

# SIMP Miracles and WIMP Dead Ends: Navigating the Freeze-Out of MeV Dark Matter

**Josef Pradler**

Discrete 2024

Ljubljana, Slovenia

Dec 04, 2024

**ÖAW**

AUSTRIAN  
ACADEMY OF  
SCIENCES



universität  
wien



European Research Council  
Established by the European Commission

**FWF**

Der Wissenschaftsfonds.

# Navigating the Freeze-Out of MeV DM

## 1 SIMPs *natural habitat: 100 MeV mass scale*

=> putting a new perspective on the “SIMP miracle”

[arXiv:2401.12283 \(PRL\)](#)

w/ Xiaoyong Chu, Marco Nikolic

## 2 WIMPs *natural habitat: EW mass scale*

=> what is the lightest WIMP mass? The WIMP “dead end”.

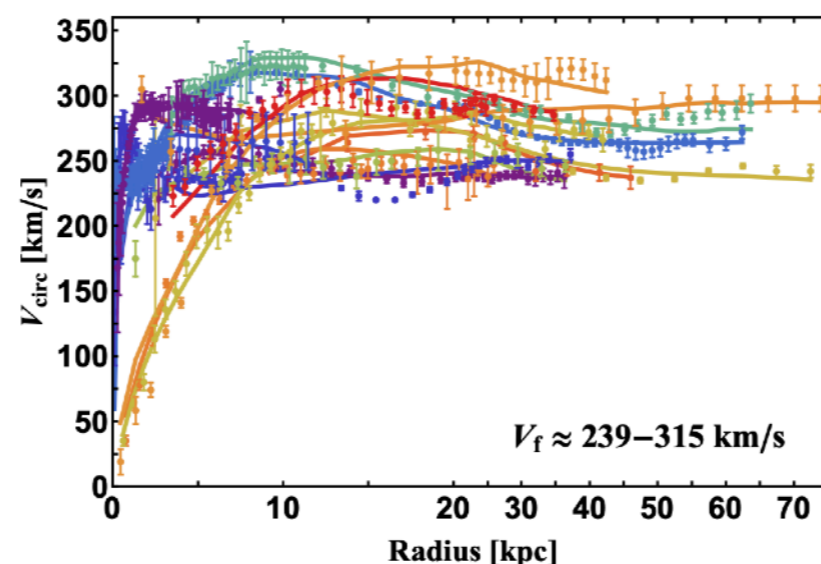
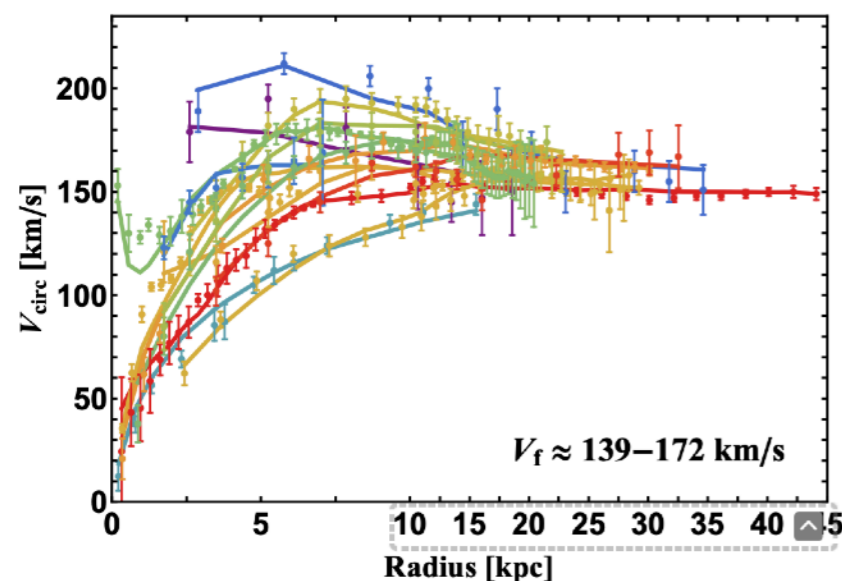
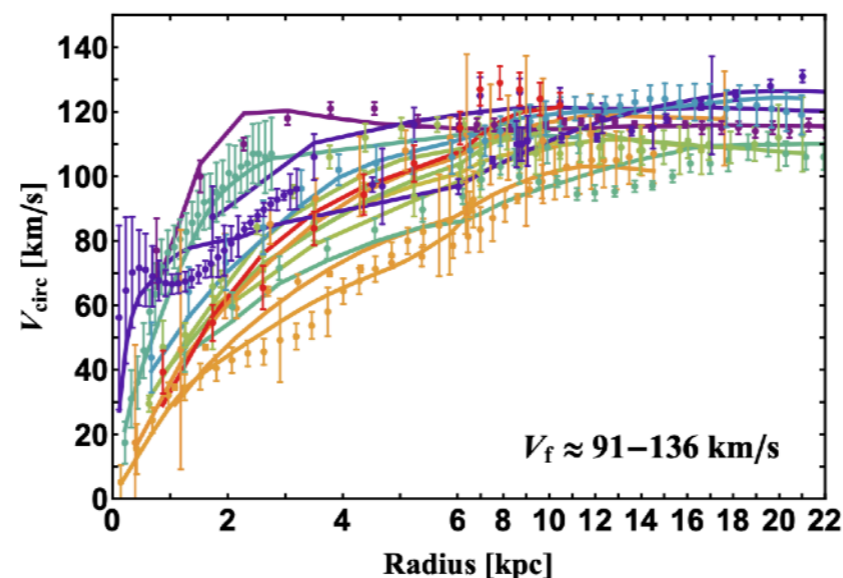
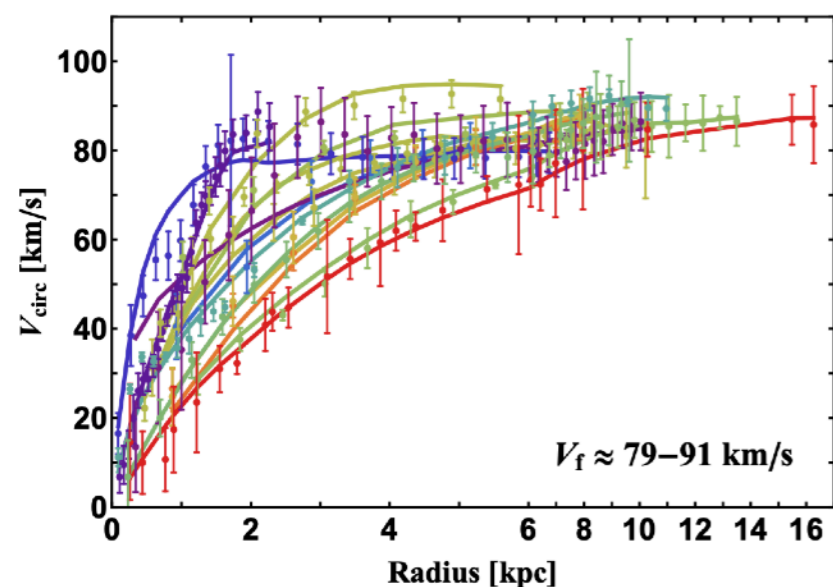
[arXiv:2205.05714 \(PRD\)](#)

[arXiv:2310.06611 \(PRD\)](#)

w/ Xiaoyong Chu, Jui-Lin Kuo

# Motivation for SIMPs

Small scale structure problems in LCDM (core-cusp, diversity, ...)



self-interactions lead to heat transfer in the halo, diversifying the halo density in the central regions of galaxies

*natural habitat:  
MeV mass scale paired  
with strong interactions*

e.g. Ren, Kwa, Kaplinghat, Yu [2019]

$$\sigma/m = 3 \text{ cm}^2/\text{g}$$

# WIMPs

“Weakly Interacting Massive Particles”

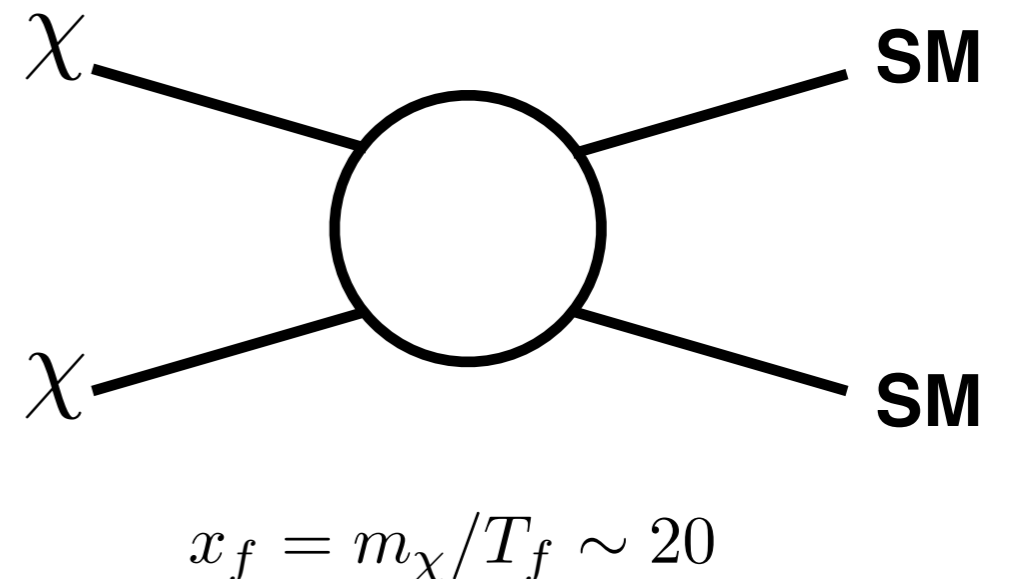
Freeze out when  $2 \rightarrow 2$  annihilation rate  $\sim$  Hubble rate

$$\Gamma_{2 \rightarrow 2}(T_f) = \langle \sigma v \rangle n_\chi(T_f) \sim H(T_f)$$

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{m_\chi^2}$$

$$n_\chi(T_f) = \frac{\rho_\chi(T_f)}{m_\chi} = \frac{T_{eq} m_\chi^2}{x_f^3}$$

$$H(T_f) \sim \frac{T_f^2}{M_P} = \frac{m_\chi^2}{x_f^2 M_P}$$



# WIMPs

“Weakly Interacting Massive Particles”

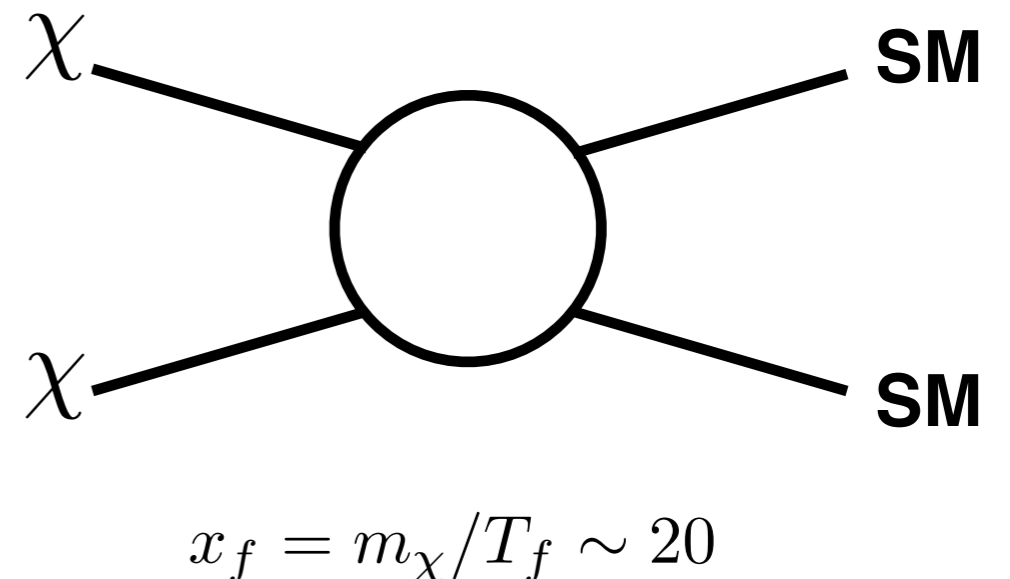
Freeze out when  $2 \rightarrow 2$  annihilation rate  $\sim$  Hubble rate

$$\Gamma_{2 \rightarrow 2}(T_f) = \langle \sigma v \rangle n_\chi(T_f) \sim H(T_f)$$

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{m_\chi^2}$$

$$n_\chi(T_f) = \frac{\rho_\chi(T_f)}{m_\chi} = \frac{T_{eq} m_\chi^2}{x_f^3}$$

$$H(T_f) \sim \frac{T_f^2}{M_P} = \frac{m_\chi^2}{x_f^2 M_P}$$



$$m_\chi \sim \frac{\alpha}{\sqrt{x_f}} \sqrt{T_{eq} M_P} \sim \alpha(30 \text{ TeV})$$

=> points to electroweak scale

# SIMPs

“Strongly Interacting Massive Particles”

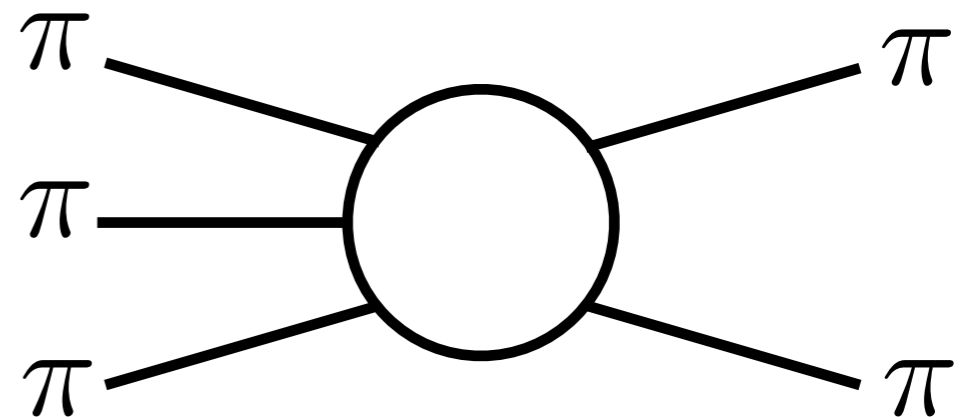
Freeze out when **3**  $\rightarrow$  **2** annihilation rate  $\sim$  Hubble rate

$$\Gamma_{3 \rightarrow 2}(T_f) = \langle \sigma v^2 \rangle n_\pi^2(T_f) \sim H(T_f)$$

$$\langle \sigma v^2 \rangle \sim \frac{\alpha^3}{m_\chi^5}$$

collision term or

“cross section” of mass dimension -5



[Hochberg et al 2015, ...]

# SIMPs

“Strongly Interacting Massive Particles”

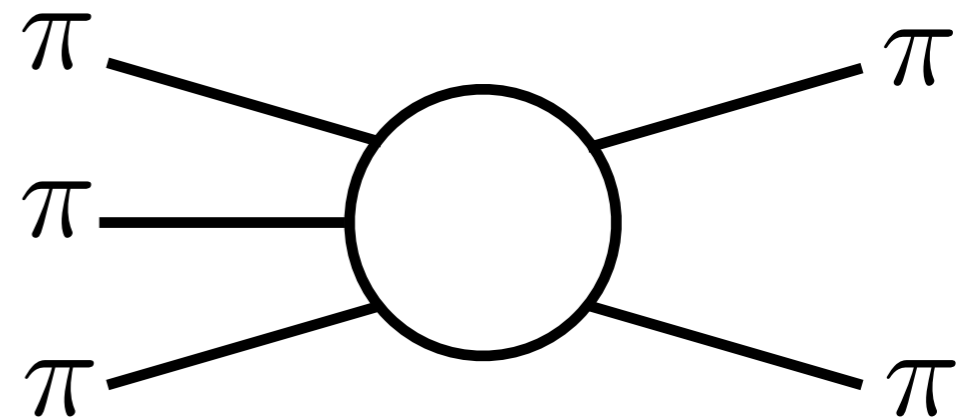
Freeze out when **3**  $\rightarrow$  **2** annihilation rate  $\sim$  Hubble rate

$$\Gamma_{3 \rightarrow 2}(T_f) = \langle \sigma v^2 \rangle n_\pi^2(T_f) \sim H(T_f)$$

$$\langle \sigma v^2 \rangle \sim \frac{\alpha^3}{m_\chi^5}$$

collision term or

“cross section” of mass dimension -5



$$m_\pi \sim \alpha(T_{eq}^2 M_P)^{1/3} \sim \alpha(100 \text{ MeV})$$

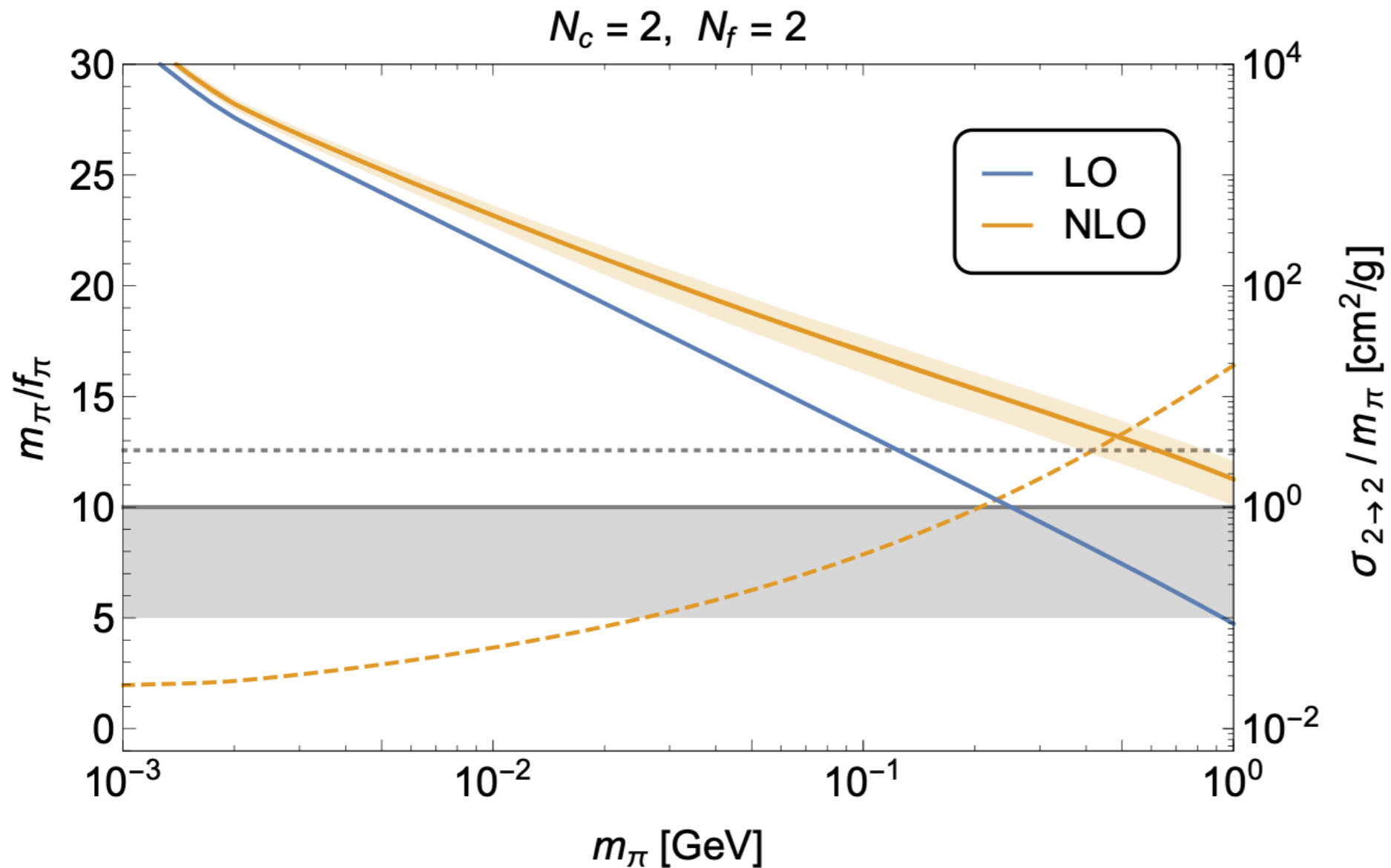
=> points to strong interactions

=> MeV scale DM

[Hochberg et al 2015, ...]

