \hat{E}_1 & 0 \\ g_s & 0 & \hat{E}_2 \end{pmatrix}, \hat{V}(r) = \hat{V}_0 + \sigma r + \gamma/r$$

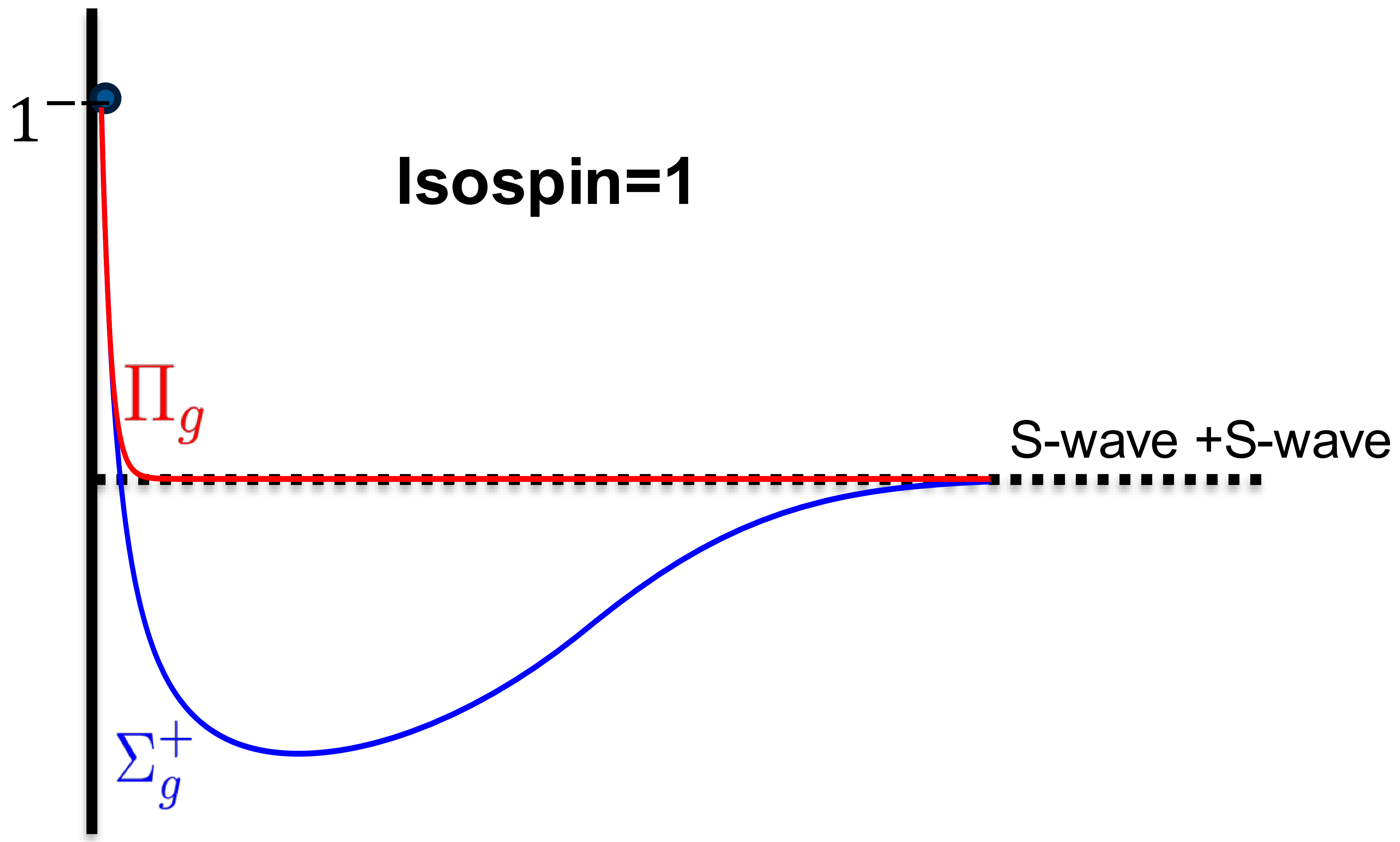
String breaking radius $\approx$ 1.22 fm

