TESTING THE PAULI EXCLUSION PRINCIPLE AND FUNDAMENTAL SYMMETRIES IN UNDERGROUND EXPERIMENTS

Fabrizio Napolitano on behalf of the VIP-2 Collaboration

fabrizio.napolitano@lnf.infn.it

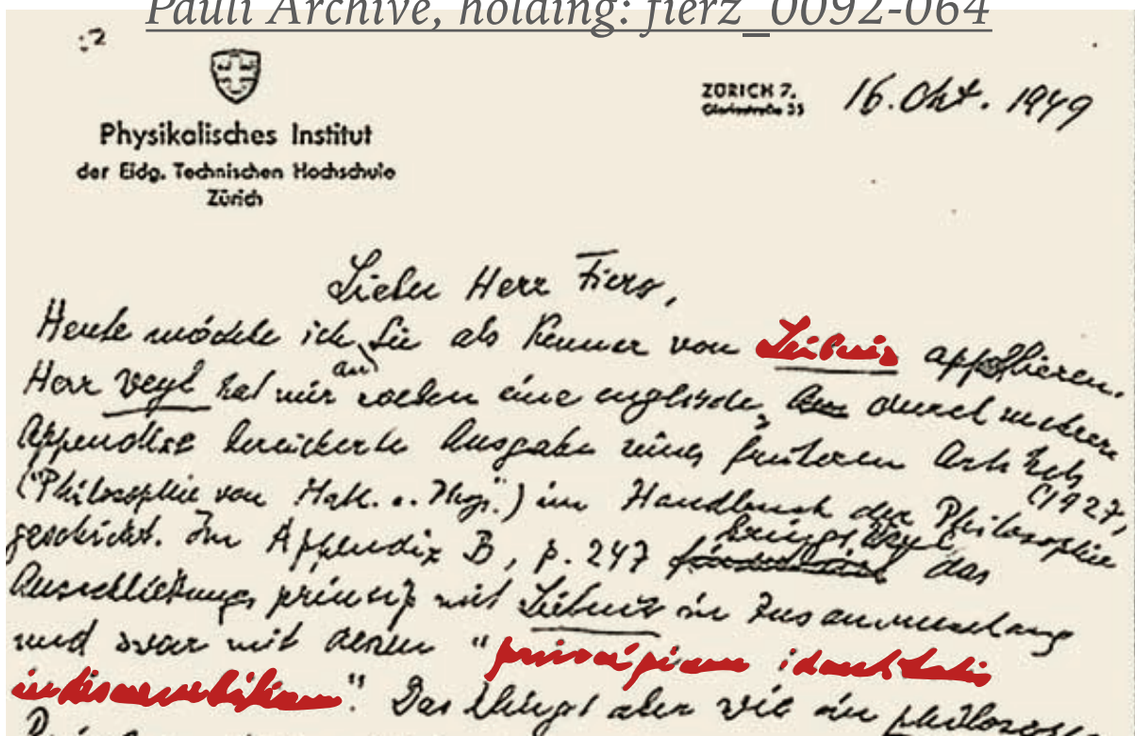
DISCRETE2024 - Ljubljana 4/12/24

Outline: Line of Research at LNGS
From the shoulders of Giants

In an atom there cannot be two or more equivalent electrons for which the values of all four quantum numbers coincide. If an electron exists in an atom for which all of these numbers have definite values, then the state is occupied.

W.Pauli, Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren, Zeitschrift für Physik 31 (1925) 765.

Pauli Archive, holding: fierz_0092-064



Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

ZÜRICH 7.
Gloriastraße 35

16. Okt. 1949

Lieber Herr Fierz,

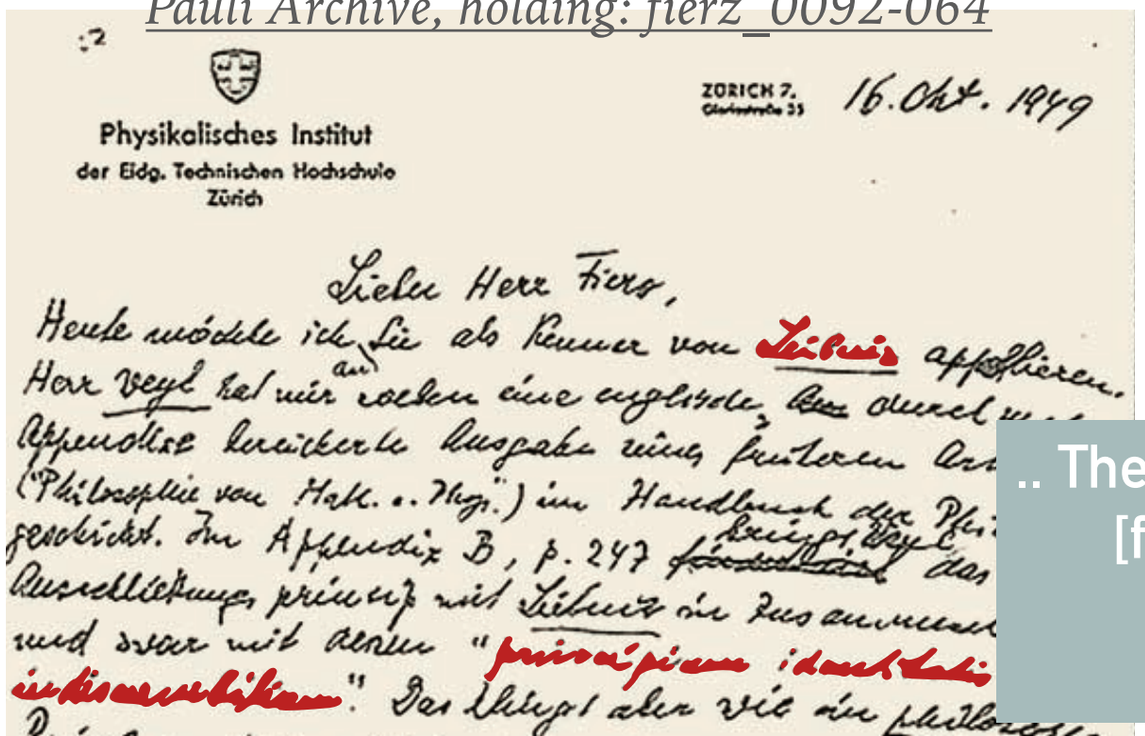
Heute möchte ich Sie als Kenner von Leibniz applizieren. Herr Veyl hat mir soeben eine englische, [illegible] Ausgabe seines früheren Artikels (Philosophie von Hatt.. Hrgb.) im Handbuch der Philosophie geschickt. Im Appendix B, p. 247 [illegible] (1927), beruft Veyl das Ausschließungsprinzip mit Leibniz in Zusammenhang und zwar mit denen "principium identitatis indiscernibilium". Das scheint aber nicht ganz richtig

Outline: Line of Research at LNGS
From the shoulders of Giants

In an atom there cannot be two or more equivalent electrons for which the values of all four quantum numbers coincide. If an electron exists in an atom for which all of these numbers have definite values, then the state is occupied.

W.Pauli, Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren, Zeitschrift für Physik 31 (1925) 765.

Pauli Archive, holding: fierz_0092-064



Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

ZÜRICH 7. Gloriastraße 35 16. Okt. 1949

Lieber Herr Fierz,

Heute möchte ich Sie als Kenner von Leibniz applizieren. Herr Veyl hat mir soeben eine englische, [illegible] Appendix erweiterte Ausgabe seines früheren Art. (Philosophie von Hak.. Nyg.) im Handbuch der Ph... geschickt. In Appendix B, p. 247 bringt Veyl das Auseinanderhalten, primär mit Leibniz in Zusammen... und zwar mit denen "principium identitatis indiscernibilium". Der Verfasser sieht die philosophi...

.. The impression that the shadow of some incompleteness [falls] here on the bright light of success of the new quantum mechanics seems to me unavoidable.
W. Pauli, Nobel lecture 1945

The Pauli Exclusion Principle (PEP)

Spin-statistic connection:

half-integer spin particles → antisymmetric wave function & Fermi-Dirac stat

Integer spin particles → symmetric wave function & Bose statistics

Lüders and Zumino: spin-statistics lays on few, general assumption:

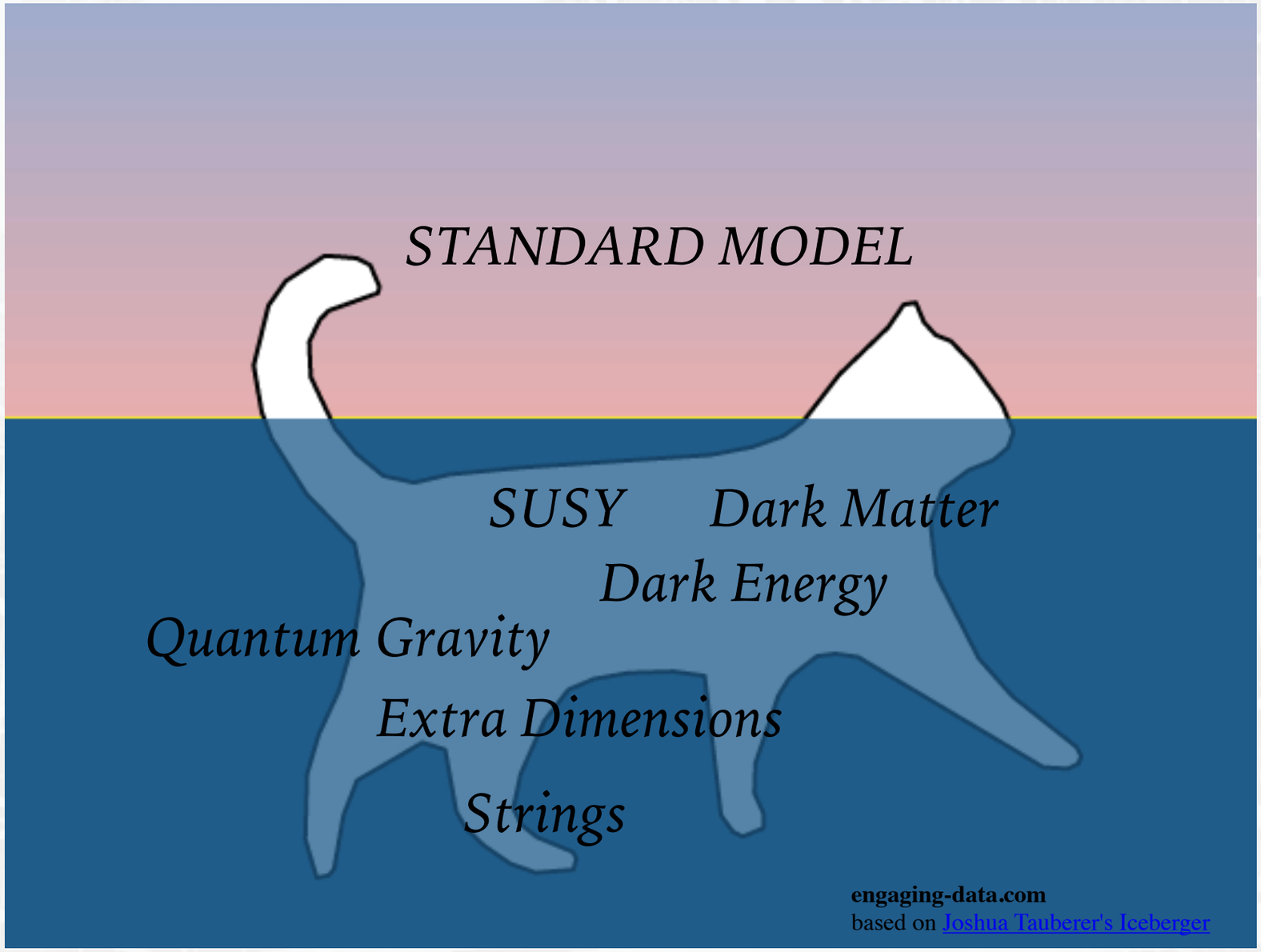
Lorentz/Poincaré Symmetry, CPT, unitarity, locality & causality

