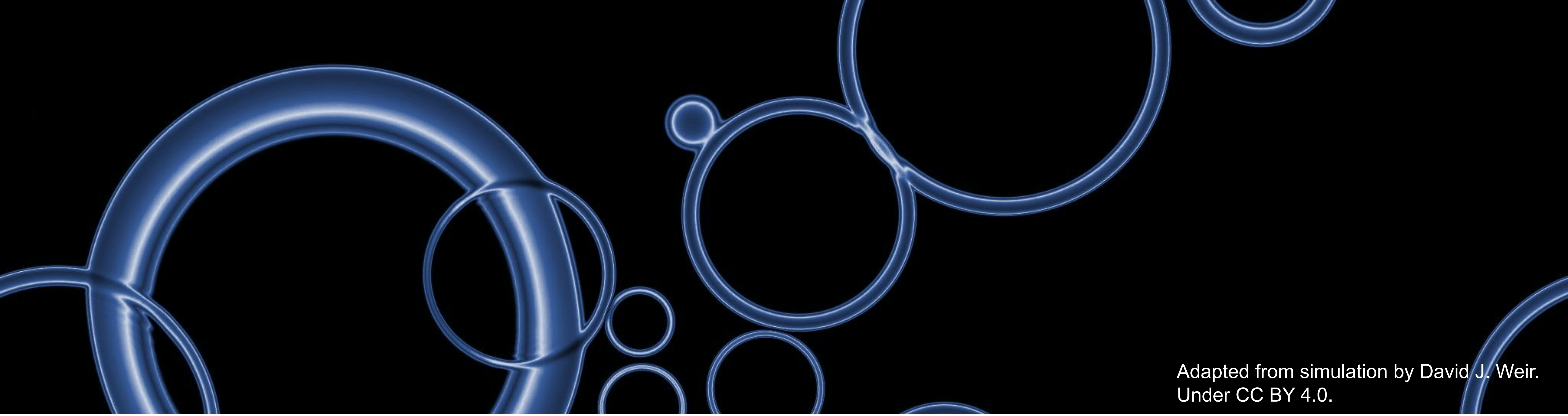


Dimensionally reduced EFTs for cosmological phase transitions

03/12/24 ·DISCRETE 2024

Javier López Miras

Based (mostly) on [2406.02667], by:

M. Chala, J. C. Criado, L. Gil and JLM

FTAE High Energy Theory

UNIVERSIDAD DE GRANADA

Junta de Andalucía

Thermal field theory

Outline of the Matsubara formalism

Field correlators

- **Generating functional** (J=0) in QFT:

$$\mathcal{Z}[0] = \langle q' \; t | q \; 0 \rangle = \langle q' \; 0 | e^{-i\mathcal{H}t} | q \; 0 \rangle = \mathcal{N} \int \mathcal{D}q \exp\left(iS\right)$$

- **Partition function** in quantum statistical mechanics:

$$\mathcal{Z}_{\mathrm{th}} = \mathrm{Tr}\left(e^{-\beta\mathcal{H}}\right) = \sum_q \langle q \; 0 | e^{-\beta\mathcal{H}} | q \; 0 \rangle = \mathcal{N} \int_{q(0)=q(-i\beta)} \mathcal{D}q \exp\left(-S_E\right)$$

Time compactification

QFT at finite temperature **=** Euclidean QFT at zero temperature with periodic time

Thermal field theory

Outline of the Matsubara formalism

- At thermal equilibrium, fields decompose in **Matsubara modes** that live in 3D Euclidean space:

Bosons

$$\phi(\tau, \mathbf{x}) \equiv T \sum_{n=-\infty}^{\infty} \phi_n(\mathbf{x}) e^{i\omega_n \tau}$$

$$\omega_n = 2\pi n T$$

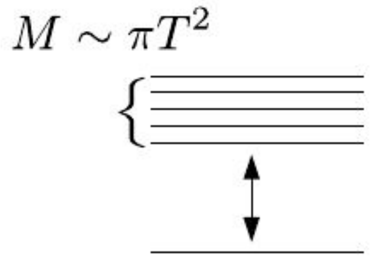
Fermions

$$\psi(\tau, \mathbf{x}) \equiv T \sum_{n=-\infty}^{\infty} \psi_n(\mathbf{x}) e^{i\omega'_n \tau}$$

$$\omega'_n = 2\pi \left(n + \frac{1}{2}\right) T$$

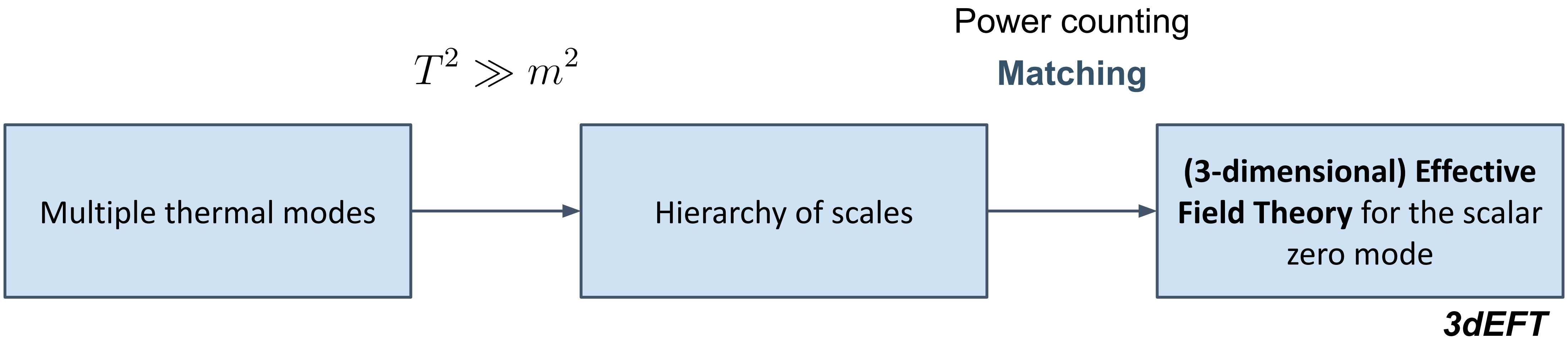
- Each mode acquires a **thermal mass** given by its **Matsubara frequency**.

