Spontaneous CP Violation and Flavor Changing Neutral Currents in Minimal SO(10)

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Based on: 2412.00196

Motivation and Introduction

Why GUT and SO(10) GUT?

- Matter Unification: $\psi_{\mathrm{SM}} + \nu_R = 16_F$.

  Explains charge quantization.

  otherwise, $Q = Q_{SM} + \epsilon \frac{B-L}{2}$ *Foot, Lew, Volkas. '93*

- Intermediate Scales: e.g. LR and QL.

  Successful gauge coupling unification.

- Profound Prediction: Proton decay.

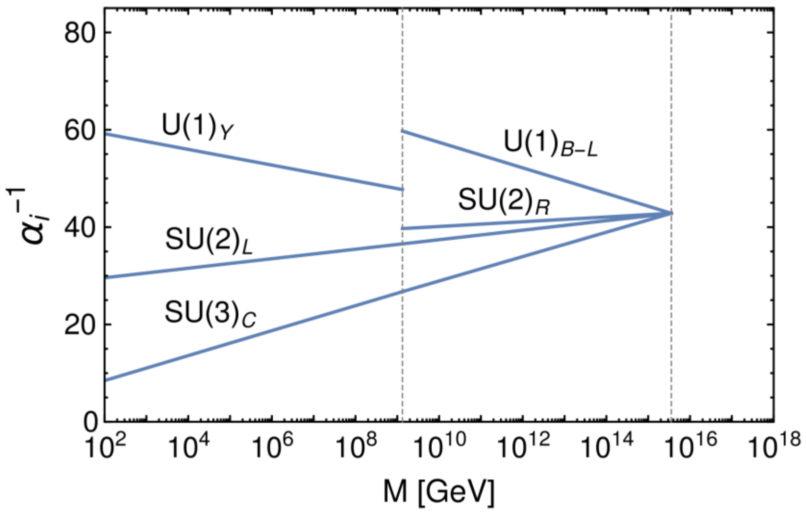
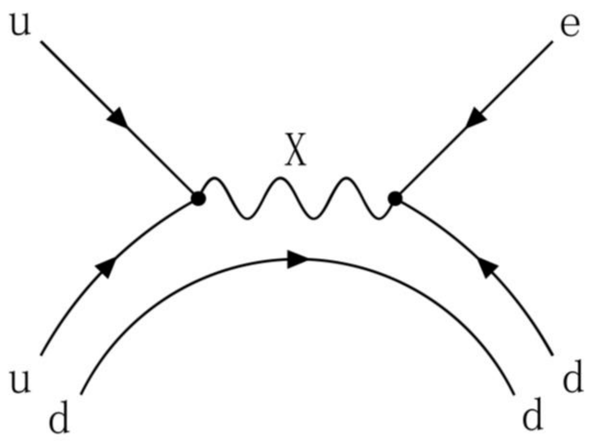
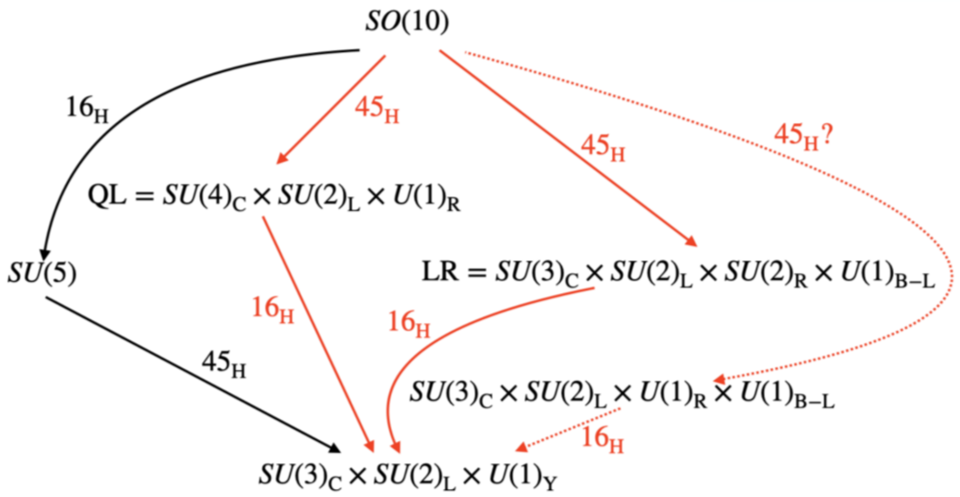


Image from Deppisch, et al.'17



Motivation and Introduction

What's Minimal SO(10) GUT?



Fermion Sector:

- $16_F = (Q_L, u_R, d_R) + (l_L, \nu_R, e_R)$.

Scalar Sector:

- Small Rep: $10_H^c, 16_H, 45_H$.
  - Light Scalars required.
  - Need Non-renormalizable operators.
- Large Rep:
  - $10_H^c, 45_H, 126_H$.
  - $10_H, 120_H, 45_H, 126_H$.

Bertolini, Di Luzio, Malinsky.'10; Preda, Senjanovic, Zantedeschi.'22 '24 (figure above);

Bajc, et al. '06; Patel.'23; Bertolini, Di Luzio, Malinsky. '12; Jarkovska, Malinsky, Susic. '23; Babu, Bajc, Saad. '17.

Motivation and Introduction

How to test renormalizable SO(10)?

$M_t \sim 10^2$ GeV → **Desert** → $M_I \gtrsim 10^{10}$ GeV, $M_{\mathrm{GUT}} \gtrsim 10^{15}$ GeV

Desert

- SM flavor parameters?
- Proton decay branching ratios?

  No very robust predictions for now.

- **Oasis** of a new flavor sector?

Oasis

The theory: Minimal SO(10)×CP

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Specified SO(10): Real couplings

- CP symmetry enhanced, broken only spontaneously.
- KM phase from complex VEVs. *T. D. Lee '73.*
- No high scale SCPV, with minimal scalar sector:

$$45_H \to 45_H, \quad 126_H \to \overline{126}_H, \quad 10_H \to 10_H^*$$

Need **two degenerate EW vacua** v_i, v_i^*
→ Two light Higgs Doublets ≲500 GeV,
→ one more fine-tuning than SM.

Mohapatra, Senjanovic. '83;
Nebot, Botella, Branco. '19;
Nierste, Tabet, Ziegler, '20;
Miró, Nebot, Queiroz. '24.

