Extending the SM with Vector-Like Quarks: consequences for CKM unitarity and CP violation

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TÉCNICO LISBOA

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Based on:

[2207.14235], [2111.15401] and **[2210.14248]**

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Motivation

- A fourth chiral generation of quarks is ruled out, but the quark sector can be extended with VLQs.

- VLQs take part in many models from GUTs, to the Nelson-Barr solutions to the strong CP problem. They have a rich phenomenology that can be used to try to explain several types of anomalies/ tensions.

- **Cabibbo Angle Anomaly (CAA):** The independent determinations of $|V_{us}|$ (semi-leptonic kaon decays), the ratio $|V_{us}/V_{ud}|$ (kaon and pion leptonic decays) and $|V_{ud}|$ (β decays) are not in agreement with each other within the framework of the CKM unitary of SM (discrepancy of $\sim 3\sigma$). These values fit best to the relation

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta^2, \quad \Delta \approx 0.04$$

- Extensions with VLQs iso-singlets naturally introduce deviations to CKM unitarity.

- **CP Violation:** The introduction of VLQs allows for extra Yukawa couplings and bare mass terms. In principle, this means more physical phases which could lead to the enhancement of CP violation in the quark sector.

Solving the CAA with an up-type VLQ iso-singlet

Introducing a **Q=2/3 VLQ iso-singlet** T=T_L+T_R with mass m_T leads to:

$$m_{u,d} = v y_{u,d}$$

- $$-\mathcal{L}_Y = y^u_{ij}\overline{Q'_{iL}}\tilde{\Phi}u'_{jR} + y^u_{i4}\overline{Q'_{iL}}\tilde{\Phi}T'_R + y^d_{ij}\overline{Q'_{iL}}\Phi d'_{jR} + \underbrace{M^u_i \overline{T}'_L u'_{iR} + M^u_4 \overline{T}'_L T'_R}_{\textbf{Bare mass terms}} + h.c.$$

$$\mathcal{M}_u = \underbrace{\begin{pmatrix} m_u \\ M_u \end{pmatrix}}_{4} \begin{array}{l} \}3 \\ \}1 \end{array}$$

- $$-\mathcal{L}_m = \begin{pmatrix} \overline{u}'_L & \overline{T}'_L \end{pmatrix} \mathcal{M}_u \begin{pmatrix} u'_R \\ T'_R \end{pmatrix} + \overline{d}'_L m_d d'_R + h.c.$$

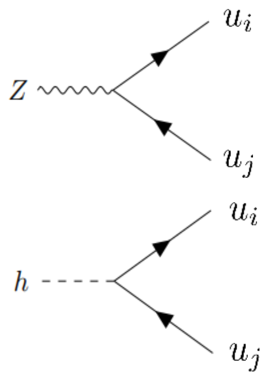
Both T_L and T_R have the same quantum numbers as u_R.

In the physical basis we get **non-unitary mixing** and **tree-level FCNCs**:

- $$\mathcal{L}_W = -\frac{g}{\sqrt{2}} W^+ \begin{pmatrix} \overline{u}_L & \overline{T}_L \end{pmatrix} V_{\text{CKM}}\, \gamma_\mu d_L + h.c. \longrightarrow$$ Non-Unitary 4x3 CKM matrix

- $$\mathcal{L}_Z = \frac{g}{2c_W} Z^\mu \left[\begin{pmatrix} \overline{u}_L & \overline{T}_L \end{pmatrix} F_u \gamma_\mu \begin{pmatrix} u_L \\ T_L \end{pmatrix} - \overline{d}_L \gamma_\mu d_L - 2 s^2_W J^\mu_{\text{EM}} \right]$$

- $$-\mathcal{L}_h = \frac{h}{v} \left[\begin{pmatrix} \overline{u}_L & \overline{T}_L \end{pmatrix} F_u \begin{pmatrix} D_u & 0 \\ 0 & m_T \end{pmatrix} \begin{pmatrix} u_R \\ T_R \end{pmatrix} + \overline{d}_L D_d d_R \right] + h.c.$$



Tree-Level FCNCs

Solving the CAA with an up-type VLQ iso-singlet

The mixing can be parametrized as:

- Auxiliary Unitary Matrix: $\mathcal{V} = \underbrace{\begin{pmatrix} A_u \\ B_u \end{pmatrix}}_{4} \begin{matrix} \}3 \\ \}1 \end{matrix}$
- Non-Unitary 4x3 CKM Matrix: $V_{\mathrm{CKM}} = A_u^\dagger$
- Matrix Controlling FCNCs: $F_u = V_{\mathrm{CKM}} V_{\mathrm{CKM}}^\dagger$

$$\mathcal{V}^\dagger = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & s_{24}e^{-i\delta_{24}} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24}e^{i\delta_{24}} & 0 & c_{24} \end{pmatrix} \begin{pmatrix} c_{14} & 0 & 0 & s_{14}e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{23} & s_{23} & 0 \\ 0 & -s_{23} & c_{23} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} & 0 \\ 0 & 1 & 0 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 & 0 \\ -s_{12} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\theta_{ij} \in [0, \pi/2]$$
$$\delta_{ij} \in [0, 2\pi]$$

We have for the first row of the mixing: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - s_{14}^2 \xRightarrow{\text{CAA}} s_{14} \approx 0.04 \sim \lambda^2$

At first glance, a "minimal" solution to the CAA could be: $s_{14} \approx 0.04$, $s_{24}, s_{34} = 0$ $\quad \delta_{14}, \delta_{24} \longrightarrow$ Factored out

We study this case in **Botella et al. [2111.15401].**

General solution and its parameter space analyzed in **Branco et al. [2103.13409**], but VLQ mass bounds **assume predominant couplings to the third generation**.