This introduces a **hierarchy of scales**.



Thermal field theory

The 3D EFT approach



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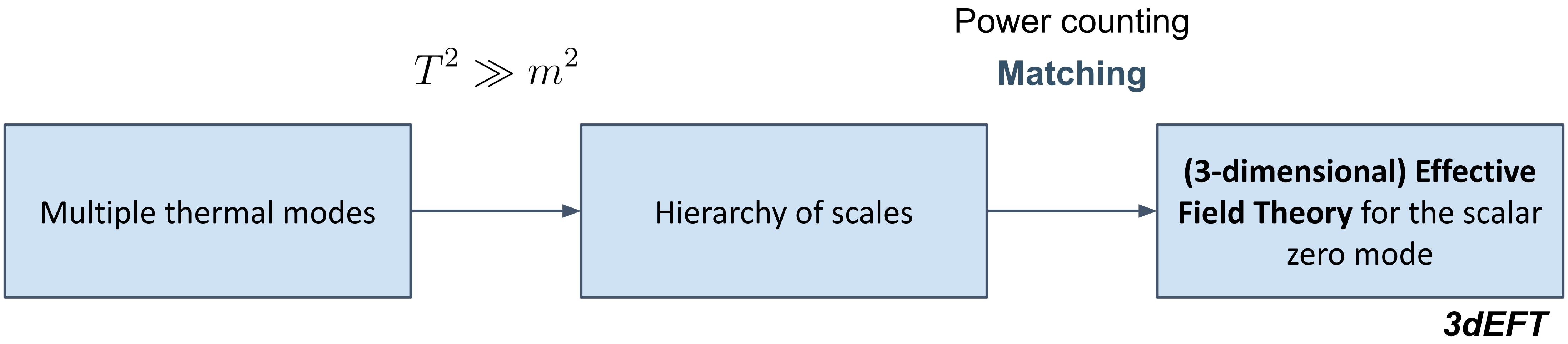

We have a simplified theory to study cosmological phase transitions in the high temperature limit

Bubbles and gravitational waves

Thermally induced field phase transitions

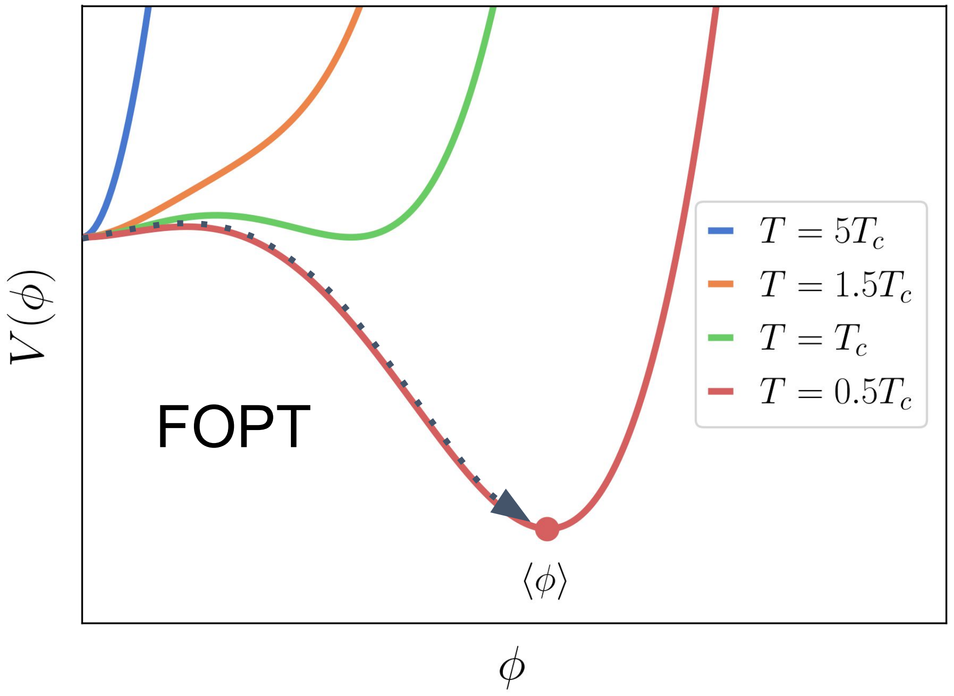

Fig. 1: Temperature evolution of scalar potential

Transition rate: $$\Gamma = A(T)e^{-S_E[\varphi_b](T)}$$

Depends on static, non-homogeneous solutions to the (Euclidean) EoMs:

$$\left.\frac{\delta S_E}{\delta \varphi}\right|_{\varphi=\varphi_b} = 0$$

These are the so-called **bounce solutions**.

