



Jožef Stefan Institute

Constraining the SMEFT extended with sterile neutrinos at FCC-ee

Patrick Bolton (Jožef Stefan Institute)

In collaboration with: Frank Deppisch (UCL), Suchita Kulkarni (NAWI Graz), Chayan Majumdar (UCL), Wenna Pei (UCL/Eötvös Loránd U.)

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Future Circular e^+e^- Collider (FCC-ee)



[European Strategy for Particle Physics Preparatory Group, Update 2020]

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 m_{N} (GeV)

HNL production and decay via $V_{\ell N}$

- HNLs are long-lived (LLPs)
- Displaced vertex search $N \rightarrow \ell W^* \rightarrow \ell j j$ [Blondel, Graverini, Serra, Shaposhnikov, 14]
- Displaced vertex search $N \rightarrow \nu \ell \ell$ [Alimena et al., 22]





Future Circular e^+e^- Collider (FCC-ee)



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Future Circular e^+e^- Collider (FCC-ee)



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vSMEFT at FCC-ee: four-fermio

d = 6 $C_i = \frac{1}{\Lambda^2}$ $\Delta L = 0$

$$\begin{array}{c|c} \psi^{4} \\ \mathcal{O}_{ll} & (\bar{L}\gamma_{\mu}L)(\bar{L}\gamma^{\mu}L) \\ \mathcal{O}_{le} & (\bar{L}\gamma_{\mu}L)(\bar{e}_{R}\gamma^{\mu}e_{R}) \\ \mathcal{O}_{lNle} & \epsilon_{ij}(\bar{L}^{i}N_{R})(\bar{L}^{j}e_{R}) \\ \mathcal{O}_{lN} & (\bar{L}\gamma_{\mu}L)(\bar{N}_{R}\gamma^{\mu}N_{R}) \\ \mathcal{O}_{eN} & (\bar{e}_{R}\gamma_{\mu}e_{R})(\bar{N}_{R}\gamma^{\mu}N_{R}) \end{array}$$

$\psi^4 H$							
\mathcal{O}_{llleH}	$\epsilon_{ij}\epsilon_{mn}(\bar{e}_R L^i)(\bar{L}^{jc}L^m)H^n$						
\mathcal{O}_{lNlH}	$\epsilon_{ij}(\bar{L}\gamma_{\mu}L)(\bar{N}^{c}_{R}\gamma^{\mu}L^{i})H^{j}$						
\mathcal{O}_{eNlH}	$\epsilon_{ij} (ar{e}_R \gamma_\mu e_R) (ar{N}^c_R \gamma^\mu L^i) H^j$						
\mathcal{O}_{lNeH}	$(ar{L}N_R)(ar{N}_R^c e_R)H$						
\mathcal{O}_{elNH}	$H^{\dagger}(ar{e}_R L)(ar{N}_R^c N_R)$						
	\mathcal{O}_{llleH} \mathcal{O}_{lNlH} \mathcal{O}_{eNlH} \mathcal{O}_{lNeH}						

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$$\begin{split} \psi^{4}H \\ d &= 7 \\ C_{i} = \frac{1}{\Lambda^{3}} \\ \Delta L &= \pm 2 \\ \end{split}$$

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on operators
$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}+N_R} + \sum_i C_i^{(d)} \mathcal{O}_i^{(d)}$$

 e^+ ν_i / N_i $C_i^{(d)} \propto N_i$
 $e^ \nu_j / N_j$

Vector:
$$C_{Ne}^{V,RR}(\bar{N}_R\gamma_{\mu}N_R)(\bar{e}_R\gamma^{\mu}e_R)$$
 $C_{Ne}^{V,RL}(\bar{N}_R\gamma_{\mu}N_R)(\bar{e}_L)$ Scalar: $C_{Ne}^{S,RR}(\bar{N}_R^c N_R)(\bar{e}_L e_R)$ $C_{Ne}^{S,RL}(\bar{N}_R^c N_R)(\bar{e}_R e_R)$ Tensor: $C_{Ne}^{T,RR}(\bar{N}_R^c \sigma_{\mu\nu}N_R)(\bar{e}_L \sigma^{\mu\nu}e_R)$



vSMEFT at FCC-ee: four-fermion operators

d = 6 $C_i = \frac{1}{\Lambda^2}$ $\Delta L = 0$

$$\begin{array}{c|c} \psi^{4} \\ \mathcal{O}_{ll} & (\bar{L}\gamma_{\mu}L)(\bar{L}\gamma^{\mu}L) \\ \mathcal{O}_{le} & (\bar{L}\gamma_{\mu}L)(\bar{e}_{R}\gamma^{\mu}e_{R}) \\ \mathcal{O}_{lNle} & \epsilon_{ij}(\bar{L}^{i}N_{R})(\bar{L}^{j}e_{R}) \\ \mathcal{O}_{lNle} & (\bar{L}\gamma_{\mu}L)(\bar{N}_{R}\gamma^{\mu}N_{R}) \\ \mathcal{O}_{eN} & (\bar{e}_{R}\gamma_{\mu}e_{R})(\bar{N}_{R}\gamma^{\mu}N_{R}) \end{array}$$