Phenomenology: the s_{14}-dominance limit

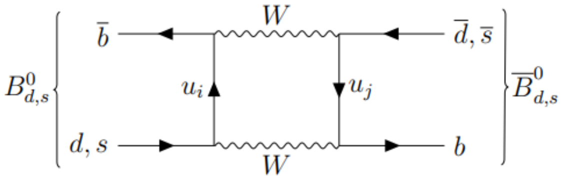
$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13}c_{14} & s_{12}c_{13}c_{14} & s_{13}c_{14}e^{-i\delta} \\ -s_{12}c_{23} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}s_{13}c_{23} & -c_{12}s_{23} - e^{i\delta}s_{12}s_{13}c_{23} & c_{13}c_{23} \\ -c_{12}c_{13}s_{14} & -s_{12}c_{13}s_{14} & -s_{13}s_{14}e^{-i\delta} \end{pmatrix} \qquad F_u = \begin{pmatrix} c_{14}^2 & 0 & 0 & -s_{14}c_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}c_{14} & 0 & 0 & s_{14}^2 \end{pmatrix}$$

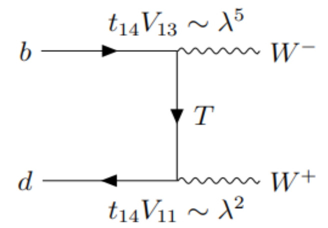
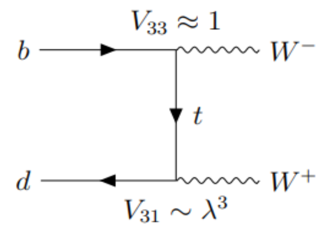
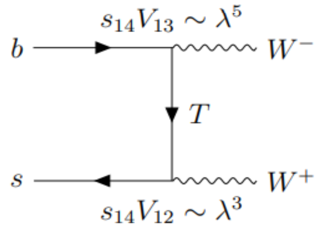
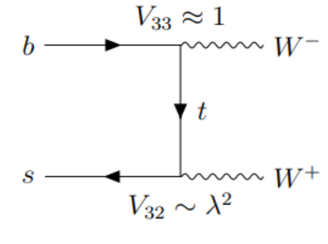
- We assume: $\mathrm{Br}\,(T \to dW^+) + \mathrm{Br}\,(T \to uZ) + \mathrm{Br}\,(T \to uh) \simeq 1 \implies m_T > 1.15\ \mathrm{TeV}$ **[2405.19862]**

  Typically **searches assume**: $\mathrm{Br}\,(T \to bW^+) + \mathrm{Br}\,(T \to tZ) + \mathrm{Br}\,(T \to th) \simeq 1 \implies m_T > 1.48\ \mathrm{TeV}$

- The NP contributions to $B^0_{d,s} - \overline{B}^0_{d,s}$ mixing are suppressed

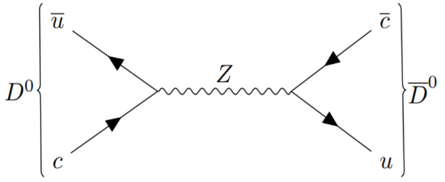
$B^0_{d,s} - \overline{B}^0_{d,s}$:



	NP	SM	NP/SM
$B^0_d - \overline{B}^0_d$:			$\sim\lambda^4$
$B^0_s - \overline{B}^0_s$:			$\sim\lambda^6$

- No NP contribution to $D^0 - \overline{D}^0$ in the s_{14}-dominance limit:

$D^0 - \overline{D}^0$:



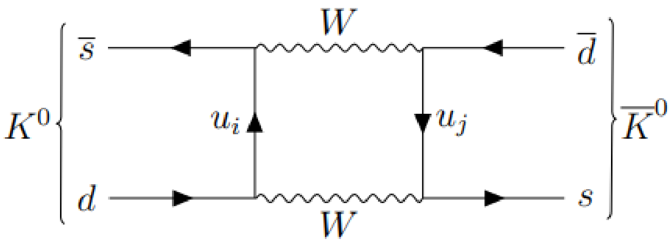
$\sim |F^u_{12}| = 0$

or to $K_L \to \pi^0 \bar\nu\nu$ or ε'/ε, since $\mathrm{Im}\,(V_{Td}V^*_{Ts}) = 0$

Phenomenology: Kaon Physics

The kaon system imposes the most stringent constraints.

$K^0 - \overline{K}^0$:



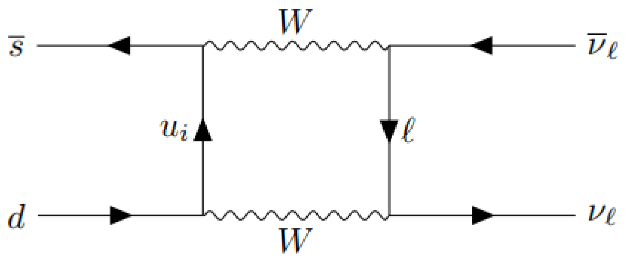
$$|\varepsilon_K^{\text{exp}}| \simeq (2.228 \pm 0.011) \times 10^{-3}$$
$$|\varepsilon_K^{\text{SM}}| \simeq (2.16 \pm 0.18) \times 10^{-3}$$
$$\Bigg\} \; |\varepsilon_K^{\text{NP}}| \approx (0.68 \pm 1.80) \times 10^{-4} \implies \begin{array}{c} s_{24} \neq 0 \\ \sin(\delta_{24} - \delta_{14}) \neq 0 \end{array}$$

$\Downarrow$

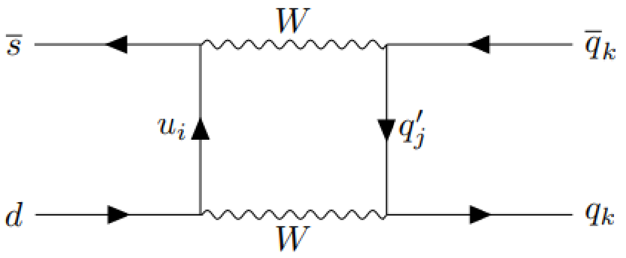
SM prediction from **Brod et al. [1911.06822]**

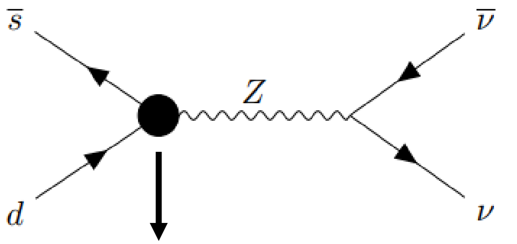
Use instead $s_{24}, s_{34} \ll s_{14}$

$K^+ \to \pi^+ \nu \overline{\nu}$:

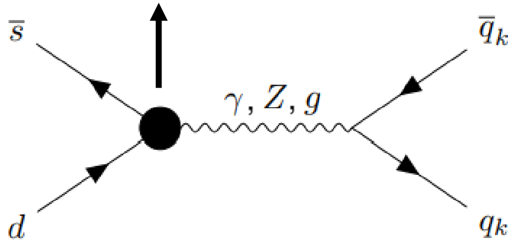


$K_L \to \pi\pi$:

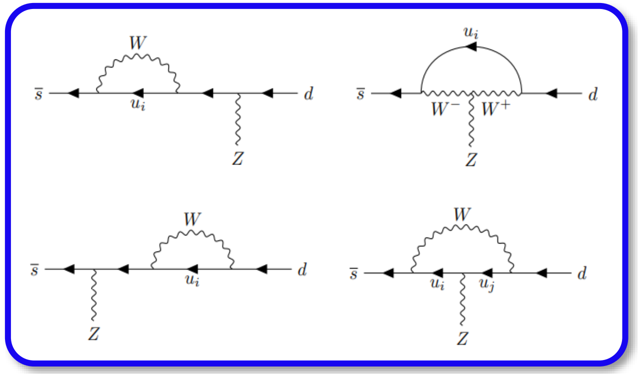




Penguin Loops



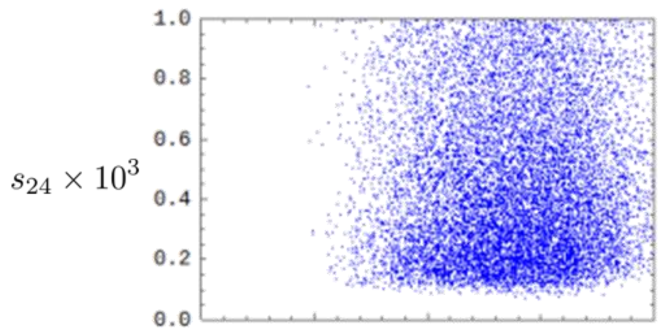
EW Penguin Loops



$$-4 \times 10^{-4} \lesssim \left(\frac{\epsilon'}{\epsilon}\right)^{\text{NP}}_{1\sigma} \lesssim 10 \times 10^{-4}$$

[Aebischer et al. 2005.0597]

Phenomenology: the s_{14}-dominance fit

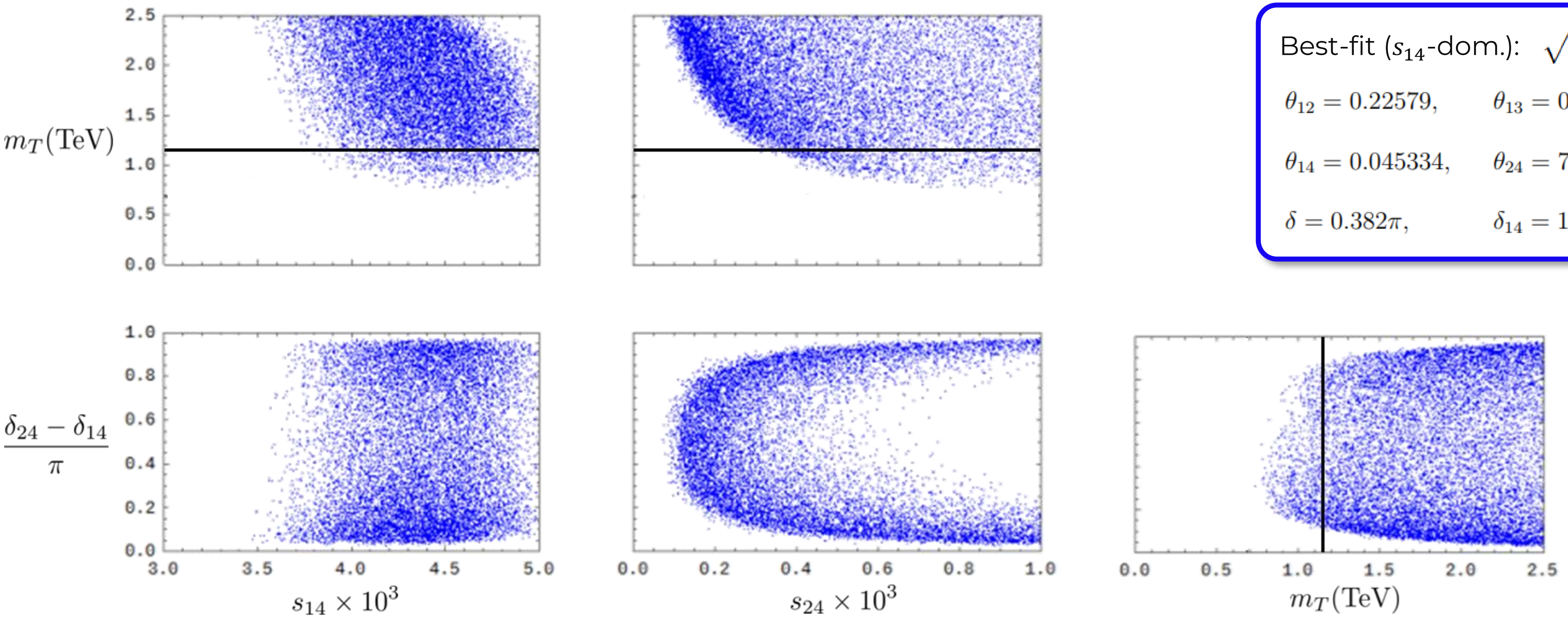


Mass Lower Bound:

$$m_T^{\text{scan}} \approx 0.80 \text{ TeV}$$

$$m_T^{\text{ATLAS}} = 1.15 \text{ TeV}$$

The assumption of **predominant coupling to the 1st gen.** allows for a more **promising bound**, than the 3rd gen. assumption.