a $\approx$ 0.063 fm

$$m_\pi \approx 200 - 340 \text{ MeV} \quad m_K \approx 440 - 480 \text{ MeV}$$

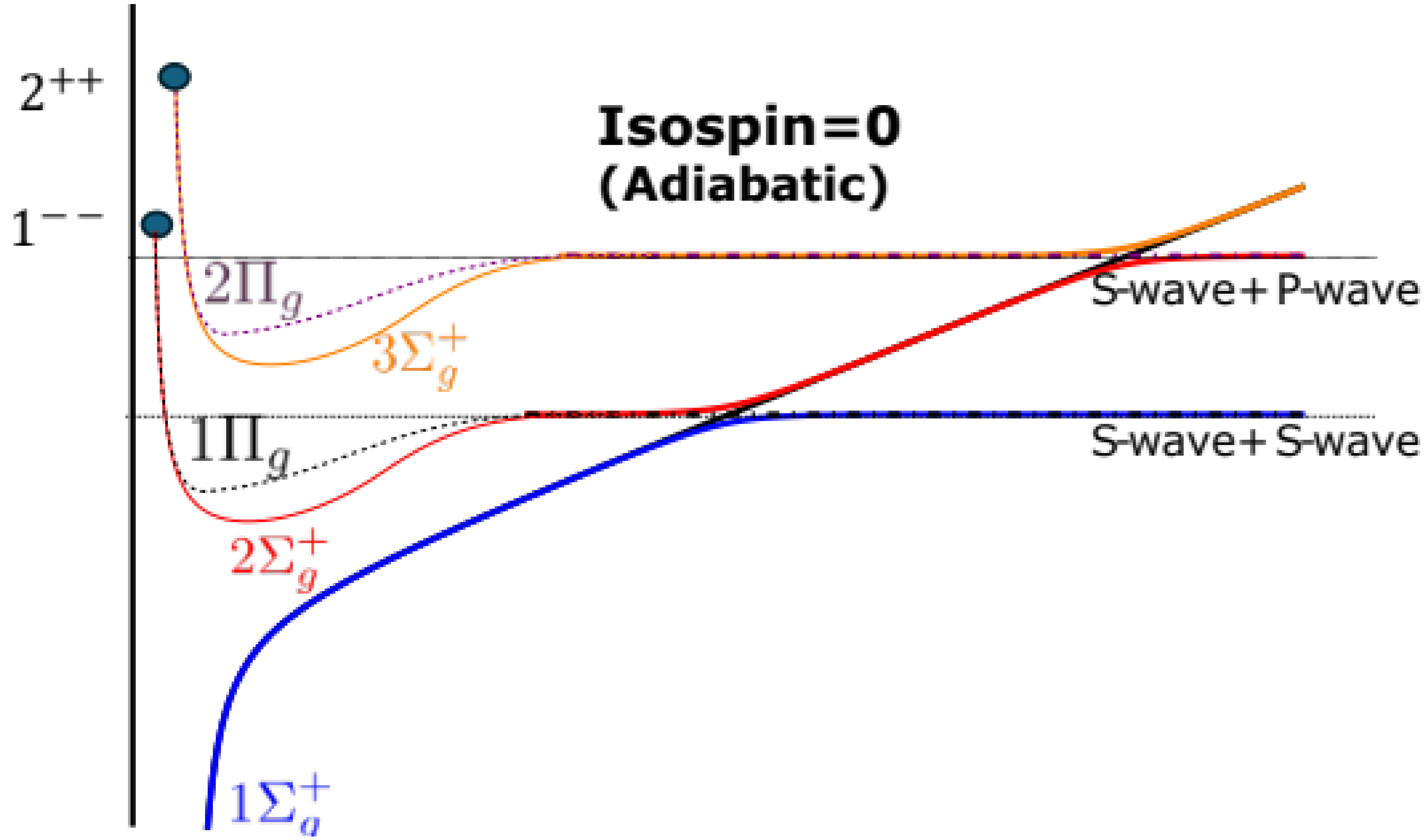
BOEFT: behaviour static energies quarkonium and tetraquarks





Behavior of tetraquark static energy:

- ❑ Adjoint meson behavior at small r $(r \to 0)$
- ❑ Heavy meson pair threshold at large r $(r \to \infty)$
- ❑ Avoided crossing with quarkonium static energy (Isopsin=0)



BOEFT: mixing between static energies with same kappa

Berwein, N.B., Mohapatra, Vairo 2408.04719

- BOEFT Lagrangian:

$$L_{\mathrm{BOEFT}} = \int d^3\boldsymbol{R} \int d^3\boldsymbol{r} \sum_{\kappa\lambda\lambda'} \mathrm{Tr}\left\{ \Psi^{\dagger}_{\kappa\lambda}(\mathbf{r},\, \mathbf{R},\, t) \left[i\partial_t\, \delta_{\lambda\lambda'} - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}(\theta,\phi)\, \frac{\boldsymbol{\nabla}^2_r}{m_Q} P^{i}_{\kappa\lambda'}(\theta,\phi) \right] \Psi_{\kappa\lambda'}(\mathbf{r},\, \mathbf{R},\, t) \right\}$$

LDF-quantum #: $\kappa = \{K^{PC}, f\}$ BO-quantum #: Λ^{σ}_{η} $\lambda = \pm\Lambda$

Projection vectors : $P^{i}_{K\lambda}(\theta,\varphi) = D^{\lambda *}_{K i}(0,\theta,\varphi)$ Wigner D matrices

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Berwein, N.B.,
Mohapatra, Vairo
2408.04719

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Projection vectors : $P^i_{K\lambda}(\theta,\varphi) = D^{\lambda *}_{Ki}(0,\theta,\varphi)$ Wigner D matrices

- **BO potentials:** **Potential between Q & $\bar{Q}$** due to LDF (light quarks, gluons).