# A SIMP miracle?

Successful relic density paired with sufficient self-scattering cross section



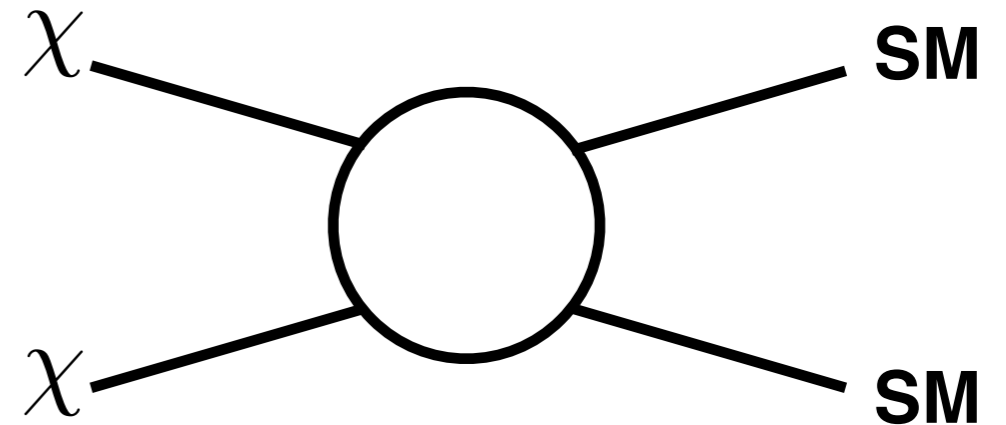
[Hansen, Langaebler, Sannino 2016]

**tension in the joint “miracle” solution**

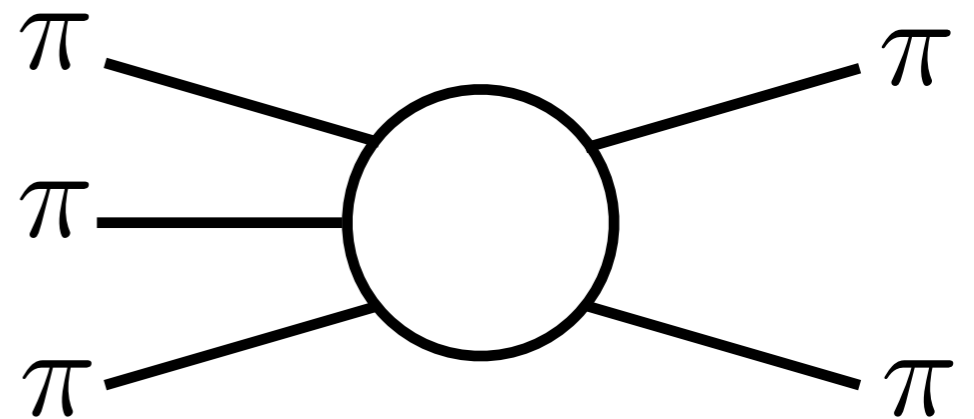


# WIMPs vs. SIMPs

$$m_\chi \sim \frac{\alpha}{\sqrt{x_f}} \sqrt{T_{eq} M_P} \sim \alpha(30 \text{ TeV})$$

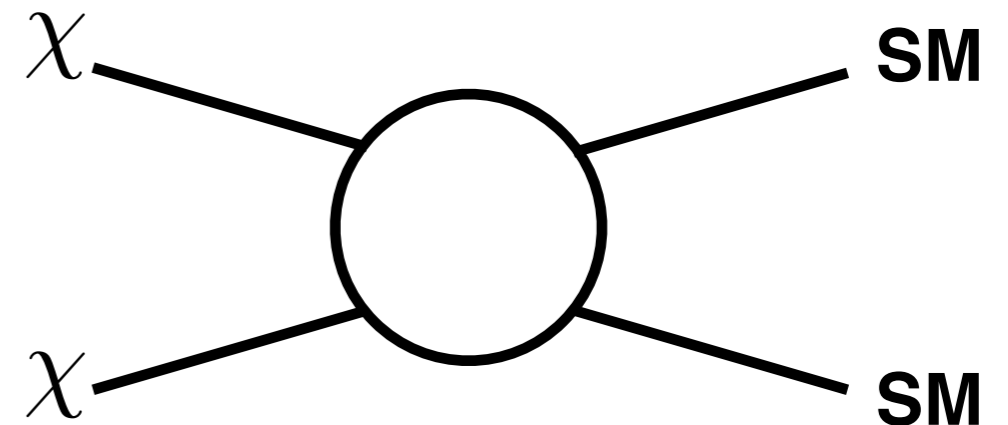


$$m_\pi \sim \alpha(T_{eq}^2 M_P)^{1/3} \sim \alpha(100 \text{ MeV})$$



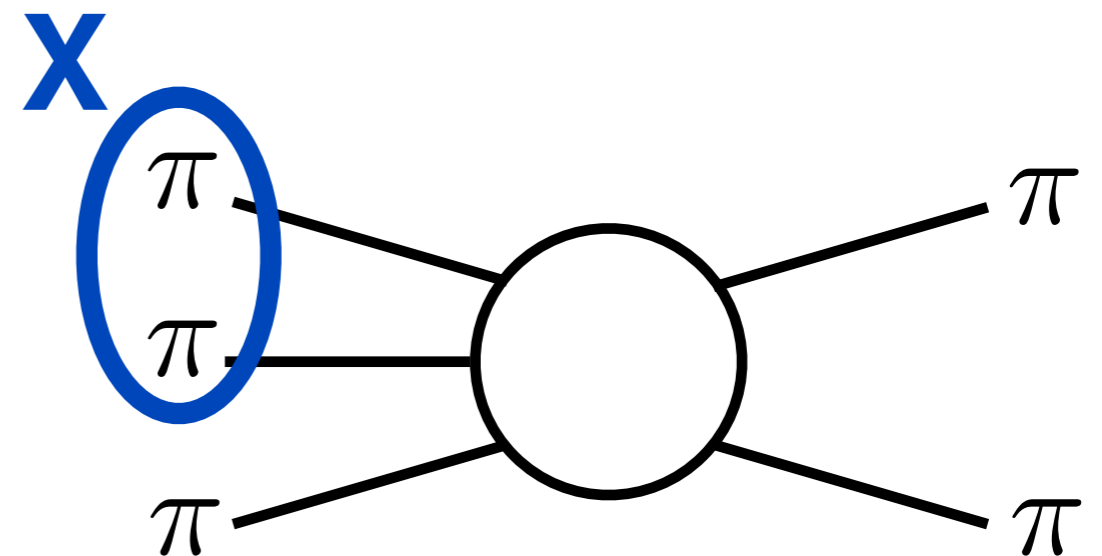
# WIMPs vs. SIMPs

$$m_\chi \sim \frac{\alpha}{\sqrt{x_f}} \sqrt{T_{eq} M_P} \sim \alpha(30 \text{ TeV})$$



what if we make a  
stable bound state?

$$m_\pi \sim \alpha(T_{eq}^2 M_P)^{1/3} \sim \alpha(100 \text{ MeV})$$



# SIMP prototype model

Dark Matter as Goldstone bosons of a confining dark sector

For example, two flavor  $N_f = 2$ ,  $Sp(4)_c$  gauge group

Kulkarni, Maas, Mee, Nikolic, JP, Zierler SciPost Phys. 14 (2023) 3, 044,

$$\mathcal{L}^{\text{UV}} = -\frac{1}{2} \text{Tr} [G_{\mu\nu} G^{\mu\nu}] + \bar{u} (\gamma_\mu D_\mu + m_u) u + \bar{d} (\gamma_\mu D_\mu + m_d) d$$

Quarks are in pseudoreal representation of color group  $(\tau^a)^T = S \tau^a S$

$$\Psi \equiv \begin{pmatrix} u_L \\ d_L \\ \sigma_2 S u_R^* \\ \sigma_2 S d_R^* \end{pmatrix} \Rightarrow \mathcal{L}_{\text{kin}}^{\text{UV},f} = i \Psi^\dagger \bar{\sigma}_\mu D^\mu \Psi. \quad \Rightarrow \text{SU}(4)$$

Flavor:

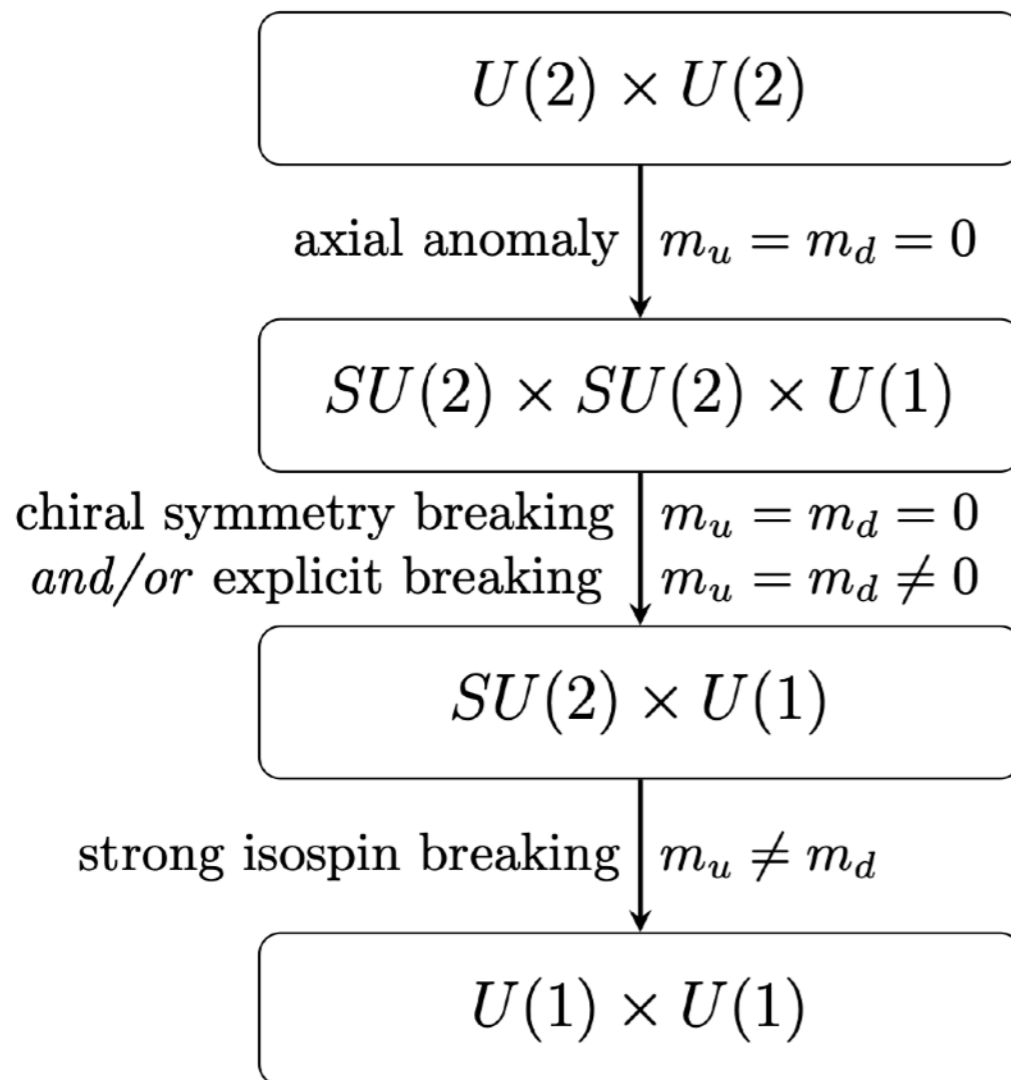
$$\bar{u}u + \bar{d}d = -\frac{1}{2} \Psi^T \sigma_2 S E \Psi + \text{h.c.} \quad (m_u = m_d)$$

$$E = \begin{pmatrix} 0 & \mathbb{1}_{N_f} \\ -\mathbb{1}_{N_f} & 0 \end{pmatrix} \quad U^T E U = E \quad \Rightarrow \text{Sp}(4)$$

# Flavor breaking pattern

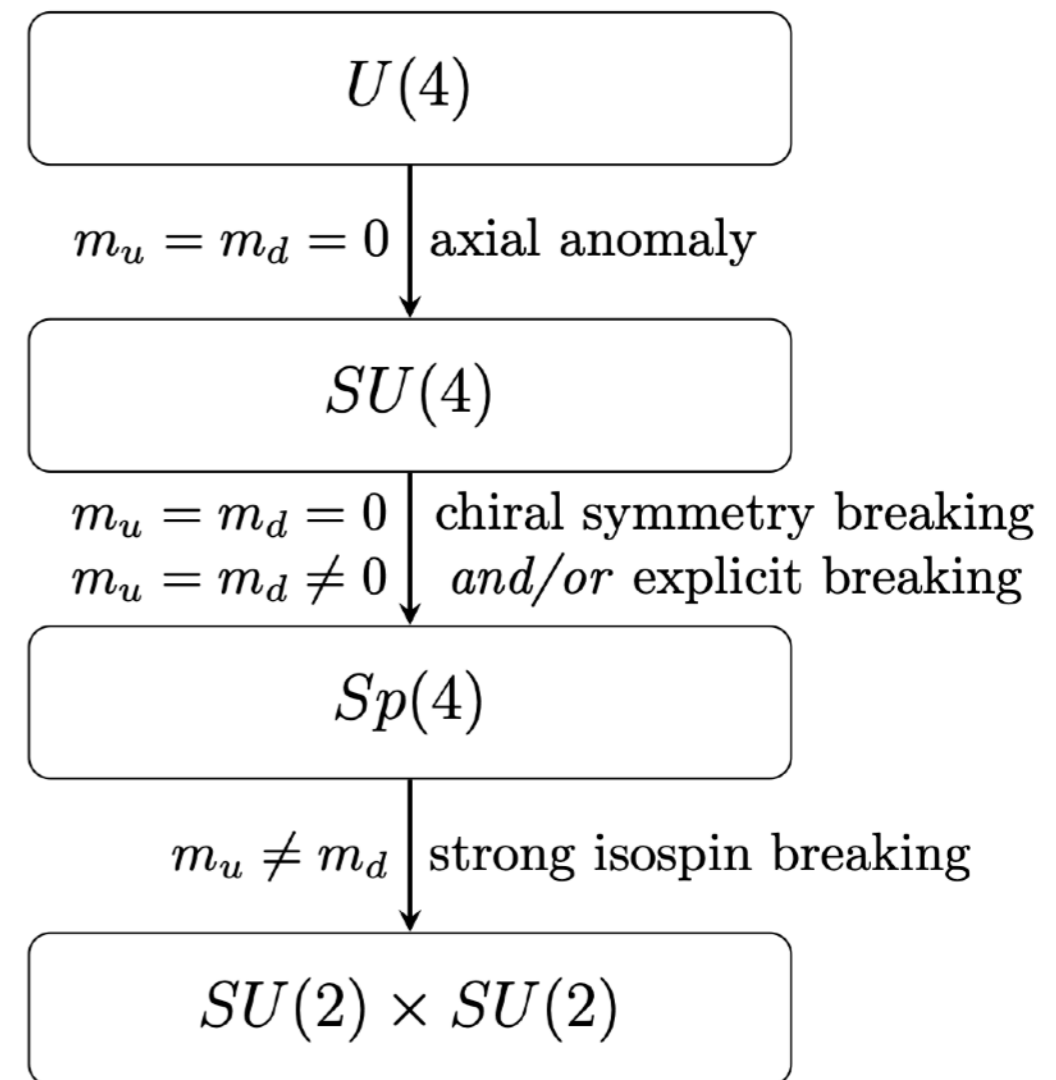
QCD-like

## COMPLEX



this example

## PSEUDOREAL



5 broken  
generators

=> 5 Goldstone bosons

# Prototype SIMP theory

## Low energy description

chiral field  $\Sigma = e^{i\pi/f_\pi} \Sigma_0 e^{i\pi^T/f_\pi}$        $\pi = \sum_{n=1} \pi_n T^n$  ← broken generators

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr} [\partial_\mu \Sigma \partial^\mu \Sigma^\dagger] - \frac{\mu^3}{2} (\text{Tr} [M \Sigma] + \text{Tr} [\Sigma^\dagger M^\dagger]) + \dots$$



expansion yields 4-point, 6-point, etc interactions

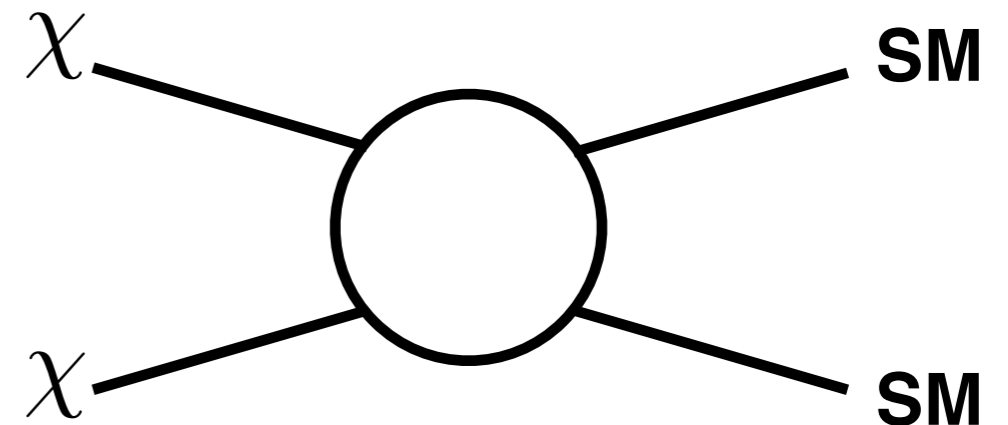
$$\mathcal{L}_{\text{int}}^{\text{even}} \supset -\frac{1}{3f_\pi^2} \text{Tr} ([\pi, \partial_\mu \pi][\pi, \partial^\mu \pi]) + \frac{m_\pi^2}{3f_\pi^2} \text{Tr} [\pi^4] \quad \text{even-numbered only}$$