[Coleman - PhysRevD.15.2929]

Bubbles and gravitational waves

[Caprini *et al.* - 1512.06239]

From PTs to GWs

As the transition rate grows, in a **first-order PT (FOPT)**:

1) Bubbles of true vacuum nucleate and expand in a hot plasma
2) Bubble fronts collide
3) Sound waves
4) Turbulence

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$$h\Omega_{\mathrm{GW}} \simeq h\Omega_{\phi} + h\Omega_{\mathrm{sw}} + h\Omega_{\mathrm{turb}}$$

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Probe for NP

How do we connect a QFT model to these GW spectra?

Bubbles and gravitational waves

From PTs to GWs

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Bubbles and gravitational waves

From PTs to GWs

How do we connect a QFT model to these GW spectra?

Nucleation temperature

$$S_3[\varphi_c] \sim 100 - 4\log\frac{T_*}{100\,\mathrm{GeV}}$$

Strength parameter

$$\alpha = \frac{\Delta\left(V_3(\varphi) - \frac{T}{4}\frac{\mathrm{d}}{\mathrm{d}T}V_3(\varphi)\right)\Big|_{T_*}}{\rho_r(T_*)}$$

Inverse duration of PT

$$\frac{\beta}{H_*} = T_*\frac{\mathrm{d}S_3[\varphi_c]}{\mathrm{d}T}\bigg|_{T_*}$$

Terminal bubble wall velocity

Not very well understood how to compute it

[Lewicki *et al.* - 2111.02393]

3D EFT approach

Building our high-T EFT

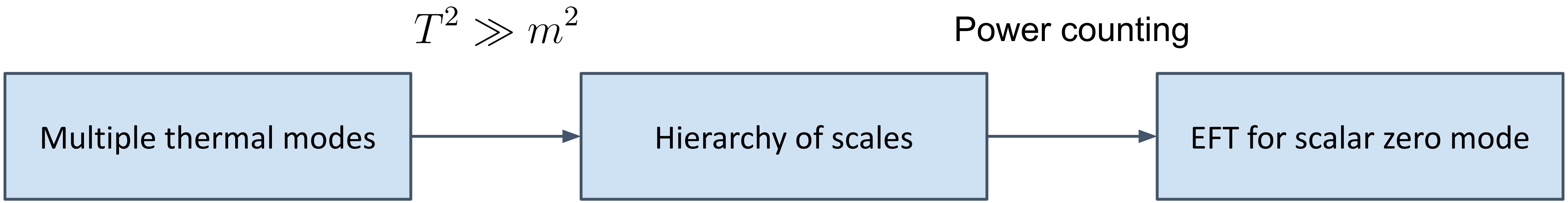
UV theory in 4D Minkowski:

$$\mathscr{L}_4 = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \kappa\phi^3 - \lambda\phi^4 + \overline{\Psi} i \partial\!\!\!/ \Psi - g\phi\overline{\Psi}\Psi$$

[Gould, Xie - 2310.02308]

At temperature T, in Euclidean space:

$$S_0 = (-i)\frac{1}{T}\int \mathrm{d}^3\mathbf{x} \sum_{n=-\infty}^{\infty}\left[\frac{1}{2}(\partial_i \varphi_n)^2 + \frac{1}{2}\left(m^2 + (2\pi n T)^2\right)\varphi_n^2 + \overline{\psi}_n \partial\!\!\!/ \psi_n + \left(2\pi\left(n+\frac{1}{2}\right)T\right)^2 \overline{\psi}_n \psi_n\right]$$



3D EFT approach

Building our high-T EFT

[Kajantie *et al.* - 9508379]
[Criado - 1901.03501]

We determine our 3D EFT by matching correlators to the full theory in the **high-temperature limit**:

$$\mathcal{L}_3 = \frac{K_3}{2}(\partial\varphi)^2 + \frac{1}{2}m_3^2\varphi^2 + \kappa_3\varphi^3 + \lambda_3\varphi^4 + \underbrace{\textbf{higher orders in } \left(\frac{\mathbf{m}}{\mathbf{T}}\right)}$$

New effective operators **(EO)**

ex. $\alpha_6\varphi^6$ or $\beta_{61}\partial^2\varphi\partial^2\varphi$

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Complete bases of EO are usually neglected because they:

- Are scale-suppressed
- Require **higher-order matching**
- Are not implemented in tools for PT computations

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How important are they in strong transitions?

Results

PT magnitudes and effective interactions

[Chala, Criado, LG, López Miras - 2406.02667]

$(m^2, \kappa, \lambda)_A = (20\,000 \text{ GeV}^2, -40 \text{ GeV}, 0.01)$ $(m^2, \kappa, \lambda)_B = (31\,643.5 \text{ GeV}^2, -71.1 \text{ GeV}, 0.045)$