Best-fit (s_{14}-dom.): $\sqrt{\chi^2} \simeq 2.25$ $\quad m_T = 1477$ GeV

$\theta_{12} = 0.22579,\quad \theta_{13} = 0.0038275,\quad \theta_{23} = 0.039524,$

$\theta_{14} = 0.045334,\quad \theta_{24} = 7.412 \times 10^{-4},\quad \theta_{34} = 2.346 \times 10^{-4},$

$\delta = 0.382\pi,\quad \delta_{14} = 1.872\pi,\quad \delta_{24} = 1.979\pi.$

SM Mixing and CPV from a VLQ model

- Consider a minimal solution where the mass matrices take the form (**Branco et al. [2207.14235]**)

$$M_d = \begin{pmatrix} m_{11}^d & m_{12}^d & 0 \\ m_{21}^d & m_{22}^d & 0 \\ 0 & 0 & m_{33}^d \end{pmatrix}$$

This form can be motivated within a 2HDM with an underlying Z_4 symmetry

$$(m_{ij}^u > 0) \quad \mathcal{M}_u = \begin{pmatrix} 0 & 0 & 0 & m_{14}^u \\ 0 & m_{22}^u & m_{23}^u & m_{24}^u e^{i\beta} \\ 0 & m_{32}^u e^{i\alpha} & m_{33}^u & 0 \\ m_{41}^u & 0 & -m_{43}^u e^{i\delta} & M \end{pmatrix} \Longrightarrow M_u^{\text{eff}} \approx \begin{pmatrix} -\frac{m_{14}^u m_{41}^u}{M} & 0 & \frac{m_{14}^u m_{43}^u}{M} e^{i\delta} \\ -\frac{m_{24}^u m_{41}^u}{M} e^{i\beta} & m_{22}^u & m_{23}^u \\ 0 & m_{32}^u & m_{33}^u \end{pmatrix}$$

$$(m_{33}^u \approx m_t, \quad M \approx m_T, \quad m_{14}^u \gg m_{24}^u)$$

- In the **decoupling limit** we have:

$$M_u^{\text{eff}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{22}^u & m_{23}^u \\ 0 & m_{32}^u & m_{33}^u \end{pmatrix} \Longrightarrow V_{\text{CKM}}^{\text{eff}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}_{\text{up}} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{13} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\text{down}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{23}c_{12} & -s_{23} \\ -s_{23}s_{12} & s_{23}c_{12} & c_{23} \end{pmatrix}$$

In the **absence of VLQ - SM quark couplings**, we have $\boldsymbol{m_u}$**=0**, $\boldsymbol{V_{ub}}$**=0** and ***J*=0**.

We obtain **$|V_{ub}|<|V_{td}|=|V_{ct}||V_{us}|$** which is a crude approximation of what we observe in the SM.

SM Mixing and CPV from a VLQ model

- In general, the mixing is described by

$$\mathcal{V}^\dagger \approx \begin{pmatrix} c_{12} & s_{12} & \frac{m^u_{14} m^u_{43} e^{-i\delta}}{m_t m_T} & \frac{m^u_{14}}{m_T} \\ -s_{12} & c_{12} & \frac{m^u_{23}}{m_t} & \frac{m^u_{24}}{m_T} e^{-i\beta} \\ -c_{12}\frac{m^u_{14} m^u_{43}}{m_t m_T} e^{i\delta} + s_{12}\frac{m^u_{23}}{m_t} & -c_{12}\frac{m^u_{23}}{m_t} & 1 & -\frac{m_t m^u_{43}}{m_T^2} e^{i\delta} \\ -c_{12}\frac{m^u_{14}}{m_T} & -c_{12}\frac{m_{23}}{m_T} e^{i\beta} - s_{12}\frac{m^u_{14}}{m_T} & \frac{m_t m_{43}}{m_T^2} e^{-i\delta} & 1 \end{pmatrix}$$

$$s_{12} = \lambda$$

$$s_{23} \approx \frac{m^u_{23}}{m_t}$$

$V_{ub} \neq 0$

$$s_{13} \approx \frac{m^u_{14} m^u_{43}}{m_t m_T}$$

CAA

$$s_{14} \approx \frac{m^u_{14}}{m_T}$$

$$s_{24} \approx \frac{m^u_{24}}{m_T}$$

$$s_{34} \approx \frac{m_t m^u_{43}}{m_T^2}$$

and the 3x3 effective mixing reduces to the SM form

$$V^{\text{eff}}_{\text{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}_{\text{up}} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{12} \end{pmatrix}_{\text{NP}} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{13} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\text{down}}$$

CPV

$$J \approx \lambda \cdot \frac{m^u_{23} m^u_{43}}{m_t^2} \cdot \frac{m^u_{14}}{m_T} \sin\delta \ll 1$$

A **non-zero m^u_{14}** coupling, crucial to **address the CAA,** allows for a **non-zero m_u, V_{ub} and J.**

The **large VLQ mass** is responsible for the **s_{12}, s_{23} >> s_{13} hierarchy** and the **CPV suppression** in the SM quark sector **J<<1**.

Enhancing CP Violation with VLQs

- We study this **Albergaria et al. [2210.14248]**.

- **Weak Basis Invariants** remain **unchanged under weak-basis transformations** (WBTs) which leave EW currents flavor-diagonal. **CP-odd WBIs point to new sources of CPV**.