The theory: Minimal SO(10)×CP

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Yukawa Sector: constrained by SO(10):

$$-\mathcal{L}_Y = Y_{10} 16_F 10_H 16_F + \widetilde{Y}_{10} 16_F 10_H^* 16_F + Y_{126} 16_F \overline{126}_H 16_F + \text{h.c.}$$

$$\supset \overline{Q_L}(Y_{10}\Phi^d_{10} - \widetilde{Y}_{10}\widetilde{\Phi}^u_{10} + Y_{126}\Phi^d_{126})d_R + \overline{Q_L}(Y_{10}\Phi^u_{10} + \widetilde{Y}_{10}\widetilde{\Phi}^d_{10} + Y_{126}\Phi^u_{126})u_R$$
$$+\overline{\ell_L}(Y_{10}\Phi^d_{10} - \widetilde{Y}_{10}\widetilde{\Phi}^u_{10} - 3Y_{126}\Phi^d_{126})e_R + \overline{\ell_L}(Y_{10}\Phi^u_{10} + \widetilde{Y}_{10}\widetilde{\Phi}^d_{10} - 3Y_{126}\Phi^u_{126})\nu_R$$
$$+\frac{1}{2}\overline{\nu^c_R}Y_{126}\Delta^0_R\nu_R + \frac{1}{2}\overline{\ell^c_L}Y_{126}\Delta_L\ell_L + \text{h.c.} \quad (1)$$

$Y_{10}, \tilde{Y}_{10}, Y_{126}$ are **real, symmetric**. Flavor structure **stable under RG**.

Numerical fit: *Patel '23.*

$$M_D = Y_{10}v^d_{10} - \widetilde{Y}_{10}v^{u*}_{10} + Y_{126}v^d_{126}, \qquad M_{\nu_R} = \tfrac{1}{2}Y_{126}\langle\Delta^0_R\rangle,$$
$$M_E = Y_{10}v^d_{10} - \widetilde{Y}_{10}v^{u*}_{10} - 3Y_{126}v^d_{126}, \qquad M_{\nu_D} = Y_{10}v^u_{10} + \widetilde{Y}_{10}v^{d*}_{10} - 3Y_{126}v^u_{126},$$
$$M_U = Y^u_{10}v^u_{10} + \widetilde{Y}_{10}v^{d*}_{10} + Y_{126}v^u_{126}, \qquad M_{\nu_L} = -M^T_{\nu_D}M^{-1}_{\nu_R}M_{\nu_D} + \tfrac{1}{2}Y_{126}\langle\Delta_L\rangle. \quad (2)$$

$$M_D = D^* m^{\text{diag}}_D D^\dagger, \quad M_U = U^* m^{\text{diag}}_U U^\dagger, \quad M_E = E^* m^{\text{diag}}_E E^\dagger, \quad M_{\nu_L} = N^* m^{\text{diag}}_{\nu_L} N^\dagger,$$
$$V_{\text{CKM}} = U^\dagger D, \qquad V_{\text{PMNS}} = E^\dagger N, \qquad \boxed{V_E = E^\dagger D.}$$

The theory: Minimal SO(10)×CP

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The low energy theory: Flavor violating 2HDM

$$-\mathcal{L}_{\Phi\overline{F}F} \supset (\frac{m_E}{v} + \epsilon Y_E^{\ell\ell'})h\overline{\ell_L}\ell'_R + (\frac{m_D}{v} + \epsilon Y_D^{qq'})\overline{d_L^q}d_R^{q'}h + (\frac{m_U}{v} + \epsilon Y_U^{qq'})\overline{u_L^q}u_R^{q'}h$$
$$+ \mathcal{Y}_E^{\ell\ell'}(H + iA)\overline{\ell_L}\ell'_R + \mathcal{Y}_D^{qq'}(H + iA)\overline{d_L^q}d_R^{q'} + \mathcal{Y}_U^{qq'}(H + iA)\overline{u_L^q}u_R^{q'} + \text{h.c.}$$

$$\begin{aligned}
Y_E &= \mathcal{C}_{EE}\frac{m_E}{v} + \mathcal{C}_{ED}V_E^*\frac{m_D}{v}V_E^\dagger + \mathcal{C}_{EU}V_E^*V_{\text{CKM}}^T\frac{m_U}{v}V_{\text{CKM}}V_E^\dagger,\\
Y_D &= \mathcal{C}_{DE}V_E^T\frac{m_E}{v}V_E + \mathcal{C}_{DD}\frac{m_D}{v} + \mathcal{C}_{DU}V_{\text{CKM}}^T\frac{m_U}{v}V_{\text{CKM}},\\
Y_U &= \mathcal{C}_{UE}V_{\text{CKM}}^*V_E^T\frac{m_E}{v}V_E V_{\text{CKM}}^\dagger + \mathcal{C}_{UD}V_{\text{CKM}}^T\frac{m_D}{v}V_{\text{CKM}} + \mathcal{C}_{UU}\frac{m_U}{v},
\end{aligned}$$

where $\mathcal{Y}_F = e^{i\alpha_c}Y_F,\ \ell,\ell' = e,\mu,\tau,\ q,q' = d,s,b$ or u,c,t

- $Y_F =$ Linear combinations of M_F.
- Flavor structure: only V_E unknown.
- Chiral Sym: $Y_F = 0$ when $m_f = 0$.
- m_t explicitly break chiral sym.