Born-Oppenheimer (BO) potential: $V_{\kappa\lambda\lambda'}(r) = \boxed{E^{(0)}_{\kappa,|\lambda|}(r)}\delta_{\lambda\lambda'} + \boxed{\frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q}} + \dots\,,$

Static Energy **Spin-dependent potentials**

Brambilla, Lai, Segovia, Castellà, Phys. Rev. D. 101, (2020)

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 99, (2019)

Soto, Valls, Phys. Rev. D 108 (2023)

Castellà , Soto Phys. Rev. D. 102, (2020)

$$X(3872)\ \&\ T_{cc}^{+}(3875)$$

X(3872) $Q\bar{Q}q\bar{q}$

Berwein, N.B., Mohapatra, Vairo 2408.04719

$Q\bar{Q}$ color state	Light spin k^{PC}	Static energies	l	J^{PC} $\{S_Q = 0, S_Q = 1\}$	Multiplets
Octet	0^{-+}	$\{\Sigma_u^-\}$	0	$\{0^{++}, 1^{+-}\}$	T_1^0
			1	$\{1^{--}, (0,1,2)^{-+}\}$	T_2^0
			2	$\{2^{++}, (1,2,3)^{+-}\}$	T_3^0
	1^{--}	$\{\Sigma_g^{+\prime}, \Pi_g\}$	1	$\{1^{+-}, (0,1,2)^{++}\}$	T_1^1
		$\{\Sigma_g^{+\prime}\}$	0	$\{0^{-+}, 1^{--}\}$	T_2^1
		$\{\Pi_g\}$	1	$\{1^{-+}, (0,1,2)^{--}\}$	T_3^1
		$\{\Sigma_g^{+\prime}, \Pi_g\}$	2	$\{2^{-+}, (1,2,3)^{--}\}$	T_4^1

Isospin-1 channel:

$Z_c(3900), Z_c(4200), Z_b(10610)$, $Z_b(10610)$**states:**

Mixing between $K^{PC} = 0^{-+}$ and $K^{PC} = 1^{--}$

Light-quark spin-symmetry !!

Voloshin, Phys. Rev. D. 93, 074011 (2016)

Braaten, Bruschini arXiv 2409.08002

Isospin-0 channel:

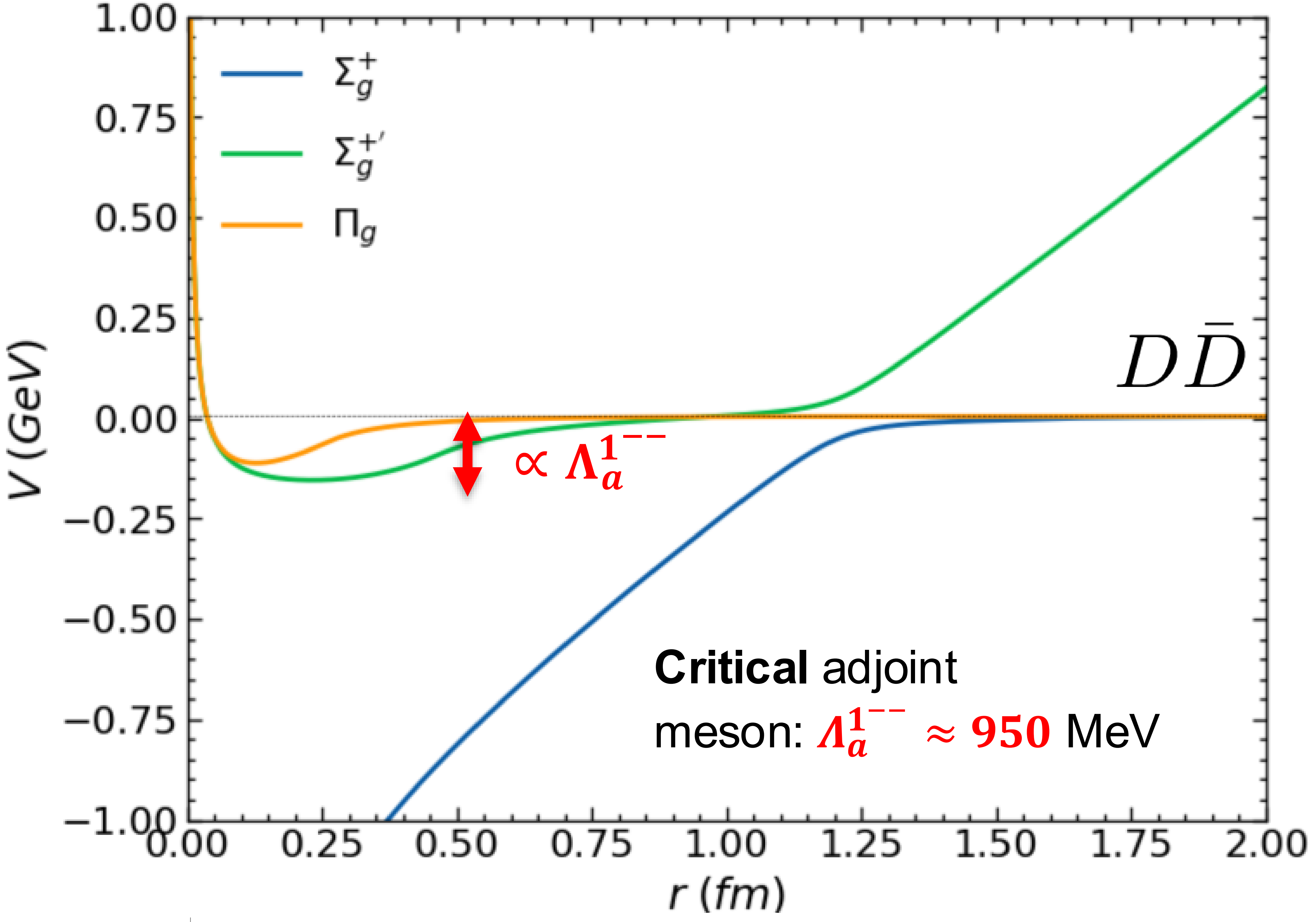
X(3872)

