Cascade topologies in rare charm decays & implications for CP violation

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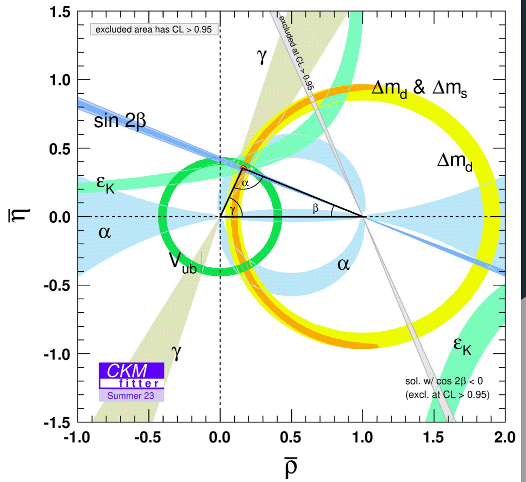
EXCELENCIA SEVERO OCHOA

Why is charm interesting?



The CKM matrix is (generally) well probed from various exp. processes: lots of processes,

only 4 independent parameters

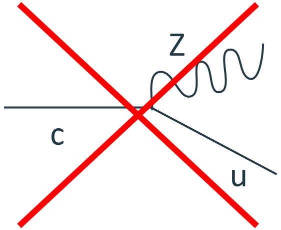
Charm is the only weakly decaying up-type quark bound in hadrons

⟶ Can still perform complementary CKM tests from the charm sector (in the future…)

Otherwise, assuming good control over CKM matrix:

⟶ Can look for rare processes where there is more room for NP to show up:

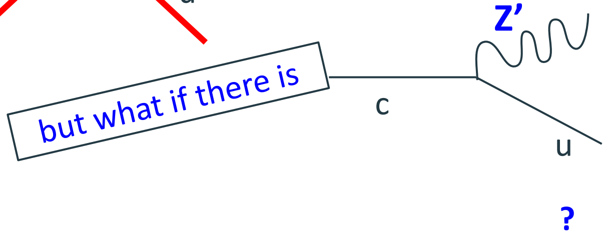
- b⟶sμμ, b⟶svv, s⟶dvv, … [lots of work there!]



In this search,

different NP scenarios can be explored by starting off from the charm quark

"No stone left unturned" approach



Rich experimental programme (LHCb, Belle II, BESIII, future facilities,...)

CP violation in $D^0 \rightarrow \pi^+\pi^-$

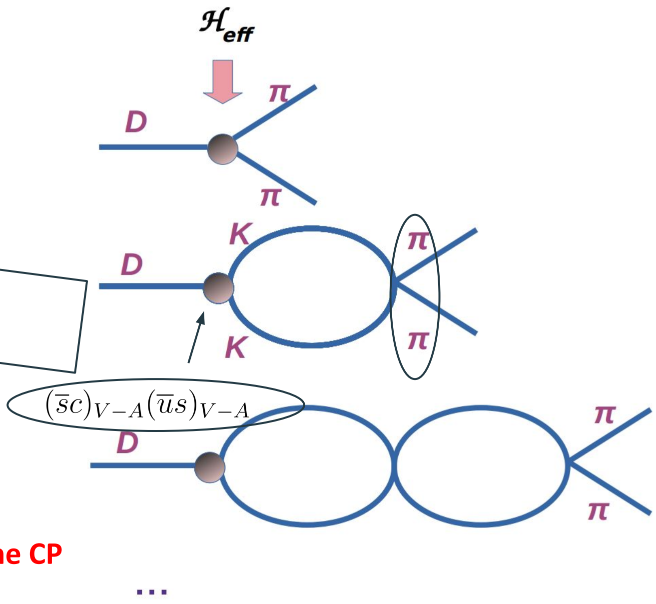
Measurement (LHCb 2019 + 2022):

$$A_{\rm CP}^{\rm direct}(\pi^-\pi^+) = (23.2 \pm 6.1) \times 10^{-4}$$

Data-driven approach [Pich, ES, Vale Silva '23]:

$$A_{\rm CP}^{\rm direct}(\pi^-\pi^+) \approx 3 \cdot 10^{-4}$$

Also Khodjamirian, Petrov '17 & Lenz , Piscopo, Rusov '23 agree



Assumptions:

- Long distance effects encoded in the final-state interactions (FSI)
- Inelastic rescattering between isospin-0 pion and kaon pairs *(and them only)*

Scrutinising the assumption of two-channel FSI: we find it is **impossible to explain the CP asymmetry**

Kubis et al: In other environments, **third channel of 4 pions is important**; couples to ππ, KK through resonances

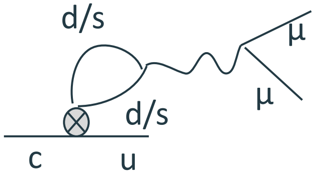
Those resonances are found around the mass of the D-meson ($f_0(1500)$, $f_0(2020)$) → could be important in the charm case

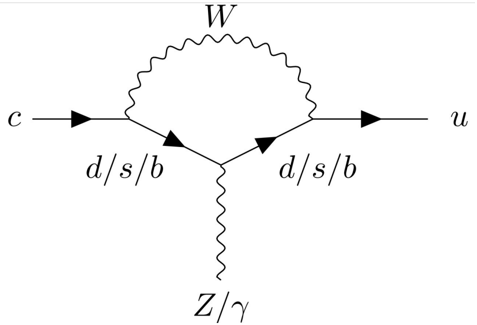
Rare decays $D^0 \to \pi^+\pi^- l^+ l^-$