The theory: Minimal SO(10)×CP

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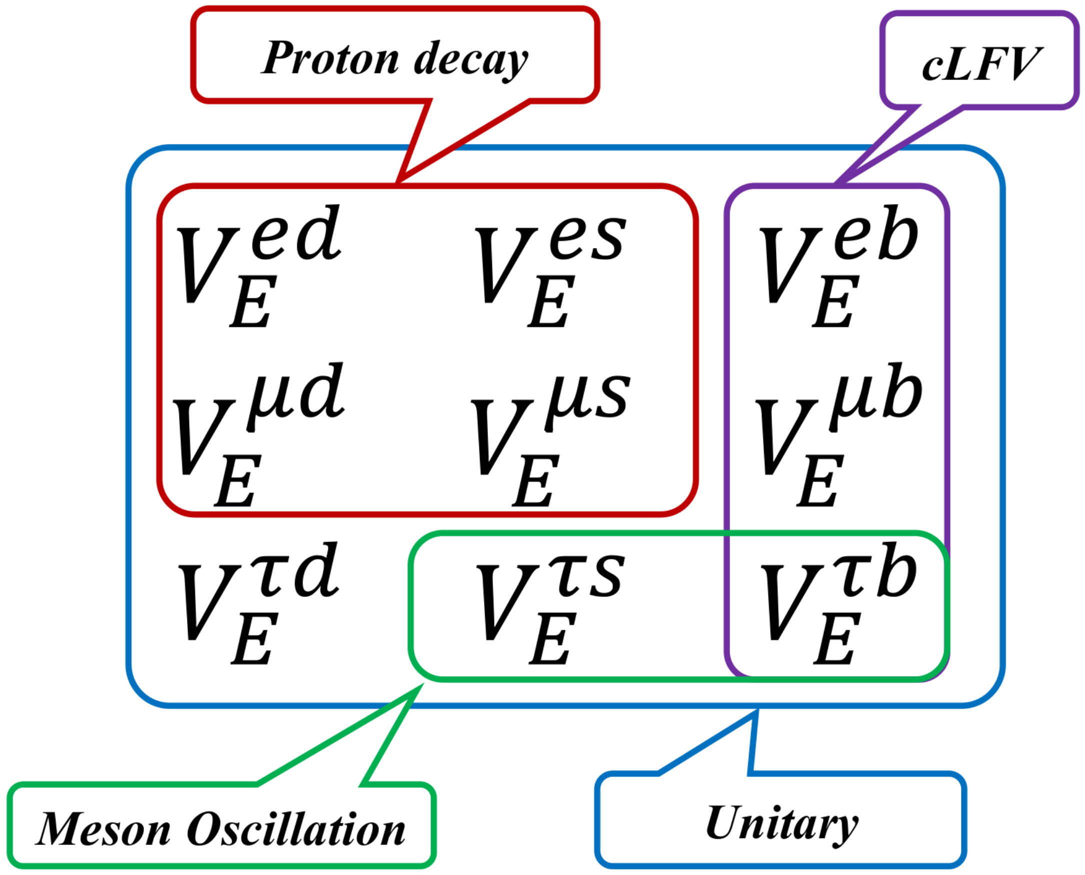
Understand the flavor structure:

- Charged Leptons: NMFV
- Down-type Quarks: MFV or NMFV
- Up-type quarks: $o(\lambda)$ corrections.

  λ~0.2, the Wolfstein parameter



$$Y_E^{\ell\ell'} \propto (V_E^{\ell b} V_E^{\ell' b})^* + o(\lambda^2)(V_E^{\ell b} + V_E^{\ell' b})^*, \quad \ell \neq \ell',$$

$$Y_D^{qq'} \propto V_E^{\tau q} V_E^{\tau q'} + \frac{c_{DU} m_t}{c_{DE} m_\tau} V_{\mathrm{CKM}}^{tq} V_{\mathrm{CKM}}^{tq'}, \quad q \neq q',$$

$$Y_U^{qq'} \propto V_E^{\tau q} V_E^{\tau q'} + \frac{c_{UD} m_b}{c_{UE} m_\tau} V_{\mathrm{CKM}}^{tq} V_{\mathrm{CKM}}^{tq'} + o(\lambda), \quad q \neq q',$$

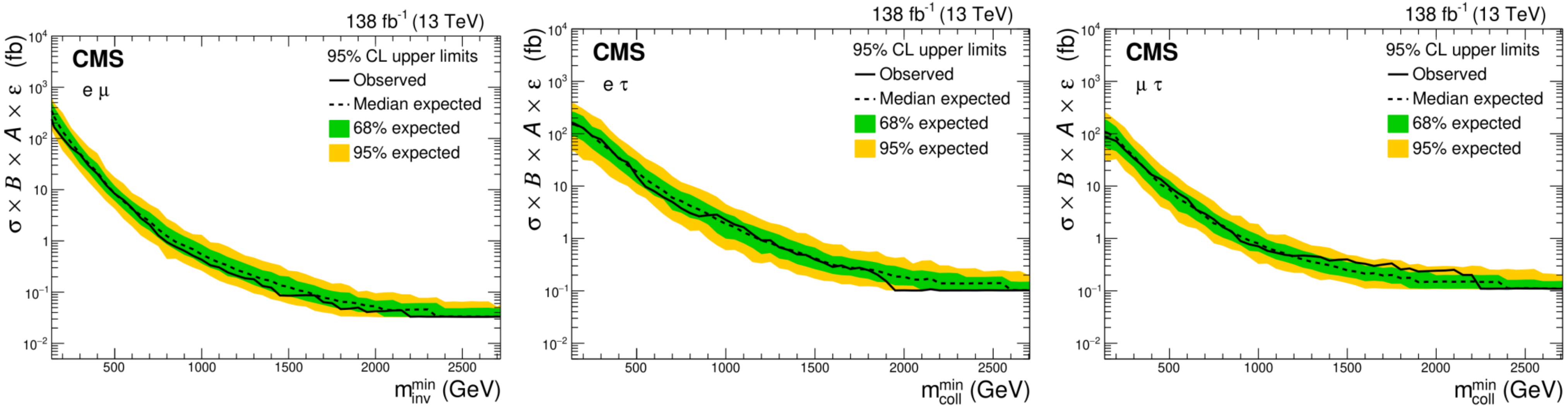
Phenomenology: A Window to Check SO(10)

LFV decay of H and A (third column of V_E)

$$\sigma(pp \to H, A) \times \mathrm{Br}(H, A \to \ell\ell') \ \propto\ \left|Y_E^{\ell\ell'}\right|^2 \ \propto\ |V_E^{\ell b} V_E^{\ell' b}|^2$$

$$\frac{N_{e\mu}}{N_{\tau\mu}} = \frac{|V_E^{eb}|^2}{|V_E^{\tau b}|^2}, \qquad \frac{N_{e\mu}}{N_{e\tau}} = \frac{|V_E^{\mu b}|^2}{|V_E^{\tau b}|^2}.$$

