HVP contribution to the muon anomaly from first principles to an accuracy of 4.6 per mil

Davide Giusti

JÜLICH Forschungszentrum

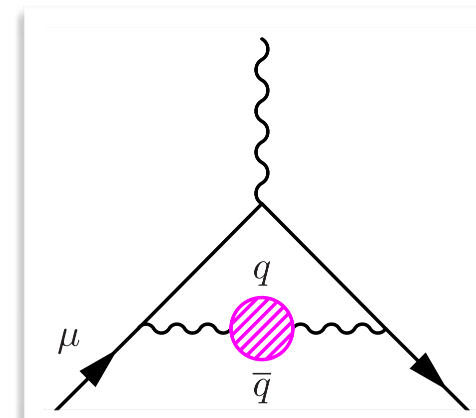
9th Symposium on Prospects in the Physics of Discrete Symmetries

Ljubljana

3rd December 2024

OUTLINE

- Introduction
- HVP from the lattice



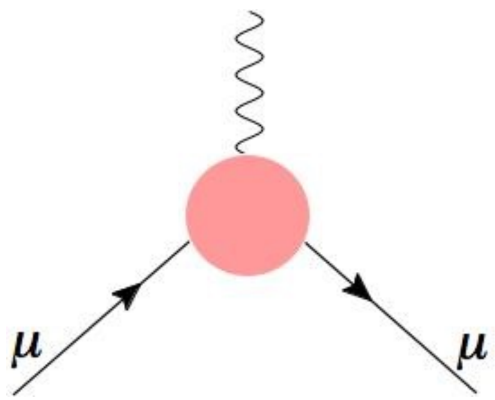
The BMW-DMZ Collaboration

A. Boccaletti, Sz. Borsanyi, M. Davier, Z. Fodor, F. Frech, A. Gérardin, D. Giusti, A.Yu. Kotov, L. Lellouch, Th. Lippert, A. Lupo, B. Malaescu, S. Mutzel, A. Portelli, A. Risch, M. Sjö, F. Stokes, K.K. Szabo, B.C. Toth, G. Wang and Z. Zhang

arXiv:2407.10913

Introduction

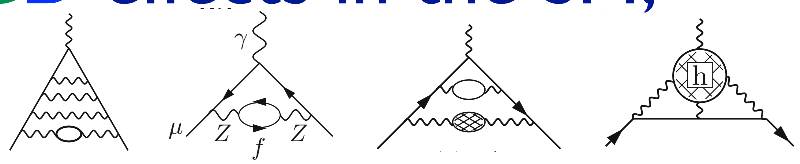
Muon magnetic anomaly



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right]u(p)$$

<u>muon anomalous magnetic moment:</u> $a_\mu \equiv \frac{g_\mu - 2}{2} = F_2(0)$

- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics

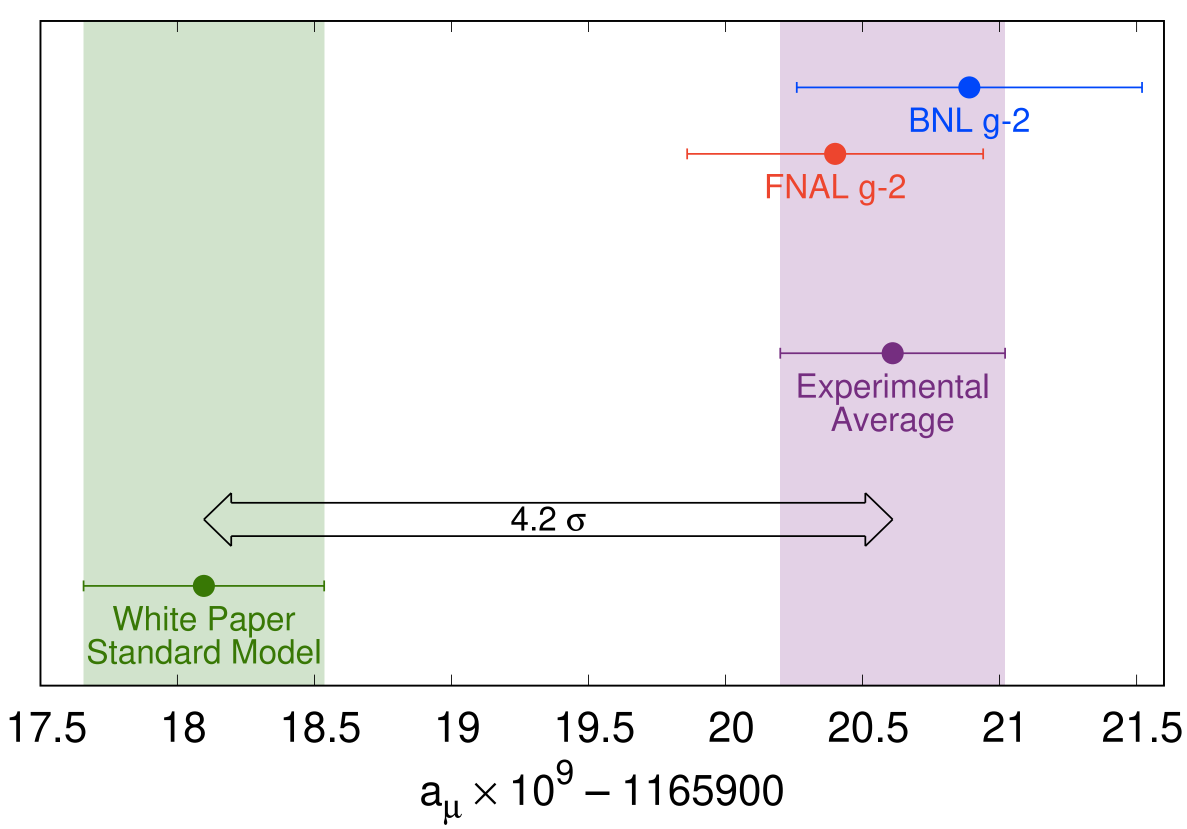


SM contributions to $a_\mu[\times 10^{10}]$

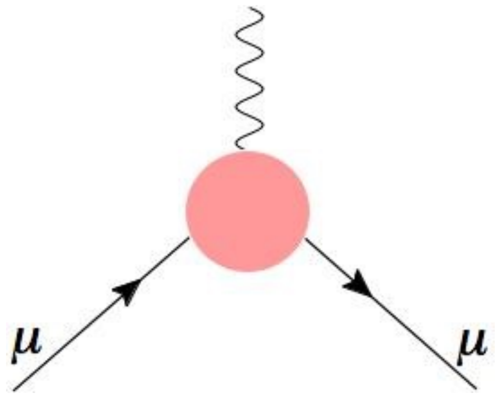
5-loop QED	11 658 471.8931(104)
2-loop EW	15.36(10)
HVP LO	693.1(4.0)
HVP NLO	-9.83(7)
HVP NNLO	1.24(1)
HLbL	9.2(1.8)

Aoyama *et al.* [WP] 2020

Theory error dominated by hadronic physics



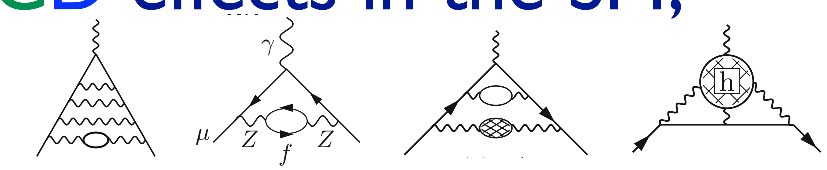
Muon magnetic anomaly

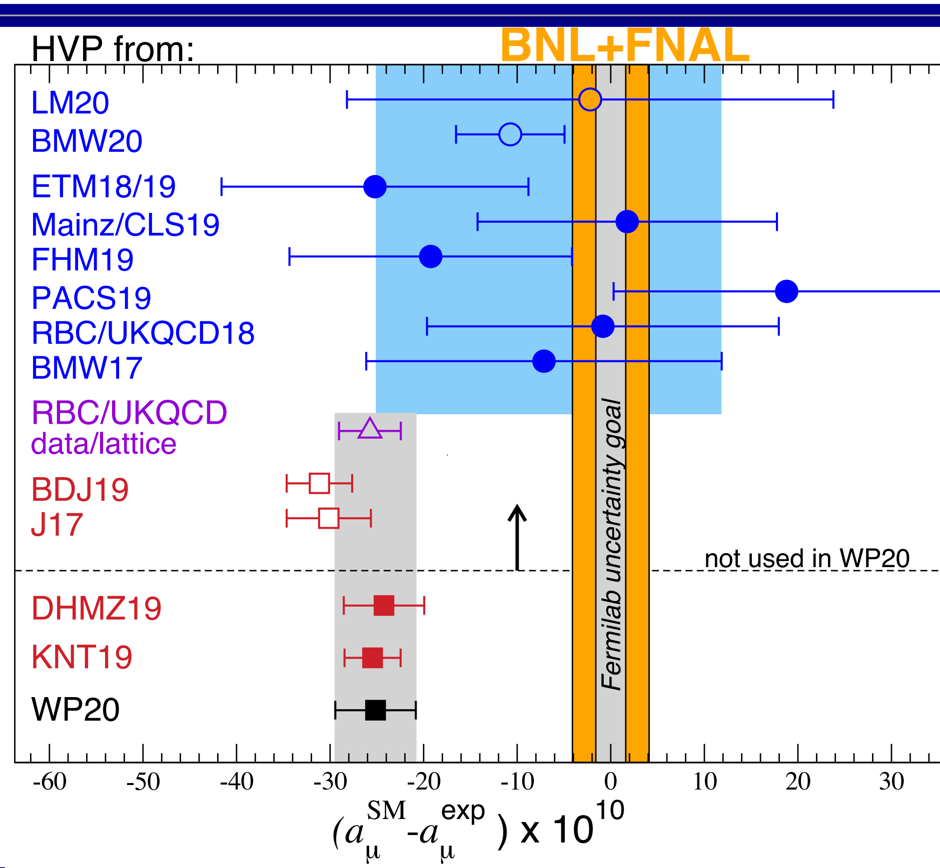


$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

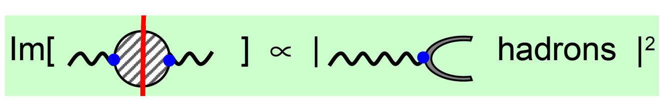
<u>muon anomalous magnetic moment:</u> $a_\mu \equiv \frac{g_\mu - 2}{2} = F_2(0)$

- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics





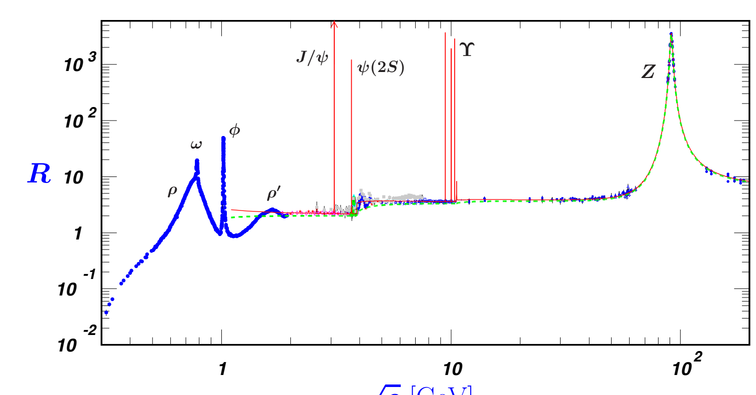
Use (Bouchiat et al '61) optical theorem (unitarity)



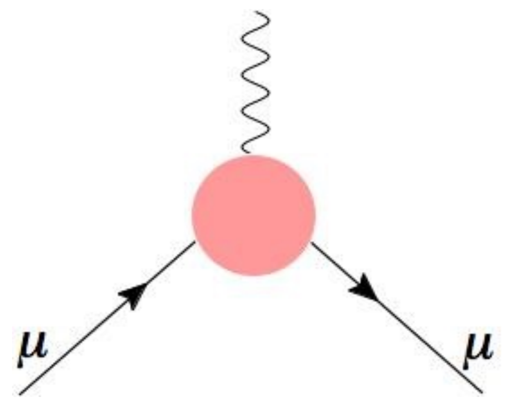
$\Rightarrow \mathrm{Im}\Pi(s) = -\frac{R(s)}{12\pi}$, $\quad R(s) \equiv \frac{\sigma(e^+e^- \to \text{had})}{4\pi\alpha(s)^2/(3s)}$

and a dispersion relation w/ data for $R(s)$ (CMD-2&3, SND, BES, KLOE '08,'10&'12, BABAR '09, etc.)