Experiment: [LHCb 1707.08377 and 2111.03327] $D^0 \to \pi^+\pi^-\mu^+\mu^-$ decay rates & plenty of angular observables (5 kin. variables)

Also recently $D^0 \to \pi^+\pi^- e^+ e^-$ (& $\Lambda_c \to p\mu\mu$)

Theory: In charm, $c \to u\, ll$ decays only driven by long-distance QCD (very effective GIM mechanism)





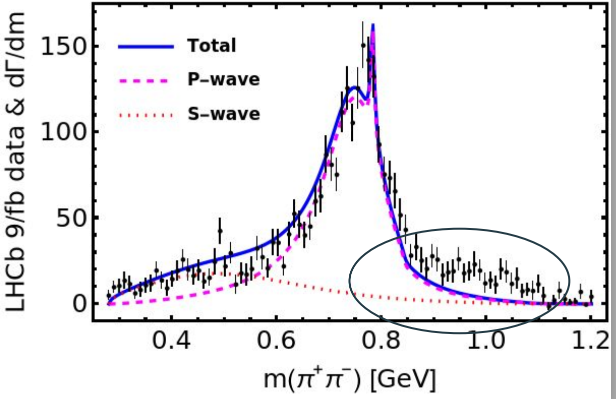
Lepton pair created from the **electromagnetic decay of a vector meson**

[Fajfer, ES, Vale Silva '23] SM prediction for the differential decay rates

& SM-dominated angular observables, estimations of SM null tests in presence of NP

[Gisbert, Hiller, Suelmann '24] More general fit of data to low-energy EFT in presence of NP

Results (found in both works): good description of data, improvement when including S-wave.



But:

- Some observables have tensions; NP cannot be the explanation
- Overall large correction of the normalisation
- Bounds on NP coefficients not competitive to other processes, yet

→ This motivates us to study further those decays; they are a ***QCD laboratory*** (Hiller/Suelmann)

and a blank canvas for NP discovery in some observables

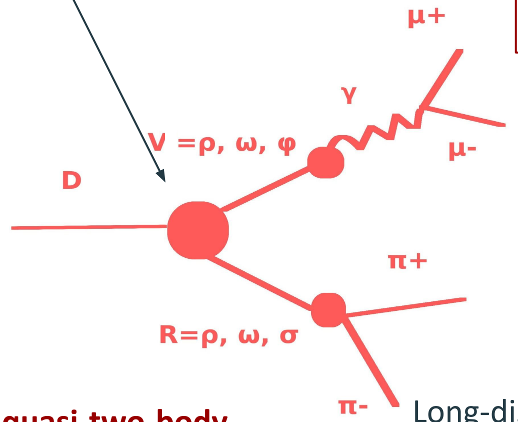
Theoretical approach to $D^0 \to \pi^+\pi^-\mu^+\mu^-$

V =ρ, ω,

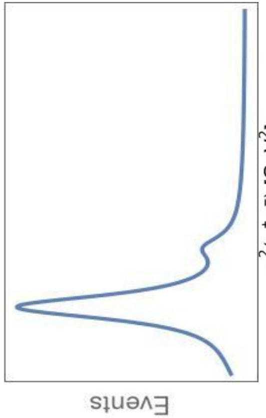
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{2} C_i(\mu) \left(\lambda_d Q_i^d + \lambda_s Q_i^s \right)$$

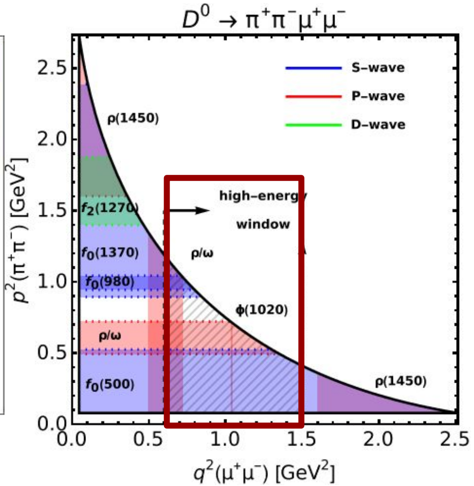
$$Q_1^d = (\bar{d}c)_{V-A}(\bar{u}d)_{V-A}$$
$$Q_2^d = (\bar{u}c)_{V-A}(\bar{d}d)_{V-A}$$
$$Q_1^s = (\bar{s}c)_{V-A}(\bar{u}s)_{V-A}$$
$$Q_2^s = (\bar{u}c)_{V-A}(\bar{s}s)_{V-A}$$