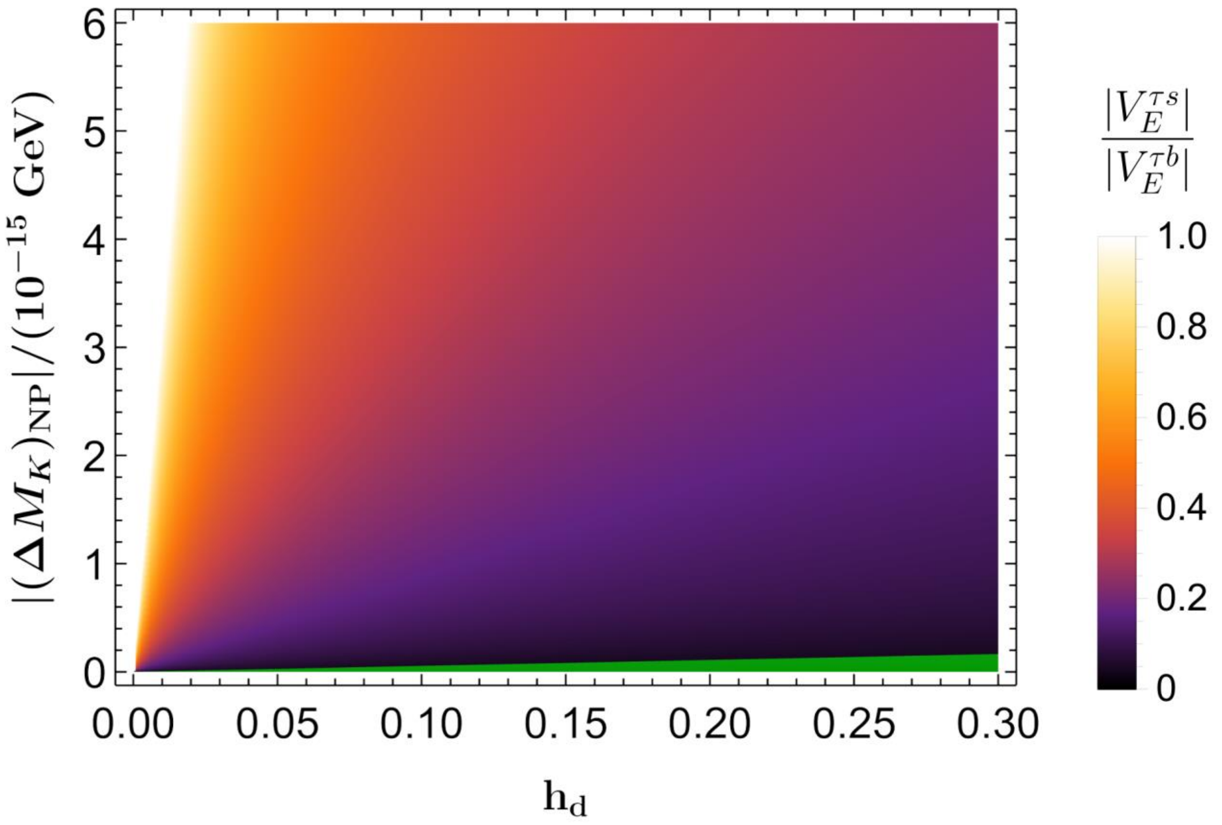
$N_{ll'}$, total # of excess evens, normalized by detection efficiency.



10 fb for m_H~500 GeV. CMS. 2205.06709

Phenomenology: A Window to Check SO(10)

Neutral meson mixing (third row of V_E)



MFV only for the narrow green band.

MFV:

$$\frac{|(\Delta M_K)_{\rm NP}| m_K}{\langle K^0|\overline{d_L}s_R\overline{d_R}s_L|\overline{K^0}\rangle} \approx \lambda^4 \cdot \frac{2\mathbf{h_d}|M_{12}^{d\rm SM}|m_{B_d}}{\langle B_d^0|\overline{b_L}d_R\overline{b_R}d_L|\overline{B_s^0}\rangle}$$

$$\approx \lambda^6 \cdot \frac{2\mathbf{h_s}|M_{12}^{s\rm SM}|m_{B_s}}{\langle B_s^0|\overline{b_L}s_R\overline{b_R}s_L|\overline{B_s^0}\rangle}.$$

NMFV:

$$\frac{|(\Delta M_K)_{\rm NP}|}{2\mathbf{h_d}|M_{12}^{d\rm SM}|\xi_B} = \frac{|V_E^{\tau s}|^2}{|V_E^{\tau b}|^2},$$

$$\xi_B = \frac{m_B\langle K^0|\overline{d_L}s_R\overline{d_R}s_L|\overline{K^0}\rangle}{m_K\langle B_d^0|\overline{b_L}d_R\overline{b_R}d_L|\overline{B_d^0}\rangle}.$$

Phenomenology: A Window to Check SO(10)

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Proton decay (2×2 top-left submatrix of V_E)

$$\frac{\Gamma(p \to \pi^+ \overline{\nu})}{\Gamma(p \to K^+ \overline{\nu})} = \frac{4\left(1 - m_K^2/m_p^2\right)^{-2} \langle \pi^+ | (du)_R d_L | p \rangle^2}{\langle K^+ | (us)_R d_L | p \rangle^2 + \lambda^2 \langle K^+ | (ud)_R s_L | p \rangle^2}$$

$$\frac{\Gamma(p \to \pi^0 e^+)}{\Gamma(p \to \pi^+ \overline{\nu})} = |V_E^{ed} + \frac{\lambda}{2} V_E^{es}|^2, \qquad \frac{\Gamma(p \to \pi^0 \mu^+)}{\Gamma(p \to \pi^+ \overline{\nu})} = |V_E^{\mu d} + \frac{\lambda}{2} V_E^{\mu s}|^2,$$

$$\frac{\Gamma(p \to K^0 e^+)}{\xi_K \Gamma(p \to K^+ \overline{\nu})} = |V_E^{es} + \lambda V_E^{ed}|^2, \qquad \frac{\Gamma(p \to K^0 \mu^+)}{\xi_K \Gamma(p \to K^+ \overline{\nu})} = |V_E^{\mu s} + \lambda V_E^{\mu d}|^2,$$

$$\text{with} \quad \xi_K = \frac{2\langle K^0 | (us)_R u_L | p \rangle^2}{\langle K^+ | (us)_R d_L | p \rangle^2 + \lambda^2 \langle K^+ | (ud)_R s_L | p \rangle^2} \approx 6.4.$$

Symmetric Yukawa couplings simplifies a lot.

Perez '04;
Nath, Perez '07.

Decay Mode	$\ell = e^+$	$\ell = \mu^+$	$\ell = \bar{\nu}$
$p \to \pi \ell$	$> 2.4 \times 10^{34}$ yr	$> 1.6 \times 10^{34}$ yr	$> 3.9 \times 10^{32}$ yr
$p \to K \ell$	$> 1.0 \times 10^{33}$ yr	$> 3.6 \times 10^{33}$ yr	$> 5.9 \times 10^{33}$ yr

Conclusion and Discussion

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A concrete prediction:

$$\frac{2\Gamma(p\to\pi^0\ell^+)}{\Gamma(p\to\pi^+\overline{\nu})}-\frac{\Gamma(p\to K^0\ell^+)}{\xi_K\Gamma(p\to K^+\overline{\nu})}$$
$$=\left(\frac{3|(\Delta M_K)_{\rm NP}|}{2\mathbf{h_d}|M_{12}^{d\rm SM}|\xi_B}-\frac{N_{e\mu}}{N_{\tau\mu}}-\frac{N_{e\mu}}{N_{e\tau}}+1\right)\left(\frac{N_{e\mu}}{N_{\tau\mu}}+\frac{N_{e\mu}}{N_{e\tau}}+1\right)^{-1}$$

- Wait for Hyper-K, HL-LHC, and more lattice QCD results.
- Hopefully, a hint for SO(10) in future.
- What we need most? Patience!