Wess-Zumino-Witten term when coset space has non-trivial fifth homotopy group

$$\mathcal{L}_{\text{int}}^{\text{odd}} = \frac{2N_c}{15\pi^2 f_\pi^5} \epsilon^{\mu\nu\rho\sigma} \text{Tr} [\pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi] \cdot \quad \text{odd-numbered}$$

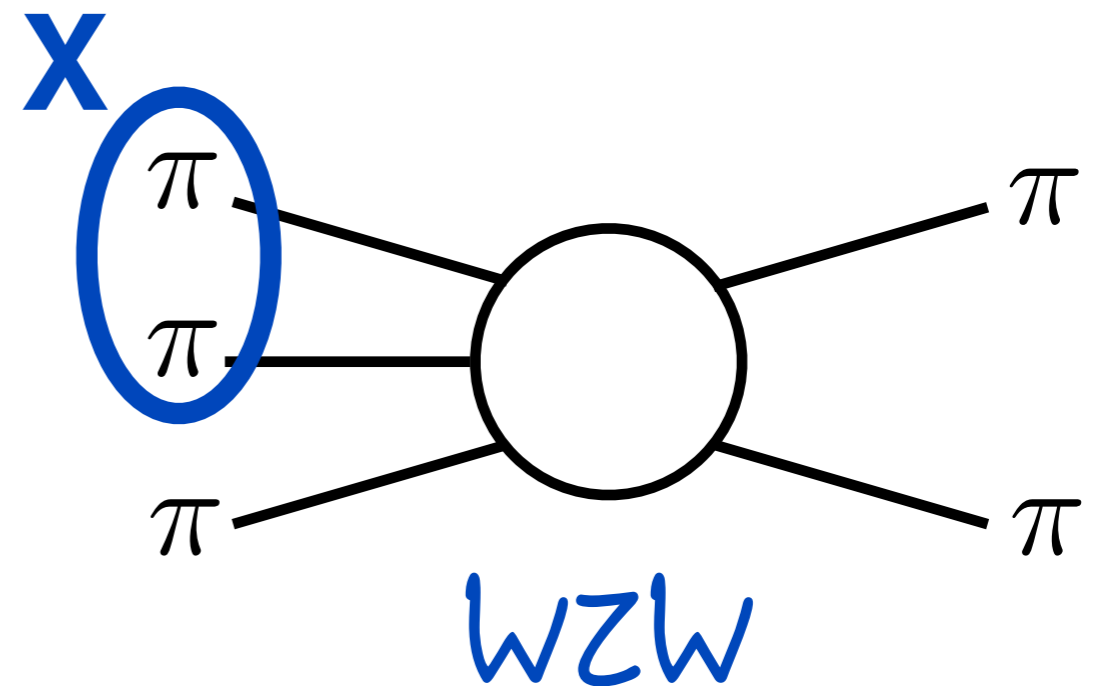
# WIMPs vs. SIMPs

$$m_\chi \sim \frac{\alpha}{\sqrt{x_f}} \sqrt{T_{eq} M_P} \sim \alpha(30 \text{ TeV})$$



what if we make a  
stable bound state?

$$m_\pi \sim \alpha(T_{eq}^2 M_P)^{1/3} \sim \alpha(100 \text{ MeV})$$



# SIMP bound states

$X = [\pi \pi]$  must exist

- considering SIMPs as pseudo-Nambu-Goldstone bosons of a strongly interacting theory we require a molecular state with negative binding energy such that  $m_X \leq 2m_\pi$

QCD with  $m_q \ll \Lambda_{\text{strong}}$  has a mass gap, hence not prospective

=> better consider a dark confining theory with  $m_q \sim \Lambda_{\text{strong}}$  and

=> make *SIMP-onium*

=> or take  $m_q \gg \Lambda_{\text{strong}}$ : Glueball dark matter  $J^{PC} = 0^{++}$  or  $0^{-+}$  e.g. [Soni, Zhang, 2016]

$$V(G) = \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left( G^4 \ln \left| \frac{G}{\Lambda_G} \right| - \frac{G^4}{4} \right) \quad \begin{array}{l} \Rightarrow \text{yields odd } G^3 \text{ interactions} \\ \Rightarrow \text{3-to-2 SIMP mechanism} \end{array}$$

=> make *Glueball-onium* for G-bound states see [Giacosa, Piloni, Trotti 2021]

- one may also use a Yukawa force with sizable coupling; options exist

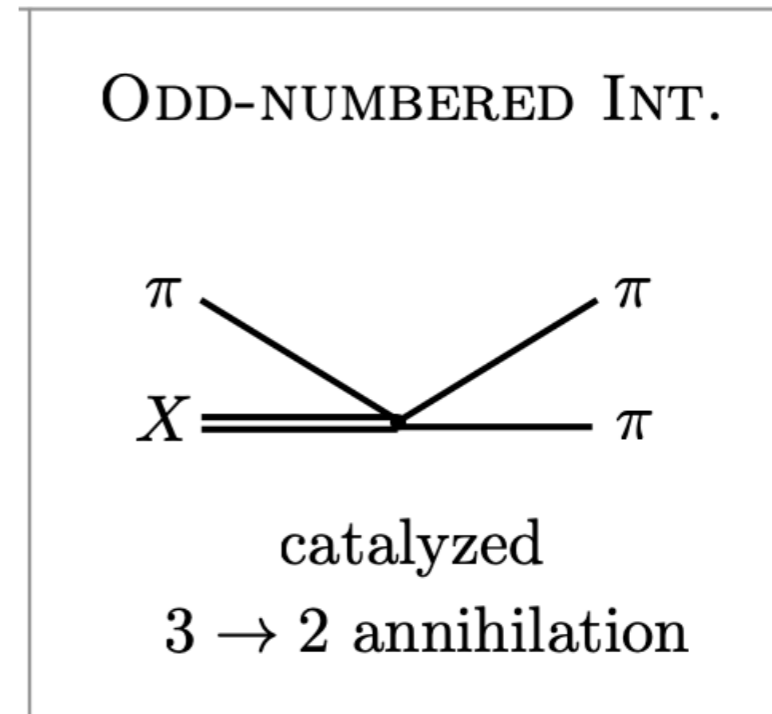
e.g. [G. Kribs and E. Neil 2016, Y. Tsai, R. McGehee, H. Murayama 2020, R. Mahbubani, M. Redi and A. Tesi 2020, ....].

# Bound-state assisted freeze-out

## Catalysis

Probability of two particles finding each other in a bound state vs. as free particles

$$\frac{n_X |\psi(0)|^2}{n_\pi^2} \approx 2\sqrt{2}\pi^{3/2} x_f^{3/2} e^{\kappa x_f} \frac{|\psi(0)|^2}{m_\pi^3}$$





# Bound-state assisted freeze-out

## Catalysis

Probability of two particles finding each other in a bound state vs. as free particles

$$\frac{n_X |\psi(0)|^2}{n_\pi^2} \approx 2\sqrt{2}\pi^{3/2} x_f^{3/2} e^{\kappa x_f} \frac{|\psi(0)|^2}{m_\pi^3}$$

---

$$\approx 10^3$$

---

$$O(1)$$

$$x_f = 20.$$

$$\kappa \equiv E_B/m_\pi \sim 0.1$$

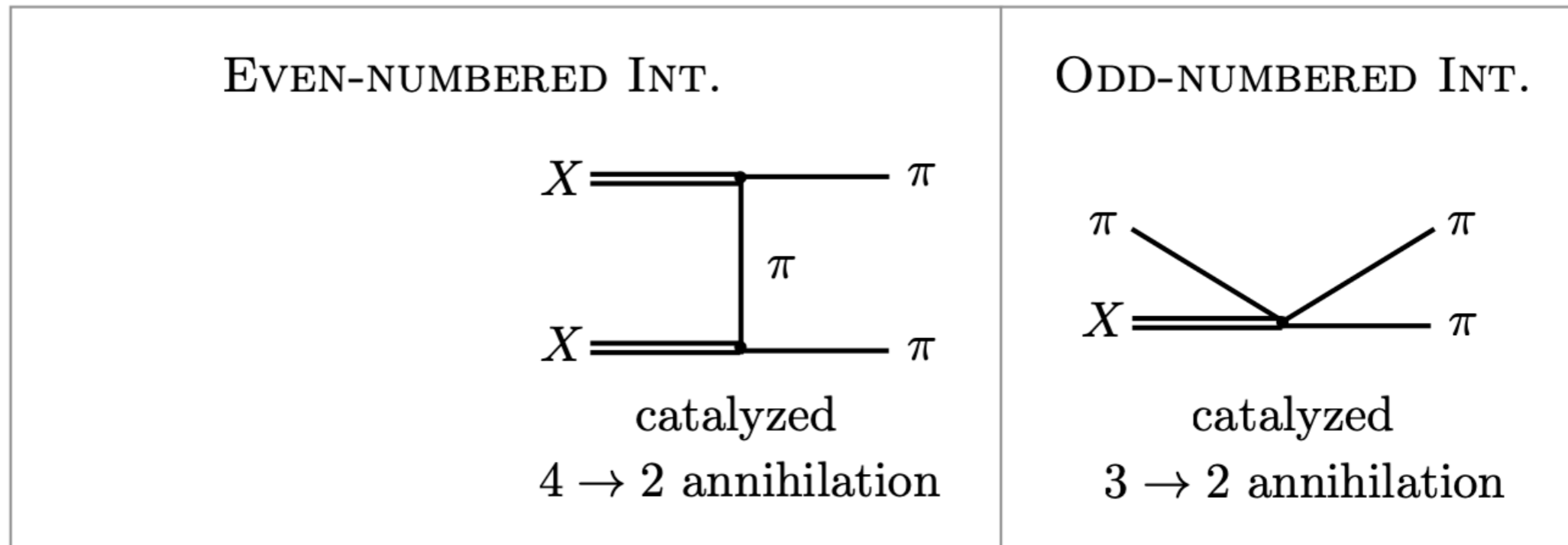
ODD-NUMBERED INT.



catalyzed

3 → 2 annihilation

# Bound-state assisted freeze-out

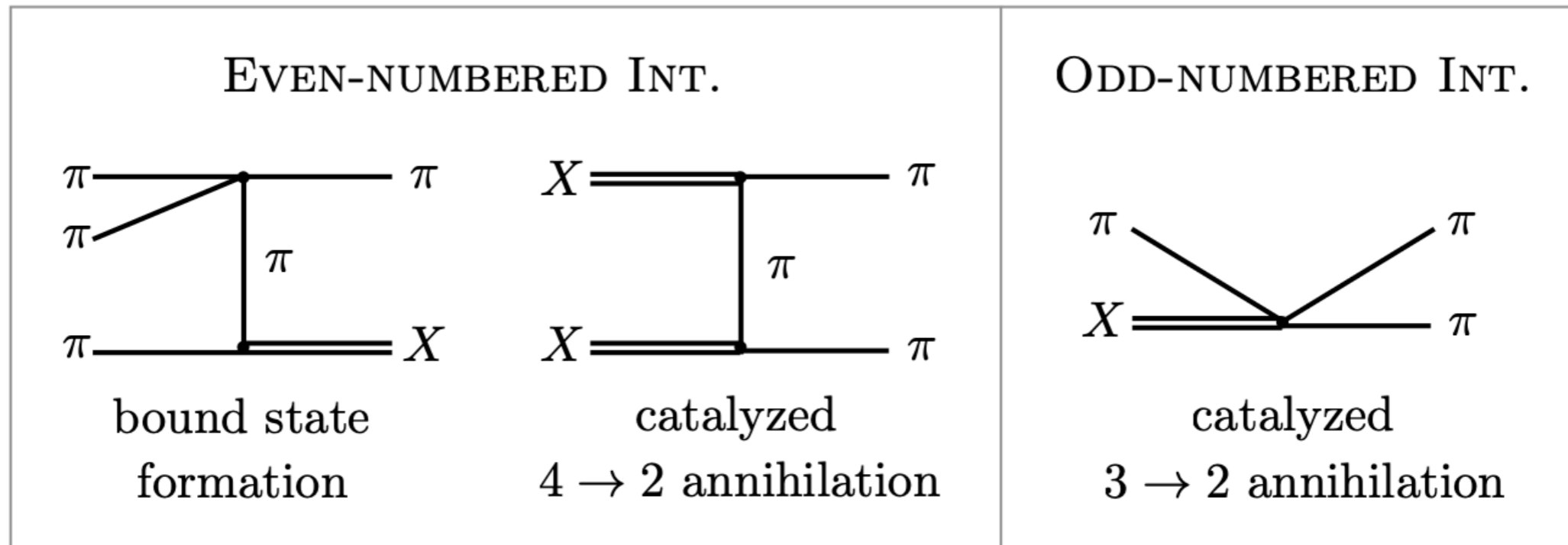


## WZW-free SIMP mechanism

self-depletion of mass density in the early Universe possible  
with even-numbered interactions only!

=> relaxes the requirement on the topological structure of the theory

# Bound-state assisted freeze-out

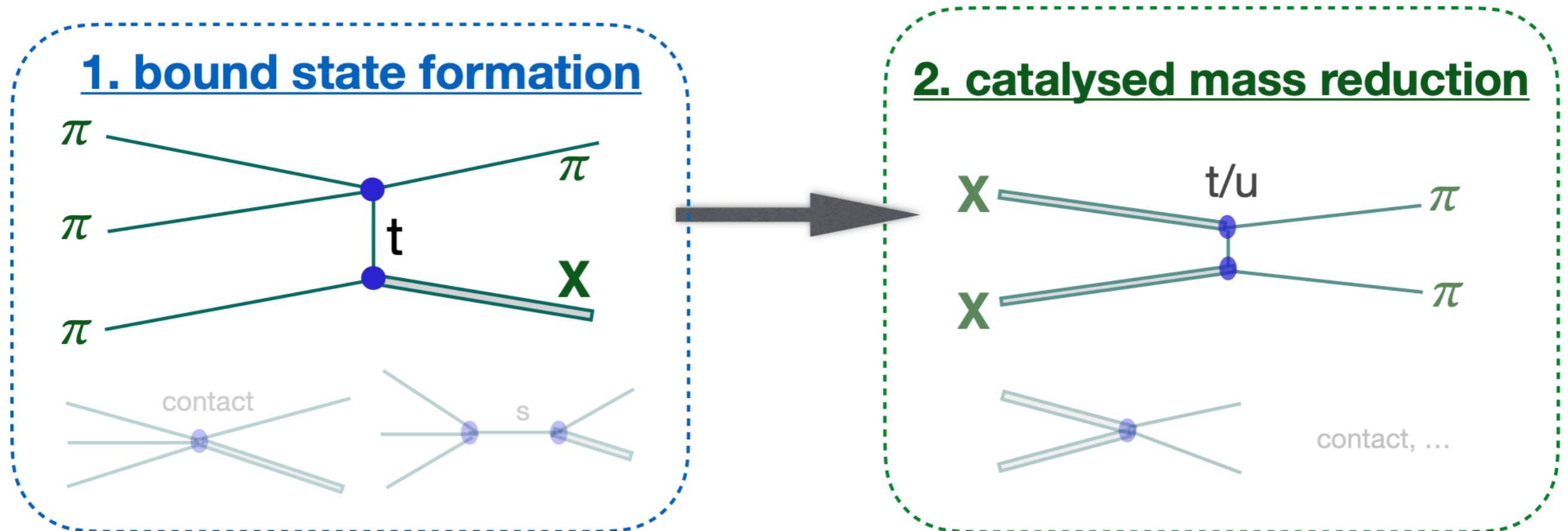


guaranteed X formation

Comparing the rates of X-formation to free

$$\frac{\Gamma_{3\pi \rightarrow X\pi}}{\Gamma_{3\pi \rightarrow 2\pi}} = \frac{\langle \sigma_{3\pi \rightarrow X\pi} v^2 \rangle}{\langle \sigma_{3\pi \rightarrow 2\pi} v^2 \rangle} \approx \frac{|\psi(0)|^2 f_\pi^2}{m_\pi^5} x_f^2. \quad \text{easily exceeds unity}$$

# CASE 1: even-numbered interactions only



## Working hypothesis:

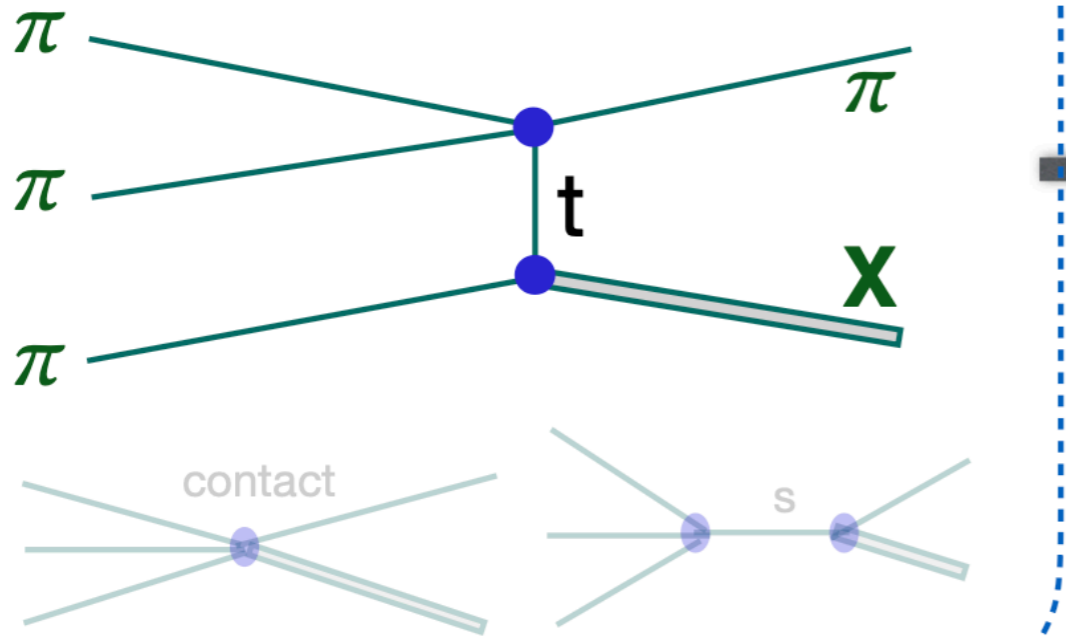
$X$  is a weakly bound (non-relativistic) state, such as a hadronic molecule

Bethe-Salpeter wave functions  $\Rightarrow$  non-relativistic Schroedinger equation

[e.g. K.Petraki, M.Postma, J.de Vries 2016, ...]

