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Unifying Quark and Lepton Flavor Observables through Modular Symmetry

Antonio Marrone, University of Bari and INFN-Bari

based on the work: Gui-Jun Ding, Eligio Lisi, A. M., S. T. Petcov, arXiv:2409.15823

Modular Symmetries introduced in the contest of the flavour problem by F. Feruglio (arXiv:1706.08749)

Motivated by string theory: the modulus τ is related to a torus compactification

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Consider the "Principal congruence group of level N"

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The Quotient group gives the "Finite Modular Group"

$\Gamma_N \equiv SL(2,\mathbb{Z})/\pm\Gamma(N)$ or $\Gamma'_N \equiv SL(2,\mathbb{Z})/\Gamma(N)$

N	2	3	4	5
Γ_N	S_3	A_4	S_4	A_5
Γ'_N	S_3	$A'_4 \equiv T'$	S'_4	A'_5

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Fields transform as $\phi(\tau) \to (c\tau + d)^{-k_\phi} \rho_\phi(\gamma)\, \phi(\tau)$

Weights

Irreps of Γ_N (Γ'_N)

Yukawa couplings are in multiplets transforming as $Y(\tau) \to (c\tau + d)^{k_Y} \rho_Y(\gamma)\, Y(\tau)$

SUSY Superpotential $W \sim \sum \left(Y_{I_1 I_2 \dots I_n} \phi_{I_1} \phi_{I_2} \dots \phi_{I_n} \right)_{\mathbf{1}}$
is invariant under the modular symmetry if

$$\begin{cases} k_Y = k_{\phi_1} + k_{\phi_2} + \ldots + k_{\phi_n} \\ \rho_Y \otimes \rho_{\phi_1} \otimes \rho_{\phi_2} \otimes \cdots \otimes \rho_{\phi_n} \supset \mathbf{1} \end{cases}$$

For a recent review and comprehensive literature see G.J. Ding and S.F King, Rept.Prog.Phys. 87 (2024) 8, 084201

Model based on the 2O group

The group 2O, the binary octahedral group, is a group of order 48 closely related to the octahedral group O, the group of rotational symmetries of a cube (or an octahedron). Specifically, 2O is the double cover of the octahedral group and is isomorphic to S_4

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2O has 8 irreducible representations

- two one-dimensional irreps $\mathbf{1}, \mathbf{1}'$
- three two-dimensional irreps $\mathbf{2}, \hat{\mathbf{2}}, \hat{\mathbf{2}}'$
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Particle content of the model

Three $SU(2)$ lepton doublets $L_i = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}$ $i = e, \mu, \tau$

Three right-handed fermions $E^c = (e^c, \mu^c, \tau^c)$ that are $SU(2)$ singlets

Three right-handed neutrinos N_i^c $i = 1{,}2{,}3$ that are $SU(2)$ singlets

Six right-handed quarks $U^c = (u^c, c^c, t^c)$ and $D^c = (d^c, s^c, b^c)$ that are $SU(2)$ singlets

Three $SU(2)$ quark doublets $Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$ $u_i = u, c, t$ $\quad d_i = d, s, b$

Two Higgs fields, H_u and H_d that are invariant singlets of 2O with zero modular weights.

Representation under the 2O modular group and weights

$$L \sim \mathbf{3}\,,\quad E_D^c \equiv (e^c, \mu^c) \sim \hat{\mathbf{2}}'\,,\quad \tau^c \sim \mathbf{1}'\,,\quad N^c \sim \mathbf{3}$$

$$k_L = -1\,,\quad k_{E_D^c} = 6\,,\quad k_{\tau^c} = 5\,,\quad k_{N^c} = 1$$

$$Q_D \equiv (Q_1, Q_2)^T \sim \mathbf{2}\,,\quad Q_3 \sim \mathbf{1}'\,,\quad U_D^c \equiv (u^c, c^c) \sim \hat{\mathbf{2}}'$$

$$t^c \sim \mathbf{1}'\,,\quad D_D^c \equiv (d^c, s^c) \sim \mathbf{2}\,,\quad b^c \sim \mathbf{1}'$$

$$k_{Q_D} = 3 - k_{U_D^c} = k_{Q_3} = 6 - k_{t^c} = 6 - k_{D_D^c} = -k_{b^c}$$

The Lagrangian of the model contains the most general superpotential with all possible singlets under 2O built with the modular forms organized in various multiplets of different weights

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$$\mathscr{W}_\nu = g H_u (N^c L)_{\mathbf{1}} + \Lambda \left(N^c N^c\right)_{\mathbf{2}} Y_{\mathbf{2}}^{(2)}$$ neutrinos

$$\mathscr{W}_E = g_1^E \left(E_D^c L\right)_{\hat{\mathbf{2}}'} Y_{\hat{\mathbf{2}}'}^{(5)} H_d + g_2^E \left(E_D^c L\right)_{\hat{\mathbf{4}}} Y_{\hat{\mathbf{4}}}^{(5)} H_d + g_3^E \left(\tau^c L\right)_{\mathbf{3}'} Y_{\mathbf{3}'}^{(4)} H_d$$ charged leptons

$$\mathscr{W}_u = g_1^u \left(U_D^c Q_D\right)_{\hat{\mathbf{4}}} Y_{\hat{\mathbf{4}}}^{(3)} H_u + g_2^u \left(t^c Q_D\right)_{\mathbf{2}} Y_{\mathbf{2}}^{(6)} H_u + g_3^u \left(t^c Q_3\right)_{\mathbf{1}} Y_{\mathbf{1}}^{(6)} H_u$$ up - quarks

$$\mathscr{W}_d = g_1^d \left(D_D^c Q_D\right)_{\mathbf{1}} Y_{\mathbf{1}}^{(6)} H_d + g_2^d \left(D_D^c Q_D\right)_{\mathbf{1}'} Y_{\mathbf{1}'}^{(6)} H_d + g_3^d \left(D_D^c Q_D\right)_{\mathbf{2}} Y_{\mathbf{2}}^{(6)} H_d + g_4^d \left(D_D^c Q_3\right)_{\mathbf{2}} Y_{\mathbf{2}}^{(6)} H_d + g_5^d \left(b^c Q_3\right)_{\mathbf{1}} H_d$$ down - quarks

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Each term multiplied by an unknown real (gCP invariance) parameter

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Light neutrino mass matrix is derived by the seesaw formula

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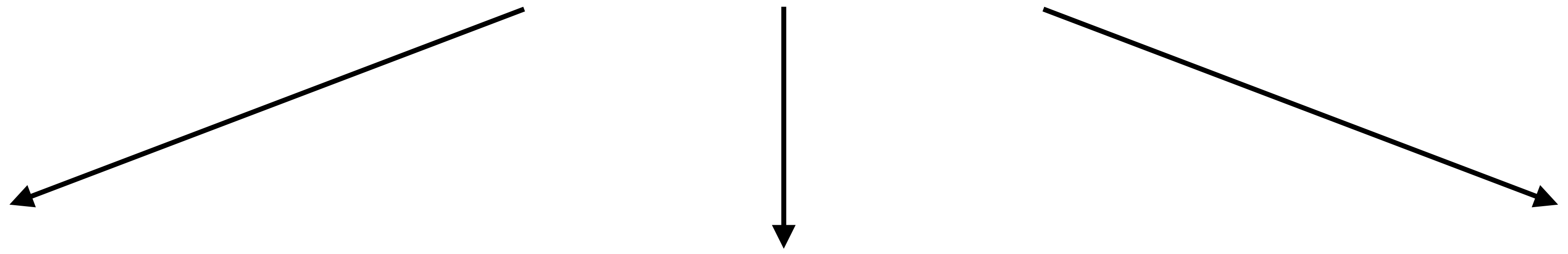
The model contains 14 unknowns



The modulus τ
2 real parameters

Lepton sector
4 real parameters

Quark sector
8 real parameters

Minimal model in the modular form literature so far

Experimental inputs and model parameters

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Model should satisfy 18 experimental constraints (only Normal Ordering of neutrino masses allowed)

$$\mathrm{O}_i = \begin{cases} \sin^2\theta_{12}, \sin^2\theta_{23}, \sin^2\theta_{13} & i = 1,2,3 & \nu \text{ mixing,} \\ \delta m^2, \Delta m^2 & i = 4,5 & \nu \text{ masses,} \\ r_{e\mu}, r_{\mu\tau}, m_\tau & i = 6,7,8 & \text{charged lepton masses,} \\ \theta^q_{12}, \theta^q_{23}, \theta^q_{13}, \delta^q_{CP} & i = 9,10,11,12 & \text{quark mixing,} \\ r_{uc}, r_{ct}, r_{ds}, r_{sb}, m_t, m_b & i = 13,\ldots,18 & \text{quark masses.} \end{cases}$$

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10 for quarks (i = 9–18)

charged fermion masses and quark mixing parameters extrapolated to the GUT scale

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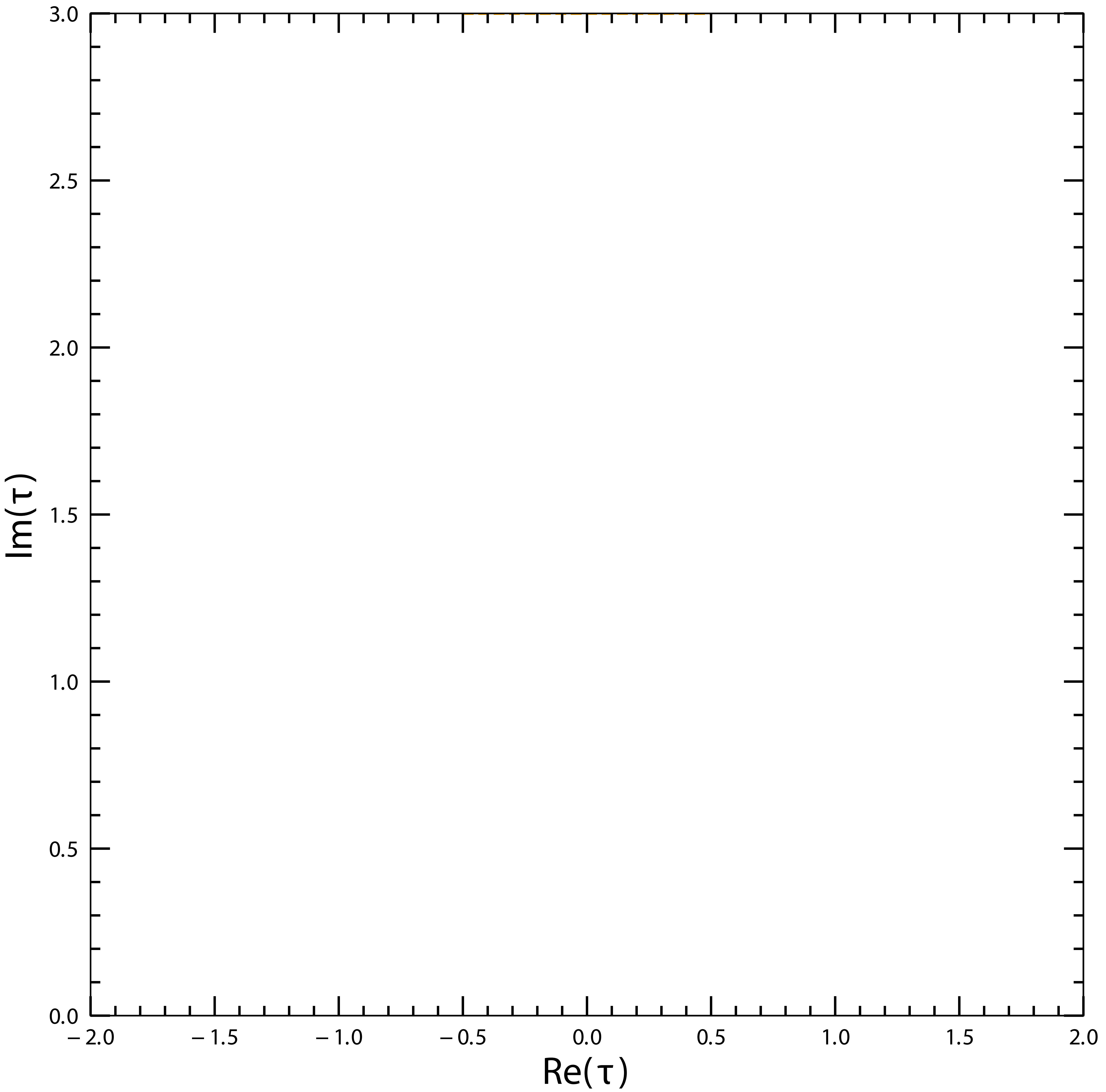
charged fermion masses and quark mixing parameters extrapolated to the GUT scale

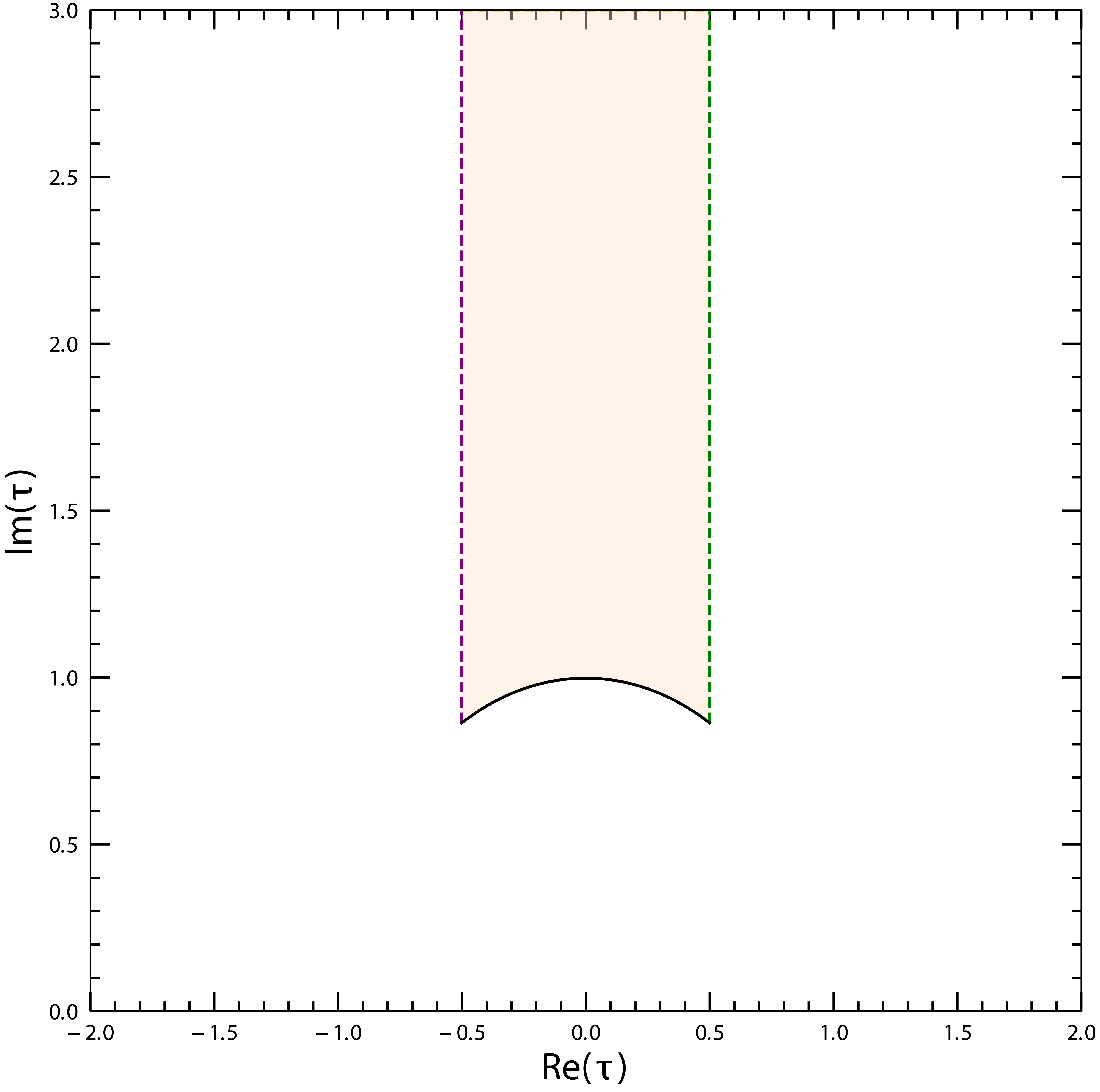
The Lagrangian of the model contains 14 real parameters, 6 for leptons, 10 for quarks, with τ in common

$$
\mathbf{P}_{\text{leptons}} = \left(\tau, g_1^E\, v_d, \frac{g_2^E}{g_1^E}, \frac{g_3^E}{g_1^E}, \frac{g\, v_u}{\sqrt{\Lambda}}\right),
$$

$$
\mathbf{P}_{\text{quarks}} = \left(\tau, g_1^u\, v_u, \frac{g_2^u}{g_1^u}, \frac{g_3^u}{g_1^u}, g_1^d\, v_d, \frac{g_2^d}{g_1^d}, \frac{g_3^d}{g_1^d}, \frac{g_4^d}{g_1^d}, \frac{g_5^d}{g_1^d}\right)
$$