Theories of Statistics Violation

O.W. Greenberg: AIP Conf.Proc.545:113-127,2004

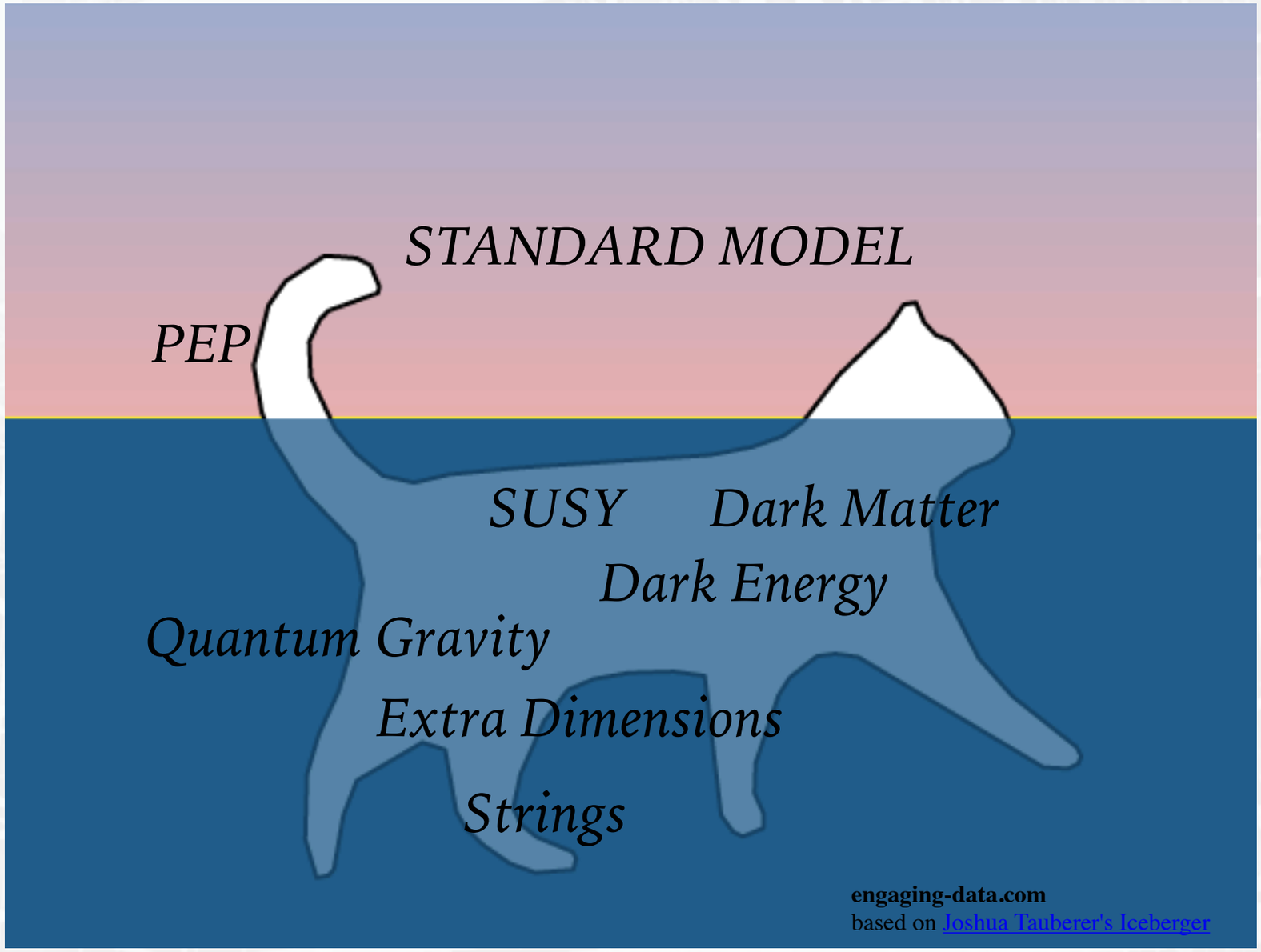
*"Possible external **motivations for violation of statistics** include: (a) violation of CPT, (b) violation of locality, (c) violation of Lorentz invariance, (d) extra space dimensions, (e) discrete space and/or time and (f) non-commutative spacetime....."*

The Pauli Exclusion Principle (PEP)



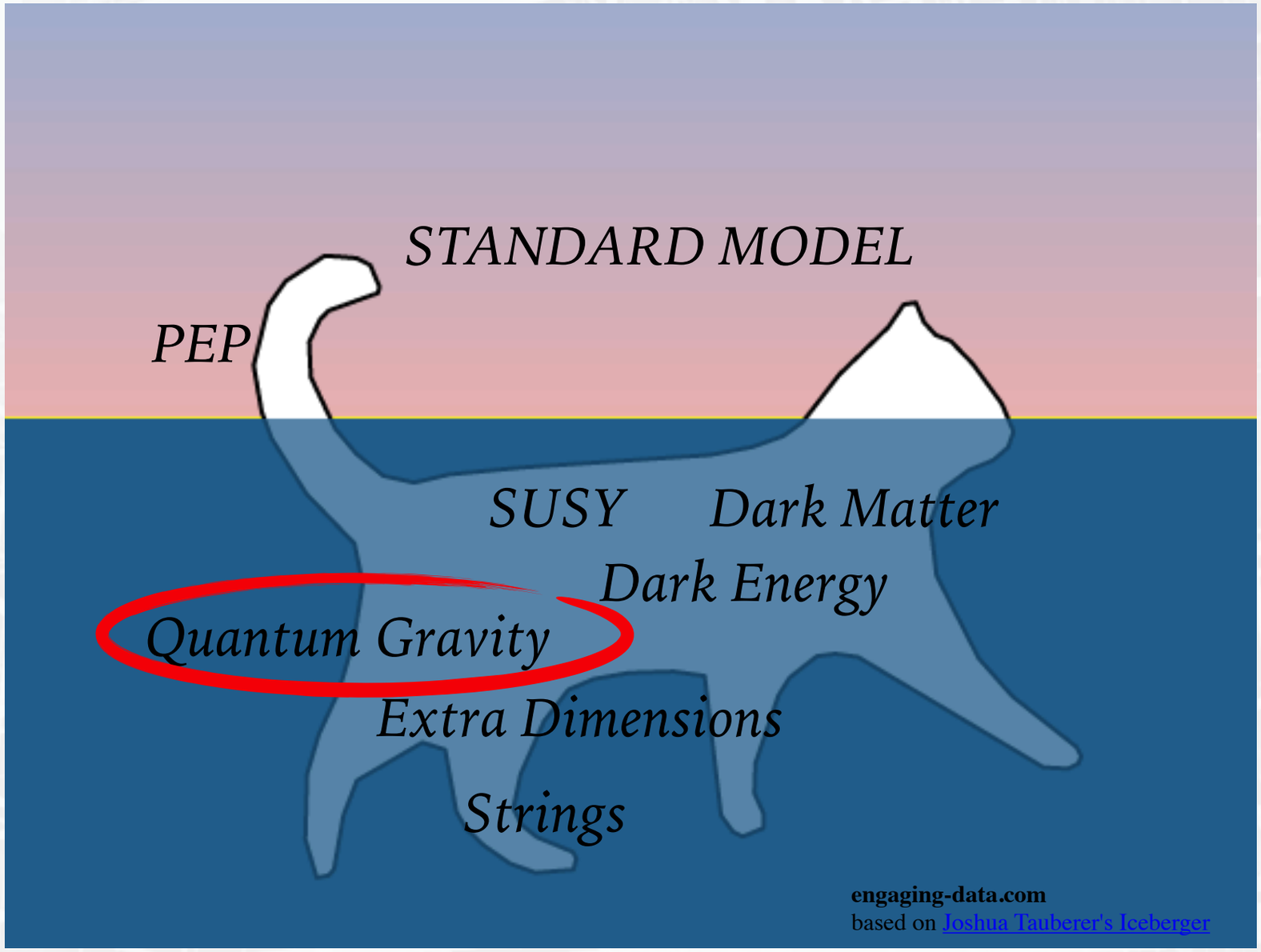
BSM theories embedding extra dimensions, non commutative and/or discrete spacetime could have effect on PEP

The Pauli Exclusion Principle (PEP)



BSM theories embedding extra dimensions, non commutative and/or discrete spacetime could have effect on PEP

The Pauli Exclusion Principle (PEP)



BSM theories embedding extra dimensions, non commutative and/or discrete spacetime could have effect on PEP

How to model PEP violations

- *Ignatiev & Kuzmin model: Fermi oscillator with a third state*

(Ignatiev, A.Y., Kuzmin, V. , Quarks '86: Proceedings of the 229 Seminar, Tbilisi, USSR, 1517 April 1986)

$$a^+|0\rangle = |1\rangle \qquad a|0\rangle = 0$$
$$a^+|1\rangle = \beta|2\rangle \qquad a|1\rangle = |0\rangle$$
$$a^+|2\rangle = 0 \qquad a|2\rangle = \beta|1\rangle$$

β quantifies the degree of violation in the transition

- *Greenberg & Mohapatra: Local Quantum Field Theory, q parameter deforms anticommutators [Phys. Rev. Lett. 1987,59,2507]:*

$$a_k a^+_l - q a^+_l a_k = \delta_{k,l}$$

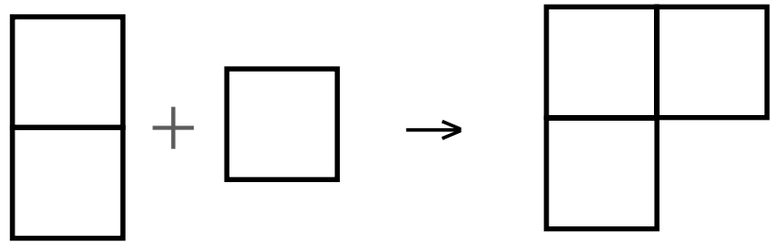
- *Rahal & Campa: global wave function of the electrons not exactly antisymmetric,*

PEP holds as long as the number of wrongly entangled pairs is small

All respect the Messiah-Greenberg super-selection rule!

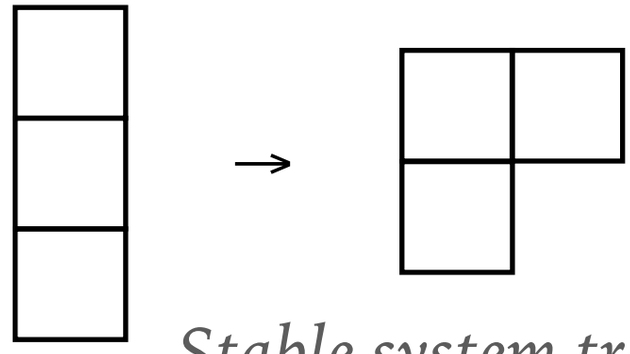
Messiah-Greenberg (M-G) Superselection Rule!

Open systems



Formation of a new state

Closed Systems



Stable system transition

Violations from deformation of anti/comm relations are restricted to open systems

Must introduce a new state from outside to form a new violating state

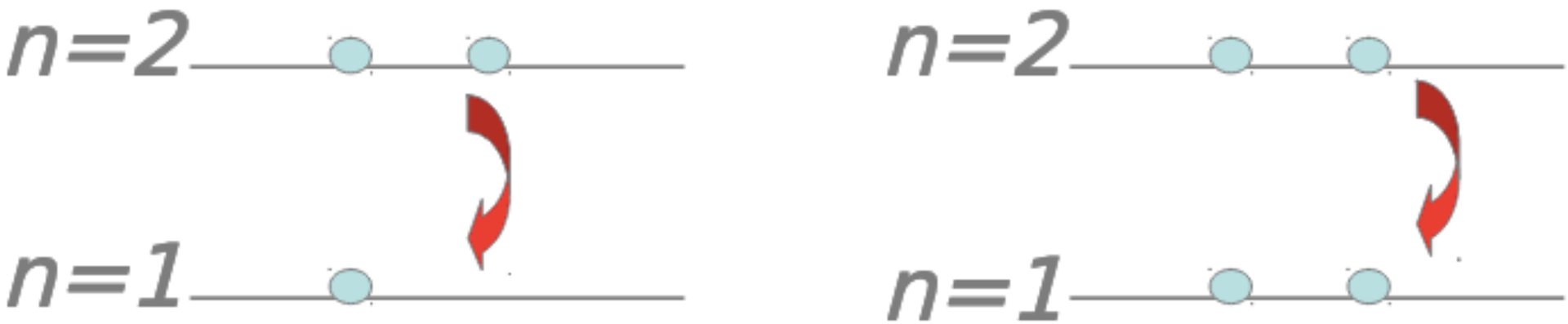
***VIP-2**: current on target to introduce new electrons*

Violation from space-time properties are NOT restricted to Open Systems

Quantum Gravity models can include space-time non commutativity

VIP-Lead

Search for anomalous X-ray transitions performed by electrons introduced in a target trough a DC current (open system)