# CASE 1: even-numbered interactions only

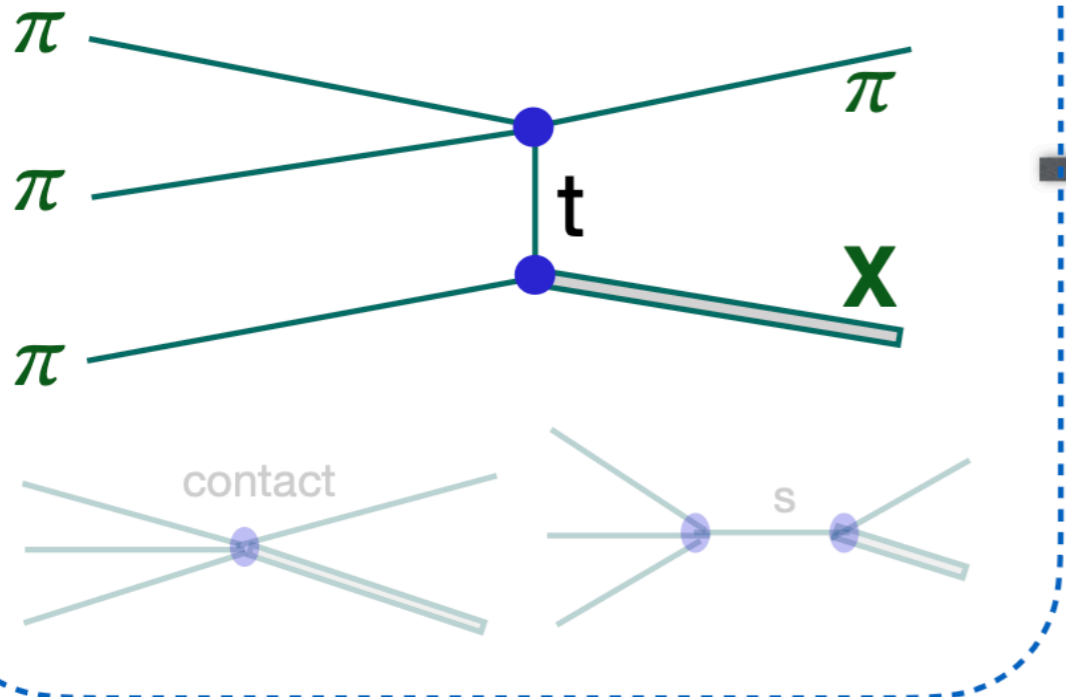
## 1. bound state formation



$$\begin{aligned}
 & i\mathcal{M}(p_1, p_2, p_3 \rightarrow k, Q)_{3\pi \rightarrow \pi X} \\
 & \simeq \frac{\sqrt{2m_X}}{2m_\pi} \int \frac{d^3q}{(2\pi)^3} \int d^3r \psi_X^*(\vec{r}) e^{-i\vec{q}\vec{r}} \times i\mathcal{M}_{(p_1, p_2, p_3 \rightarrow k, Q/2+q, Q/2-q)}^{\text{free}}
 \end{aligned}$$

# CASE 1: even-numbered interactions only

## 1. bound state formation



In the non-relativistic limit, one obtains a t-channel **resonance**:

$$\frac{s}{t - m_\pi^2} \propto \frac{m_\pi^2}{m_X^2 - 4m_\pi^2} \propto \frac{m_\pi}{E_B} \gg 1$$

radial wave function of X (s-wave)

$$\langle \sigma_{3\pi \rightarrow \pi X} v^2 \rangle \simeq \frac{57\,041}{1\,310\,720 \sqrt{3} \pi^2} \frac{R_S^2(0)}{f_\pi^8} \left( \frac{m_\pi}{E_B} \right)^{3/2}$$

↑ additional t-channel enhancement

# CASE 1: even-numbered interactions only

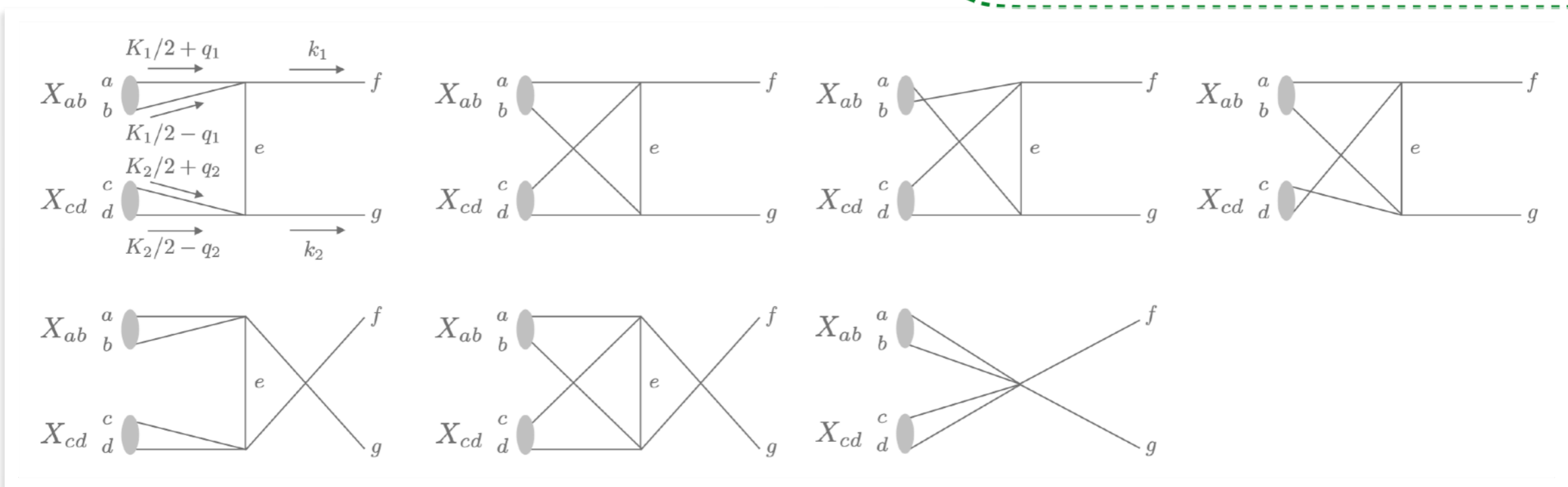
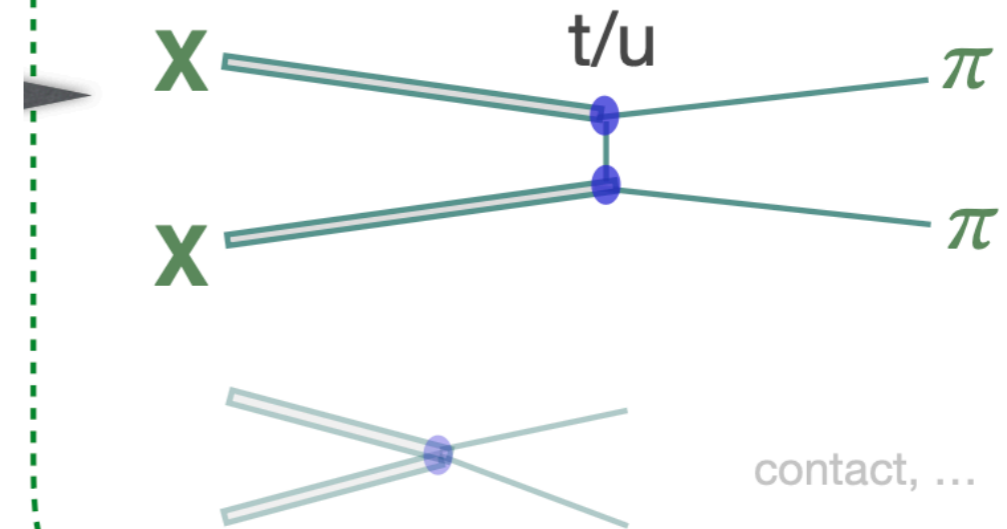
Mass-reduction rate

$$\Gamma_{XX \rightarrow \pi\pi} = \frac{n_X^2 \langle \sigma_{XX \rightarrow \pi\pi v} \rangle}{n_\pi}$$

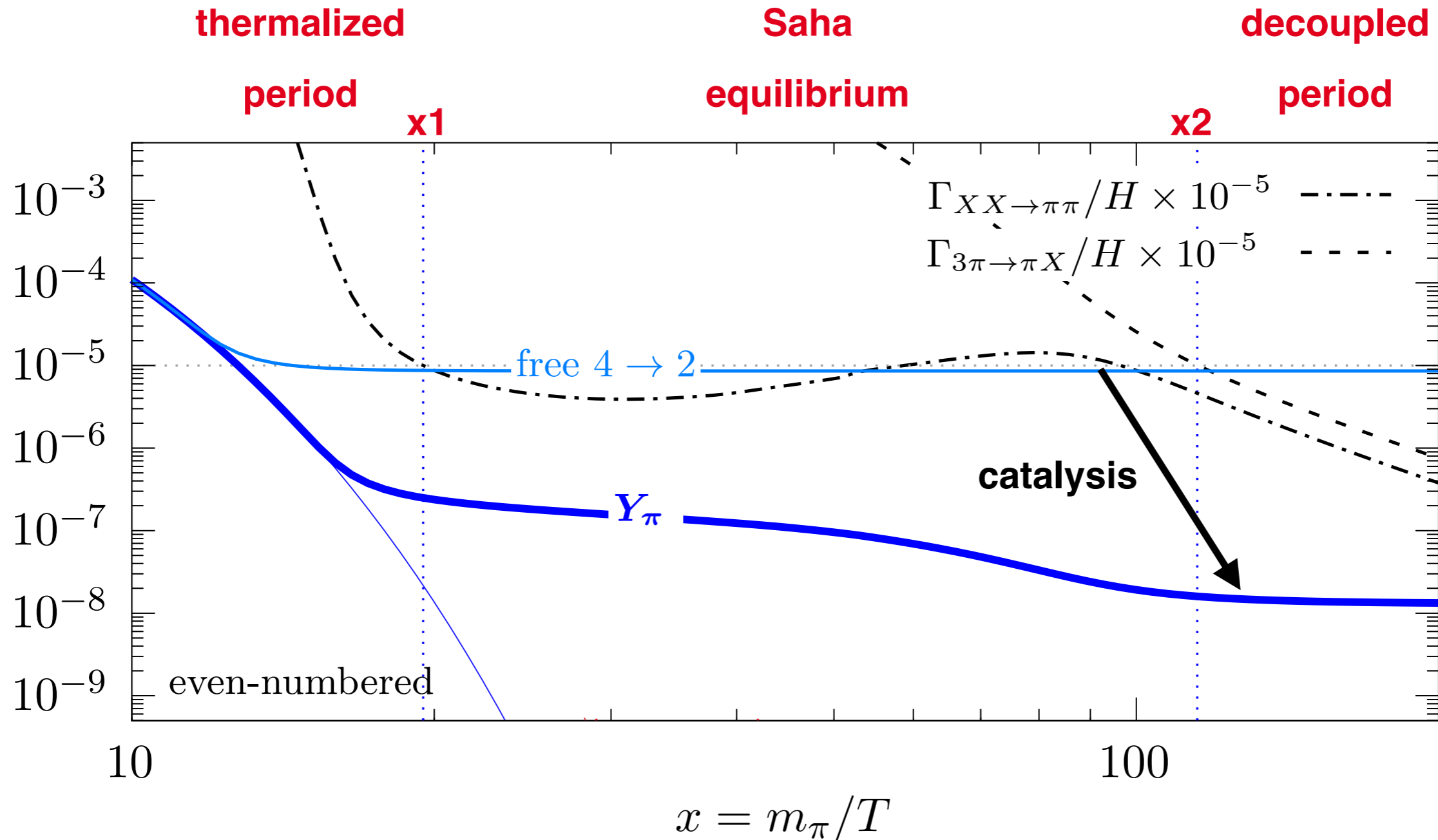
Cross section is s-wave

$$\langle \sigma_{XX \rightarrow \pi\pi v} \rangle \simeq \frac{2\,529\,757}{424\,673\,280\sqrt{3}\pi^3} \frac{R_S^4(0)}{f_\pi^8}$$

## 2. catalysed mass reduction



# CASE 1: even-numbered interactions only



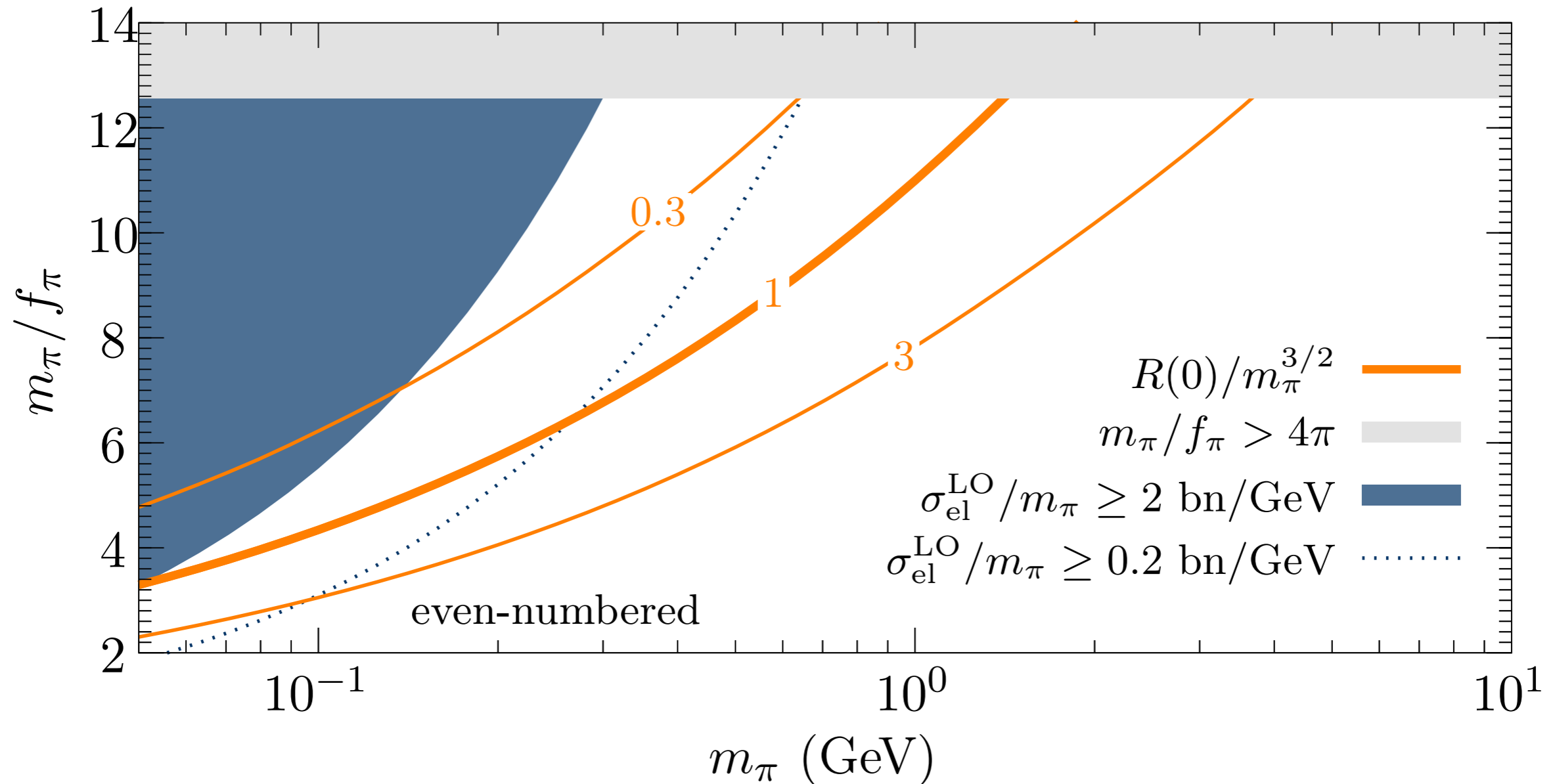
$$\Omega_\pi^{\text{even}} \sim 0.2 \left( \frac{10^3}{\kappa^4 e^{\kappa x_2}} \frac{\text{bn/GeV}}{\langle \sigma_{XX \rightarrow \pi\pi} v \rangle / m_\pi} \frac{m_\pi}{\text{GeV}} \right)^{1/3}$$