Until when we introduce spin all states in the X multiplet are degenerate

X(3872)

Berwein, N.B.,Mohapatra, Vairo 2408.04719

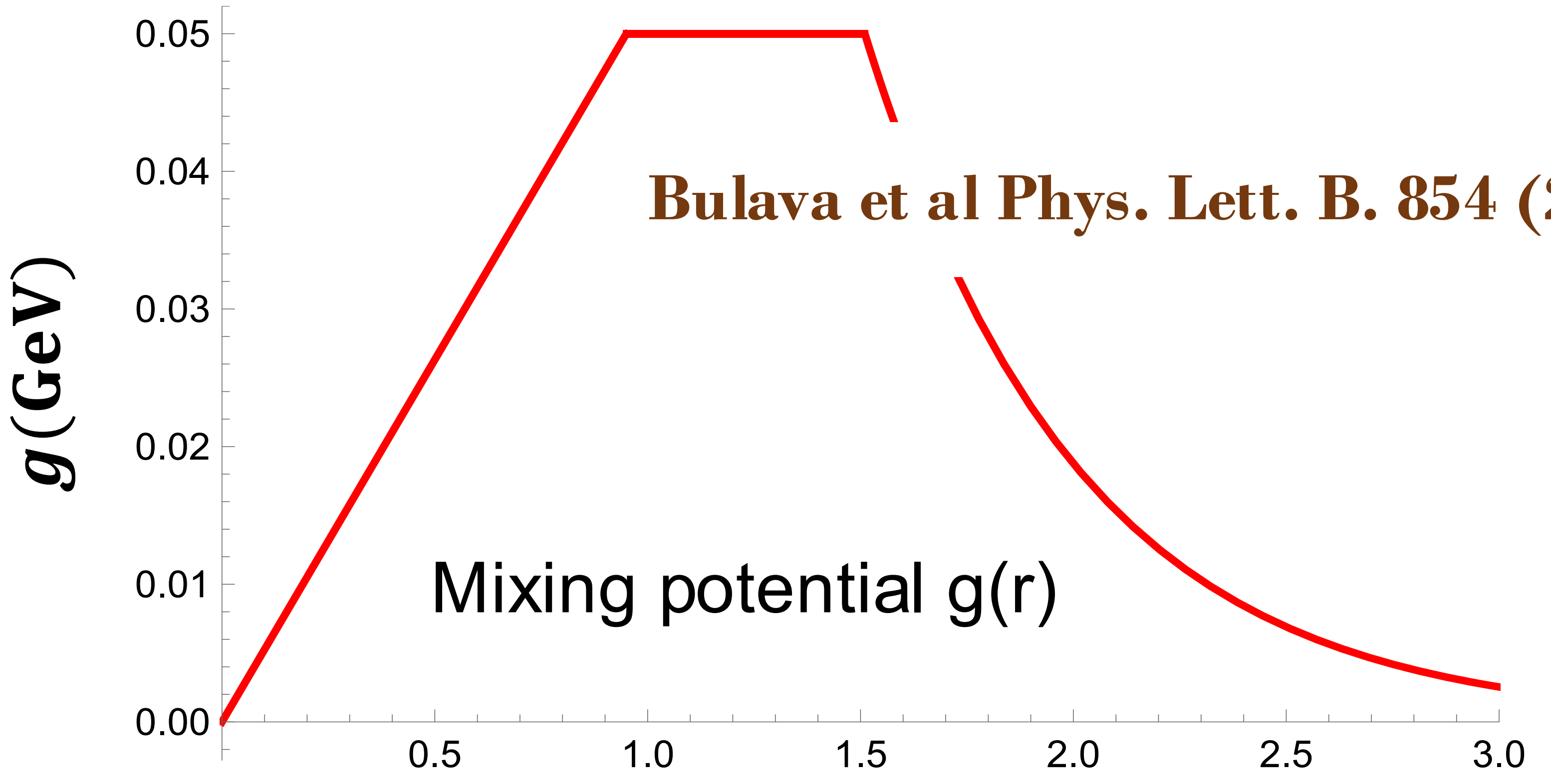
N.B. Mohapatra, Scirpa, Vairo 2411.14306



Coupled-channel Equations:

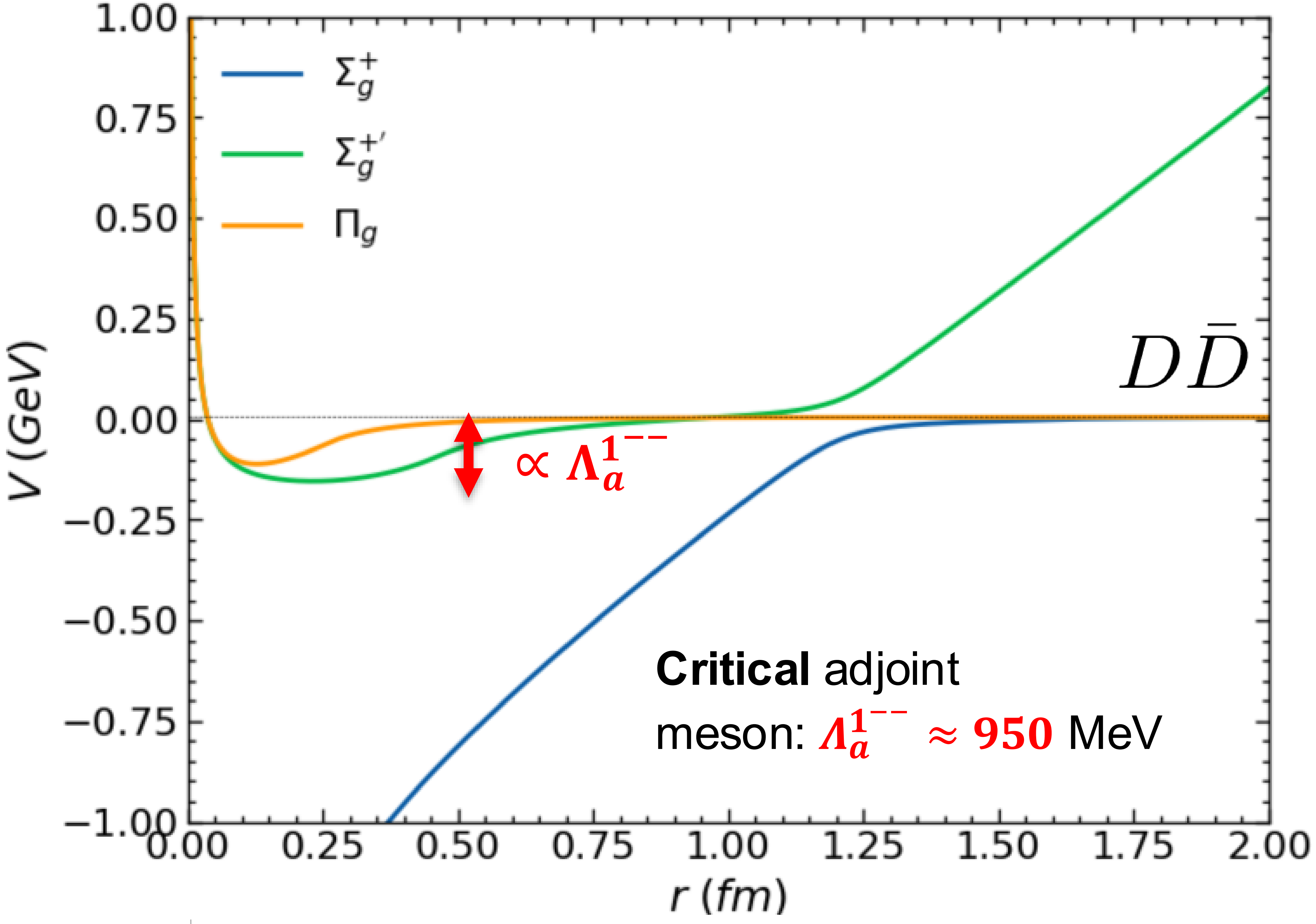
$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l\,(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix}$$