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 $C^{V,LL}_{
u e}(ar{
u}_L\gamma_\mu
u_L)(ar{e}_L\gamma^\mu e_L)$ $C^{V,LR}_{
u e}(ar{
u}_L\gamma_\mu
u_L)(ar{e}_R\gamma^\mu e_R)$ Vector: Scalar: $C_{\nu e}^{S,LL}(\bar{\nu}_{L}^{c}\nu_{L})(\bar{e}_{R}e_{L}) \qquad C_{\nu e}^{S,LR}(\bar{\nu}_{L}^{c}\nu_{L})(\bar{e}_{L}e_{R})$ Tensor: $C_{\nu e}^{T,LL}(\bar{\nu}_{L}^{c}\sigma_{\mu\nu}\nu_{L})(\bar{e}_{R}\sigma^{\mu\nu}e_{L})$ 8



vSMEFT at FCC-ee: four-fermion operators

d = 6 $C_i = \frac{1}{\Lambda^2}$ $\Delta L = 0$

$$\begin{array}{c|c} \psi^{4} \\ \mathcal{O}_{ll} & (\bar{L}\gamma_{\mu}L)(\bar{L}\gamma^{\mu}L) \\ \mathcal{O}_{le} & (\bar{L}\gamma_{\mu}L)(\bar{e}_{R}\gamma^{\mu}e_{R}) \\ \mathcal{O}_{lNle} & \epsilon_{ij}(\bar{L}^{i}N_{R})(\bar{L}^{j}e_{R}) \\ \mathcal{O}_{lN} & (\bar{L}\gamma_{\mu}L)(\bar{N}_{R}\gamma^{\mu}N_{R}) \\ \mathcal{O}_{eN} & (\bar{e}_{R}\gamma_{\mu}e_{R})(\bar{N}_{R}\gamma^{\mu}N_{R}) \end{array}$$

$$\begin{split} \psi^{4}H \\ d &= 7 \\ C_{i} &= \frac{1}{\Lambda^{3}} \\ \Delta L &= \pm 2 \\ \end{bmatrix} \begin{matrix} \mathcal{O}_{llleH} \\ \mathcal{O}_{lNlH} \\ \mathcal{O}_{eNlH} \\ \mathcal{O}_{eNlH} \\ \mathcal{O}_{eNlH} \\ \mathcal{O}_{enneH} \\ \mathcal{O}_{en$$

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Vector:
$$C_{\nu N e}^{V,RR}(\bar{\nu}_{L}^{c}\gamma_{\mu}N_{R})(\bar{e}_{R}\gamma^{\mu}e_{R})$$
 $C_{\nu N e}^{V,RL}(\bar{\nu}_{L}^{c}\gamma_{\mu}N_{R})(\bar{e}_{L}\gamma^{\mu}e_{R})$
Scalar: $C_{\nu N e}^{S,RR}(\bar{\nu}_{L}N_{R})(\bar{e}_{L}e_{R})$
Tensor: $C_{\nu N e}^{T,RR}(\bar{\nu}_{L}\sigma_{\mu\nu}N_{R})(\bar{e}_{L}\sigma^{\mu\nu}e_{R})$

Note: Weak basis EFT operators \rightarrow mass basis No mixing between $\nu\nu$, νN , NN operators for $V_{\ell N} \rightarrow 0$





ν SMEFT at FCC-ee: effective neutral- and charged-currents

$$\begin{split} \psi^2 H^2 D \\ d &= 6 \qquad \mathcal{O}_{Hl}^{(1)} \quad (\bar{L}\gamma_{\mu}L)(H^{\dagger}i\overleftrightarrow{D}^{\mu}H) \\ \mathcal{O}_{Hl}^{(3)} \quad (\bar{L}\gamma_{\mu}\tau^I L)(H^{\dagger}i\overleftrightarrow{D}^{I\mu}H) \\ \mathcal{O}_{HN}^{(3)} \quad (\bar{N}_R\gamma_{\mu}N_R)(H^{\dagger}i\overleftrightarrow{D}^{\mu}H) \\ \Delta L &= 0 \qquad \mathcal{O}_{HNe} \quad (\bar{N}_R\gamma_{\mu}e_R)(\tilde{H}^{\dagger}i\overleftrightarrow{D}^{\mu}H) \end{split}$$