$$\hat{\Pi}(Q^2) = \int_0^\infty ds\, \frac{Q^2}{s(s+Q^2)}\frac{1}{\pi}\mathrm{Im}\Pi(s) = \frac{Q^2}{12\pi^2}\int_0^\infty ds\,\frac{1}{s(s+Q^2)}R(s)$$



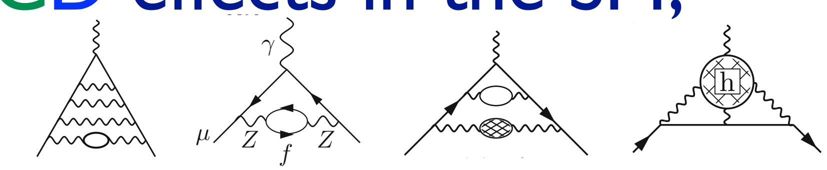
Muon magnetic anomaly

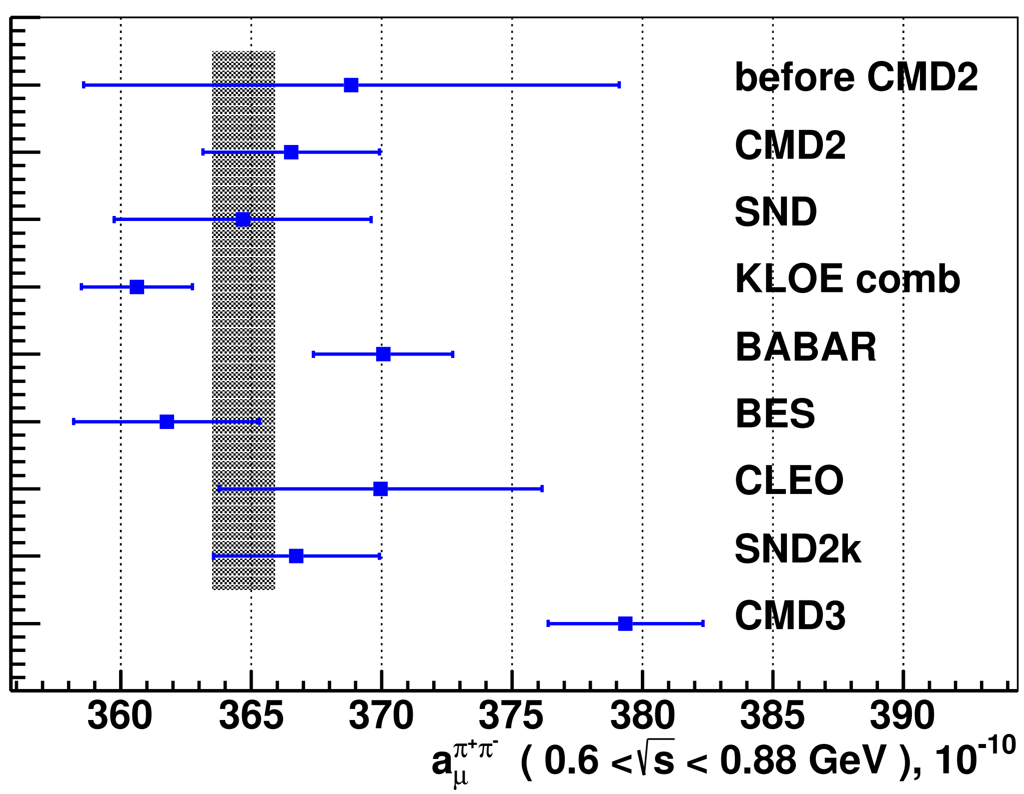


$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

muon anomalous magnetic moment: $a_\mu \equiv \frac{g_\mu - 2}{2} = F_2(0)$

- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics



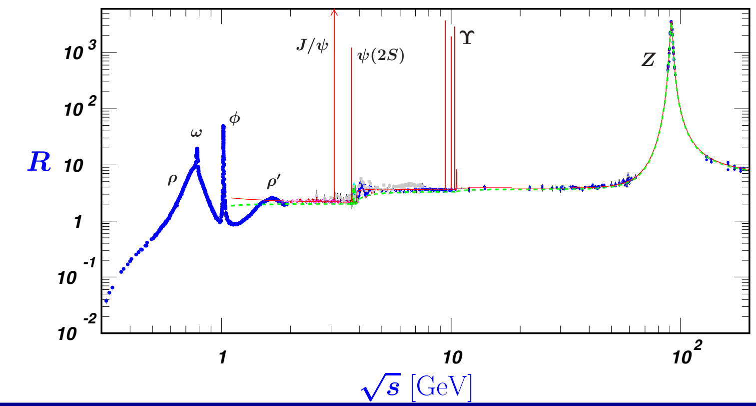


Use (Bouchiat et al '61) optical theorem (unitarity)

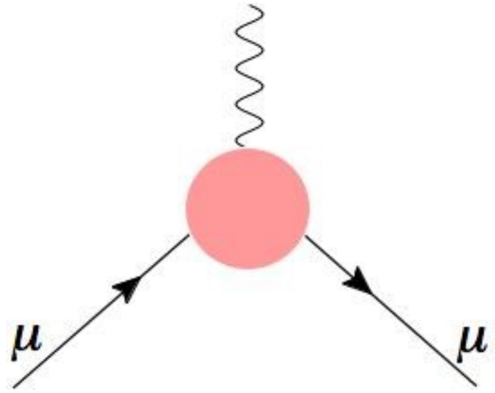
Im[ ] ∝ | hadrons $|^2$ $\Rightarrow \mathrm{Im}\Pi(s) = -\frac{R(s)}{12\pi}$, $R(s) \equiv \frac{\sigma(e^+e^- \to \mathrm{had})}{4\pi\alpha(s)^2/(3s)}$

and a dispersion relation w/ data for $R(s)$ (CMD-2&3, SND, BES, KLOE '08,'10&'12, BABAR '09, etc.)

$$\hat{\Pi}(Q^2) = \int_0^\infty ds\, \frac{Q^2}{s(s+Q^2)}\frac{1}{\pi}\mathrm{Im}\Pi(s) = \frac{Q^2}{12\pi^2}\int_0^\infty ds\, \frac{1}{s(s+Q^2)}R(s)$$



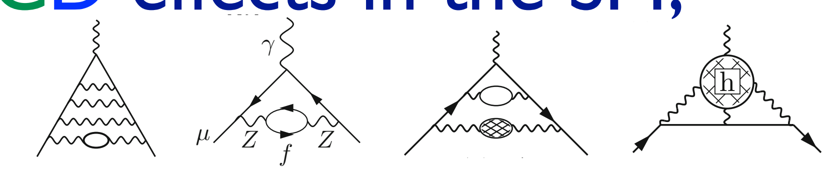
Muon magnetic anomaly



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

<u>muon anomalous magnetic moment:</u> $a_\mu \equiv \frac{g_\mu - 2}{2} = F_2(0)$

- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics

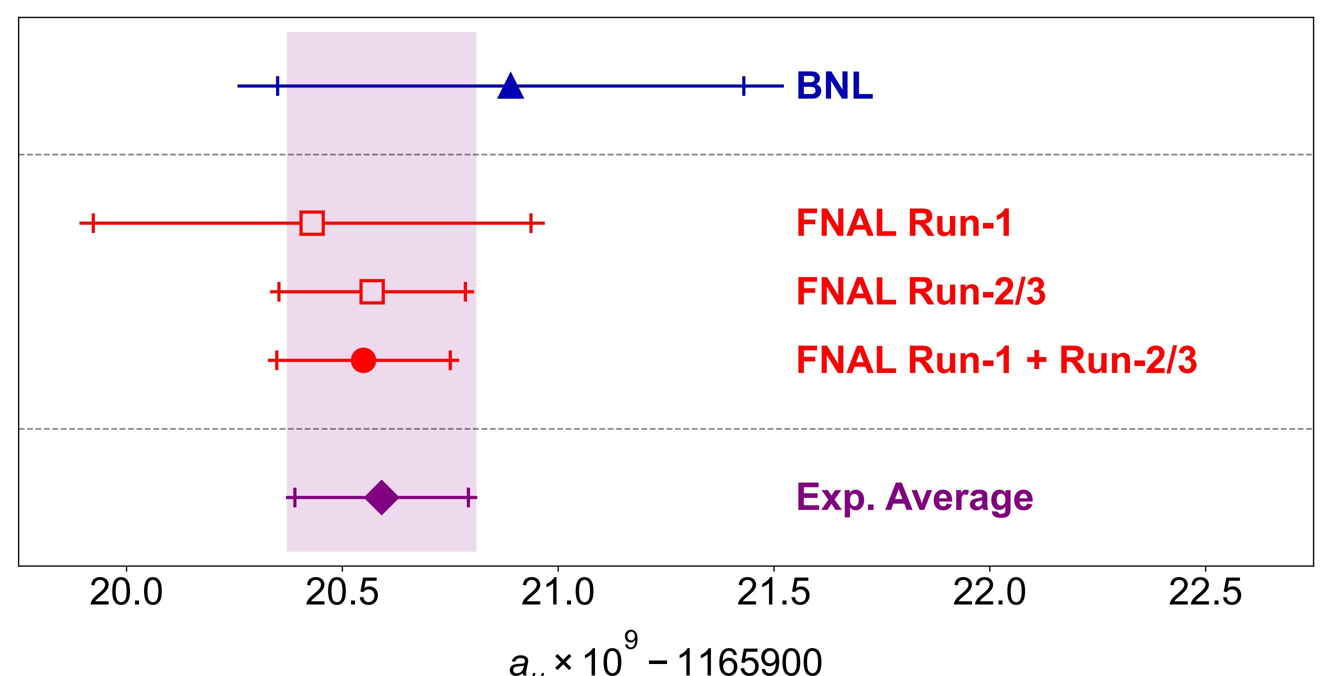


SM contributions to $a_\mu [\times 10^{10}]$

5-loop QED	11 658 471.8931(104)
2-loop EW	15.36(10)
HVP LO	693.1(4.0)
HVP NLO	-9.83(7)
HVP NNLO	1.24(1)
HLbL	9.2(1.8)

Aoyama *et al.* [WP] 2020

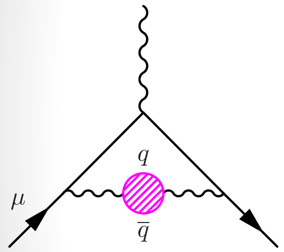
Precision goal for Fermilab ×4 better implies knowing HVP at 0.2-0.3% accuracy



Muon g-2 2023

HVP from the lattice

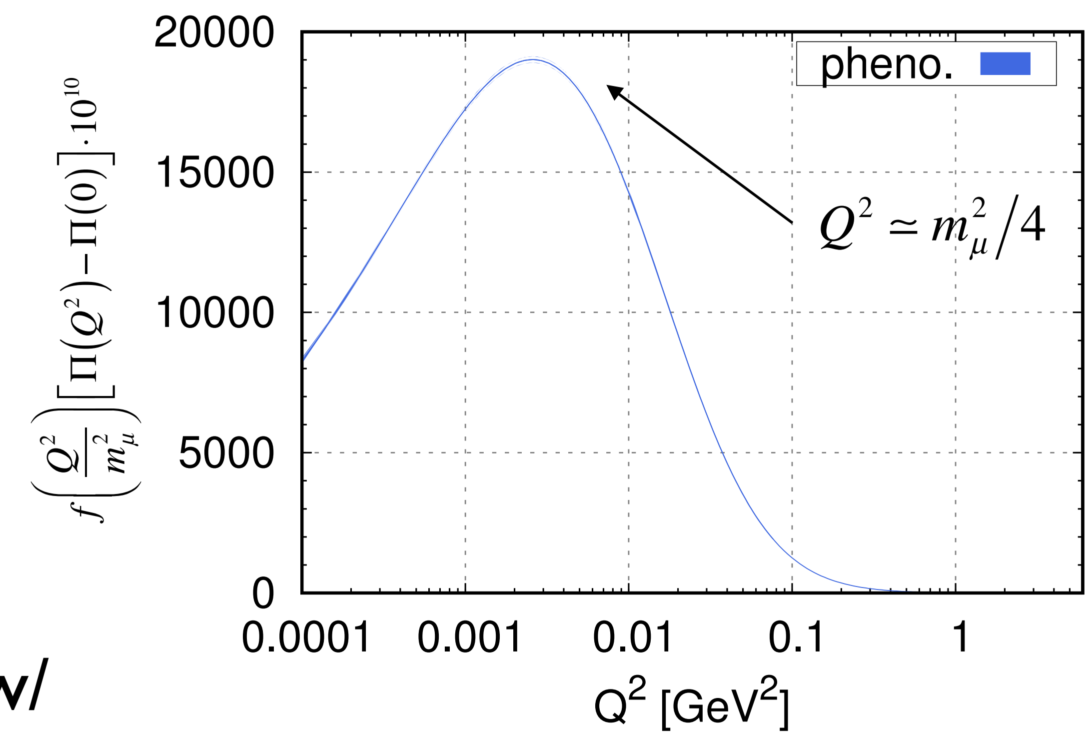
HVP from LQCD



$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0)\rangle = \left[\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu\right]\Pi(Q^2)$$

