Probing CP-violation using lattice QCD

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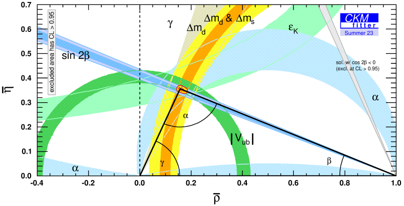
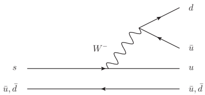
This talk: CP-violating hadronic processes within standard model



- Kaons: direct ($K \to \pi\pi$), indirect ($K - \overline{K}$ mixing)
- D mesons: direct ($D \to KK$, $D \to \pi\pi$), indirect ($D - \overline{D}$ mixing)

$K \to \pi\pi$, setup



$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}.$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3) \times 10^{-4} \quad \text{(experiment)}$$

measure of direct CPV — measure of indirect CPV

KTeV, NA48

Main challenges of lattice calculation:

- Kinematics require high-precision control of excited states in LQCD
- $\pi\pi$ scattering in finite-volume requires multi-operator approach
- Scheme matching between lattice and $\overline{\mathrm{MS}}$
- Wilson coefficients of $\Delta S = 1$ effective Hamiltonian
- Statistical noise
- Isospin-breaking corrections could be enhanced ($\Delta I = 1/2$ rule, need QCD+QED)

$K \to \pi\pi$, a brief history of the lattice approach (1/2)

- 1991: Lüscher formalism relating FV spectra and scattering (Nucl.Phys.B 354 (1991) 531-578)

- 2001: Lellouch-Lüscher formalism relating FV matrix elements to IV (Commun.Math.Phys.219(2001)31-44)

- 2011: RI/SMOM operator renormalization and 1-loop finite-terms for matching to $\overline{\mathrm{MS}}$ (Phys.Rev.D 84 (2011) 014001)

- 2011: threshold computation at $m_\pi > m_\pi^{\mathrm{phys}}$ (Phys.Rev.D 84 (2011) 114503)

- 2012: first-principles reproduction and deconstruction of $\Delta I = 1/2$ rule ($I = 0$ final state dominance, Phys.Rev.Lett. 110 (2013) 15, 152001)

- 2012: $I = 2$ final state at m_π^{phys} no $a \to 0$ limit (Phys.Rev.Lett. 108 (2012) 141601, Phys.Rev.D 86 (2012) 074513)

- 2015: $I = 2$ final state at m_π^{phys} with $a \to 0$ limit (Phys.Rev.D 91 (2015) 7, 074502)

$K \to \pi\pi$, a brief history of the lattice approach (2/2)

- 2015: $I = 0$ final state at $m_\pi^{\rm phys}$ no $a \to 0$ limit, G-parity BC (Phys.Rev.Lett. 115 (2015) 21, 212001)

  Re(ε'/ε)= 1.38(5.15)(4.59)10^{-4} (2.1σ tension with experiment)

- 2020: Multi-operator update of 2015 result (Phys.Rev.D 102 (2020) 5, 054509), systematic effect uncovered

  Re(ε'/ε)= $21.7(2.6)_{\rm stat}(6.2)_{\rm syst}(5.0)_{\rm IB}10^{-4}$ (tension resolved)

- 2021: Detailed study of multi-operator effect on phase shifts (Phys.Rev.D 104 (2021) 11, 114506)

- 2023: $I = 0, 2$ $\pi\pi$ scattering with periodic BC as alternative approach (Phys.Rev.D 107 (2023) 9, 094512)

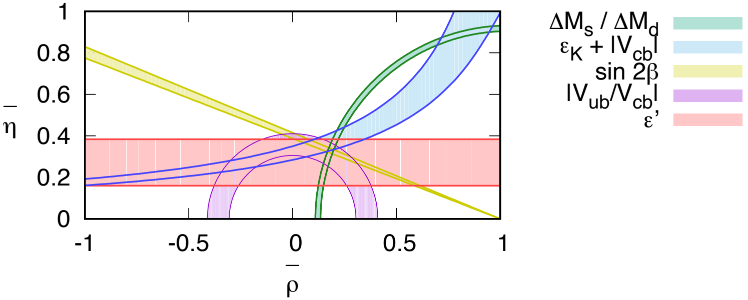
- 2023: $I = 0, 2$ final states with periodic BC (Phys.Rev.D 108 (2023) 9, 094517)

