NON-VANILLA AXION SOLUTIONS TO THE STRONG CP PROBLEM

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Based on
*Kivel, Laux, FY, JHEP **11** (2022) 88, [2207.08740];*
*Elahi, Elor, Kivel, Laux, Najjari, FY, PRD Letters **108** (2023) 3, [2301.08760];*
*FY, Annalen der Physik **536** (2024) 1, [2308.08612]*

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Intro: Longstanding strong CP problem

- Search for nEDM indicates absence of CPV in strong interactions
  - $|d_n| < 1.8 \cdot 10^{-26} e\ cm$ Abel, et. al. [nEDM collaboration], 2001.11966
    - For low energy SM, $d_n = C_{EDM}\, e\, \bar{\theta}$, with $C_{EDM} = 2.4\,(1.0) \cdot 10^{-16}$ and $\bar{\theta} = \theta + \arg\det Y_u Y_d$ Pospelov, Ritz, hep-ph/9908508
  - Get severe constraint on $|\bar{\theta}| < 10^{-10}$
    - Naïvely, QCD θ is O(1) vacuum angle parameter and unknown Y_u and Y_d matrices give rise to $J_{CKM} = (3.08^{+0.15}_{-0.13}) \cdot 10^{-5}$
- Traditionally, three classes of solutions
  - Massless up quark (now excluded by lattice)
  - Nelson-Barr: CPV only arises spontaneously
  - Peccei-Quinn mechanism and QCD axion

Review: see FY, AdP **538**, [2308.08612]

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  - spontaneously Review: see FY, AdP **538**, [2308.08612]
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Also, current unresolved claims about strong CP fundamentally
See, *e.g.* Ai, Cruz, Garbrecht, Tamarit Phys. Lett. B **822** (2021) [2001.07152]

Model-building strong CP solutions

- Promote θ to a spurion, a = “axion”
  - Generate potential for *a* via QCD topological susceptibility χ
  - Vafa-Witten ensures *a* settles into CP-conserving minimum
  - Necessarily need U(1) current to be *chiral* and have non-trivial $U(1) \times SU(3)^2$ anomaly

Peccei, Quinn PRL 38, 1440 (1977)

- Minimal (vanilla) benchmark models to generate the color anomaly

Kim (1979), Shifman, Vainshtein, Zakharov (1980); Dine, Fischler, Srednicki (1981), Zhitnitsky (1980)

  - **KSVZ** – PQ scalar field couples to SU(3) via heavy VLQ
  - **DFSZ** – PQ scalar field couples to 2HDM which have Yukawa interactions with SM quarks

In both cases, global PQ symmetry is exact and accidental at renormalizable level

Model-building strong CP solutions

- Why consider non-minimal models?
  - Strong CP is a flavor physics problem
  - Vacuum angles are of central importance in CP studies
  - Non-decoupling effects make the effective description non-trivial
  - Connections between EW and NP chirality is understudied
  - Phenomenological consequence: address quality problem, expand axion parameter space, expand axion cosmology (better connect CP observables to cosmology)

Model-building strong CP solutions

- Vacuum angles and non-decoupling
  - Chiral anomalies rotate vacuum angles
  - Consider extension of color gauge group to $SU(3)_1 \times SU(3)_2$: high energy theory has two independent vacuum angles θ_1 and θ_2
  - Consider further embedding $SU(3)_1 \times SU(3)_2$ into $SU(6)$
    - Known that (semi-simple) Lie groups only have on non-trivial topological vacuum angle
- Can bootstrap $U(1)_{PQ}$ from QCD scale to UV scale and get non-decoupling enhancement of axion potential

Color-Unified SU(6) x SU(3)' model

Gaillard, Gavela, Houtz, Quilez, del Rey, Eur. Phys. J. C **78** (2018) [1805.064365]

- Embed $SU(3)_c$ into SU(6) at high scales, SU(6) has massless Q fermion to solve θ_6
  - Use SU(3)' to make SM-charged exotic multiplets decouple at Λ_{CUT} scale by bifundamental scalar Δ

$$SU(6) \times SU(3') \xrightarrow{\Lambda_{\rm CUT}} SU(3)_c \times SU(3)_{\rm diag} \xrightarrow{v_{\rm diag}} SU(3)_c$$

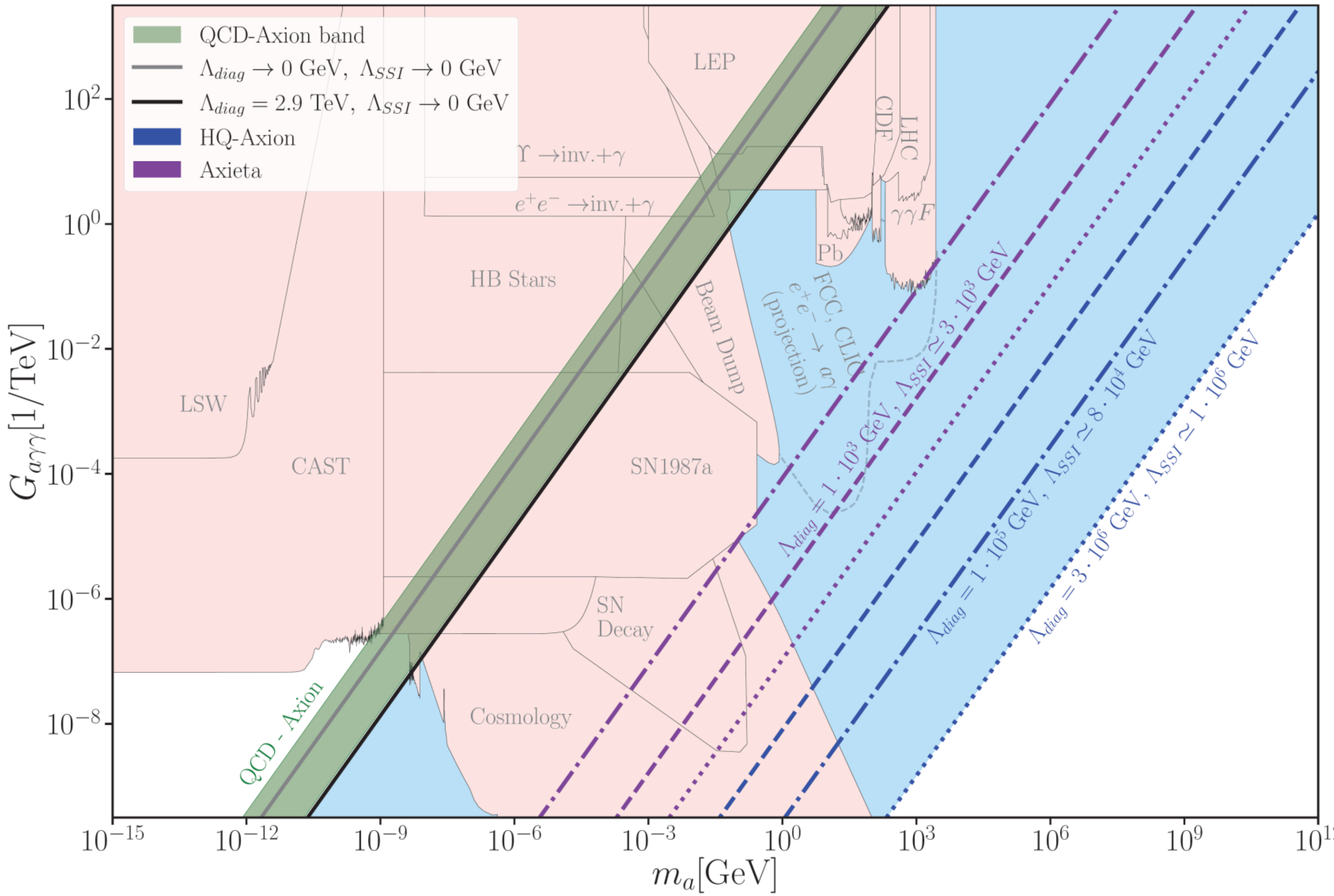
– M1 (KSVZ-like) or M2 (DFSZ-like) variants solve θ'

	$SU(3)_{\rm diag}$	$SU(3)_c$	$SU(3')$	$SU(6)$
Q	$\bar{\square}$	$\square$	1	20
q	$\square$	1	$\square$	1

