# Thermodynamics from the S-matrix reloaded

based on arXiv: 2408.06729 w Emanuele Gendy & Joan Elias Miró

Belica 2024

# Thermal Field Theory

## why?

- Systems that are at the same time in (1) particle physics regime (i.e. relativity + QM) and (2) statistical physics regime, i.e. with temperature T or chemical potential $\mu$ turned on, where

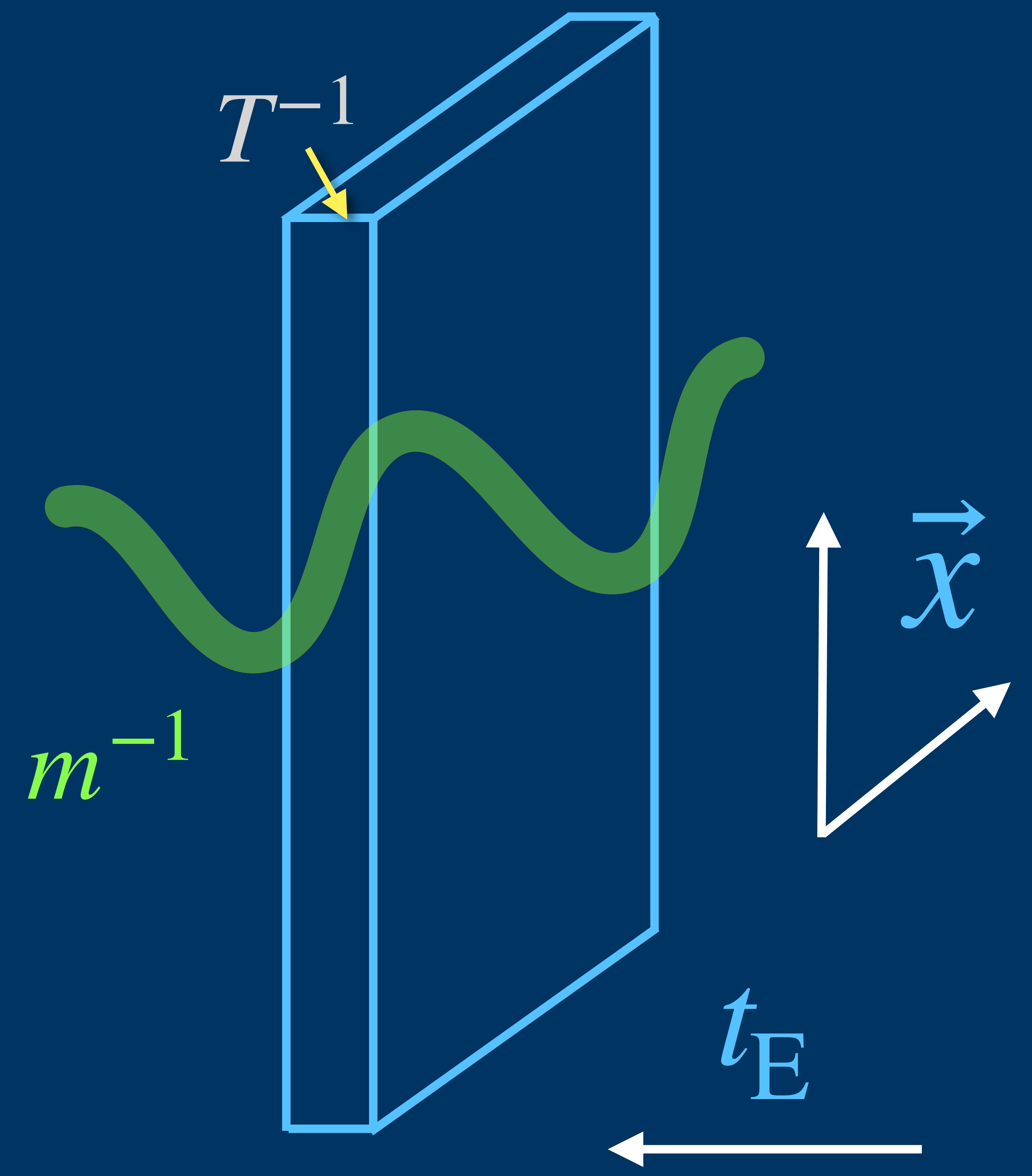
$$T \text{ or } \mu \gg m$$

- Early Universe physics
- Heavy ion collisions
- Extremely dense stellar objects or configurations (e.g. supernovae)

# Thermal Field Theory

## Overview

- Two standard methods to treat this regime, both based on a representation of the partition function in terms of a Euclidean path integral