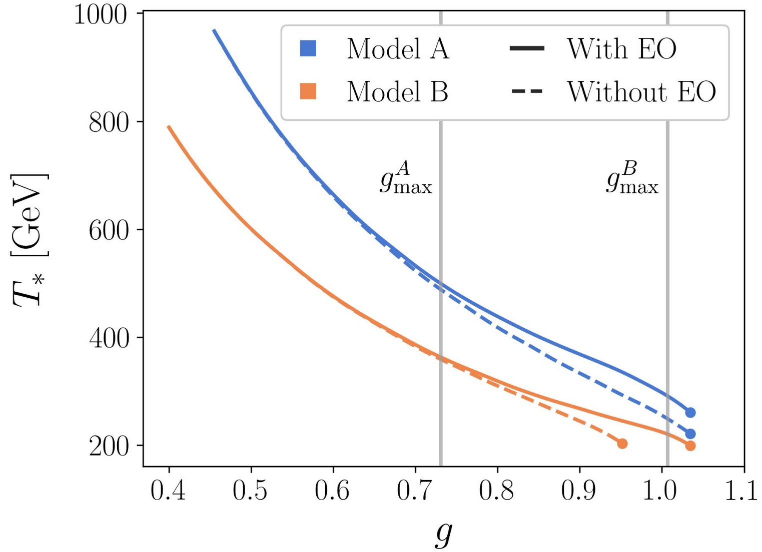

Fig. 5: PT magnitudes in two models, with and w/o effective operators

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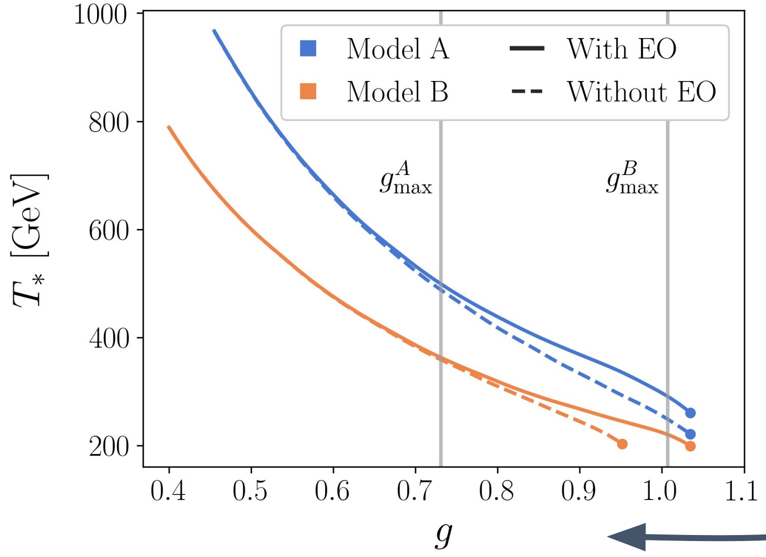

Fixed

Each point is the result for a specific model **(4 parameters)**

Variable

Fig. 5: PT magnitudes in two models, with and w/o effective operators

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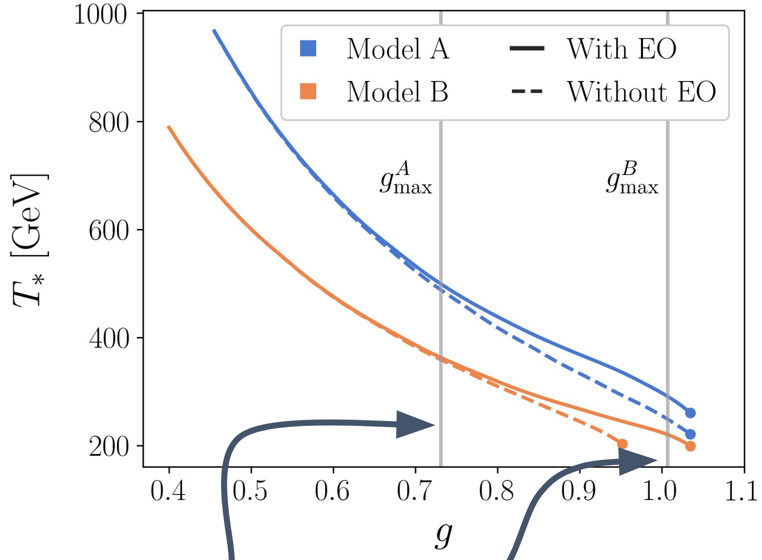

3dEFT not valid beyond the corresponding line!