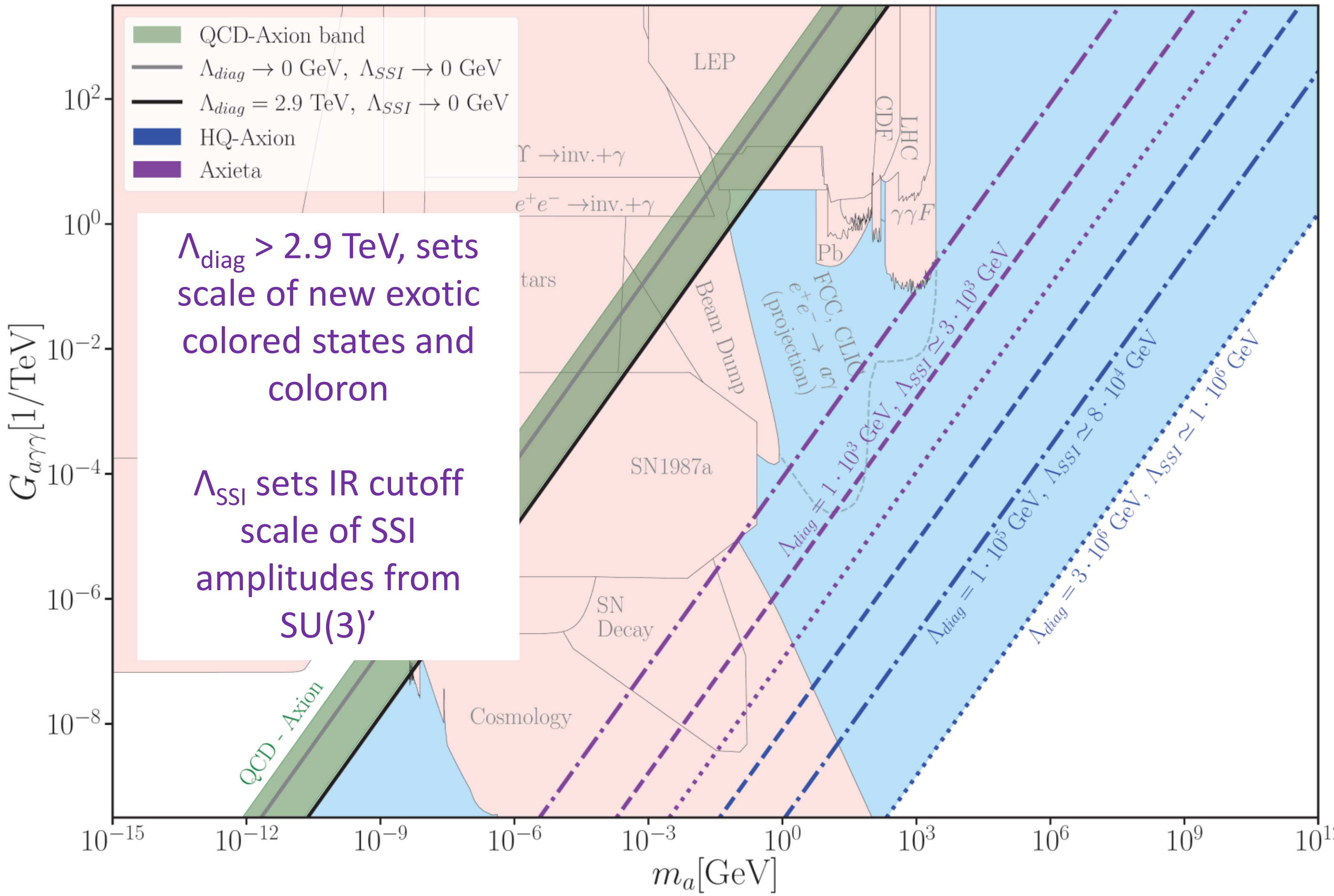
	$SU(3)_{\rm diag}$	$SU(3)_c$	$SU(3')$	$SU(6)$
Q	$\bar{\square}$	$\square$	1	20
Δ_2	-	-	$\bar{\square}$	$\square$

– v_{diag} separates exotic colored states from EW scale

SSI-driven mass enhancements – M1 variant



SSI-driven mass enhancements – M1 variant



$\Lambda_{\text{diag}} > 2.9$ TeV, sets scale of new exotic colored states and coloron

Λ_{SSI} sets IR cutoff scale of SSI amplitudes from SU(3)’

Model-building strong CP solutions

- Consider explicit – ***soft!*** – breaking of the $U(1)_{PQ}$ symmetry
  - DFSZ already has two incommensurate variants: $\lambda_3 \, S \, H_u H_d, \lambda_4 \, S^2 \, H_u H_d$
  - Having both λ_3 and λ_4 couplings nonzero (NB: global symmetries should be *accidental*) causes PQ mechanism to generally fail
- Study anarchic axion where PQ symmetry is softly broken
  - Also dictates effective description where PQ symmetry is non-linearly realized in full effective description (including EOMs)

The anarchic axion

- Tadpole of axion = observable $|\theta_{\text{eff}}|$
- Axion mass arises from canonical cubic DFSZ contribution and soft-breaking B_μ piece

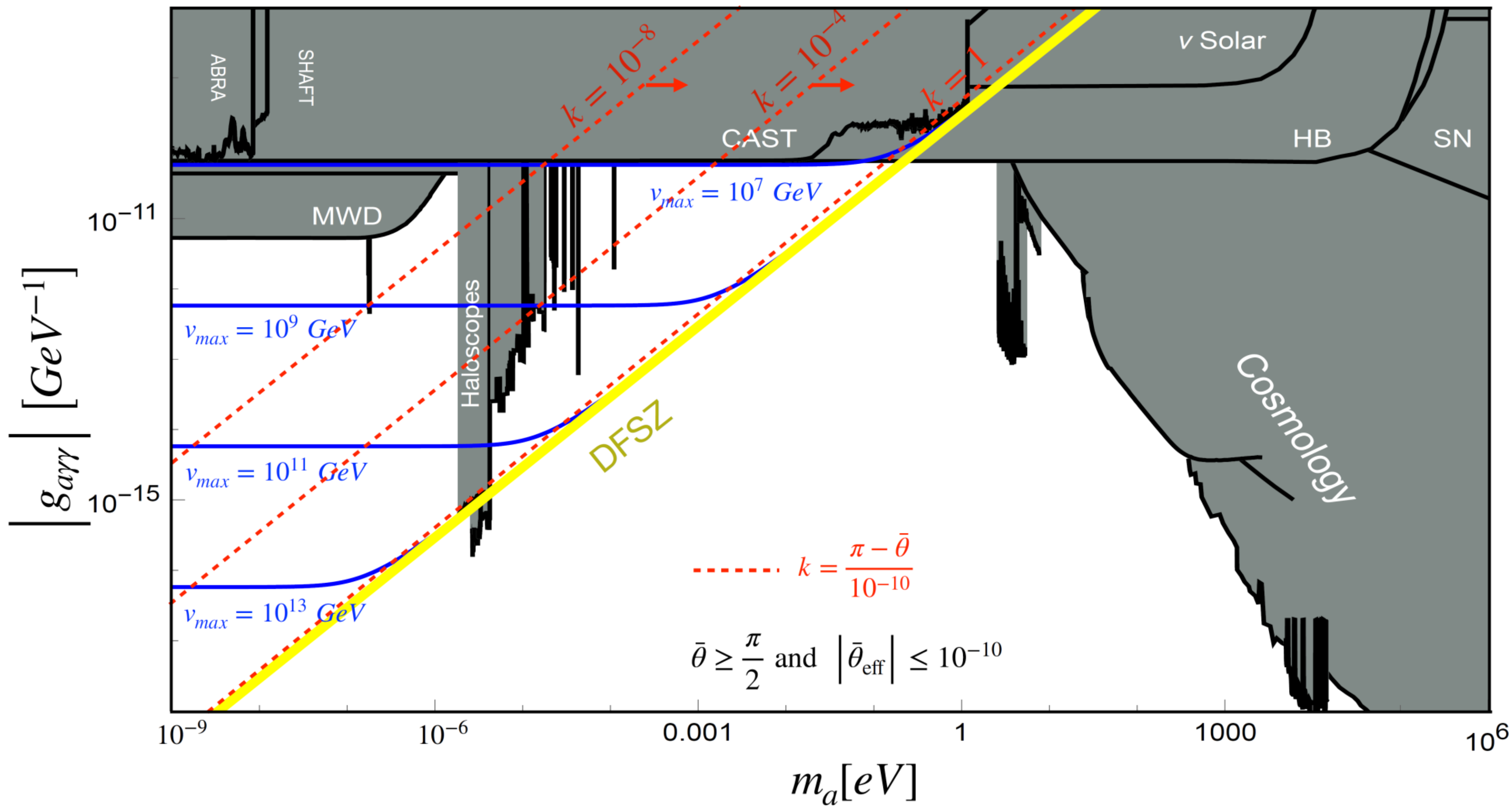
$$m_a^2 = \frac{\Lambda_{\text{QCD}}^4}{v_a^2}\left(N_g^2 \cos\left(N_g \bar{\theta}_{\text{eff}}\right) + \frac{v_a}{v_{\max}} \cos\left(\bar{\theta} - \bar{\theta}_{\text{eff}}\right)\right)$$

$$\frac{1}{f_a} \equiv \frac{N_g}{v_a} = -\frac{\cos\left(\bar{\theta} - \bar{\theta}_{\text{eff}}\right)}{2 N_g v_{\max} \cos\left(N_g \bar{\theta}_{\text{eff}}\right)} + \sqrt{\frac{m_a^2}{\Lambda_{\text{QCD}}^4 \cos\left(N_g \bar{\theta}_{\text{eff}}\right)} + \left(\frac{\cos\left(\bar{\theta} - \bar{\theta}_{\text{eff}}\right)}{2 N_g v_{\max} \cos\left(N_g \bar{\theta}_{\text{eff}}\right)}\right)^2} \,. \tag{11}$$