**x2 - dependent!**

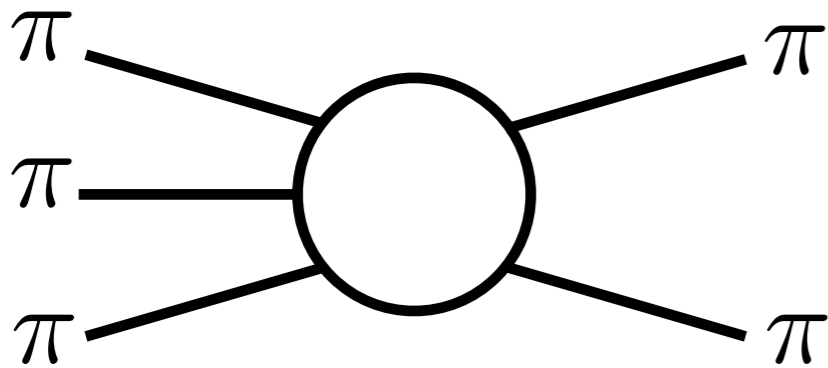


# Even SIMP miracles are possible!

coincidence of correct relic density + interesting self scattering ballpark



# CASE 2: odd-numbered interactions



standard WZW annihilation (**d-wave**)

$$\langle \sigma_{3\pi \rightarrow 2\pi} v^2 \rangle = \frac{\sqrt{5} N_c^2 m_\pi^3 T^2}{12800 \pi^5 f_\pi^{10}}$$

p-wave X are available through collisional excitation



catalyzed

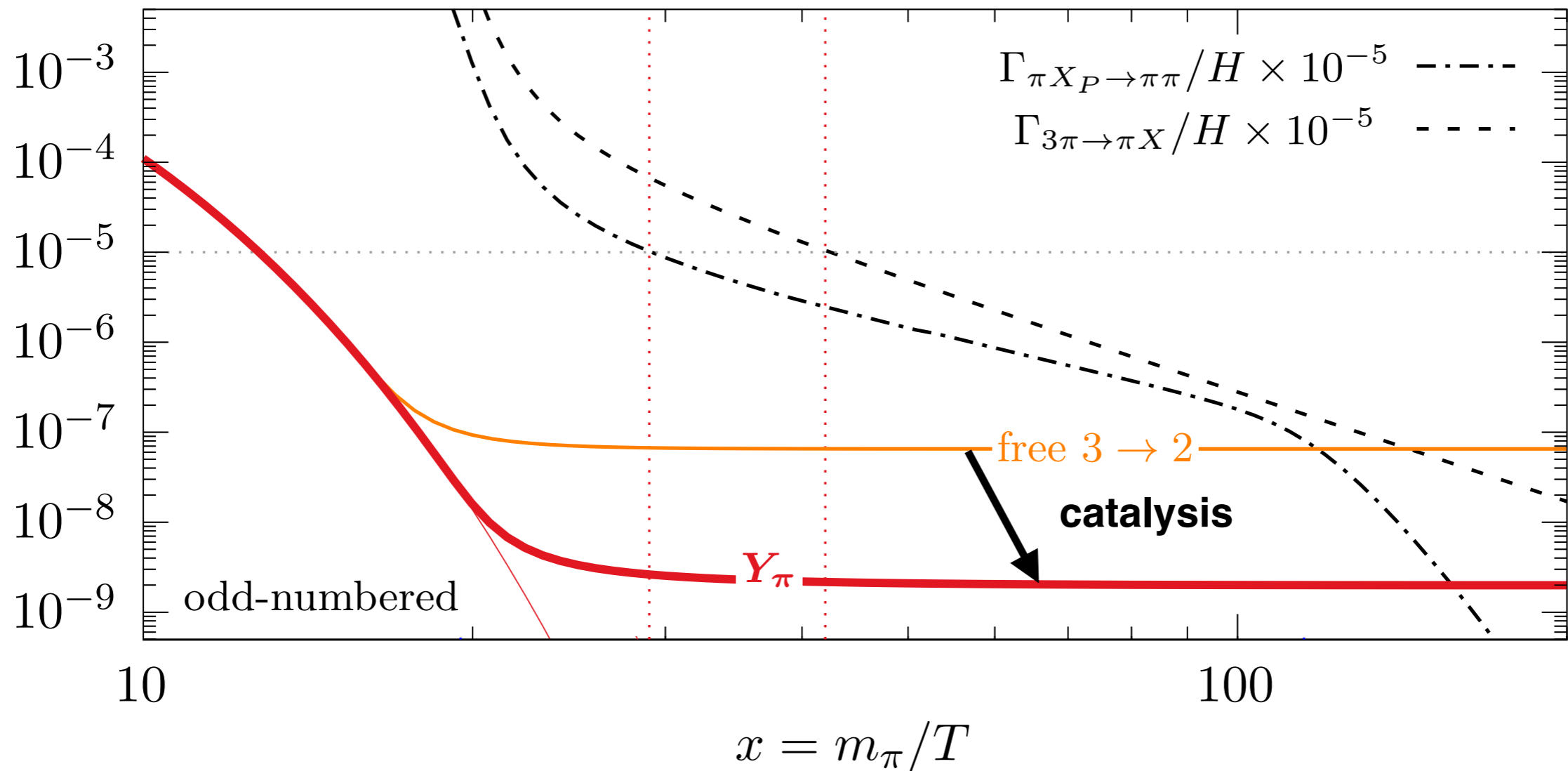
3 → 2 annihilation

derivative of radial wave function of X (**p-wave**)

$$\langle \sigma_{\pi X \rightarrow 2\pi} v \rangle = \frac{\sqrt{5} N_c^2 R'(0)^2 m_\pi^2 T}{512 \pi^6 f_\pi^{10}}$$

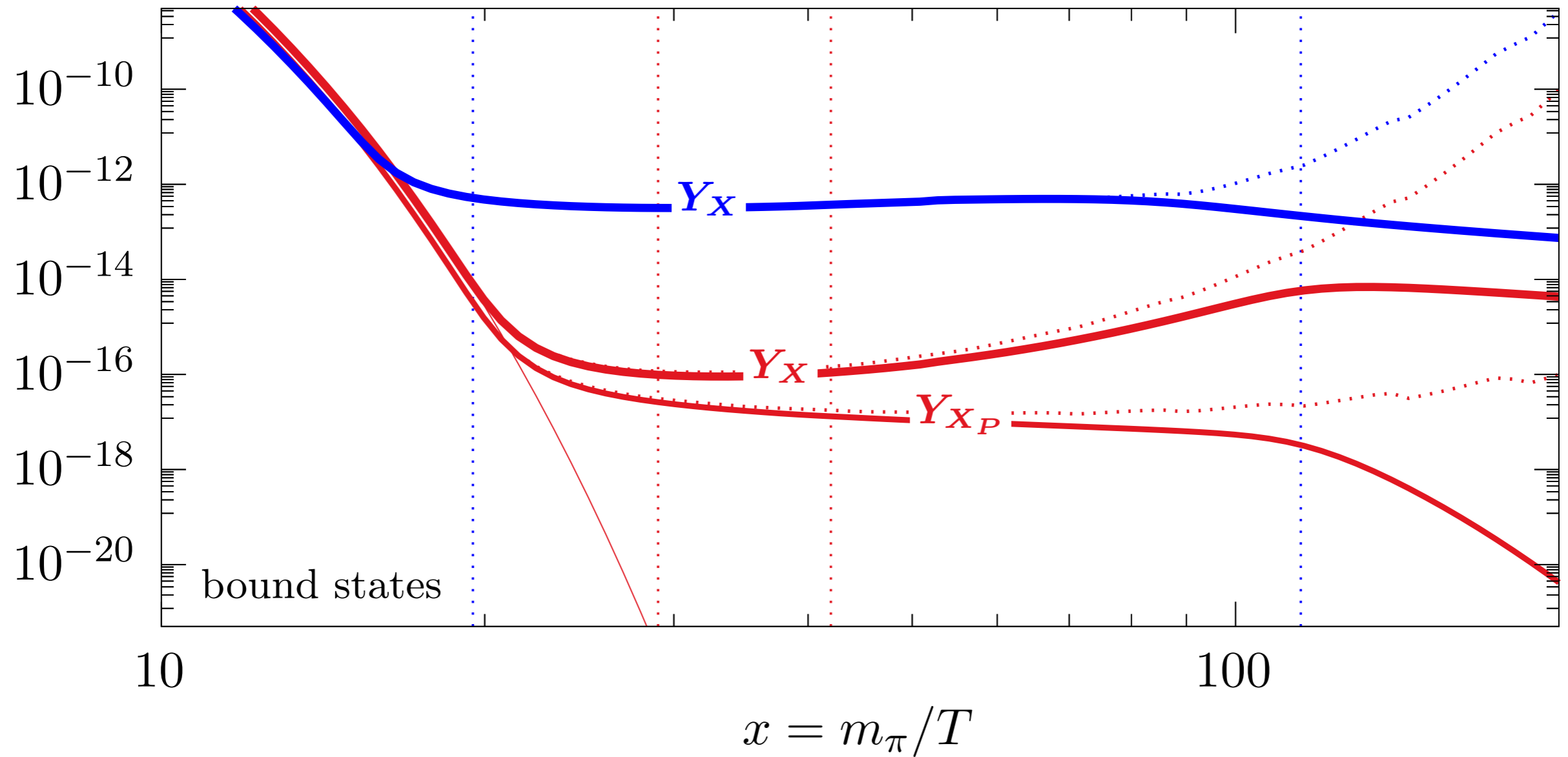
$$\frac{n_{X_P}}{n_{X_S}} = 3e^{-|E_S - E_P|T/m_\pi}$$

# CASE 2: odd-numbered interactions



$$\Omega_\pi^{\text{odd}} \simeq 0.2 \left( \frac{x_1}{20} \right)^{5/4} \left( \frac{e^{-\kappa_P x_1} 10^{-3} \text{ bn/GeV}}{\langle \sigma_{\pi X_P \rightarrow \pi\pi} v \rangle / m_\pi} \right)^{1/2}$$

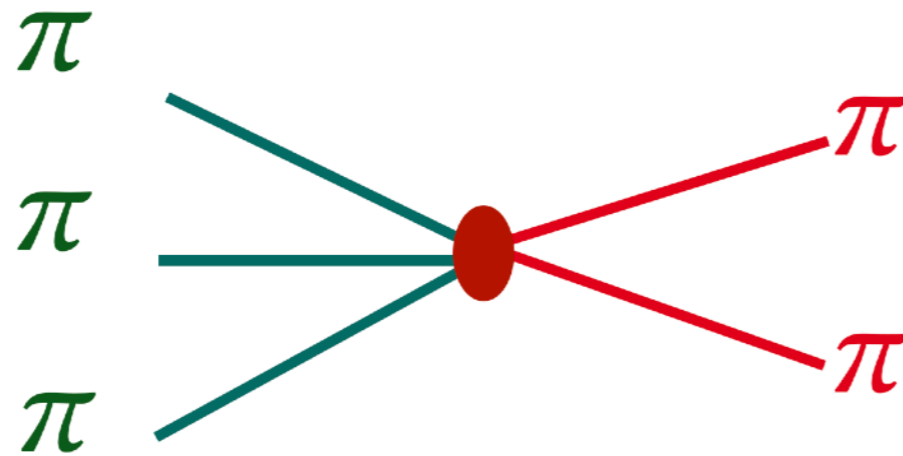
# What bound states do?



two-body process remains efficient even after pions are frozen out

$$n_X \langle \sigma_{XX \rightarrow \pi\pi v} \rangle > H(x_2)$$

# Coupling to Standard Model



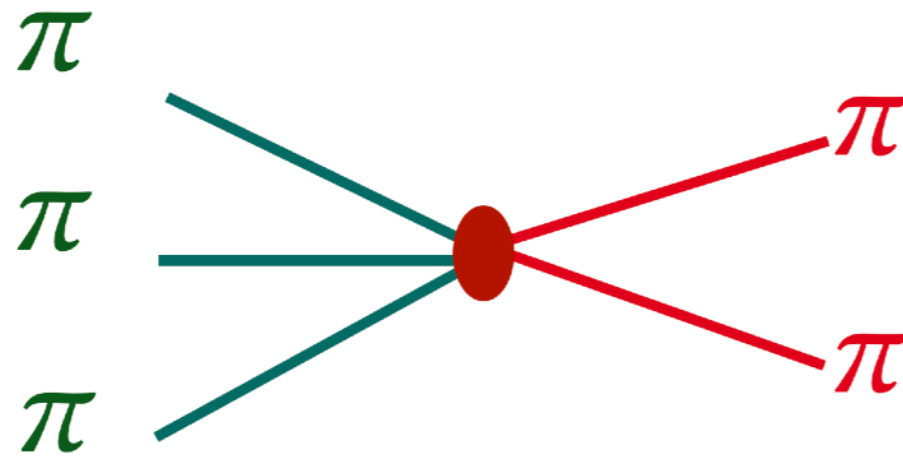
**SIMPs in isolation lead to  
HOT dark matter (excluded)**

SIMPs must come into kinetic equilibrium with the SM plasma (=share the same temperature)

$$\pi \text{ SM}_i \rightarrow \pi \text{ SM}_i \quad \text{with} \quad \Gamma_{\pi \text{ SM}} = \langle \sigma_{\pi \text{ SM}} c \rangle n_i > H$$

=> typically enables  $\pi\pi \rightarrow \text{SM}_i \overline{\text{SM}_i}$  but OK, because  $n_i/n_\pi \gg 1$

# Coupling to Standard Model



**SIMPs in isolation lead to  
HOT dark matter (excluded)**

SIMPs must come into kinetic equilibrium with the SM plasma (=share the same temperature)

$$\pi \text{ SM}_i \rightarrow \pi \text{ SM}_i \quad \text{with} \quad \Gamma_{\pi \text{ SM}} = \langle \sigma_{\pi \text{ SM}} c \rangle n_i > H$$

=> typically enables  $\pi\pi \rightarrow \text{SM}_i \overline{\text{SM}}_i$  but OK, because  $n_i/n_\pi \gg 1$

**HERE: destabilizes the bound state**

$$X = [\pi\pi] \rightarrow \text{SM}_i \overline{\text{SM}}_i$$

# Meta-stability of $X$

$$X = [\pi\pi] \rightarrow \text{SM}_i \overline{\text{SM}}_i$$

Noting that  $|\psi(0)|^2 v$  has units of particle flux  $\Rightarrow \Gamma_X \sim |\psi(0)|^2 (\sigma_{\text{ann}} v)$

$$\Gamma_X / H < 1 \quad \Rightarrow \quad \sigma_{\text{ann}} v \lesssim 10^{-3} \text{pb} \, x^{-2} \left( \frac{m_\pi}{100 \text{ MeV}} \right)^2 \frac{\text{MeV}^3}{|\psi(0)|^2}$$

# Meta-stability of X

$$X = [\pi\pi] \rightarrow \text{SM}_i \overline{\text{SM}}_i$$

Noting that  $|\psi(0)|^2 v$  has units of particle flux  $\Rightarrow \Gamma_X \sim |\psi(0)|^2 (\sigma_{\text{ann}} v)$

$$\Gamma_X / H < 1 \quad \Rightarrow \quad \sigma_{\text{ann}} v \lesssim 10^{-3} \text{pb} \, x^{-2} \left( \frac{m_\pi}{100 \text{ MeV}} \right)^2 \frac{\text{MeV}^3}{|\psi(0)|^2}$$

Taking  $\sigma_{\pi \text{SM} C} \sim \sigma_{\text{ann}} v$ , the stability requirement (X lives beyond freeze out) imposes upper limit on the elastic scattering rate that is needed to make Dark Matter “cold”.

$$1 \lesssim \frac{\Gamma_{\pi \text{SM}}}{H} \lesssim \frac{10^6}{x^3} \left( \frac{m_\pi}{100 \text{ MeV}} \right)^3 \frac{\text{MeV}^3}{|\psi(0)|^2}$$

**$\Rightarrow$  can easily be satisfied: retain kinetic equilibrium while maintaining sufficient longevity of X, paired with sub-Hubble two-body annihilation**

**$\Rightarrow$  no escalated model building requirements in comparison to original works on the SIMPs**

**$\Rightarrow$  previously explored phenomenology remains in place**



# X-catalyzed SIMP mechanism

When coupled to SM

additional X formation and breakup reactions may open

=> the detailed balancing condition

$$Y_X = \frac{Y_\pi^2 Y_X^{\text{eq}}}{(Y_\pi^{\text{eq}})^2} \quad \text{remains unaltered}$$

=> If the new processes dominate over  $3\pi \leftrightarrow \pi X$ , detailed balancing retains its validity longer

=>  $x_2$  will be larger, and **relic density smaller**

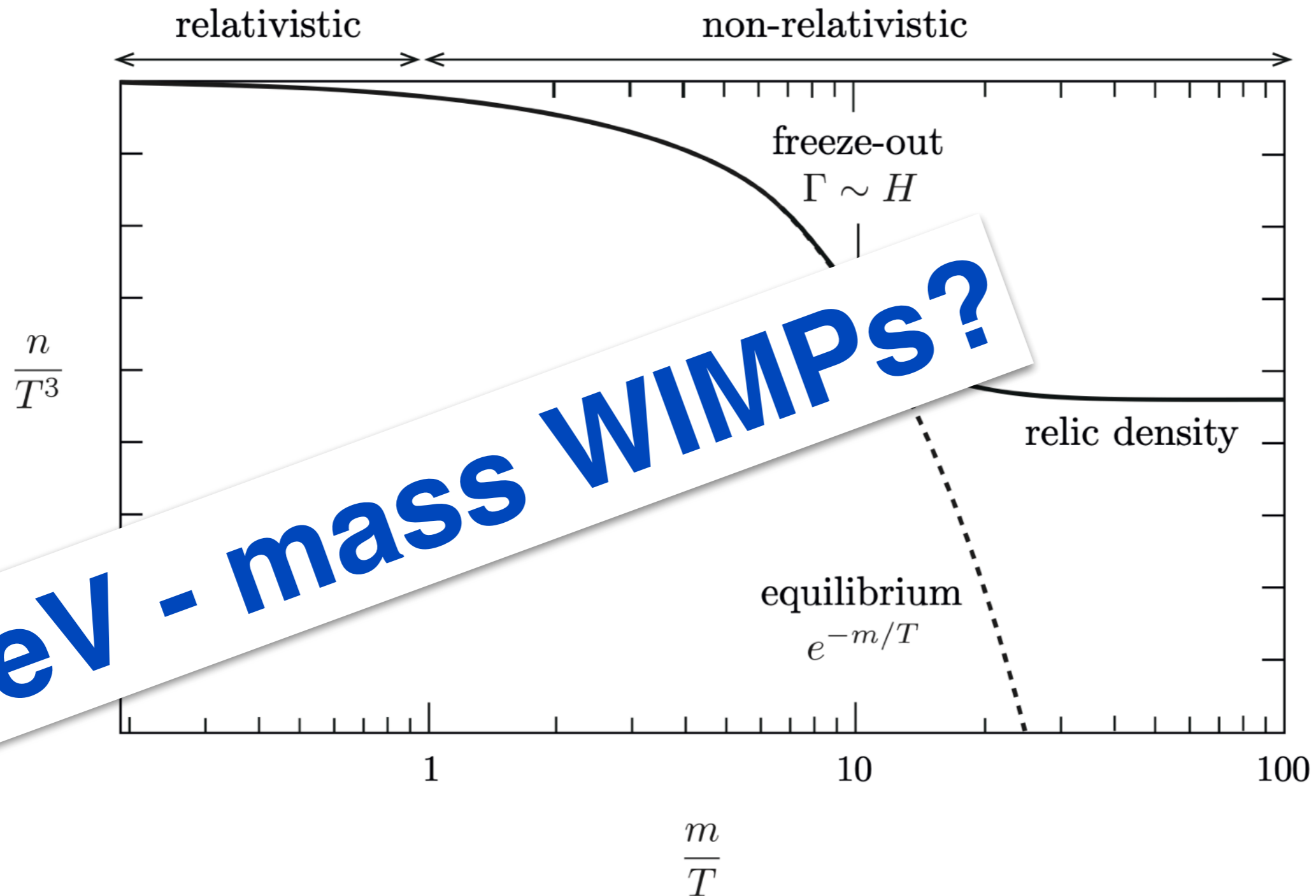
Introduction of **SM-interactions** harbor the potential to make **X-assisted freeze-out even more efficient**, without jeopardizing the overall picture!

# 2. WIMP dead end

Chu, Kuo, JP, PRD 2022

Chu, JP PRD 2024

What is the lightest thermal DM mass?