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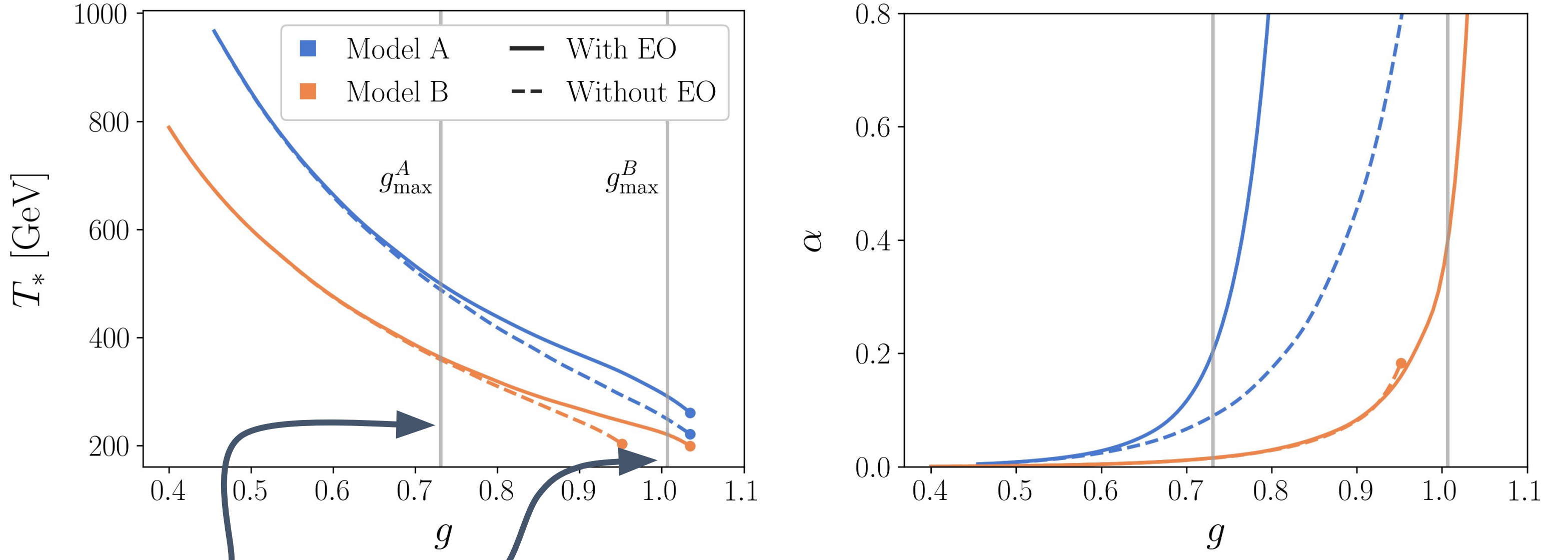

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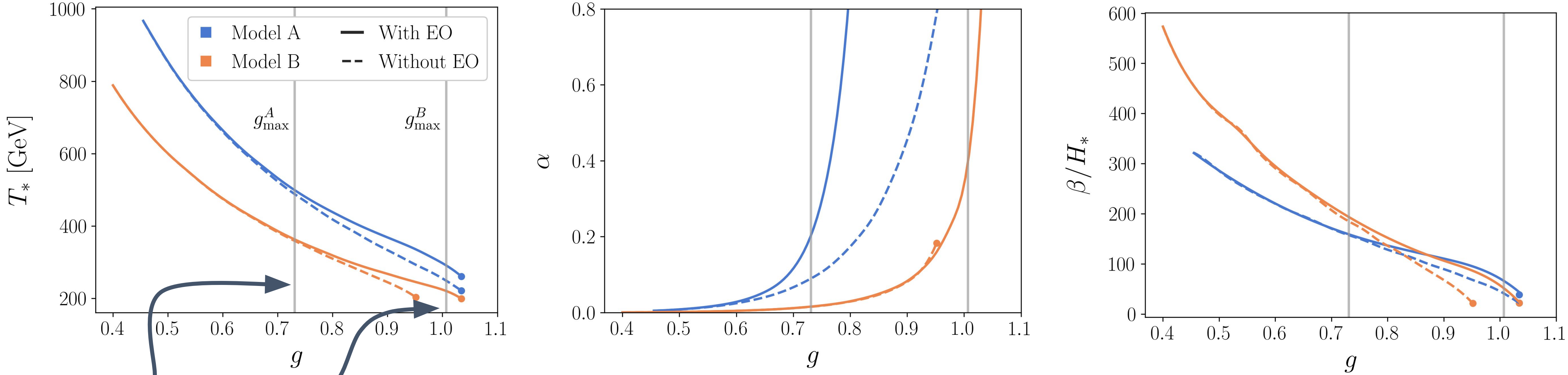

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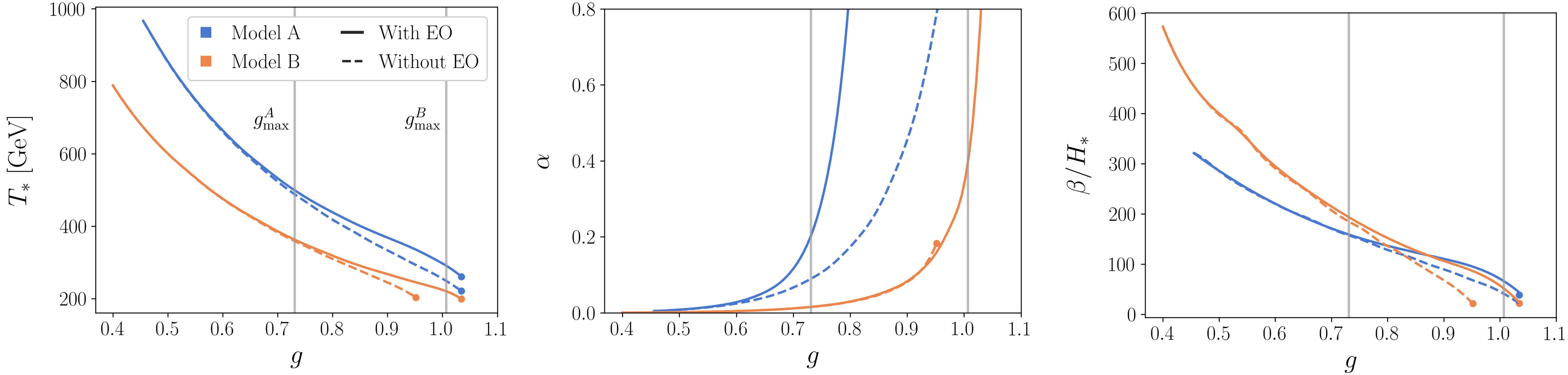