Thanks

Additional Slides

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Part of the scalar potential

Bertolini, Di Luzio, Malinsky. '12

ϕ: 45_H Σ: 126_H

$$V = V_{45} + V_{126} + V_{\rm mix}\,, \tag{1}$$

where

$$V_{45} = -\frac{\mu^2}{2}(\phi\phi)_0 + \frac{a_0}{4}(\phi\phi)_0(\phi\phi)_0 + \frac{a_2}{4}(\phi\phi)_2(\phi\phi)_2\,, \tag{2}$$

$$\begin{aligned} V_{126} = & -\frac{\nu^2}{5!}(\Sigma\Sigma^*)_0 \\ & + \frac{\lambda_0}{(5!)^2}(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 + \frac{\lambda_2}{(4!)^2}(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 \\ & + \frac{\lambda_4}{(3!)^2(2!)^2}(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 + \frac{\lambda_4'}{(3!)^2}(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} \\ & + \frac{\eta_2}{(4!)^2}(\Sigma\Sigma)_2(\Sigma\Sigma)_2 + \frac{\eta_2^*}{(4!)^2}(\Sigma^*\Sigma^*)_2(\Sigma^*\Sigma^*)_2\,, \end{aligned} \tag{3}$$

$$\begin{aligned} V_{\rm mix} = & \frac{i\tau}{4!}(\phi)_2(\Sigma\Sigma^*)_2 + \frac{\alpha}{2\cdot 5!}(\phi\phi)_0(\Sigma\Sigma^*)_0 \\ & + \frac{\beta_4}{4\cdot 3!}(\phi\phi)_4(\Sigma\Sigma^*)_4 + \frac{\beta_4'}{3!}(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} \\ & + \frac{\gamma_2}{4!}(\phi\phi)_2(\Sigma\Sigma)_2 + \frac{\gamma_2^*}{4!}(\phi\phi)_2(\Sigma^*\Sigma^*)_2\,. \end{aligned} \tag{4}$$

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Estimating maximal radiative corrections:

Assuming the Yukawa Sector is not much fine-tuned:

$$(\theta^L_{bq}-\theta^R_{bq}),(\theta^L_{tq'}-\theta^R_{tq'}) \lesssim \frac{y_t^2}{16\pi^2}|V_E^{\tau q}V_E^{\tau b}|\log(M_{\rm GUT}/M_H) \sim 5\% \times |V_E^{\tau q}V_E^{\tau b}|,$$

$$(\theta^L_{\tau\ell}-\theta^R_{\tau\ell}) \lesssim \frac{y_t^2}{16\pi^2}|V_E^{\ell b}V_E^{\tau b}|\log(M_{\rm GUT}/M_H) \sim 5\% \times |V_E^{\ell b}V_E^{\tau b}|.$$

$$(\theta^L_{uc}-\theta^R_{uc}) \lesssim \frac{y_t^2}{16\pi^2}|V_E^{\tau d}V_E^{\tau s}|\log(M_{\rm GUT}/M_H) \sim 5\% \times |V_E^{\tau d}V_E^{\tau s}|,$$

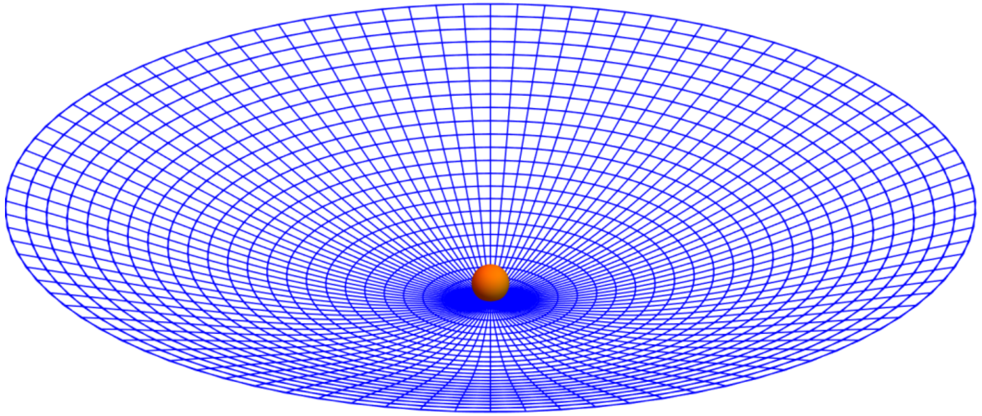
$$(\theta^L_{ds}-\theta^R_{ds}) \lesssim \frac{y_t^2}{16\pi^2}|V_{\rm CKM}^{ts}V_E^{\tau d}|\frac{m_b}{m_s}\log(M_{\rm GUT}/M_H) \sim 9\% \times |V_E^{\tau d}|,$$

$$(\theta^L_{e\mu}-\theta^R_{e\mu}) \lesssim \frac{y_t^2}{16\pi^2}|V_E^{eb}V_E^{\mu b}|\frac{m_\tau}{m_\mu}\log(M_{\rm GUT}/M_{\nu_R}) \sim 19\% \times |V_E^{eb}V_E^{\mu b}|.$$

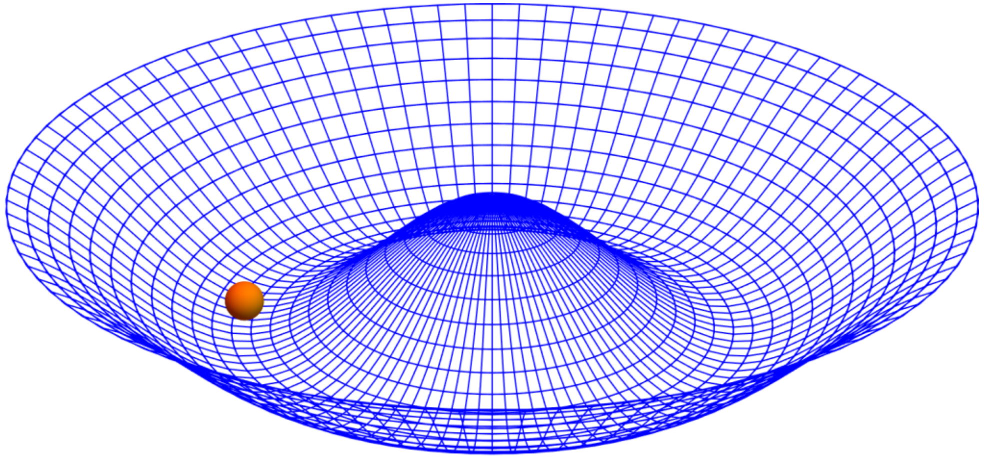
Just estimate an upper limit here. Long way to the complete GUT Yukawa texture.