In the SM:
$\mathrm{tr}\,[h_u, h_d]^3 \propto J$

- WBTs with one up-type VLQ iso-singlet:

$$\underbrace{Q'_L \to W_L Q'_L}_{\text{3x3 unitary}}, \quad T'_L \to e^{i\varphi} T'_L, \quad \underbrace{d'_R \to W^d_R d'_R}_{\text{3x3 unitary}}, \quad \underbrace{\begin{pmatrix} u'_R \\ T'_R \end{pmatrix} \to W^u_R \begin{pmatrix} u'_R \\ T'_R \end{pmatrix}}_{\text{4x4 unitary}}$$

- Hermitian "building blocks" (all transforming as $H \to W_L^\dagger H W_L$)

$$h_d^n = (m_d m_d^\dagger)^n \qquad h_u^n = (m_u m_u^\dagger)^n \qquad h_u^{(n)} = m_u (m_u^\dagger m_u + M_u^\dagger M_u)^{n-1} m_u^\dagger$$

built from the up-sector mass matrix: $\mathcal{M}_u = \begin{pmatrix} m_u \\ M_u \end{pmatrix}$

- More **CP violating phases imply more independent CP-odd WBI**. The WBI of lowest mass dimension is:

$$\mathrm{tr}\left([h_u, h_d]\, h_u^{(2)} \right) = 2i \sum_{i=1}^{3} \sum_{\alpha,\beta=1}^{4} m_{d_i}^2 m_{u_\alpha}^4 m_{u_\beta}^2 \,\mathrm{Im}\left(F^u_{\alpha\beta} V^*_{\alpha i} V_{\beta i} \right) \sim \mathrm{M}^8$$

Enhancing CP Violation with VLQs

- CP violation should depend on dimensionless quantities such as

$$I_{\text{SM}} = \text{tr}\left[y_u y_u^\dagger, y_d y_d^\dagger\right]^3 = \frac{\overbrace{\text{tr}\left[h_u, h_d\right]^3}^{\sim M^{12}}}{v^{12}} \sim 10^{-25} \qquad I_{\text{VLQ}} = \frac{\text{tr}\left(\overbrace{\left[h_u, h_d\right] h_u^{(2)}}^{\sim M^{8}}\right)}{v^6 m_T^2} \qquad I'_{\text{VLQ}} = \frac{\text{tr}\left(\overbrace{\left[h_u^2, h_d\right] h_u^{(2)}}^{\sim M^{10}}\right)}{v^8 m_T^2}$$

- These WBIs in VLQ extensions can be significantly larger than the SM one:

Best-fit (s_{14}-dom.): $\sqrt{\chi^2} \simeq 2.25$ $\quad m_T = 1477$ GeV

$$\theta_{12} = 0.22579, \quad \theta_{13} = 0.0038275, \quad \theta_{23} = 0.039524,$$
$$\theta_{14} = 0.045334, \quad \theta_{24} = 7.412 \times 10^{-4}, \quad \theta_{34} = 2.346 \times 10^{-4},$$
$$\delta = 0.382\pi, \quad \delta_{14} = 1.872\pi, \quad \delta_{24} = 1.979\pi.$$

$$I_{\text{VLQ}} = \frac{\text{tr}\left(\left[h_u, h_d\right] h_u^{(2)}\right)}{v^6 m_T^2} \simeq 2.02 \times 10^{-10} \qquad I'_{\text{VLQ}} = \frac{\text{tr}\left(\left[h_u^2, h_d\right] h_u^{(2)}\right)}{v^8 m_T^2} \simeq 1.16 \times 10^{-10}$$

Important for Baryogenesis??

- With VLQs we can even obtain CP violation in the **limit of extremely high energies** (extreme chiral limit) where **m_u=m_c=m_d=m_s=0** and **I_{SM}=0** (also pointed out in **del Aguila et al. [hep-ph/9703410]**).

$$\text{tr}\left(\left[h_u, h_d\right] h_u^{(2)}\right) = 2i\ m_b^2 m_t^2 m_T^2 (m_T^2 - m_t^2) I_{\text{ECL}}$$
$$I_{\text{ECL}} = c_{23} c_{14}^2 c_{24}^2 c_{34} s_{23} s_{24} s_{34} \sin \delta_{\text{ECL}}$$

$$V_{\text{CKM}} = \begin{pmatrix} c_{14} & 0 & 0 \\ -s_{14} s_{24} & c_{23} c_{24} & s_{23} c_{24} \\ -s_{14} c_{24} s_{34} e^{i\delta_{\text{CL}}} & -s_{23} c_{34} - c_{23} s_{24} s_{34} e^{i\delta_{\text{CL}}} & c_{23} c_{34} - s_{23} s_{24} s_{34} e^{i\delta_{\text{CL}}} \\ -s_{14} c_{24} c_{34} e^{i\delta_{\text{CL}}} & s_{23} s_{34} - c_{23} s_{24} c_{34} e^{i\delta_{\text{CL}}} & -c_{23} s_{34} - s_{23} s_{24} c_{34} e^{i\delta_{\text{CL}}} \end{pmatrix}$$

Summary/Conclusions

- Extension with VLQs can provide very simple solutions to the CAA.

- The s_{14}-dominance limit is particularly safe in relation to a large variety of pheno. constraints and is related to an unusual decay pattern for the VLQ connected to the most favorable mass bounds for singlets.