Fig. 5: PT magnitudes in two models, with and w/o effective operators

Two observations

- EO **can allow for PTs in a wider range** of values of the Yukawa
- Including EO yields **very different estimations** at large Yukawas

Results

GW power spectra and effective interactions

[Chala, Criado, LG, López Miras - 2406.02667]

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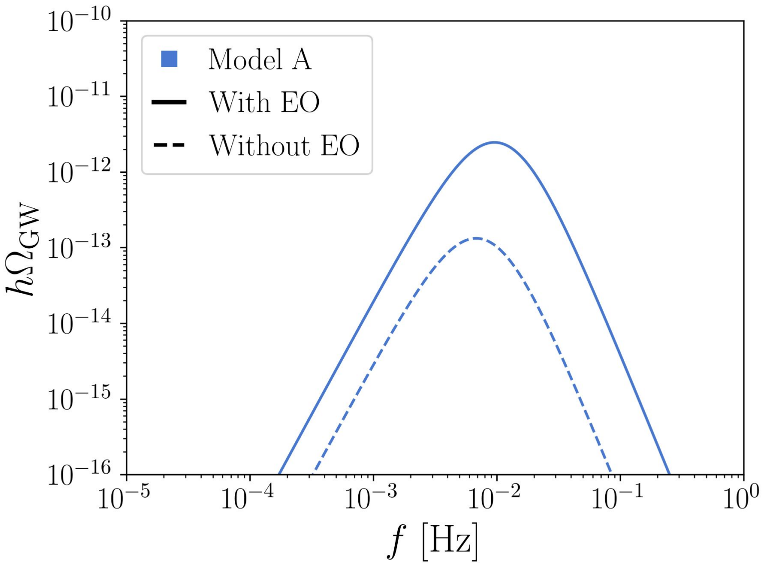
$g \lesssim g_{\text{max}}$



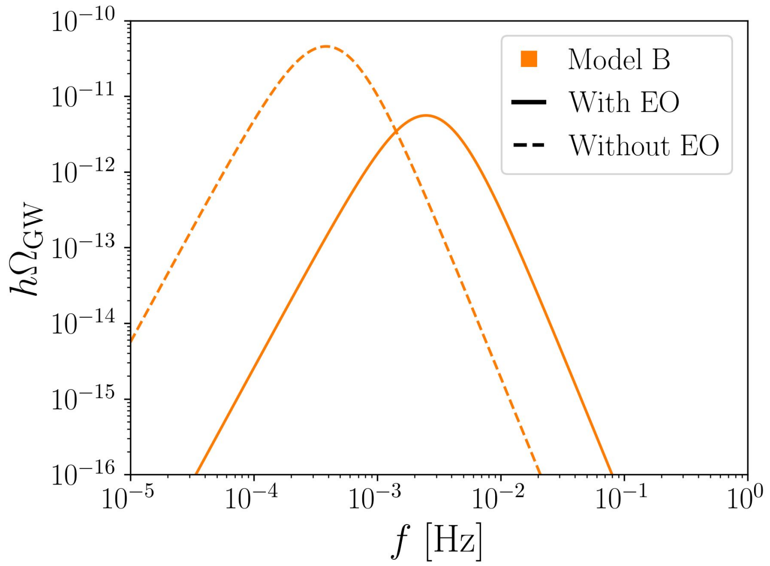


Fig. 6: GW power spectra in two models, with and w/o effective operators

(*) Generated with **PTPlot**

[Caprini *et al.* - 1910.13125]

Results

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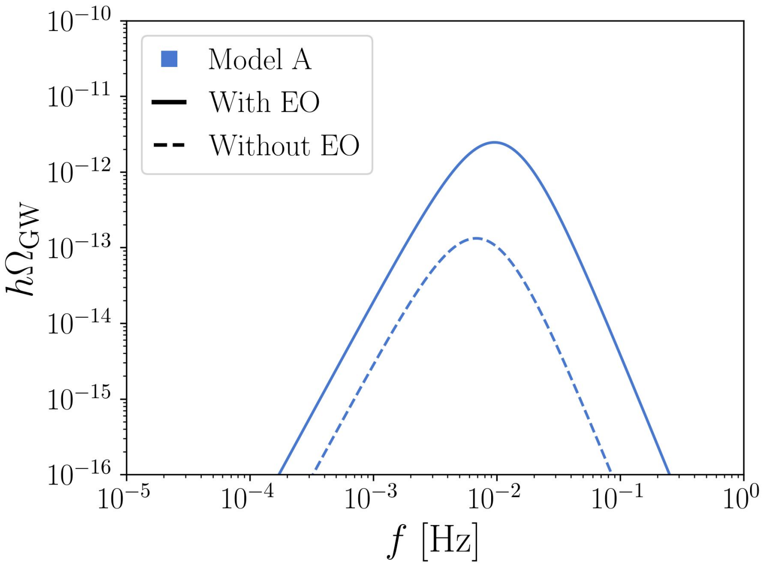