Allowed regions in the τ plane
(contours at 1σ,2σ,3σ)





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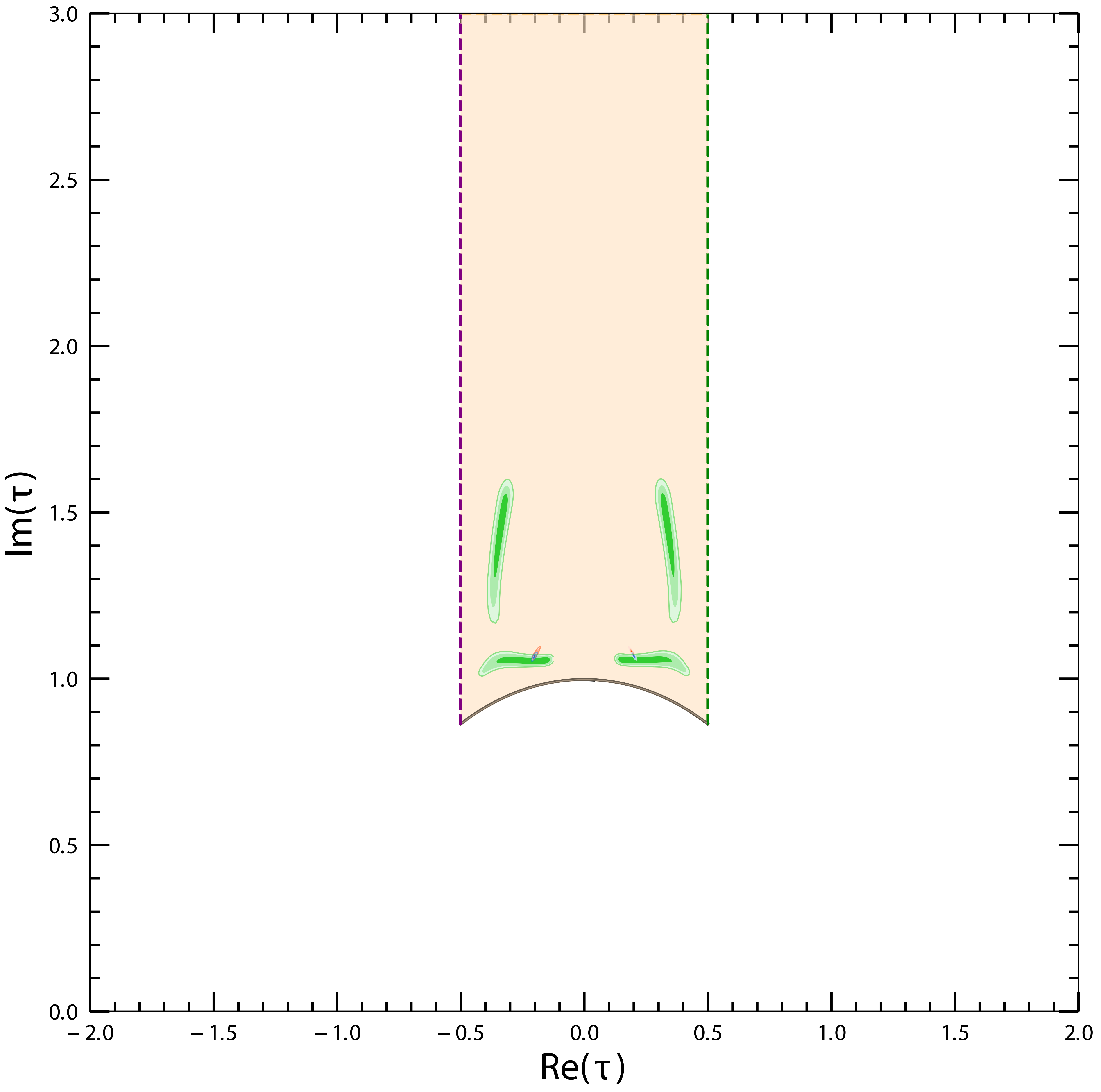
Explore the fundamental domain of τ

$$\mathscr{F} \ : \ \ |\mathrm{Re}(\tau)| \leq \frac{1}{2} \ , \ \ \mathrm{Im}(\tau) > 0 \ , \ \ |\tau| \geq 1 \ .$$



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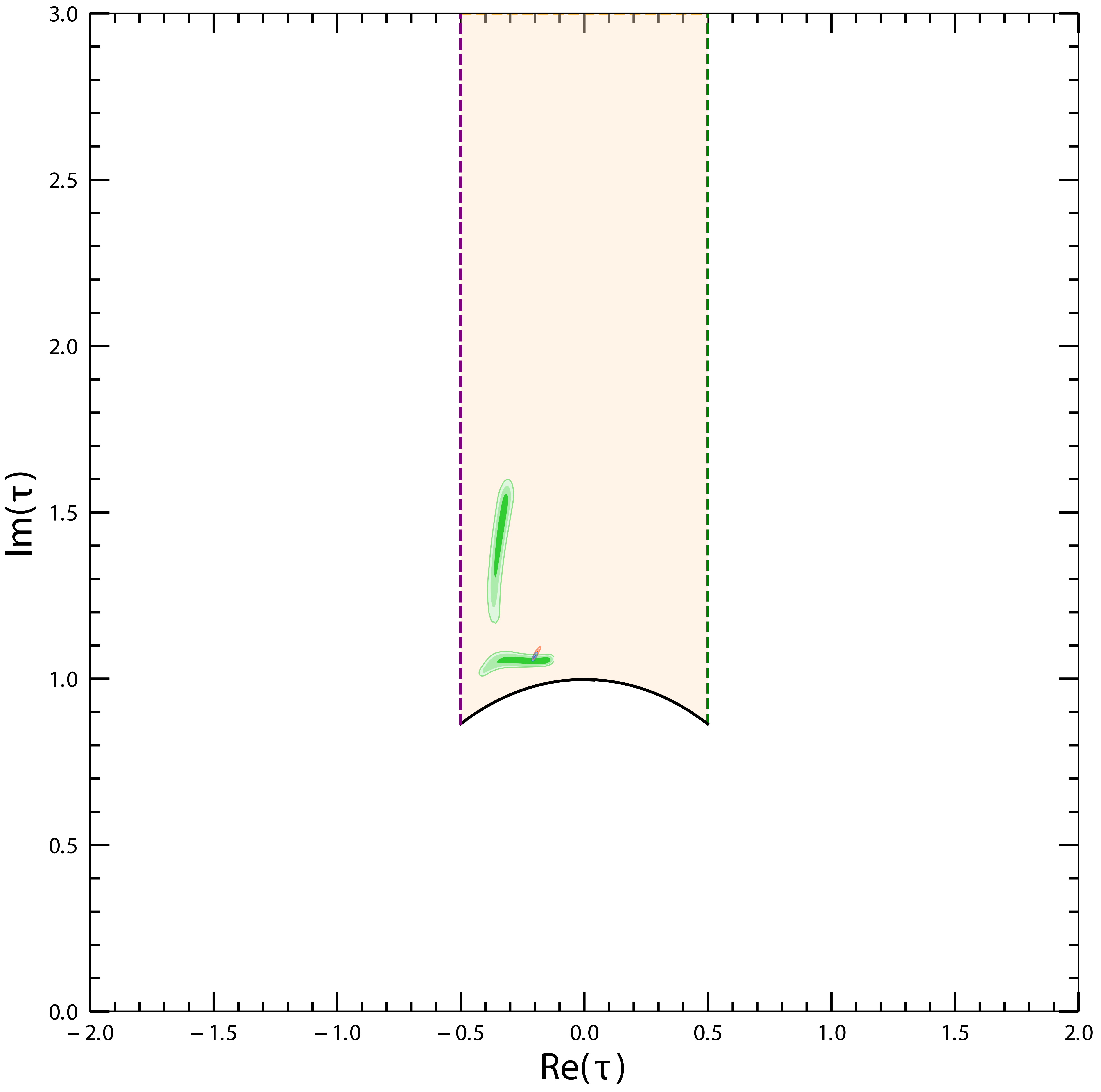
We perform the analysis in three steps:

- Quark only → large green regions
- Lepton only → smaller red regions
- Combined quarks+leptons → small blue regions



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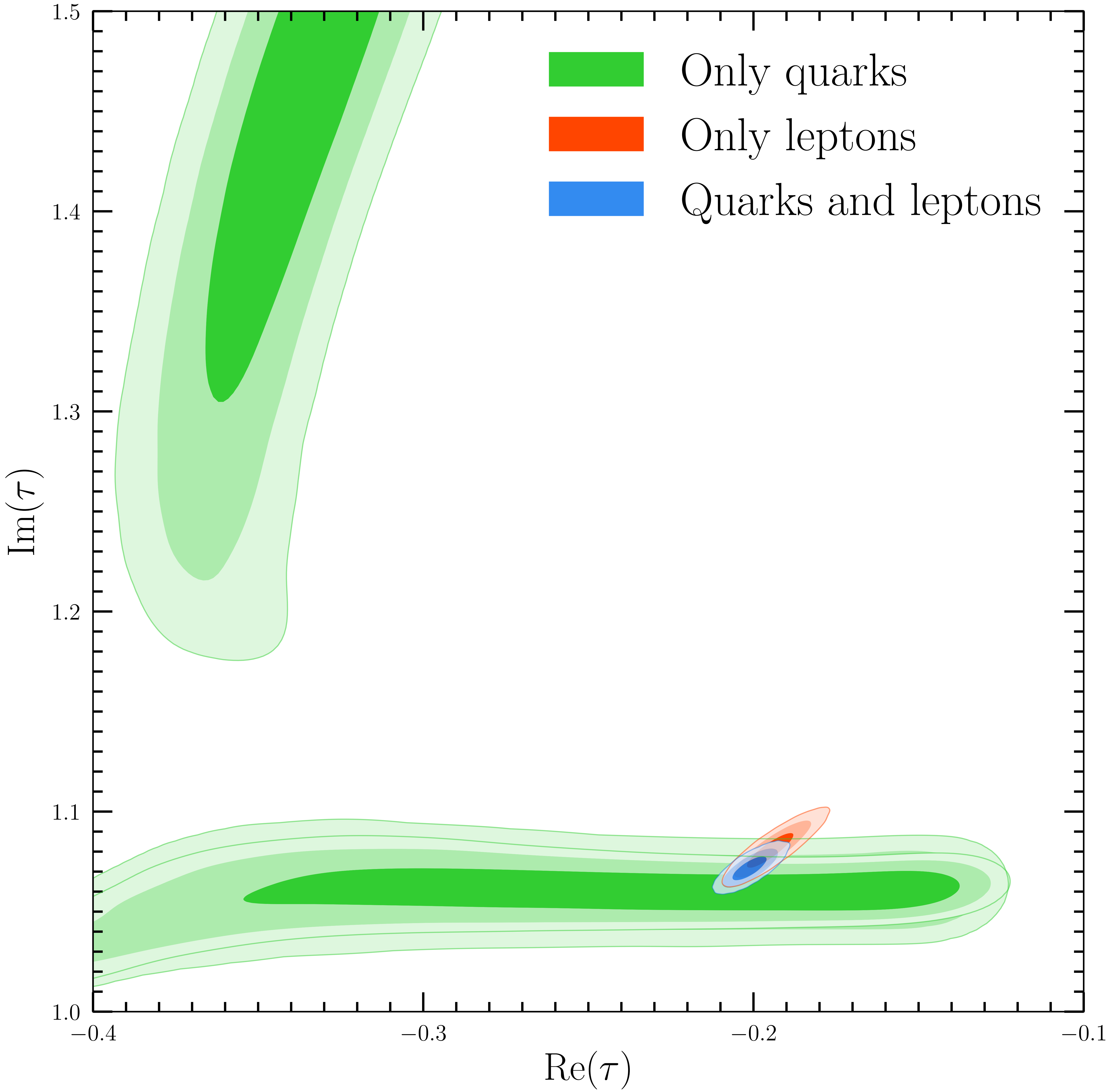
global CP invariance ⇒ symmetry with respect to the imaginary τ axis



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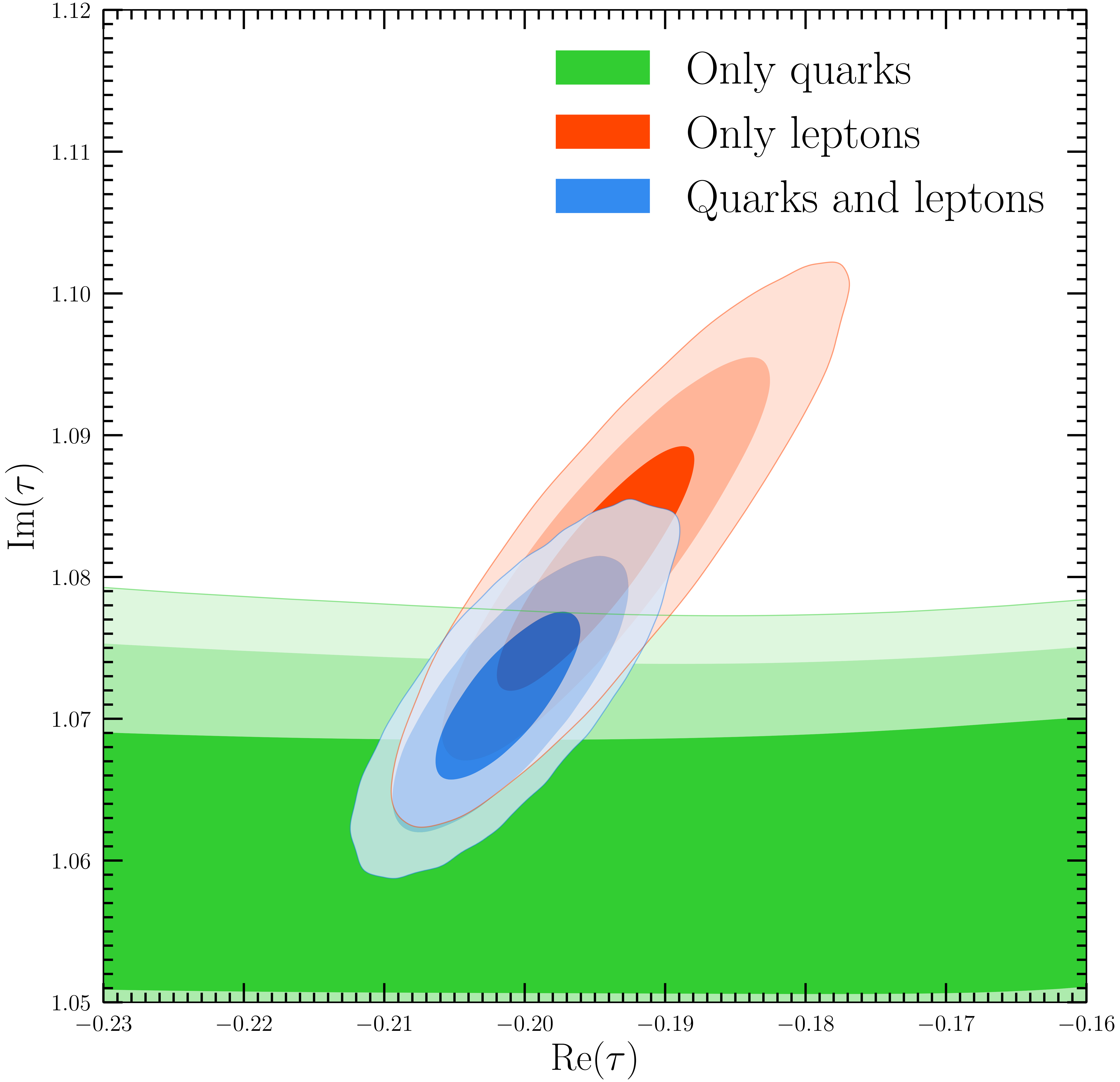
Only in the lower left quark allowed region there is an overlap