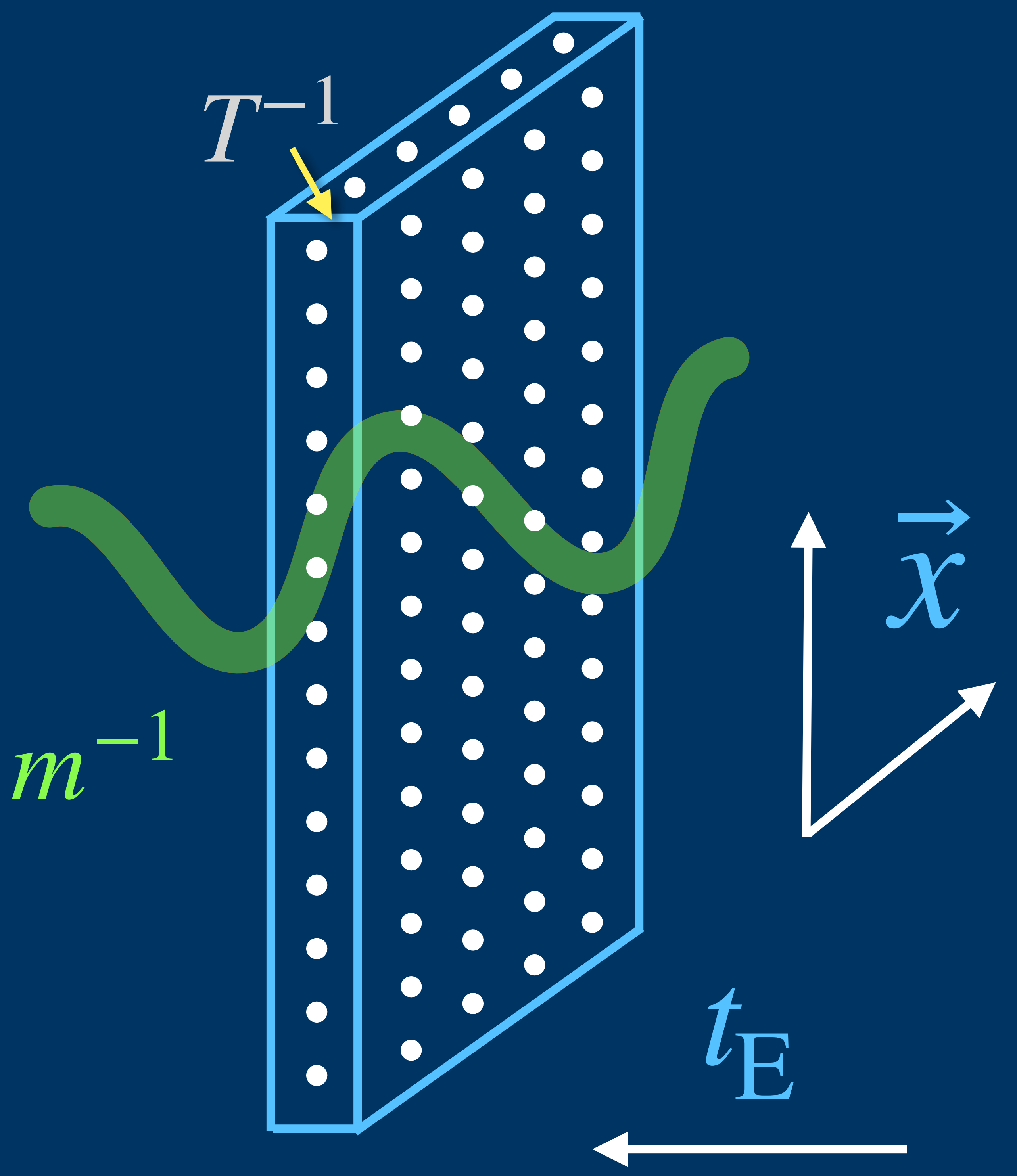



- Time is an interval $[0, T^{-1}]$ and fields have periodic (bosons) or anti-periodic (fermions) b.c.
- Perturbative expansion with Feynman diagrams having modified propagator to account for the geometry of spacetime

# Thermal Field Theory

## Overview

- Two standard methods to treat this regime, both based on a representation of the partition function in terms of a Euclidean path integral




- Spacetime is replaced by a lattice, with the Euclidean time direction having fewer points to mimic finite temperature
- The path integral is then performed numerically in a non-perturbative fashion