Elahi, Elor, Kivel, Laux, Najjari, FY PRD Letters **108** (2023) 3, [2301.08760]

Anarchic axion deviates from DFSZ band

Elahi, Elor, Kivel, Laux, Najjari, FY PRD Letters **108** (2023) 3, [2301.08760]



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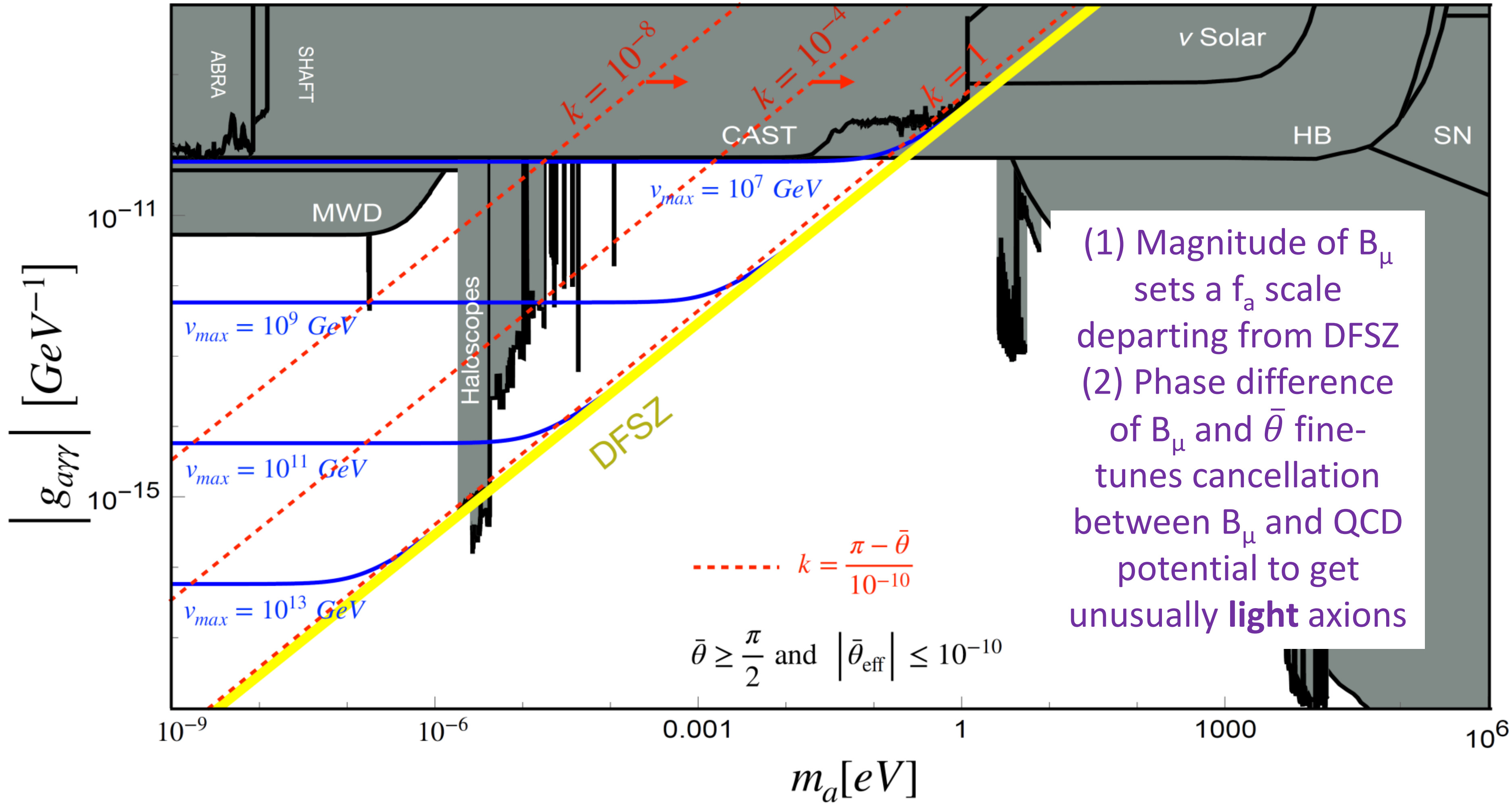


(1) Magnitude of B_μ sets a f_a scale departing from DFSZ

(2) Phase difference of B_μ and $\bar\theta$ fine-tunes cancellation between B_μ and QCD potential to get unusually **light** axions

Conclusions

- Axion field theory is rich with phenomenological applications and diverse model-building tools
  - Viable strong CP axion models with SSIs enhance axion mass into collider reach and beyond
  - Soft-breaking PQ symmetry motivates unusually light axions – prime targets for host of experiments
- Axion field theory offers more powerful insights into non-decoupling connections between vacuum angles, symmetry breaking, and flavor physics
  - Interesting targets for cosmological probes

Topological field configurations

- Instantons are a type of topological soliton, i.e. a field configuration that carries a topological index
  - Example of a topological soliton: scalar field in a double well potential in 1+1 dimensions interpolating between the two wells
  - $SU(3)_C$ instantons are gauge field configurations characterized integer winding numbers
    - Arising from the homotopy classification $\Pi_3(S_3) = \mathbb{Z}$
- Original phenomenological calculation by ‘t Hooft in the context of the U(1) problem and the η’ meson

Computation of the quantum effects due to a four-dimensional pseudoparticle*

G. ’t Hooft†
Physics Laboratories, Harvard University, Cambridge, Massachusetts 02138
(Received 28 June 1976)

A detailed quantitative calculation is carried out of the tunneling process described by the Belavin-Polyakov-Schwarz-Tyupkin field configuration. A certain chiral symmetry is violated as a consequence of the Adler-Bell-Jackiw anomaly. The collective motions of the pseudoparticle and all contributions from single loops of scalar, spinor, and vector fields are taken into account. The result is an effective interaction Lagrangian for the spinors.

Axion decay width vs. stability

- The primary target for QCD axion detection is the diphoton coupling

$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu} = g_{a\gamma\gamma}\, a\, \mathbf{E}\cdot\mathbf{B}$$

- Gives a decay width of (using E/N = 0)

$$\Gamma_{a\to\gamma\gamma} = \frac{g_{a\gamma\gamma}^2 m_a^3}{64\pi} = 1.1\times 10^{-24} s^{-1} \left(\frac{m_a}{eV}\right)^5$$

- Axion lives longer than age of universe for $m_a \lesssim 20\ eV$
  – Very cold dark matter (can have coherent oscillations with negligible velocity dispersion)

SSI-driven mass enhancements – M1 variant

Mass terms, parameters and notation

$$
\begin{aligned}
\mathcal{L} \supset\ & \frac{1}{2}\left(\frac{\eta_d}{F_a}\right)^2 4\left(K' v_{\rm diag}^3 + \frac{v_{\rm diag}^6}{2K_{\rm diag}^2}\right) \\
&+ \frac{1}{2}\left(\frac{a}{F_a}\right)^2 6\left(\frac{v_{\rm diag}^6}{2K_{\rm diag}^2} + \frac{v_{\rm diag}^3 v^9}{4K^8} + \frac{v^6 v_{\rm diag}^3}{2K^8} m_u \Lambda_u^2 + \frac{v^6 v_{\rm diag}^3}{2K^8} m_d \Lambda_d^2\right) \\
&+ \frac{1}{2}\left(\frac{\eta'}{F_{\eta'}}\right)^2 \left(m_u v^3 + m_d v^3 + \frac{v_{\rm diag}^3 v^9}{K^8} + \frac{v^6 v_{\rm diag}^3}{2K^8} m_u \Lambda_u^2 + \frac{v^6 v_{\rm diag}^3}{2K^8} m_d \Lambda_d^2\right) \\
&+ \frac{1}{2}\left(\frac{\pi^0}{F_{\pi^0}}\right)^2 \left(m_u v^3 + m_d v^3 + \frac{v^6 v_{\rm diag}^3}{2K^8} m_u \Lambda_u^2 + \frac{v^6 v_{\rm diag}^3}{2K^8} m_d \Lambda_d^2\right) \\
&+ \left(\frac{\eta_d}{F_a}\right)\left(\frac{a}{F_a}\right)\left(2\sqrt{6}\frac{v_{\rm diag}^6}{2K_{\rm diag}^2}\right) \\
&+ \left(\frac{a}{F_a}\right)\left(\frac{\eta'}{F_{\eta'}}\right)\left(\sqrt{6}\right)\left(\frac{v_{\rm diag}^3 v^9}{2K^8} + \frac{v^6 v_{\rm diag}^3}{2K^8} m_u \Lambda_u^2 + \frac{v^6 v_{\rm diag}^3}{2K^8} m_d \Lambda_d^2\right) \\
&+ \left(\frac{\eta'}{F_{\eta'}}\right)\left(\frac{\pi^0}{F_{\pi^0}}\right)\left(m_u v^3 - m_d v^3\right) ,
\end{aligned}
$$

$$
m_+ = m_u + m_d , \quad m_- = m_d - m_u , \quad \mu = \frac{m_u m_d}{m_u + m_d} ,
$$

$$
\mu L^2 = m_u \Lambda_u^2 + m_d \Lambda_d^2 , \quad \Lambda_{\rm inst}^3 = \frac{L^2}{4K^8} v^6 v_{\rm diag}^3 , \quad \Lambda_{\eta'}^4 = \frac{v_{\rm diag}^3 v^9}{4K^8}
$$

$$
\Lambda_{\rm SSI}^4 = K' v_{\rm diag}^3 , \quad \Lambda_{\rm diag}^4 = \frac{v_{\rm diag}^6}{2K_{\rm diag}^2} .
$$