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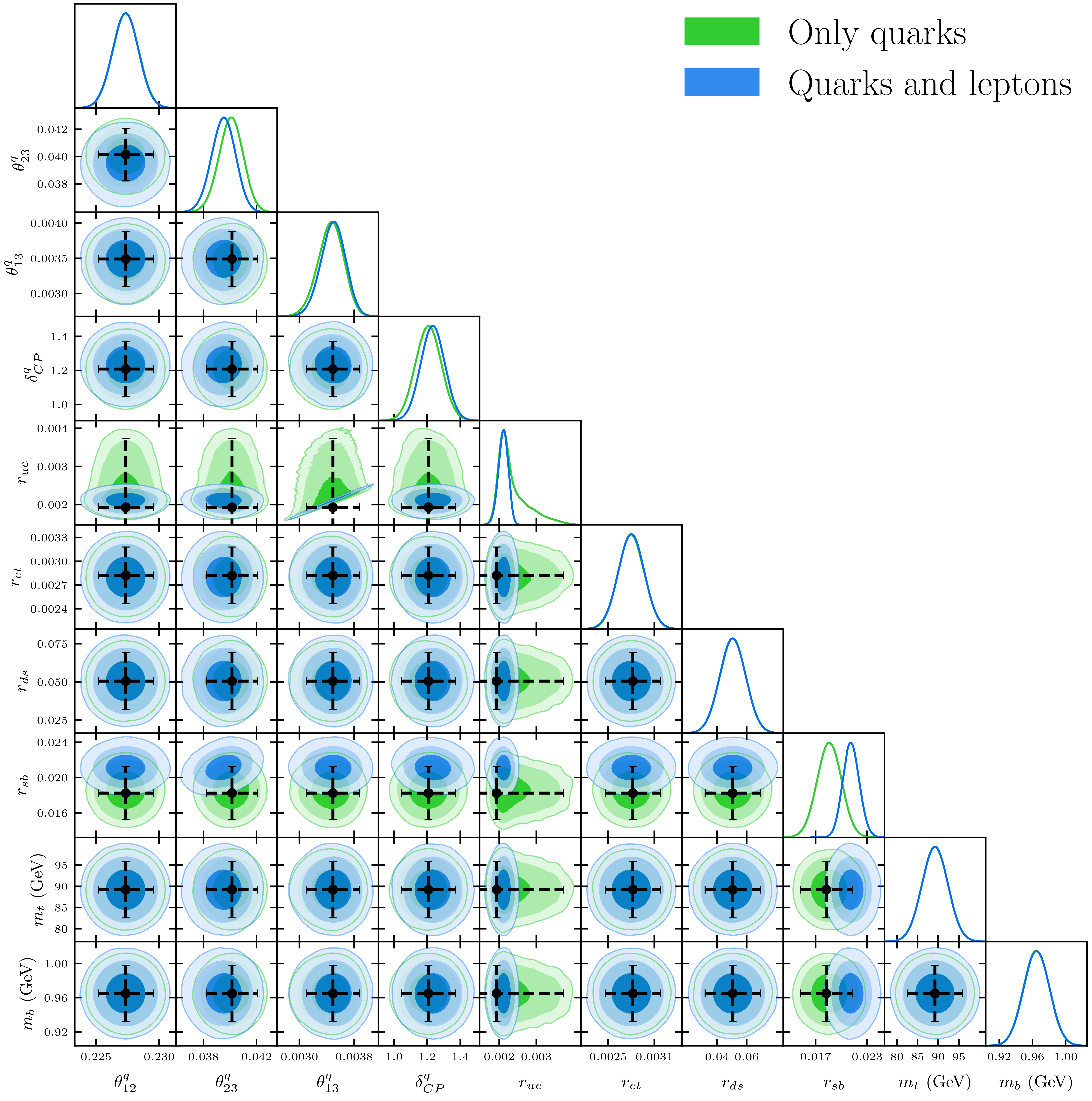
Combined analysis result is not the superposition of the quark and lepton separate allows regions

Observables in the quark sector



Observables in the quark sector

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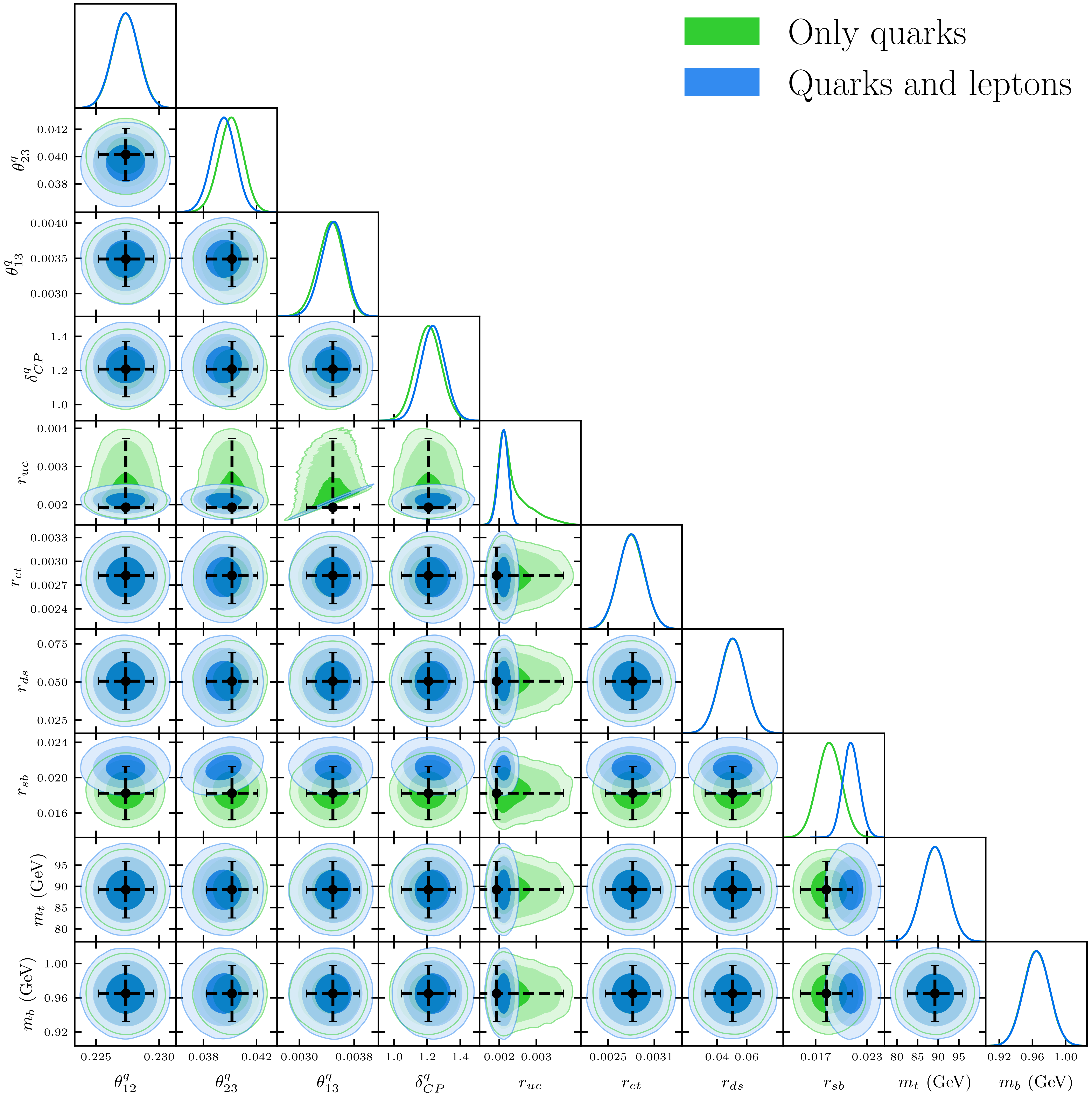
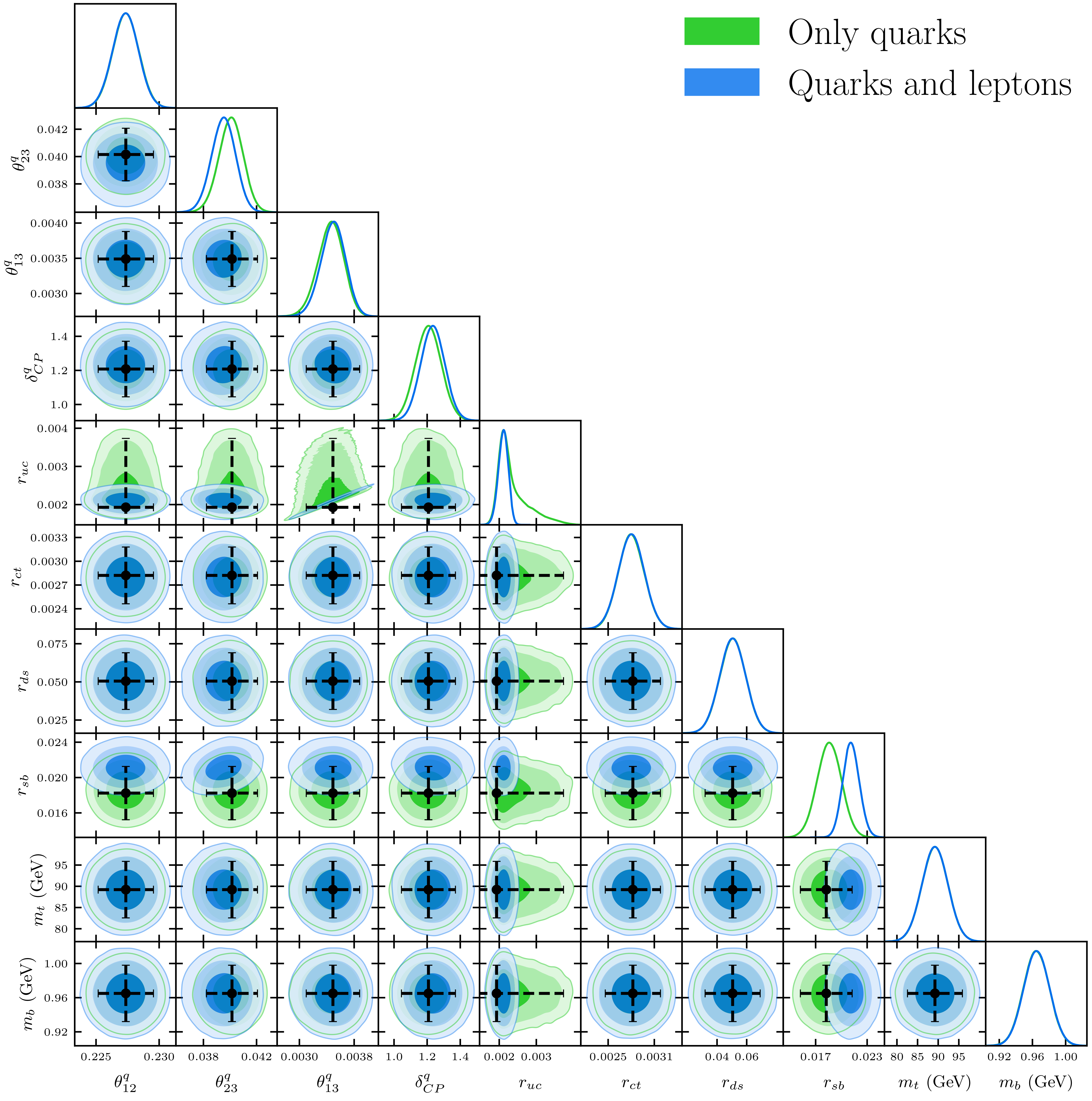


Points with error bars are the experimental measured values and the 3σ errors



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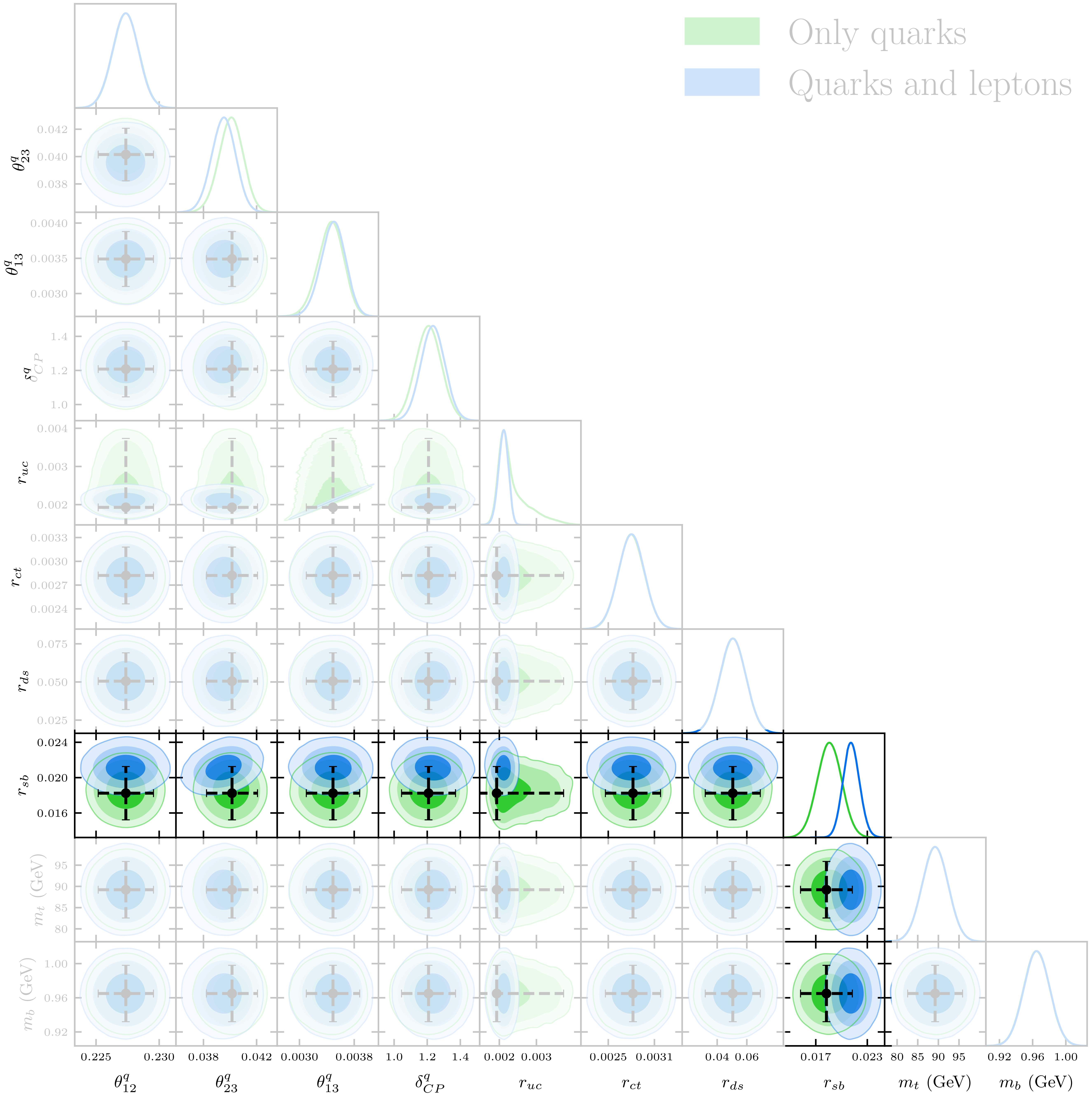
Quark only analysis (green) in perfect agreement