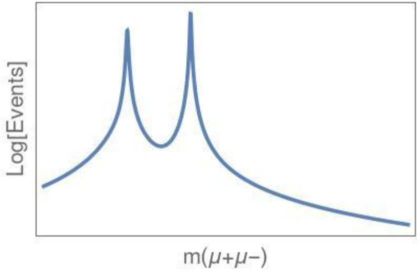
V = φ



The picture looks like this:







quasi-two-body (Q2B) topologies

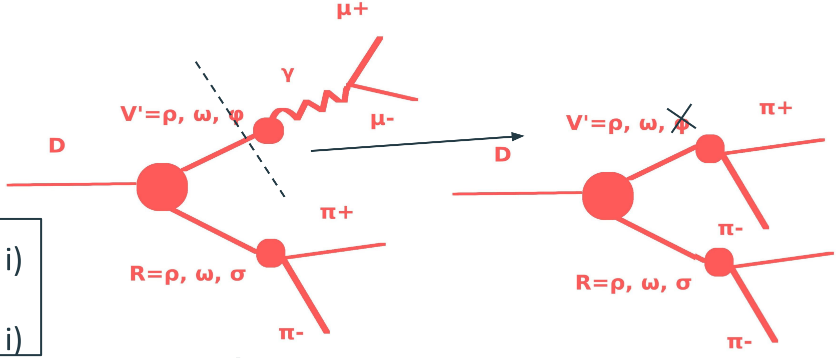
Long-distance QCD effects encoded in the line shapes of the resonances (Gounaris-Sakurai, Bugg…)
Additional normalisation factor & constant phase assigned at each D⟶ RV vertex:
account for *further long-distance QCD effects*

Hadronic decays $D^0 \to \pi^+\pi^-\pi^+\pi^-$ & how they may help

Large Br (~ 7 * 10^{-3})

Amplitude analyses available [CLEO 1703.08505, BES-III 2312.02524]

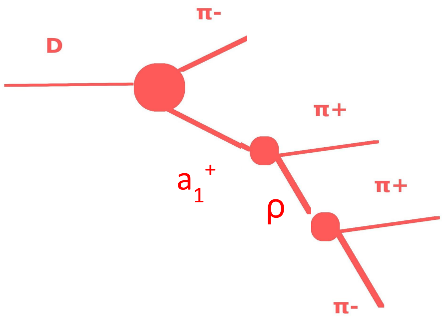
Approach:



Total amplitude = Sum_i (intermediate state amplitude i)

x (complex coefficient i)

- Essentially the same idea implemented in the rare-decay approach [normalisation& phase]
- **The same intermediate topologies of D→ππμμ are present in the 4-body hadronic decays**
- (In this model) the long-distance QCD effects for a given intermediate state should be *comparable* between the hadronic and the rare decays (as they are assigned in the same D→ RV vertex) [roughly; more topologies present in D→4π]



Both amplitude analyses (CLEO, BES-III) find the largest contribution to the Br to come from the cascade topology $D^0 \to a_1^+(1260)(\to \rho(\to \pi^+\pi^-)\,\pi^+)\,\pi^-$

Partially explained because it comes with C1=1.2, compared to C2=~-0.4

Is then the cascade topology also important in the rare decays?

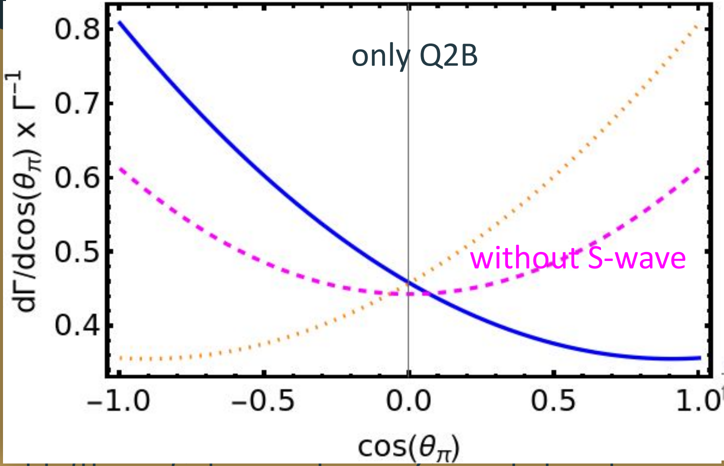
Cascade decay: qualitative picture (kinematics)

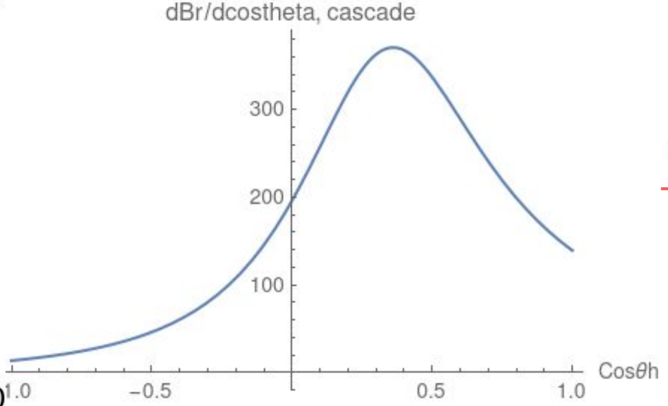
a_1^+ carries the momentum of the π^+ AND of the lepton pair
$\longrightarrow$ its momentum is a function of the kinematical variables p^2=m(ππ), q^2=m(μμ) and cosθh (dihadron angle)
$\longrightarrow$ no clear resonant peak in the p^2 distribution, but DOES contribute over a wide range of energies

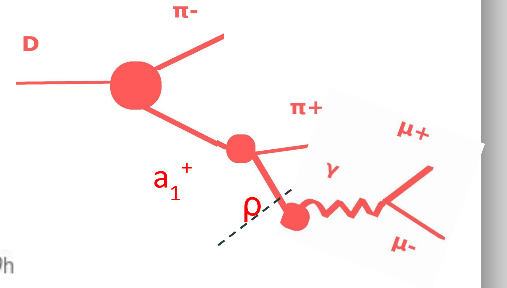
$\longrightarrow$ typical ρ-resonant peak in the q^2 distribution (will be added on top of the quasi-two-body topologies)