Supplemental definitions

$$v_1 = v \sin\phi \,, \quad v_2 = v\cos\phi \,, \quad v_3 = v_a \sin\beta = v_A \cos\beta \,, \quad v_a \cos\beta = v \sin\phi \cos\phi$$

$$\tan\phi = v_1/v_2, \; \tan\beta = v_A/v_a$$

- Goldstones aligned with their vevs

$$\begin{pmatrix} G \\ a \\ A \end{pmatrix} = \begin{pmatrix} s_\phi c_\gamma & -c_\phi c_\gamma & -s_\gamma \\ c_\phi c_\beta - s_\phi s_\beta s_\gamma & s_\phi c_\beta + c_\phi s_\beta s_\gamma & -s_\beta c_\gamma \\ c_\phi s_\beta + s_\phi c_\beta s_\gamma & s_\phi s_\beta - c_\phi c_\beta s_\gamma & c_\beta c_\gamma \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$\begin{aligned} V_{\rm ang} &= -|B_\mu| \left[\prod_{i=1}^{2} (v_i + h_i)\right] \cos\left(\sum_{i=1}^{2} \frac{a_i}{v_i} - \theta_\mu\right) - \frac{|C_\lambda|}{\sqrt{2}} \left[\prod_{i=1}^{3} (v_i + h_i)\right] \cos\left(\sum_{i=1}^{3} \frac{a_i}{v_i} - \theta_\lambda\right) \\ &\xrightarrow{O(3)} -|B_\mu| \left[\prod_{i=1}^{2} (v_i + h_i)\right] \cos\left(\frac{a}{v_a} + \frac{A}{v_A} \tan^2\beta - \theta_\mu\right) - \frac{|C_\lambda|}{\sqrt{2}} \left[\prod_{i=1}^{3} (v_i + h_i)\right] \cos\left(\frac{A}{v_A} \sec^2\beta - \theta_\lambda\right) \end{aligned}$$

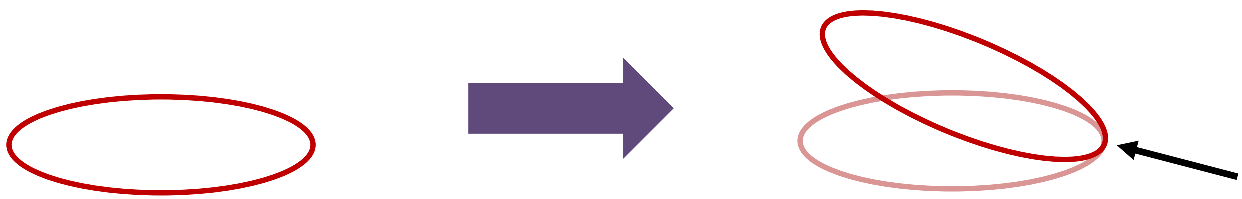
Recap of standard PQ mechanism

- Instanton-induced potential is consequence of $U(1)_{PQ} \times SU(3)_C^2$ anomaly

't Hooft; Callan, Dashen Gross; Weinberg; Wilczek

$$\mathcal{L} = \left(\frac{a}{f_a} - \bar{\theta}\right)\frac{1}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

  – Below Λ_{QCD}, evaluating instanton diagrams leads to approximate cosine potential for PNGB of $U(1)_{PQ}$



$$\bar{\theta} = \frac{\langle a \rangle}{f_a}$$

PNGB flat potential

Instanton-induced tilted potential

    - Classical shift symmetry becomes periodic with f_a scale
    - Diverse cosmological implications from possible axion misalignment and strings from multiple PQ vacua

Current status of vanilla QCD axions

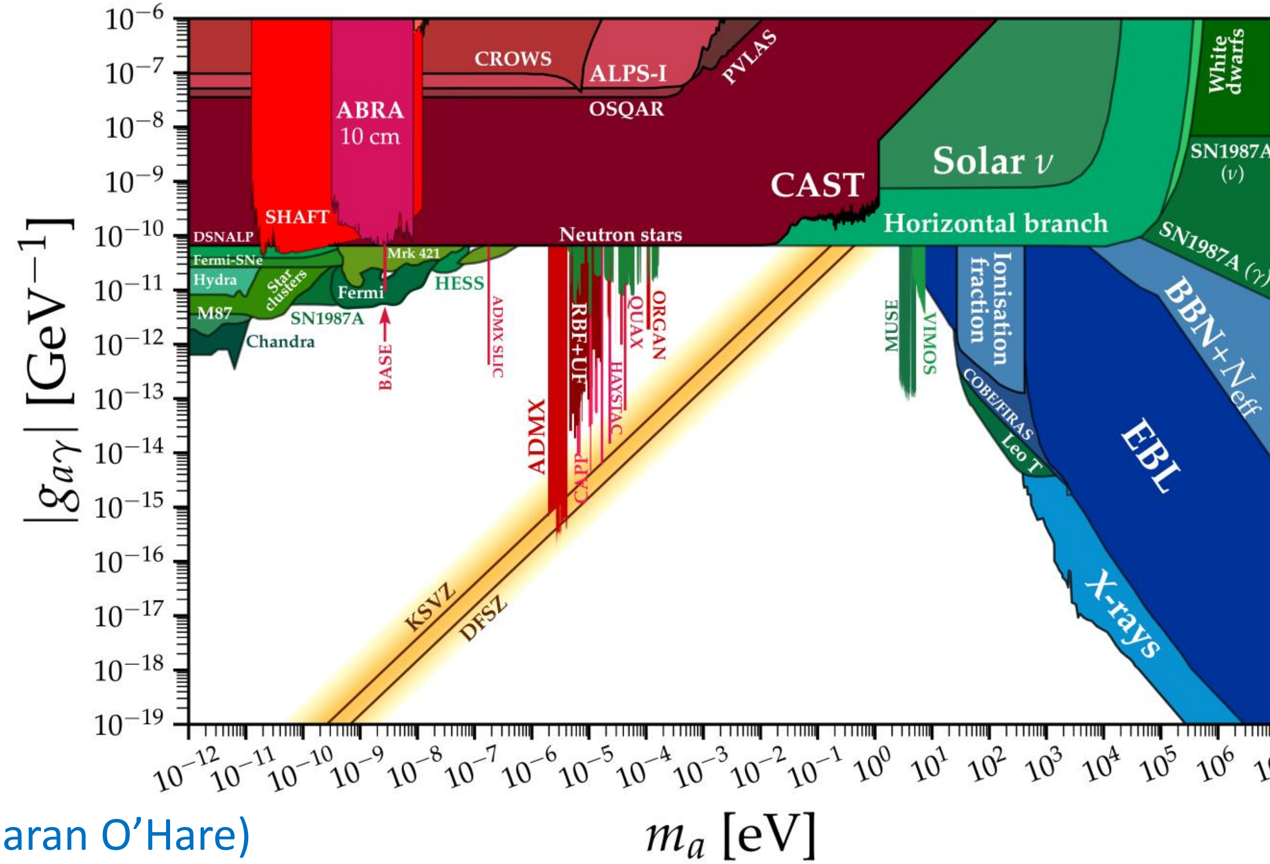
$$m_a = 5.70 \pm 0.06 \pm 0.04\,\mu\text{eV}\left(\frac{10^{12}\text{GeV}}{f_a}\right)$$

$$g_{a\gamma\gamma} = \frac{\alpha_{EM}}{2\pi f_a}\left(\frac{E}{N} - 1.92(4)\right)$$

m_a: quark mass and higher order uncertainties
$g_{a\gamma\gamma}$: O(α) in QED and NNLO in χPT
Gorghetto, Villadoro [1812.01008]

The standard QCD axion band focuses on sub-eV axion masses and tiny axion-photon couplings

Width of band roughly given by E/N = 44/3 and E/N = 5/3 as boundary values



PDG (from AxionLimits by Ciaran O'Hare)

Small-size instanton effects

- Key insight: Use axion sensitivity to UV physics to enhance quality of axion solution

Agrawal, Howe [1710.04213, 1712.05803]