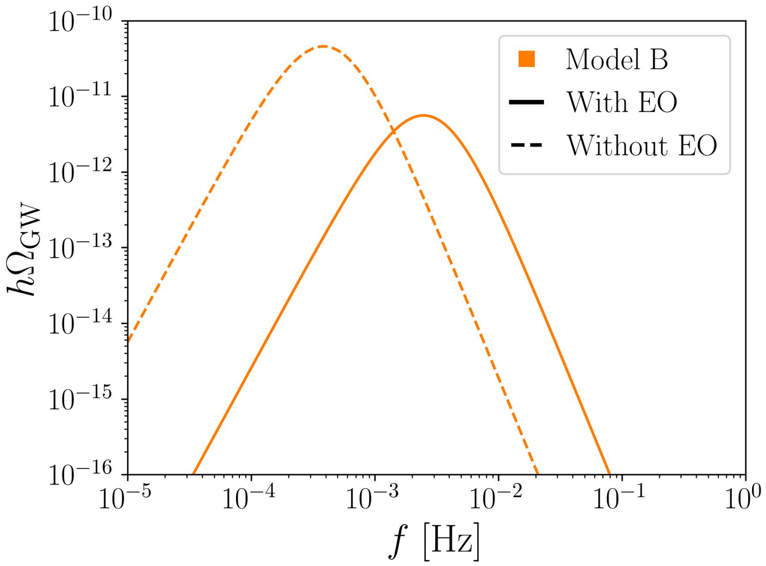

Fig. 6: GW power spectra in two models, with and w/o effective operators

Including EO can change the peak amplitude and frequency by **orders of magnitude**

Take home messages

We have learned

- Strong FOPTs occur in regions of parameter space **close the limit of validity** of the 3D EFT
- Higher-order-operator corrections in the 3dEFT **are thus relevant in the estimation of PT-related magnitudes** (order of magnitude differences in GW spectra)

Future work

- **Currently working on**: full order corrections to assess dependence on unphysical parameters
  →Matching at 2-loops (3-loops?), 4D and 3D RGEs and effective action
- Application to high-temperature **SMEFT**

Thank you for your attention!

Results

What we expect

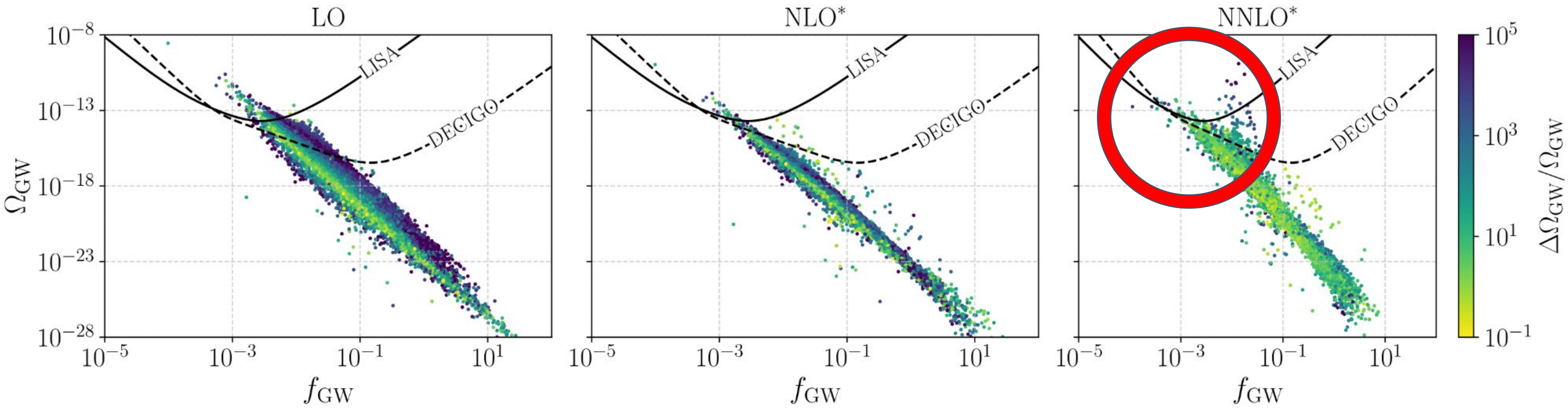

Fig. A: Gravitational wave peak amplitude and frequency from parameter scan at different perturbative orders. Extracted from Fig. 3 in [Gould et al. - 2411.08951]

3D EFT approach

Building our high-T EFT

[Kajantie *et al.* - 9508379]
[Criado - 1901.03501]

We find an off-shell basis of operators up to (4D) dim-8 with the help of *BasisGen* and match at 1-loop (fermion only loops)[1]:

$$\mathscr{L}_3 = \frac{1}{2}K_3(\partial\varphi)^2 + \frac{1}{2}m_3^2\varphi^2 + \kappa_3\varphi^3 + \lambda_3\varphi^4$$
$$+ \alpha_{61}\varphi^6 + \beta_{61}\partial^2\varphi\partial^2\varphi + \beta_{62}\varphi^3\partial^2\varphi$$
$$+ \alpha_{81}\varphi^8 + \alpha_{82}\varphi^2\partial_\mu\partial_\nu\varphi\partial^\mu\partial^\nu\varphi + \beta_{81}\varphi\partial^6\varphi + \beta_{82}\varphi^3\partial^4\varphi + \beta_{83}\varphi^2\partial^2\varphi\partial^2\varphi + \beta_{84}\varphi^5\partial^2\varphi$$

[1] Scalar loop contributions to effective operators are subleading (see Appendix A in [2406.02667]).

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dim-6

Measure eff. ops. relevance

dim-8

Control EFT validity

[1] Scalar loop contributions to effective operators are subleading (see Appendix A in [2406.02667]).