  Re(ε'/ε)= $29.4(5.2)_{\rm stat}(11.1)_{\rm syst}(5.0)_{\rm IB}10^{-4}$

$K \to \pi\pi$, status of G-parity effort (1/2) (driven by Chris Kelly at BNL)

Latest paper (Phys.Rev.D 102 (2020) 5, 054509)

Re(ε'/ε)$= 21.7(2.6)_{\mathrm{stat}}(6.2)_{\mathrm{syst}}(5.0)_{\mathrm{IB}}10^{-4}$ (tension resolved)



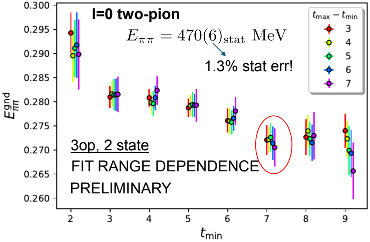
Errors dominated by Wilson coefficients ($\approx 12\%$ uncertainty), IB ($\approx 23\%$), discretization errors of A_0 ($\approx 12\%$)

$K \to \pi\pi$, status of G-parity effort (2/2) (driven by Chris Kelly at BNL)

Discretization error: so far only single lattice spacing $a^{-1} = 1.38$ GeV, currently generate second lattice spacing $a^{-1} = 1.73$ GeV; expect to have target statistics soon

Significant work invested to remove computational overhead of G-parity setup compared to simpler periodic BC

First look at new lattice ensemble:



(more details can be found in Chris Kelly's talk at Lattice 2024)

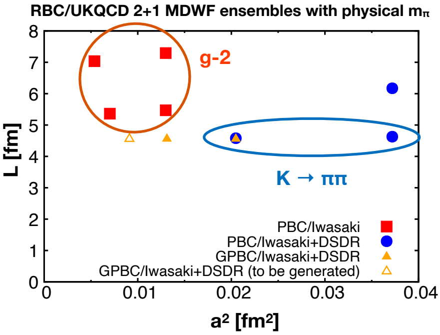
$K \to \pi\pi$, status of periodic BC effort (1/2) (driven by Masaaki Tomii at UConn/RBRC)

Latest paper (Phys.Rev.D 108 (2023) 9, 094517)

$$\text{Re}(\varepsilon'/\varepsilon) = 29.4(5.2)_{\text{stat}}(11.1)_{\text{syst}}(5.0)_{\text{IB}}10^{-4}$$

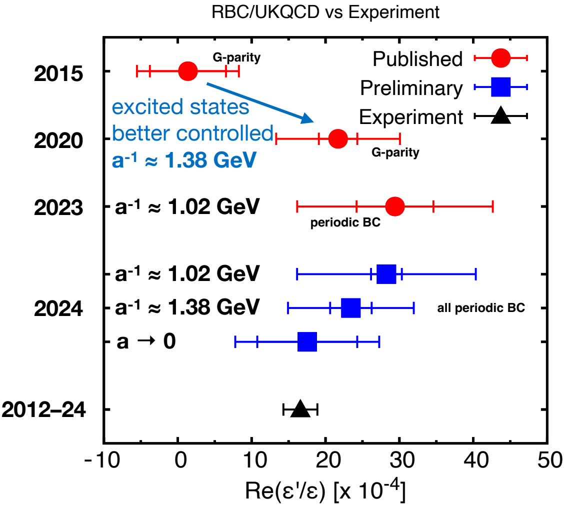
Advantages over G-parity BC:

- Can re-use existing data for other projects such as muon g-2



- Significantly simplifies inclusion of QED corrections

$K \to \pi\pi$, status of periodic BC effort (2/2) (driven by Masaaki Tomii at UConn/RBRC)



(more details can be found in Masaaki Tomii's talk at Lattice 2024)

$K \to \pi\pi$, towards including IB corrections
Phys.Rev.D 106 (2022) 1, 014508

$\pi - \pi$ scattering, QED and finite-volume quantization

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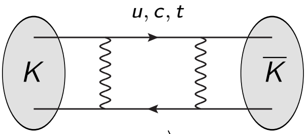
Abstract

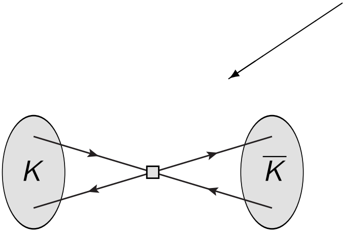
Using the Coulomb gauge formulation of QED we present a lattice QCD procedure to calculate the $\pi^+\pi^+$ scattering phase shift including the effects of the Coulomb potential which appears in this formulation. The approach described here incorporates the effects of relativity and avoids finite-volume corrections that vanish as a power of the volume in which the lattice calculation is performed. This is the first step in developing a complete lattice QCD calculation of the electromagnetic and isospin-breaking light-quark mass contributions to ε', the parameter describing direct CP violating effects in $K_L \to \pi\pi$ decay.