# Thermal Field Theory

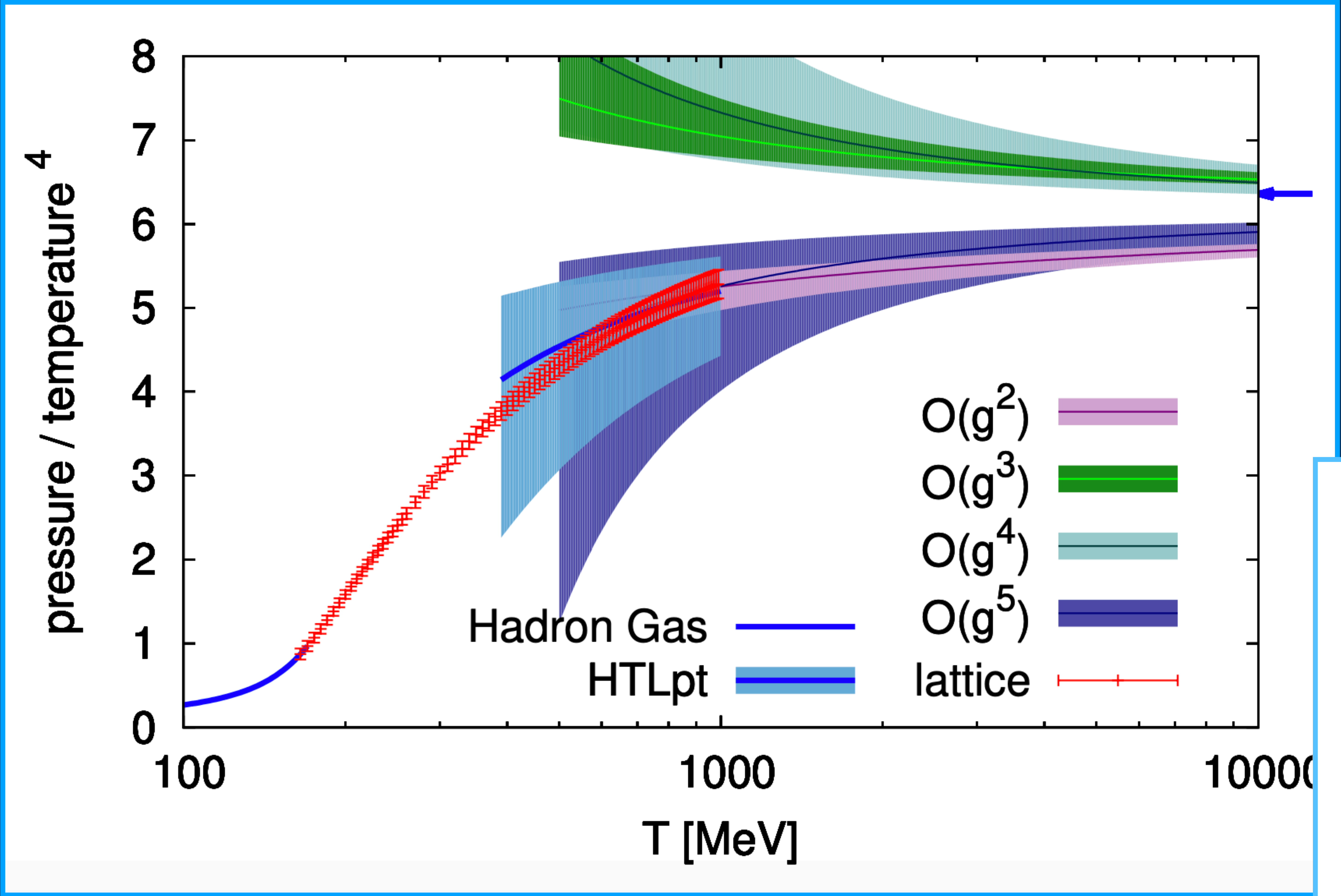
## Open questions

### Perturbative

- In realistic models like QCD the perturbative series is found to be badly behaved
- Several attempts to get a better convergence have been tried with some success
- **Linde problem**: starting from 4 loops strongly coupled cromo-magnetic modes enter and render the perturbative expansion meaningless