  – Embed $SU(3)_C$ in larger UV gauge group
    - Confinement of extended color gauge group will give additional instanton-induced potential terms to light axion
    - Requires non-trivial book-keeping to trace PQ symmetry from IR to UV
  – Practical consequence: Extended color groups typically involve coloron/axigluon color octet vectors and additional collider signatures

See, e.g. Dobrescu, FY [1306.2629]

Refresher: SM instanton effects

- SM + axion Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{\partial_\mu a}{F_a}\left(\sum_{i=1}^{N_f} c_1^i \bar{q}_i \gamma_\mu \gamma_5 q_i\right) - \left(\sum_{i=1}^{N_f} m_i \bar{q}_L^i e^{i c_2^i a/F_a} q_R^i + \text{h.c.}\right)$$

$$- \frac{a}{F_a}\left(c_3^G \frac{g_s^2}{32\pi^2} G\tilde{G} + c_3^W \frac{g^2}{32\pi^2} W\tilde{W} + c_3^B \frac{g'^2}{32\pi^2} B\tilde{B}\right) ,$$

Kim, Carosi (2008)

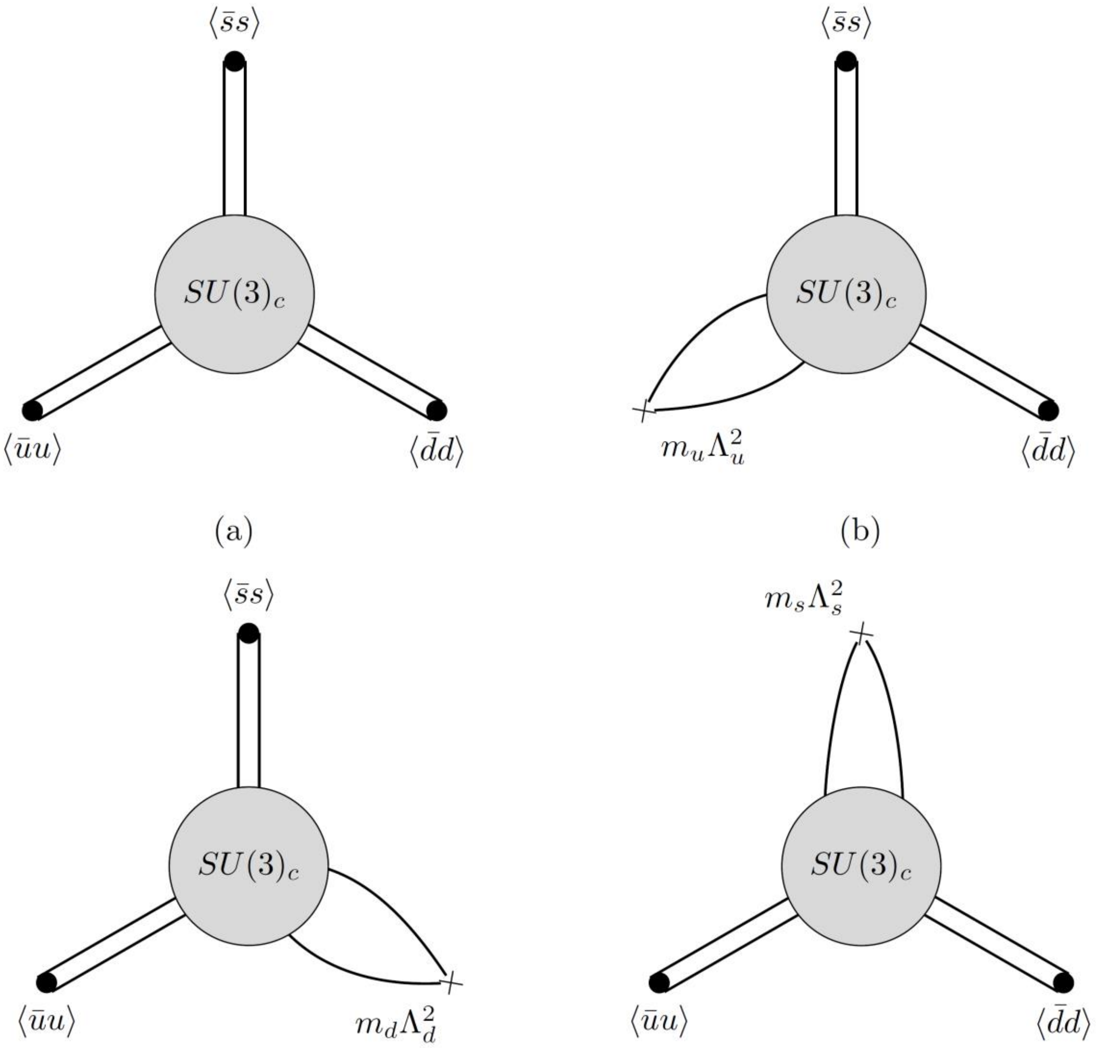
- Axion mass is generated by gluon operator
  - Via index theorem, instanton action induces 't Hooft determinantal operator according to PQ color anomaly

$$\mathcal{L}_{\text{det}} = (-1)^{N_f} K^{4-3N_f} \left(\prod_{i=1}^{N_f} \det\left(\bar{q}_L^i q_R^i\right)\right) e^{-i c_3^G \frac{a}{F_a}} + \text{h.c.}$$

Refresher: SM instanton effects

- Calculate leading determinantal operators
  - Correspond to "instanton flower" diagrams
  - Power counting in chiral symmetry breaking parameters

$$\mathcal{L}_{\text{det}} = -\frac{1}{K^5}\sum_i A_i$$



Refresher: SM instanton effects

- Operators lead to mixing of π^0, η, η' and axion

$$A_1 = \left(\prod_i \det(\bar{q}_{i,L}\, q_{i,R})\right) e^{-ic_3^G \theta_a} + \text{ h.c. },$$

$$\sim \left(\frac{v^3}{2} \exp\big(i(\theta_{\pi^0} + \theta_{\eta'})\big)\right) \left(\frac{v^3}{2} \exp\big(i(-\theta_{\pi^0} + \theta_{\eta'})\big)\right) \left(\frac{v^3}{2}\right) e^{-ic_3^G \theta_a} + \text{ h.c. },$$

$$= \frac{v^9}{8} \left(\exp\big(i(2\theta_{\eta'} - c_3^G \theta_a)\big) + \text{h.c.}\right) = \frac{v^9}{4} \cos\big(2\theta_{\eta'} - c_3^G \theta_a\big) ,$$

$$A_2 = \frac{v^6}{2} m_u \Lambda_u^2 \cos\big(\theta_{\pi^0} + \theta_{\eta'} - c_3^G \theta_a\big) ,$$

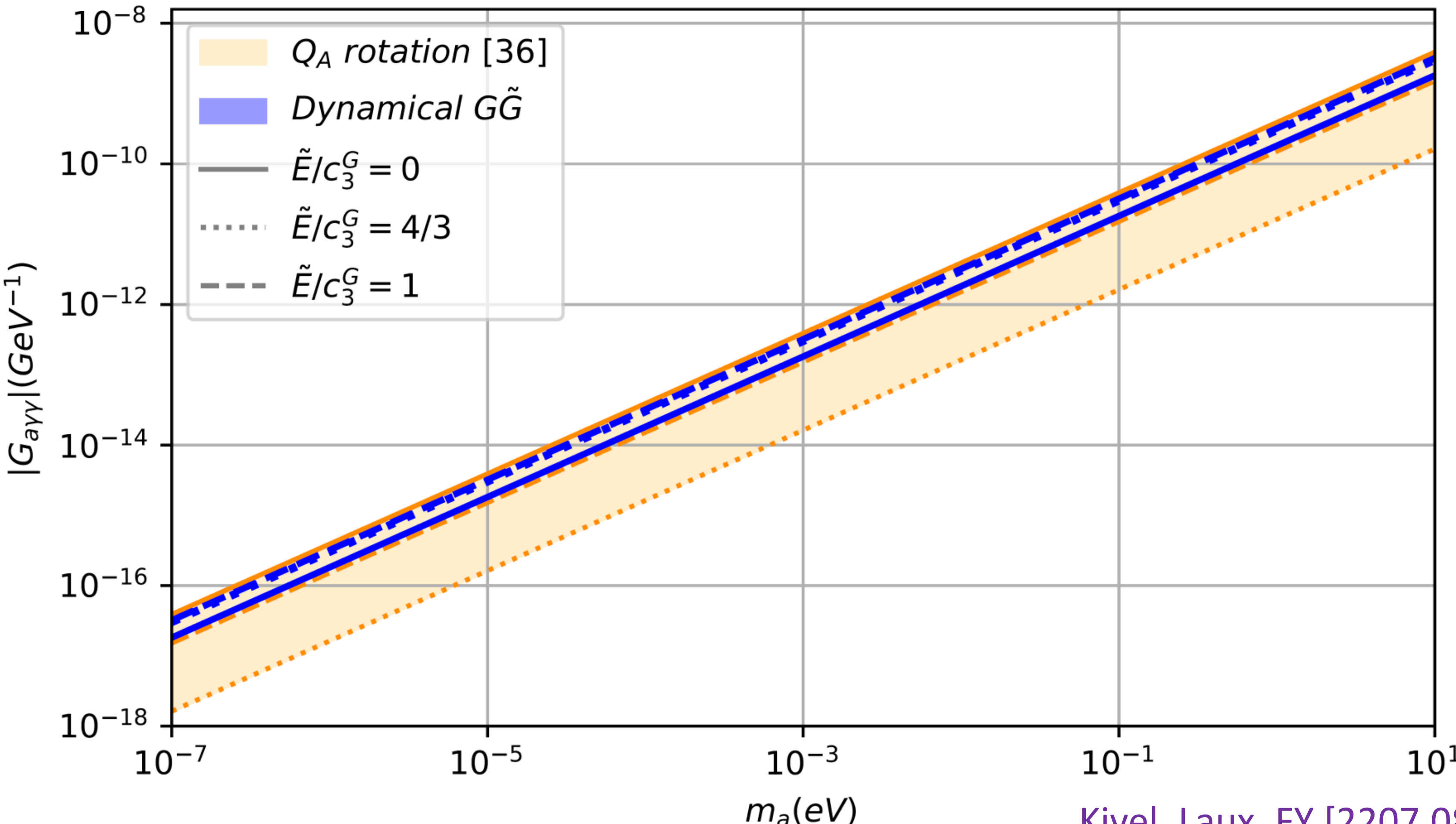
$$A_3 = \frac{v^6}{2} m_d \Lambda_d^2 \cos\big(-\theta_{\pi^0} + \theta_{\eta'} - c_3^G \theta_a\big) ,$$