Recent SU(5) work: Patel, Shukla. '24; Shukla. '24

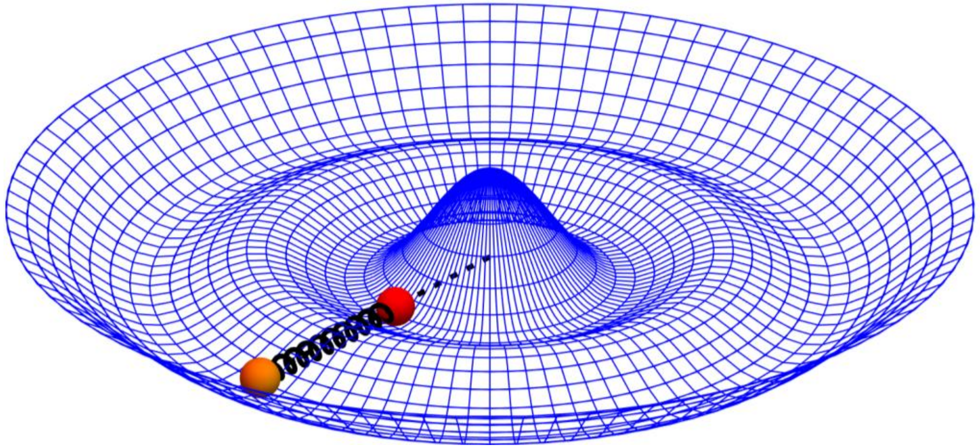
The theory: Minimal SO(10)×CP



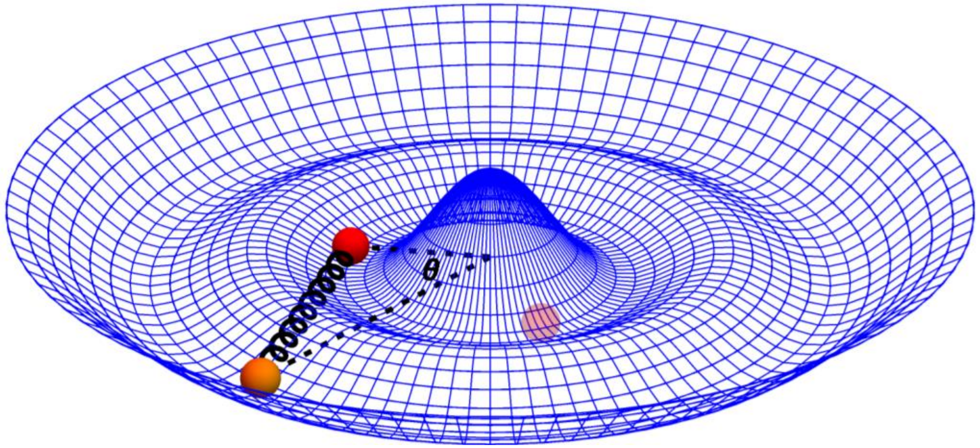
(a) No SSB



(b) SM SSB: Mexican Hat



(c) Unstable vacuum



(d) Spontaneous CPV

Based on the planar diagrams in Lee ’73

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For careful audience: Domain Walls

- Spontaneous breakdown of a discrete symmetry leads to Domain Walls. Disastrous for cosmology. *Zeldovich, et al. '74*
- **Way out?**
  - Natural idea: symmetry non-restoration at High temperature.
    Weinberg '74, Mohapatra, Senjanovic '79, Dvali, Senjanovic '95
    However, one needs at least a third light Higgs doublet.
    Another fine tuning.
    Mohapatra, Senjanovic' 79, Dvali, et al. '96
  - Biased term: tiny CP odd perturbation. *Vilenkin, '81, Gelmini, et al. '89*
    Seemingly, gravitational effects violate CP. *Rai, Senjanovic '94*

The theory: Minimal SO(10)×CP

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$$
\begin{aligned}
SO(10) \times CP \quad & \xrightarrow[M_{\text{GUT}}]{\langle(1,1,1,0)\rangle \in 45_H} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times CP \\
& \xrightarrow[M_R]{\langle(1,1,3,1)\rangle \in 126_H} SU(3)_C \times SU(2)_L \times U(1)_Y \times CP \\
& \xrightarrow[M_W]{\langle(1,2,2,0)\rangle \in 126_H, 10_H} SU(3)_c \times U(1)_{\text{EM}}.
\end{aligned}
$$

- Low scale SCPV: unavoidable FCNC, but not ruled out.
- Perturbative expansion may fail.

$$a_0^{max} = 0.46 < \frac{1}{2}$$

Jarkovska, Malinsky, Susic. '23; Milagre, Lavoura. '24.

- No domain wall problem, with a biased term.

Gravity may softly break CP.

Vilenkin, '81, Gelmini, et al. '89; Rai, Senjanovic. '94.

Additional Slides

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Scalar Sector: Poorly constrained by SO(10) *Jarkovska, et al. '23*

- We assume the most general CP invariant Higgs Potential.
- Physical effects of general 2HDM below a few TeV: *Branco, et al. '11*
  - 125 GeV h may slightly deviate from SM predictions.
  - New neutral states H, A, and a charged scalar H^+.
- Unknown parameters:
  - A complex mixing $\epsilon = \alpha_{hH} + i\alpha_{hA} \lesssim \frac{\mathrm{O}(\mathrm{v}^2)}{\mathrm{m}_\mathrm{H}^2}$. *Gunion, Haber '03*
    - α_{hH}, α_{hA} can't be too large. The discovered 125 GeV h is SM-like.
  - A CPV mixing α_{HA}. *Hayashi, et al. '94*
  - Vacua configuration $v_i = \left(v_{10}^d, v_{10}^{u*}, v_{126}^d, v_{126}^{u*}\right)$.

Phenomenology: Is the Theory Safe?