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Quark only analysis (green) in perfect agreement

Combined analysis (blue): slight tension (3σ) for $r_{sb} = m_s/m_b$

Observables in the lepton sector

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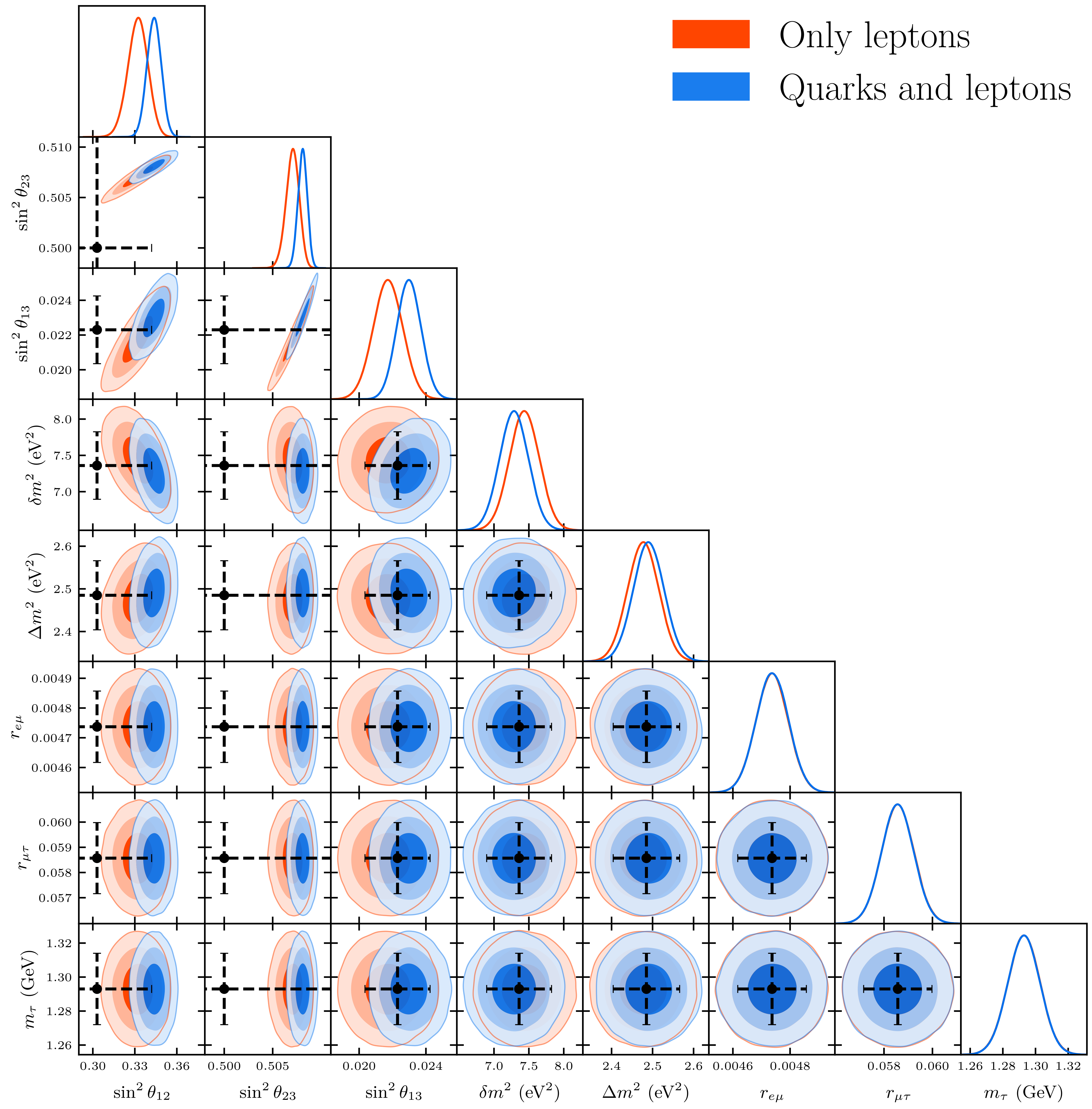

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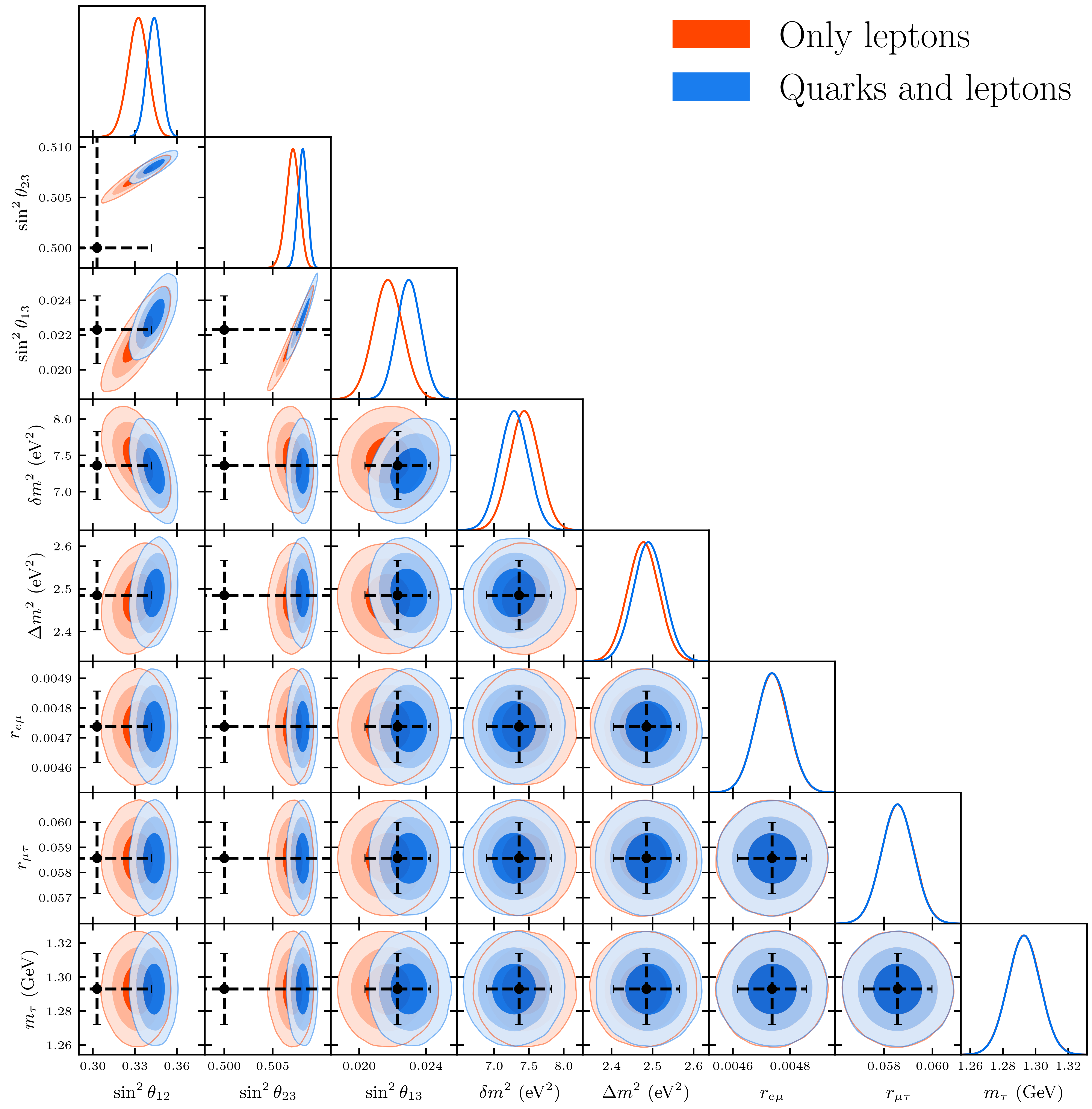
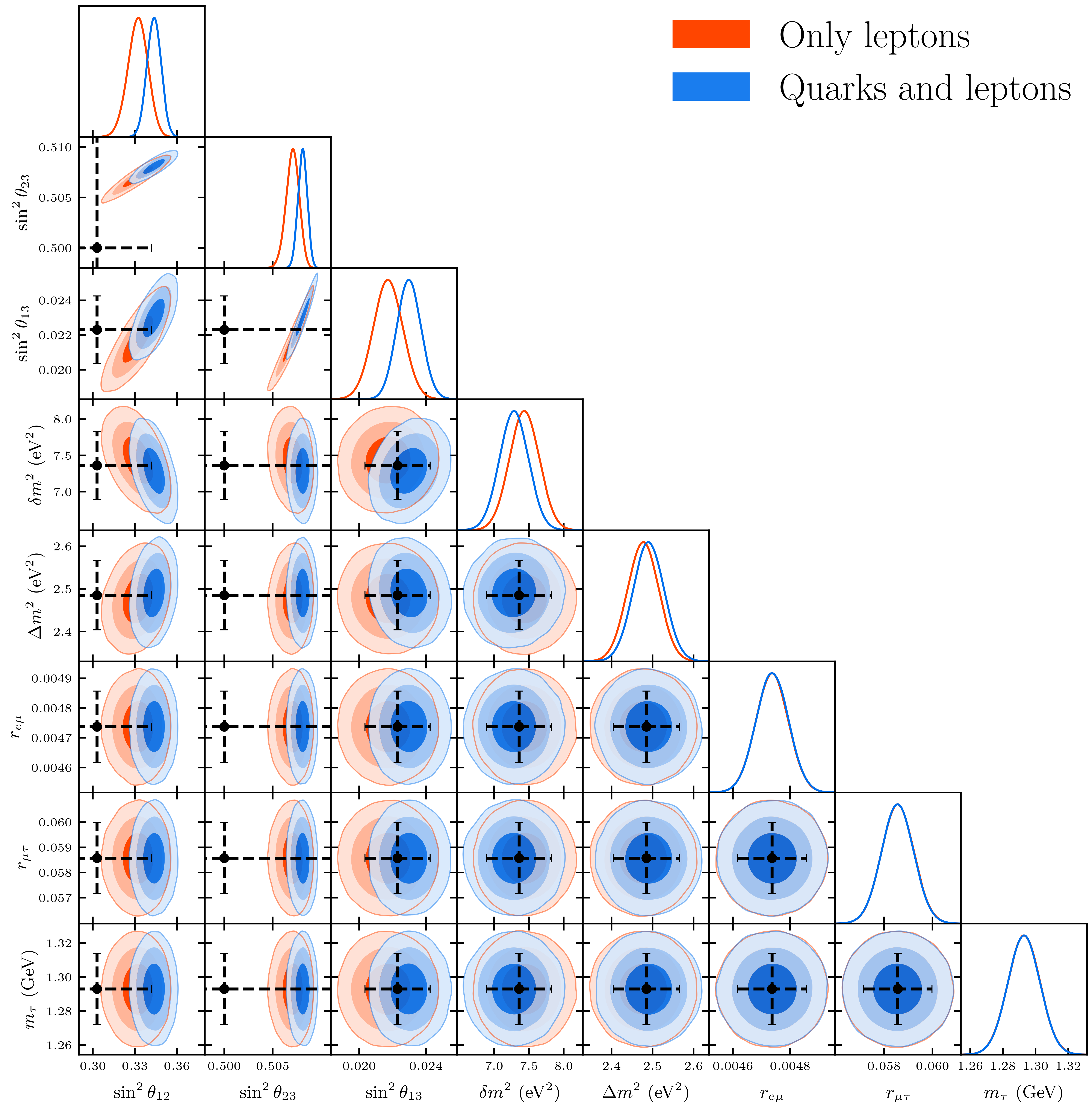


Lepton only (red) and combined (blue) analyses in perfect agreement with experimental results on $(\Delta m^2, r_{e\mu}, r_{\mu\tau}, m_\tau)$