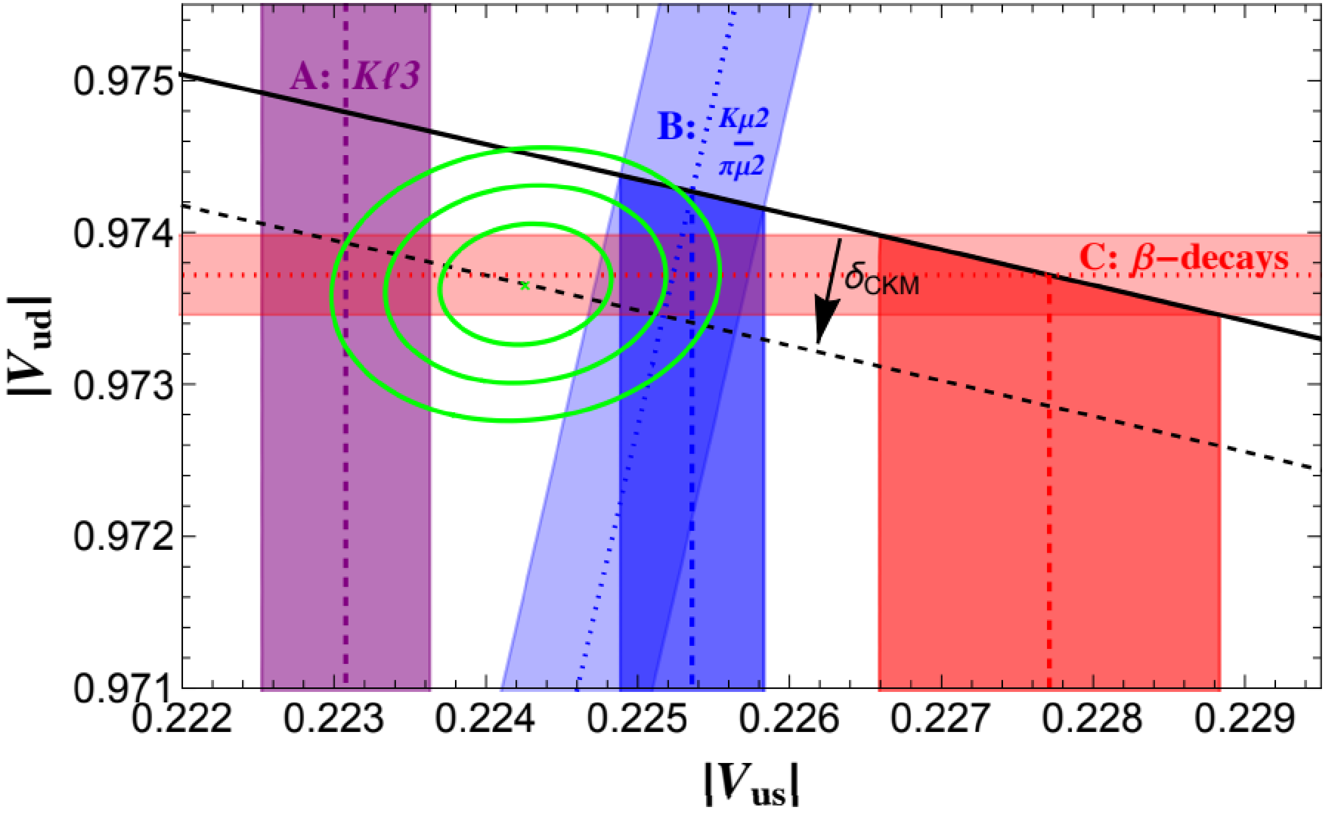
- Within the framework of s_{14}-dominance we show how some of the structure of CKM, hierarchies and CPV suppression may emerge from mixing with VLQs.

- The introduction of VLQs to the theory could enhance CP violation in the quark sector and even achieve CP violation at very high energies. This is also a consequence of CKM non-unitarity.

Thank You!

CAA

B. Belfatto and S. Trifinopoulos [2302.14097]



From unitarity:

$$|V_{us}|_A = 0.22308(55)$$

$$|V_{us}|_B = 0.22536(47)$$

$$|V_{us}|_C = 0.2277(11)$$

$|V_{us}|_{A+B} = 0.22440(51)$ vs $|V_{us}|_C = 0.2277(11)$ — CAA1: $\sim 2.7\sigma$

$|V_{us}|_A = 0.22308(55)$ vs $|V_{us}|_B = 0.22536(47)$ — CAA2: $\sim 3.1\sigma$

$|V_{us}|_A = 0.22308(55)$ vs $|V_{us}|_C = 0.2277(11)$ — $\sim 3.7\sigma$

Best fit: $\delta_{\rm CKM} \approx 1.7 \times 10^{-3}$

$$\delta_{\rm CKM} \equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2$$

Neutral Meson Mixings

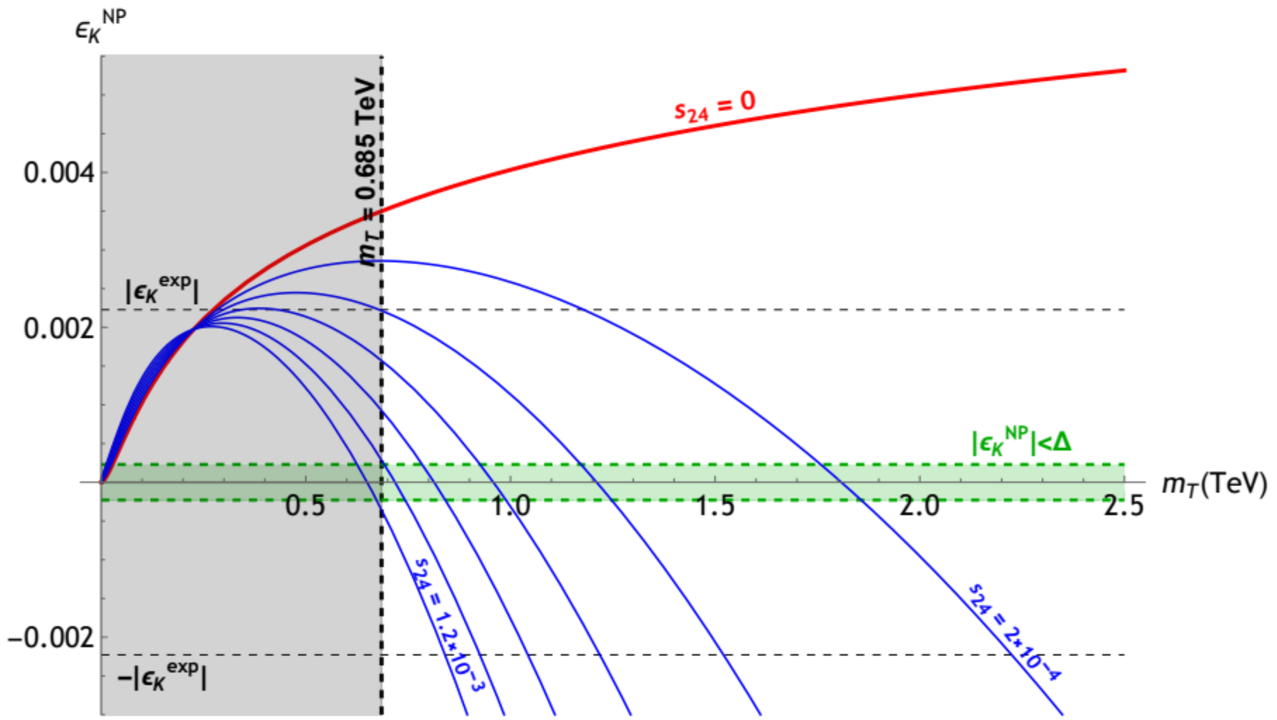
$$\Delta m_N^{\rm NP} \simeq \frac{G_F^2 M_W^2 m_N f_N^2 B_N}{6\pi^2} |2\eta_{cT}^N S_{cT}\lambda_c^N\lambda_T^N + 2\eta_{tT}^N S_{tT}\lambda_t^N\lambda_T^N + \eta_{TT}^N S_T(\lambda_T^N)^2|$$