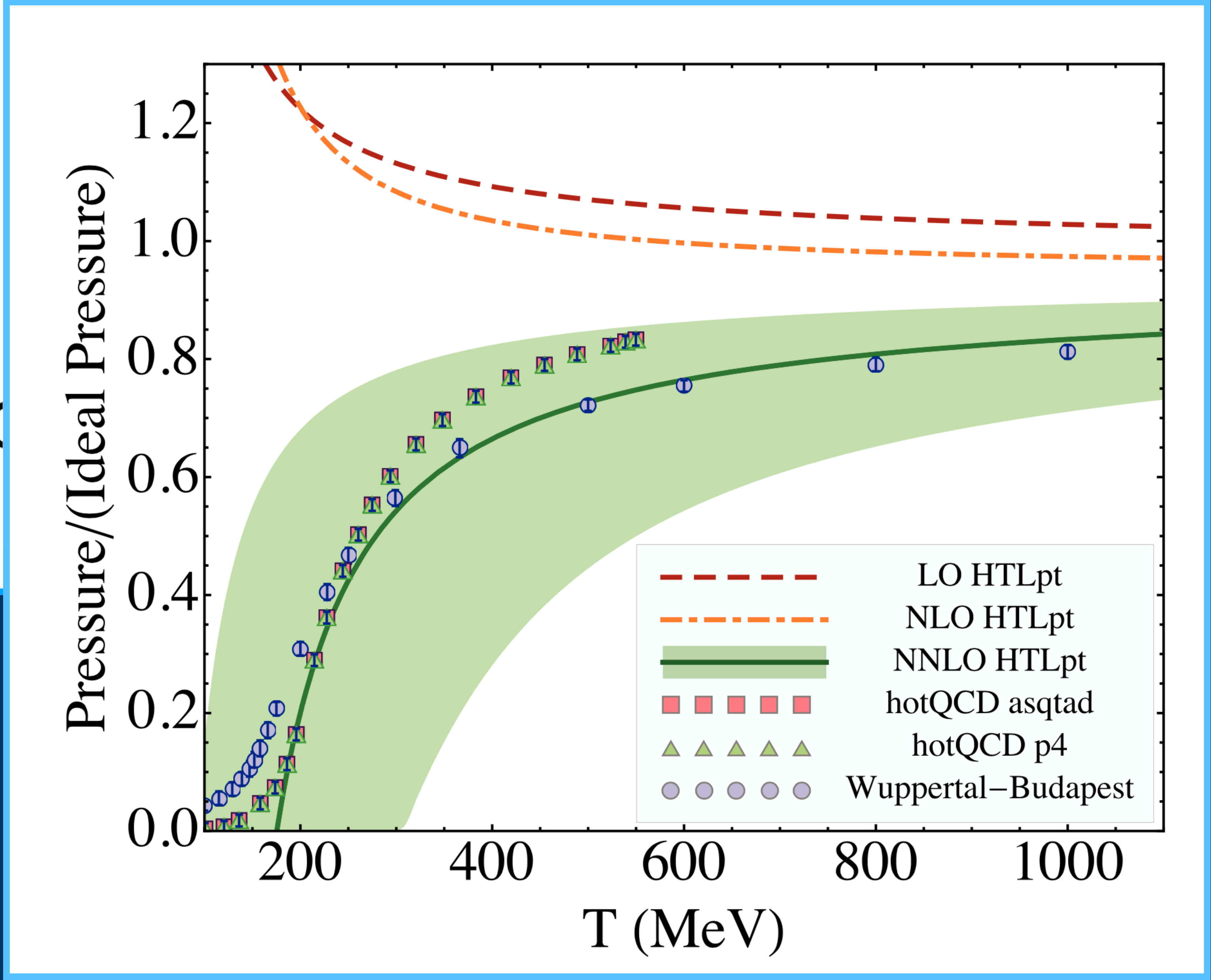
### Lattice

- **Sign problem**: main limitation of the lattice approach, as it renders the inclusion of a chemical potential numerically untreatable
- Measure of the path integral not positive definite (oscillatory behaviour of the integrand)




Borsanyi's slides 2018

Andersen et al 2011



# Thermal Field Theory

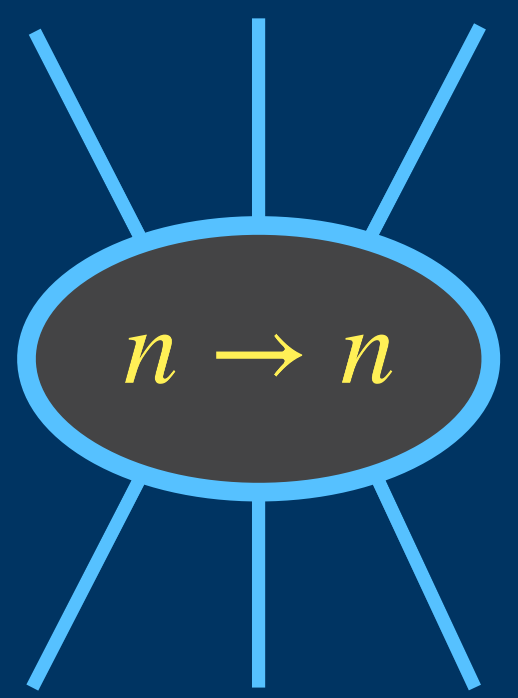
## A third way?