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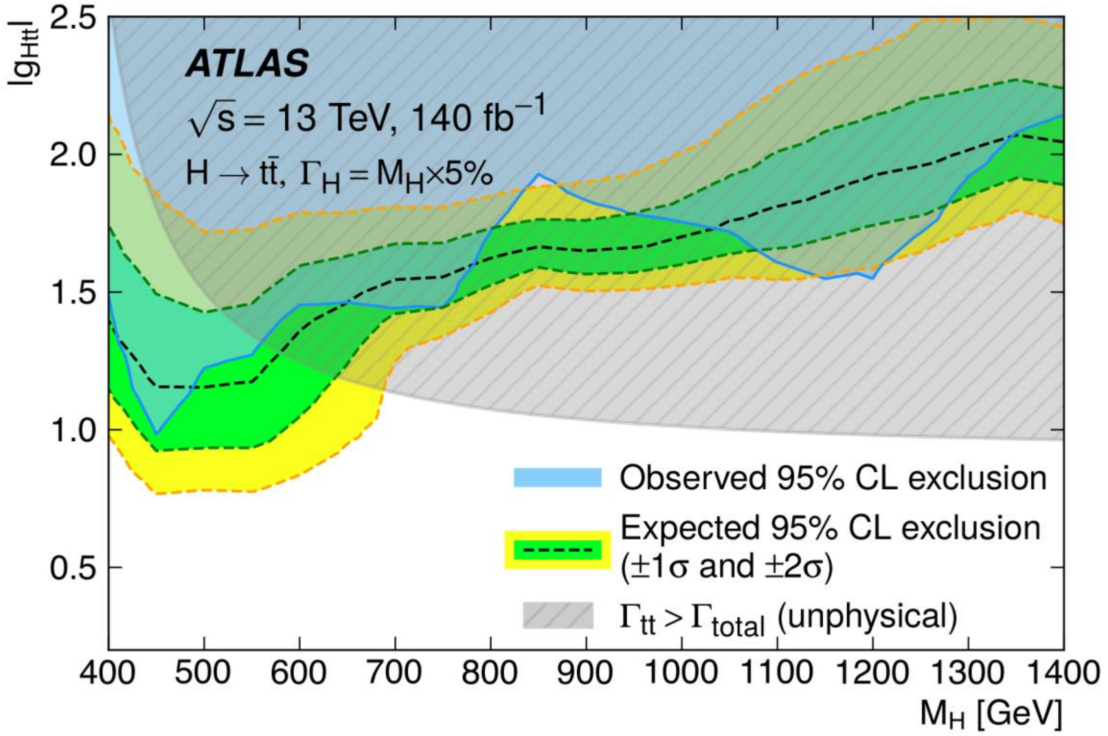
Direct collider search for H, A:

- $Y_U^{tt} \sim Y_E^{\tau\tau} \sim o(1)$: excluded (MSSM-like with large $tan\beta$).
- Smaller $Y_E^{\tau\tau}$, mainly decay to $\bar{t}t$ and $\bar{b}b$. Survive due to large background at hadron colliders.
- VBF production and $ZZ + WW$ decay: suppressed by $|\epsilon|^2$.



ϵ, mixing angle with h^{SM}
($c_{\alpha\beta}$ in benchmark 2HDM).
$\epsilon = 0$ iff h^{125} is exactly SM-like.

e.g. survive when $Y_U^{tt} < 1$.
Atlas. 2404.18986

Phenomenology: Is the Theory Safe?

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Why FCNC is safe?

$$\begin{pmatrix} \mathcal{C}_{EE} & \mathcal{C}_{ED} & \mathcal{C}_{EU} \\ \mathcal{C}_{DE} & \mathcal{C}_{DD} & \mathcal{C}_{DU} \\ \mathcal{C}_{UE} & \mathcal{C}_{UD} & \mathcal{C}_{UU} \end{pmatrix} = \frac{v}{u} \begin{pmatrix} u_1 & -u_2 & -3u_3 \\ u_1 & -u_2 & u_3 \\ u_2^* & u_1^* & u_4^* \end{pmatrix} \begin{pmatrix} v_{10}^d & -v_{10}^{u*} & -3v_{126}^d \\ v_{10}^d & -v_{10}^{u*} & v_{126}^d \\ v_{10}^u & v_{10}^{d*} & v_{126}^u \end{pmatrix}^{-1}$$

- Naively, all $\mathcal{C}_{FF'}$ at $o(1)$. Strictly, not predicted.
- $\mathcal{C}_{EU} = \mathcal{C}_{DU}$. No other correlations.
- B_s mixing constrains $\mathcal{C}_{DU}$.
- MFV: B_d, K mixing less constrained.
- $\Delta F = 1$ processes: suppressed by loop factor or additional chirality flipping.

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ (v_i + \rho_i + i\eta_i)/\sqrt{2} \end{pmatrix},$$

$$u_i = v_i^* - \left(\sum_{j=1}^{4} v_j^{*2}\right)\frac{v_i}{v}, \quad u = \sum_{i=1}^{4} |u_i|^2.$$

$$|\mathcal{C}_{DU}| \lesssim \left(\frac{v}{m_t} \cdot \frac{1}{|V_{\rm CKM}^{ts} V_{\rm CKM}^{tb}|}\right) \times \frac{m_H/\sqrt{2}}{10^3 \text{ TeV}}$$
$$\approx 0.013 \times \frac{m_H}{500 \text{ GeV}}$$

Phenomenology: A Window to Check SO(10)

Neutral meson mixing (third row of V_E)

$$H^q_{\rm NP} = -\frac{1}{2m_H^2}\left(Y_D^{bq}\overline{b_L}q_R + Y_D^{qb*}\overline{b_R}q_L\right)^2 - \frac{1}{2m_A^2}\left(iY_D^{bq}\overline{b_L}q_R - iY_D^{qb*}\overline{b_R}q_L\right)^2$$
$$\approx -\frac{2|Y_D^{bq}|^2}{m_H^2}\overline{b_L}q_R\overline{b_R}q_L, \quad q = d, s.$$

$\overline{b_L}q_R\overline{b_L}q_R$ and $\overline{b_R}q_L\overline{b_R}q_L$ vanish in the limit $m_H = m_A$