**MeV - mass WIMPs?**

# 2. Thermal MeV DM

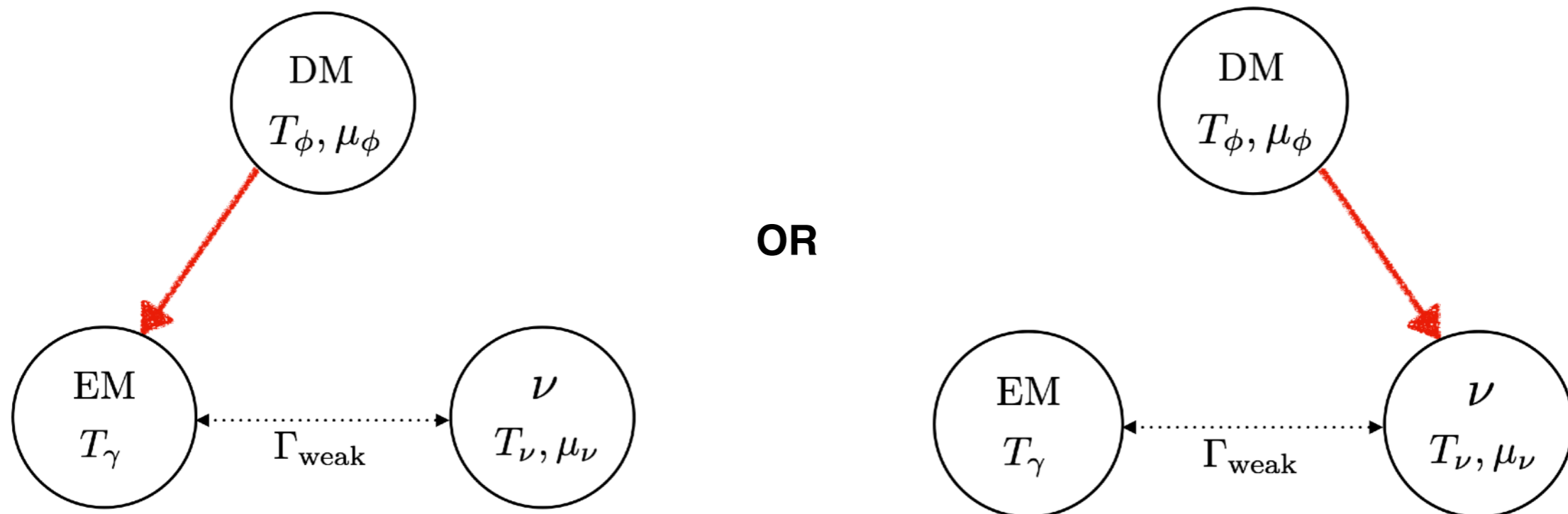
Chu, Kuo, JP, PRD 2022

Chu, JP PRD 2024

## What is the lightest thermal DM mass?

Well known that MeV-DM subject to  $N_{\text{eff}}$  bound from heating by annihilation

**Previous treatments** had to assume a branching either into EM-sector OR neutrinos:



see e.g. [M. Escudero 2019, Depta et al. 2019, Sabti et al. 2020]

# 2. Thermal MeV DM

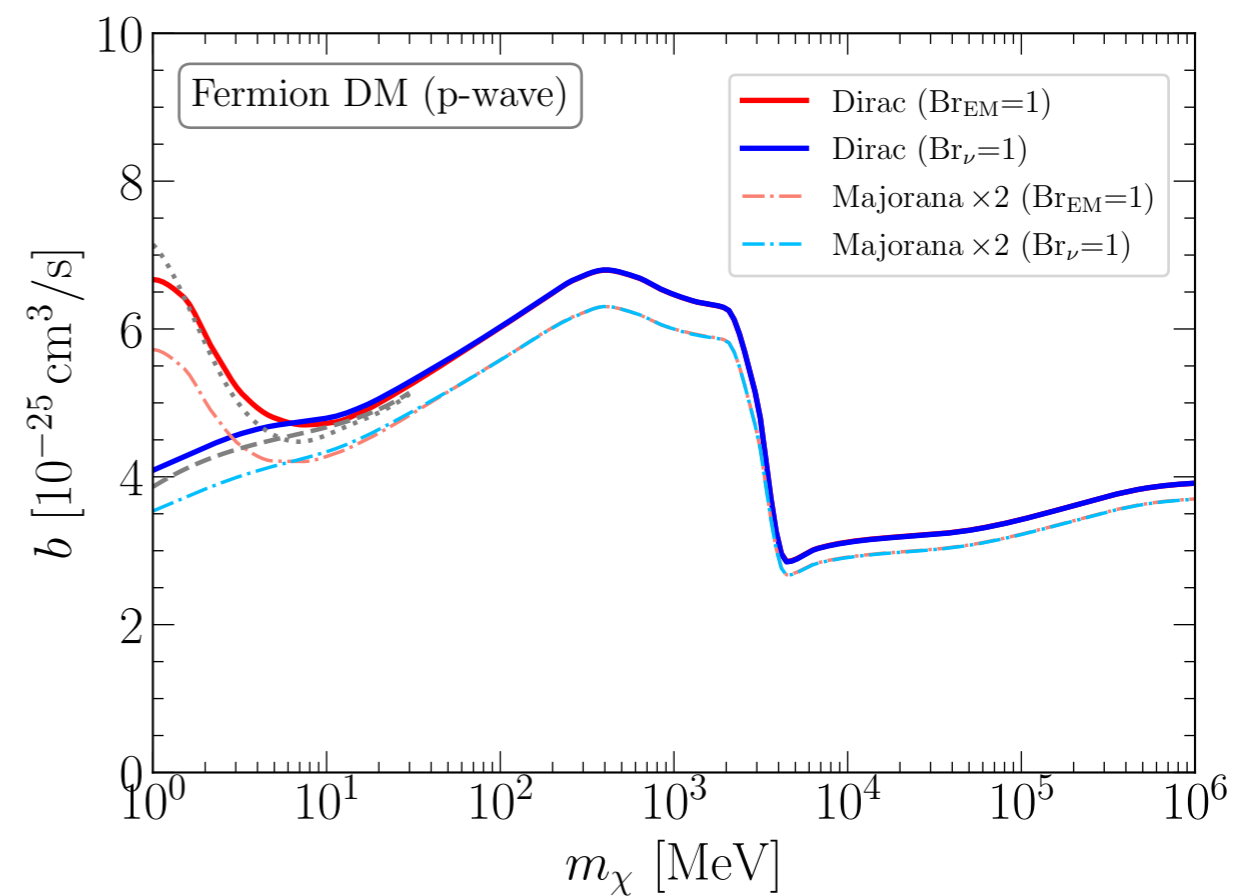
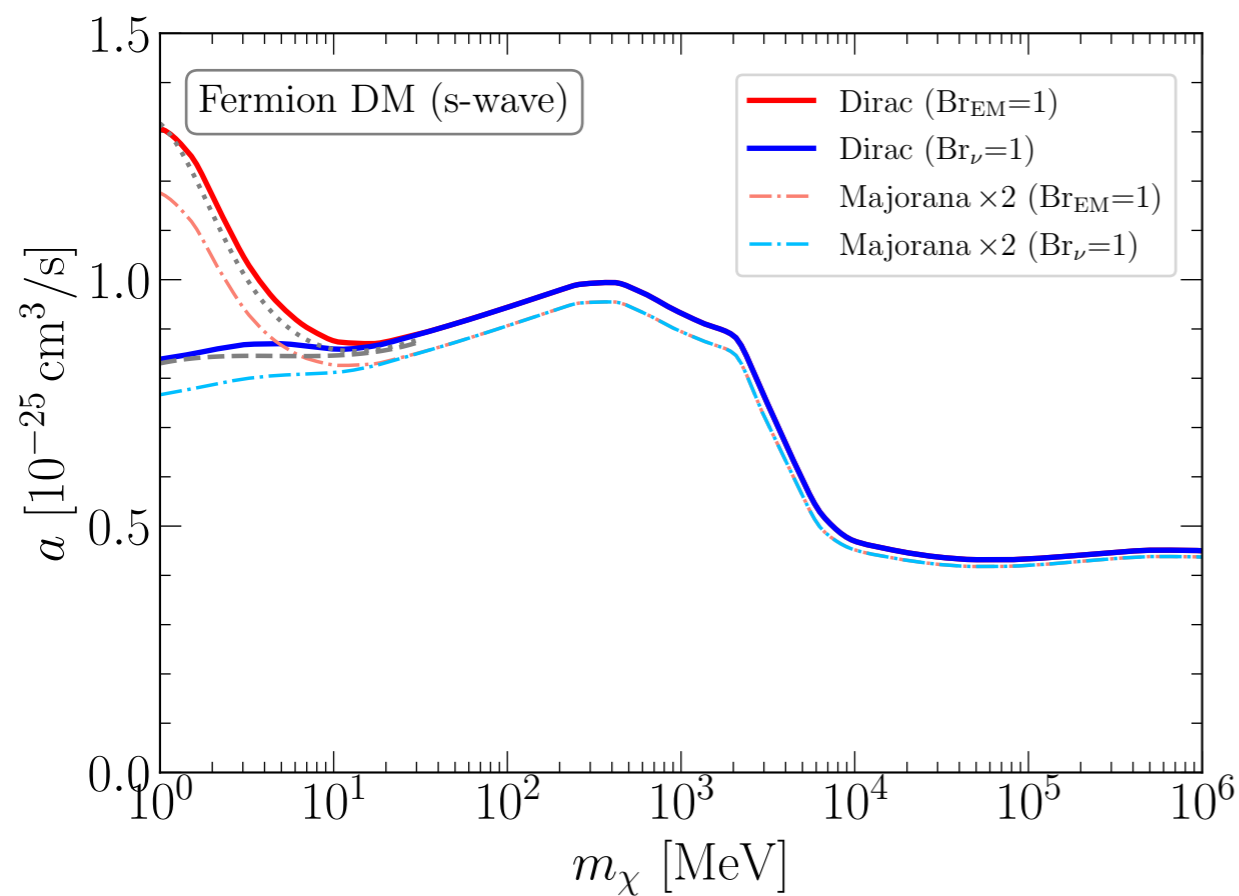
Chu, Kuo, JP, PRD 2022

Chu, JP PRD 2024

## What is the lightest thermal DM mass?

Well known that MeV-DM subject to  $N_{\text{eff}}$  bound from heating by annihilation

**Previous treatments** had to assume a branching either into EM-sector OR neutrinos:



$$\langle \sigma_{\text{ann}} v \rangle = a + b (6T/m_{\phi,\chi})$$

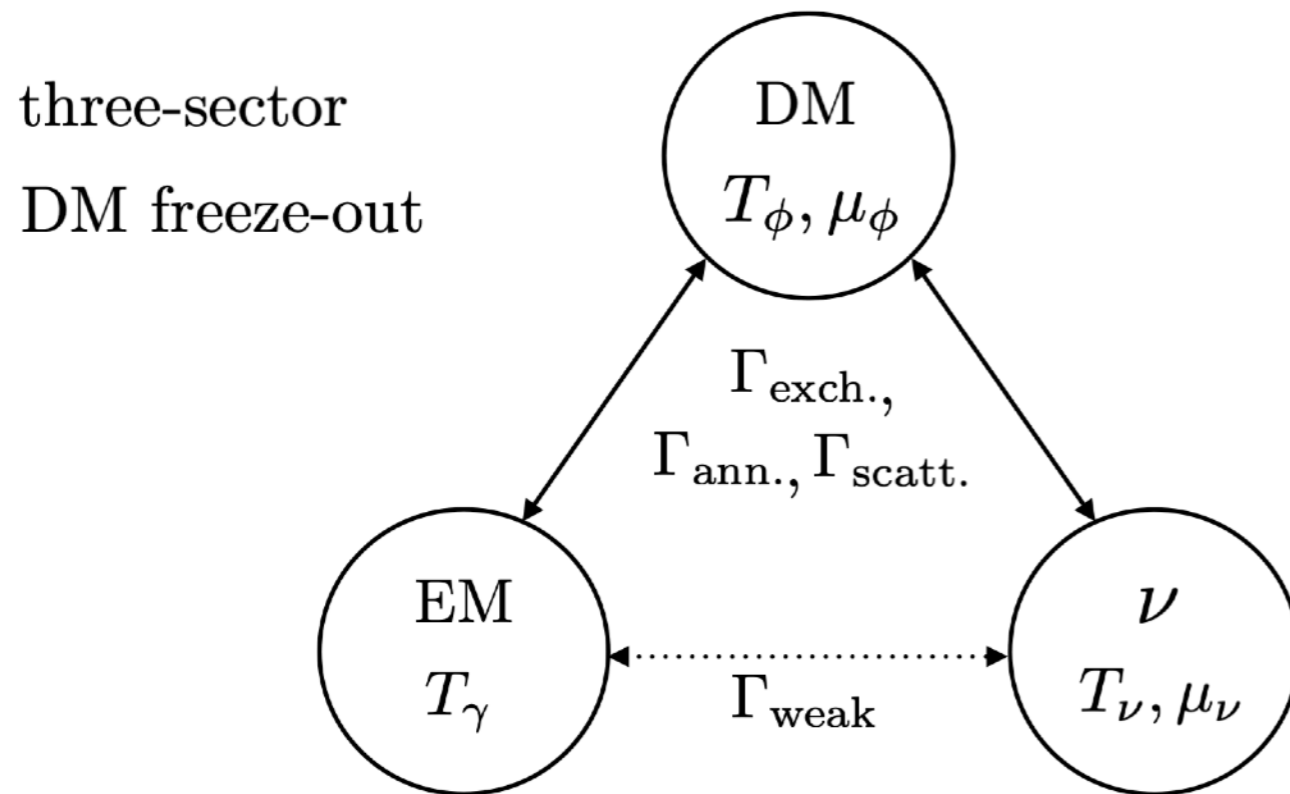
# 2. Thermal MeV DM

Chu, Kuo, JP, PRD 2022  
Chu, JP PRD 2024

## What is the lightest thermal DM mass?

Well known that MeV-DM subject to Neff bound from heating by annihilation

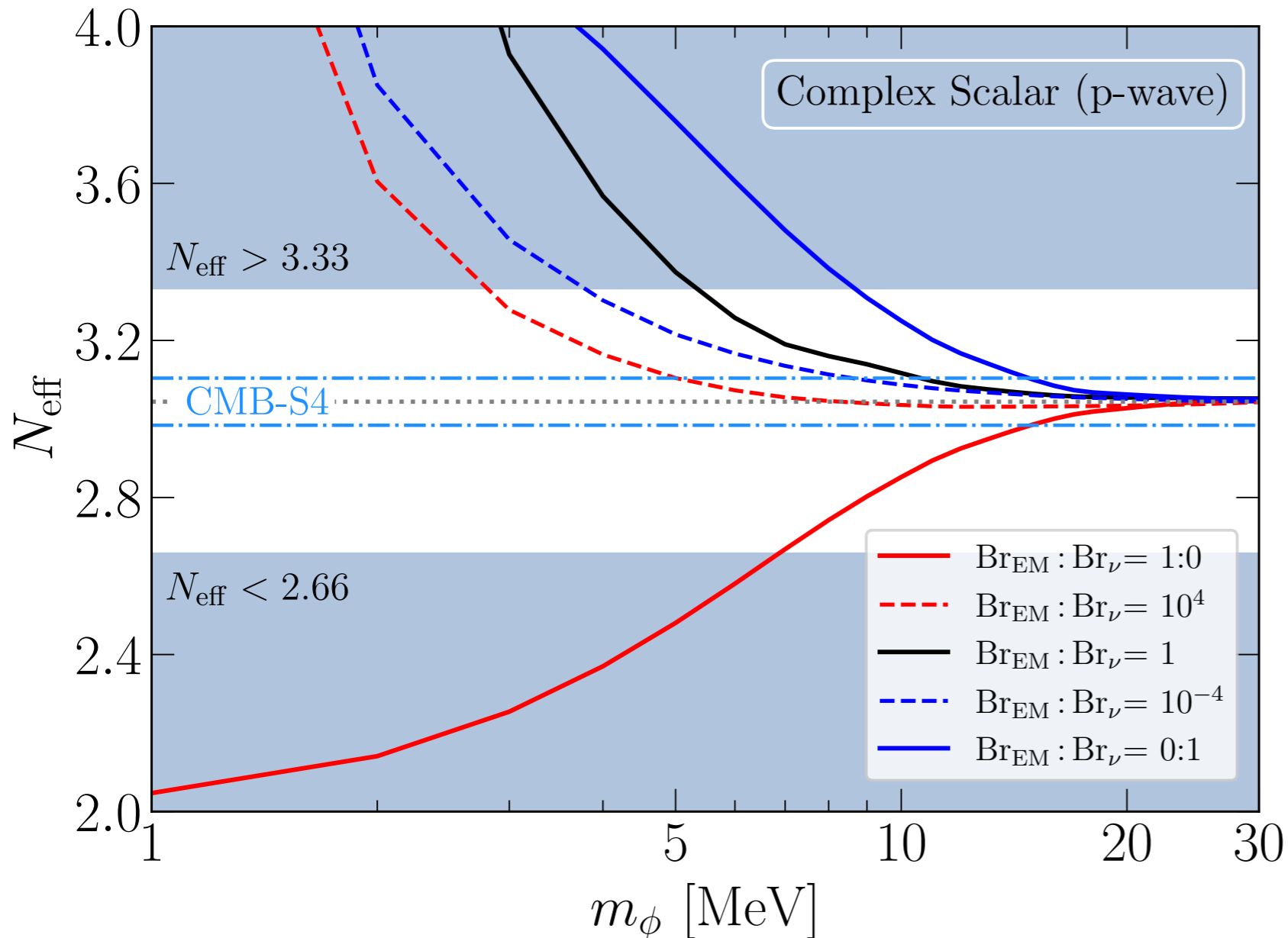
**In the full picture**, joint treatment of the three coupled sectors is necessary



$$\Gamma_{\text{weak}} \equiv n_e G_F^2 T_\gamma^2,$$
$$\Gamma_{\text{ann.}} \equiv n_\phi \langle \sigma_{\text{ann.}} v \rangle,$$
$$\Gamma_{\text{exch.}, i} \equiv n_\phi^2 \langle \sigma_{\text{ann.}, i} v \delta E \rangle / \rho_i,$$
$$\Gamma_{\text{scatt.}, i} \equiv n_i \langle \sigma_{\text{scatt.}}^{\phi i} v \rangle.$$

# Light DM freeze out

What is the lightest thermal DM mass?