$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \, \frac{1}{m_\mu^2} f\left(\frac{Q^2}{m_\mu^2}\right)\left[\Pi(Q^2) - \Pi(0)\right]$$

B. E. Lautrup *et al.*, 1972



F. Jegerlehner, "alphaQEDc17"

FV & $a \neq 0$: A. discrete momenta ($Q_{\min} = 2\pi/T > m_\mu/2$); B. $\Pi_{\mu\nu}(0) \neq 0$ in FV contaminates $\Pi(Q^2) \sim \Pi_{\mu\nu}(Q)/Q^2$ for $Q^2 \to 0$ w/ very large FV effects; C. $\Pi(0) \sim \ln(a)$

Time-Momentum Representation

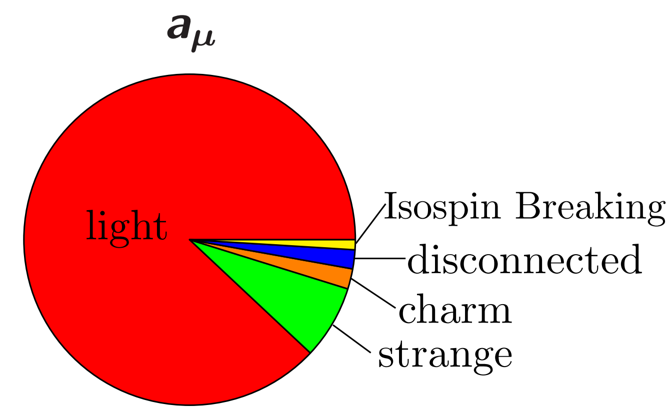
$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t)\, V(t)$$

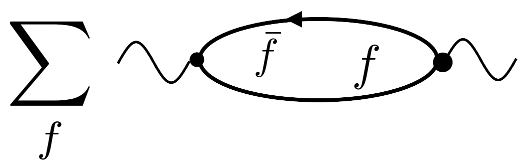
$$V(t) \equiv \frac{1}{3}\sum_{i=1,2,3}\int d\vec{x}\, \langle J_i(\vec{x},t) J_i(0)\rangle$$

D. Bernecker and H. B. Meyer, 2011

Time-Momentum Representation

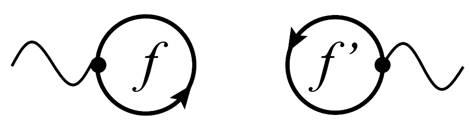
- Hadronic quantities used to calibrate the simulation ($M_\pi, M_K, M_{nucl}, \dots$)
- Can perform an explicit quark flavor separation of $a_\mu^{\mathrm{HVP,LO}}$



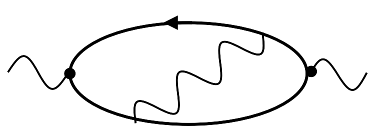


light-quark connected $a_\mu^{\mathrm{HVP,LO}}(\mathrm{ud}) \sim 90\,\%$ of total

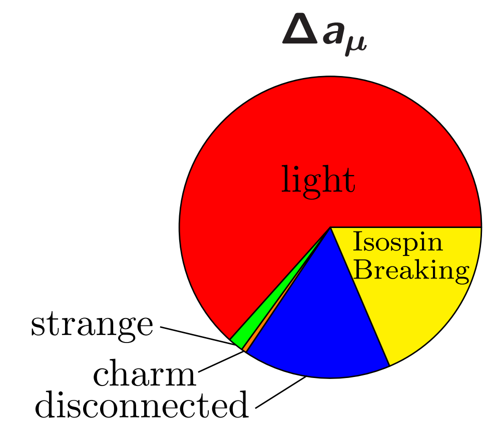
s,c-quark connected $a_\mu^{\mathrm{HVP,LO}}(\mathrm{s,c}) \sim 8\,\%\,,2\,\%$ of total



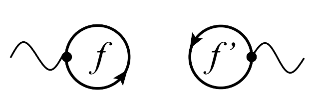
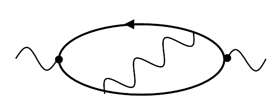
disconnected $a_{\mu,disc}^{\mathrm{HVP,LO}} \sim 2\,\%$ of total

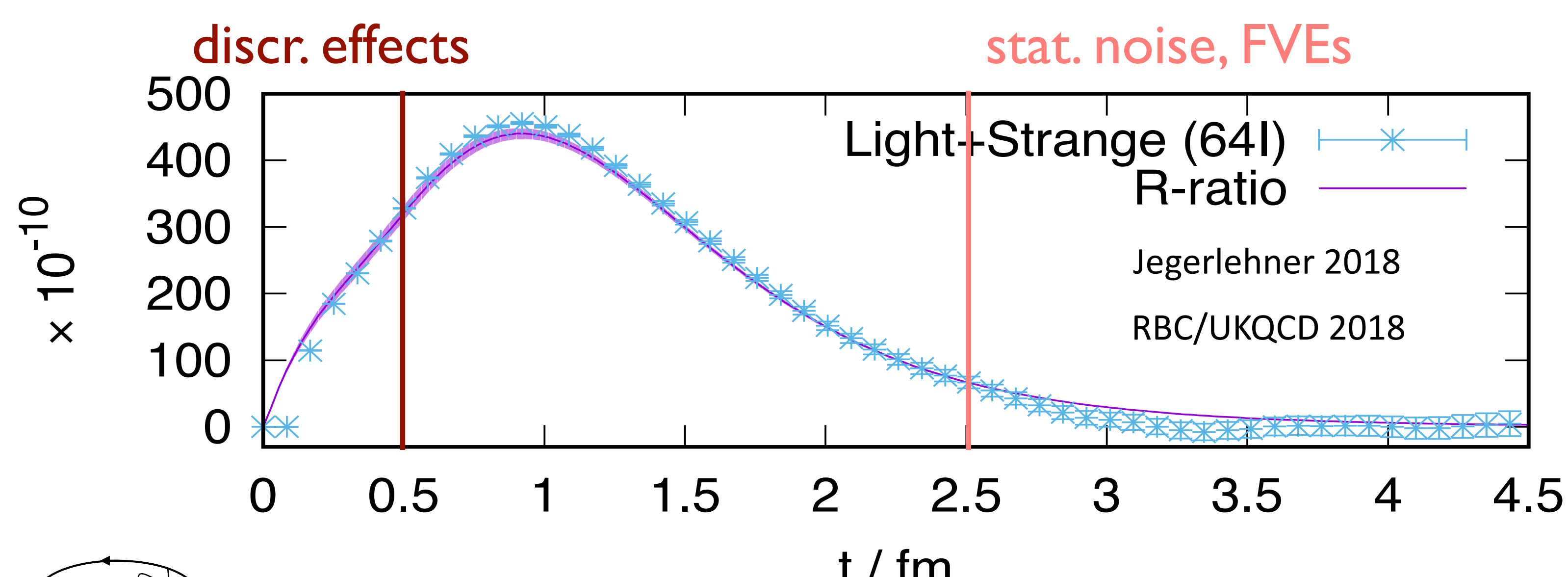


IB ($m_u \neq m_d + \mathrm{QED}$) $\delta a_\mu^{\mathrm{HVP,LO}} \sim 1\,\%$ of total



Challenges:

- sub-percent stat. precision
  exp. growing StN ratio in $V(t)$ as $t \to \infty$
- correct for FVEs, control discr. effects (scale setting and continuum extrap.)
- quark-disconn. diagrams 
  control stat. & stochastic noise
- isospin-breaking: $m_u \neq m_d$, $\alpha_{em} \neq 0$ 



Windows "on the g-2 mystery"

Restrict integration over Euclidean time to sub-intervals

→ reduce/enhance sensitivity to systematic effects

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD}$$

$$a_\mu^{SD}(f;t_0,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[1 - \Theta\left(t, t_0, \Delta\right)\right]$$

$$a_\mu^{W}(f;t_0,t_1,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[\Theta\left(t, t_0, \Delta\right) - \Theta\left(t, t_1, \Delta\right)\right]$$

$$a_\mu^{LD}(f;t_1,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t)\, \Theta\left(t, t_1, \Delta\right)$$

$$\Theta\left(t, t', \Delta\right) = \frac{1}{1 + e^{-2(t-t')/\Delta}}$$

"Standard" choice:

$$t_0 = 0.4 \text{ fm} \quad t_1 = 1.0 \text{ fm}$$

$$\Delta = 0.15 \text{ fm}$$

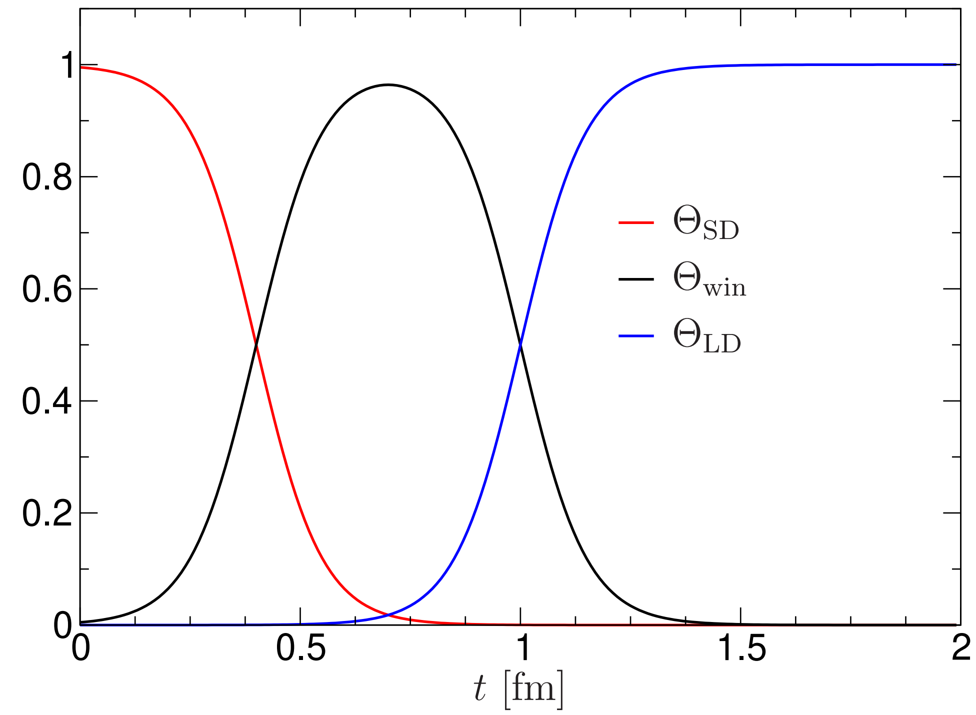
RBC/UKQCD 2018

Intermediate window

- Reduced FVEs
- Much better StN ratio

→ Precision test of different lattice calculations

→ Commensurate uncertainties compared to dispersive evaluations

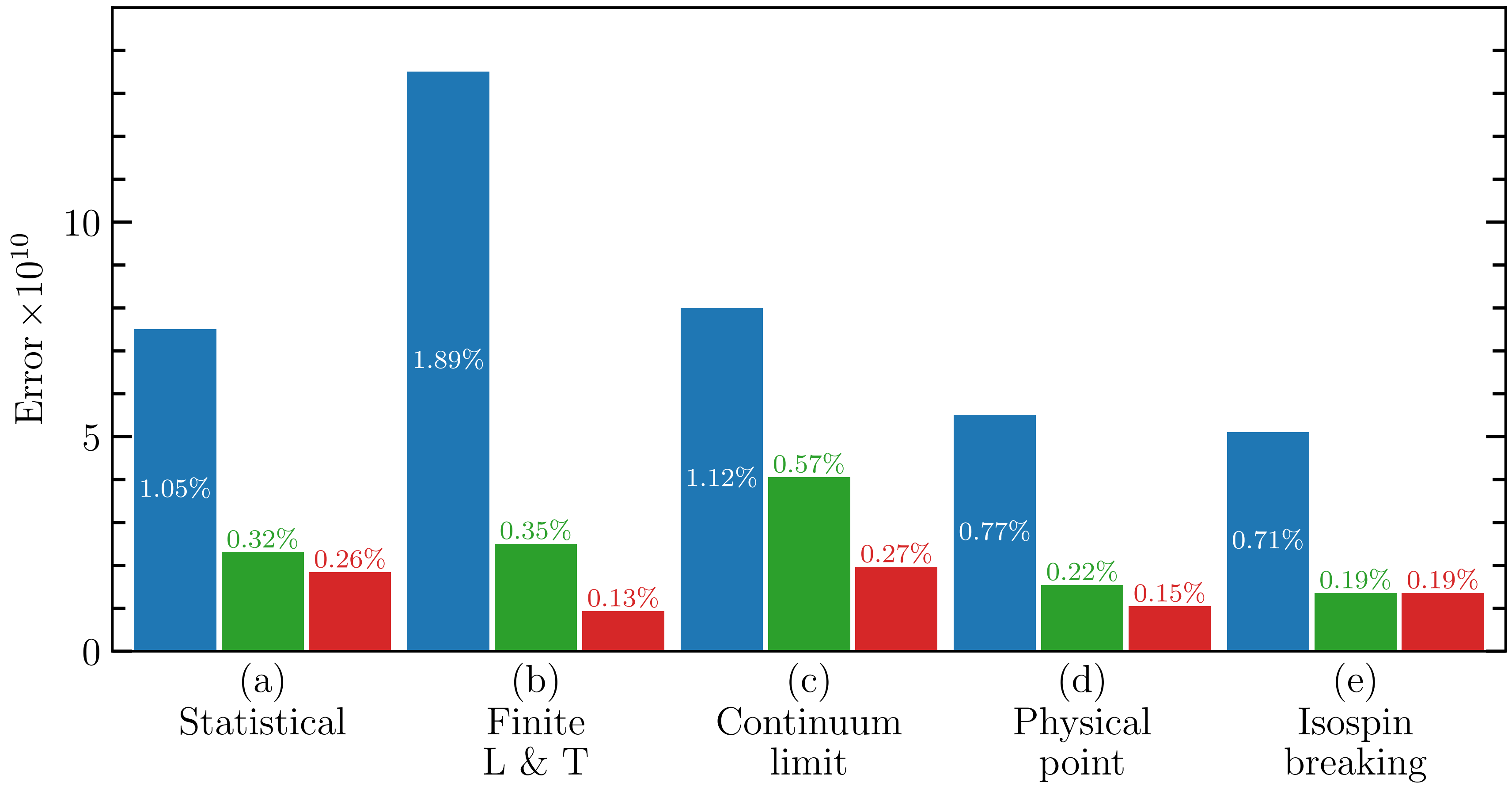


BMW-DMZ '24 calculation

High precision calculation of the hadronic vacuum polarisation contribution to the muon anomaly

arXiv:2407.10913

A. Boccaletti[1,2], Sz. Borsanyi[1], M. Davier[3], Z. Fodor[1,4,5,2,6,7,*], F. Frech[1], A. Gérardin[8], D. Giusti[2,9], A.Yu. Kotov[2], L. Lellouch[8], Th. Lippert[2], A. Lupo[8], B. Malaescu[10], S. Mutzel[8,11], A. Portelli[12,13], A. Risch[1], M. Sjö[8], F. Stokes[2,14], K.K. Szabo[1,2], B.C. Toth[1], G. Wang[8], Z. Zhang[3]



- New lattice spacing $a = 0.048$ fm (same cost as all of BMWc '20) $\longrightarrow$ divides a^2 effects by 2
- Over 30,000 gauge configurations, 10's of millions of measurements

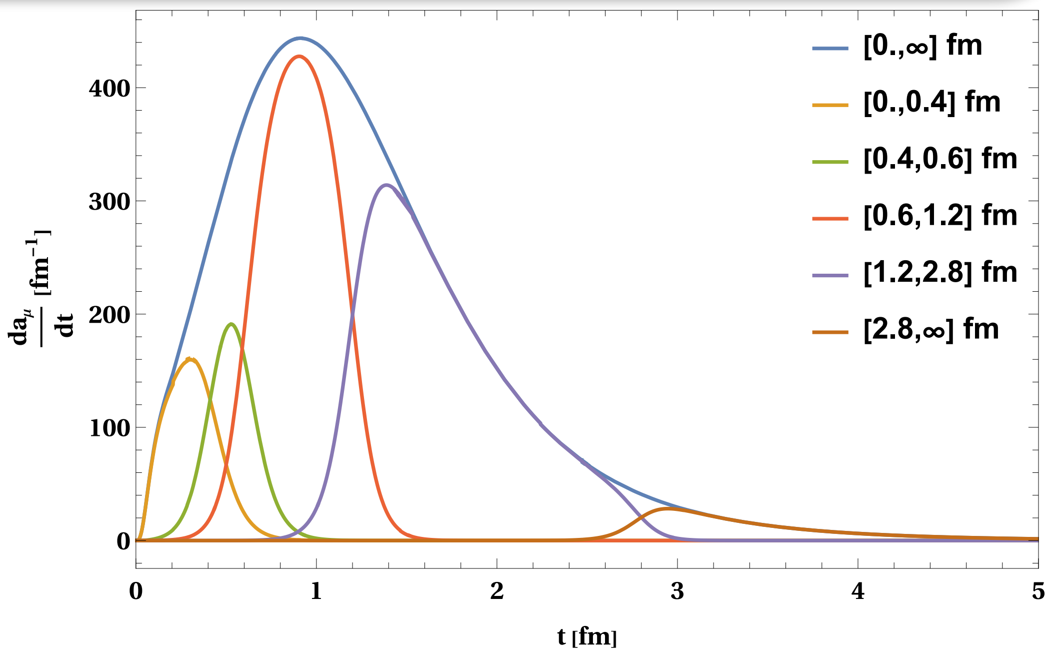
Strategy for improvement

- New simulations on finer lattice spacing: $128^3 \times 192$ w/ $a = 0.048\,\mathrm{fm}$
- Completely revamped analysis vs BMWc '20
- Break up analysis into optimized set of windows: $0-0.4$, $0.4-0.6$, $0.6-1.2$, $1.2-2.8\,\mathrm{fm}$
- Combined fit to $a_{\mu,\text{win},04\text{-}06}^{\text{LO-HVP}}$, $a_{\mu,\text{win},06\text{-}12}^{\text{LO-HVP}}$, $a_{\mu,\text{win},12\text{-}28}^{\text{LO-HVP}}$
- Continuum extrapolate $I = 0$ instead of disconnected

$\to$ reduces statistical uncertainty
$\to$ reduces $a \to 0$ error

- Data-driven evaluation of tail: $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ (proposed and used w/ $1\,\mathrm{fm} \to \infty$ [RBC/UKQCD '18])

$\to$ reduces FV effect $18.5(2.5) \to 9.3(9)$, i.e. cv $\div 2$ & err $\div 3$
$\to$ reduces LD noise
$\to$ reduces LD taste breaking and $a \to 0$ error



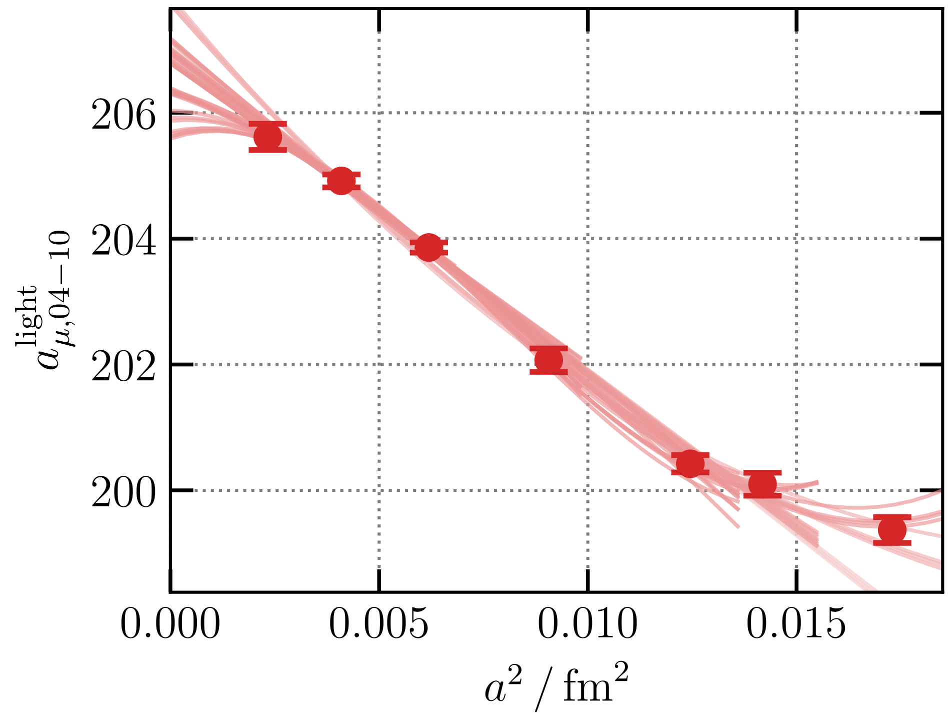
[plot made w/ KNT '18 data set]

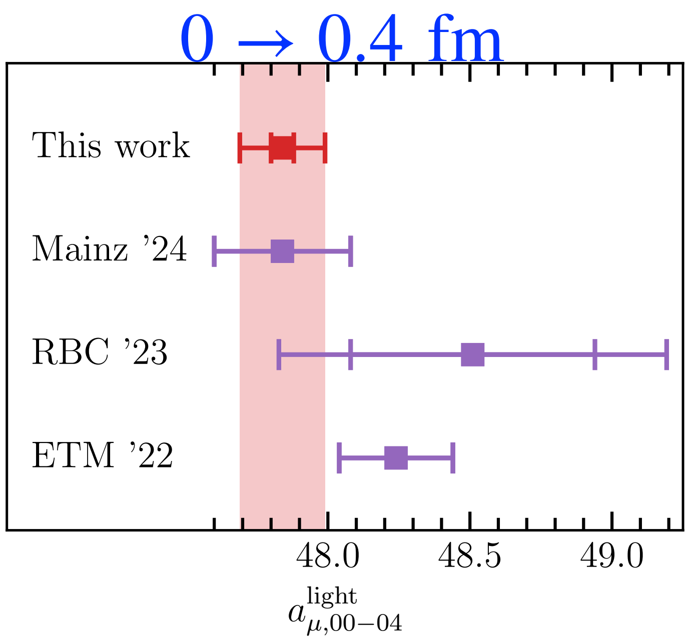
Fully blinded analysis:

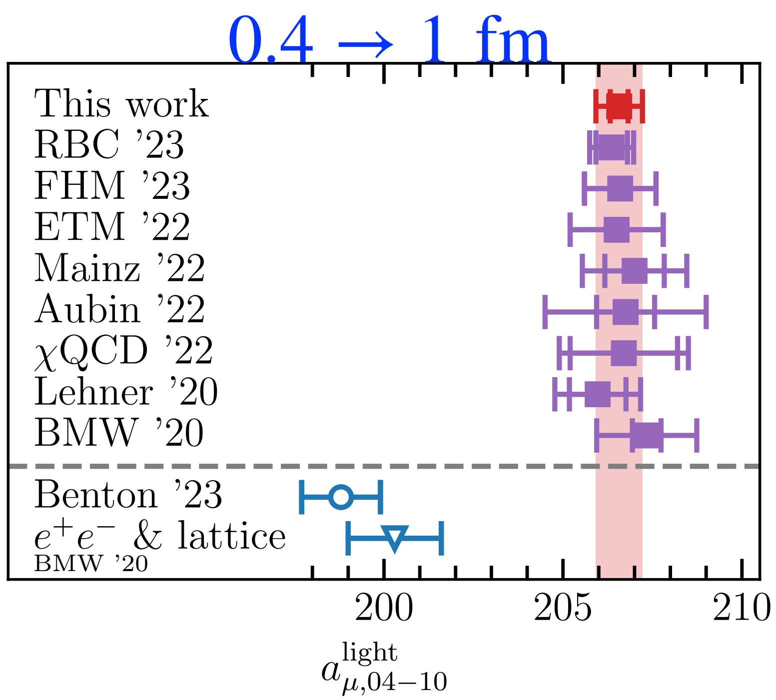
- Independent blinding by factor $\pm 3\%$ on correlator for each window and component, including data-driven tail
- $\gtrsim 2$ independent analyses of all blinded $a_{\mu}^{\text{LO-HVP}}$ contributions (and of other aspects)
- Once agreement reached, partial unblinding to allow sum of contributions
- Full unblinding on July 12, 2024, w/ automatic script that made appropriate changes in all figures and text
- Paper submitted to arXiv on July 15, 2024