Normal 2p → 1s transition

~ 8.05 keV in Cu

2p → 1s transition violating Pauli principle

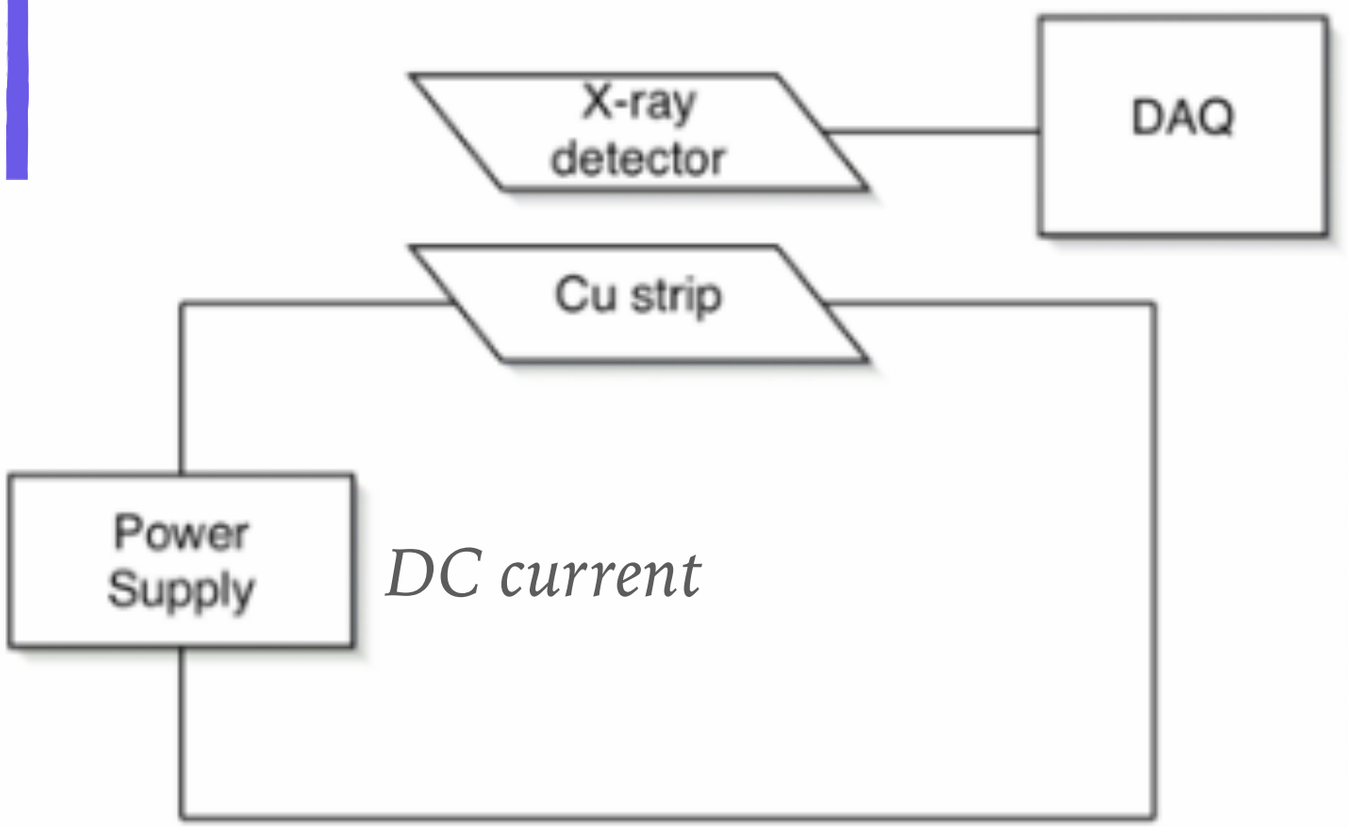
~ 7.7 keV in Cu

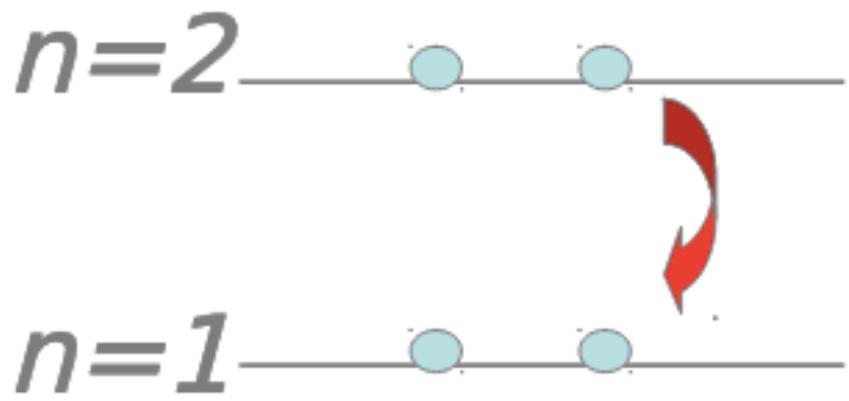
Paul Indelicato (Ecole Normale Supérieure et Université Pierre et Marie Curie)

Multiconfiguration Dirac-Fock approach

Accounts for the shielding of the two inner electrons

Search for anomalous X-ray transitions performed by electrons introduced in a target trough a DC current (open system)





2p → 1s transition violating Pauli principle

~ 7.7 keV in Cu

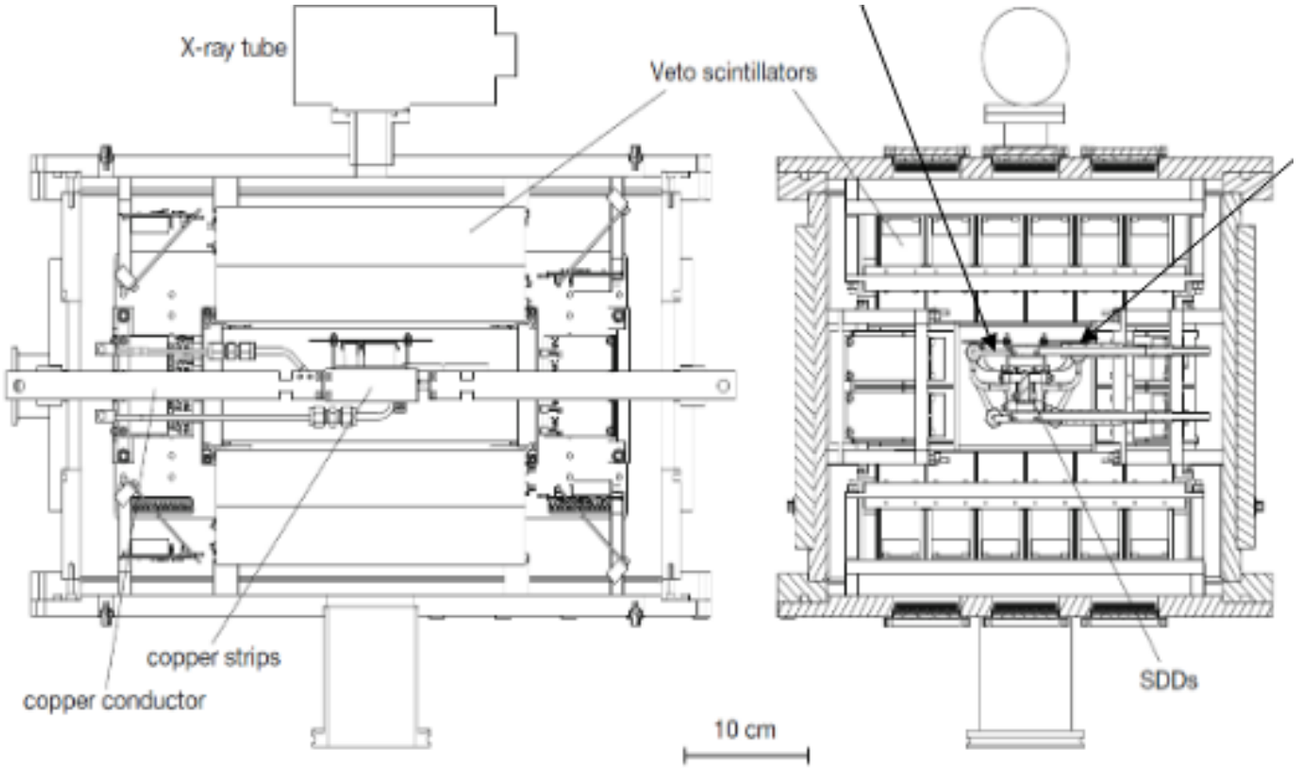
Paul Indelicato (Ecole Normale Supérieure et Université Pierre et Marie Curie)

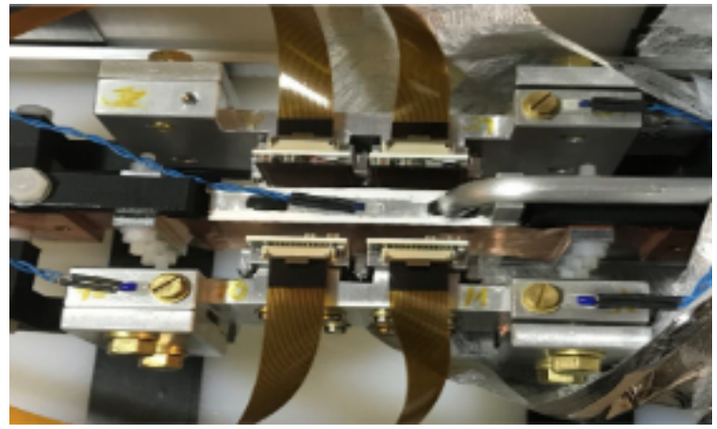
Multiconfiguration Dirac-Fock approach

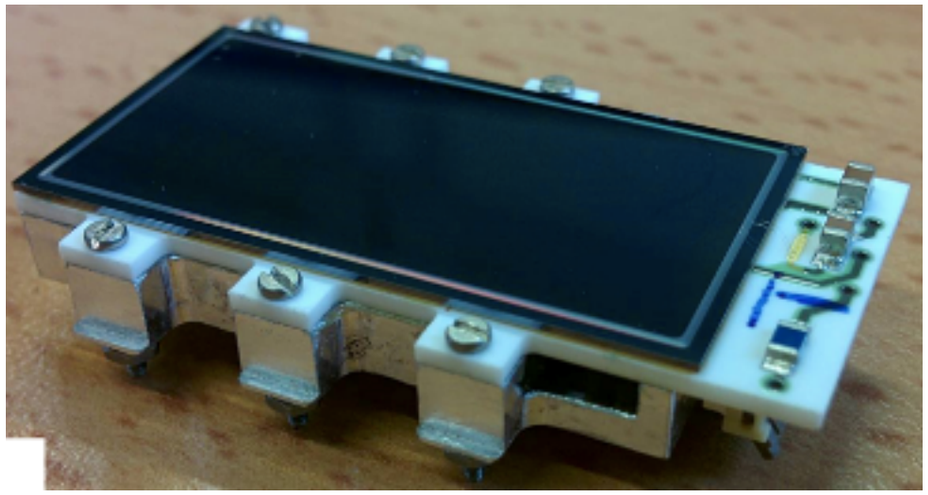
Accounts for the shielding of the two inner electrons

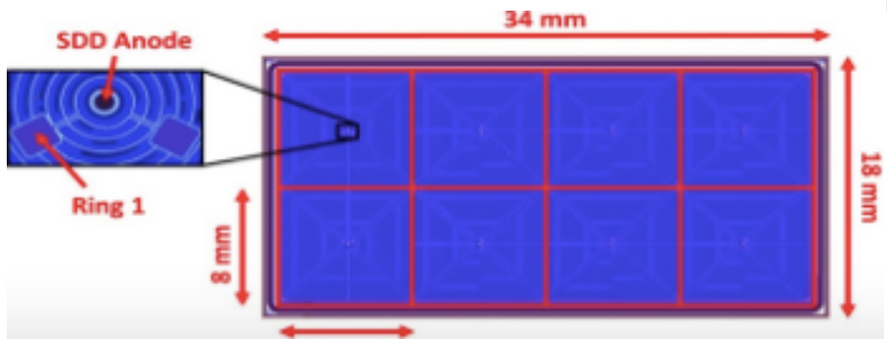
The VIP-2 Experiment

Silicon Drift Detectors (SDDs) higher resolution (190 eV FWHM at 8.0 →keV), faster (triggerable) detectors. 4 arrays of 2 x 4 SDDs 8mm x 8mm each, liquid argon closed circuit cooling 170 °C