 $\psi^2 H^3 D$

d = 7	\mathcal{O}_{Nl1}	$\left \ \epsilon_{ij} (\bar{N}^c_R \gamma_\mu L^i) (i D^\mu H^j) (H^\dagger H) ight.$
$C_i = \frac{1}{\sqrt{3}}$	\mathcal{O}_{Nl2}	$\epsilon_{ij}(\bar{N}^c_R\gamma_\mu L^i)H^j(H^\dagger i\overleftrightarrow{D}^\mu H)$
Λ^{J} $\Lambda I = \pm 2$	\mathcal{O}_{leHD}	$\epsilon_{ij}\epsilon_{mn}(\bar{L}^{ic}\gamma_{\mu}e_{R})H^{j}H^{m}D^{\mu}H^{n}$
$\Delta L - \perp Z$		

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 e^{-}



ν SMEFT at FCC-ee: effective neutral- and charged-currents

$$d = 6$$

$$C_{i} = \frac{1}{\Lambda^{2}}$$

$$d = 7$$

$$C_{i} = \frac{1}{\Lambda^{3}}$$

$$C_{i$$

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strained by Electroweak Precision Data (EWPD);



HNL production cross sections



Calculated at LO, including all interference terms, neglecting m_{ρ}

For cut-based analysis, EFT operators implemented in Feynrules and simulated in MadGraph_aMC@NLO



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Monophoton analysis

HNLs produced and decay via EFT operators:

- HNLs long-lived and decay outside detector, monophoton plus E_{miss} signal
- Can be used to constrain ν SMEFT operator coefficients C_i
- Apply cuts to minimise SM background ($e^+e^- \rightarrow \nu \bar{\nu} \gamma$)
- Require HNL not to decay inside detector ($\beta\gamma c\tau > 5$ m)



Consider bounds on 'diagonal' and 'off-diagonal' couplings

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\sqrt{s}	Cuts
91.2	$ \cos(\theta) < 0.4$ and $ \cos(\theta) > 0.8$
240	$\begin{aligned} \cos(\theta) < 0.95 \\ E_{\gamma} < 40 \text{ GeV} \end{aligned}$

$$N_{\rm sig} = \mathcal{L} \cdot \sigma(s) \cdot \epsilon_{\rm cuts} \cdot \epsilon_{\rm decay}$$





Displaced vertex analysis

If the HNLs are produced and decay via EFT operators:

- HNLs long-lived and decay inside detector, displaced vertex (DV) signal
- Complimentary constraint on ν SMEFT operator coefficients C_i
- Decays considered: $N_2 \rightarrow N_1 e^- e^+$ and $N_2 \rightarrow \nu e^- e^+$
- Cuts on e^{\pm} track transverse impact parameter $|d_0|$ minimise SM backgrounds



 $N_{\rm sig} = \mathcal{L} \cdot \sigma(s) \cdot \operatorname{Br}(N_2 \to N_1 e^- e^+) \cdot \epsilon_{\rm cuts} \cdot P_{\rm decay}$

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Constraints: four-fermion operation



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$$ators \quad \mathcal{O}_{eN} = (\bar{e}_R \gamma_\mu e_R) (\bar{N}_R \gamma^\mu N_R) \quad C_{Ne}^{V,RR} = C_{eN} = \frac{1}{\Lambda^2}$$

 $\delta = 10^{-2}$









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[PDB, Deppisch, Kulkarni, Majumdar, Pei, 24]



Summary and conclusions

We consider the production and decay of Dirac/Majorana HNLs via EFT operators at FCC-ee

- Possible if active-sterile mixing is suppressed in the relevant HNL mass range
- d = 6 and $d = 7 \nu$ SMEFT operators with possible tree-level UV completions are considered

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Summary and conclusions

We consider the production and decay of Dirac/Majorana HNLs via EFT operators at FCC-ee • Possible if active-sterile mixing is suppressed in the relevant HNL mass range • d = 6 and $d = 7 \nu$ SMEFT operators with possible tree-level UV completions are considered

For long-lived HNLs, two interesting signatures:

- Monophoton plus E_{miss} signature ($e^+e^- \rightarrow \nu\nu\gamma$, $e^+e^- \rightarrow \nuN\gamma$ and $e^+e^- \rightarrow NN\gamma$)
- Displaced vertex ($e^+e^- \rightarrow \nu N\gamma$ and $e^+e^- \rightarrow NN'\gamma$ followed by $N \rightarrow N'e^-e^+$ or $N \rightarrow \nu e^-e^+$)

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Summary and conclusions

We consider the production and decay of Dirac/Majorana HNLs via EFT operators at FCC-ee • Possible if active-sterile mixing is suppressed in the relevant HNL mass range • d = 6 and $d = 7 \nu$ SMEFT operators with possible tree-level UV completions are considered

For long-lived HNLs, two interesting signatures:

- Monophoton plus E_{miss} signature ($e^+e^- \rightarrow \nu\nu\gamma$, e^+e^-
- Displaced vertex ($e^+e^- \rightarrow \nu N\gamma$ and $e^+e^- \rightarrow NN'\gamma$ for

The two analyses provide complimentary constraints in the (m_N, C_i) parameter space:

- Monophoton constraints generally weaker, but applicable for $m_N \rightarrow 0$
- Displaced vertex stronger in a specific mass range

Conclusion: FCC-ee can <u>considerably</u> constrain ν SMEFT operators involving HNLs in small $V_{\ell N}$ scenario Going forwards, combine with other probes to constrain NP

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$$e^- \rightarrow \nu N \gamma$$
 and $e^+ e^- \rightarrow N N \gamma$)
Followed by $N \rightarrow N' e^- e^+$ or $N \rightarrow \nu e^- e^+$)

Thank you for your attention!

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Bonus slides

HNL production cross sections



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Constraints: effective charged-current $\mathcal{O}_{HNe} = (\bar{N}_R \gamma_\mu e_R) (\tilde{H}^\dagger i \vec{D}^\mu H)$



[Fernández-Martínez et al., 23]

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[PDB, Deppisch, Kulkarni, Majumdar, Pei, 24]



Monophoton constraints - Splitting dependence



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Tree-level UV complete scenarios

		Scalar \mathcal{S}			$\mathcal{S}_1 \qquad arphi$		arphi	[1]		Ξ_1					
	_	Ir	rep.	(1,	$, 1)_{0}$	(1,	$(1)_{1}$	(1,	$2)_{\frac{1}{2}}$	(1,	$, 3)_{0}$	(1,	$3)_{1}$		
Fe	rmio	n	\mathcal{N}		E		Z	Δ_1		Δ_3	5	Σ	2	Σ_1	
Ι	rrep.		(1, 1)	0	(1, 1)	-1	(1, 2	2)	<u> (1</u>	, 2)	$-\frac{3}{2}$	(1, 3)	$B)_{0}$	(1, 3).	-1
	Vect	or	B	3	\mathcal{B}	1	И	2	\mathcal{W}	, 1	\mathcal{L}_{1}	1		\mathcal{L}_3	
	Irre	p.	(1, 1))0	(1, 1)	$)_{1}$	(1, 3)	B) ₀	(1, 3)	$)_{1}$	(1, 2	$\left(\frac{1}{2}\right)$	(1,	$2)_{-\frac{3}{2}}$	