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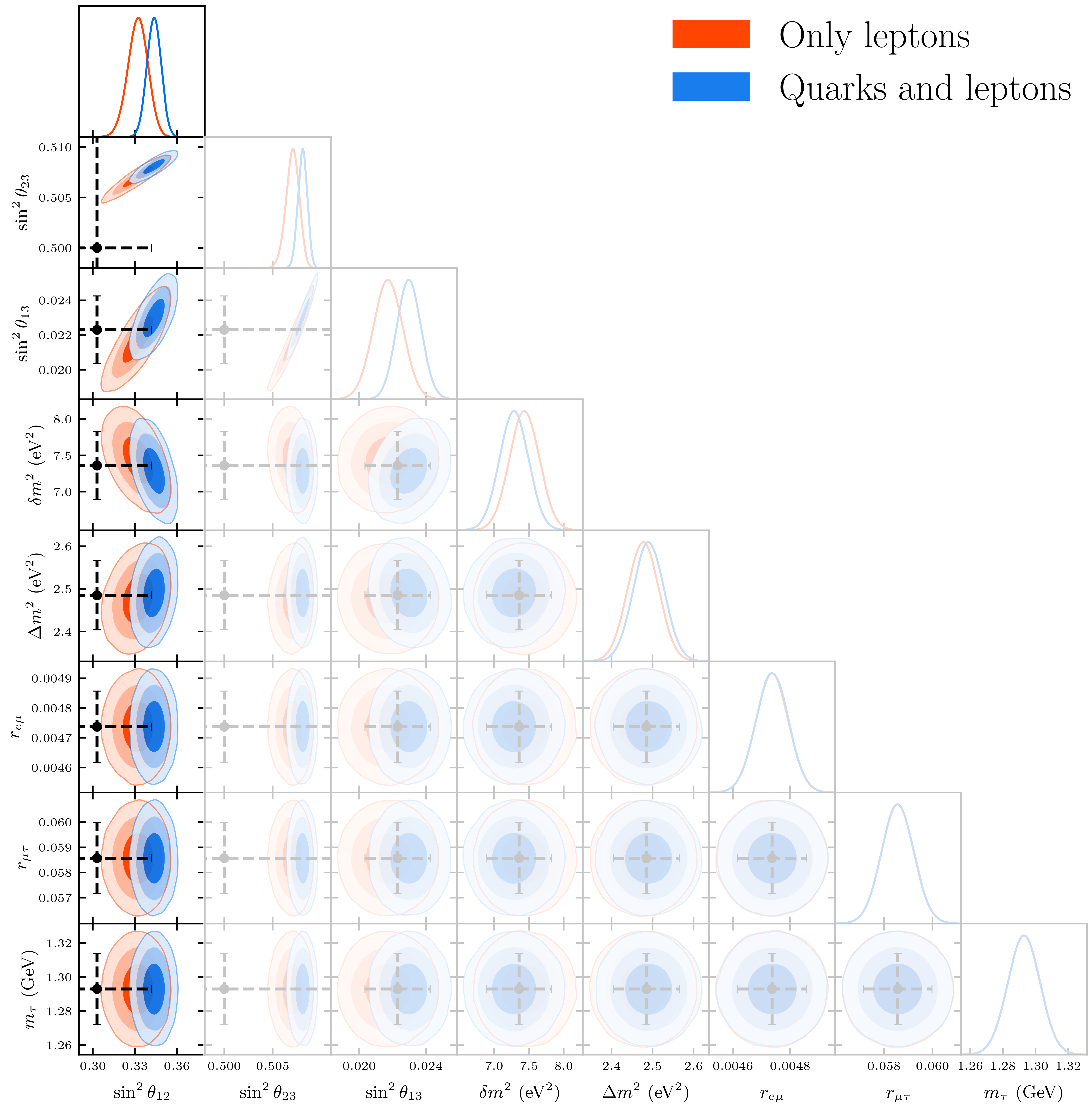
For the "atmospheric" mixing $\sin^2\theta_{23}$, due to the octant ambiguity still persistent in experimental measurements, we fix the experimental value at 0.5 with the 1σ error equal to 1/6 of the 3σ allowed region

Given the error on $\sin^2\theta_{23}$, $\sigma_{\exp} \sim 0.1$, the model prediction on the atmospheric mixing angle is well inside the experimentally allowed 1σ region



Observables in the lepton sector

Lepton only (red) and combined (blue) analyses in perfect agreement with experimental results on $(\Delta m^2, r_{e\mu}, r_{\mu\tau}, m_\tau)$

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Slight tension ($\sim 3\sigma$) for $\sin^2\theta_{12}$

How good is the Fit?

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	Combined		**Leptons only**		**Quarks only**	
Observable	Best fit	χ^2 breakdown	Best fit	χ^2 breakdown	Best fit	χ^2 breakdown
$\sin^2\theta_{12}$	0.344	10.2	0.329	3.99	-	-
$\sin^2\theta_{23}$	0.508	0.0714	0.506	0.0438	-	-
$\sin^2\theta_{13}$	0.0231	1.41	0.0219	0.462	-	-
δm^2 (eV^2)	7.28×10^{-5}	0.246	7.43×10^{-5}	0.183	-	-
Δm^2 (eV^2)	0.00249	0.0636	0.00248	0.0497	-	-
$r_{e\mu}$	0.00474	2.60×10^{-5}	0.00474	8.18×10^{-6}	-	-
$r_{\mu\tau}$	0.0586	4.66×10^{-5}	0.0586	4.66×10^{-8}	-	-
m_τ (GeV)	1.29	5.52×10^{-8}	1.29	2.94×10^{-9}	-	-
$\min(\chi^2_{\text{leptons}})$	11.94		4.74		-	
θ^q_{12}	0.227	5.68×10^{-4}	-	-	0.227	1.52×10^{-7}
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δ^q_{CP}	1.23	0.221	-	-	1.21	1.63×10^{-8}
r_{uc}	0.00212	0.100	-	-	0.00196	0.0024
r_{ct}	0.00282	2.61×10^{-6}	-	-	0.00282	2.90×10^{-8}
r_{ds}	0.0505	5.84×10^{-5}	-	-	0.0505	3.08×10^{-8}
r_{sb}	0.0209	7.12	-	-	0.0183	2.62×10^{-5}
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All observables very well fitted

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The two observables $\sin^2\theta_{12}$ and r_{sb} contribute with $\sim$ 17.3 units to the total χ^2.

Benefits of a true combined Quarks + Leptons Analysis

Benefits of a true combined Quarks + Leptons Analysis

Modulus

$$\mathscr{L}_{\text{mass}} = \mathscr{L}_{\text{mass}}\left(\text{Re}(\tau), \text{Im}(\tau), \; g_1^E v_d, \frac{g_2^E}{g_1^E}, \frac{g_3^E}{g_1^E}, \frac{g\, v_u}{\sqrt{\Lambda}}, \; g_1 v_u, \frac{g_2^u}{g_1^u}, \frac{g_3^u}{g_1^u}, g_1^d v_d, \frac{g_2^d}{g_1^d}, \frac{g_3^d}{g_1^d}, \frac{g_4^d}{g_1^d}, \frac{g_5^d}{g_1^d}\right)$$

Leptons

Quarks

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Leptons

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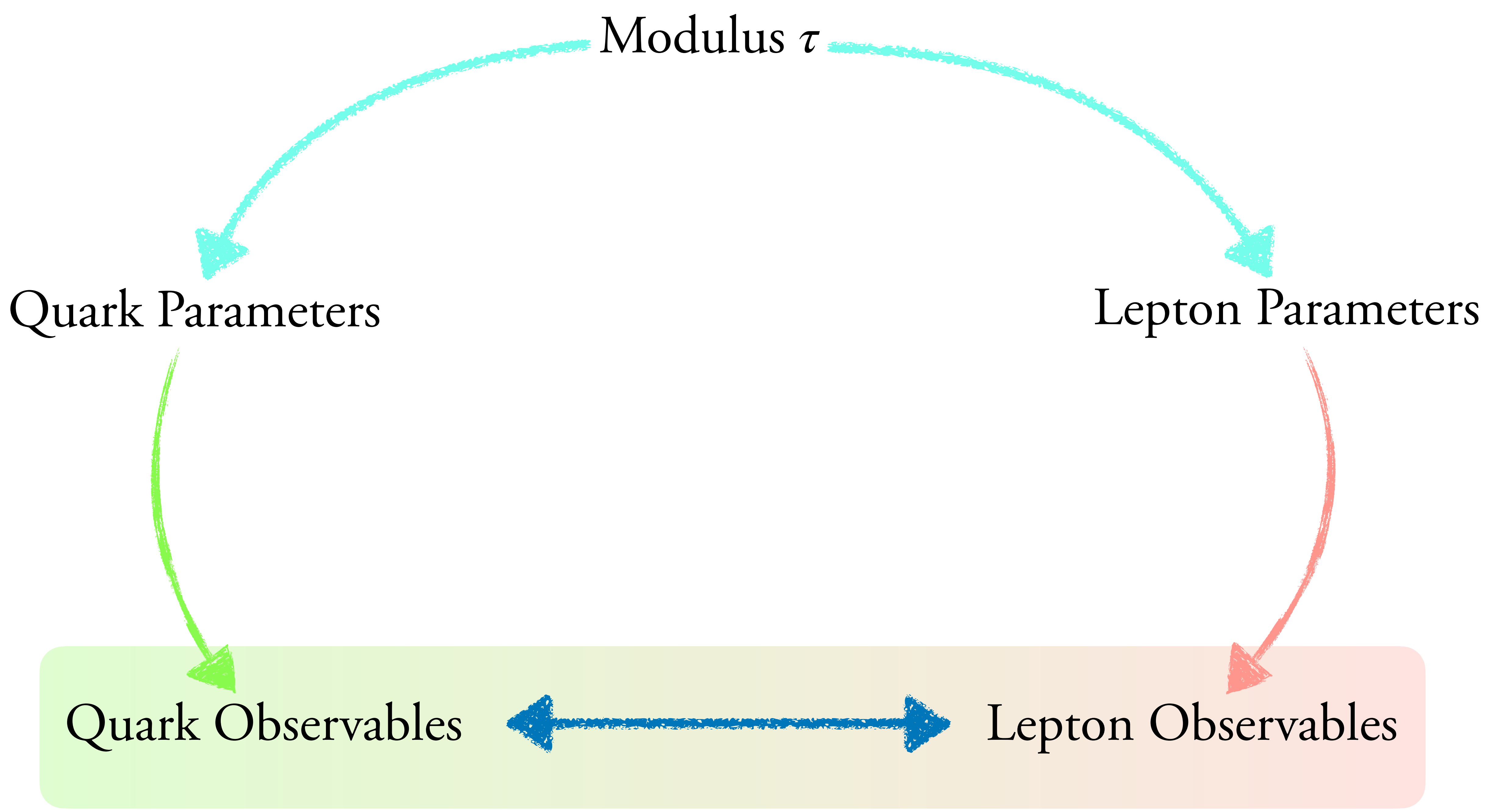

The modulus τ a common parameter of the quark and lepton sectors

The parameters of the models in the two sectors are correlated to τ. Though their correlation to the model parameters observables of the two sectors become correlated

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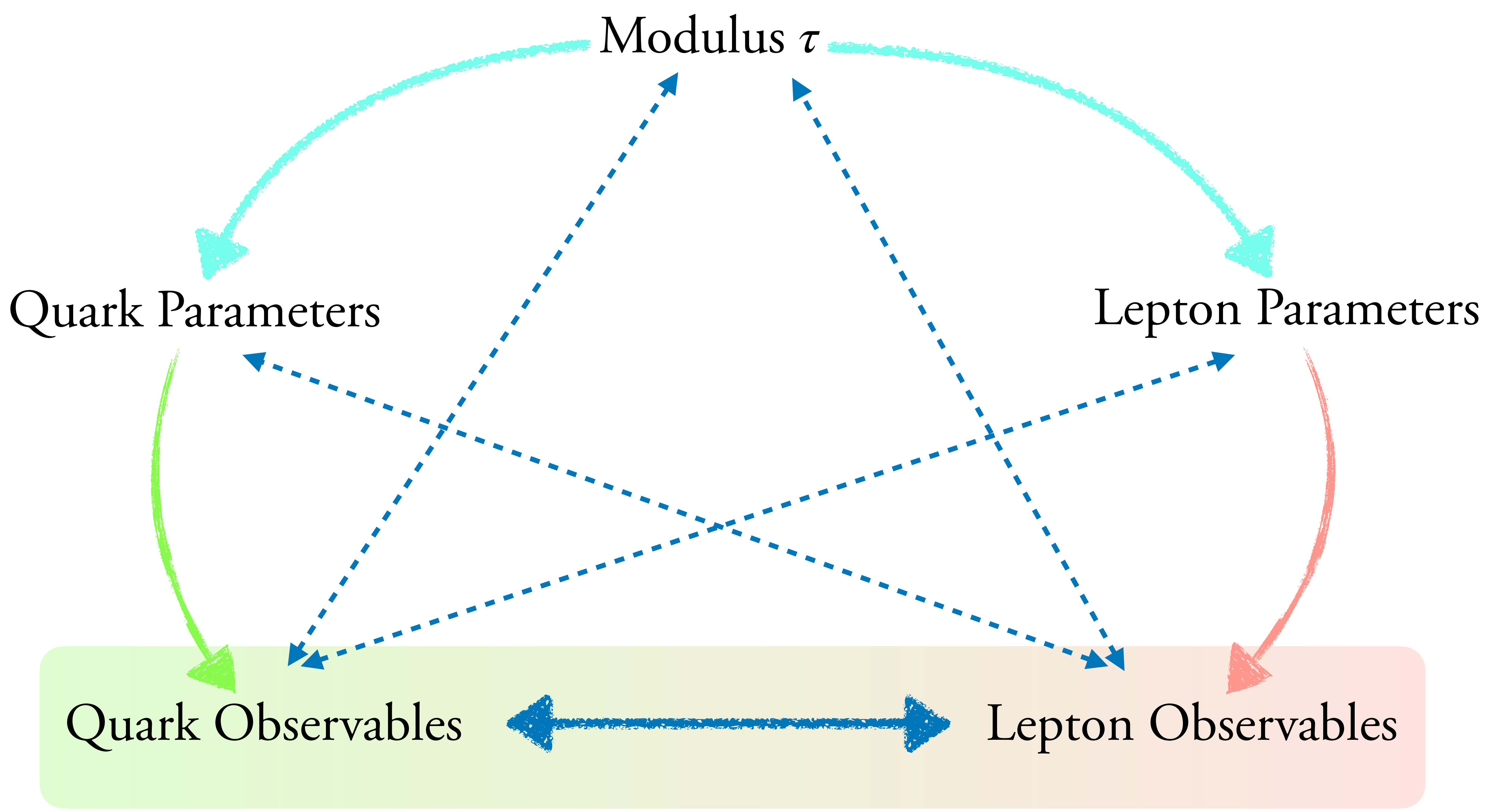

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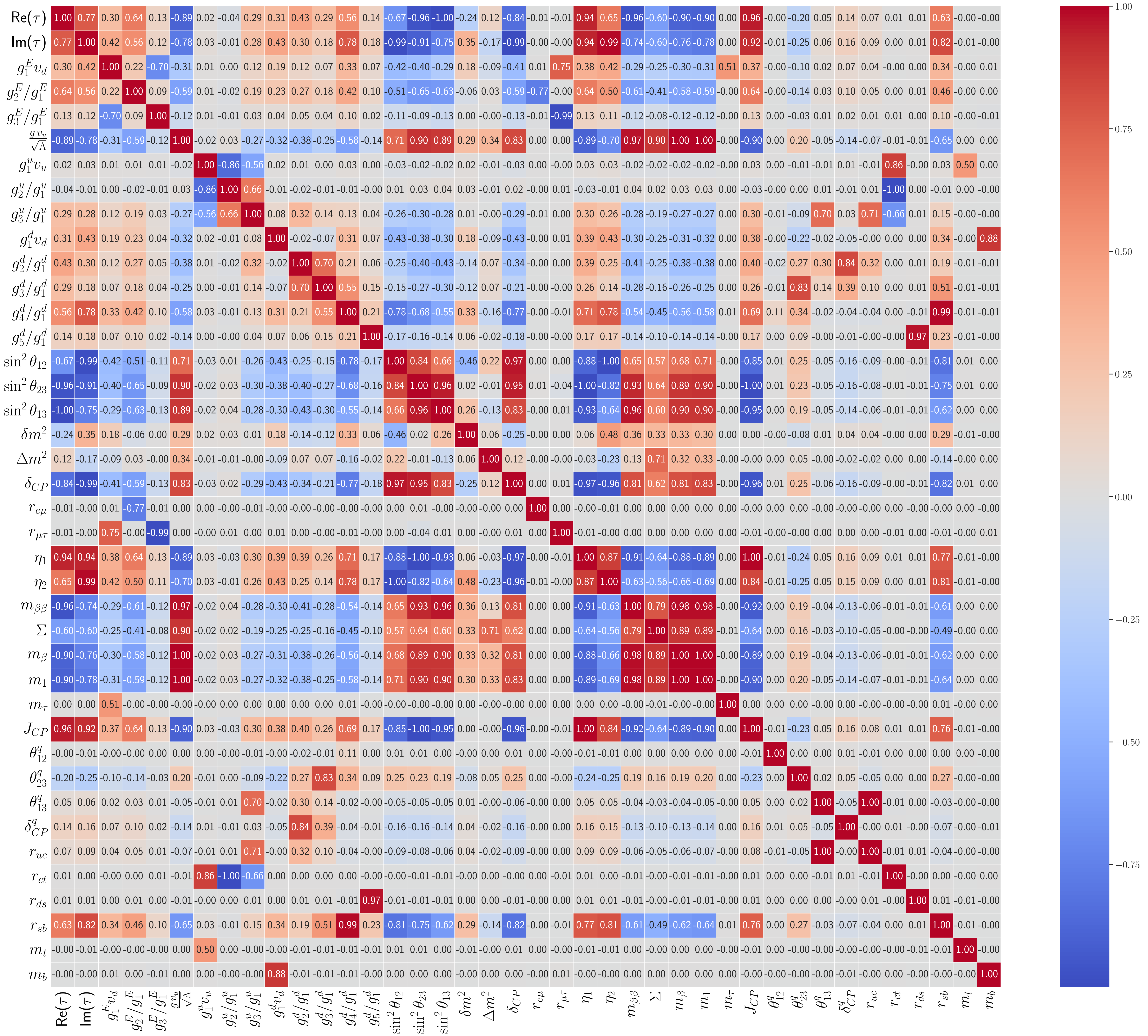
Actually, correlations can be studied in many different ways