[Fajfer, ES, Vale Silva '23]

Signature shape: in the distribution over cosθh (yet unmeasured)







Differential distribution: preliminary results

Need: $D \to \pi$ form factors, a_1 decay constant, $a_1 \to \rho\pi$ coupling: take the combination of these from the amplitude analysis of the $D \to 4\pi$ decays

Add the rest of the Q2B contributions as per our last work

New: overall normalisation is NOT a free parameter;

the cascade component is NOT fitted

⟶ Without a need for rescaling, **the cascade decay predicts**

the points which previously showed large tensions

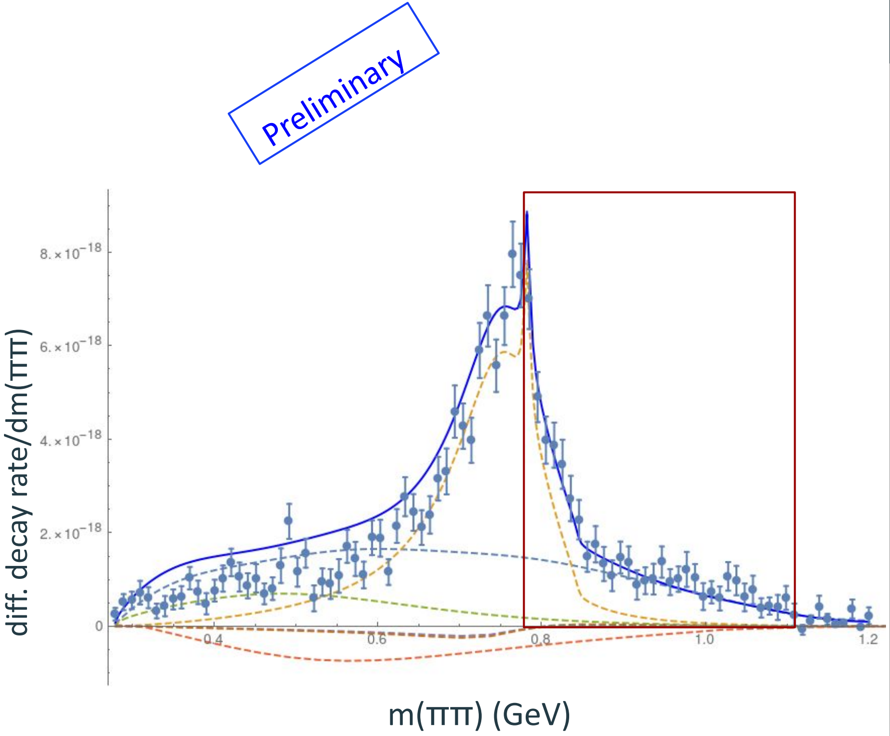
⟶ Large percentage of the total branching fraction from cascade

⟶ At low p^2 some adjustments needed;

interference effects between cascade and S, P-wave Q2B

⟶ Also contributes to the ang. observables I8, I9

(*some* previous tensions might be alleviated)



Implications for CP violation in D⟶PP

[Roig, ES, Vale Silva, in preparation]

Data-driven approach (with multi-channel Omnes dispersion relations) requires as input the phases and inelasticities of each channel present; OK for ππ and KK, data available (up to around the mass of the D)

3rd channel in this approach is **unfeasible**: 4-body final state, no data available for 4 pions over invariant mass

⟶ Some model-dependent approach needed [Kubis et al.]; approximate the 4π channel as a two-body one

If $D^0 \to a_1^+ \pi^-$ is the predominant contribution to D⟶ 4π then the focus should be on it (and not e.g. ρρ)

In this endeavour, D⟶ ππll serves as an extra cross-check for how the $a_1 \pi$ works: how big are inelastic effects? how does it compare to other processes with a_1 decays?

Open question: how to incorporate both $a_1 \pi$ and the non-negligible component ρρ in the effective description of the 4π

Conclusions

Long-distance contributions are crucial in most weak decays of the charm mesons

But because of this, many processes are complementary for checking how QCD works in charm

Modelling $D \rightarrow \pi\pi ll$ with Q2B topologies still shows discrepancies with the data that cannot be explained with NP

Analyses of $D \rightarrow 4\pi$ show a very sizeable contribution of the intermediate cascade decay with $a_1(1260)$

We include the same topology in the $D \rightarrow \pi\pi\mu\mu$

<u>Results:</u> description of $D \rightarrow \pi\pi ll$ data **improves significantly** when including the cascade

in the differential distribution over $m(\pi\pi)$ and the overall prediction for the Br; angular observables to be checked but promising

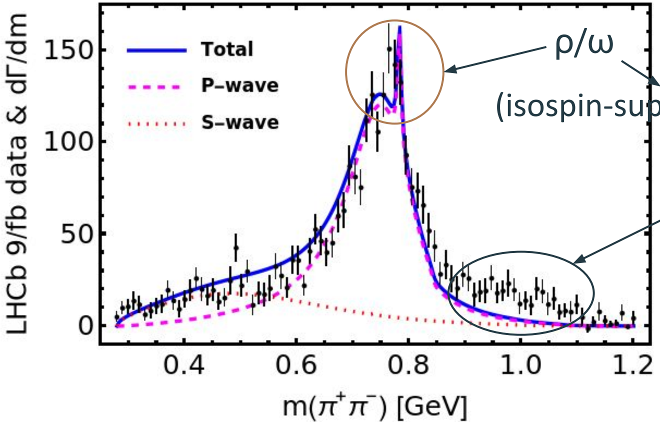
$a_1 \pi$ might be the best way to quantify CP violation in $D \rightarrow \pi\pi$, $D \rightarrow KK$ in the presence of a 3rd rescattering channel

Ultimate goal: combining all the processes, probe NP in charm with more certainty

(see if *"LHCb discovered NP in 2019" [U. Nierste]* !)