$$A_4 = \frac{v^6}{2} m_s \Lambda_s^2 \cos\big(2\theta_{\eta'} - c_3^G \theta_a\big) .$$

  - Encapsulate relevant instanton effects via chiral insertions

Cross-check: no SSIs

- Even neglecting η-mixing, we reproduce QCD axion band



Kivel, Laux, FY [2207.08740]

Adding small-size instanton effects

- Determinantal approach readily accounts for SSIs
  - In contrast to χEFT, we capture alignment of explicit and instanton PQ breaking by different PNGBs
  - Can embed vacuum angle θ in different ways in UV
    - Thought experiment: SU(3) x SU(3) vs. SU(6)
    - SU(3) × SU(3) – original model by Agrawal, Howe
    - SU(6) × SU(3)' – model by Gaillard, et. al.
      - Embeds $SU(3)_C$ into SU(6) and adopts massless up' solution for SU(6)
    - Intuitively, the mixing of PNGBs induced by $SU(3)_C$ instantons and small-size instantons determines the embedding of PQ symmetry from low to high scales

Gaillard, Gavela, Houtz, Quilez, del Rey [1805.064365]

M1 variant

- QCD axion inherited from M1 composite axion and axieta admixture
  - Exotic quark fields after SU(6) breaking to $\mathrm{SU}(3)_{\mathrm{diag}}$

$$\mathcal{L} \supset \bar{Q}_{I,i}\left(i\delta_{IJ}\ \delta_{ij}\ \partial\!\!\!/ - g_{\mathrm{diag}}\ T^A_{IJ} A\!\!\!/^A_{\mathrm{diag}} \delta_{ij} - g_s\ \delta_{IJ} T^a_{ij} A\!\!\!/^a\right) Q_{J,j}$$
$$+ \bar{q}_{I,i'}\left(i\delta_{IJ}\ \delta_{i'j'} \partial\!\!\!/ - g_{\mathrm{diag}}\ T^A_{IJ} A\!\!\!/^A_{\mathrm{diag}} \delta_{i'j'} - g'\ \delta_{IJ} T^b_{i'j'} A\!\!\!/'^{,b}\right) q_{J,j'}$$
$$+ \theta_{\mathrm{diag}} \frac{\alpha_{\mathrm{diag}}}{8\pi} G_{\mathrm{diag}} \tilde{G}_{\mathrm{diag}} + \bar{\theta}\frac{\alpha_s}{8\pi} G\tilde{G} + \theta' \frac{\alpha'}{8\pi} G'\tilde{G}' + \frac{(g')^2\, \Lambda^2_{\mathrm{CUT}}}{2} A'_\mu A'^\mu$$

- Treat all field strength duals via ‘t Hooft determinantal operators to calculate instanton effects

M1 variant – SSI amplitudes

$$\mathcal{L}_{\rm det} = -K' A_1 - \frac{1}{K^2_{\rm diag}} A_2 - \frac{1}{K^8}(A_3 + A_4 + A_5)$$

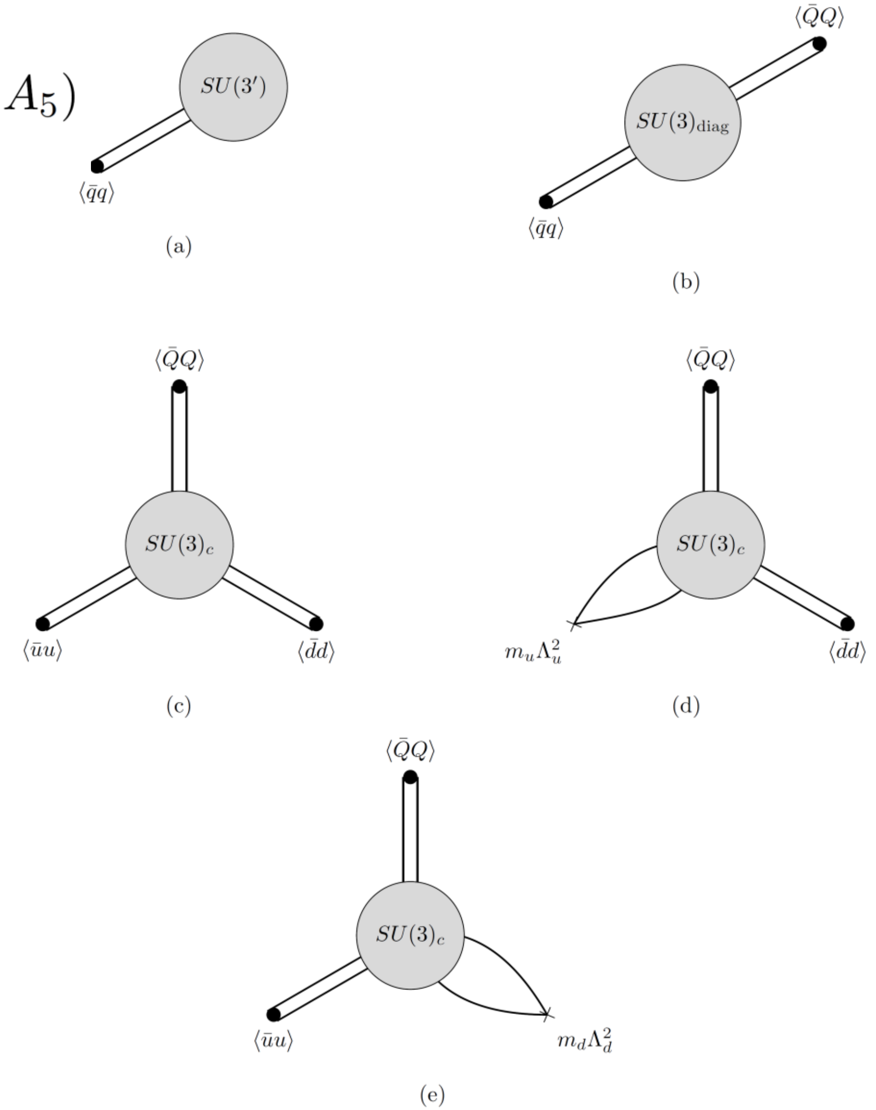
$$A_1 = v^3_{\rm diag} \cos\left(\frac{2\eta_d}{F_a}\right) \,,$$

$$A_2 = \frac{v^6_{\rm diag}}{2} \cos\left(\frac{2\eta_d}{F_a} + \frac{\sqrt{6}a}{F_a}\right) \,,$$

$$A_3 = \frac{v^3_{\rm diag} v^9}{4} \cos\left(\frac{\sqrt{6}a}{F_a} + 2\frac{\eta'}{F_{\eta'}}\right) \,,$$