$$|\epsilon_K^{\rm NP}| \simeq \frac{G_F^2 M_W^2 m_K f_K^2 B_K \kappa_\epsilon}{12\sqrt{2}\pi^2\Delta m_K} \left|{\rm Im}\left[2\eta_{cT}^K S_{cT}\lambda_c^K\lambda_T^K + 2\eta_{tT}^K S_{tT}\lambda_t^K\lambda_T^K + \eta_{TT}^K S_T(\lambda_T^K)^2\right]\right| = \frac{G_F^2 M_W^2 m_K f_K^2 B_K \kappa_\epsilon}{12\sqrt{2}\pi^2\Delta m_K}\mathcal{F}$$

$$\lambda_i^K = V_{id}V_{is}^*$$

$$\lambda_i^{B_d} = V_{id}V_{ib}^*$$

$$\lambda_i^{B_s} = V_{is}V_{ib}^*$$



- $s_{14}\sim\lambda^2, s_{24} = 0$: $\mathcal{F} \approx 2\eta_{tT}^K S_{tT} s_{12} s_{14}^2 s_{13} s_{23}\sin\delta$

- $s_{14}\sim\lambda^2, s_{24}\sim\lambda^4$:

$$\mathcal{F} \approx 2 s_{12} s_{14}^2\left(\eta_{tT}^K S_{tT} s_{13} s_{23}\sin\delta - \eta_{TT}^K S_{TT} s_{14} s_{24}\sin\delta'\right)$$

$$\delta' = \delta_{24} - \delta_{14}$$

Kaon Decays

- $$\frac{\mathrm{Br}(K^+ \to \pi^+ \overline{\nu}\nu)}{\mathrm{Br}(K^+ \to \pi^+ \overline{\nu}\nu)_{\mathrm{SM}}} = \left| \frac{\lambda_c^K X^{\mathrm{NNL}}(x_c) + \lambda_t^K X(x_t) + \lambda_T^K X(x_T) + A_{ds}}{\lambda_c^K X^{\mathrm{NNL}}(x_c) + \lambda_t^K X(x_t)} \right|^2$$

$$A_{ds} = \sum_{i,j=c,t,T} V_{is}^* \left(F^u - I\right)_{ij} V_{jd} N(x_i, x_j) \simeq -\frac{x_T}{8} c_{14}^2 c_{24}^2 \lambda_T^K$$

$$N(x_i, x_j) = \frac{x_i x_j}{8} \left(\frac{\log x_i - \log x_j}{x_i - x_j} \right)$$

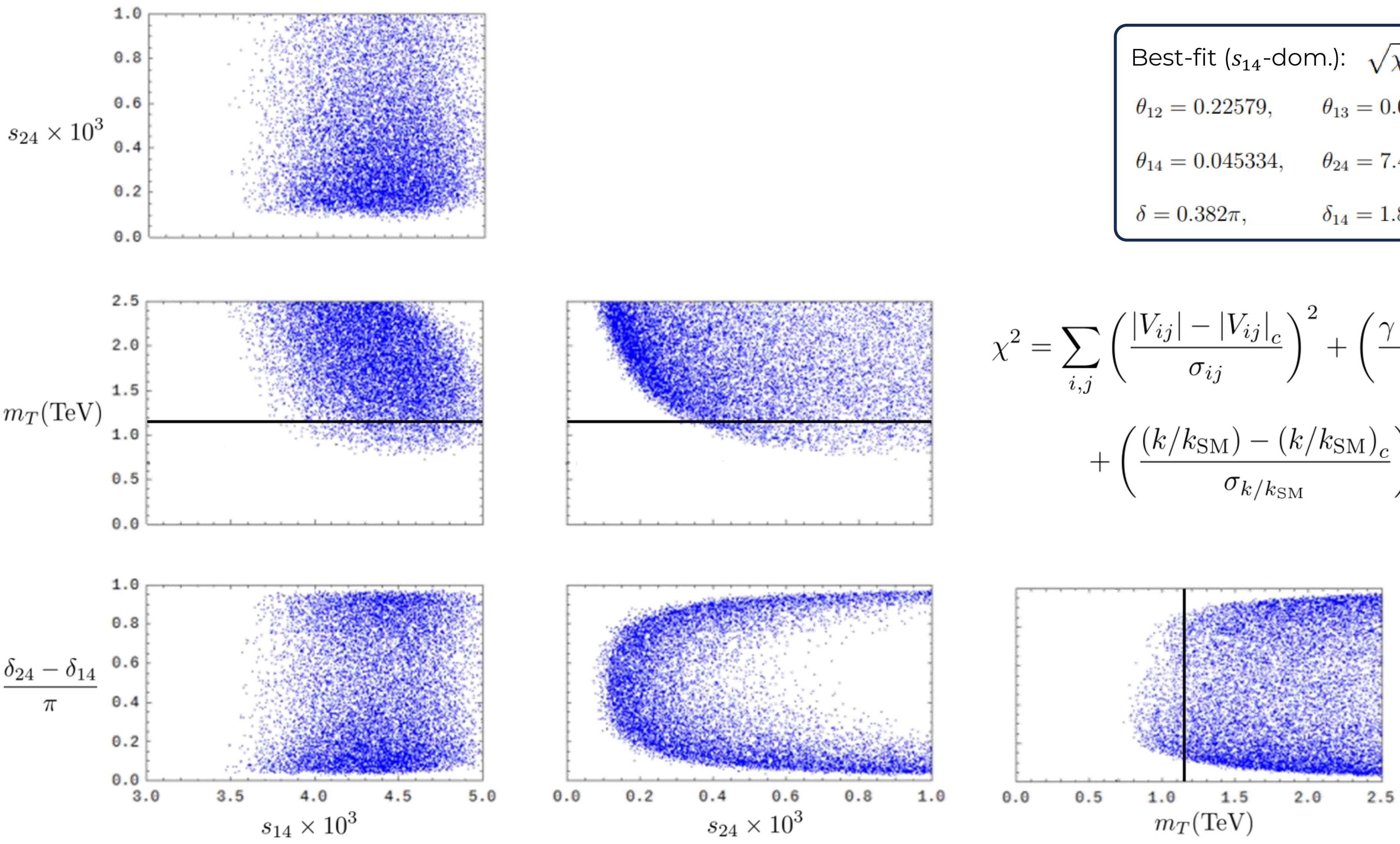
$$N(x_i, x_i) \equiv \lim_{x_j \to x_i} N(x_i, x_j) = \frac{x_i}{8}$$