BACKUP

Q2B: diff. branching ratios

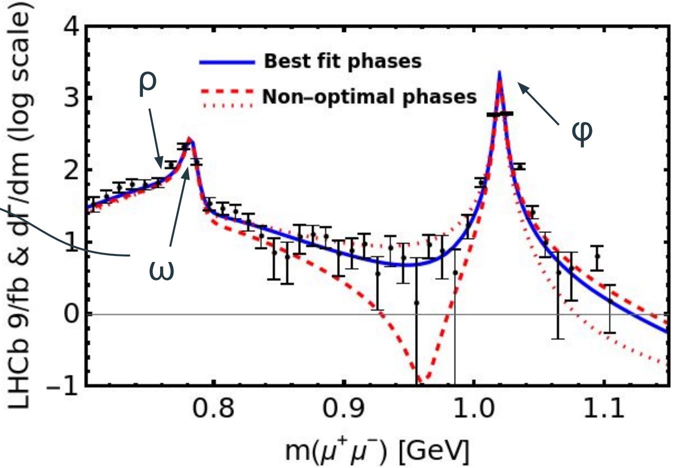


ρ/ω

(isospin-suppressed)

KK threshold,
f0(980),...
(not taken into account)

from
D→σω



Fair agreement with m(ππ) data

Significant improvement with S-wave inclusion (~**20% of total Br**)

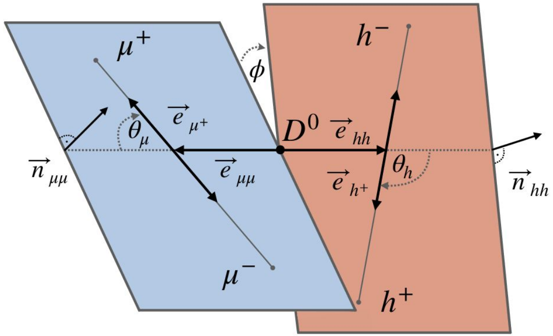
Fit prefers suppression of D→σφ - similar to $D^0 \to \pi^+\pi^- K^+K^-$

- Good agreement with m(μμ) data in the considered region
- Relative phases important for fitting in the inter-resonant region
- Different phases there result in <u>different predictions for the high-energy region</u> *(important for future NP searches)*

using [LHCb *Phys.Rev.Lett.* 119 (2017) 18, 181805]

Overall normalisation factor (1 in perfect naive factorisation) around 1.8

Setup of 4-body decays



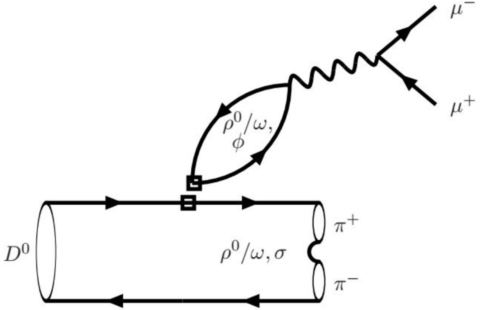
$\langle I_i \rangle_-$			
i	S-wave	Null test	WCs
1	✓		$C_9^{\text{eff}:S}\,(C_9^{\text{eff}:P})^*$
2	✓		$C_9^{\text{eff}:S}\,(C_9^{\text{eff}:P})^*$
$4^\dagger$	×		$\lvert C_9^{\text{eff}:P}\rvert^2$
$5^\dagger$	×	yes	$\text{Re}\left[C_9^{\text{eff}:P}\, C_{10}^*\right]$
$7^\dagger$	×	yes	$\text{Re}\left[C_9^{\text{eff}:P}\, C_{10}^*\right]$
$8^\dagger$	×		$\lvert C_9^{\text{eff}:P}\rvert^2$

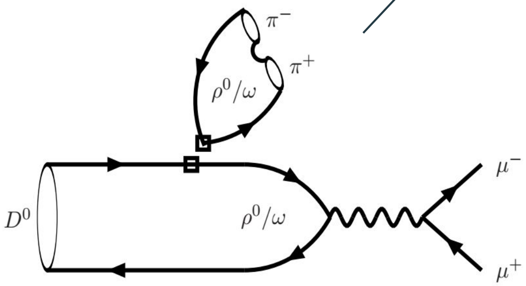
$$\frac{3}{2}\langle I_8\rangle_- = \frac{1}{2}\left[\text{Re}\left(\mathcal{F}_P\mathcal{F}_\perp^*\right)\,\text{Im}\rho_2^+ - \text{Im}\left(\mathcal{F}_P\mathcal{F}_\perp^*\right)\,\text{Re}\rho_2^-\right]$$

$$\langle I_9\rangle_+ = \frac{2}{3}\left[\text{Re}(\mathcal{F}_\perp\mathcal{F}_\parallel^*)\,\text{Im}\rho_2^+ + \text{Im}(\mathcal{F}_\perp\mathcal{F}_\parallel^*)\,\text{Re}\rho_2^-\right]$$