- In both approaches one encounters a problem with a name
- Due to the limitations of lattice methods in treating the $\mu \neq 0$ regime, it would be interesting to have a better control on perturbative methods
- <u>We follow a third path</u>
- Instead of starting from a Lagrangian, we consider a formulation of statistical mechanics where the dynamical information is encoded in **scattering amplitudes among the constituents of the system** (Dashen, Ma and Bernstein)

# Thermal Field Theory from S-matrix

## A third way

- Similar to the perturbative method, since one expands in $M_{n \to n}$
- It can be shown for simple scalar theories that




$\propto g^{2n-2}$

Vacuum bubbles with thermal propagators $\equiv$ Forward amplitudes averaged over Bose or Fermi distributions

[arXiv:2408.00729]

- For gauge theories the amplitude approach is powerful in that $M$ is gauge invariant (no ghosts, only physical polarisations)

# Thermal Field Theory from S-matrix

## A third way

- DMB approach neatly disentangles zero temperature dynamical information and the effect of temperature or chemical potential
- Recycle all the knowledge about QCD amplitudes (known to high loop orders and particle multiplicities)
- Different perspective on the same problem might suggest a different resummation method of the perturbative series
- In 2408.06729 we (1) show the power of the method at LO in QCD and (2) push the formalism to NLO and NNLO in a (1+1)-dimensional model of “flux tube long strings”

# S-matrix approach

## DMB master formula

Sum over a complete set of states

Only ``connected histories``

$$F - F_0 = -\frac{1}{2\pi i}\int_0^\infty \mathrm{d}E \; e^{-\beta E} \sum_\alpha \delta(E - E_\alpha)\, T^c_{\alpha\alpha}(E) + \dots$$

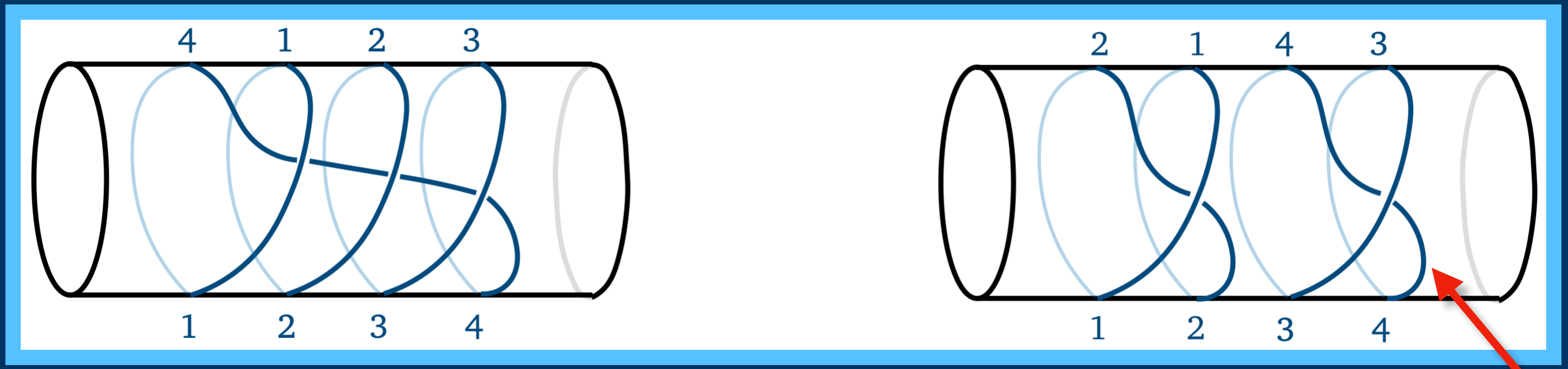
Boltzmann suppression

T-matrix elements in the forward limit

# S-matrix approach

## Ideal gas

- The formula can be unpacked by evaluating the trace on asymptotic states with increasing number of particles
- Keep only connected contributions to the trace to extract F