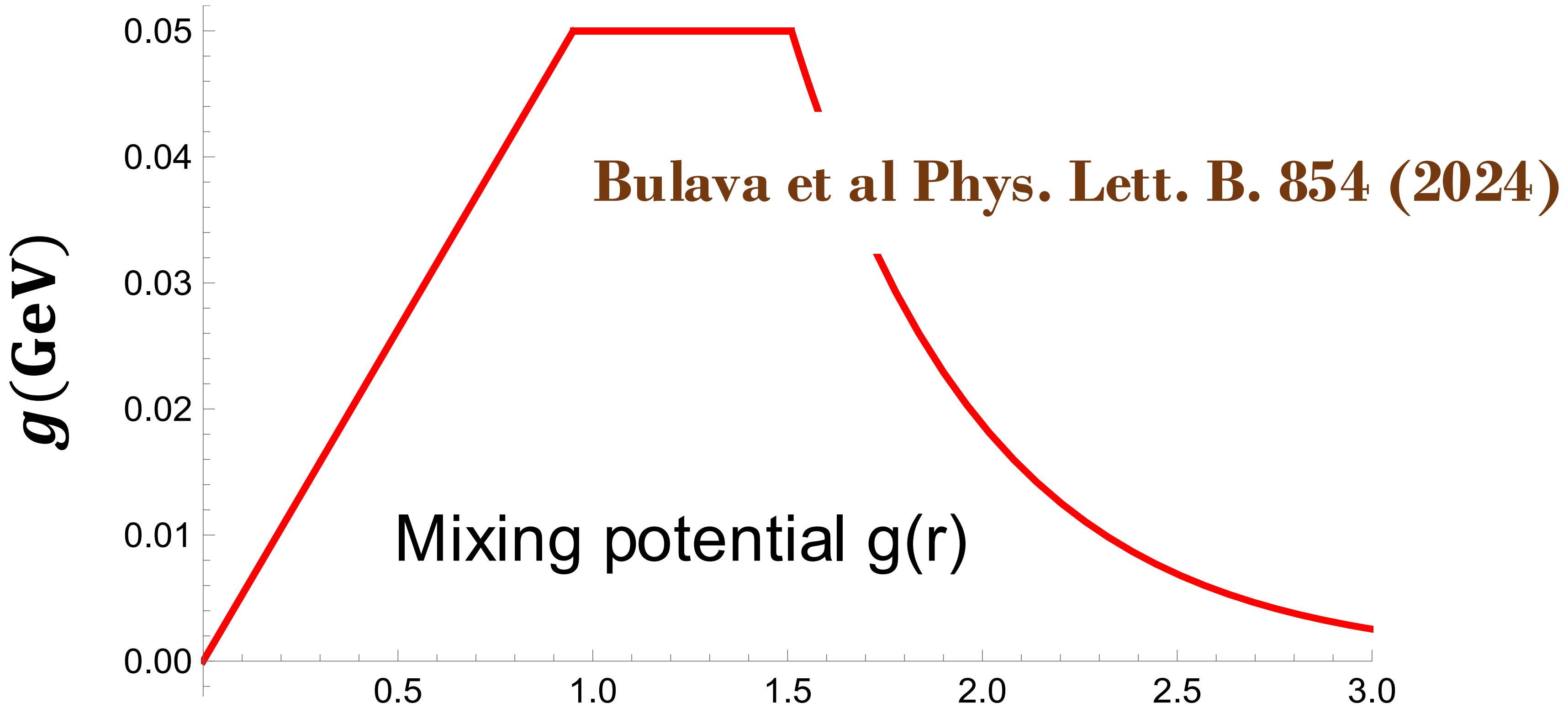
$l = 1$



X(3872)

Berwein, N.B.,Mohapatra, Vairo 2408.04719
N.B. Mohapatra, Scirpa, Vairo 2411.14306





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$l = 1$

1) Quarkonium percentage: $|\psi_\Sigma|^2 \sim 6\ \%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \sim 35\ \%,\ |\psi_\Pi|^2 \sim 59\ \%$
3) Radius ~ 14 fm. (and a) Binding energy 90 keV
4) Deeper bound state in bottom sector: 15 MeV below spin-isospin averaged $B\bar{B}$ threshold.

clear prediction of BOEFT-> the adjoint does not depend on the flavor

5) we found also a deeper bound state 400 MeV below DDbar_> spin average 1P quarkonium (3529 MeV)

Critical adjoint meson: $\Lambda_a^{1^{--}} \approx$ **950** MeV:
No other bound states in higher multiplets T_2^1, T_3^1, T_4^1 ....

X(3872)

Berwein, N.B.,Mohapatra, Vairo 2408.04719

N.B. Mohapatra, Scirpa, Vairo 2411.14306

Radiative decays

$$\mathcal{R}_{\gamma\psi} = \frac{\Gamma_{\chi_{c1}(3872)\to\gamma\psi(2s)}}{\Gamma_{\chi_{c1}(3872)\to\gamma J\psi}},$$

We find: $R_{\gamma\psi} = 2.95 \pm 2.28$

In agreement within errors with LHCb

This is thanks to the quarkonium component-> may work also for production, gives the correct order of magnitude for compositeness

X(3872)

Berwein, N.B.,Mohapatra, Vairo 2408.04719
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Multiplet T_1^1: $\{1^{+-}, (0,1,2)^{++}\}$

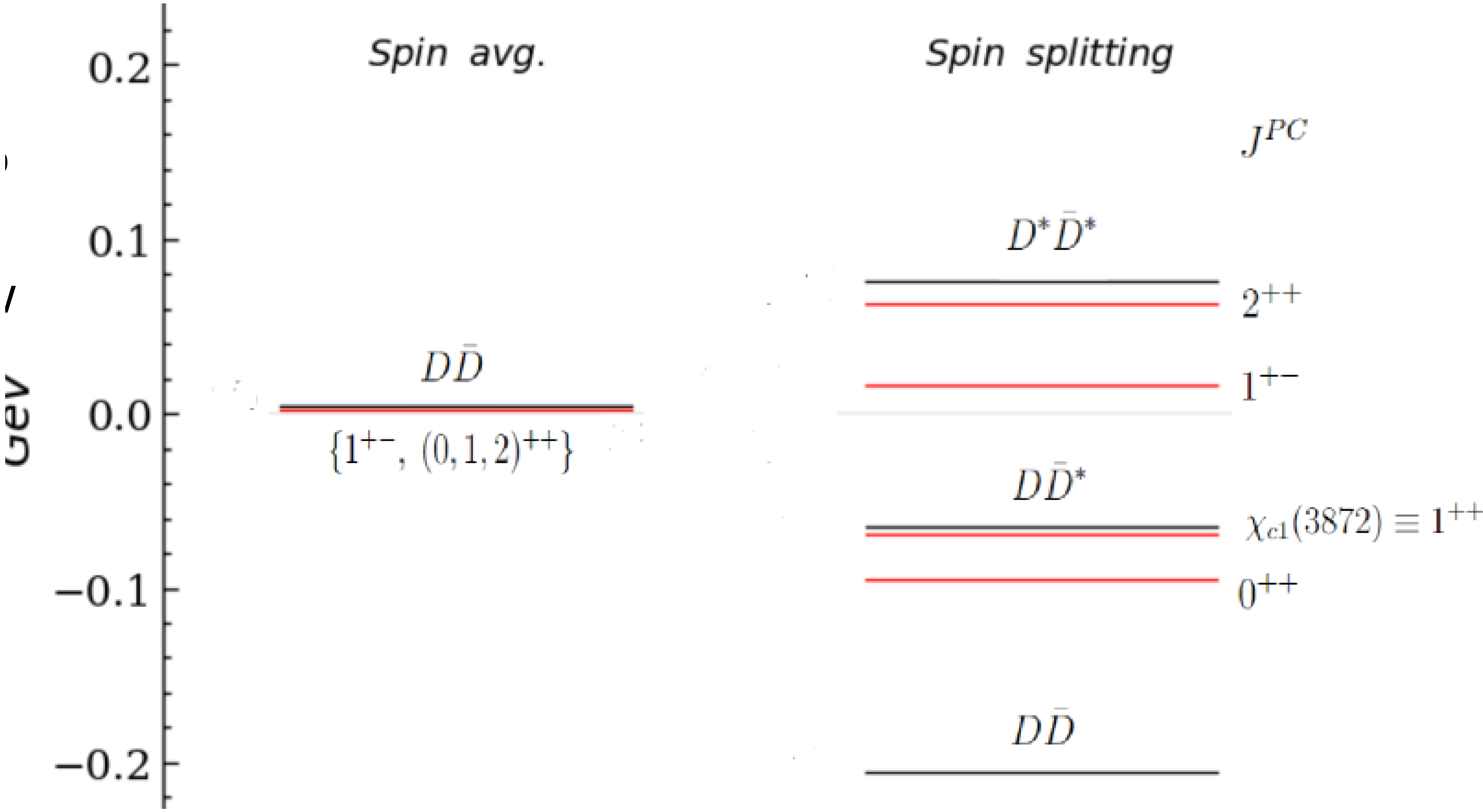
1^{++} state: Identified with $\chi_{c1}(3872)$