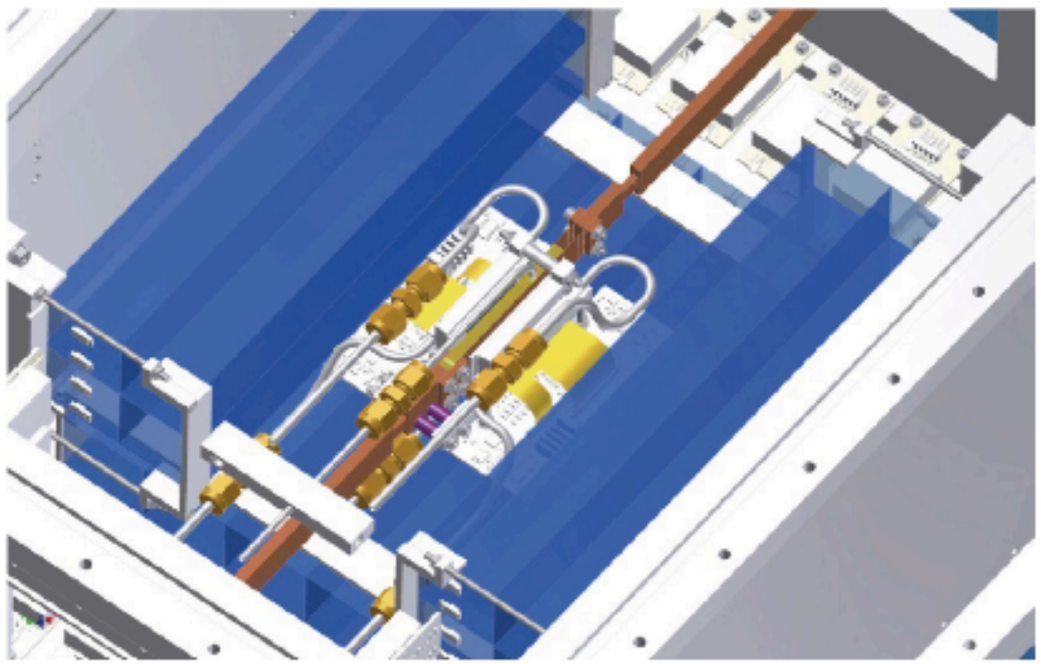
The VIP-2 Experiment

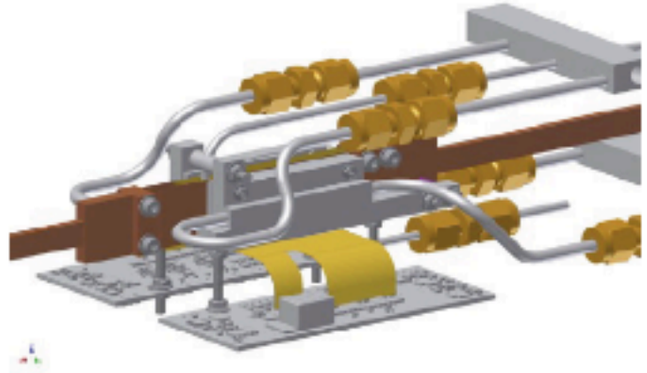
2 strip shaped Cu targets (25 um x 7 cm x 2 cm) more compact target → higher acceptance, thinner → higher efficiency
DC current supply to Cu bars

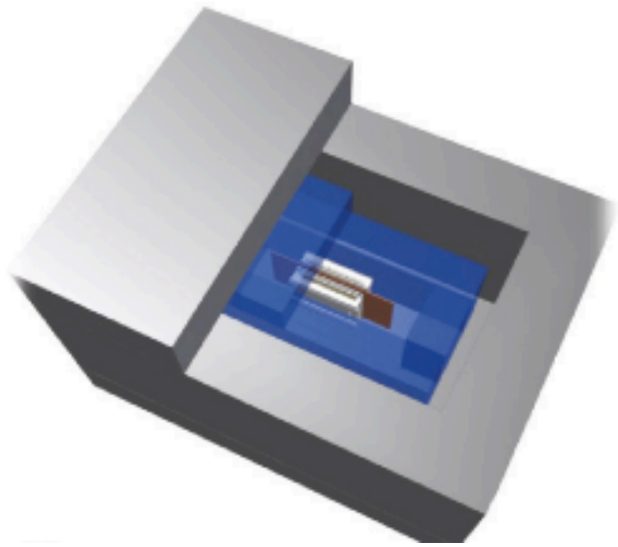
Cu strips cooled by a closed Fryka chiller circuit →higher current

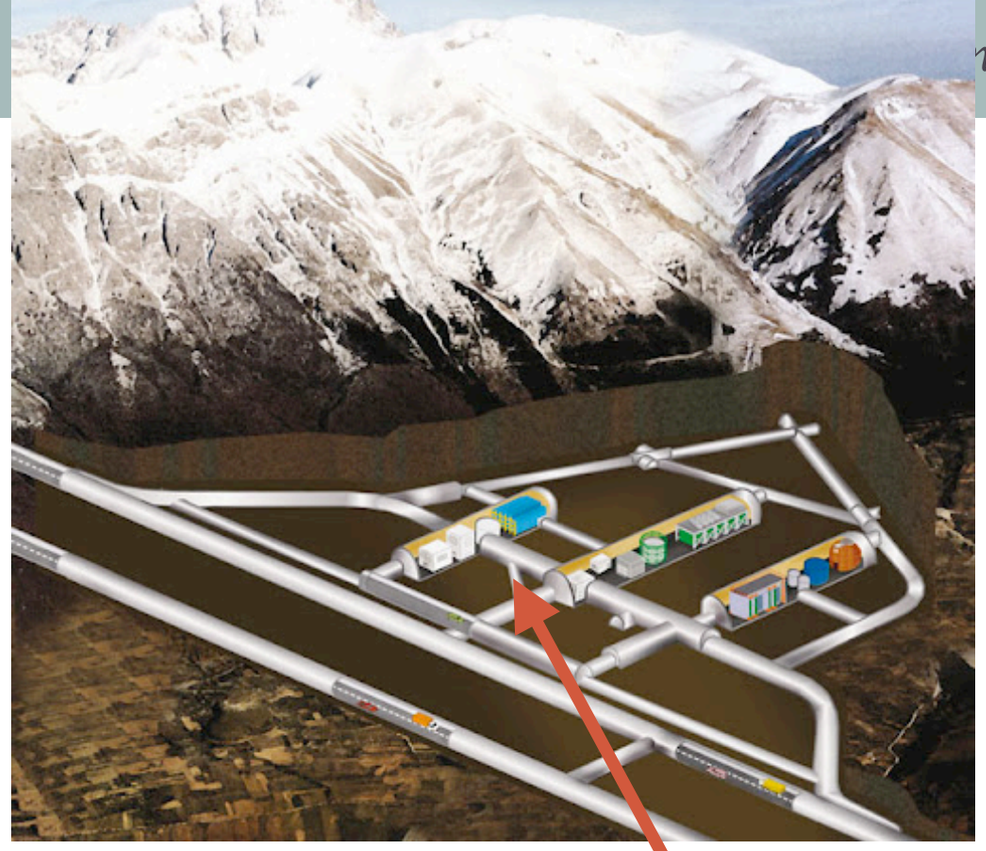
(100 A) @ 20 °C of Cu target implies 1 °K heating in SDDs

Sketch of the VIP2 Setup:



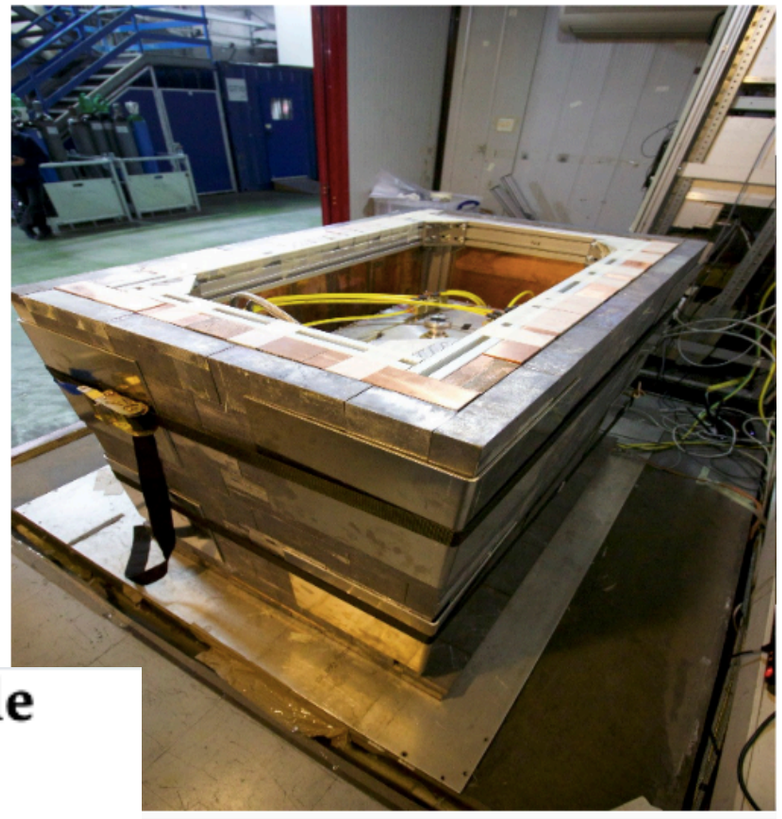




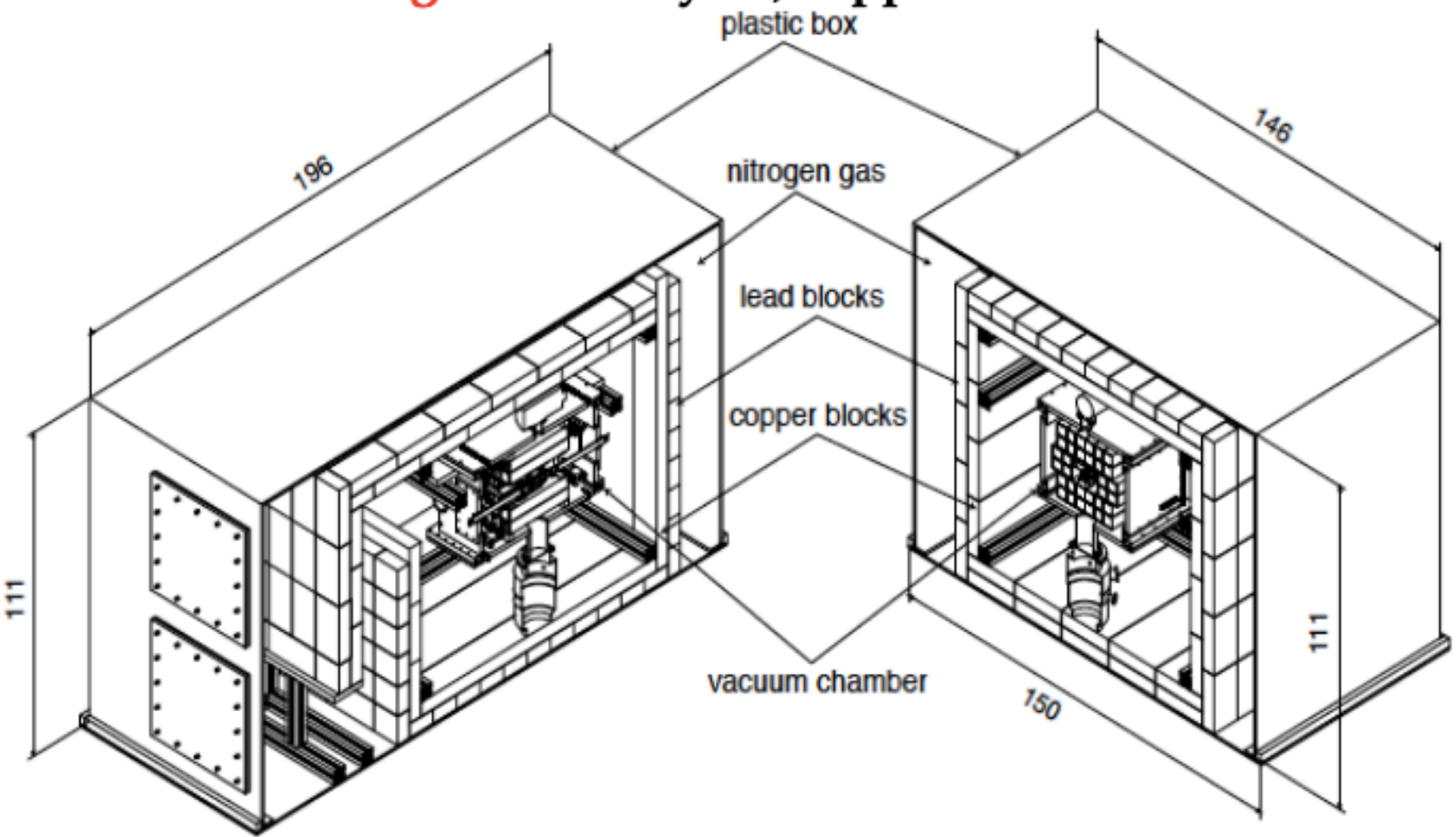


1400 m rock coverage

Upgrade concluded in April 2019:



Passive scielding → two layers, copper inside lead outside

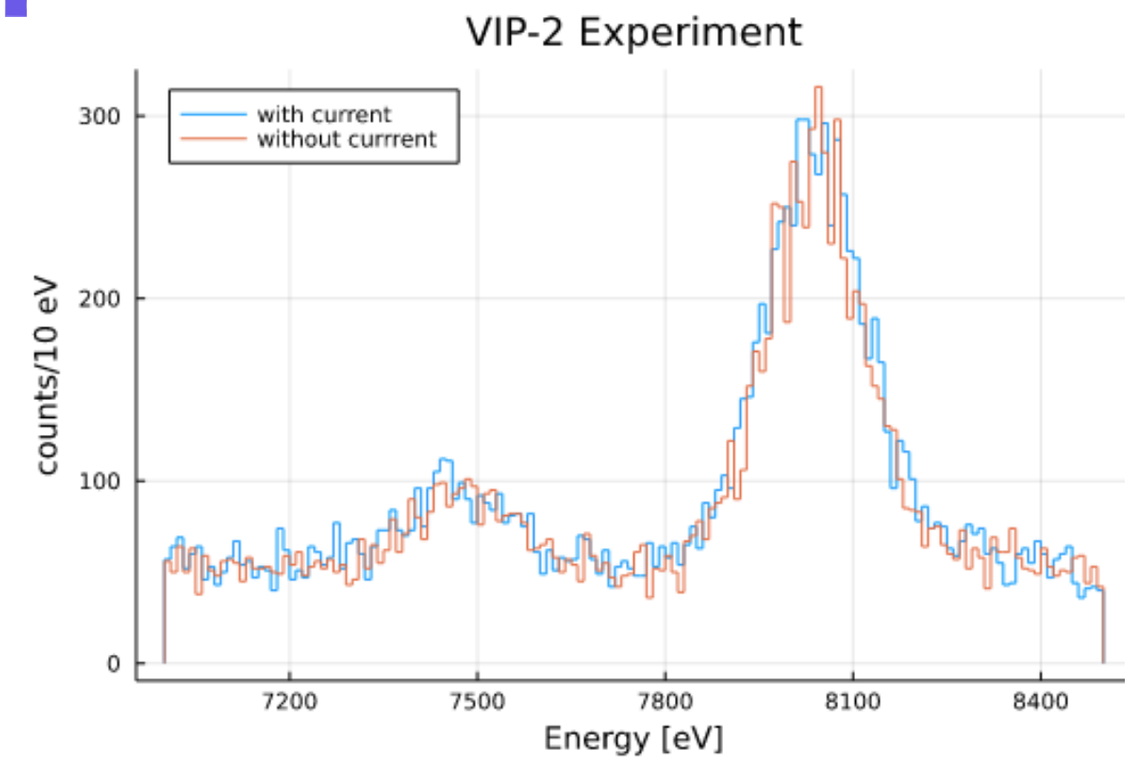


Symmetry **2022**, *14*(5), 893;
https://doi.org/10.3390/sym14050893

Article

Testing the Pauli Exclusion Principle with the VIP-2 Experiment

Fabrizio Napolitano [1,*], Sergio Bartalucci [1], Sergio Bertolucci [2], Massimiliano Bazzi [1], Mario Bragadireanu [1,3], Cesidio Capoccia [1], Michael Cargnelli [4], Alberto Clozza [1], Luca De Paolis [1], Raffaele Del Grande [1,5,6], Carlo Fiorini [7], Carlo Guaraldo [1], Mihail Iliescu [1], Matthias Laubenstein [8], Johann Marton [1,4], Marco Miliucci [1], Edoardo Milotti [9], Federico Nola [10], Kristian Piscicchia [1,5], Alessio Porcelli [1,4], Alessandro Scordo [1], Francesco Sgaramella [1], Hexi Shi [4], Diana Laura Sirghi [1,3], Florin Sirghi [1,3], Oton Vazquez Doce [1], Johann Zmeskal [4] and Catalina Curceanu [1,3]



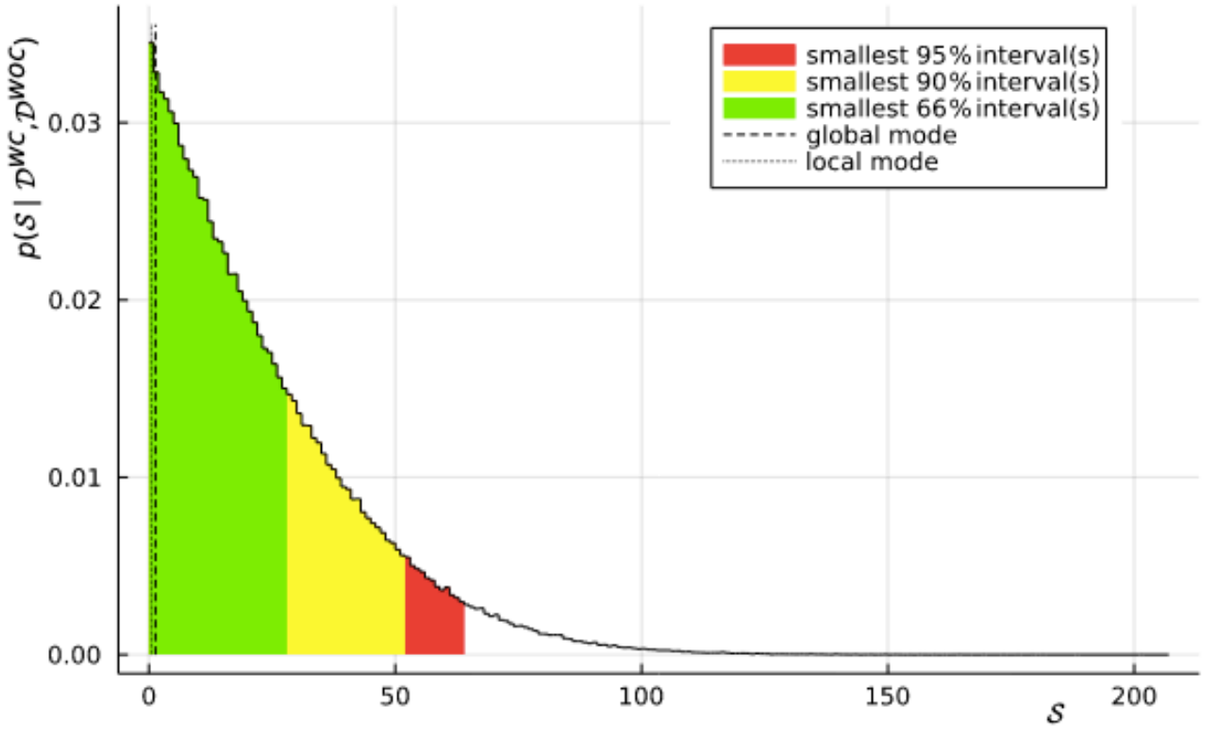


Figure 4. The posterior distribution for the signal yield S obtained by marginalization on all the parameters. Red, yellow, and green show the 95%, 90%, and 66% intervals, respectively.

$$\beta^2/2 \leq 6.8 \times 10^{-43} \text{ (Bayesian)}$$

New paradigm for VIP-2

Quantum gravity models can embed PEP violating transitions!

PEP is a consequence of the spin statistics theorem based on: Lorentz/Poincaré and CPT symmetries; locality; unitarity and causality. Deeply related to the very same nature of space and time

↓

most effective theories of QG foresee the non-commutativity of the space-time quantum operators (e.g. *k*-Poincarè, θ-Poincarè)

↓

non-commutativity induces a deformation of the Lorentz symmetry and of the locality → naturally encodes the violation of PEP

S. Majid, Hopf algebras for physics at the Planck scale, Class. Quantum Grav. 5 (1988) 1587.
S. Majid and H. Ruegg, Bicrossproduct structure of Kappa Poincare group and noncommutative geometry, Phys. Lett. B 334 (1994) 348, hep-th/9405107.
M. Arzano and A. Marciano, Phys. Rev. D 76, 125005 (2007) [arXiv:0707.1329].
G. Amelino-Camelia, G. Gubitosi, A. Marciano, P. Martinetti and F. Mercati, Phys. Lett. B 671, 298 (2009) [arXiv:0707.1863].
A. Addazi, A. Marcianò International Journal of Modern Physics A Vol. 35, No. 32, 2042003 (2020)

↓

PEP violation is suppressed with $(E/\Lambda)^n$, n depends on the specific model, E is the energy of the PEP violating transition, Λ is the scale of the space-time non-commutativity emergence.

Theoretical prediction *Int.J.Mod.Phys.A* 35 (2020) 32, 2042003

specific calculation of atomic levels transitions probabilities for θ-Poincaré

$$W \simeq W_0 \phi_{PEPV}\,, \qquad \phi_{\rm PEPV} = \delta^2 \simeq \frac{D}{2}\frac{E_N}{\Lambda}\frac{\Delta E}{\Lambda} \qquad \phi_{\rm PEPV} = \delta^2 \simeq \frac{C}{2}\frac{\bar{E}_1}{\Lambda}\frac{\bar{E}_2}{\Lambda}$$

for non-vanishing (vanishing) electric like components of the θμν tensor.

Connection with quon algebra (in the case of quon fields however the q factor does not show any energy dependence):

$$q(E) = -1 + 2\delta^2(E)$$

An experimental bound on the probability that PEP may be violated in atomic transition processes, straightforwardly translates into a bound on the new physics scale Λ, consistently with the choice of the θ_{0i} components.

Experimental Setup

High purity Ge detector measurement:

- **high purity co-axial p-type germanium detector (HPGe), diameter of 8.0 cm, length of 8.0 cm, surrounded by an inactive layer of lithium-doped germanium of 0.075 mm.**
- **The target material is composed of three cylindrical sections of radio-pure Roman lead, completely surrounding the detector.**

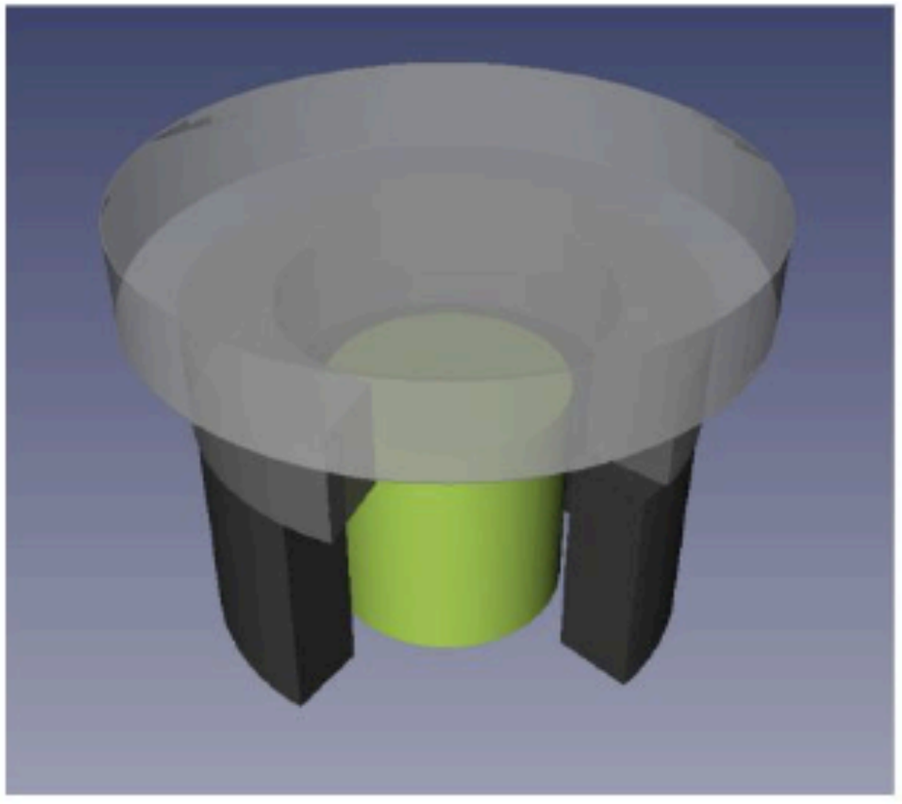
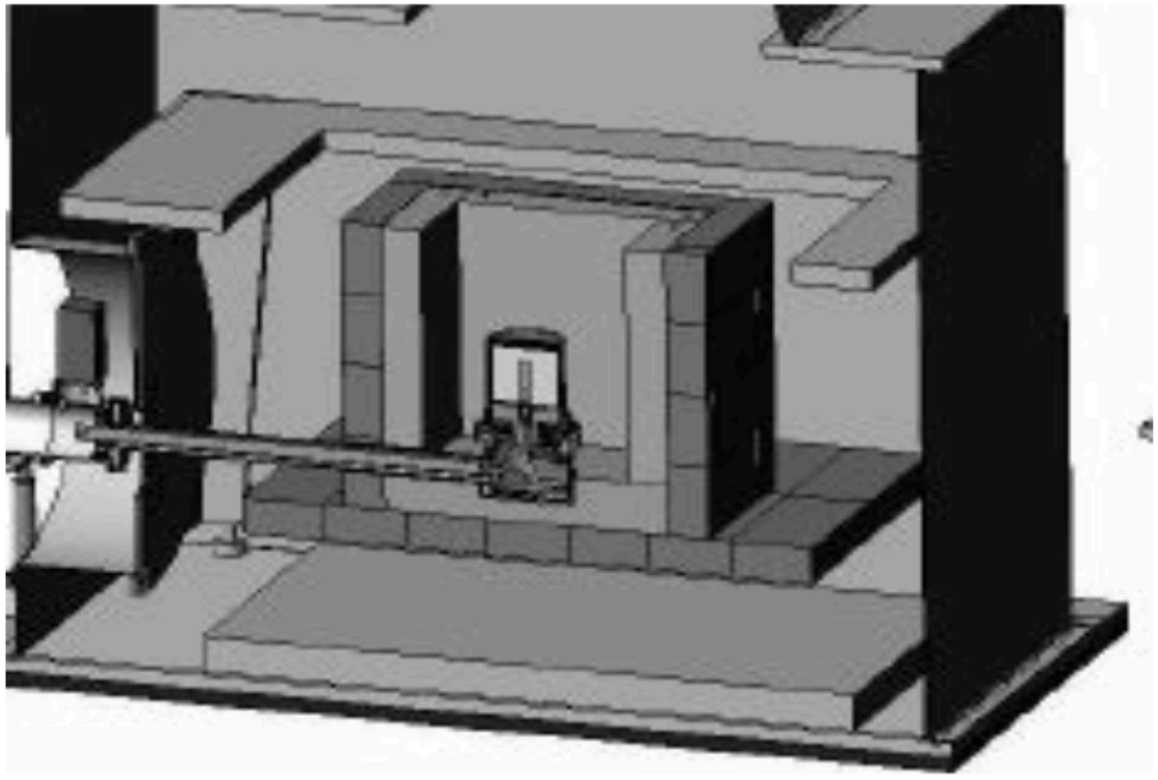


Fig. 1 Schematic representation of the Ge crystal (in green) and the surrounding lead target cylindrical sections (in grey)

Experimental Setup

- **Passive shielding:**
  outer part lead (30 cm from the bottom and 25 cm from the sides). Inner layer (5 cm) electrolytic copper.
  On the bottom and on the sides 5 cm thick 10B-polyethylene plates reduce the neutron flux towards the detector.