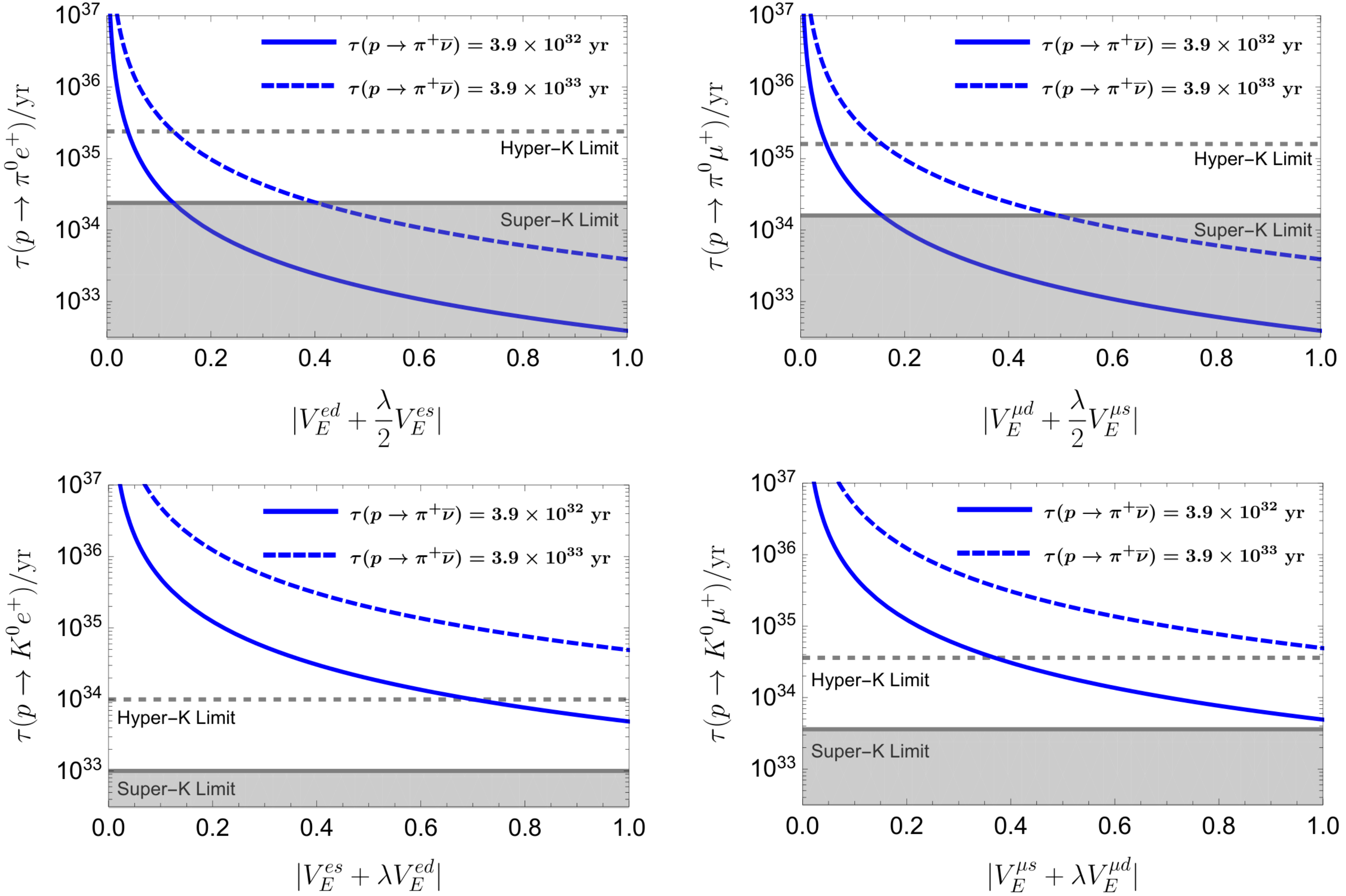
Observable	Current limit	Future sensitivity
$\mathbf{h_d}$	0.26	0.049 (0.038)
$\mathbf{h_s}$	0.12	0.044 (0.031)
$\lvert(\Delta M_K)_{\rm NP}\rvert$	5.2×10^{-15} GeV	0.2×10^{-15} GeV

$$\frac{\langle B_q^0|H^q_{\rm NP}|\overline{B_q^0}\rangle}{\langle B_q^0|H^q_{\rm SM}|\overline{B_q^0}\rangle} = \mathbf{h_q}e^{i\sigma_q}$$

$$(\Delta M_K)_{\rm NP} = \frac{1}{m_K}\langle K^0|H^K_{\rm NP}|\overline{K^0}\rangle$$

CKMfit, 2006.04824; KTEV, 1011.0127;
Bai, et al. '14; Wang '23.

Phenomenology: A Window to Check SO(10)



Combining π and *K* modes, λ goes away.

$$\frac{2\Gamma(p \to \pi^0 e^+)}{\Gamma(p \to \pi^+ \bar{\nu})} - \frac{\Gamma(p \to K^0 e^+)}{\xi_K \Gamma(p \to K^+ \bar{\nu})} = 2|V_E^{ed}|^2 - |V_E^{es}|^2,$$

$$\frac{2\Gamma(p \to \pi^0 \mu^+)}{\Gamma(p \to \pi^+ \bar{\nu})} - \frac{\Gamma(p \to K^0 \mu^+)}{\xi_K \Gamma(p \to K^+ \bar{\nu})} = 2|V_E^{\mu d}|^2 - |V_E^{\mu s}|^2.$$

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More on V_E:

$$V_E^T(V_{\rm PMNS}^* m_{\nu_L}^{\rm diag} V_{\rm PMNS}^\dagger - k_1 m_E^{\rm diag}) V_E \;=\; -k_1 m_D^{\rm diag} + k_2 (V_{\rm CKM}^T m_U^{\rm diag} V_{\rm CKM}) + k_3 M_T,$$

$$k_1 \;=\; \frac{8(v_4^*)^2}{v_3 \langle \Delta_R^0 \rangle} - \frac{\langle \Delta_L \rangle}{8 v_3}, \quad k_2 \;=\; \frac{16 v_4^*}{\langle \Delta_R^0 \rangle}, \quad k_3 \;=\; \frac{8 v_3}{\langle \Delta_R^0 \rangle}.$$

$$M_T \;=\; V_{\rm CKM}^T m_U^{\rm diag} V_{\rm CKM} (V_E^T m_E^{\rm diag} V_E - m_D^{\rm diag})^{-1} V_{\rm CKM}^T m_U^{\rm diag} V_{\rm CKM}.$$

M_T is ‘diagonal’ when $m_t \to \infty$ and $V_{\rm CKM} \to \mathbf{1}$

- $k_1 m_\tau \gg m_{\nu_L} \sim 0.1$ eV, $V_E \sim \mathbb{1}$.
- $k_1 m_\tau \lesssim m_{\nu_L} \sim 0.1$ eV, $V_E \sim V_{\rm PMNS}$.