Correlations



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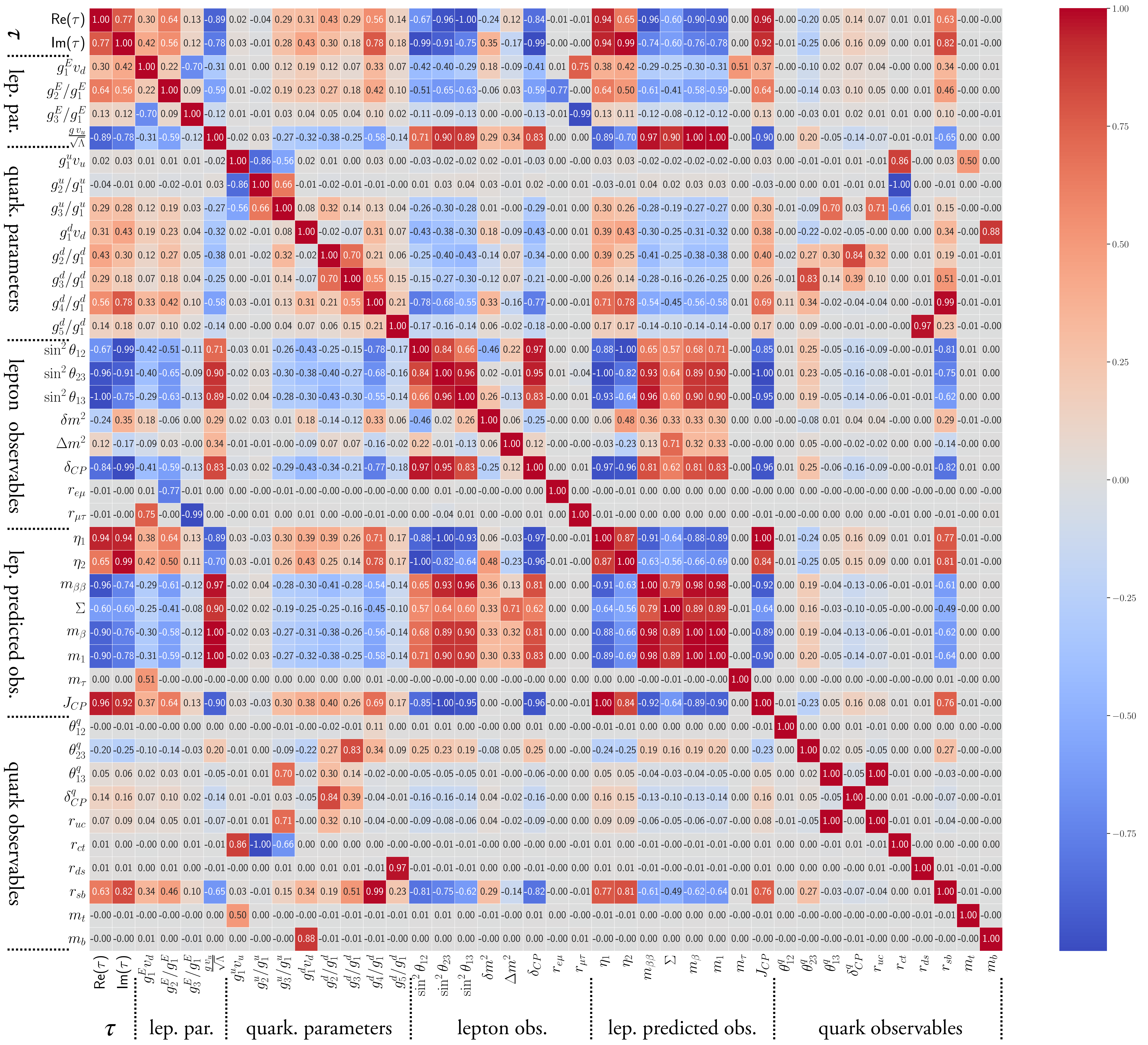
New information from a true combined analysis



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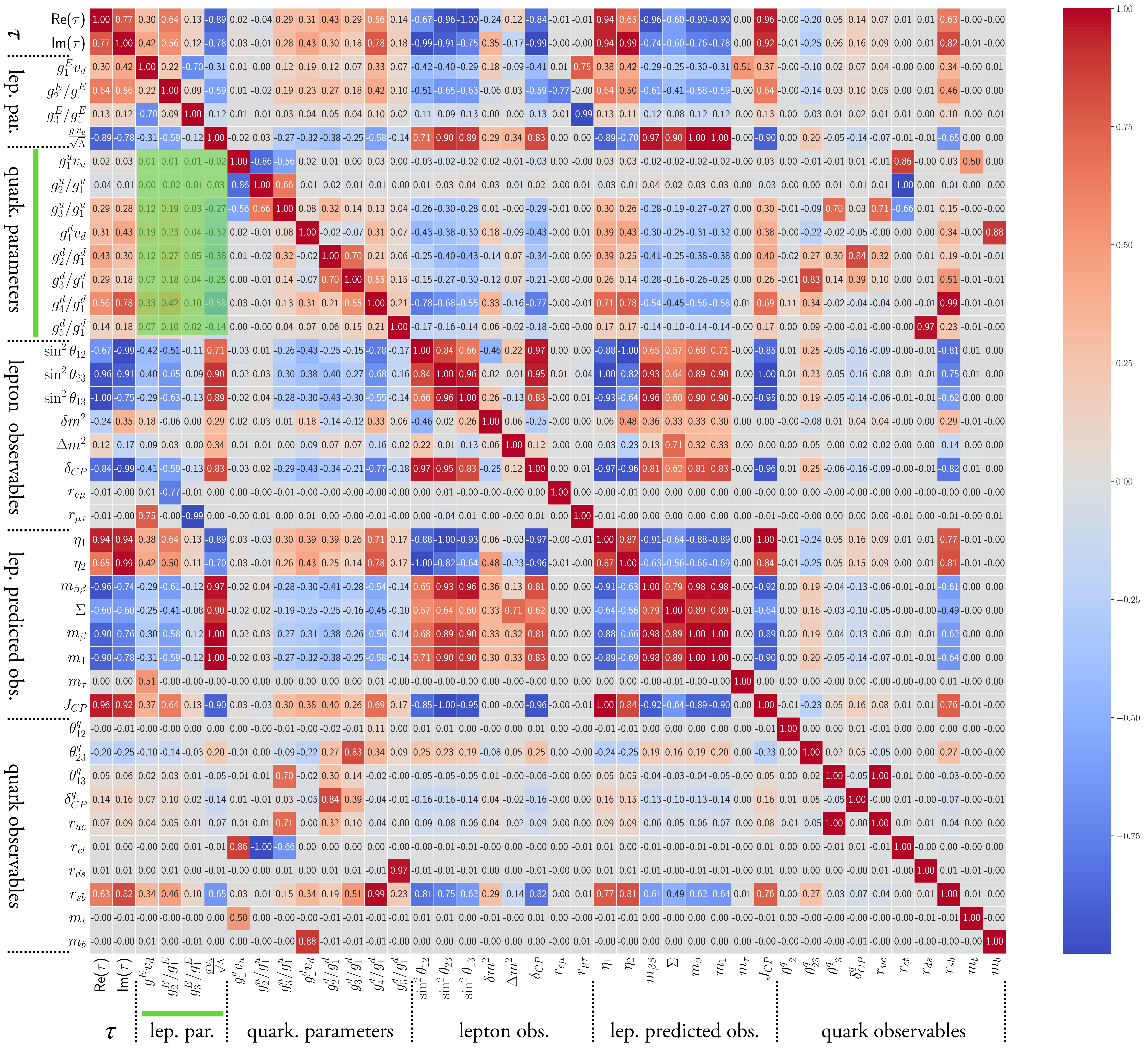
Correlations between lepton and quark parameters



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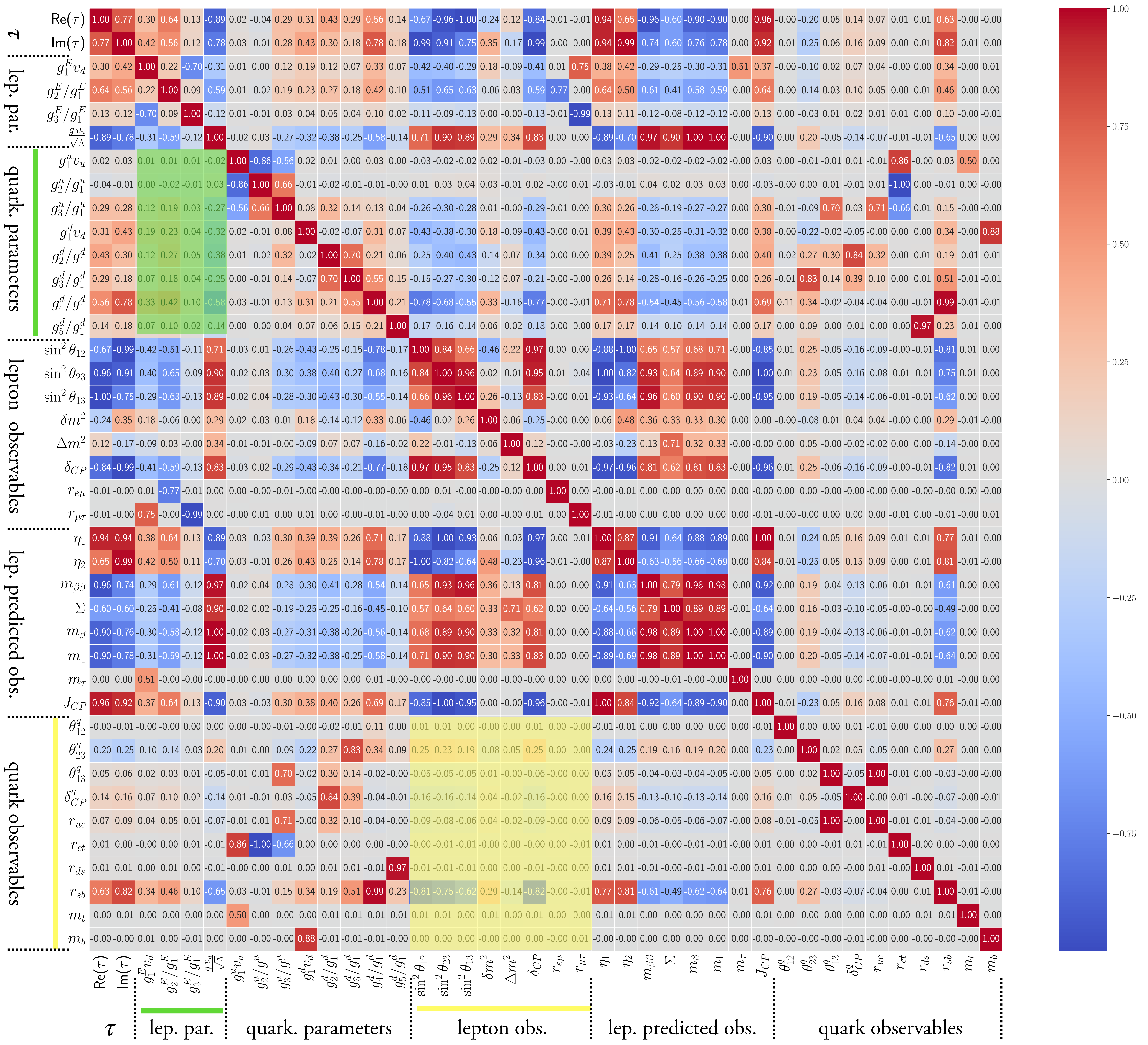

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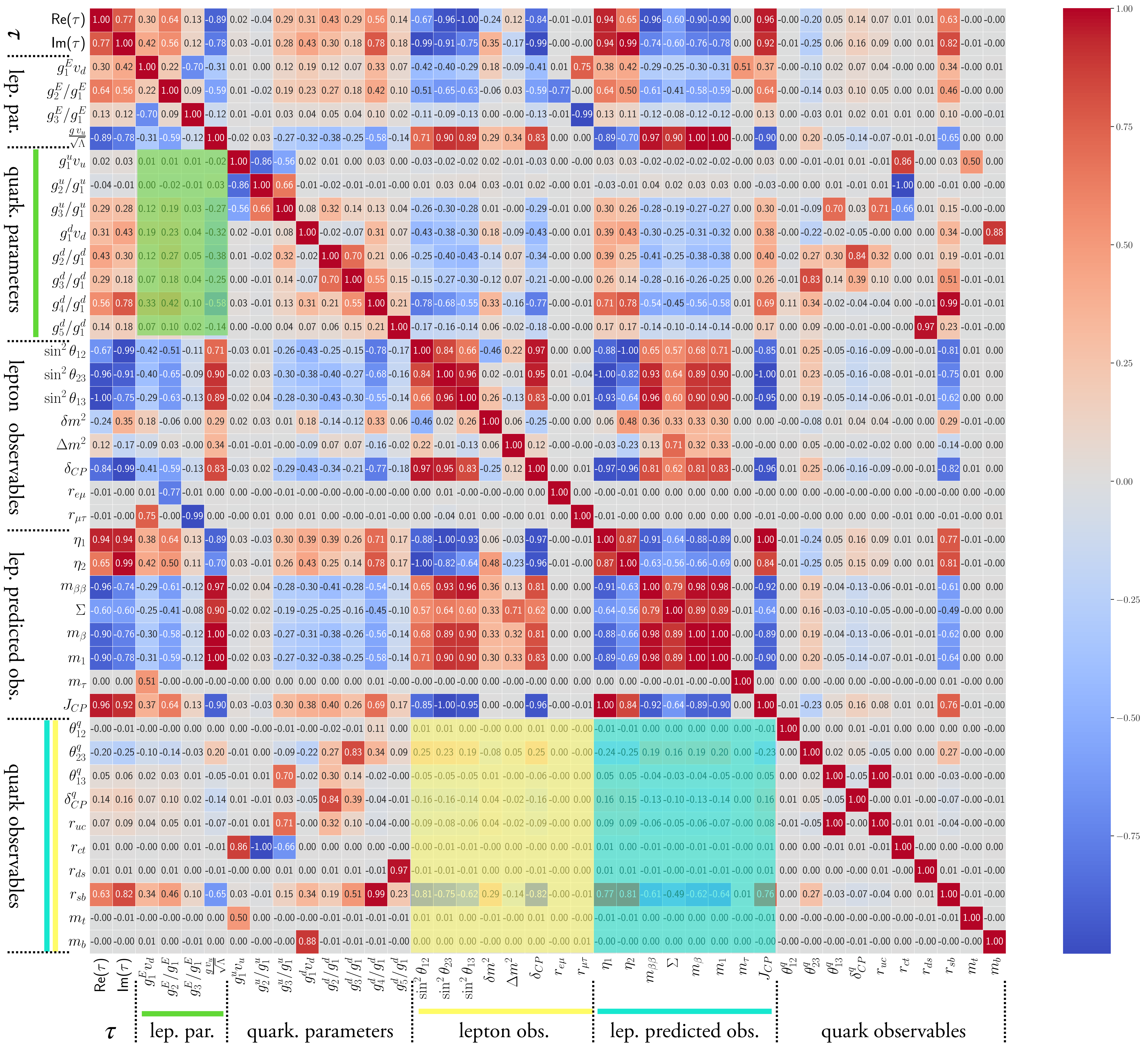