- **shield + cryostat enclosed in air tight steel housing flushed with nitrogen to avoid contact with external air (thus radon).**



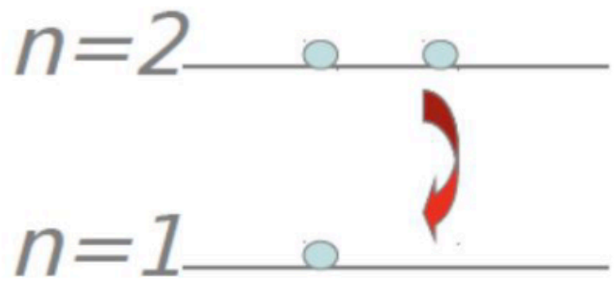
- **Whole detector is characterised and all of its components have been put into a validated Monte Carlo (MC) code based on GEANT4.**

- **Acquisition time $\Delta t \approx 70d \approx 6.1\ 10^6 s$**

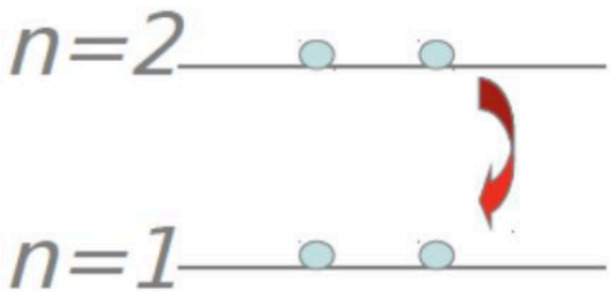
K. P. et al., Eur. Phys. J. C (2020) 80: 508
https://doi.org/10.1140/epjc/s10052-020-8040-5

- **Aim of the measurement: search for the X-rays signature of PEP-violating K_α and K_β transitions in Pb, when the 1s level is already occupied by two electrons.**

- **Transitions are shifted with respect to the standard ones due to additional shielding.**



Normal 2p → 1s transition



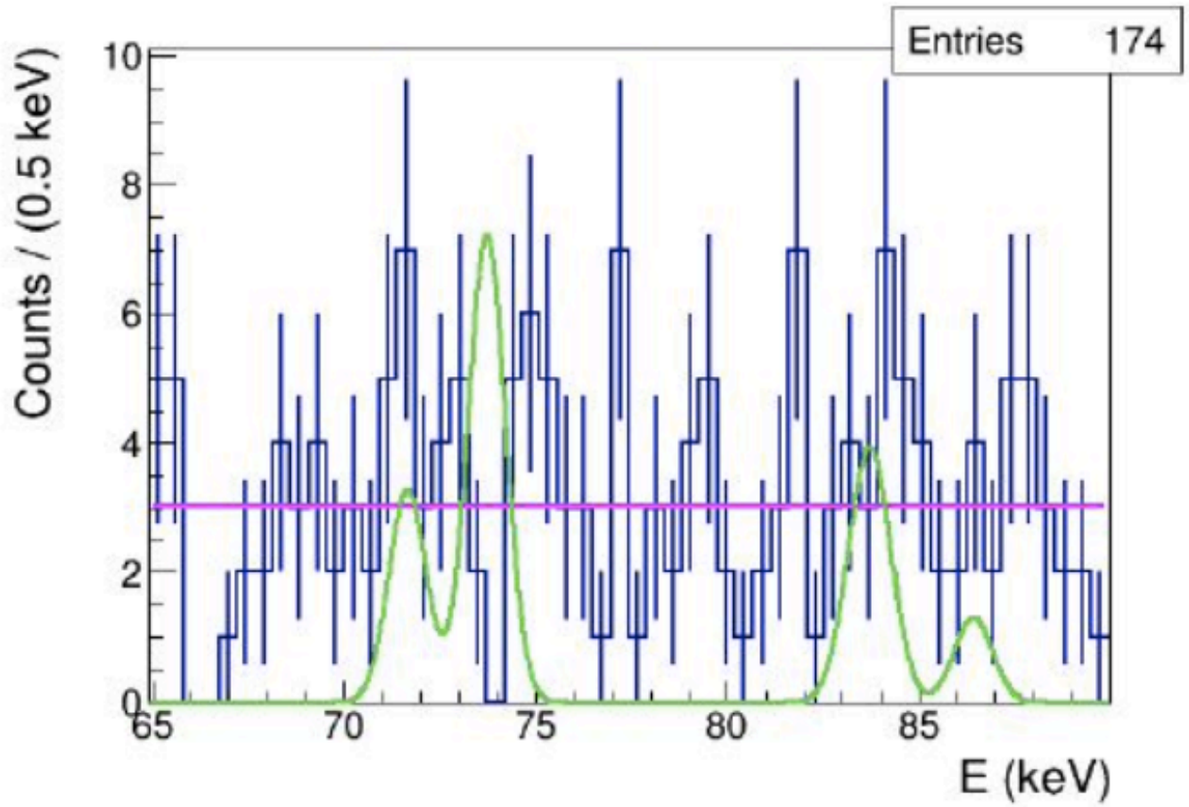
2p → 1s transition violating Pauli principle

- **Deformation of the algebra preserves, at the first order, standard atomic transition probabilities, the violating transition probabilities being dumped by factors $\delta^2(E)$ -> transitions to the 1s level from levels higher then 4p can be neglected.**

- **PEP violating K lines energies based on multi configuration Dirac-Fock and General Matrix Elements numerical code.**

Transitions in Pb	allow. (keV)	forb. (keV)
1s - $2p_{3/2}$ $K_{\alpha 1}$	74.961	73.713
1s - $2p_{1/2}$ $K_{\alpha 2}$	72.798	71.652
1s - $3p_{3/2}$ $K_{\beta 1}$	84.939	83.856
1s - $4p_{1/2(3/2)}$ $K_{\beta 2}$	87.320	86.418
1s - $3p_{1/2}$ $K_{\beta 3}$	84.450	83.385

Results



Background

Signal from violating transitions in Lead

From which an upper limit on the non-commutativity scale is obtained (90% Probability):

θ_{0i}	$\bar{S}$	**lower limit on Λ (Planck scales)**
$\theta_{0i} = 0$	13.2990	$6.9 \cdot 10^{-2}$
$\theta_{0i} \neq 0$	18.1515	$2.6 \cdot 10^{2}$

PHYSICAL REVIEW LETTERS 129, 131301 (2022)

PHYSICAL REVIEW D 107, 026002 (2023)

Conclusions

- *VIP collaboration tests PEP violation in Open Systems (VIP-2) and Closed Systems (VIP-Lead)*
- *VIP-2 setting most stringent limits in Open Systems*
  - *VIP-3 soon to be installed at LNGS*
- *Effective Theories of Quantum Gravity (NCQG) predict PEP violation in Closed Systems through non-commutativity of space-time and thus Lorentz symmetry / locality*
- *Using High-Purity Germanium Detectors, we have set strong bounds on theta Poincaré, excluding beyond the Planck scale the non vanishing electric-like case $\theta_{i,0} \neq 0$ and strongly constrained the vanishing case*

Thank you for your attention!

Questions?

PHYSICAL REVIEW LETTERS **129,** 131301 (2022)

Strongest Atomic Physics Bounds on Noncommutative Quantum Gravity Models

Kristian Piscicchia,[2,3] Andrea Addazi,[1,3,*] Antonino Marcianò,[4,3,†] Massimiliano Bazzi,[3] Michael Cargnelli,[5,3] Alberto Clozza,[3] Luca De Paolis,[3] Raffaele Del Grande,[6,3] Carlo Guaraldo,[3] Mihail Antoniu Iliescu,[3] Matthias Laubenstein,[7] Johann Marton,[5,3] Marco Miliucci,[3] Fabrizio Napolitano,[3] Alessio Porcelli,[5,3] Alessandro Scordo,[3] Diana Laura Sirghi,[3,8] Florin Sirghi,[3,8] Oton Vazquez Doce,[3] Johann Zmeskal,[5,3] and Catalina Curceanu[3,8]

[1]*Center for Theoretical Physics, College of Physics Science and Technology, Sichuan University, 610065 Chengdu, China*
[2]*Centro Ricerche Enrico Fermi—Museo Storico della Fisica e Centro Studi e Ricerche Enrico Fermi, 00184 Roma, Italy, EU*
[3]*Laboratori Nazionali di Frascati INFN, 00044 Frascati (Rome), Italy, EU*
[4]*Center for Field Theory and Particle Physics & Department of Physics Fudan University, 200438 Shanghai, China*
[5]*Stefan Meyer Institute for Subatomic Physics, Austrian Academy of Science, 1030 Vienna, Austria, EU*
[6]*Technische Universitt Mnchen, Physik Department E62, 85748 Garching, Germany, EU*
[7]*Laboratori Nazionali del Gran Sasso INFN, 67100 Assergi (L'Aquila), Italy, EU*
[8]*IFIN-HH, Institutul National pentru Fizica si Inginerie Nucleara Horia Hulubei, 077125 Măgurele, Romania, EU*

(Received 25 March 2022; accepted 22 August 2022; published 19 September 2022)

Investigations of possible violations of the Pauli exclusion principle represent critical tests of the microscopic space-time structure and properties. Space-time noncommutativity provides a class of universality for several quantum gravity models. In this context the VIP-2 lead experiment sets the strongest bounds, searching for the Pauli exclusion principle violating atomic transitions in lead, excluding the θ-Poincaré noncommutative quantum gravity models far above the Planck scale for nonvanishing $\theta_{\mu\nu}$ electriclike components, and up to 6.9×10^{-2} Planck scales if $\theta_{0i} = 0$.

DOI: 10.1103/PhysRevLett.129.131301

DOI: 10.1103/PhysRevLett.129.131301

Proof of spin-statistics theorem by Lüders and Zumino

Postulates:

- *The theory is invariant with respect to the proper inhomogeneous Lorentz group (includes translations, does not include reflections)*
- *Two operators of the same field at points separated by a spacelike interval either commute or anticommute (locality – microcausality)*
- *The vacuum is the state of lowest energy*
- *The metric of the Hilbert space is positive definite*
- *The vacuum is not identically annihilated by a field*

From these postulates it follows that (pseudo)scalar fields commute and spinor fields anticommute.