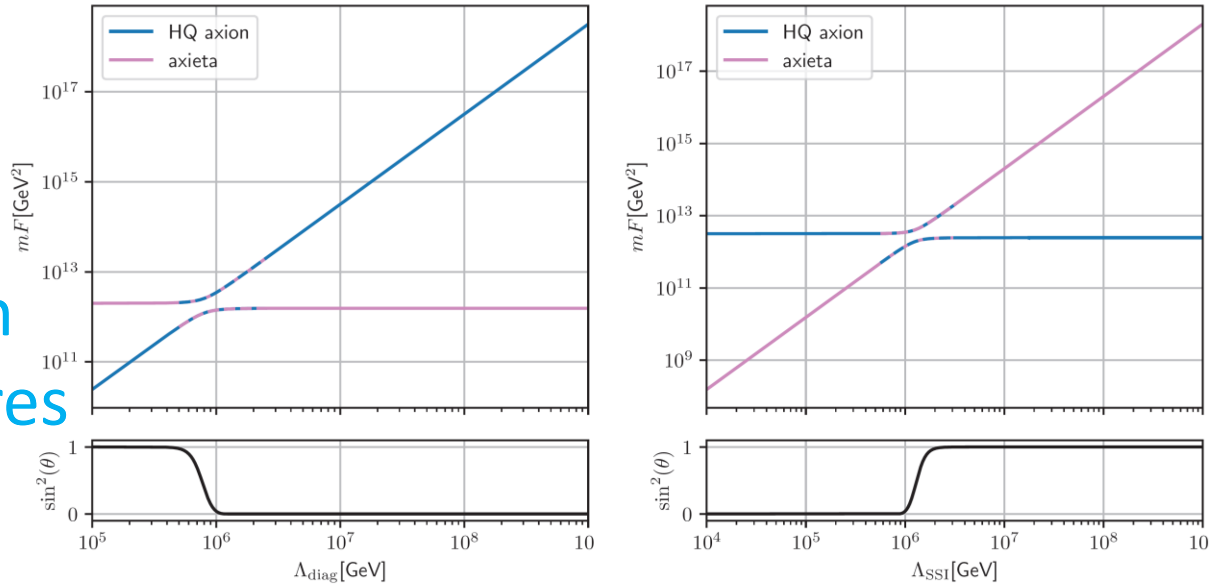
$$A_4 = \frac{v^3_{\rm diag} v^6}{2} m_u \Lambda_u^2 \cos\left(\frac{\sqrt{6}a}{F_a} + \frac{\eta'}{F_{\eta'}} + \frac{\pi^0}{F_{\pi^0}}\right)$$

$$A_5 = \frac{v^3_{\rm diag} v^6}{2} m_d \Lambda_d^2 \cos\left(\frac{\sqrt{6}a}{F_a} + \frac{\eta'}{F_{\eta'}} - \frac{\pi^0}{F_{\pi^0}}\right)$$



Axion and axieta masses and level repulsion

- After diagonalizing PNGB matrix, get eigenvalues of axion and axieta admixtures with non-trivial $\Lambda_{\rm SSI}$ dependence



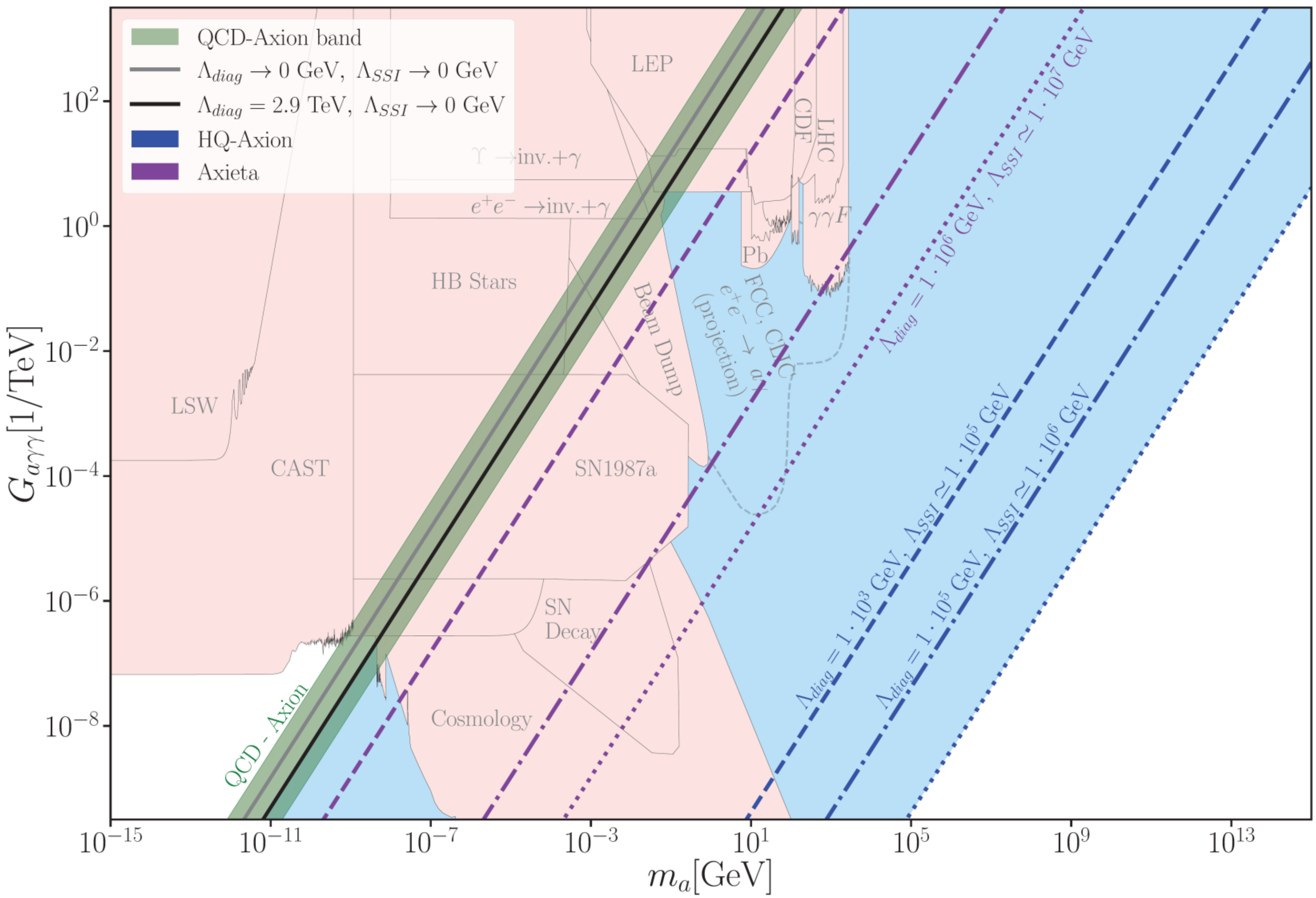
$$m_a^2 F_a^2 = 4(\Lambda_{\rm diag}^4 + \Lambda_{\rm SSI}^4) - 24\Lambda_{\rm diag}^8$$
$$\times \left| 2\Lambda_{\rm SSI}^4 - \Lambda_{\rm diag}^4 - 3(m_a^2 F_a^2)^{\rm KSVZ} - \sqrt{\left(2\Lambda_{\rm SSI}^4 - \Lambda_{\rm diag}^4 - 3(m_a^2 F_a^2)^{\rm KSVZ}\right)^2 + 24\Lambda_{\rm diag}^8} \right|^{-1}$$

$$m_{\eta_d}^2 F_a^2 = 2\Lambda_{\rm SSI}^4 + 5\Lambda_{\rm diag}^4 + 3(m_a^2 F_a^2)^{\rm KSVZ} + \sqrt{\left(2\Lambda_{\rm SSI}^4 - \Lambda_{\rm diag}^4 - 3(m_a^2 F_a^2)^{\rm KSVZ}\right)^2 + 24\Lambda_{\rm diag}^8} \, ,$$