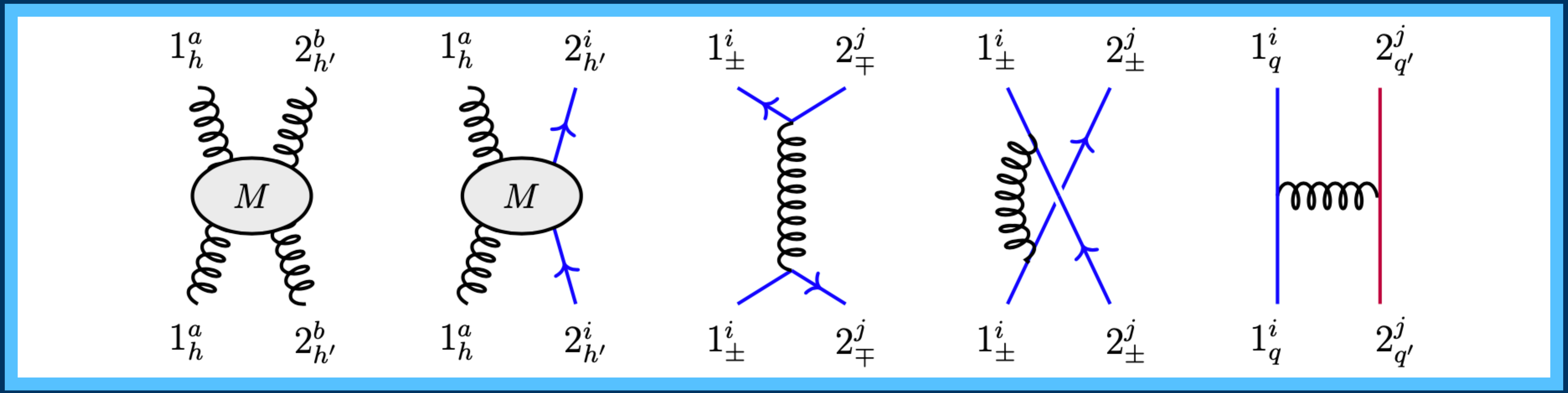

Connected and disconnected contributions to the free theory

to be discarded

# LO effects in QCD

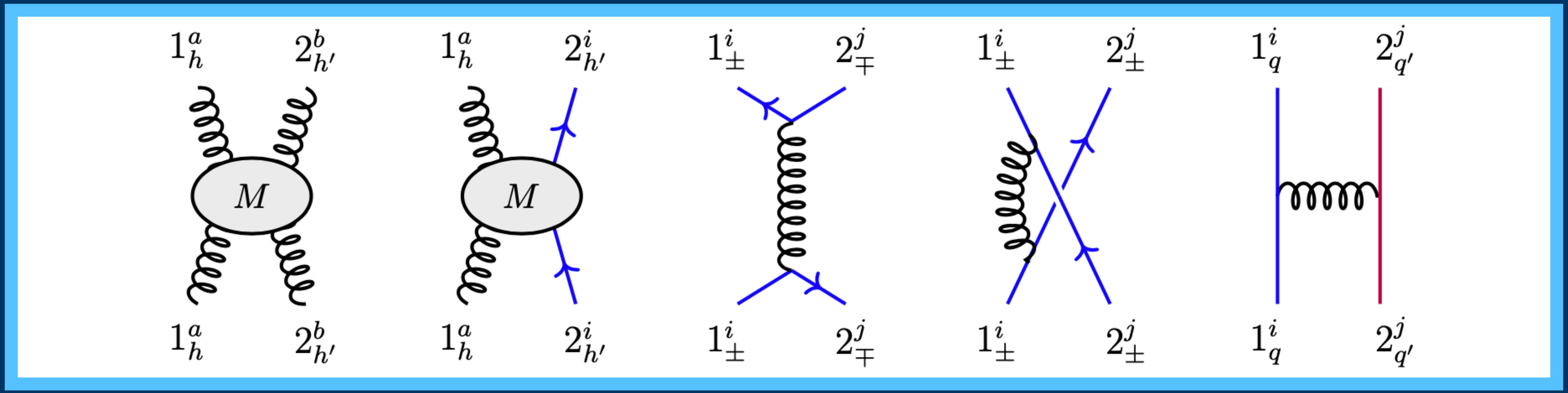
$$M(1^a_-, 2^b_-, 3^c_+, 4^d_+) = -2g_s^2 \langle 12 \rangle^4 \left[ \frac{f^{abe} f^{cde}}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} + \frac{f^{ace} f^{bde}}{\langle 13 \rangle \langle 32 \rangle \langle 24 \rangle \langle 41 \rangle} \right]$$

Parke-Taylor formula



- Compact expressions for amplitudes (Parke-Taylor formula for $gg \to gg$)
- They admit a simple forward limit independent on the momenta
- Problems coming from $\lim_{t \to 0} s/t$ are avoided by first summing over colours