(G. Lüders and B. Zumino, Phys. Rev. 110 (1958) 1450)

Models of Pauli Exclusion Principle (PEP) Violations

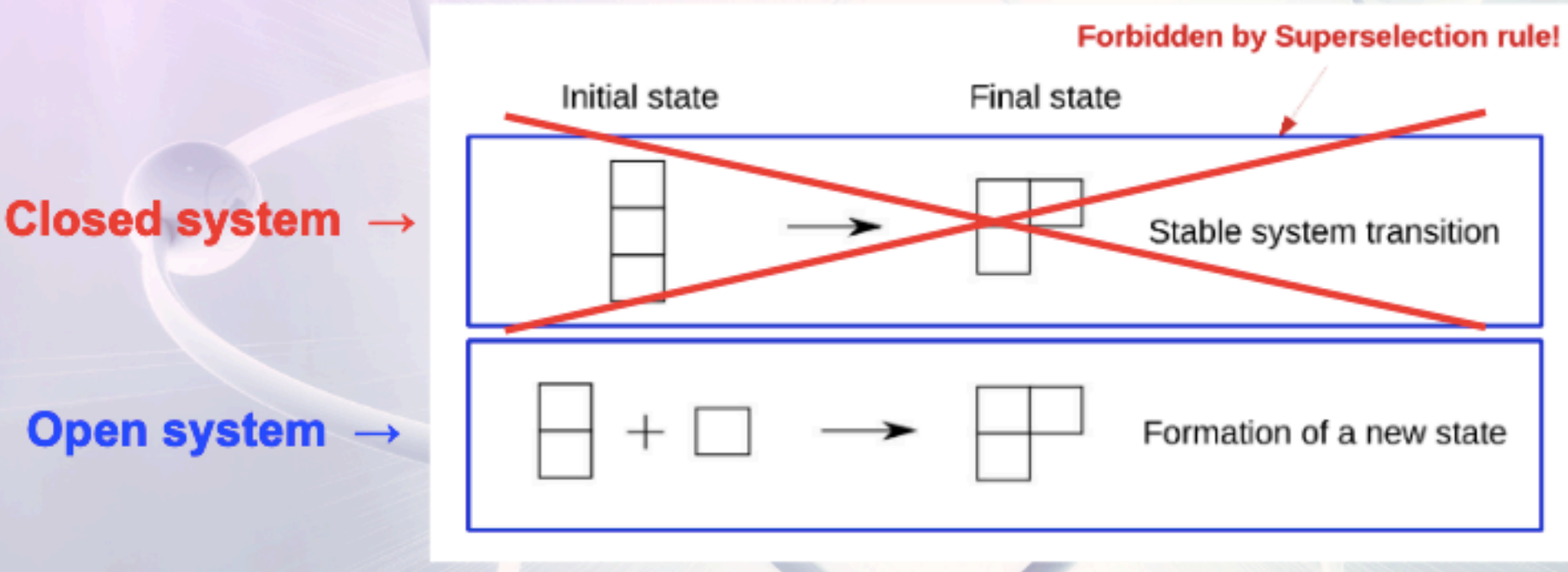
Some more PEP Violating models:

Greenberg, O.W. Mohapatra, R.N. Physical Review Letters 1987, 59, 2507
Govorkov, A. Physica A: Statistical Mechanics and its Applications 1994, 203, 655
Rahal, V.; Campa, A. , Physical Review A (1988) 38, 3728

Messiah – Greenberg superselection rule

Superpositions of states with different symmetry are not allowed →
transition probability between two symmetry states is ZERO

Messiah-Greenberg superselection rule :



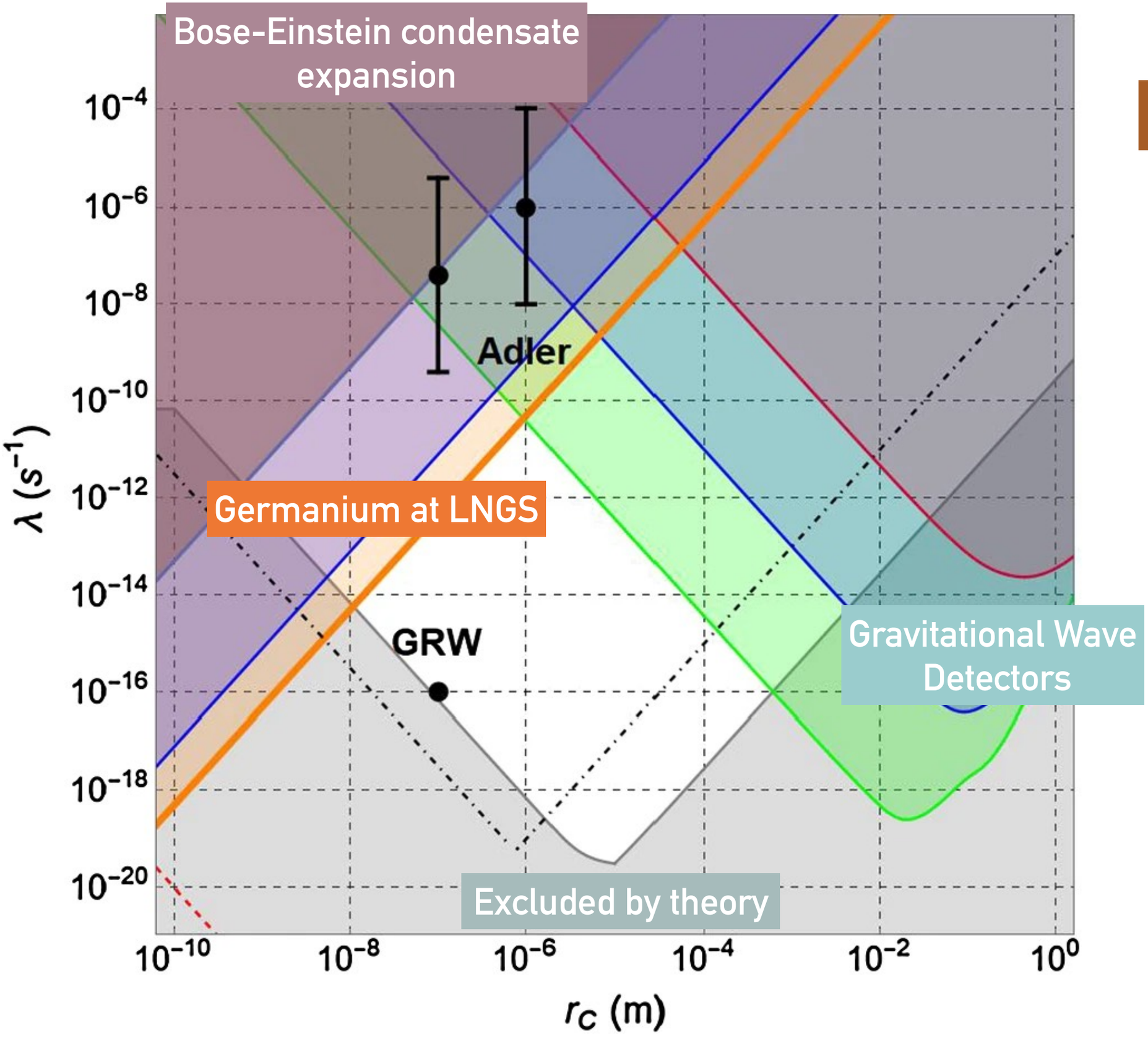
VIP-open systems sets the best limit on PEP violation
for an elementary particle
respecting the M-G superselection rule

VIP-2 experiment goal

(Upper limit not using Close Encounters (CE) treatment)

As reference for past experiments

Experiment	Target	Upper limit of $\beta^2/2$	reference
Ramberg-Snow	Copper	1.7×10^{-26}	[5]
S.R. Elliott et al.	Lead	1.5×10^{-27}	[14]
VIP(2006)	Copper	4.5×10^{-28}	[12]
VIP(2012)	Copper	4.7×10^{-29}	[13]
VIP2(goal)	Copper	$\times 10^{-31}$	[15]

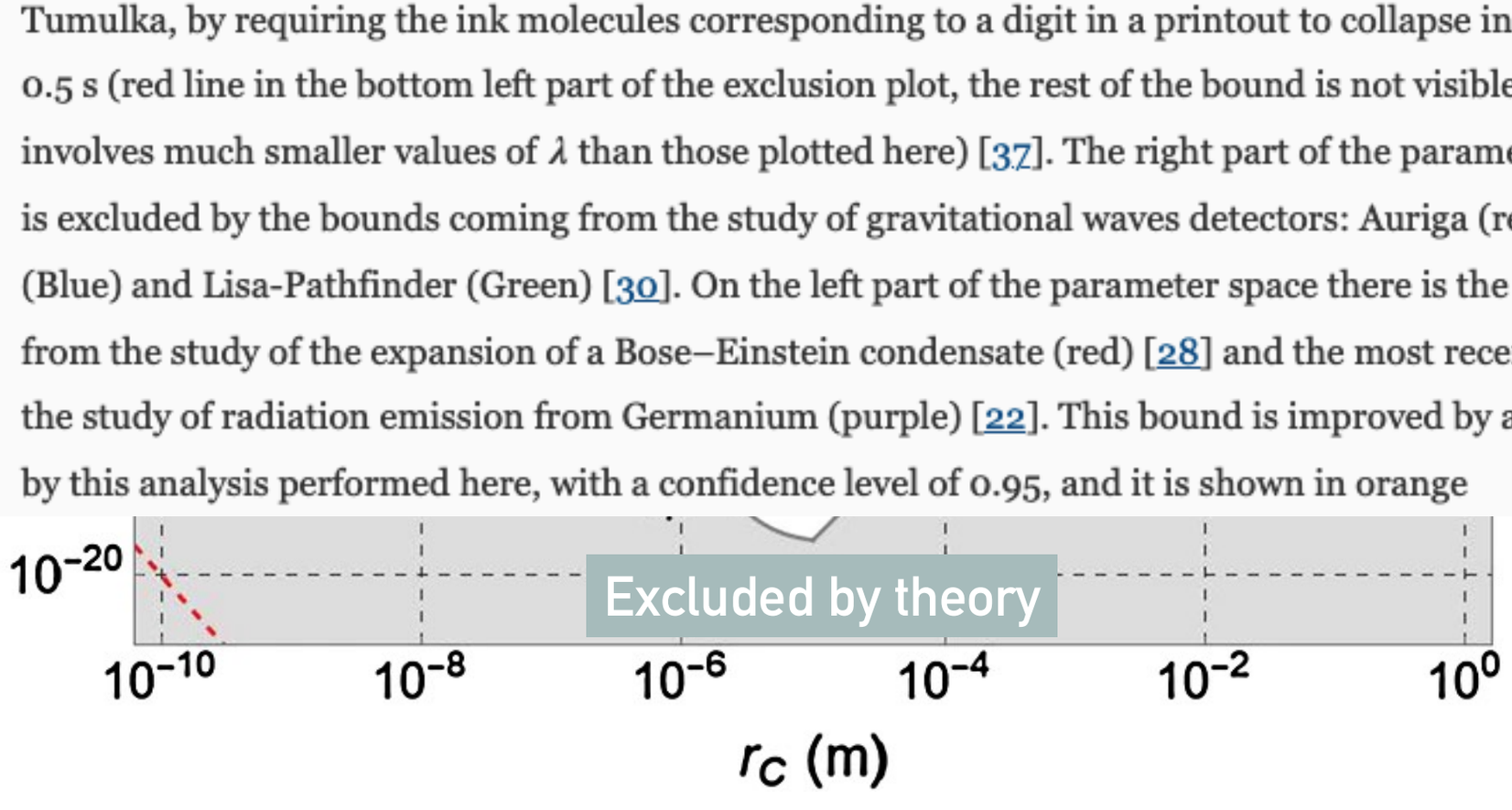


Donadi, S., Piscicchia, K., Del Grande, R. *et al.*. *Eur. Phys. J. C* 81, 773 (2021).

Bose-Einstein condensate expansion

Model

Mapping of the $\lambda - r_C$ CSL parameters: the proposed theoretical values (GRW [6], Adler [24, 25]) are shown as black points. The region excluded by theoretical requirements is represented in gray, and it is obtained by imposing that a graphene disk with the radius of 10 μm (about the smallest possible size detectable by human eye) collapses in less than 0.01 s (about the time resolution of human eye) [31]. Contrary to the bounds set by experiments, the theoretical bound has a subjective component, since it depends on which systems are considered as "macroscopic". For example, it was previously suggested that the collapse should be strong enough to guarantee that a carbon sphere with the diameter of 4000 Å should collapse in less than 0.01 s, in which case the theoretical bound is given by the dash-dotted black line [36]. A much weaker theoretical bound was proposed by Feldmann and Tumulka, by requiring the ink molecules corresponding to a digit in a printout to collapse in less than 0.5 s (red line in the bottom left part of the exclusion plot, the rest of the bound is not visible as it involves much smaller values of λ than those plotted here) [37]. The right part of the parameter space is excluded by the bounds coming from the study of gravitational waves detectors: Auriga (red), Ligo (Blue) and Lisa-Pathfinder (Green) [30]. On the left part of the parameter space there is the bound from the study of the expansion of a Bose–Einstein condensate (red) [28] and the most recent from the study of radiation emission from Germanium (purple) [22]. This bound is improved by a factor 13 by this analysis performed here, with a confidence level of 0.95, and it is shown in orange



Donadi, S., Piscicchia, K., Del Grande, R. *et al.*. *Eur. Phys. J. C* 81, 773 (2021).