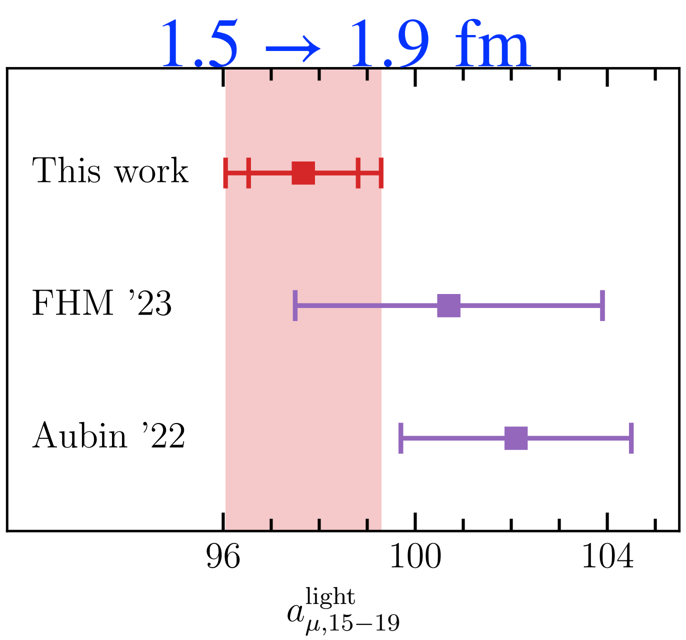
July 12, 2024: unblinding

Benchmarking of lattice calculations: windows







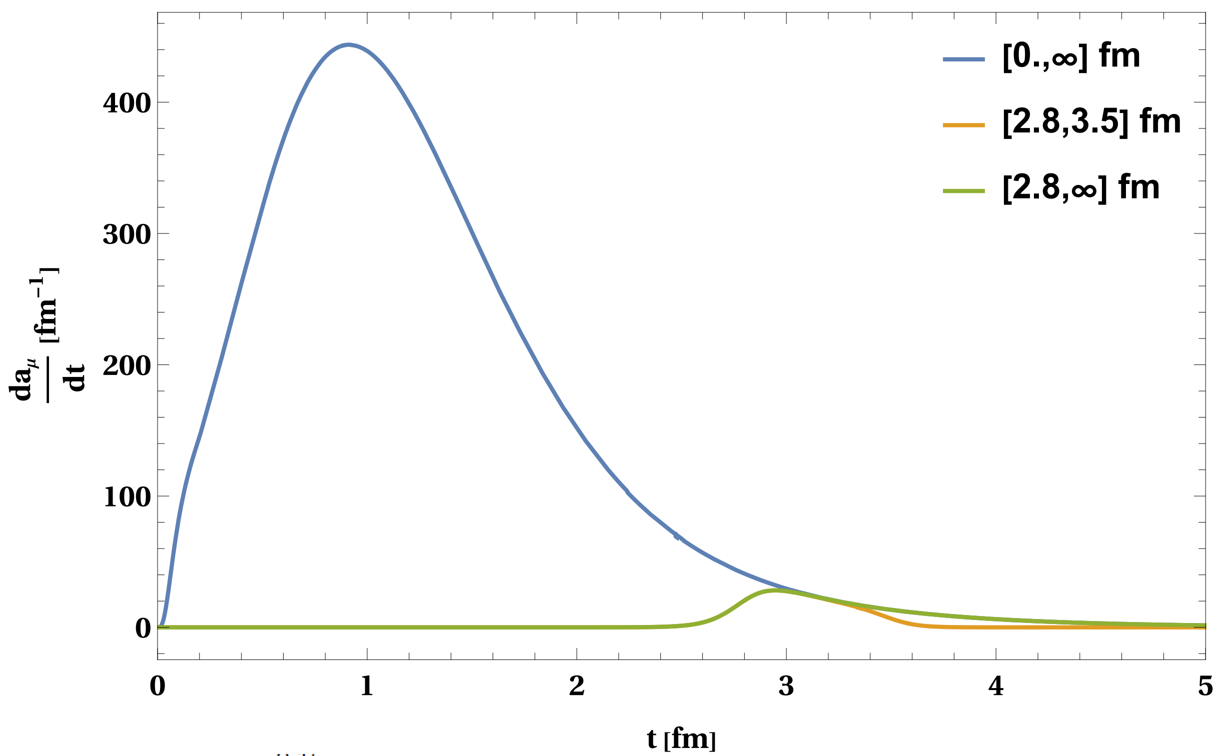


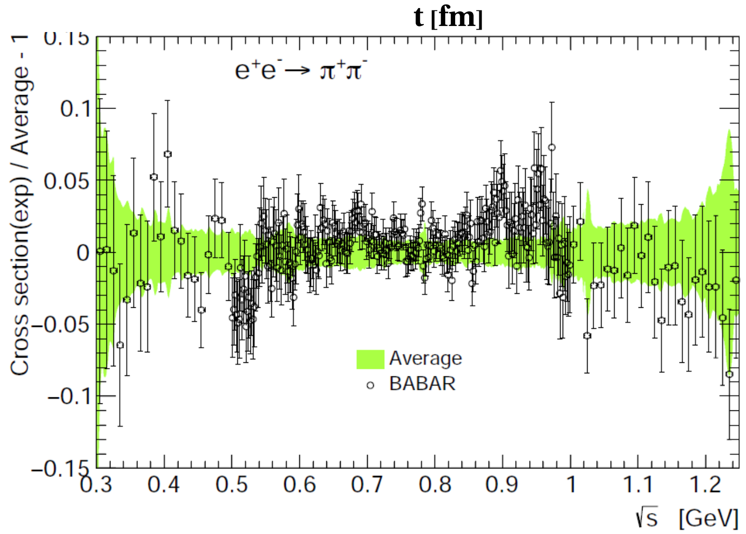
light = *ud* contribution to the long-distance window ($1 \rightarrow \infty$ fm):

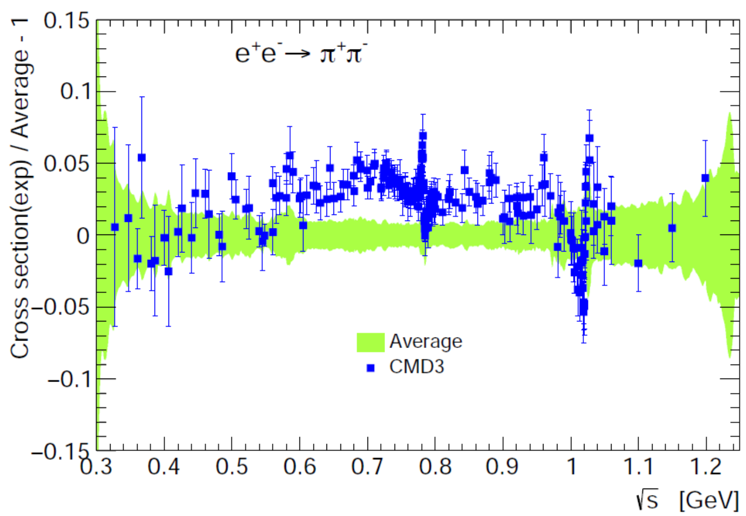
411.4(4.9) [RBC/UKQCD '24] ; 410.7(5.9) [Mainz '24]

compatible w/ BMW-DMZ '24

Tail contribution







- Lattice computation up to $t = 2.8$ fm : $> 95\,\%$ of final result for $a_\mu^{\mathrm{LO-HVP}}$

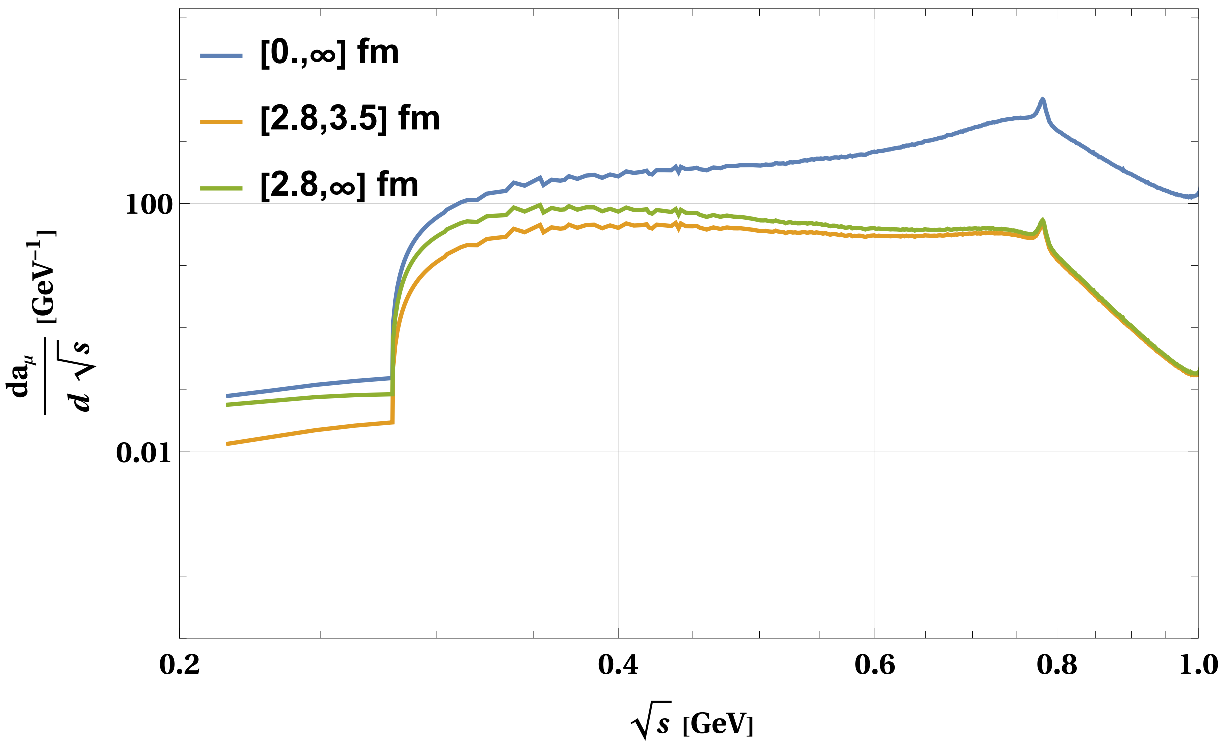
- Tail $a_{\mu,28-\infty}^{\mathrm{LO-HVP}}$ computed using $e^+e^- \to$ hadrons for $t > 2.8$ fm : $\lesssim 5\,\%$ of final result for $a_\mu^{\mathrm{LO-HVP}}$

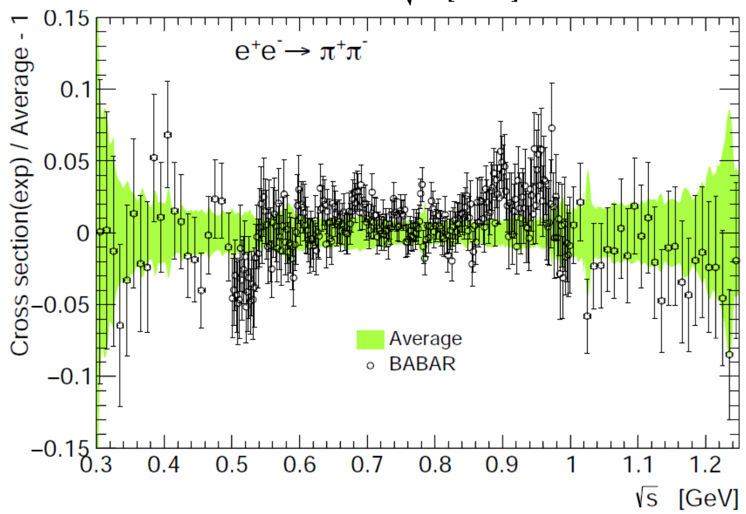
- Tail dominated by cross section below ρ peak: $\sim 75\,\%$ for $\sqrt{s} \leq 0.63$ GeV