$\langle I_i \rangle_+$			
i	S-wave	Null test	WCs
$1^\dagger$	○		$\lvert C_9^{\text{eff}:S}\rvert^2,\ \lvert C_9^{\text{eff}:P}\rvert^2$
$2^\dagger$	○		$\lvert C_9^{\text{eff}:S}\rvert^2,\ \lvert C_9^{\text{eff}:P}\rvert^2$
$3^\dagger$	×		$\lvert C_9^{\text{eff}:P}\rvert^2$
4	✓		$C_9^{\text{eff}:S}\,(C_9^{\text{eff}:P})^*$
5	✓	yes	$C_9^{\text{eff}:S}\,C_{10}^* + C_{10}\,(C_9^{\text{eff}:P})^*$
$6^\dagger$	×	yes	$\text{Re}\left[C_9^{\text{eff}:P}\, C_{10}^*\right]$
7	✓	yes	$C_9^{\text{eff}:S}\,C_{10}^* + C_{10}\,(C_9^{\text{eff}:P})^*$
8	✓		$C_9^{\text{eff}:S}\,(C_9^{\text{eff}:P})^*$
$9^\dagger$	×		$\lvert C_9^{\text{eff}:P}\rvert^2$

Effective C9

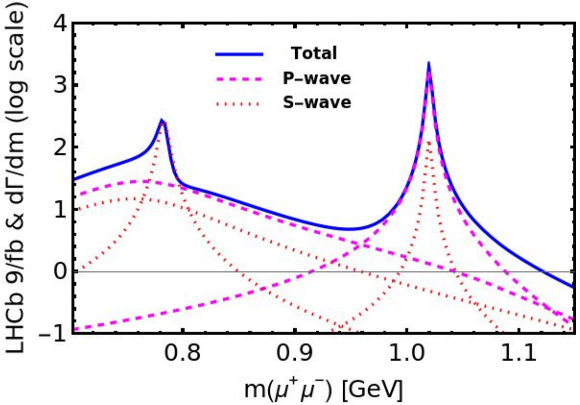




only for P-wave

cannot be written as

$$\langle \pi^+\pi^- | (\bar{u}c)_{V-A} | D^0 \rangle \times \langle \ell^+\ell^- | (\bar{\ell}\ell)_V(x) | 0 \rangle$$



For S-wave (D→σ V), we can write

$$C_9^{\mathrm{eff}:S}(\mu;q^2) = 8\pi^2 C_2(\mu)\left(\frac{f_{\rho^0}^2}{2P_{\rho^0}(q^2)}B_{\rho^0}^{(S)}e^{i\delta_{\{\sigma,\rho^0\}}} - \frac{f_\omega^2}{6P_\omega(q^2)}B_\omega^{(S)}e^{i\delta_{\{\sigma,\omega\}}} - \frac{f_\phi^2}{3P_\phi(q^2)}B_\phi^{(S)}e^{i\delta_{\{\sigma,\phi\}}}\right)$$

For P-wave (D→ρ/ω V), we can *approximately* write

$$C_9^{\mathrm{eff}:P}(\mu;q^2) \approx 8\pi^2 C_2(\mu)\left(\frac{f_{\rho^0}^2}{P_{\rho^0}(q^2)}B_{\rho^0}^{(P)}e^{i\delta_{\{\rho^0/\omega,\rho^0\}}} - \frac{f_\phi^2}{3P_\phi(q^2)}B_\phi^{(P)}e^{i\delta_{\{\rho^0/\omega,\phi\}}}\right)$$

No D→ ρ ω because of competing contributions

$$b_{\rho^0}\left(1 + a_\omega\,\mathrm{RBW}_\omega(p^2)\right)\left(-\frac{1}{6}\right)\frac{f_\omega^2}{P_\omega(q^2)}$$

vs

$$b_{\rho^0}\left(1 - a_\omega\,\mathrm{RBW}_\omega(p^2)\right)\frac{1}{6}\frac{f_\omega^2}{P_\omega(q^2)}$$

Isospin & unitarity

$$A(D^0 \to \pi^+\pi^-) = -\frac{1}{\sqrt{6}}|A_{\pi\pi}^{I=0}|e^{i\delta_{\pi\pi,0}} - \frac{1}{2\sqrt{3}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^0 \to \pi^0\pi^0) = -\frac{1}{\sqrt{6}}|A_{\pi\pi}^{I=0}|e^{i\delta_{\pi\pi,0}} + \frac{1}{\sqrt{3}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^+ \to \pi^+\pi^0) = \frac{\sqrt{3}}{2\sqrt{2}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^0 \to K^-K^+) = \frac{1}{2}\left(|A_{KK}^{I=1}|e^{i\delta_{KK,1}} - |A_{KK}^{I=0}|e^{i\delta_{KK,0}}\right)$$

$$A(D^0 \to \overline{K}^0 K^0) = \frac{1}{2}\left(-|A_{KK}^{I=1}|e^{i\delta_{KK,1}} - |A_{KK}^{I=0}|e^{i\delta_{KK,0}}\right)$$