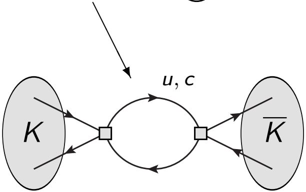
$K \to \pi\pi$, future directions

- Going through mass thresholds non-perturbatively (current work on integrating out charm)
- Eventually, lattice can also eliminate all perturbative truncation errors in α_s by simulating very fine lattices and integrating out EW physics with fully non-perturbative QCD (Phys.Rev.D 97 (2018) 7, 074509)
- For now it would be great to have NNLO accuracy for Wilson coefficients. Work started by Cerda-Sevilla, Gorbahn, Jäger, Kokulu (J.Phys.Conf.Ser. 800 (2017) 1, 012008, Acta Phys.Polon.B 49 (2018) 1087-1096).

K–$\overline{K}$ mixing (B_K)



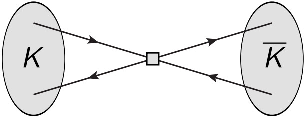


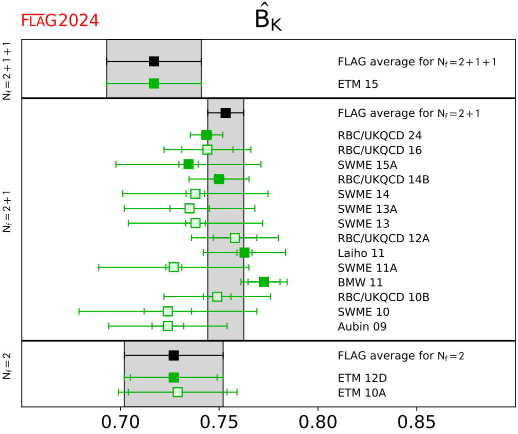


- Established lattice methodology, see next slide

[PRL113(2014)112003]

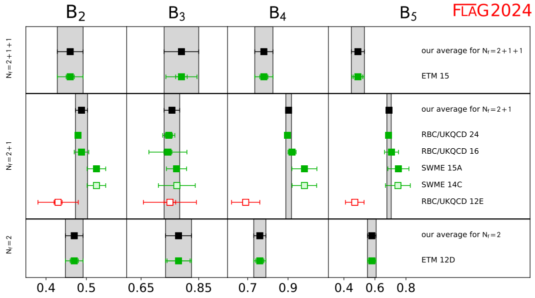
- “Long-distance contribution” under active research
- Estimated to yield $\approx 5\%$ correction to ε_K
- Non-local (bi-local) methodology has broad impact



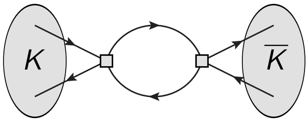


Above is FLAG24; after FLAG24 also NNLO matching appeared (Gorbahn, Jäger, Kvedaraite, arXiv:2411.19861)

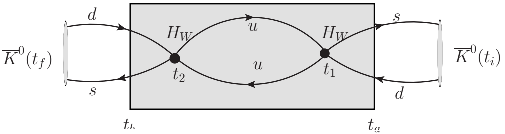
Excursion BSM contributions, tension



Tensions between results using RI/MOM and RI/SMOM for B_4 and B_5



Long-distance methodology [PRL113(2014)112003]



$$\mathcal{A} = \frac{1}{2}\sum_{t_1,t_2=t_a}^{t_b} \langle 0|T\{\overline{K}^0(t_f)H_W(t_2)H_W(t_1)\overline{K}^0(t_i)\}|0\rangle$$

Inserting a complete set of states, $T = t_b - t_a + 1 \Rightarrow$ 2nd order PT expression is accessible

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle\overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{m_K - E_n}\left(-T + \frac{e^{(M_K-E_n)T}-1}{M_K-E_n}\right)$$

Challenges:

- Finite-volume effects
- Exponentially growing contributions
- Short-distance subtraction

Applications:

- ΔM_K
- ε_K (B_K)
- rare K decays

Long-distance contribution to K–$\overline{K}$ mixing, current status:

Phys.Rev.D 109 (2024) 5, 054501

The unphysical quark masses and single lattice spacing used in our calculation make the present result an unreliable long-distance correction to ϵ_K. Nevertheless it is of interest to compare the size of this correction to the current short-distance result for ϵ_K:

$$\epsilon_K^{LD}(\mu_{\mathrm{RI}} = 2.11\ \mathrm{GeV}) = 0.195(0.077)e^{i\phi_\epsilon} \times 10^{-3} \quad (63)$$

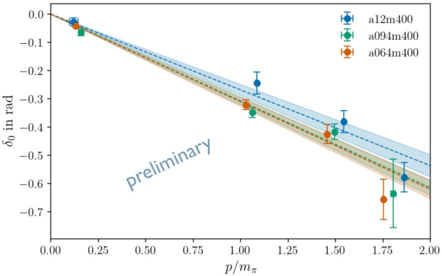
$$\epsilon_K^{SD} = 1.446(0.154)e^{i\phi_\epsilon} \times 10^{-3} \quad (64)$$

$$\epsilon_K^{\mathrm{RI}\to\overline{\mathrm{MS}}}(\mu_{\mathrm{RI}} = 2.11\ \mathrm{GeV}) = -0.086e^{i\phi_\epsilon} \times 10^{-3}. \quad (65)$$

Improved calculation currently underway using two ensembles at physical pion mass and $a^{-1} = 2.359$ GeV as well as $a^{-1} = 2.7$ GeV. Ensemble with $a^{-1} = 3.5$ GeV currently being generated that will be crucial as well!

Hadronic D meson decays, first steps

- Problem of many open channels for physical kinematics, application of Luscher method very challenging
- Simplified at SU(3) symmetric point (PoS LATTICE2022 (2023) 063), where spectrum is manageable (on reasonable volumes 5-10 states lighter than $m_D \approx 4m_\pi$)
- First results for scattering phase shift (plot from MT Hansen Lattice 2023)



- Lellouch-Lüscher factor also available (generalization by Hansen and Sharpe Phys.Rev.D 86 (2012), 016007)
- Going to physical pion masses very challenging

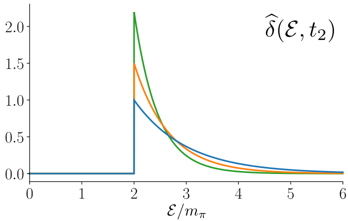
Hadronic D meson decays, towards physical pion mass

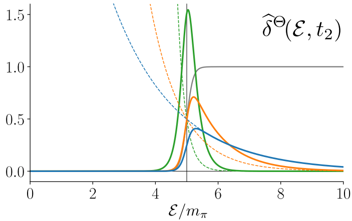
Bruno-Hansen alternative approach JHEP 06 (2021) 043 may be viable.

Idea: introduce smearing kernel in energy that can be approximated well by weighted sum over Euclidean correlators.

$$\widehat{\rho}(\overline{\omega}, \Delta, L) \equiv \int_0^\infty d\omega \, \widehat{\delta}_\Delta(\omega, \overline{\omega}) \, \rho(\omega, L) \,,$$

Introduces additional scale (smearing width Δ) that needs to be carefully removed after infinite-volume limit $L \to \infty$ is taken.





D–$\overline{D}$ mixing

Remember expression for kaon case, need to subtract all intermediate states lighter than the D meson:

$$\mathcal{A} = N_K^2 e^{-M_K(t_f - t_i)} \underline{\sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n}} \left(-T + \frac{e^{(M_K - E_n)T} - 1}{M_K - E_n} \right)$$

This is currently intractible at physical pion mass and reasonably large volumes. Again, smeared kernels can help at the cost of a careful double limit procedure.

Summary of CP-violating hadronic processes within standard model

- More than a decade of effort in $K \to \pi\pi$, steady progress. Methods are refined but significant challenges remain for next precision frontier! Now there are two competing methods (G-parity and periodic BC) that are being carried out in parallel by RBC/UKQCD.

- Long-distance contribution to K–$\overline{K}$ mixing has well established methodology but discretization errors still seem large. Costly calculation to address this is in progress!

- Hadronic D decays can be studied with similar methodology at SU(3) symmetric point but going to physical masses may require a change in methodology using smeared kernels.

- D–$\overline{D}$ mixing also facing similar issues at physical pion masses and also in this case smeared kernel methods may be the fastest path towards robust results.