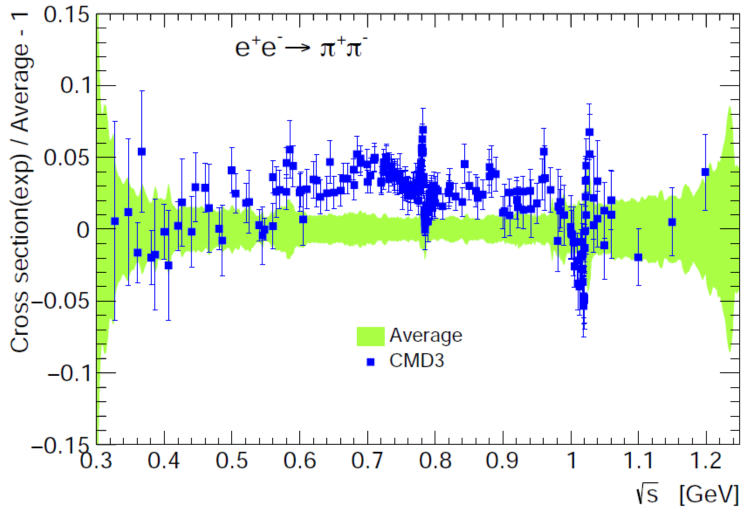
- All measurements agree to within 1.4σ for $\sqrt{s} \lesssim 0.55$ GeV. Tensions that plague $a_\mu^{\mathrm{LO-HVP}}$ & $a_{\mu,\mathrm{win}}^{\mathrm{LO-HVP}}$ not present here

- Partial tail $a_{\mu,28-35}^{\mathrm{LO-HVP}}$ for comparison with lattice; dominated by cross section below ρ peak: $\sim 70\,\%$ for $\sqrt{s} \leq 0.63$ GeV

Tail contribution







- Lattice computation up to $t = 2.8$ fm : $> 95\,\%$ of final result for $a_\mu^{\rm LO-HVP}$

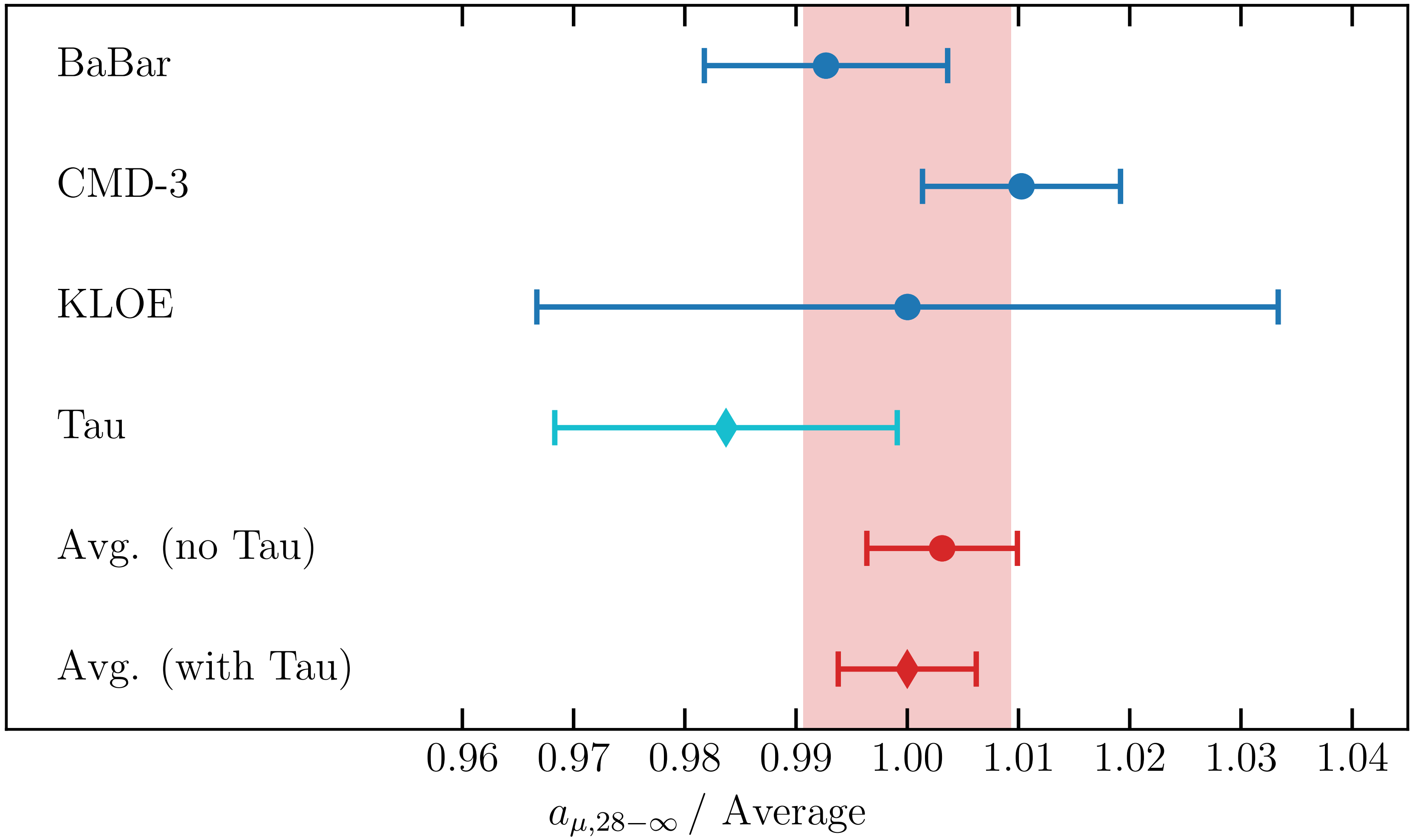
- Tail $a_{\mu,28-\infty}^{\rm LO-HVP}$ computed using $e^+e^- \to$ hadrons for $t > 2.8$ fm : $\lesssim 5\,\%$ of final result for $a_\mu^{\rm LO-HVP}$

- Tail dominated by cross section below ρ peak: $\sim 75\,\%$ for $\sqrt{s} \leq 0.63$ GeV

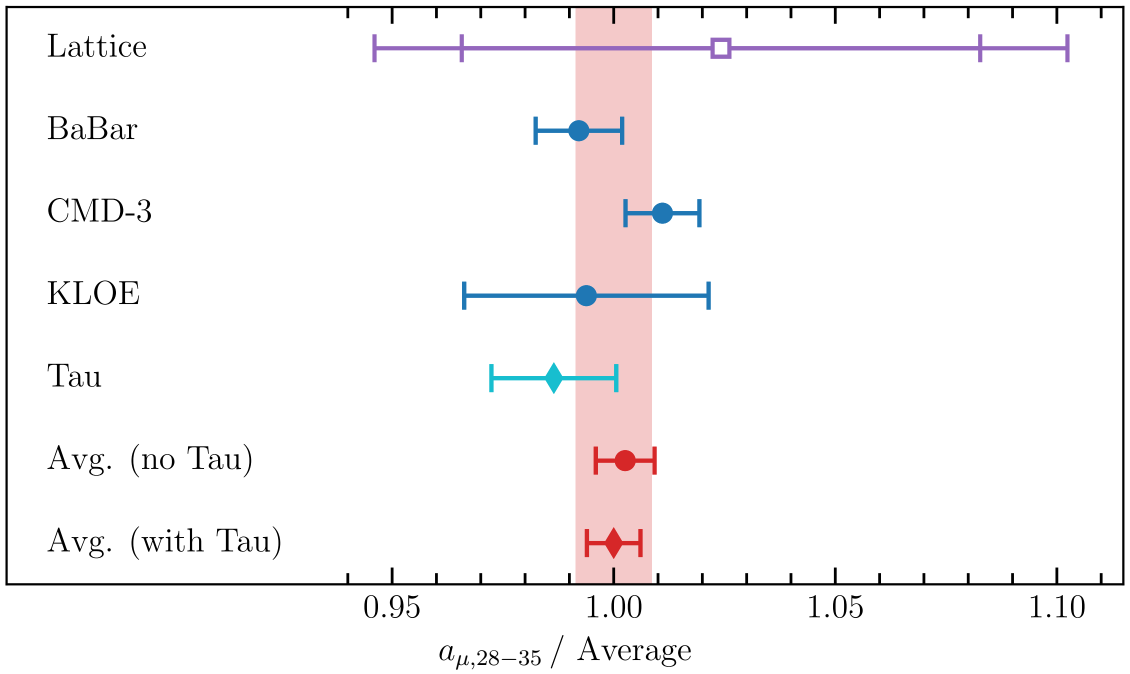
- All measurements agree to within 1.4σ for $\sqrt{s} \lesssim 0.55$ GeV. Tensions that plague $a_\mu^{\rm LO-HVP}$ & $a_{\mu,{\rm win}}^{\rm LO-HVP}$ not present here

- Partial tail $a_{\mu,28-35}^{\rm LO-HVP}$ for comparison with lattice; dominated by cross section below ρ peak: $\sim 70\,\%$ for $\sqrt{s} \leq 0.63$ GeV

Data-driven tail



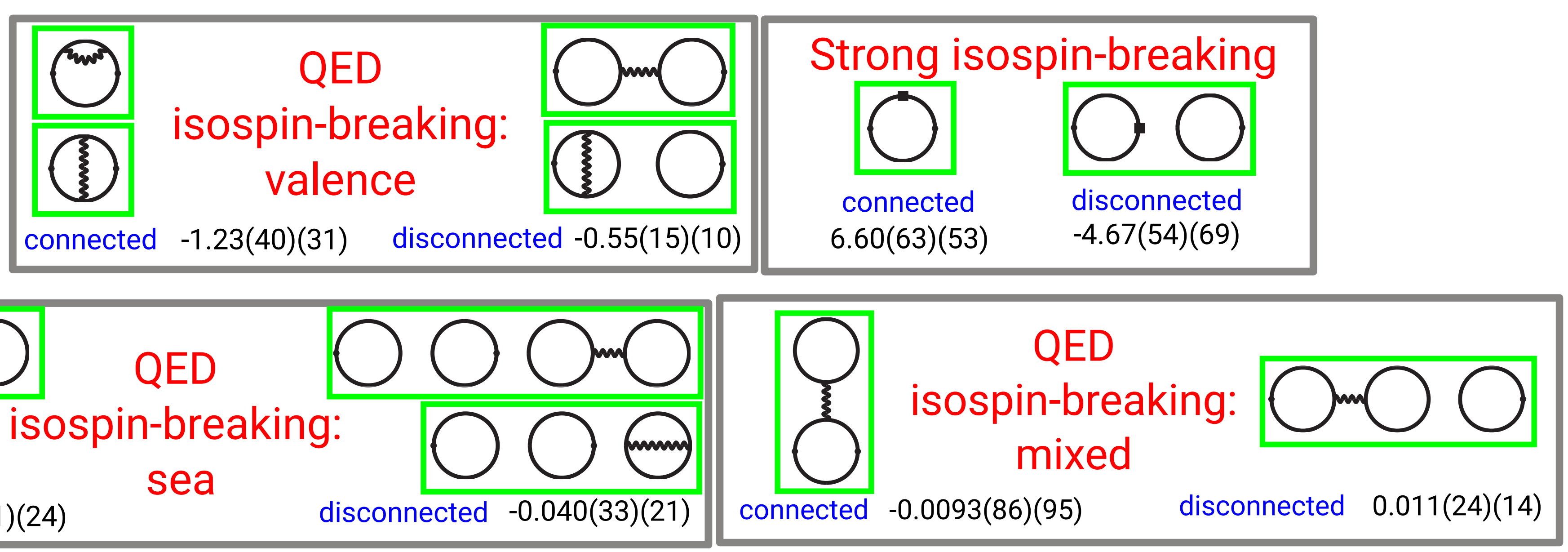
- Only $\lesssim$ 5% of final result for a_μ
- Contributes $\sim$ 65% to total squared uncertainty improvement: 5.5 $\rightarrow$ 3.3