$$A(D^+ \to \overline{K}^0 K^+) = |A_{KK}^{I=1}|e^{i\delta_{KK,1}}$$

Both ππ and KK have an isospin-zero component

Isospin=1, 2: only KK, ππ channels respectively

The S-matrix is unitary

In *isospin-zero*, *spin-zero*, the strong S-submatrix is also unitary

$$\begin{pmatrix} A(D\to\pi\pi) \\ A(D\to KK) \end{pmatrix} = \underbrace{\begin{pmatrix} S_0(\pi\pi\to\pi\pi) & S_0(\pi\pi\to KK) \\ S_0(KK\to\pi\pi) & S_0(KK\to KK) \end{pmatrix}}_{\text{strong-interaction-driven}} \cdot \begin{pmatrix} A^*(D\to\pi\pi) \\ A^*(D\to KK) \end{pmatrix}$$

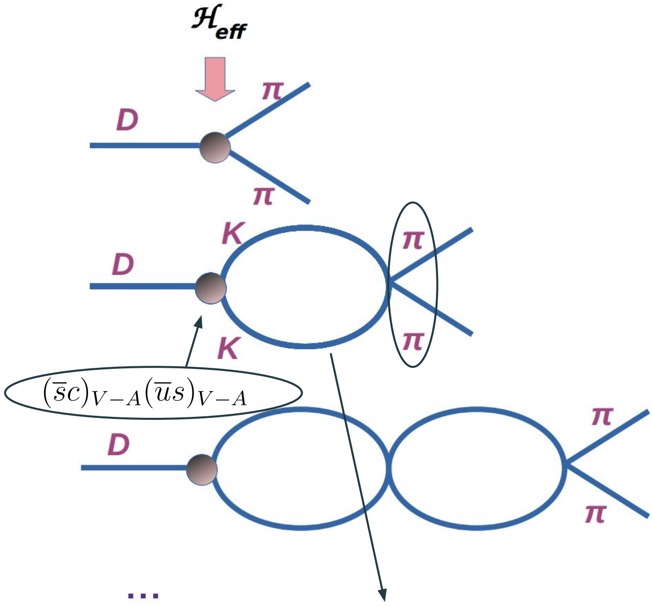
$$S_0 = \begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix}$$

Two-channel case

$$|A(D \to \pi\pi)(s)| = A(s_0) \cdot exp\{\frac{s-s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} dz \frac{\delta_1(z)}{(z-s_0)(z-s)}\}$$

now becomes

$$\begin{pmatrix} \mathcal{A}(D \to \pi\pi) \\ \mathcal{A}(D \to KK) \end{pmatrix} = \Omega \cdot \begin{pmatrix} \mathcal{A}_{(\text{large } N_C)}(D \to \pi\pi) \\ \mathcal{A}_{(\text{large } N_C)}(D \to KK) \end{pmatrix}$$



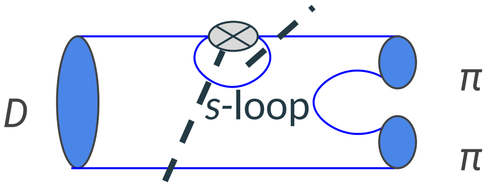
Ω is a 2-by-2 matrix that has to be found **numerically**

by solving the two-channel dispersion relation

- In the language of hadronic matrix elements:

  Non-diagonal Ω creates
  $$\langle \pi\pi(I=0)|Q_i^s|D\rangle \neq 0$$
  $$\langle KK(I=0)|Q_i^d|D\rangle \neq 0$$

"Long-distance penguin"



The ACP measurements

$$A_{\rm CP}(f) \equiv \frac{\Gamma(D^0 \to f) - \Gamma(\overline{D^0} \to \overline{f})}{\Gamma(D^0 \to f) + \Gamma(\overline{D^0} \to \overline{f})}$$

$$\Delta A_{\rm CP} = A_{\rm CP}(K^-K^+) - A_{\rm CP}(\pi^-\pi^+) = (-15.4 \pm 2.9) \times 10^{-4}$$

[LHCb 2019]

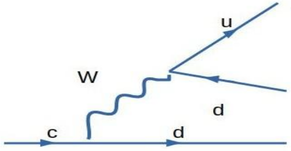
$$A_{\rm CP}(K^-K^+) = [6.8 \pm 5.4\ ({\rm stat})\ \ \pm 1.6\ ({\rm syst})\ ] \times 10^{-4}$$

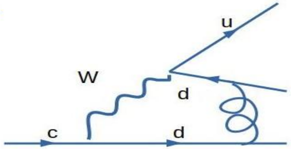
[LHCb 2022]

↓

$$A_{\rm CP}^{\rm direct}(\pi^-\pi^+) = (23.2 \pm 6.1) \times 10^{-4}$$

How CP violation arises



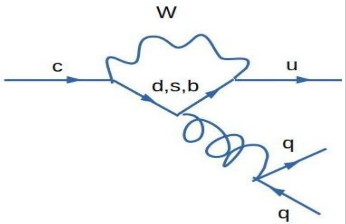


$$\mathcal{A}(D^0 \to f) = A(f) + ir_{CKM}B(f)$$
$$\mathcal{A}(\overline{D^0} \to f) = A(f) - ir_{CKM}B(f)$$

where $r_{CKM} = \mathrm{Im}\frac{V_{cb}^* V_{ub}}{V_{cd}^* V_{ud}} \approx 6.5 \times 10^{-4}$