3D EFT approach

Matching

[Chala, Criado, LG, López Miras - 2406.02667]

$$K_3 = 1 + \frac{g^2}{12\pi^2}\,, \qquad m_3^2 = m^2 + \frac{g^2T^2}{6}\,, \qquad \kappa_3 = \kappa\sqrt{T}\,, \qquad \lambda_3 = \lambda T\,;$$

$$\alpha_{61} = -\frac{7\zeta(3)g^6}{192\pi^4}\,, \qquad \beta_{61} = -\frac{7\zeta(3)g^2}{384\pi^4T^2}\,, \qquad \beta_{62} = \frac{35\zeta(3)g^4}{576\pi^4T}\,;$$

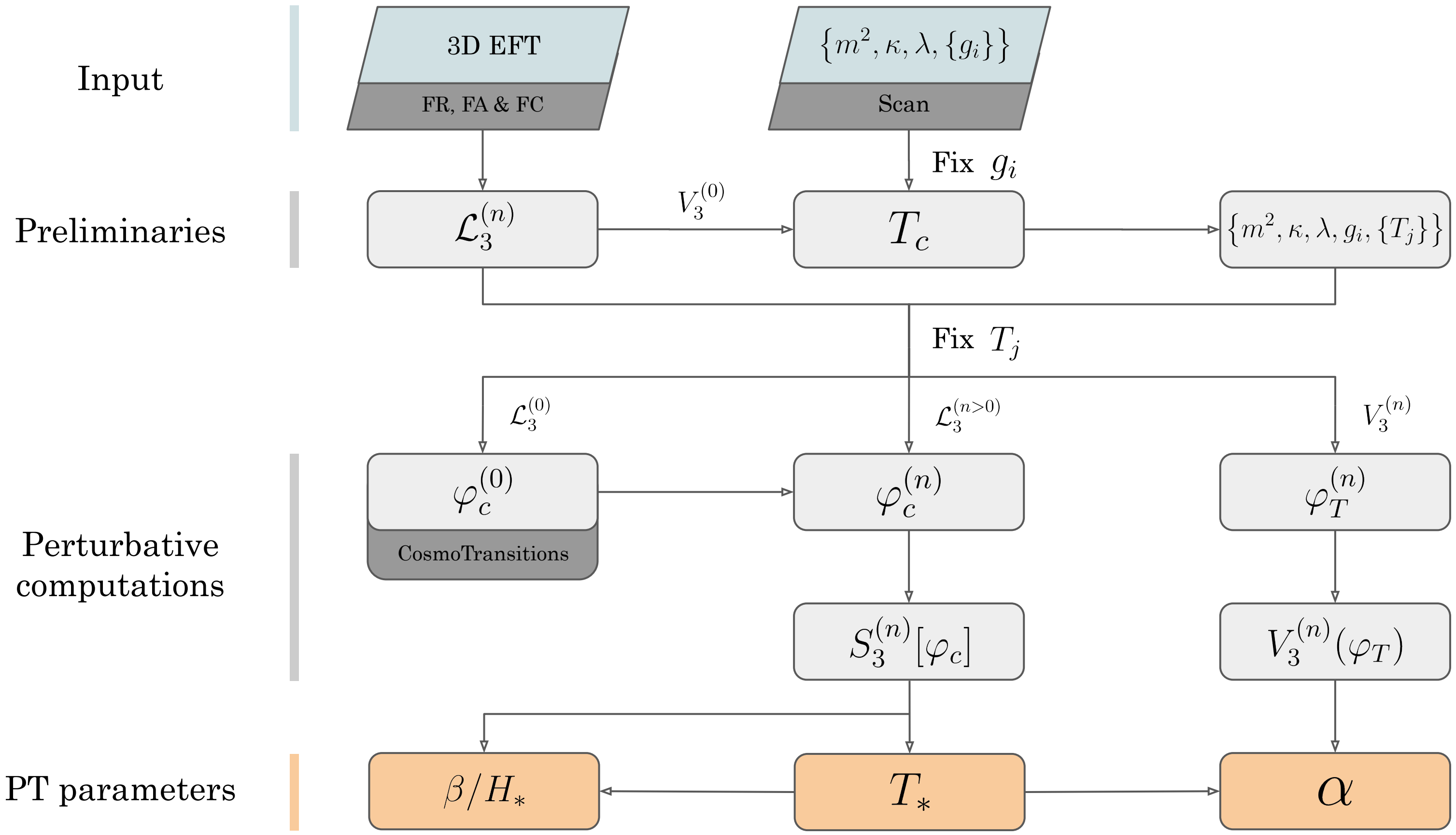
$$\alpha_{81} = \frac{31\zeta(5)g^8}{2048\pi^6T}\,, \qquad \alpha_{82} = -\frac{31\zeta(5)g^4}{10240\pi^6T^3}\,, \qquad \beta_{81} = -\frac{31\zeta(5)g^2}{10240\pi^6T^4}\,,$$

$$\beta_{82} = \frac{217\zeta(5)g^4}{20480\pi^6T^3}\,, \qquad \beta_{83} = \frac{279\zeta(5)g^4}{20480\pi^6T^3}\,, \qquad \beta_{84} = -\frac{217\zeta(5)g^6}{5120\pi^6T^2}\,.$$

3D EFT approach

The algorithm

[Chala, Criado, LG, López Miras - 2406.02667]



Results

GW power spectra and matching scale dependence