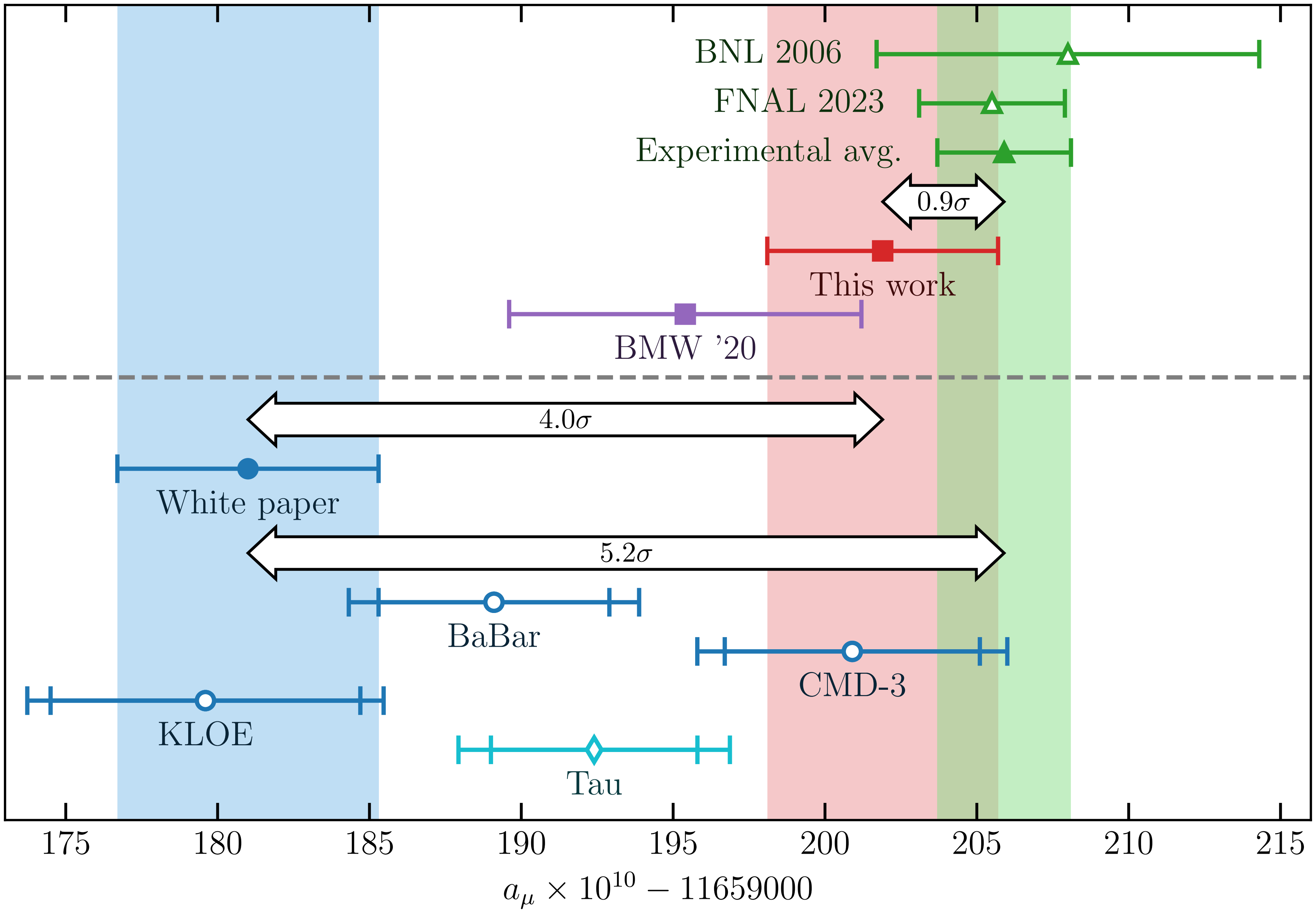
Summary of all contributions [BMW-DMZ ’24]

light and disconnected $00-28$	$618.6(1.9)(2.3)[3.0]$	this work, Equation (34)
strange $00-28$	$53.19(13)(16)[21]$	this work, Equation (37)
charm $00-28$	$14.64(24)(28)[37]$	this work, Equation (40)
light qed	$-1.57(42)(35)$	[5], Table 15 corrected in Equation (45)
light sib	$6.60(63)(53)$	[5], Table 15
disconnected qed	$-0.58(14)(10)$	[5], Table 15
disconnected sib	$-4.67(54)(69)$	[5], Table 15
disconnected charm	$0.0(1)$	[31], Section 4 in Supp. Mat.
strange qed	$-0.0136(86)(76)$	[5], Table 15
charm qed	$0.0182(36)$	[43]
bottom	$0.271(37)$	[44]
tail from data-driven $28-\infty$	$27.59(17)(9)[26]$	this work, Equation (50)
total	$714.1(2.2)(2.5)[3.3]$	

$$a_\mu^{\text{LO-HVP}} \times 10^{10} = 714.1(2.2)(2.5)[3.3] \quad [0.46\%]$$



BMW-DMZ '24 vs g-2 experiment



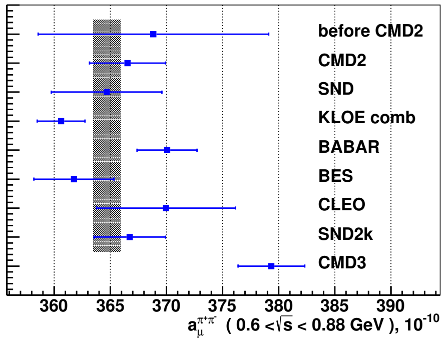
Indicates Standard Model confirmed to 0.32 ppm!

Podcast (generated by AI) on the current status of muon g-2:

https://drive.google.com/file/d/1aAi9CWSPVEYv2SMMxuGQT3l3KmEKGwKu/view?usp=d

Summary and Outlook

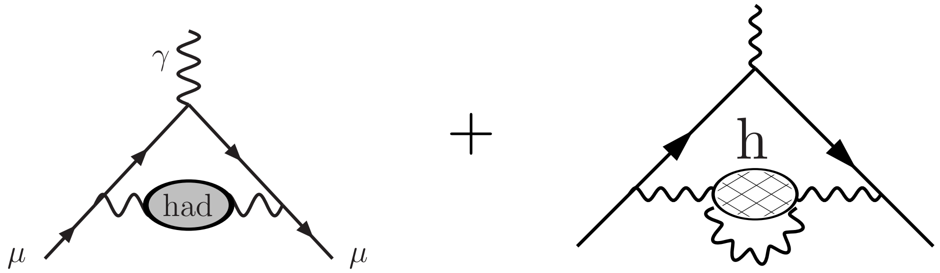
- Tremendous progress in lattice calculations of HVP (and HLbL!) contributions
- New BMW-DMZ calculation to 0.46% w/ fully blinded analysis, confirming the SM to 0.32 ppm. It needs confirmation by other groups
- Good agreement between lattice calculations for various windows
- An update of the White Paper is aimed for the beginning of 2025
- Awaiting Fermilab ~ 0.1 ppm measurement of a_μ in 2025 and J-PARC entirely new method measurement
- Awaiting new BaBar, KLOE, BESIII, Belle II, CMD3, SND2 data/analysis to clarify tensions in $\pi^+\pi^-$



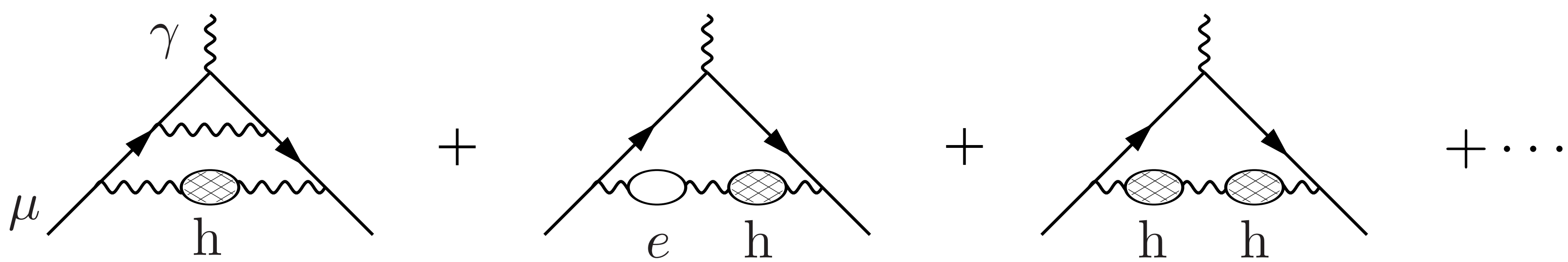
- $\mu e \rightarrow \mu e$ experiment MUonE very important for experimental cross-check and complementarity w/ LQCD