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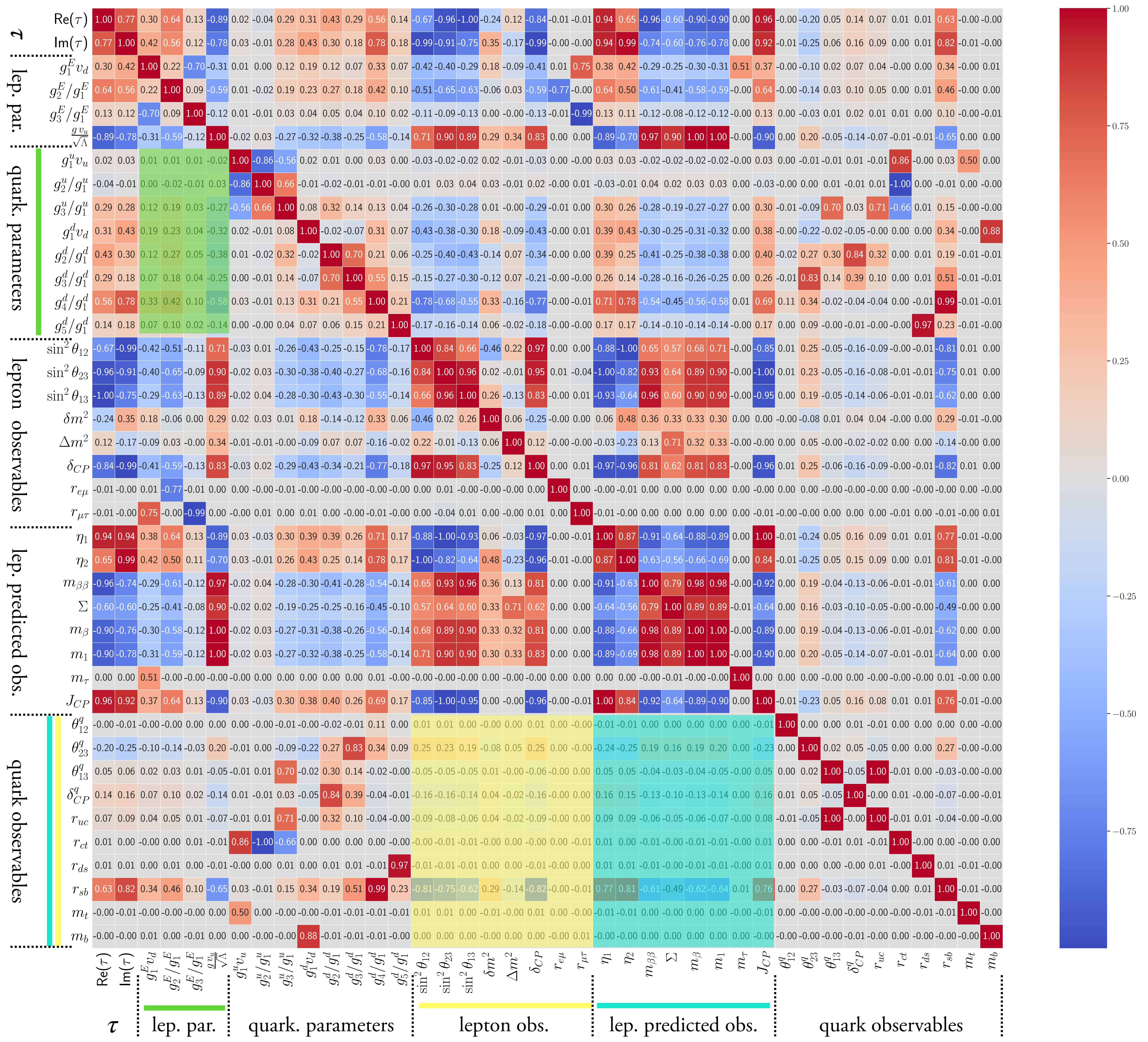
Tested by possible future experiments



Example of correlations

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	Re(τ)	Im(τ)
Re(τ)	1.00	0.77
Im(τ)	0.77	1.00
$g_1^E v_d$	0.30	0.42
g_2^E / g_1^E	0.64	0.56
g_3^E / g_1^E	0.13	0.12
$\frac{g\, v_u}{\sqrt{\Lambda}}$	-0.89	-0.78
$g_1^u v_u$	0.02	0.03
g_2^u / g_1^u	-0.04	-0.01
g_3^u / g_1^u	0.29	0.28
$g_1^d v_d$	0.31	0.43
g_2^d / g_1^d	0.43	0.30
g_3^d / g_1^d	0.29	0.18
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Leptons $\rightarrow \frac{g v_u}{\sqrt{\Lambda}}$ (negative correlation)

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$\frac{g\,v_u}{\sqrt{\Lambda}}$	-0.89	-0.78
$g_1^u v_u$	0.02	0.03
g_2^u/g_1^u	-0.04	-0.01
g_3^u/g_1^u	0.29	0.28
$g_1^d v_d$	0.31	0.43
g_2^d/g_1^d	0.43	0.30
g_3^d/g_1^d	0.29	0.18
g_4^d/g_1^d	0.56	0.78
g_5^d/g_1^d	0.14	0.18

Leptons $\rightarrow \frac{g v_u}{\sqrt{\Lambda}}$ (negative correlation)

Quarks $\rightarrow \frac{g_4^d}{g_1^d}$ (positive correlation)

Example of correlations

Example of correlations

We can directly verify the correlation among quark and lepton parameters

Example of correlations

We can directly verify the correlation among quark and lepton parameters

$\frac{g_4^d}{g_1^d}$ and $\frac{g v_u}{\sqrt{\Lambda}}$ are, indeed the most (anti)correlated parameters among the two different sectors

	$g_1^E v_d$	g_2^E/g_1^E	g_3^E/g_1^E	$\frac{g\,v_u}{\sqrt{\Lambda}}$	$g_1^u v_u$	g_2^u/g_1^u	g_3^u/g_1^u	$g_1^d v_d$	g_2^d/g_1^d	g_3^d/g_1^d	g_4^d/g_1^d	g_5^d/g_1^d
$g_1^E v_d$	1.00	0.22	-0.70	-0.31	0.01	0.00	0.12	0.19	0.12	0.07	0.33	0.07
g_2^E/g_1^E	0.22	1.00	0.09	-0.59	0.01	-0.02	0.19	0.23	0.27	0.18	0.42	0.10
g_3^E/g_1^E	-0.70	0.09	1.00	-0.12	0.01	-0.01	0.03	0.04	0.05	0.04	0.10	0.02
$\frac{g\,v_u}{\sqrt{\Lambda}}$	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14
$g_1^u v_u$	0.01	0.01	0.01	-0.02	1.00	-0.86	-0.56	0.02	0.01	0.00	0.03	0.00
g_2^u/g_1^u	0.00	-0.02	-0.01	0.03	-0.86	1.00	0.66	-0.01	-0.02	-0.01	-0.01	-0.00
g_3^u/g_1^u	0.12	0.19	0.03	-0.27	-0.56	0.66	1.00	0.08	0.32	0.14	0.13	0.04
$g_1^d v_d$	0.19	0.23	0.04	-0.32	0.02	-0.01	0.08	1.00	-0.02	-0.07	0.31	0.07
g_2^d/g_1^d	0.12	0.27	0.05	-0.38	0.01	-0.02	0.32	-0.02	1.00	0.70	0.21	0.06
g_3^d/g_1^d	0.07	0.18	0.04	-0.25	0.00	-0.01	0.14	-0.07	0.70	1.00	0.55	0.15
g_4^d/g_1^d	0.33	0.42	0.10	-0.58	0.03	-0.01	0.13	0.31	0.21	0.55	1.00	0.21
g_5^d/g_1^d	0.07	0.10	0.02	-0.14	0.00	-0.00	0.04	0.07	0.06	0.15	0.21	1.00

Example of correlations

We can directly verify the correlation among quark and lepton parameters

$\frac{g_4^d}{g_1^d}$ and $\frac{g v_u}{\sqrt{\Lambda}}$ are, indeed the most (anti)correlated parameters among the two different sectors

We trace back their anti correlation going back to their correlation to the real and imaginary parts of τ

	$g_1^E v_d$	g_2^E/g_1^E	g_3^E/g_1^E	$\frac{g\,v_u}{\sqrt{\Lambda}}$	$g_1^u v_u$	g_2^u/g_1^u	g_3^u/g_1^u	$g_1^d v_d$	g_2^d/g_1^d	g_3^d/g_1^d	g_4^d/g_1^d	g_5^d/g_1^d
$g_1^E v_d$	1.00	0.22	-0.70	-0.31	0.01	0.00	0.12	0.19	0.12	0.07	0.33	0.07
g_2^E/g_1^E	0.22	1.00	0.09	-0.59	0.01	-0.02	0.19	0.23	0.27	0.18	0.42	0.10
g_3^E/g_1^E	-0.70	0.09	1.00	-0.12	0.01	-0.01	0.03	0.04	0.05	0.04	0.10	0.02
$\frac{g\,v_u}{\sqrt{\Lambda}}$	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14
$g_1^u v_u$	0.01	0.01	0.01	-0.02	1.00	-0.86	-0.56	0.02	0.01	0.00	0.03	0.00
g_2^u/g_1^u	0.00	-0.02	-0.01	0.03	-0.86	1.00	0.66	-0.01	-0.02	-0.01	-0.01	-0.00
g_3^u/g_1^u	0.12	0.19	0.03	-0.27	-0.56	0.66	1.00	0.08	0.32	0.14	0.13	0.04
$g_1^d v_d$	0.19	0.23	0.04	-0.32	0.02	-0.01	0.08	1.00	-0.02	-0.07	0.31	0.07
g_2^d/g_1^d	0.12	0.27	0.05	-0.38	0.01	-0.02	0.32	-0.02	1.00	0.70	0.21	0.06
g_3^d/g_1^d	0.07	0.18	0.04	-0.25	0.00	-0.01	0.14	-0.07	0.70	1.00	0.55	0.15
g_4^d/g_1^d	0.33	0.42	0.10	-0.58	0.03	-0.01	0.13	0.31	0.21	0.55	1.00	0.21
g_5^d/g_1^d	0.07	0.10	0.02	-0.14	0.00	-0.00	0.04	0.07	0.06	0.15	0.21	1.00

Correlation Parameters - Observables

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All neutrino observables are positively correlated to $\frac{g v_u}{\sqrt{\Lambda}}$, (in particular $\sin^2\theta_{12}$ and the other two mixing angles)

	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	δ_{CP}	$r_{e\mu}$	$r_{\mu\tau}$
$\frac{g\,v_u}{\sqrt{\Lambda}}$	0.71	0.90	0.89	0.29	0.34	0.83	0.00	0.00

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The quark parameter most correlated to quark observables is $\frac{g_4^d}{g_1^d}$

	θ_{12}^q	θ_{23}^q	θ_{13}^q	δ_{CP}^q	r_{uc}	r_{ct}	r_{ds}	r_{sb}	m_t	m_b
g_4^d/g_1^d	0.11	0.34	-0.02	-0.04	-0.04	0.00	-0.01	0.99	-0.01	-0.01

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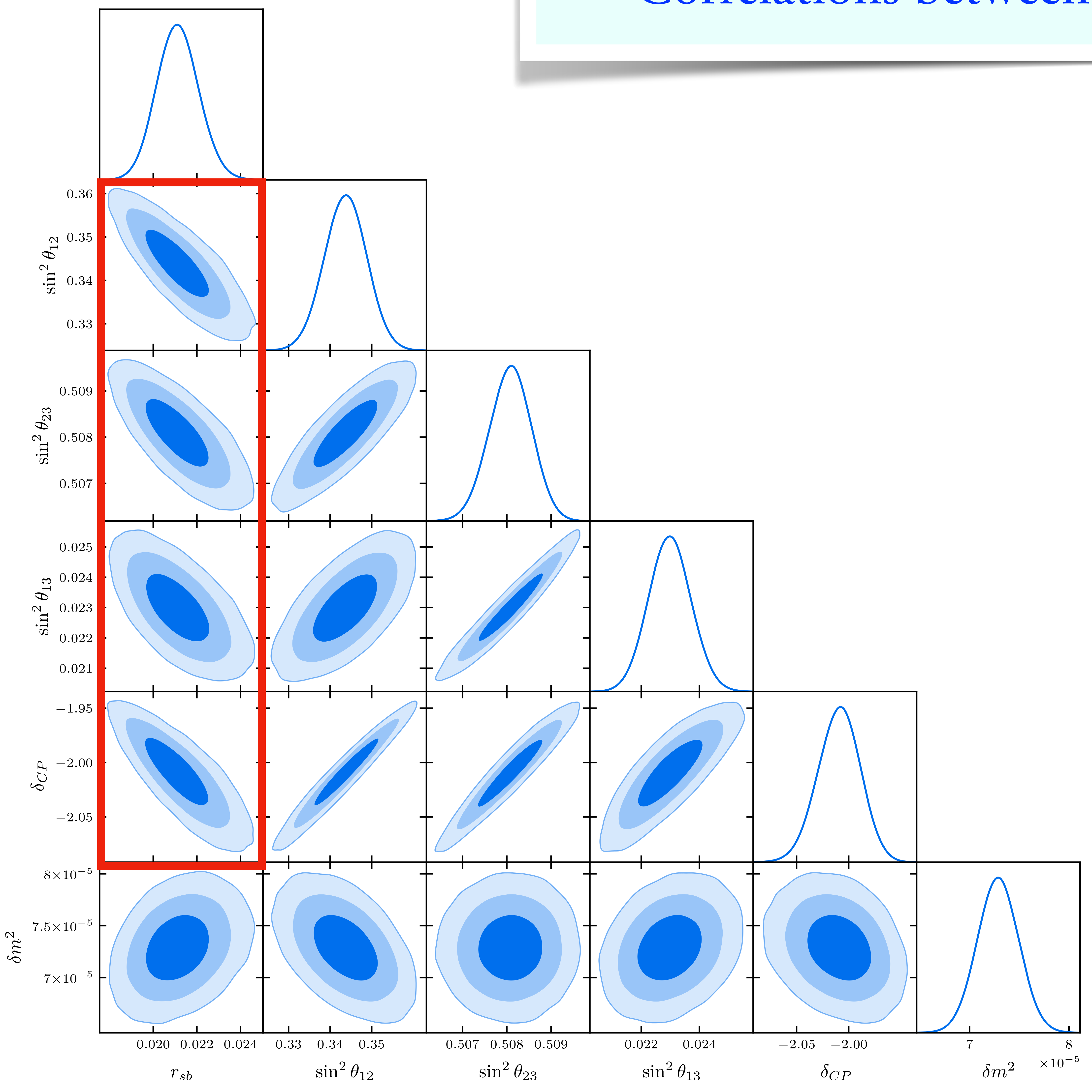
→ We can presume the presence of a significative anticorrelation between r_{sb} and all neutrino mixing angles