# LO effects in QCD



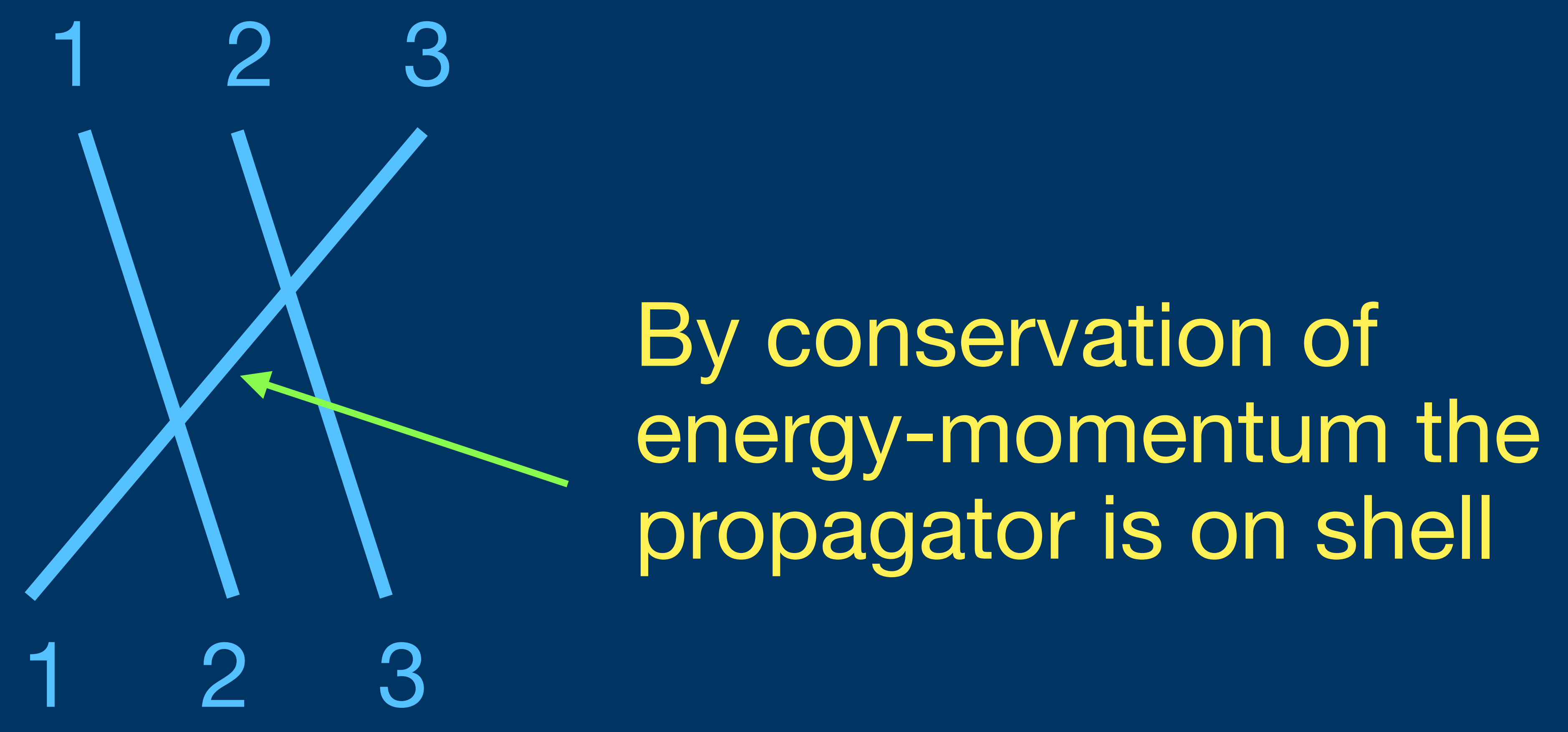

After summing over all diagrams,
we reproduce the textbook result

$$f_{\mathrm{QCD}} = \alpha_s (N_c^2 - 1) \frac{\pi T^4}{36} \left( N_c + \frac{5}{4} N_f \right)$$

# NLO effects

## Forward divergences

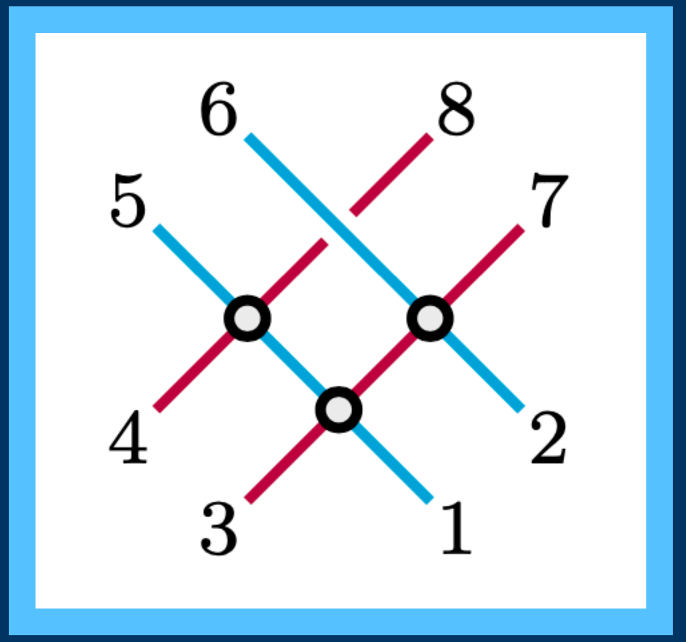
- At NLO one encounters quite generically $3 \to 3$ diagrams that are singular in the forward limit (it's enough to have nonzero $2 \to 2$)