1^{+-} state: Mass around 3.956 (11) GeV.

2^{++} state: Mass around 3.996 (11) GeV.

0^{++} state: Mass around 3.838 (11) GeV.

Identified with X(3940) ?



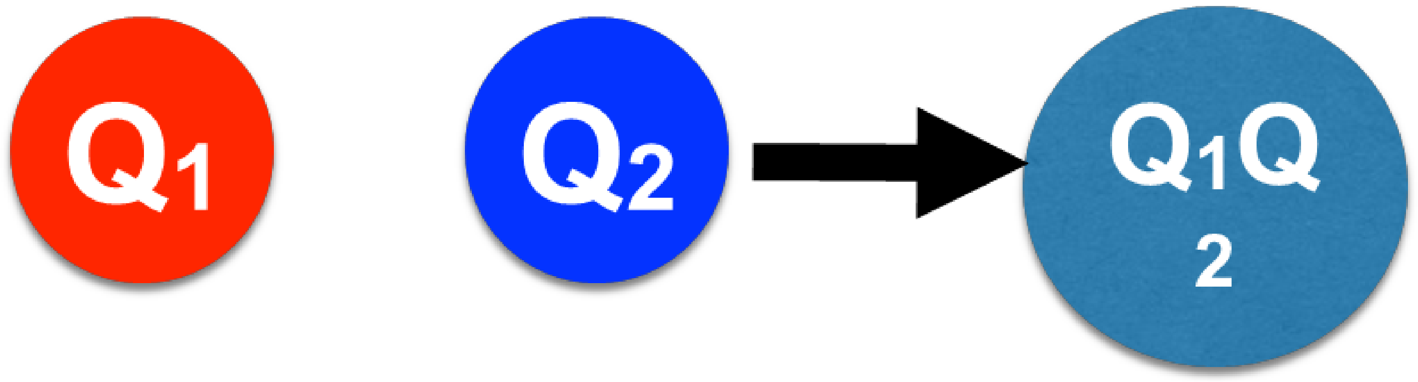
Also indicated in the lattice calculations: Prelovsek et al JHEP 06 (2021) 035.

$T_{cc}^{+}(3875)$ BOEFT: $QQ\bar{q}\bar{q}$ multiplets

Berwein, Brambilla, AM, Vairo,
arXiv 2408.04719

doubly heavy core

spin: $1/2 \otimes 1/2 = 0 \oplus 1$

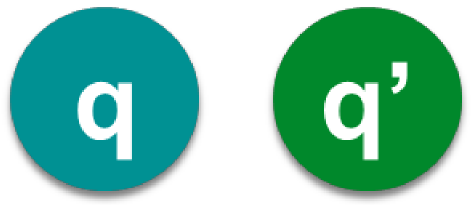


color: $3 \otimes 3 = 6 \oplus 3^*$

flavor: $\{Q_1Q_2\}$ $[Q_1Q_2]$

J^P: 1^+ 0^+

light antiquarks



{qq'}, 1^+ [qq'], 0^+

Defines the Born-Oppenheimer static potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