Correlations between quark and lepton observables

Correlations between quark and lepton observables



Significative anticorrelation between r_{sb} and $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}, \delta_{CP}$

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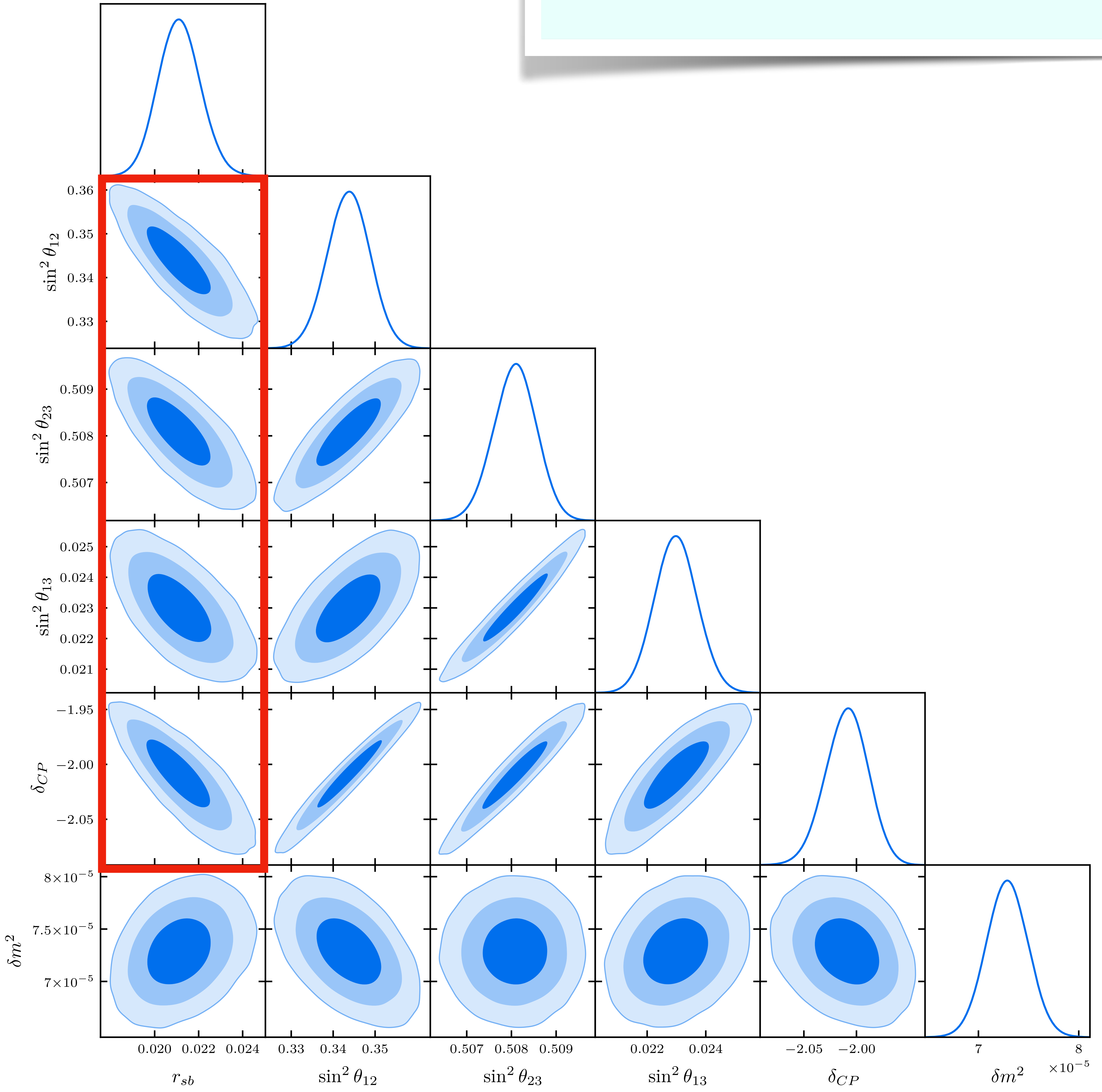


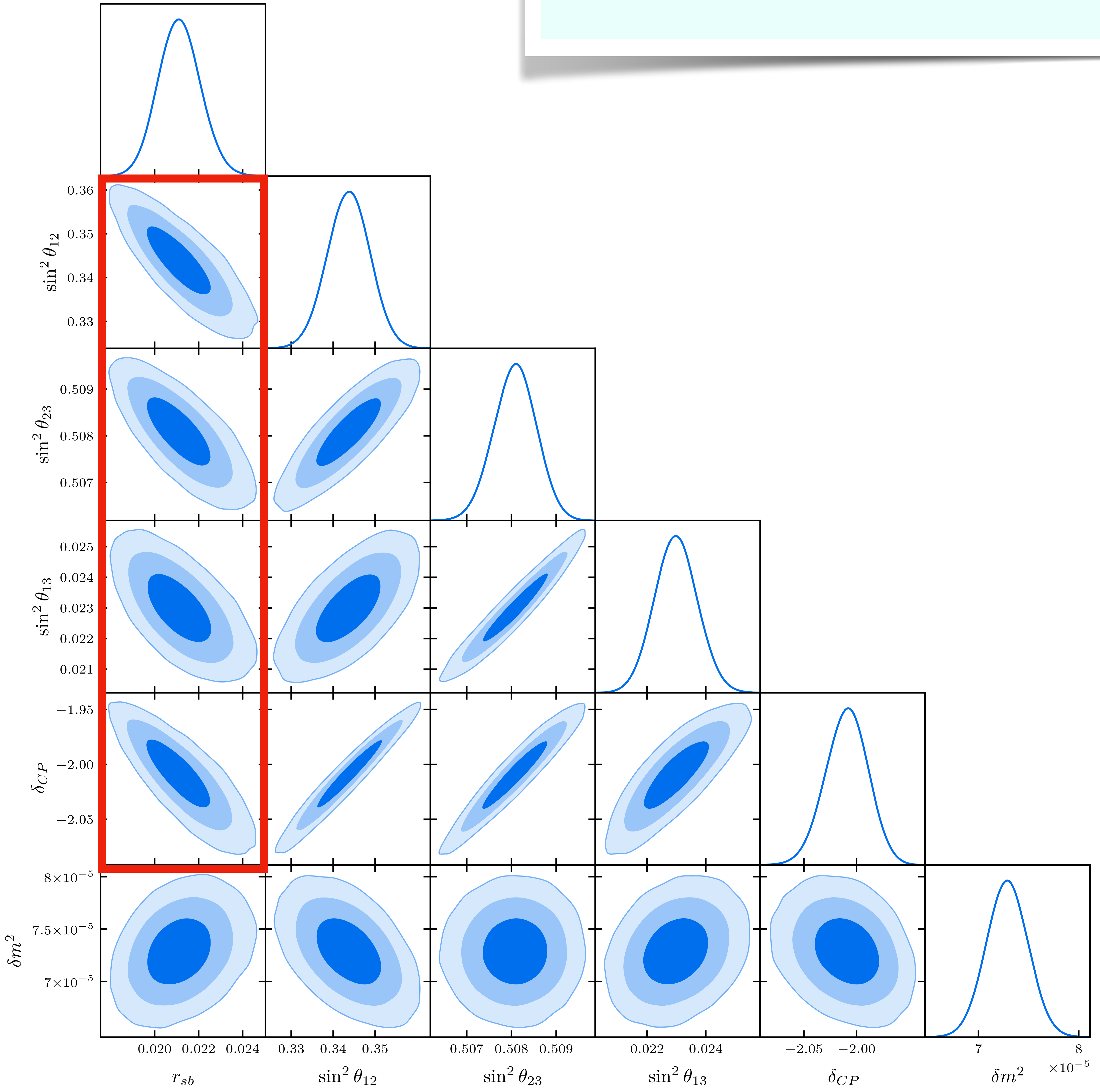
Significative anticorrelation between r_{sb} and $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, δ_{CP}

	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	δ_{CP}	$r_{e\mu}$	$r_{\mu\tau}$
θ^q_{12}	0.01	0.01	0.00	-0.00	0.00	0.01	0.00	-0.00
θ^q_{23}	0.25	0.23	0.19	-0.08	0.05	0.25	0.00	-0.00
θ^q_{13}	-0.05	-0.05	-0.05	0.01	-0.00	-0.06	-0.00	0.00
δ^q_{CP}	-0.16	-0.16	-0.14	0.04	-0.02	-0.16	-0.00	0.00
r_{uc}	-0.09	-0.08	-0.06	0.04	-0.02	-0.09	-0.00	0.00
r_{ct}	-0.00	-0.01	-0.01	-0.00	0.00	-0.00	0.00	-0.01
r_{ds}	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.00	0.00
r_{sb}	-0.81	-0.75	-0.62	0.29	-0.14	-0.82	0.00	-0.01
m_t	0.01	0.01	0.00	-0.01	-0.00	0.01	0.00	-0.00
m_b	0.00	0.00	0.00	-0.00	0.00	0.00	-0.00	0.01

Going back to the results of the fit we understand how the 3σ tension on the values of $\sin^2\theta_{12}$ predicted by the model (0.344) and the measured one (0.303) induces a new tension on r_{sb} in the combined fit

Overestimating $\sin^2\theta_{12}$ at 3σ causes an underestimation of r_{sb} at $\sim 2.7\sigma$

Correlations between quark and lepton observables



Significative anticorrelation between r_{sb} and $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}, \delta_{CP}$

	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	δ_{CP}	$r_{e\mu}$	$r_{\mu\tau}$
θ^q_{12}	0.01	0.01	0.00	-0.00	0.00	0.01	0.00	-0.00
θ^q_{23}	0.25	0.23	0.19	-0.08	0.05	0.25	0.00	-0.00
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Overestimating $\sin^2\theta_{12}$ at 3σ causes an underestimation of r_{sb} at $\sim 2.7\sigma$

The existence of correlations of this type among lepton and quark observables may be a strong indication in favor of modular flavour symmetry in particle physics

Conclusions

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Combined Analysis: Quark and lepton observables studied together in modular symmetry-based flavor model

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The model, based on the 2O group, describes 22 observables with 14 real parameters, including a single complex modulus

Successfully describes quark masses, CKM mixing angles, and lepton flavor data.

Tensions with data: $\sin^2\theta_{12}, r_{sb}$ at $\sim 3\sigma$ from the experimental value $\rightarrow$ JUNO and other experiments will critically test predictions, especially $\sin^2\theta_{12}$

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Correlations between quark and lepton observables

Experimental Predictions for δ_{CP}, the Majorana phases, $m_1, \Sigma, m_{\beta\beta}, m_\beta$

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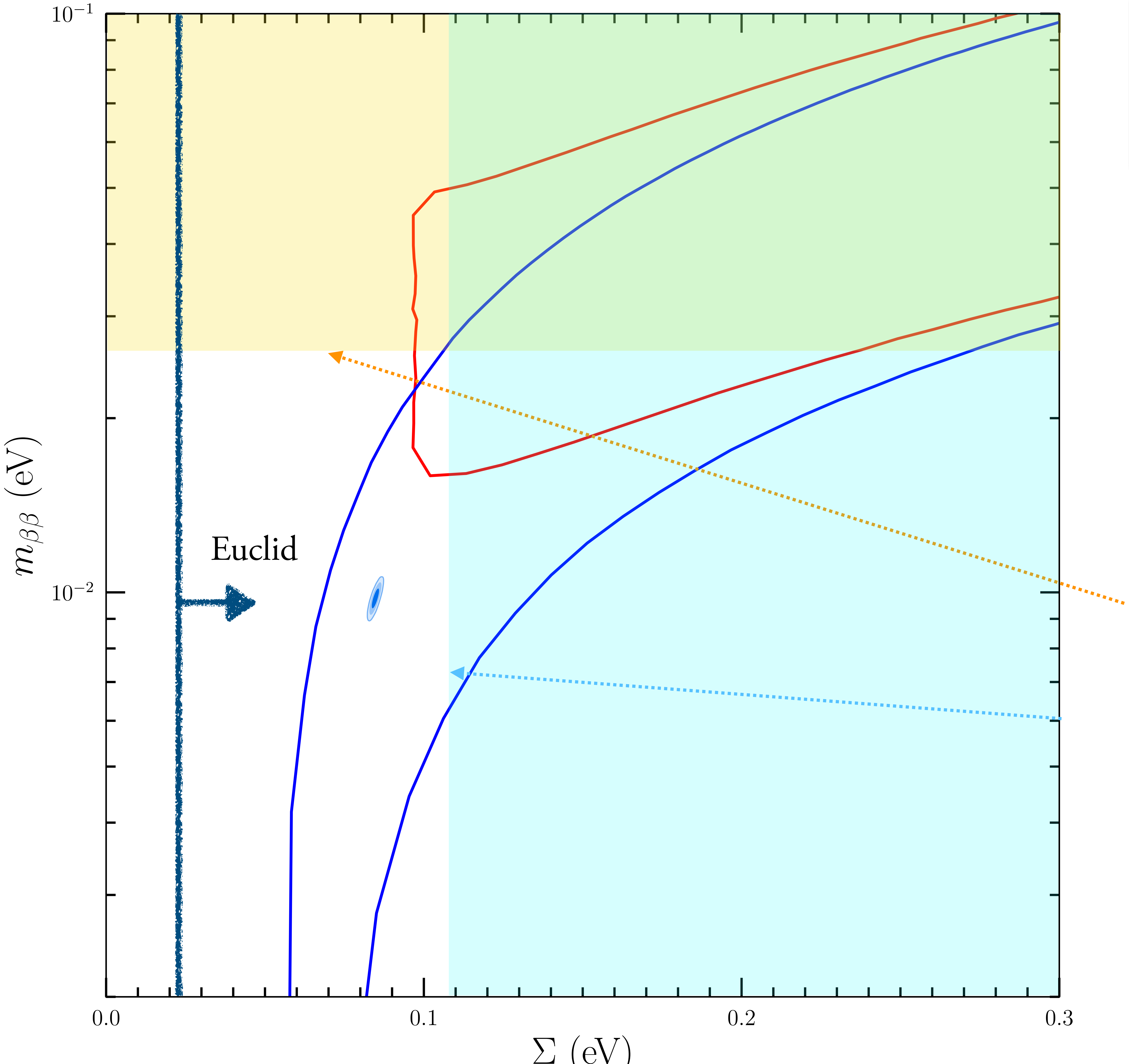
Modular symmetries offer a unified and predictive framework, possibly setting the stage for advancing our understanding of flavor physics

Backup Slides



Predicted ν observables

δ_{CP} not included as input since the measurement still not robust

The model predicts $\delta_{CP} \sim -0.64\pi$, to be compared to $\delta_{CP} \sim -\pi/2$ preferred by global analyses

m_β a factor ~ 20 below the KATRIN upper limit, $m_\beta < 0.45$ eV at 90% C.L. (talk by M. Schlösser @NOW2024)

$m_{\beta\beta}$ a factor from 4 to 12 below the next generation projects (Cupid, LEGEND, nEXO) depending on the Nuclear Matrix Element

Σ below the present cosmological bounds (at the level of ~ 0.12 eV, depending on the analysis) Planck Collaboration: Aghanim, N. et al. 2020, A&A, 641, A6, [Erratum: A&A 652, C4 (2021)]

To be tested by Euclid in combination with other cosmological and astrophysical dataset ($\Sigma > 0.023$ eV)
Euclid preparation. Sensitivity to neutrino parameters arXiv:2405.06047