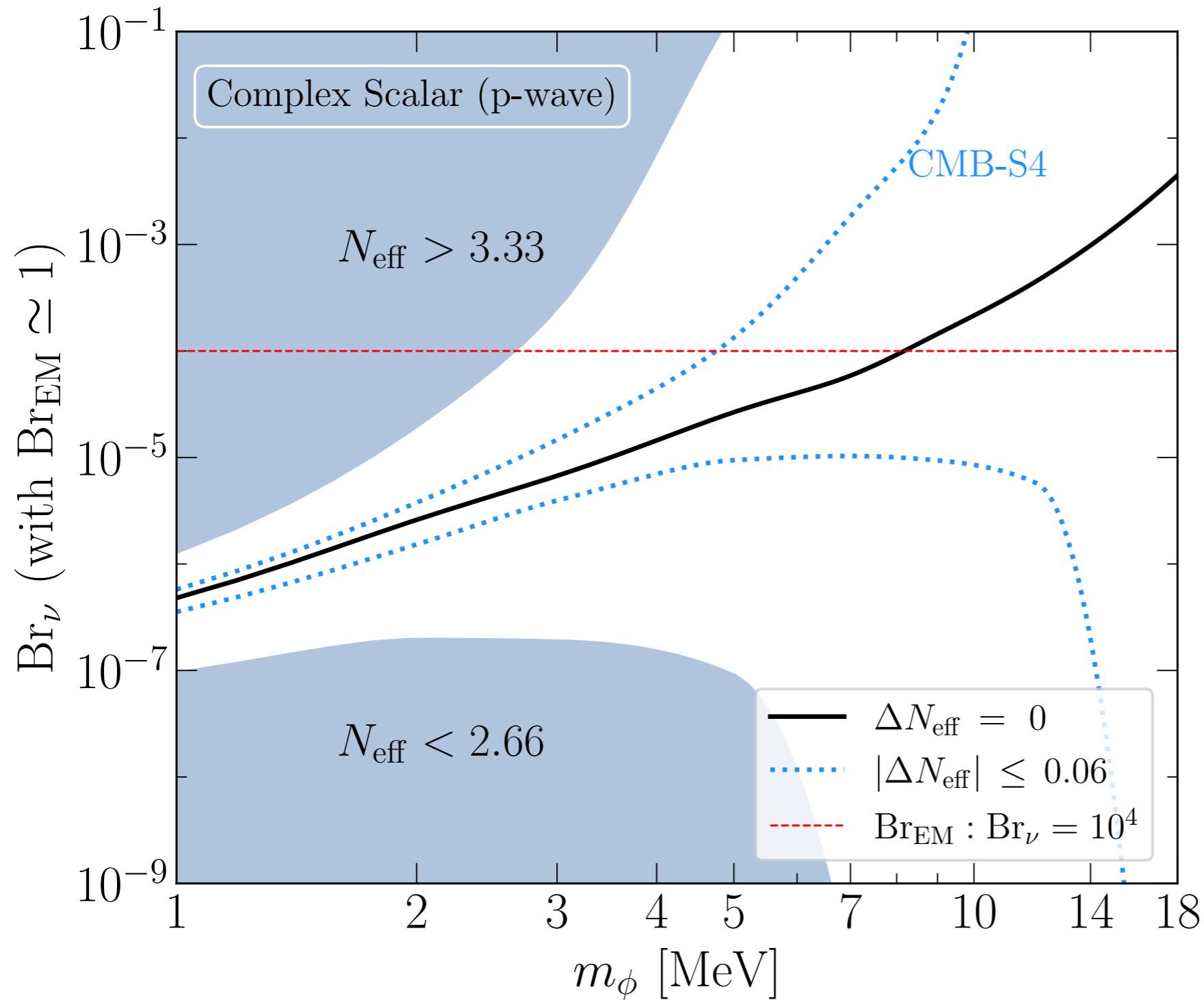
Example: p-wave annihilation

$$\rho_{\text{rad}} = \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_{\gamma}$$



# Evading Neff bound

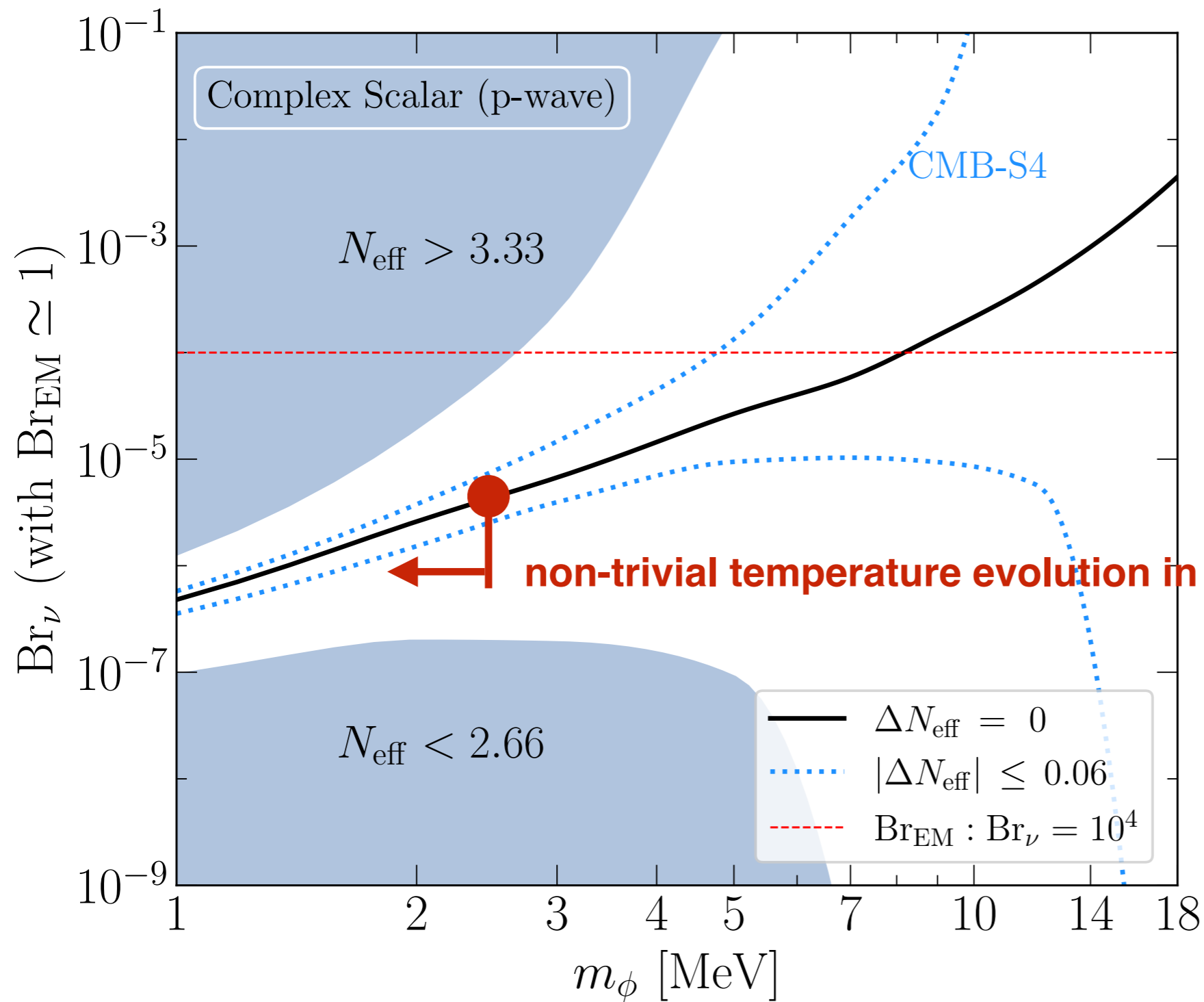
How low can you go?



Fine-tuned branching  
into neutrinos evades  
Neff constraint.

# Evading Neff bound

How low can you go?



Fine-tuned branching  
into neutrinos evades  
Neff constraint.

## Application:

thermal DM affecting  
21cm cosmology  
with millicharged DM  
(=> see paper)



# Summary

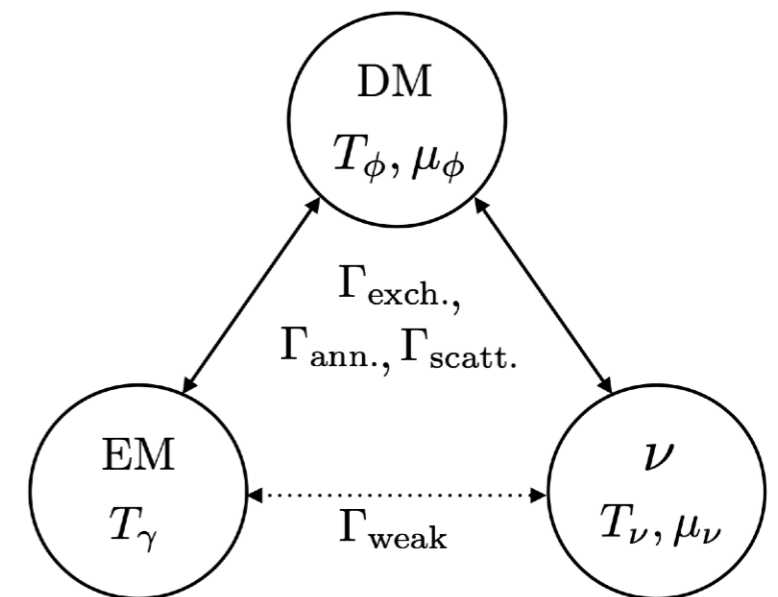
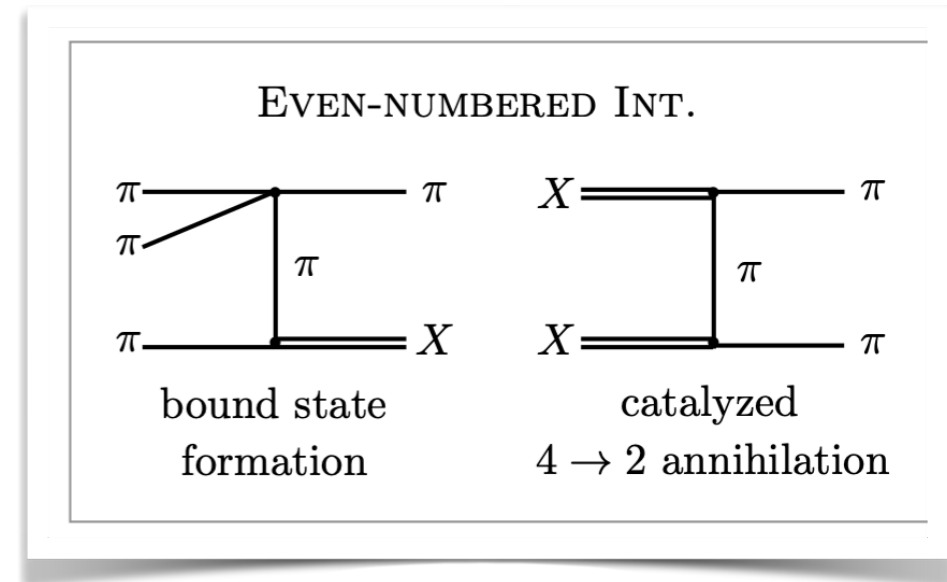
## Freeze-out of MeV-mass DM candidates

- Small-scale structure problems pertinent to LCDM may be a hint for DM self-interactions, naturally realized in theories with strongly interacting particles (SIMPs)
- When SIMPs regulate their relic abundance in  $N \rightarrow 2$  processes, bound states — should they exist — significantly alter the standard picture.
- Even-numbered SIMP-mechanism is possible

.....

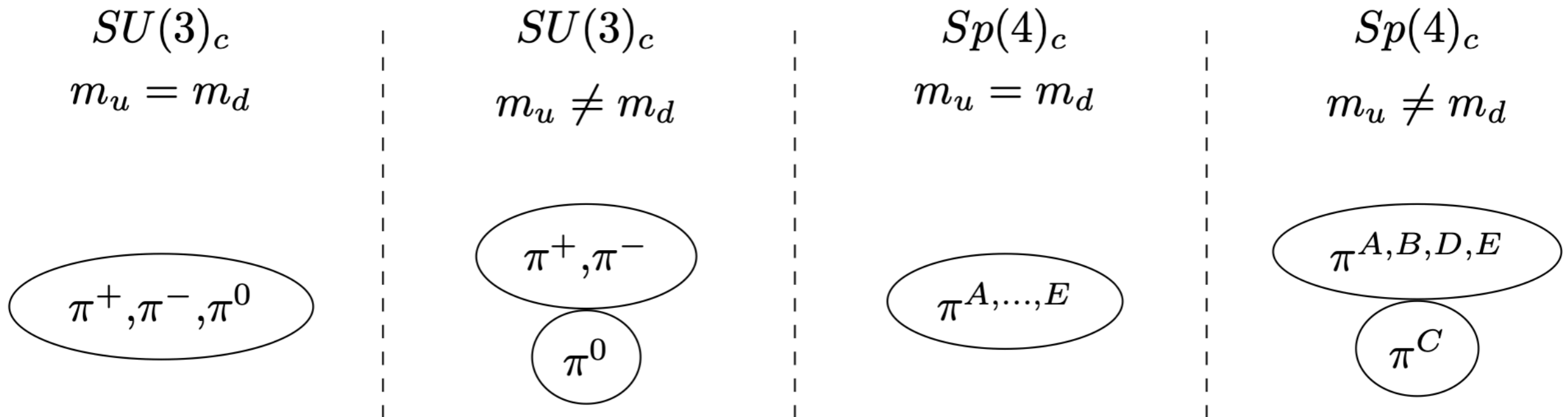
- A comprehensive assessment of thermal MeV-scale DM necessitates a three-sector treatment of vastly changing rates => systematic formulation
- nice application for DM affecting 21cm cosmology

**Thank you**



# Backup slides

# Meson multiplet structure



$$\pi^D = \bar{d} \gamma_5 S C \bar{u}^T$$

$$\pi^E = d^T S C \gamma_5 u$$

$$\pi = \sum_{i=1, \dots, 5} \pi_a T^a = \sum_{N=A, \dots, E} \pi_N T^N = \frac{1}{2} \begin{pmatrix} \pi^C & \pi^B & 0 & \pi^E \\ \pi^A & -\pi^C & -\pi^E & 0 \\ 0 & -\pi^D & \pi^C & \pi^A \\ \pi^D & 0 & \pi^B & -\pi^C \end{pmatrix}$$

=> 5 Goldstone bosons

# Bound-state assisted freeze-out

Expectations/guesses for  $|\psi(0)|^2$

In analogy to QED, one may posit a scale  $a_B$  “Bohr radius”

For perturbative couplings  $\alpha$   $a_B \sim 1/(\alpha\mu) = 2/(\alpha m_\pi) \geq 2/m_\pi$

Radial profiles (for  $n=1$ )

$$R_s(r) \simeq R_s(0) e^{-(r/2a_B)}, \quad R_p(r) \simeq R'_p(0) r e^{-(r/2a_B)},$$

s-wave ( $l=0$ )

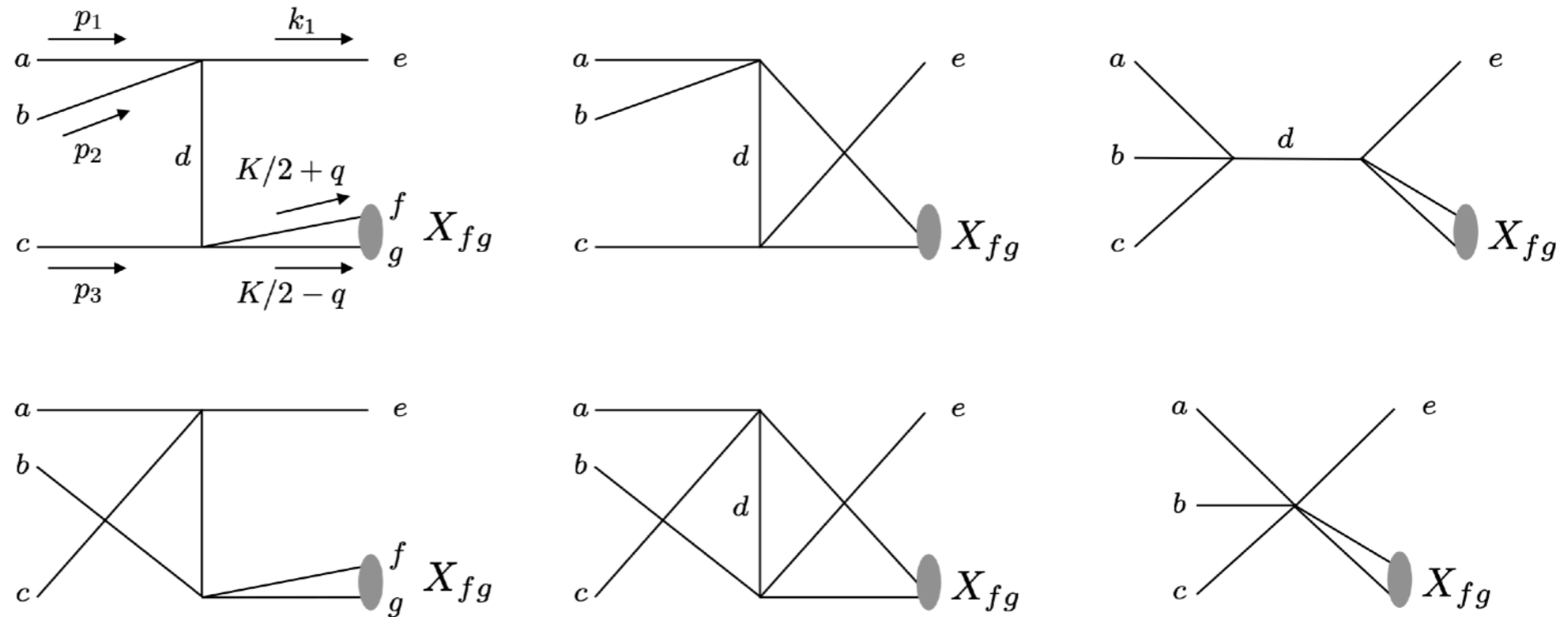
p-wave ( $l=1$ )

$$R_s(0) = \frac{1}{\sqrt{2a_B^3}} \sim 0.25(\alpha m_\pi)^{3/2}, \quad R'_p(0) = \frac{1}{\sqrt{24a_B^5}} \sim 0.035(\alpha m_\pi)^{5/2}$$

$$\Rightarrow |\psi(0)|/m_\pi^{3/2} \sim 0.9\alpha^{3/2}$$

# Bound state formation

## X-formation



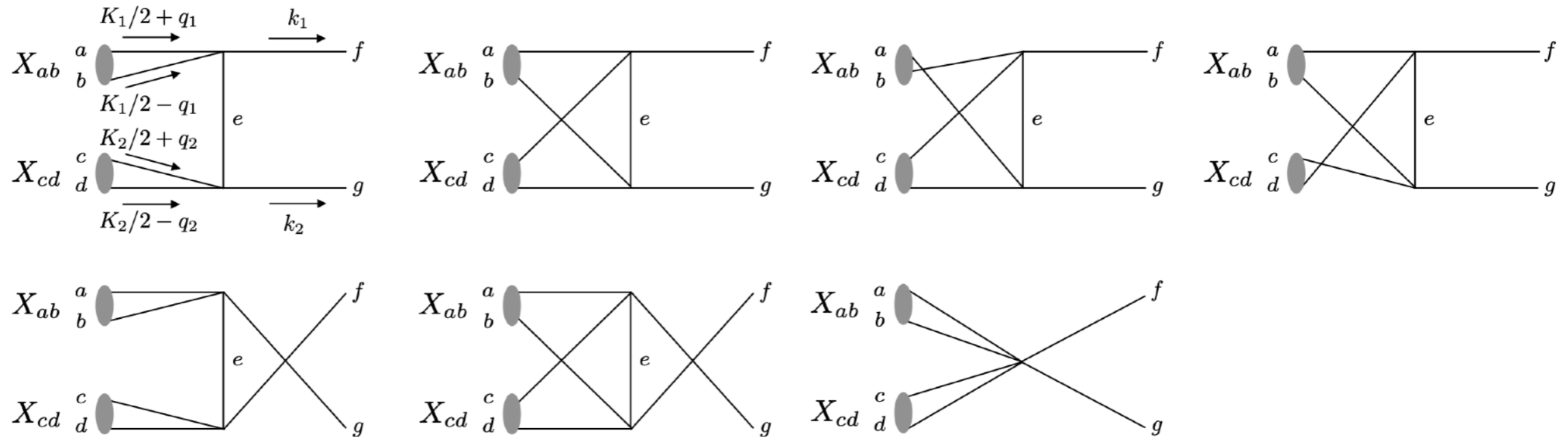
radial wave function of X (s-wave)

$$\langle \sigma_{3\pi \rightarrow \pi X} v^2 \rangle \simeq \frac{57\,041}{1\,310\,720 \sqrt{3} \pi^2} \frac{R_S^2(0)}{f_\pi^8} \left( \frac{m_\pi}{E_B} \right)^{3/2}$$

↑  
additional t-channel enhancement

# Bound state-assisted annihilation

## XX annihilation



$$\langle \sigma_{XX \rightarrow \pi\pi\nu} \rangle \simeq \frac{2\,529\,757}{424\,673\,280\sqrt{3}\pi^3} \frac{R_S^4(0)}{f_\pi^8}$$

## catalyzed 3- $\rightarrow$ 2

$$\langle \sigma_{\pi X \rightarrow \pi\pi\nu} \rangle \simeq \frac{\sqrt{5}N_c^2 m_\pi^3}{512\pi^6 f_\pi^{10} x} R_P^{\prime 2}(0)$$

derivative of X wavefunction in p-state!