doubly heavy tetraquarks

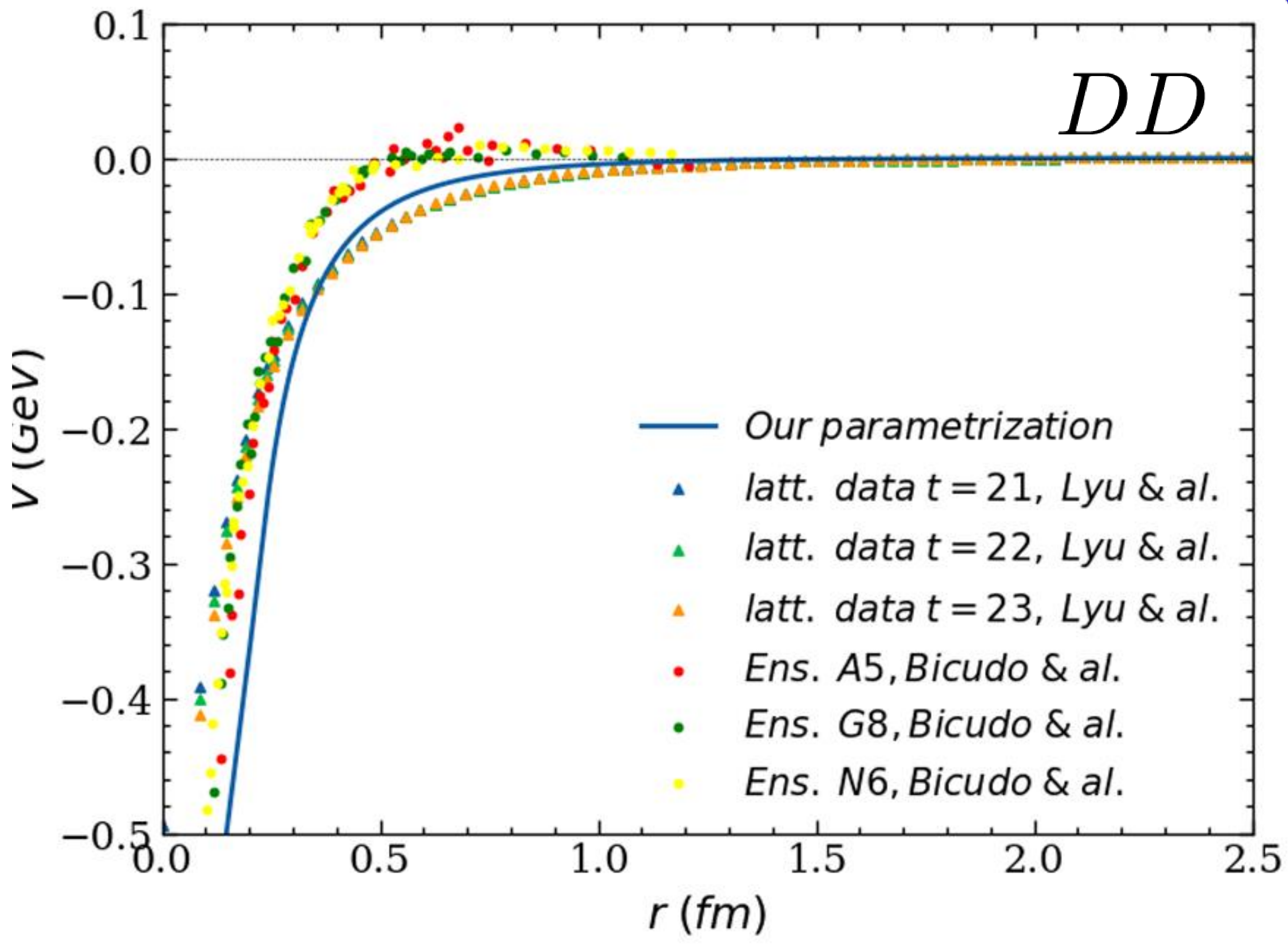
QQ color state	Light spin K^{PC}	Static energies	Isospin I	l	J^P $S_Q=0$	J^P $S_Q=1$
anti-triplet $\bar{3}$	0^+	$\{\Sigma_g^+\}$	0	0	—	1^+
				1	1^-	—
	1^+	$\{\Sigma_g^-, \Pi_g\}$	1	0	0^-	—
				1	1^-	$(0,1,2)^+$

J^P for T_{cc}^+

$T_{cc}^{+}(3875)$

Berwein, N.B.,Mohapatra, Vairo 2408.04719

N.B. Mohapatra, Scirpa, Vairo 2411.14306



Critical triplet meson: $\Lambda_t^{0^+} \approx 650$ MeV

Lyu, Aoki, Doi, Hatsuda, Ikeda, Meng, Phys. Rev. Lett. 131, 161901 (2023)

Bicudo, Marinkovic, Mueller, Wagner, arXiv 2409.10786

Schrödinger equation

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{m_Q r^2} + V_{\Sigma_g^+}\right]\psi_{\Sigma_g^+} = \mathcal{E}_N\,\psi_{\Sigma_g^+}\,.$$

$l = 0$

1) T_{cc} state : 320 keV below DD threshold
2) Radius ~ 8 fm or larger.
3) Deeper bound state in bb sector: T_{bb} 110 MeV below DD threshold. r=0.29 fm
4) Deeper bound state in bc sector: T_{bc} 20 MeV below BD threshold. r=0.79 fm

   Both 0+ 1+ degenerate

$a = \hbar/\sqrt{m_c E_b} = 7.95$ fm, In agreement with LHCb

HALQCD collaboration: pion mass 146 MeV: T_{cc} a virtual state. $E_{\text{pole}} = -59^{+53\,+2}_{-99\,-67}$ keV

physical pion mass 135 MeV: T_{cc} a bound state

Lyu et al, Phys. Rev. Lett. 131, 161901 (2023)

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  at the short distance scale we have control of the perturbative calculation
  at the large distance scale we need lattice calculations of few gauge invariant universal correlators
  still the structure of the EFT allows for model independent predictions

- Once the lattice input is there the BOEF allows applications to domain in general not directly accessible to a lattice calculation (decay, production, medium propagation)

- The results on the X and the Tcc gives is an idea of their nature: not simple molecules nor compact tetraquarks: results come from a conspiracy between short and long range behaviour of the potential constrained by symmetry

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Combining BOEFT + open quantum systems one can attempt to study the X Y Z in heavy ion collisions