- $$\left(\frac{\epsilon'}{\epsilon}\right)^{\mathrm{NP}} \simeq \tilde{F}(x_i)\mathrm{Im}(\lambda_T^K) \simeq -\tilde{F}(x_i) c_{12}^2 s_{14} s_{24} \sin\delta'$$

$$\tilde{F}(x_T) \equiv F(x_T) - \tfrac{x_T}{8}\left(P_X + P_Y + P_Z\right)$$

$$F(x_i) = P_0 + P_X X(x_i) + P_Y Y(x_i) + P_Z Z(x_i) + P_E E(x_i)$$

Phenomenology: s_{14}- dominance Fit



Best-fit (s_{14}-dom.): $\sqrt{\chi^2} \simeq 2.25$ $m_T = 1477$ GeV

$\theta_{12} = 0.22579,$ $\theta_{13} = 0.0038275,$ $\theta_{23} = 0.039524,$

$\theta_{14} = 0.045334,$ $\theta_{24} = 7.412 \times 10^{-4},$ $\theta_{34} = 2.346 \times 10^{-4},$

$\delta = 0.382\pi,$ $\delta_{14} = 1.872\pi,$ $\delta_{24} = 1.979\pi.$

$$\chi^2 = \sum_{i,j} \left(\frac{|V_{ij}| - |V_{ij}|_c}{\sigma_{ij}} \right)^2 + \left(\frac{\gamma - \gamma_c}{\sigma_\gamma} \right)^2 + \left(\frac{|\varepsilon_K^{\mathrm{NP}}| - |\varepsilon_K^{\mathrm{NP}}|_c}{\sigma_\varepsilon} \right)^2$$
$$+ \left(\frac{(k/k_{\mathrm{SM}}) - (k/k_{\mathrm{SM}})_c}{\sigma_{k/k_{\mathrm{SM}}}} \right)^2 + \left(\frac{(\varepsilon'/\varepsilon)^{\mathrm{NP}} - (\varepsilon'/\varepsilon)^{\mathrm{NP}}_c}{\sigma_{\varepsilon'/\varepsilon}} \right)^2$$

$$k \equiv \mathrm{Br}\left(K^+ \to \pi^+ \bar{\nu}\nu\right)$$

Constraints:

- $(\Delta m_K^{\mathrm{NP}})_{\mathrm{SD}} < \Delta m_K^{\mathrm{exp}}$
- $\sqrt{\chi^2} < 3$
- $s_{24}, s_{34} \in [0, 0.001]$

SM Mixing and CPV from a VLQ model

$$\mathcal{M}_u = \begin{pmatrix} 0 & 0 & 0 & m^u_{14} \\ 0 & m^u_{22} & m^u_{23} & m^u_{24}e^{i\beta} \\ 0 & m^u_{32}e^{i\alpha} & m^u_{33} & 0 \\ m^u_{41} & 0 & -m^u_{43}e^{i\delta} & M \end{pmatrix}$$

$$M_d = \begin{pmatrix} m^d_{11} & m^d_{12} & 0 \\ m^d_{21} & m^d_{22} & 0 \\ 0 & 0 & m^d_{33} \end{pmatrix}$$

$$\phi \longrightarrow \phi, \qquad \phi_2 \longrightarrow i\phi_2, \qquad S \longrightarrow -iS,$$

$$\overline{Q}_{L1} \longrightarrow -\overline{Q}_{L1}, \quad \overline{Q}_{L2} \longrightarrow i\overline{Q}_{L2}, \quad \overline{Q}_{L3} \longrightarrow \overline{Q}_{L3},$$

$$U_{R,L} \longrightarrow iU_{R,L} \quad d_{R1} \longrightarrow -d_{R1}, \quad d_{R2} \longrightarrow -d_{R2},$$

Up-type quarks	u_{R1} (1)	u_{R2} (1)	u_{R3} (1)	U_R $(-i)$
$\overline{Q}_{L1}$ (-1)	0	0	0	$y^u_{13}\tilde{\phi}_2$
$\overline{Q}_{L2}$ (i)	$y^u_{21}\tilde{\phi}_2$	$y^u_{22}\tilde{\phi}_2$	$y^u_{23}\tilde{\phi}_2$	$y^u_{24}\tilde{\phi}$
$\overline{Q}_{L3}$ (1)	$y^u_{31}\tilde{\phi}$	$y^u_{32}\tilde{\phi}$	$y^u_{33}\tilde{\phi}$	0
$\overline{U}_L$ (i)	$y^u_{41}S$	$y^u_{42}S$	$y^u_{43}S$	M

Down-type quarks	d_{R1} (-1)	d_{R2} (-1)	d_{R3} (1)
$\overline{Q}_{L1}$ (-1)	$y^d_{11}\phi$	$y^d_{12}\phi$	0
$\overline{Q}_{L2}$ (i)	$y^d_{21}\phi_2$	$y^d_{22}\phi_2$	0
$\overline{Q}_{L3}$ (1)	0	0	$y^d_{33}\phi$