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			$\psi^2 H$	[³		ψ^4			
		<i>(</i>)	φ, E_{z}	$, \Delta_1, \Sigma, \Sigma$	$\Sigma_1, \mathcal{O}_{ll}$	$ig \mathcal{S}_1, \Xi_1, \mathcal{B}, \mathcal{W}$			
	$\psi^2 H^2$	\mathcal{O}_{lNH}		$(\mathcal{S},\mathcal{N})$	\mathcal{O}_{le}	$arphi, \mathcal{B}, \mathcal{L}_1, \mathcal{L}_3$			
	$\mathcal{O}_5 \mid \Xi_1, \mathcal{N}, \Sigma$		$\psi^2 H^2$	^{2}D	\mathcal{O}_{lNle}	$\mathcal{S}_1,arphi$			
	$\mathcal{O}_N \mid \mathcal{S}, \Delta_1$	$\mathcal{O}_{Hl}^{(1),(3)}$	$ $ $\mathcal{N},$	E, Σ, Σ	\mathcal{O}_{lN}	$arphi, \mathcal{B}, \mathcal{L}_1$			
	i	\mathcal{O}_{HN}		$\Delta_1, {\cal B}$	\mathcal{O}_{eN}	$\mathcal{S}_1,\mathcal{B},\mathcal{B}_1$			
		\mathcal{O}_{HNe}		$\Delta_1, {\cal B}_1$					
						· ·			
	$\psi^2 H^4$					$\psi^4 H$			
\mathcal{O}_{lH}	Ξ_1,\mathcal{N},Σ	2		Om		$\mathcal{N}, \Sigma,$			
	$\mathcal{S},$				$(\mathcal{S}_1, arphi),$	$(\mathcal{S}_1,\Delta_3),(arphi,\Xi_1)$, $(\Xi_1, \Delta$		
${\cal O}_{NH}$	$(\varphi, \Delta_1), (\Xi, \Delta_1), (\Xi,$	Σ), (Ξ_1 , \angle	$\Delta_1),$		$(\mathcal{S}_1, arphi),$	$(\mathcal{S}_1, E), \ (\mathcal{S}_1, \Delta_1)$), $(\varphi, \Xi_1$		
	$(\Xi_1,\Sigma_1),~(\mathcal{N},\Delta_1),~(\Delta_1)$	$_{1},\Sigma),$ (Δ_{1}	$,\Sigma_{1})$	<i>(</i>)	$(\varphi, \mathcal{N}), (\varphi, \Sigma), (\Xi_1, \Delta_1), (\Xi_1, \Sigma_1)$				
	$\psi^2 H^3 D$	\mathcal{O}_{lNlH}	$(\mathcal{N},\mathcal{B}), (\mathcal{N},\mathcal{L}_1), (E,\mathcal{L}_1), (\Delta_1,\mathcal{B})$						
	$(\mathcal{S},\mathcal{N}),(\mathcal{S},\Delta_1),(\mathcal{S},\Delta_1)$	$\mathcal{L}_1), \ (\Xi, \Delta$	(1),		$ $ $(\Delta_1, \mathcal{W}), (\Sigma, \mathcal{W}), (\Sigma, \mathcal{L}_1), (\Sigma_1, \mathcal{L}_1)$				
	$(\Xi,\Sigma), (\Xi,\mathcal{L}_1), (\Xi_1,\mathcal{L}_2)$	$\Delta_1), (\Xi_1, \Sigma_2)$	$\Sigma_1),$		$(\mathcal{S}_1, arphi),$	$(\mathcal{S}_1,\mathcal{N}),(\mathcal{S}_1,\Delta_3)$), $(arphi, \Delta_1$		
$\mathcal{O}_{Nl1(2)}$	$(\Xi_1,\mathcal{L}_1),(\mathcal{N},\Delta_1),(\mathcal{N},\Delta_1)$	$(\mathcal{B}), \ (\Delta_1)$	$,\Sigma),$	\mathcal{O}_{eNlH}	$(\mathcal{N},\mathcal{B}),$ ($(\mathcal{N},\mathcal{B}_1),(\Delta_1,\mathcal{B}),$, $(\Delta_1, \mathcal{B}_1)$		
	$(\Delta_1, \Sigma_1), (\Delta_1, \mathcal{B}), (\Delta_1$	$,\mathcal{B}_{1}),(\Delta_{1}$	$,\mathcal{W}),$		$(\Delta_1, \mathcal{L}_1),$	$(\Delta_1, \mathcal{L}_3), (\Delta_3, \mathcal{L})$	$_{1}),(\Delta_{3},$		
	$(\Delta_1,\mathcal{W}_1),(\Sigma,\mathcal{W}),$, $(\Sigma_1, \mathcal{W}_1)$)	0	$(\mathcal{S}, arphi),$	$(\mathcal{S}, E), (\mathcal{S}, \Delta_1),$	$(\mathcal{S}_1, arphi),$		
()	$\mathcal{N}, \Sigma,$			\bigcup_{lNeH}	$(\mathcal{S}_1,$	E), $(\mathcal{S}_1, \Delta_1)$, $(\varphi$	(λ,Δ_1)		
O_{leHD}	$(\Xi_1,\Delta_1),(\Delta_1,\mathcal{B}_1)$), $(\mathcal{B}_1,\mathcal{L}_3)$		0	$(\mathcal{S}, arphi),$	$(\mathcal{S},\Delta_1),(arphi,\Delta_1)$	$, (\mathcal{S}_1, E)$		
				U_{elNH}	(E, t)	$(\Delta_1,\mathcal{B}_1),$	$({\mathcal L}_1, {\mathcal L}_1)$		