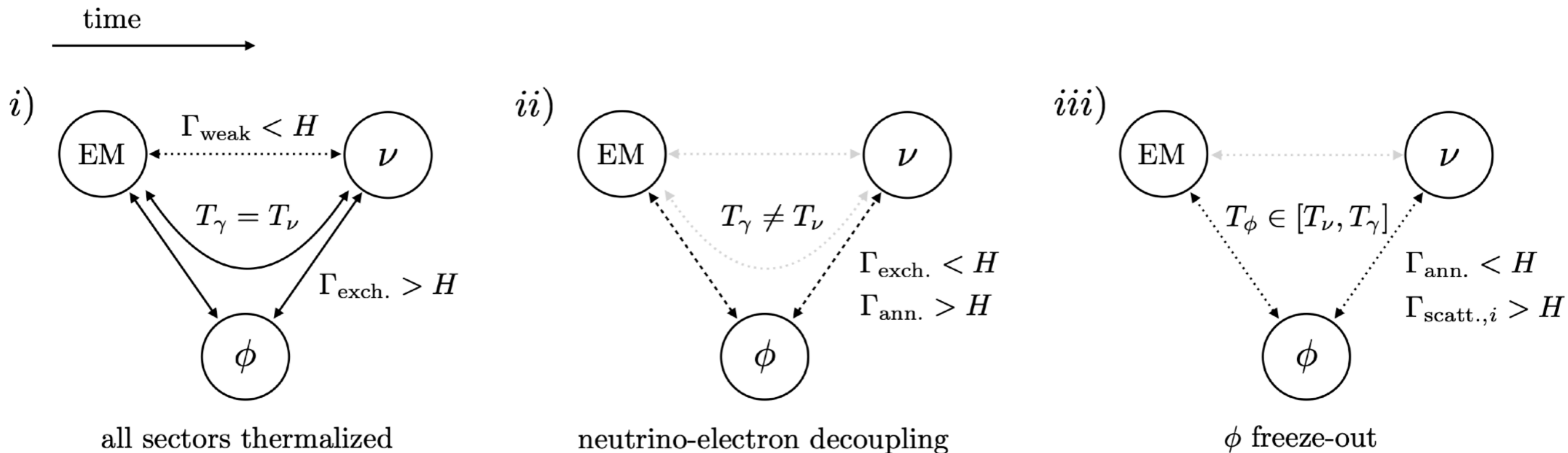
# 2. Thermal MeV DM

Chu, Kuo, JP, PRD 2022  
Chu, JP PRD 2024

**OR: what is the lightest thermal DM mass?**

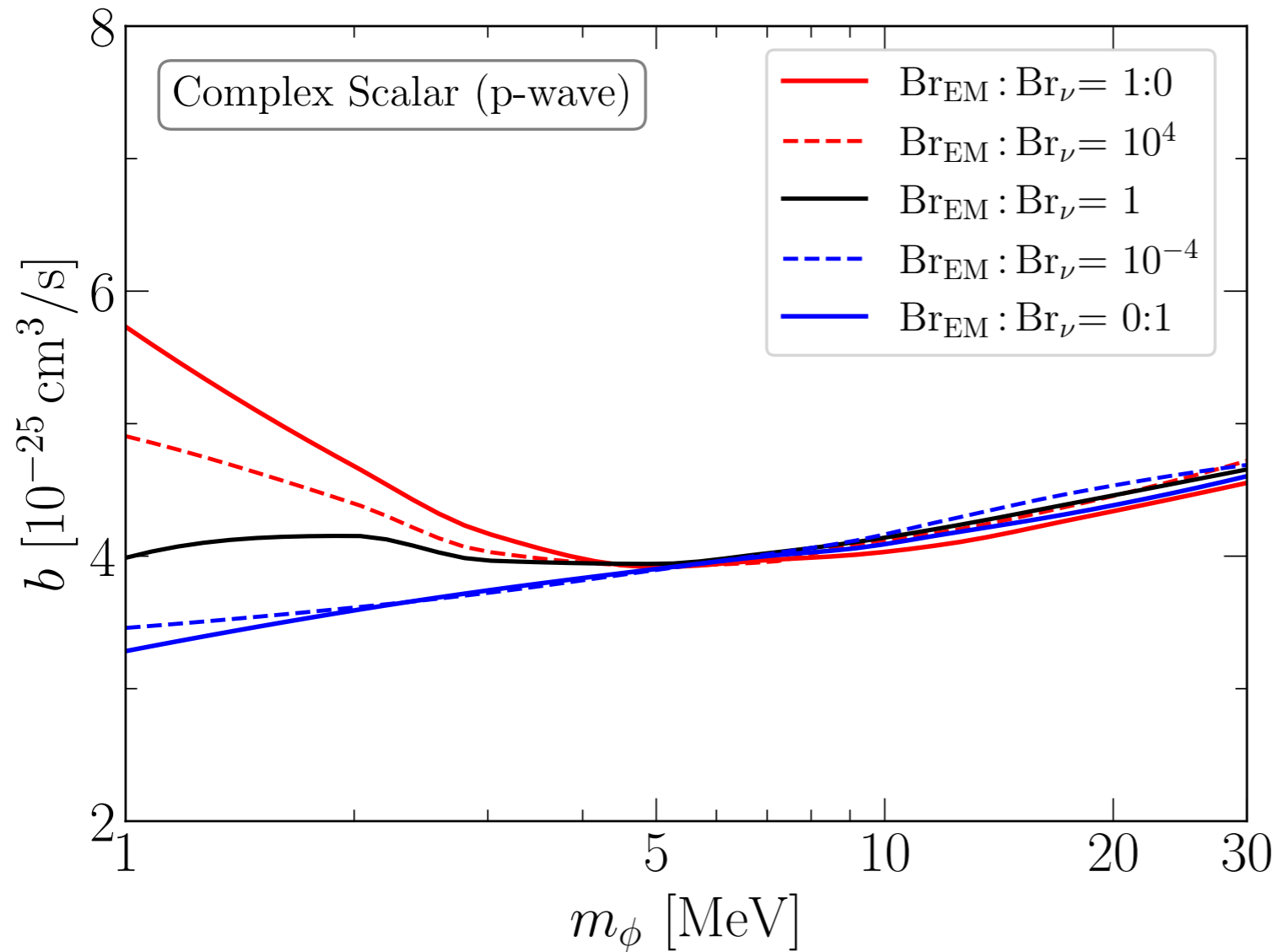
Well known that MeV-DM subject to Neff bound from heating by annihilation

**In the full picture,** joint treatment of the three coupled sectors is necessary



# Light DM freeze out

Thermal cross section



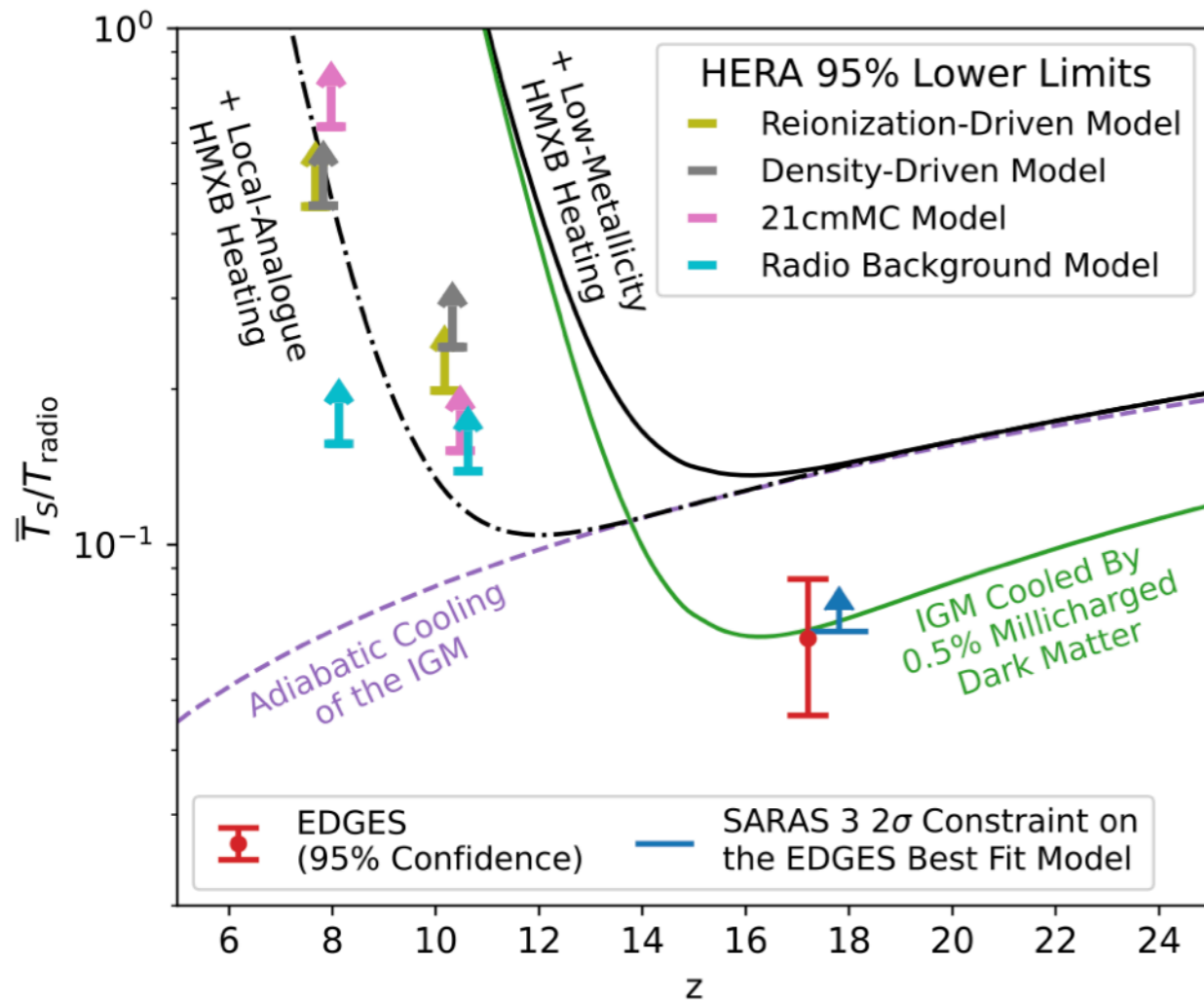
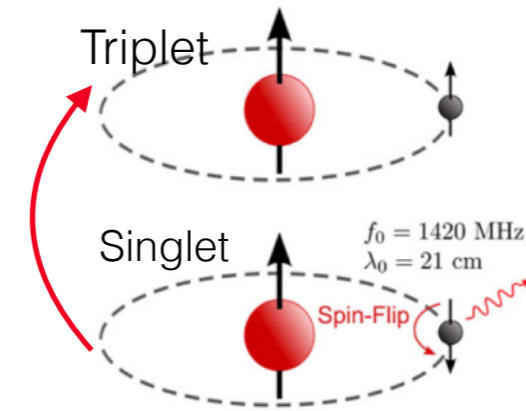
Example: p-wave annihilation

$$\mathcal{L}_{Z'}^{\text{int}} = g_\phi^2 Z'^\mu Z'_\mu \phi^* \phi - ig_\phi Z'^\mu (\phi^* \overleftrightarrow{\partial}_\mu \phi) - g_l Z'^\mu \bar{l} \gamma_\mu l.$$



# 21cm application

Weaker mass bounds from Neff can be useful:



[HERA 2210.04912]

21cm sensitivity to baryon cooled down from scattering off MeV dark matter after CMB

=> use  $1/\text{velocity}^4$  enhancement of Rutherford-type scattering of baryons on DM

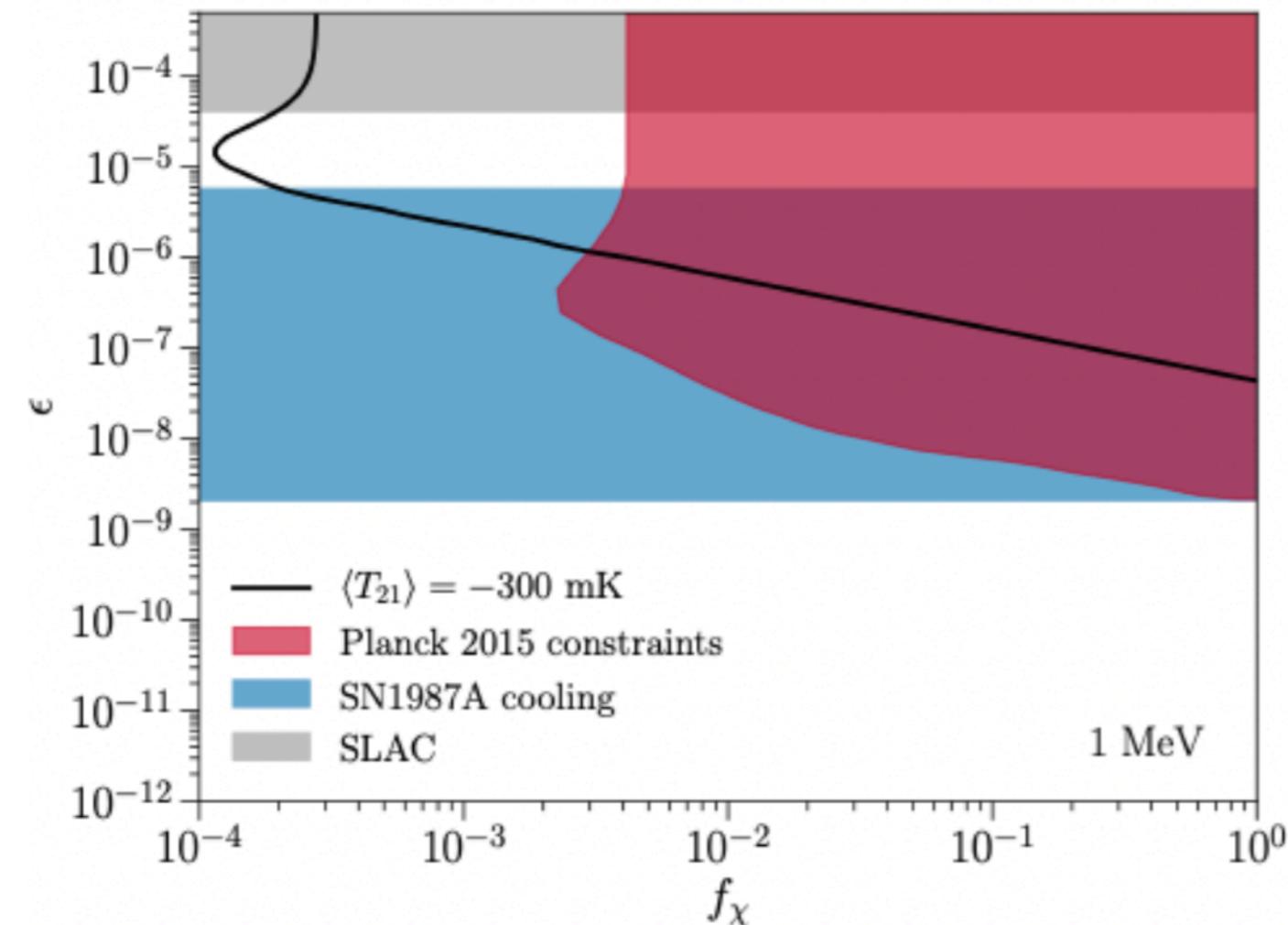
**Models of milli-charged sub-% dark matter can do this but:**

- **no valid thermal freeze-out;**
- **strong bounds from CMB/BBN;**

[e.g. Kovetz, Poulin, Gluscevic, Boddy, Barkana, Kamionkowski 1807.11482]

# 21cm application

Weaker mass bounds from  $N_{\text{eff}}$  can be useful:



21cm sensitivity to baryon cooled down from scattering off MeV dark matter after CMB

=> use  $1/\text{velocity}^4$  enhancement of Rutherford-type scattering of baryons on DM

**Models of milli-charged sub-% dark matter can do this but:**

- **no valid thermal freeze-out;**
- **strong bounds from CMB/BBN;**

[e.g. Kovetz, Poulin, Gluscevic, Boddy, Barkana, Kamionkowski 1807.11482]

# 21cm application

Provide a valid model: **add neutrino annihilation**

$$i\epsilon e\gamma^\mu J_\chi + iy_A A(\bar{\chi}\gamma_5\chi) + iy_\nu A(\bar{\nu}_l\gamma_5\nu_l)$$

**millicharge**

**(effective) neutrino interactions**

**annihilation**

$$\text{Br}_{\text{EM}} : \text{Br}_\nu \sim 10^{-2}$$

**=> dominant annihilation into neutrinos,  
makes thermal sub-% dark matter component**

	$N_{\text{eff}}$	$\Delta N_{\text{eff}}$	$\Delta(D/H)$	$Y_p \times 10$	$\Delta Y_p$	viable?
SBBN	3.044	–	–	2.478	–	✓
$m_\chi = 10$ MeV	3.119	0.075	+1.0%	2.488	+0.4%	✓
$m_\chi = 9$ MeV	3.171	0.127	+1.5%	2.493	+0.6%	✓
$m_\chi = 8$ MeV	3.193	0.149	+1.8%	2.496	+0.7%	✓
<del><math>m_\chi = 7</math> MeV</del>	<del>3.268</del>	<del>0.224</del>	<del>+2.8%</del>	<del>2.503</del>	<del>+1.0%</del>	<del>?</del>
<del><math>m_\chi = 6</math> MeV</del>	<del>3.352</del>	<del>0.308</del>	<del>+3.8%</del>	<del>2.512</del>	<del>+1.4%</del>	<del>✗</del>

**=> working model affecting 21cm**

**passing all constraints**

**(incl.  $\nu$  mfp,  $\nu$  self-interactions,  
constraints on mediator from  
flavor physics,...)**