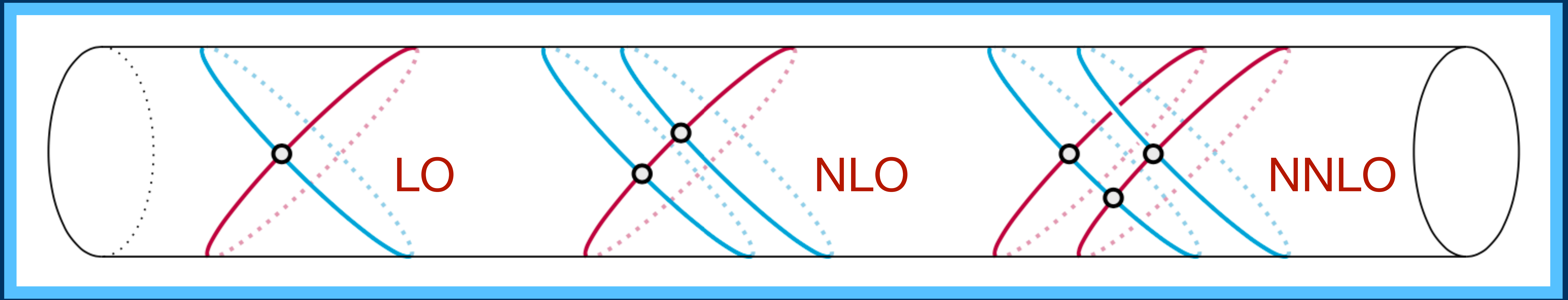

- We studied the problem in the simpler setup of a derivatively coupled (1+1)-dimensional model (`Flux Tube`, another regime of QCD) to NNLO

# Flux Tube theory

## NNLO

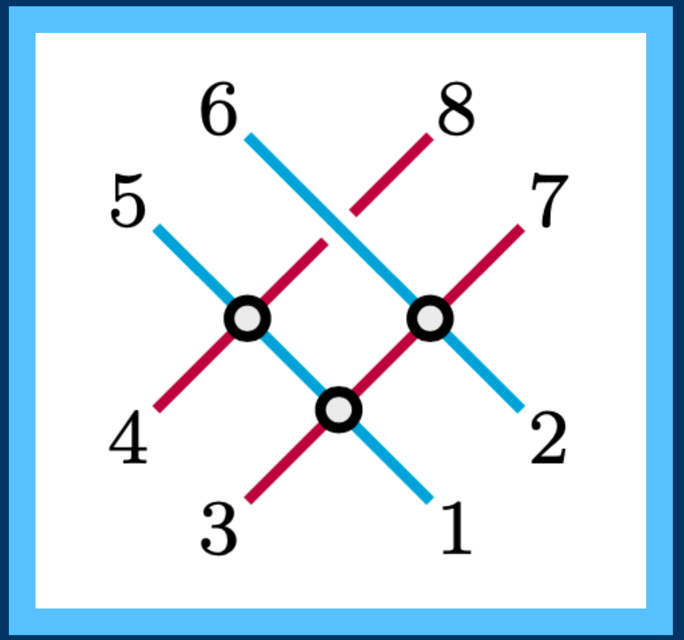



*265 Feynman diagrams




- Some of the forward diagrams computing NNLO free energy
- Integrable 1+1 theory with ``left-`` or ``right-movers``
- Many diagrams condensed in a single object*
- The Flux Tube theory can be studied at higher orders including integrability breaking effects

# Overview and Future directions




*265 Feynman diagrams

- S-matrix based methods provide a consistent framework for computing thermodynamics in a perturbative expansion
- Example with no intrinsic IR divergences fully under control
- Building blocks are already structured, being a clever sum of Feynman diagrams* (when these are available)
- Push the QCD equation of state to higher orders
- The method could provide new indications on how to cure the bad behaviour of thermal QCD perturbative series