Diósi-Penrose (DP) Collapse model

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar}\hat{H}dt + \sqrt{\frac{G}{\hbar}} \int d\mathbf{x}(\hat{\mu}(\mathbf{x}) - \langle\hat{\mu}(\mathbf{x})\rangle)dW_t(\mathbf{x}) - \right.$$

$$\left. - \frac{G}{2\hbar} \int d\mathbf{x}d\mathbf{y} \frac{(\hat{\mu}(\mathbf{x})-\langle\hat{\mu}(\mathbf{x})\rangle)(\hat{\mu}(\mathbf{y})-\langle\hat{\mu}(\mathbf{y})\rangle)}{|\mathbf{x}-\mathbf{y}|} \right] |\psi_t\rangle$$

Schrödinger

Specific dynamics for the collapse

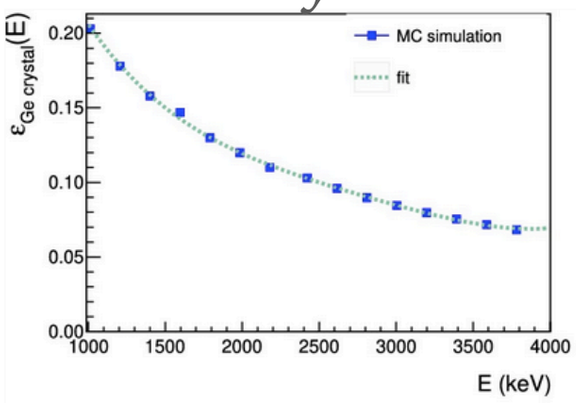
Collapse in position, no superluminal signals and amplification mechanism

$$\tau^{-1} = \frac{G}{2\hbar} \int d\mathbf{x}d\mathbf{y} \frac{(\hat{\mu}_a(\mathbf{x})-\hat{\mu}_b(\mathbf{x}))(\hat{\mu}_a(\mathbf{y})-\hat{\mu}_b(\mathbf{y}))}{|\mathbf{x}-\mathbf{y}|}$$

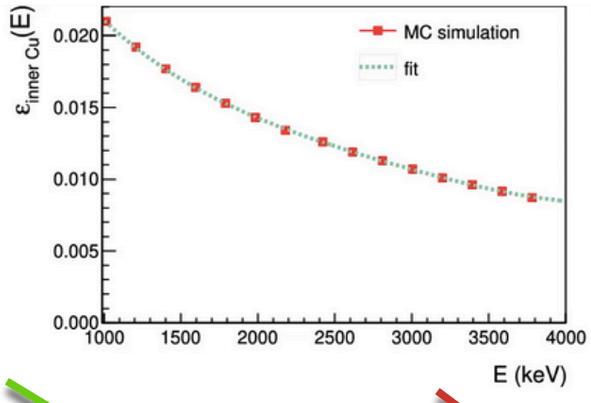
R. Penrose, Found. Phys. 44, 557-575 (2014), R. Penrose, Gen. Relativ. Gravit. 28, 581-600 (1996), L. Diósi, Phys. Rev. A **40**, 1165-1174 (1989).

Measurement and MC validation

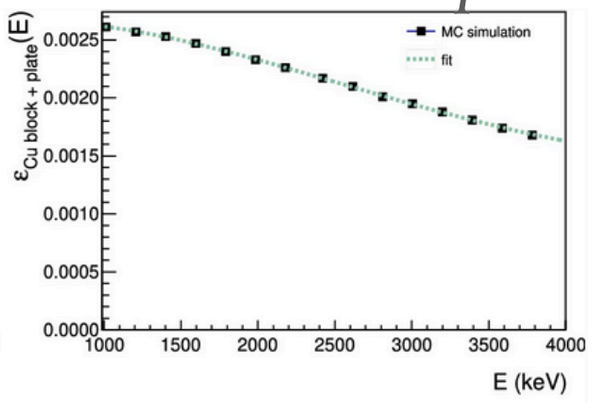
Ge Crystals



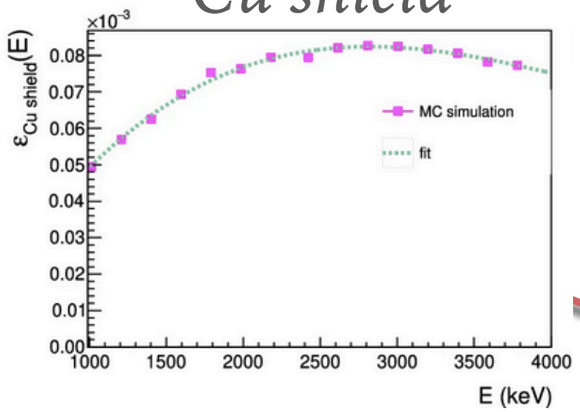
Inner Cu



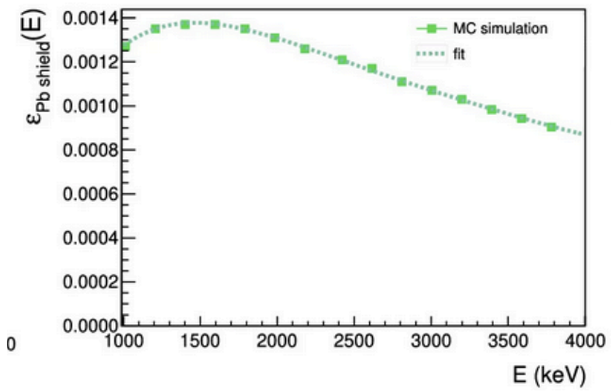
Cu block and plate

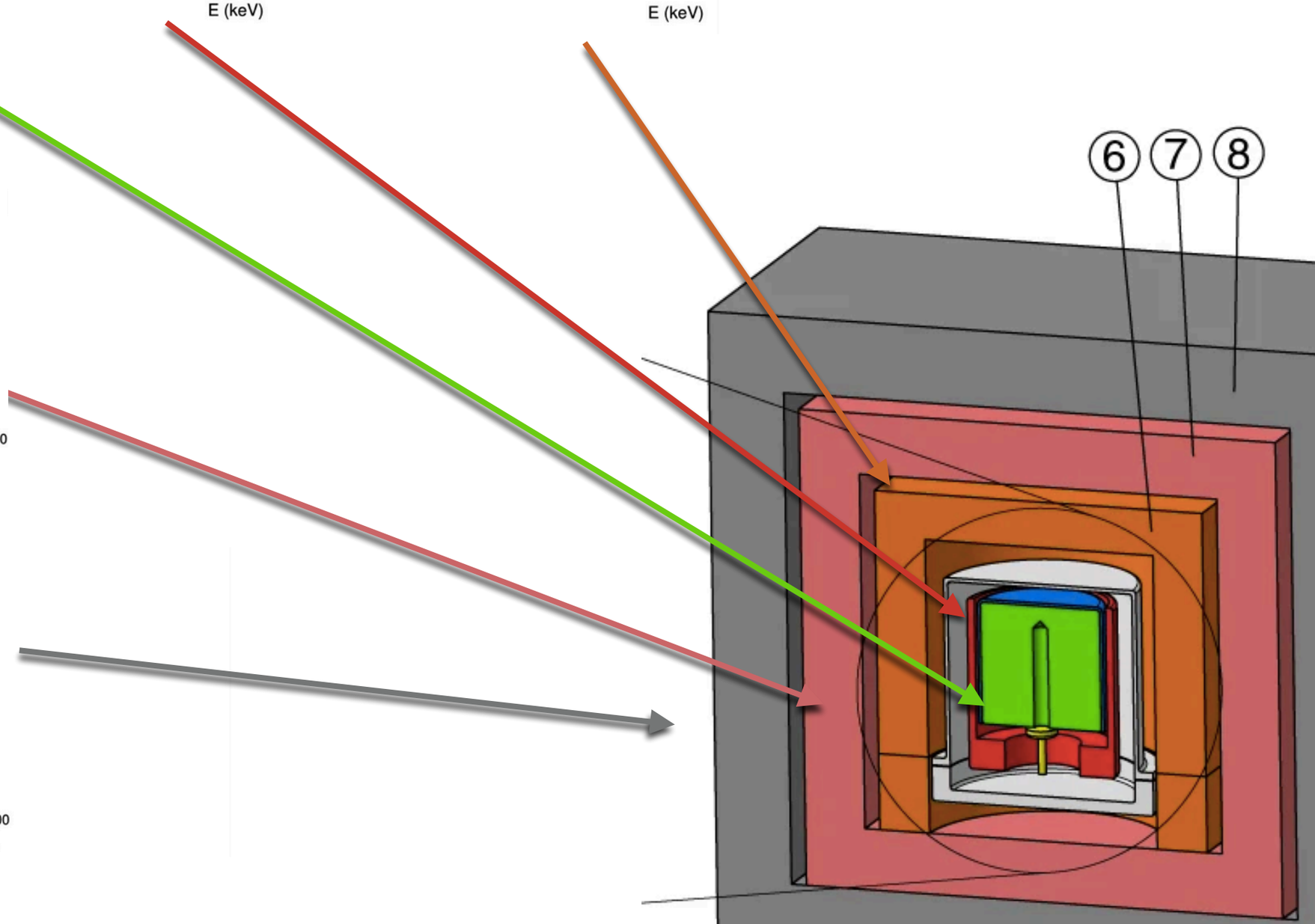


Cu shield



Pb shield





Models of Pauli Exclusion Principle (PEP) Violations

Theories of Statistics Violation

O.W. Greenberg: AIP Conf.Proc.545:113-127,2004

*"Possible external **motivations for violation of statistics** include: (a) violation of CPT, (b) violation of locality, (c) violation of Lorentz invariance, (d) extra space dimensions, (e) discrete space and/or time and (f) non-commutative spacetime....."*

Ignatiev & Kuzmin model: Fermi oscillator with a third state

(Ignatiev, A.Y., Kuzmin, V. , Quarks '86: Proceedings of the 229 Seminar, Tbilisi, USSR, 1517 April 1986)

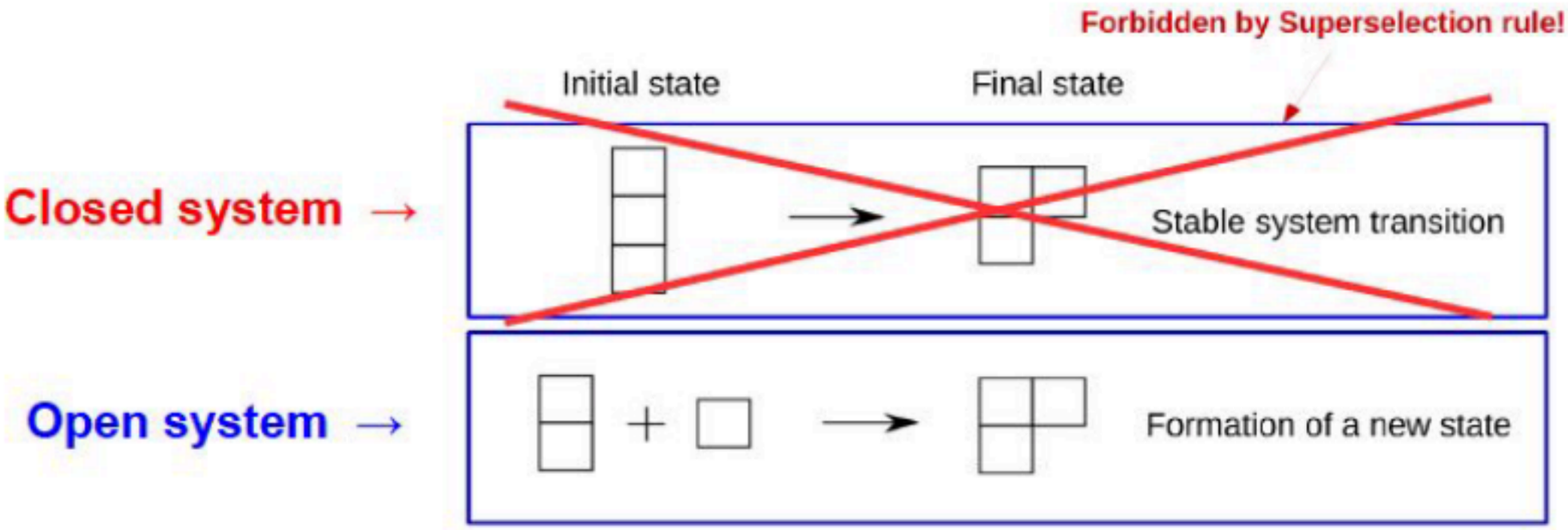
$$a^+|0\rangle = |1\rangle \qquad a|0\rangle = 0$$
$$a^+|1\rangle = \beta|2\rangle \qquad a|1\rangle = |0\rangle$$
$$a^+|2\rangle = 0 \qquad a|2\rangle = \beta|1\rangle$$

β quantifies the degree of violation in the transition

Messiah-Greenberg super-selection rule:

Superposition of states with different symmetry are not allowed →

Transition probability between two symmetry states is ZERO



VIP-2 Experiment: best limits on PEP violation of an elementary particle respecting the Messiah-Greenberg super-selection rule

New paradigm for VIP-2

Are Quantum Gravity models experimentally testable?

A. Addazi (Chengdu Univ.) A. Marcianò (Fudan University)

VIP-2 underground experiment as a *Crash-Test* of Non-Commutative Quantum Gravity

Pauli Exclusion Principle (PEP) violations induced from non-commutative space-time can be searched VIP-2 experiment set-up. We show that the limit from VIP-2 experiments on non-commutative space-time scale Λ, related to energy dependent PEP violations, are severe: κ-Poincaré non-commutativity is ruled-out up to the Planck scale. In the next future θ-Poincaré will be probed until the Grand-Unification scale! This highly motivates Pauli Exclusion Principle tests from underground experiments as a test of quantum gravity and space-time microscopic structure.

See also A. Addazi et al., 2018 Chinese Phys. C 42 094001, arXiv:1712.08082 [hep-th]