Generally: at least **2 interfering amplitudes**

$$A_{\rm CP}^{\rm direct} \approx 2 \underbrace{r_{CKM}}_{\text{weak phases}} \frac{|B(f)|}{|A(f)|} \cdot \sin \underbrace{\arg \frac{A(f)}{B(f)}}_{\text{strong phases}}$$



$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \left[\Sigma_{i=1}^2 C_i(\mu) \left(\lambda_d Q_i^d(\mu) + \lambda_s Q_i^s(\mu)\right) - \lambda_b(\Sigma_{i=3}^6 C_i(\mu) Q_i(\mu) + C_{8g}(\mu) Q_{8g}(\mu))\right]$$

and consequently

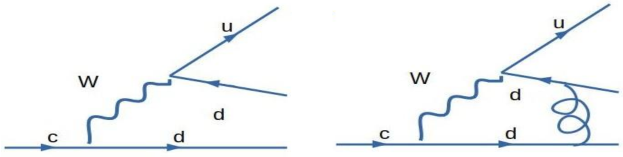
current-current operators → affect branching ratios & aCP's

penguin operators → affect only aCP's

$$\lambda_q = V_{cq}^* V_{uq}\,, \quad q = d, s, b.$$
$$|\lambda_d| \approx |\lambda_s| = \mathcal{O}(\lambda)$$
$$\lambda_d + \lambda_s + \lambda_b = 0$$
$$|C_{3-6}| < 0.1 C_2,\ 0.03 C_1$$

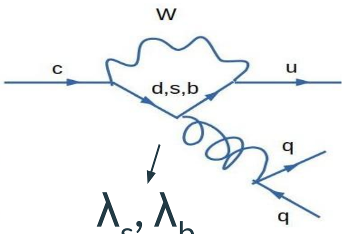
Challenge: to calculate $\langle P^+ P^- | Q_i | D^0 \rangle,\ P = \pi, K$

Sources of CP violation



λ_d

vs



λ_s, λ_b

At the level of amplitudes:

Recall: ***different weak phases & strong phases needed***

For D→ ππ (similarly for D→KK):

One I=2 amplitude

At the quark level (full theory):

$$\lambda_d \cdot \langle \pi\pi_{I=2} | (\bar{d}c)(\bar{u}d) | D \rangle$$

(current-current operators implied)

and several I=0 amplitudes

$$\lambda_d \cdot \langle \pi\pi_{I=0} | (\bar{d}c)(\bar{u}d) | D \rangle + \lambda_s \cdot \langle \pi\pi_{I=0} | (\bar{s}c)(\bar{u}s) | D \rangle - \lambda_b \cdot \langle \pi\pi_{I=0} | \text{penguin operators} | D \rangle$$

"Long-distance penguin"

Short-distance penguin

(significant for Q_6 operator-annihilation topology)

If ππ did not rescatter to KK:

$$\langle \pi\pi_{I=0} | (\bar{s}c)(\bar{u}s) | D \rangle = 0 \quad \text{AND}$$

$$\arg\langle \pi\pi_{I=0} | \text{penguin operators} | D \rangle = \arg\langle \pi\pi_{I=0} | (\bar{d}c)(\bar{u}d) | D \rangle$$

(Watson's theorem)

→ Only source ***would be*** interference of I=2 vs I=0 short-distance penguin

Instead: **more sources of CP violation now** ; no significant cancellations between different CPV sources

CP asymmetries in the rare decays

The unnormalised CP-asymmetric observables e.g. from the P-wave go as

$$\mathrm{Im}(\lambda_s \lambda_d^*)\mathrm{Im}(C_{9d}^P C_{9s}^{P*}) \tag{1}$$

where roughly

$$C_{9d}^P = \frac{e^{i\delta_{\rho\rho}}}{P_\rho(q^2)} \tag{2}$$

$$C_{9s}^P = \frac{e^{i\delta_{\rho\phi}}}{P_\phi(q^2)} \tag{3}$$

This on top of the resonance gives $3 \cdot 10^{-5}$ (from the CKM) x (up to 500).

On the other hand, the observables are normalised to the decay rates, which go as

$$|\lambda_d|^2 |C_{9d}^P + C_{9d}^S|^2 \tag{4}$$

which gives $5 \cdot 10^{-2}$ (from the CKM) x (up to $5 \cdot 10^4$).

Thus the effect on top of the resonances is very small. On the contrary, away from the resonances there are some comparative enhancement patterns. Still because of the typical CKM suppression factor $6.4 \cdot 10^{-4}$ of charm decays the overall, normalised CP-asymmetrical observables are expected to be very small, less than per mille.

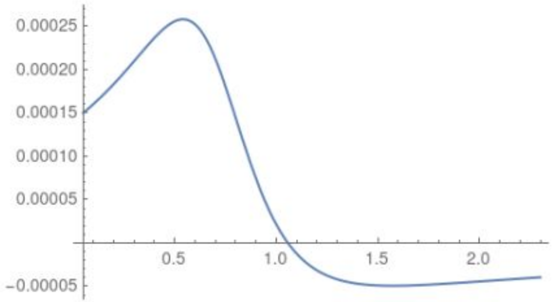


Figure 1: Generic CP-asymmetric observable A over generic CP-symmetric observable S/differential decay rate, as a function of the invariant mass of the dimuon. CKM factors included.