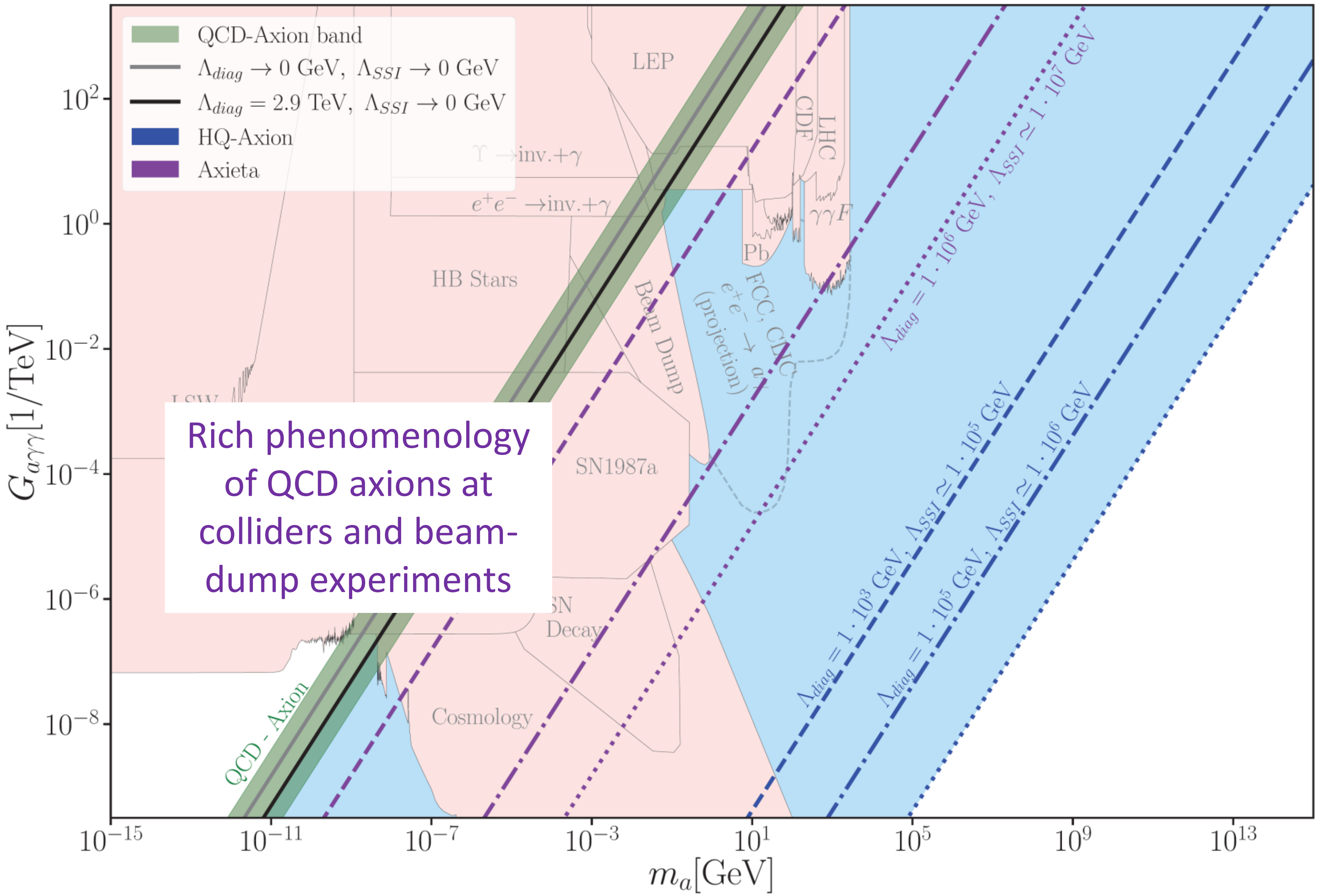
Anarchic axion

- Revisit fundamental origin of PQ symmetry: anomalous global symmetry
  - In general, global symmetries stem from field content multiplicity, explicit breaking by Lagrangian interaction terms
    - Not necessarily renormalizable
- Vanilla axion: sole source of PQ breaking arises from $U(1)_{PQ} \times SU(3)^2$ anomaly
  - But no generally rigorous embedding between low-scale PQ symmetry and UV PQ symmetries = quality problem
- Consequence of soft PQ breaking understudied

SSI-driven mass enhancements – M2 variant



SSI-driven mass enhancements – M2 variant



Rich phenomenology of QCD axions at colliders and beam-dump experiments

Building anarchy

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$\mathbb{Z}_5$	$U(1)_{PQ}$
Q_L^i	3	2	1/6	0	X_Q
u_R^i	3	1	2/3	1	X_Q-X_1
d_R^i	3	1	-1/3	0	X_Q-X_2
L_L^i	1	2	-1/2	0	X_L
e_R^i	1	1	-1	0	X_L-X_2
H_1	1	2	-1/2	4	X_1
H_2	1	2	1/2	0	X_2
Φ	1	1	0	1	X_3

- Begin with DFSZ axion
- Scalar potential (1a) leaves global $U(1)_{H1} \times U(1)_{H2} \times U(1)_\Phi$ symmetry charges undefined
- Canonical DFSZ scalar term (1b) defines PQ charges

$$V = \sum_{i=1,2} \left(\mu_i^2|H_i|^2 + \lambda_i|H_i|^4\right) + \lambda|H_1|^2|H_2|^2 + \lambda'|H_1H_2|^2$$
$$+ \mu_3^2|\Phi|^2 + \lambda_3|\Phi|^4 + \lambda_{13}|H_1|^2|\Phi|^2 + \lambda_{23}|H_2|^2|\Phi|^2\,, \tag{1a}$$

$$V_{\text{break}}^{C_\lambda} = -C_\lambda H_1 H_2 \Phi + \text{h.c.}\,, \tag{1b}$$

Building anarchy

$$V = \sum_{i=1,2} \left(\mu_i^2|H_i|^2 + \lambda_i|H_i|^4\right) + \lambda|H_1|^2|H_2|^2 + \lambda'|H_1H_2|^2$$
$$+ \mu_3^2|\Phi|^2 + \lambda_3|\Phi|^4 + \lambda_{13}|H_1|^2|\Phi|^2 + \lambda_{23}|H_2|^2|\Phi|^2 \,, \tag{1a}$$

$$V_{\text{break}}^{C_\lambda} = -C_\lambda H_1 H_2 \Phi + \text{h.c.} \,, \tag{1b}$$

- With global PQ symmetry set, now add PQ soft breaking term

$$V_{\text{break}}^{B_\mu} = -B_\mu H_1 H_2 + \text{h.c.} \,,$$

- Two sources of PQ breaking: color anomaly and B_μ
  - Three neutral Goldstones: one for SM Z, one 2HDM A, one axion
  - "Standard" 2HDM potential terms forbidden by Z_5 symmetry

Building anarchy

- Define angular fields $\alpha \equiv a/v_a,\ \alpha' \equiv A/v_A$
- Use C_λ to effectively decouple 2HDM A

$$
\begin{aligned}
-V_{\rm ang} = \ & \Lambda_{\rm QCD}^4 \cos\left(N_g\left(\alpha + \alpha'\delta^2\right)\right) \\
& + \Lambda_{\rm QCD}^4 \frac{v_a}{v_{\rm max}} \cos\left(\alpha + \alpha'\delta^2 + \bar{\theta}\right) \\
& + \frac{|C_\lambda| v v_A^2}{\sqrt{2}\delta(1+\delta^2)} \cos\left(\alpha'(1+\delta^2)\right)
\end{aligned}
$$

$$
\begin{aligned}
v_{\rm max} &\equiv \frac{\Lambda_{\rm QCD}^4}{|B_\mu| v}\sqrt{1+\delta^2} \\
\delta &= v_A/v_a
\end{aligned}
$$

  – Second line is effect of finite B_μ
  – v_{max} = maximal value of PQ vev v_a

Building anarchy

- UV phases shuffled into $N_g\,\bar{\theta} = \theta_{SM} - N_g\theta_\mu$
- Tadpole of axion = observable $|\theta_{\text{eff}}|$
- Axion mass arises from canonical DFSZ contribution and soft-breaking B_μ piece

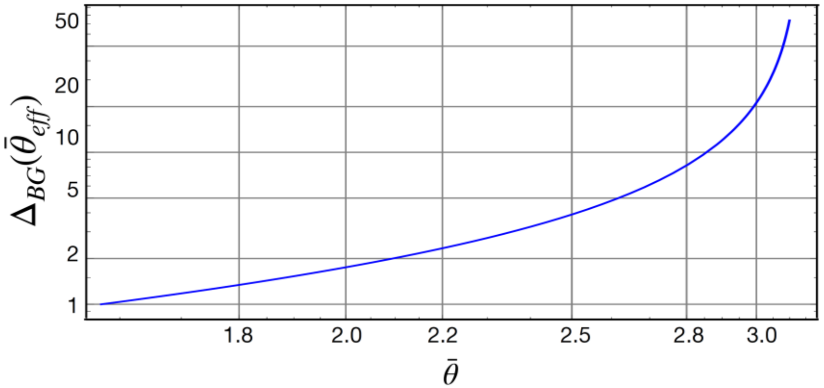
$$m_a^2 = \frac{\Lambda_{\text{QCD}}^4}{v_a^2}\left(N_g^2 \cos\left(N_g\bar{\theta}_{\text{eff}}\right) + \frac{v_a}{v_{\max}}\cos\left(\bar{\theta} - \bar{\theta}_{\text{eff}}\right)\right)$$

$$\frac{1}{f_a} \equiv \frac{N_g}{v_a} = -\frac{\cos\left(\bar{\theta} - \bar{\theta}_{\text{eff}}\right)}{2N_g v_{\max}\cos\left(N_g\bar{\theta}_{\text{eff}}\right)} \qquad (11)$$
$$+\sqrt{\frac{m_a^2}{\Lambda_{\text{QCD}}^4\cos\left(N_g\bar{\theta}_{\text{eff}}\right)} + \left(\frac{\cos\left(\bar{\theta} - \bar{\theta}_{\text{eff}}\right)}{2N_g v_{\max}\cos\left(N_g\bar{\theta}_{\text{eff}}\right)}\right)^2}\,.$$

Fine-tuning measure

- Since finite residual nEDM is *calculable*, can use Giudice-Barbieri fine-tuning measure

$$\Delta_{\mathrm{BG}}(\bar{\theta}_{\mathrm{eff}}) \equiv \left| \frac{\bar{\theta}}{\bar{\theta}_{\mathrm{eff}}} \frac{\partial \bar{\theta}_{\mathrm{eff}}}{\partial \bar{\theta}} \right|$$



- Open model-building question whether more complicated UV model can reduce fine-tuning
  - Ongoing work with Nelson-Barr origin of B_μ term
- Anarchic axion model serves as target toy effective description aiming for light axions