Backup slides

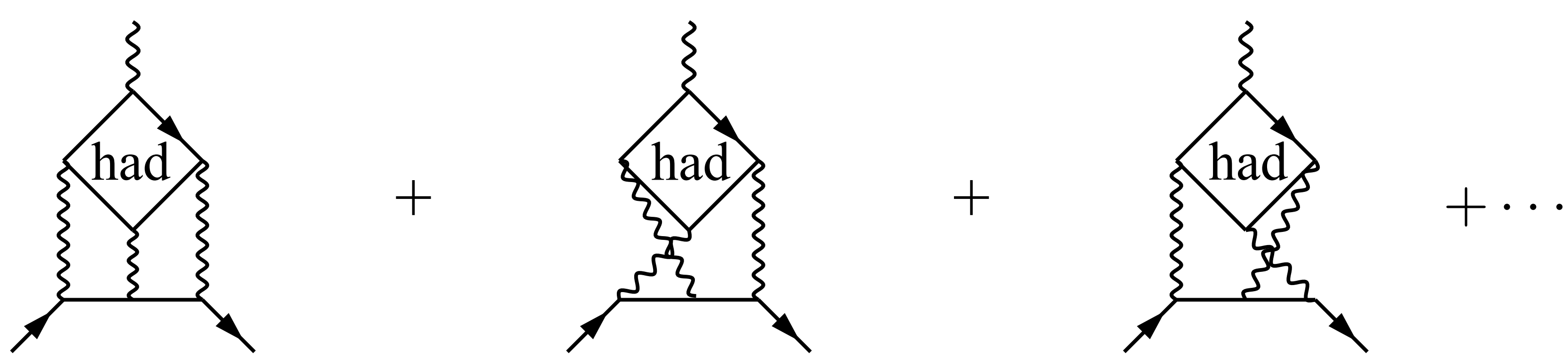
Hadronic contributions: diagrams



$\rightarrow a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$

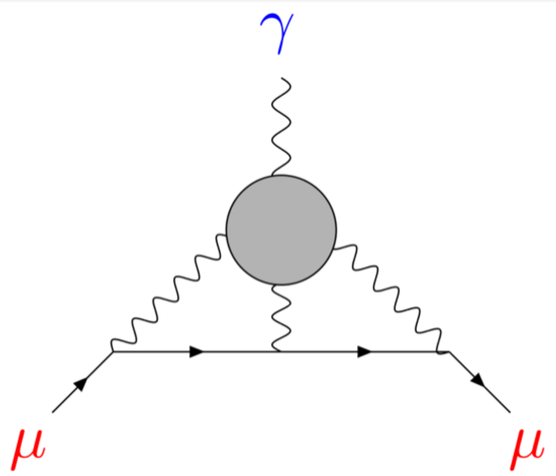


$\rightarrow a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

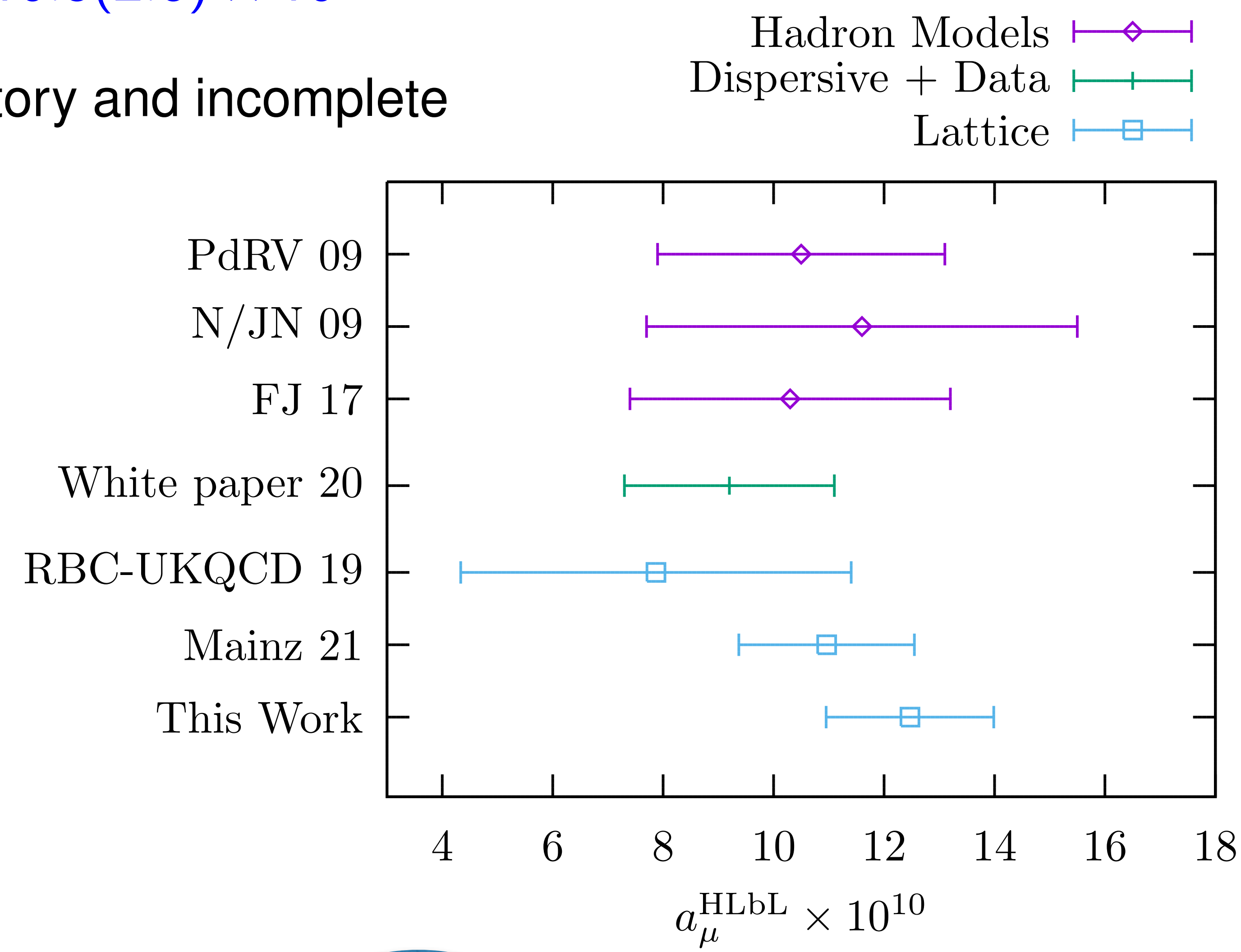


$\rightarrow a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

Hadronic light-by-light



- HLbL much more complicated than HVP, but ultimate precision needed is $\simeq$ 10% instead of $\simeq$ 0.2%
- For many years, only accessible to models of QCD w/ difficult to estimate systematics (Prades et al '09): $a_\mu^{\text{HLbL}} = 10.5(2.6) \times 10^{-10}$
- Also, lattice QCD calculations were exploratory and incomplete
- Tremendous progress in past 5 years:
  - → Phenomenology: rigorous data driven approach [Colangelo, Hoferichter, Kubis, Procura, Stoffer,... '15-'20]
  - → Lattice: first two solid lattice calculations
- All agree w/ older model results but error estimate much more solid and will improve
- Agreed upon average w/ NLO HLbL and conservative error estimates [WP '20]



[Blum et al '23]

NEW

BMWc '24

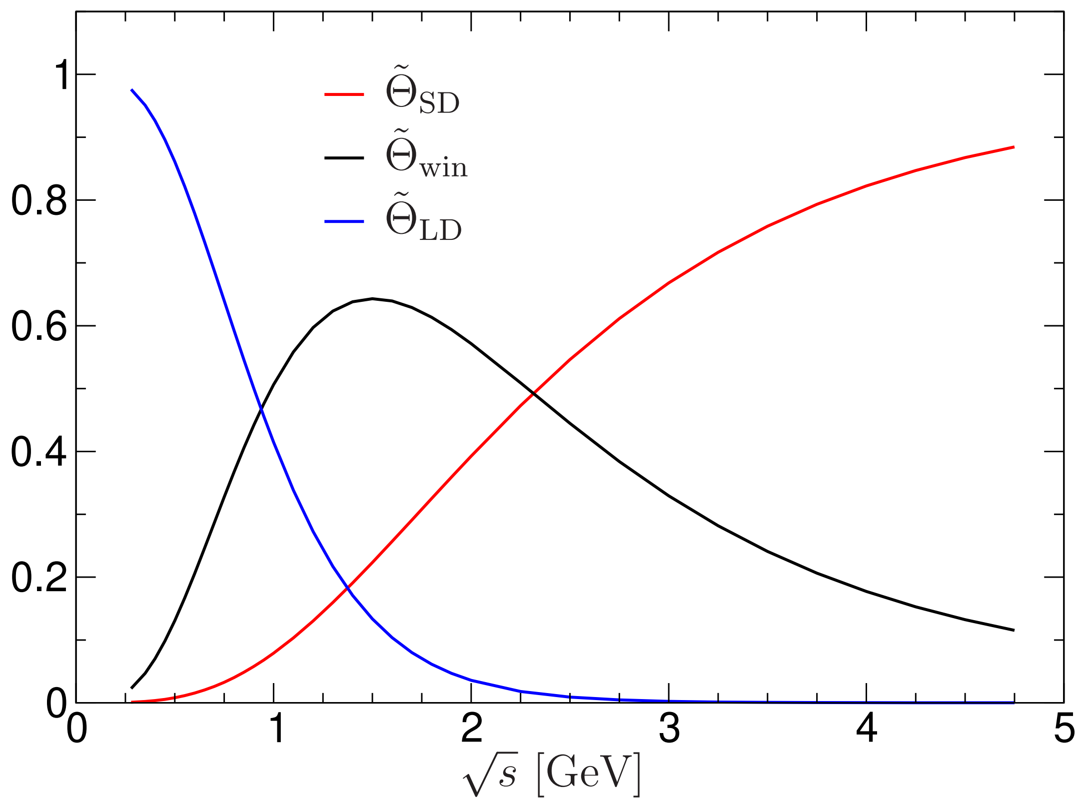
$$a_\mu^{\text{HLbL}} = 12.6(1.2) \times 10^{-10}$$

Comparison with *R*-ratio

$$V(t) = \frac{1}{12\pi^2}\int_{M_{\pi^0}}^{\infty} d(\sqrt{s})\, R(s)\, s\, e^{-\sqrt{s}t} \qquad R(s) = \frac{3s}{4\pi\alpha_{em}^2}\sigma(s, e^+e^- \to hadrons)$$

Insert $V(t)$ into the expression for TMR

$$a_{\mu,win}^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_{M_{\pi^0}}^{\infty} d(\sqrt{s}) R(s)\, \frac{1}{12\pi^2}\, s \int_0^{\infty} dt\, \tilde{f}(t)\, \Theta_{win}(t)\, e^{-\sqrt{s}t}$$



Colangelo *et al.* 2022

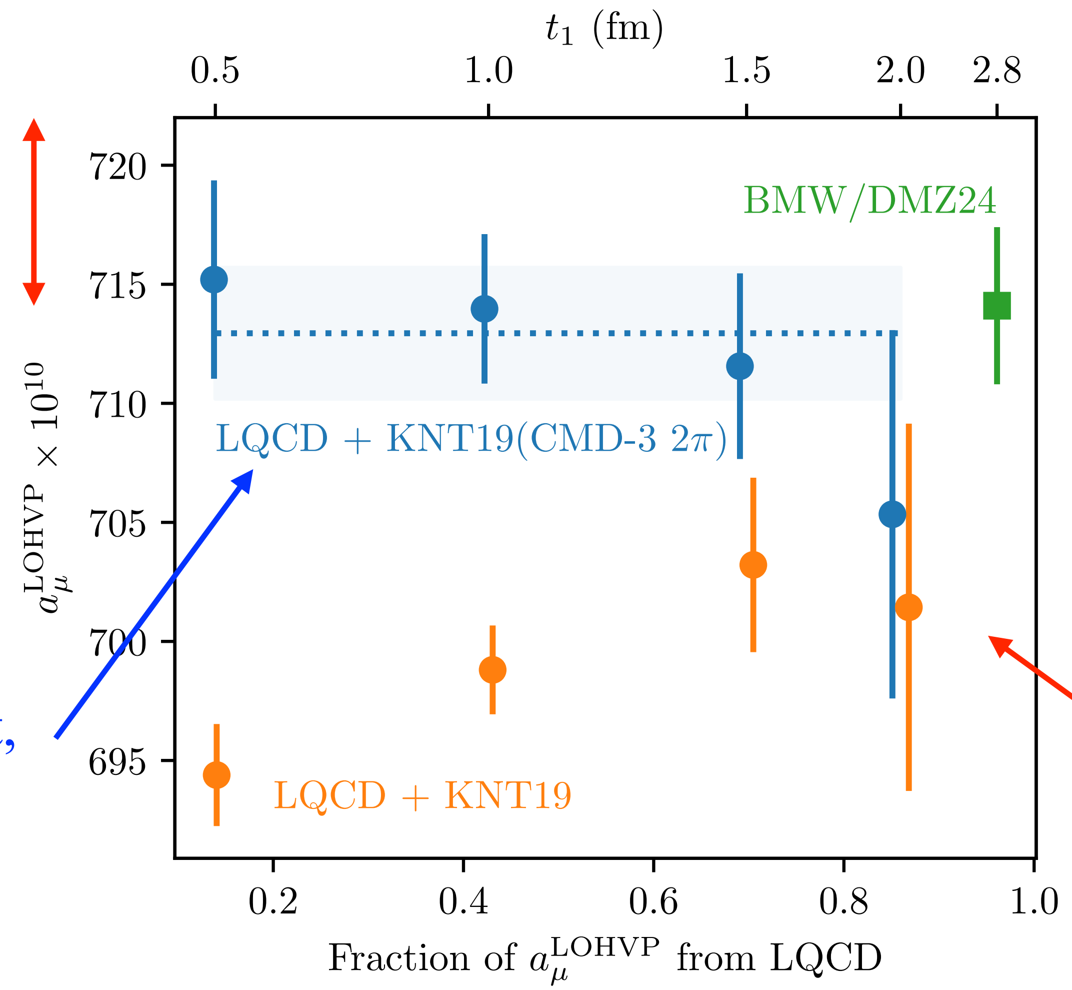
	$a_{\mathrm{SD}}^{\mathrm{HVP}}$	$a_{\mathrm{int}}^{\mathrm{HVP}}$	$a_{\mathrm{LD}}^{\mathrm{HVP}}$	$a_{\mathrm{total}}^{\mathrm{HVP}}$
All channels	68.4(5)	229.4(1.4)	395.1(2.4)	693.0(3.9)
	[9.9%]	[33.1%]	[57.0%]	[100%]
2π below 1.0 GeV	13.7(1)	138.3(1.2)	342.3(2.3)	494.3(3.6)
	[2.8%]	[28.0%]	[69.2%]	[100%]
3π below 1.8 GeV	2.5(1)	18.5(4)	25.3(6)	46.4(1.0)
	[5.5%]	[39.9%]	[54.6%]	[100%]
White Paper [1]	–	–	–	693.1(4.0)
RBC/UKQCD [24]	–	231.9(1.5)	–	715.4(18.7)
BMWc [36]	–	236.7(1.4)	–	707.5(5.5)
BMWc/KNT [7, 36]	–	229.7(1.3)	–	–
Mainz/CLS [99]	–	237.30(1.46)	–	–
ETMC [100]	69.33(29)	235.0(1.1)	–	–

talk by C. Davies @ Lattice 2024

Pragmatic hybrid strategy for further full HVP results

Thanks to A. Keshavarzi and P. Lepage

No new physics in muon g-2



Fit to a constant, χ^2/dof =0.6

Use LQCD in one-sided time window up to t_1 . Add in data-driven result for t_1 to ∞.

Totals should agree for different t_1
- test of validity of data-driven (and LQCD)
- choose smallest error or fit to a constant

Using 2019 FHM LQCD results for one-sided windows (2207.04765):
- totals are flat in t_1 for CMD3 2π
- total w. CMD-3 agrees with BMW/ DMZ '24 for all values of t_1
- newer lattice data have much better uncertainties for $t_1 \gtrsim$ 2fm

Smaller t_1 : reduces lattice stat. and finite vol. error but increases input from data-driven tail

Larger t_1 : CMD3/KNT19 tension falls: <0.3% total HVP for $t_1 \geq 2.5$ fm

Hybrid strategy best to optimise uncertainty on total HVP?